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Mechanisms of Beam-Halo Formation in High-Intensity Linacs

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An rms-mismatched beam with space-charge forces can evolve rapidly, and during the course of evolution its emittance grows and a diffuse halo forms. The halo is of particular concern for long-term operation of linacs such as those envisioned for accelerator transmutation of waste (ATW) because too much particle impingement on the accelerating structures would cause enough radioactivation to prohibit routine hands-on maintenance. Thus, halo suppression is a design consideration for these linacs. In this paper, we summarize both the physical processes associated with the rapid evolution of the rms-mismatched beam and a semianalytic formalism which was recently developed to account for them. We also provide representative results derived from the formalism.

INTRODUCTION

Space-charge forces in high-intensity linacs are known to complicate the beam dynamics significantly. If at injection the beam is mismatched, charge redistribution takes place. This injects free energy associated with the mismatch in rms beam size into a spectrum of collective modes. As individual particles interact with the time-dependent global field and with clumps of particles within it, their orbits can be chaotic. The ensemble of particle orbits rapidly tends to cover the accessible phase space in a coarse-grained manner, and its range of energies widens. In this sense, the collective interactions cause violent relaxation toward a stationary state which leads to quasi-thermalization of the free energy and of the beam. This scenario, which has been seen in both laboratory and numerical experiments, results in particles being ejected into high-amplitude orbits, causing the emittance to grow and a halo to form.

Much of the recent work on this problem concerns the interaction of single particles with an oscillating uniform-density core [1, 2]. In this simplified model, single particles with the requisite initial conditions are rapidly ejected from the core through parametric resonance [3], and so halo formation is not a surprising phenomenon. However, a proper treatment must include the evolving mode spectrum to self-consistently account for the rapid changes in the electrostatic self-field of the beam. We recently developed a self-consistent, semianalytic formalism which uses the Fokker-Planck and Poisson equations to govern the rapid evolution of the coarse-grained distribution function in the single-particle phase space [4, 5]. In what follows, we shall briefly summarize the physics of the problem and present representative results for ATW-class linacs calculated with the semianalytic formalism.

EVOLUTIONARY PHASES

A beam which is matched at injection is stationary because it is in equilibrium and

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characterized by the Maxwell-Boltzmann distribution function. If the beam is mismatched, it will evolve through a complicated evolutionary sequence toward Maxwell-Boltzmann. Ref. [5] includes a detailed account of this sequence, and here we list highlights extracted from the reference.

A mismatched beam will first undergo charge redistribution, a phase lasting about one quarter of the beam's plasma period t_p . If the beam is rms-matched, it then becomes quasi-stationary, and most of the free energy associated with mismatch of the shape of the density profile appears as emittance growth. The relaxation time for further evolution, t_r , which is the time needed for a typical particle to lose memory of its initial orbit, is of the order of the very long time scale t_b associated with binary Coulomb interactions, *i.e.*, collisions in the absence of strong multiparticle correlations. On the other hand, if the beam is rms-mismatched, it continues to oscillate. If the Debye length λ_D is much larger than the beam size a , then space charge is unimportant, the beam oscillates in response to the external focusing force, and the free energy associated with mismatch thermalizes on the time scale t_b so that $t_r \sim t_b$. If λ_D is comparable to or smaller than a , then collective space-charge forces enter. Free energy associated with rms mismatch then distributes itself in a spectrum of collective modes, and these modes are subject to Landau damping. Modes with wavenumber $k \gtrsim \lambda_D^{-1}$ damp on a time scale $\sim t_p$, and modes for which $k \ll \lambda_D^{-1}$ are long-lived. Thus, if $\lambda_D \gtrsim a$ (a warm beam), we can expect the modes to damp within a few core oscillations, but if $\lambda_D \ll a$ (a cool, space-charge-dominated beam), the large-scale global modes will persist.

There are many degrees of freedom available by which a space-charge-dominated beam may release the free energy of rms mismatch. It will pass through a state of strong turbulence triggered by wave breaking associated with charge redistribution if $\Delta E_m/E_t \gg \Delta k^2 \lambda_D^2$, where ΔE_m is the free energy per particle due to rms mismatch, E_t is the thermal energy per particle, and Δk is the effective half-width of the mode spectrum. As a rule of thumb, this inequality will generally be satisfied for cold space-charge-dominated beams.

Localized collective modes in a strongly turbulent beam release their free energy very quickly (on a heating time scale $t_h \sim t_p$) via Landau damping. The short-wavelength modes dissipate, and single particles efficiently gain energy by resonant interactions with the dissipating modes. The particles continue to interact with the residual weak turbulence, and the beam relaxes on a short time scale compared to t_b . For example, in three-dimensional beams which are strongly turbulent, the average "collision frequency" is $\sim g^{-1}$ times larger than in a quiescent beam, where $g \equiv 1/n\lambda_D^3$ is the plasma parameter. In space-charge-dominated beams g^{-1} is large and interactions between particles and localized fluctuations are important. In weak turbulence the average collision frequency is $\sim g^{1/2}$ times smaller than in strong turbulence.

An increase in energy spread, or "heating", reveals itself in the relative growth of the rms beam size $\tilde{x}$ and rms emittance $\tilde{\epsilon}$. For example, suppose the injected beam is in thermodynamic equilibrium at temperature T_0 and the ejected beam is in thermodynamic equilibrium at temperature T_∞ . Then the ratio of emittance to beam size is proportional to the rms velocity $\tilde{v} \propto \sqrt{T}$ at both $t = 0$ and $t \rightarrow \infty$, so $\tilde{\epsilon}_0/\tilde{x}_0 \propto \sqrt{T_0}$ and $\tilde{\epsilon}_\infty/\tilde{x}_\infty \propto \sqrt{T_\infty}$. If heating were absent, $\tilde{\epsilon}$ and $\tilde{x}$ would evolve together in lock-step, but when heating occurs, $\tilde{\epsilon}$ grows to a relatively larger amplitude than $\tilde{x}$, *i.e.*, by the factor $(T_\infty/T_0)^{1/2}$. This circumstance typifies what is observed in laboratory and numerical experiments, and heating ($T_\infty > T_0$) is prevalent. Using the principle of conservation of energy, Reiser developed a recipe for

estimating the final rms size $\tilde{x}_\infty$ and rms emittance $\tilde{\epsilon}_\infty$ from the free energy of mismatch [6], and this theory provides the basis for calculating T_∞/T_0 from the accelerator parameters and the degree of mismatch.

It is clear from these considerations that both systematic global oscillation of the potential and transient, local stochastic fluctuations can influence the violent relaxation of a beam. These processes increase the energy spread of the particles, and they inject some particles into large-amplitude orbits, yet the ways they influence the particle distribution are qualitatively different.

Calculations of the orbits of test particles in the electric potential of a globally oscillating core show that some particles gain energy, but the process is self-limiting [1-3]. This is easily explained as a resonance phenomenon. A particle will gain energy if it enters the core when the core is large and leaves the core when the core is small. The space-charge force of the core then imparts a net boost to the particle, and the process repeats itself as long as the particle orbits in phase with the core oscillations. This oscillating-core/single-particle interaction ejects resonant particles to large amplitudes in a short time, *i.e.*, within a few core oscillations [5]. However, as the particle increases its energy, its orbital period changes, the orbit eventually falls out of phase with the core oscillation, and the energy gain is accordingly self-limiting. There is consequently a maximum amplitude which a particle can attain from this systematic process.

By contrast, interactions with a broad spectrum of modes are not self-limiting. A statistically small sample of particles may be expected to interact in phase with many localized fluctuations over several orbital periods, and the orbits of these particles would thereby achieve very large amplitudes. Moreover, these interactions will divert some particles from orbits which are nonresonant with respect to the core oscillation to resonant orbits, and these particles will then experience the self-limiting energy gain. Inasmuch as these mode-particle interactions are stochastic, the attendant dynamics is very difficult to analyze, but it is clear that particles can quickly lose memory of their initial conditions. There is both experimental and numerical evidence for ongoing population of the halo by particles having small initial energy [1, 7].

FOKKER-PLANCK-POISSON FORMALISM

The Fokker-Planck-Poisson formalism is based on a two-component model of the distribution function of particles in the phase-space of a single particle. The Fokker-Planck equation governs the evolution of the coarse-grained component of the distribution function and drives it toward Maxwell-Boltzmann equilibrium. Although there is no guarantee that violent relaxation takes the beam all the way to thermodynamic equilibrium, we nevertheless assume it does as a working hypothesis. The coefficients of dynamical friction and diffusion in velocity space result from fluctuations about the coarse-grained distribution function. Turbulence excited as a consequence of charge redistribution enhances these coefficients and converts free energy due to mismatch into thermal energy. Dynamical friction and diffusion generate emittance growth and halo by injecting particles into higher-amplitude orbits. They also dissipate any fine structure present in the beam at injection. Poisson's equation provides the coarse-grained space-charge force.

The spectrum of electric-field fluctuations determines the friction and diffusion coef-

ficients. In general, the diffusion coefficient and relaxation rate may be expected to be functions of position, velocity, and time. For simplicity, and since we do not know these functions *a priori*, we ignore the position and velocity dependencies and model the beam as a fluctuating fluid in which particles execute Brownian motion. The diffusion coefficient is expressed as $D = \beta T/m$, in which β is the relaxation rate, m is the single-particle mass, and T is the “diffusive temperature” in energy units. We adopt a physically plausible phenomenological model of the diffusion coefficient by letting $T = T_\infty + (T_0 - T_\infty) \exp(-\beta_s t)$. Starting from temperature T_0 , the beam strives to reach a Maxwell-Boltzmann distribution with temperature T_∞ , and the heating occurs at the rate $\beta_s \geq \beta$ associated with “strong” turbulence.

To solve the coupled Fokker-Planck and Poisson equations self-consistently, we decompose the coarse-grained distribution function into complete sets of orthogonal polynomials [5]. The decomposition results in an infinite set of first-order, nonlinear differential equations for the time-dependent expansion coefficients which is fully equivalent to the Fokker-Planck-Poisson equations. Upon solving this set of equations, we have a self-consistent expression for the distribution function from which we may calculate any desired moment as a function of time, including the particle-density profile, the rms beam size, and the rms emittance.

We have written a FORTRAN code to solve for the time-dependent expansion coefficients associated with a cylindrically symmetric beam. The code has been benchmarked against a closed-form, analytic solution of the Fokker-Planck equation in which the orbits are all modeled as harmonic oscillators. It has also been verified to provide the correct final distribution function corresponding to thermodynamic equilibrium and for which the density profile can be calculated numerically directly from Poisson’s equation. The solution process involves truncating the series of equations, solving the truncated series, then increasing the number of equations and solving the bigger series. If the solutions substantially agree, then one knows that a sufficient number of terms has been retained in the truncation.

We provide here an example which is representative of ATW-class linacs. We consider a linear focusing channel into which a beam with a Gaussian particle-density profile and Maxwellian velocity distribution is injected. The rms radius of the Gaussian beam is 1.2, where the unit of length is $(2k_B T_0/m\omega^2)^{1/2}$, and ω is the angular betatron frequency without space charge. The rms radius of the equilibrium (matched) beam is 1.458. This example therefore resembles a transition to weaker focusing. The input beam is therefore modestly (20%) rms-mismatched, and it is believed that existing techniques for accelerator design would keep mismatches at transitions to about this level [8]. The ratio of the average Debye length [9] of the matched beam to its rms radius is 0.471, which indicates explicitly that space-charge forces are important to the beam dynamics, and therefore the beam is also mismatched in shape. The ratio of final-to-initial temperature is calculated from Reiser’s theory to be $T_\infty/T_0 = 1.211$. Because the mismatch is modest, we assume the turbulence is always weak and take $\beta_s = \beta$. Furthermore, we let $\beta = 0.05\omega$, a relaxation rate compatible with results of numerical simulations.

Evolution of the rms radius and rms emittance normalized to their initial values is shown in Figure 1. For comparison, curves calculated analytically by modeling all the orbits as harmonic oscillators are also shown. Space charge clearly has a significant quantitative effect on the evolution. It tends in this example to push particles out, increasing the size of the core both in configuration space and in phase space.

Evolution of the "halo" is illustrated in Figure 2. This figure shows the number of particles lying outside fixed radii equal to 1, 1.5, and 2. Curves calculated analytically with model harmonic-oscillator orbits are also shown, and once again it is seen that the nonlinear space-charge forces tend to push particles farther away from the beam axis, as is consistent with Figure 1. It is also seen that, in this example, the process is somewhat more prominent in the earlier stages of evolution than in the later stages.

In summary, we have developed a semianalytic formalism for calculating emittance growth and halo formation in high-current beams for which space charge is important. Unlike oscillating-core/single-particle models, this formalism strives for self-consistency. It also leads to relatively fast computation of the transient dynamics compared to N-body simulations. However, it incorporates an oversimplified model of the very complicated microscopic dynamics which involves at least one free parameter representing the relaxation rate. Thus, while the formalism accounts for all halo-formation mechanisms, its predictive accuracy is limited, particularly with regard to the detailed halo structure. Accordingly, future work should focus on improvements with the ultimate goal of including a self-consistent calculation of the Fokker-Planck coefficients.

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Fig. 1. Rms radius, emittance normalized to $t = 0$. Self-consistent solutions are denoted "S". Analytic solutions with harmonic-oscillator orbits are denoted "A".

Fig. 2. Fractional number of particles outside $r=1$ (top), $r=1.5$ (center), and $r=2$ (bottom). "S": self-consistent solutions; "A": analytic solutions.

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