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OPERATIONAL REACTIVITY ACCOUNTING BY ONE-GROUP THEORY

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OPERATIONAL REACTIVITY ACCOUNTING BY ONE-GROUP THEORY

I. INTRODUCTION AND SUMMARY

The reactivity accounting and calculating techniques currently used in operational physics work are almost entirely based upon the flux squared weighting theorem. This theorem has some very serious limitations as the flattening of the reactors is increased. As a consequence many of the calculating techniques and results are theoretically unsound. Unexplainable discrepancies from pile to pile, and in reactivity data compared to original startup data and lattice physics measurements have been noted in the past. This report describes a method of reactivity evaluation based upon a one-group diffusion theory and a multi-region cylindrical geometry approximation. Results are presented which indicate that the methods outlined provide satisfactory accuracy. It is recommended that the methods outlined be adopted for routine reactivity evaluations as soon as the necessary machine computations have been made and standard procedures have been developed. Specific recommendations and results will be published as supplements to this report as soon as data are available.

II. DISCUSSION

A. The Need for Improved Reactivity Accounting Methods

The reactivity accounting techniques currently in use in operational physics may be briefly described as follows: (See reference 8 for a complete treatment.)

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II. DISCUSSION (Continued)

1. The reactivity effect of xenon is computed from tables based upon the best available data and theoretical techniques.
2. The reactivity effect of control rods is evaluated from data obtained during rod calibration tests. The method of use of the available data is questionable in some cases.
3. The reactivity effect of flattening columns and other poisonous materials is evaluated by applying the flux squared weighting theorem to pile startup test results. The method used, while essentially correct for small loading changes, is not valid for estimating the total reactivity effects of the flattening columns under present loading conditions. The perturbations resulting from poisoning large regions of the reactor are so large that the flux squared weighting theorem is invalid from a macroscopic point of view.
4. The reactivity effect of enriched loadings is evaluated by applying the flux squared weighting theorem to startup and lattice physics test results. Here again the method is not sound due to the limitations of the flux squared weighting theorem. The errors resulting from incorrect evaluation of enrichment effects are particularly serious in that they lead to unexplainable reactivity effects and misconceptions about the proper use of radial enrichment. These items will be discussed more thoroughly later.
5. The reactivity effect of temperature changes is evaluated as follows:
 - a. Evaluate the hot pile reactivity by summing the applicable items mentioned above.
 - b. Evaluate the cold pile reactivity (usually during minimum outages) and all of the known changes which take place in going from the hot to the cold condition by the methods discussed above. The reactivity change left over is assigned to the "power" coefficients.

It is obvious that the evaluation of reactivity effects of temperature changes is subject to the same errors as the items which enter into its evaluation.

6. The reactivity of the pile at any time is computed by estimating the changes which have occurred between the time of interest and the most recently known reactivity condition.

The primary objection to the system described above is that it is based upon an improper extension of the flux squared weighting theorem, and hence is theoretically unsound. Therefore, it cannot accurately predict the effect of gross loading changes.

The growth and development of the system described above is quite natural. In the operational physics business results are needed at a moment's

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II. DISCUSSION (Continued)

notice. Hence a very simple and usable system is vital. Since the weighting theorem is very simple and easy to use it is a natural for the conditions. After a loading pattern is established for equilibrium operation, only very small changes are usually needed. Since the flux weighting technique is satisfactory for these small perturbations about a fixed condition, it seems natural to accept its overall validity.

Regardless of the reasons for the development of the system in current use, it has had some rather serious consequences over the years. First of all, it encourages statistical analysis and discourages basic theoretical studies to an unreasonable degree; calculational results have often been compared with other calculational results rather than with startup test results and basic theory. An illustration of this is the treatment of the HCR reactivity effects. The HCR's in the nine-rod reactors were calculated to be worth ~ 600 ih before B pile startup. B pile startup tests confirmed the calculations. Over the years the rod strengths were fudged to fit "observed" results until the evaluated system strength was ~ 350 ih. A rod calibration test performed in 1956 provided the surprising result that the total system strength was still ~ 600 ih.

The system of evaluation used leads to differences in assigned pile "cold clean" reactivities which are usually due to differences in bookkeeping techniques. A good illustration of this is the K, C, H "ghost reactivity loss" phenomena. The K, C, H lattice material bucklings, and hence reactivities, were measured during pile startup tests. During the course of time it was observed that the "calculated" cold clean unenriched pile reactivities were decreasing with time. This unexplainable decrease continued until the "ghost loss" had reached about 200-350 ih. This "ghost" loss was actually the result of using the flux squared weighting theorem to calculate the reactivity worth of radial enrichment. Multi-region cylindrical geometry buckling calculations were used to calculate the pile reactivities for C, H, and the K's during the times when the "ghost" loss existed. The cold material bucklings obtained from startup and exponential tests were used. The result (see Section G) was that the observed pile reactivities fit the computed values very well. Hence one must conclude that the lattice material bucklings have not changed since startup, and that the discrepancies observed are due to an unsound reactivity accounting scheme.

For these reasons it is imperative that the accounting schemes in routine use be dropped, and new schemes which are theoretically sound be adopted as soon as possible. A theoretically sound technique is described, and its adoption is proposed in the remainder of this report. The technique proposed is based on the one-group diffusion theory approximation. A multi-region cylindrical geometry is used.

B. The Flux Squared Weighting Theorem

The reactivity accounting and calculating techniques which are currently used by reactor plant assistance physicists are based primarily upon the flux squared weighting theorem. Therefore, it seems appropriate to discuss this theorem briefly before proposing new calculating and accounting techniques. The derivation which follows is taken largely from references 3 and 12.

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II. DISCUSSION (Continued)

B. Derivation

Suppose we start with a uniformly loaded reactor which is just critical. The flux distribution (one-group diffusion theory) is described by solutions of the expression:

$$(1) \quad \nabla^2 \phi_0 + B_0 \phi_0 = 0$$

$$B_0 = \frac{K_0 - 1}{M^2}, \quad K_0 = K_{00}$$

If we poison some region of the reactor down such that $K_1 < K_0$, the reactor will be subcritical. However, if we raise the K of the rest of the pile up to some value of $K = K_2$ the critical condition will be restored. The problem is to find the required K_2 .

$$\text{Let } B_1 = B_{10} + \delta B_1$$

In the poisoned region:

$$(2) \quad \delta B_1 = B_1 - B_0 = \frac{K_1 - 1}{M^2} - \frac{K_0 - 1}{M^2} = \frac{K_1 - K_0}{M^2}$$

In the unpoisoned region:

$$(3) \quad \delta B_2 = \frac{K_2 - K_0}{M^2}$$

If δB is small compared to B_0 :

$$(4) \quad \nabla^2 \phi + B\phi \approx \nabla^2 \phi + (B_0 + \delta B)\phi = 0$$

If $\phi \approx \phi_0 + \phi'$ (ϕ' small compared to ϕ_0), then:

$$(5) \quad \nabla^2 \phi + B\phi = \nabla^2 \phi_0 + \nabla^2 \phi' + B_0 \phi_0 + \delta B \phi_0 + B_0 \phi' + \delta B \phi' = 0$$

Since $\nabla^2 \phi_0 + B_0 \phi_0 \approx 0$,

$$(6) \quad \nabla^2 \phi + B\phi = \nabla^2 \phi' + B_0 \phi' + \delta B \phi_0 + \delta B \phi' = 0$$

Since $\phi' \ll \phi_0$, $\delta B \ll B_0$, $\delta B \phi' \approx 0$.

Thus the expression reduces to:

$$(7) \quad \nabla^2 \phi' + B_0 \phi' + \delta B \phi_0 \approx 0.$$

The expression $\nabla^2 \phi + B\phi$ has a set of solutions, ϕ_1 , all of which vanish at the surface of the pile. For each ϕ_1 there is a corresponding B_1 . A pair of such expressions are:

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II. DISCUSSION (Continued)

B. Derivation (Cont'd)

$$(8) \nabla^2 \phi_j + B_j \phi_j = 0$$

$$(9) \nabla^2 \phi_k + B_k \phi_k = 0$$

Multiplying the equations by ϕ_k and ϕ_j , respectively:

$$(10) \phi_k \nabla^2 \phi_j + B_j \phi_j \phi_k = \phi_j \nabla^2 \phi_k + B_k \phi_k \phi_j = 0$$

Grouping like terms and integrating over the pile:

$$(11) \int_V (\phi_j \nabla^2 \phi_k - \phi_k \nabla^2 \phi_j) dv = \int_V (B_j - B_k) \phi_j \phi_k dv$$

From Green's theorem:

$$(12) \int_V (\phi_j \nabla^2 \phi_k - \phi_k \nabla^2 \phi_j) dv = \int_S (\phi_j \nabla \phi_k - \phi_k \nabla \phi_j) dA$$

Since ϕ_j and ϕ_k vanish at the surface of the pile,

$$(13) \int_V (B_j - B_k) \phi_j \phi_k dv = \int_S (\phi_j \nabla \phi_k - \phi_k \nabla \phi_j) dA \equiv 0.$$

Since $\nabla^2 \phi' + B \phi'$ has a set of solutions, ϕ' may be expressed as a series expansion:

$$(14) \phi' = \sum_1 A_1 \phi_1$$

Since $\nabla^2 \phi_1 = -B_1 \phi_1$,

$$(15) \nabla^2 \phi' = -\sum_1 A_1 B_1 \phi_1.$$

From $\nabla^2 \phi' + B_0 \phi' + \delta B \phi_0 = 0$,

$$(16) -\sum_1 A_1 B_1 \phi_1 + B_0 \sum_1 A_1 \phi_1 + \delta B \phi_0 = 0.$$

Multiplying by ϕ_0 and integrating over the pile:

$$(17) \int_V \sum (B_0 - B_1) A_1 \phi_0 \phi_1 dv + \int_V \delta B \phi_0^2 dv = 0.$$

From (13) $\int_V (B_j - B_k) \phi_j \phi_k dv = 0$,

$$(18) \int_V \phi_0 \phi_1 = 0 \text{ for } i \neq 0.$$

$$(B_0 - B_1) = 0 \text{ for } i = 0.$$

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II. DISCUSSION (Continued)

B. Derivation (Cont'd)

Therefore:

$$(19) \int_V \sum (B_0 - B_1) A_1 \phi_0 \phi_1 dv = 0$$

and

$$(20) \int_V \delta B \phi_0^2 dv = 0.$$

From expressions 2 and 3 for δB :

$$(21) \int_V \delta B \phi_0^2 dv = \delta B_1 \int_1 \phi_0^2 dv + \delta B_2 \int_2 \phi_0^2 dv = 0.$$

$$(22) \frac{K_1 - K_0}{M^2} \int_1 \phi_0^2 dv + \frac{K_2 - K_0}{M^2} \int_2 \phi_0^2 dv = 0$$

$$\frac{K_2 - K_0}{K_0 - K_1} = \frac{\int_1 \phi_1^2 dv}{\int_2 \phi_1^2 dv}$$

$$(23) \delta K_2 = \delta K_1 \frac{\int_1 \phi_0^2 dv}{\int_2 \phi_0^2 dv}$$

Further, suppose a reactor is composed of two regions with bucklings B_1 and B_2 and we wish to find the effective buckling of the reactor (B).

Since $\int_V \delta B \phi^2 dv = 0$,

$$(24) \int_{R_1} (B_1 - B) \phi^2 dv + \int_{R_2} (B_2 - B) \phi^2 dv = 0$$

$$(25) B_1 \int_{R_1} \phi^2 dv - B \int_{R_1} \phi^2 dv + B_2 \int_{R_2} \phi^2 dv - B \int_{R_2} \phi^2 dv = 0.$$

$$(26) B_1 \int_{R_1} \phi^2 dv + B_2 \int_{R_2} \phi^2 dv = B \int_{\text{Pile}} \phi^2 dv$$

$$(27) B = B_1 \frac{\int_{R_1} \phi^2 dv}{\int_{\text{Pile}} \phi^2 dv} + B_2 \frac{\int_{R_2} \phi^2 dv}{\int_{\text{Pile}} \phi^2 dv}$$

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II. DISCUSSION (Continued)

B. Derivation (Cont'd)

$$\frac{\int_{R_1} \phi^2 dv}{\int_{\text{Pile}} \phi^2 dv} \quad \text{is known as the statistical, or flux squared, weight, of Region } R_1.$$

It must be admitted at the outset that the flux squared weighting theorem is very simple and useful. Unfortunately, it is so simple and useful that the restrictions for its proper use are usually ignored. These restrictions are: (See equations 4 and 5 in the derivation.)

1. δB must be small compared to B_1 .

The perturbation must be small compared to the original buckling. "Small" may be indefinite, but it surely implies that δB must be less than $\sim 0.1 B_0$.

2. $\phi \equiv \phi_0$.

The flux distribution is virtually unchanged by the perturbation.

A ground rule which should be applied to the flux squared weighting theorem is: Use the ϕ^2 weighting method IF and only IF the flux distribution is known and will be virtually unchanged by the perturbation.

C. "Flux Matching," or "Buckling" Calculations

Since reactor loadings are non-uniform, the condition where the flux distribution is accurately known is rare. Normal buckling perturbations made to supply a reasonable operating flux distribution are quite large. Therefore, the ϕ^2 weighting technique is of limited value in reactor physics calculations. The pile reactivities, or "effective material bucklings," and flux distributions must be calculated by "flux matching" techniques.

"Buckling," or "flux matching," calculations are basically quite simple, but usually involve laborious iteration techniques. They are, therefore, ideally suited to machine computation. The basic technique of buckling calculations may be described as follows:

- (1) Write out the solutions to the expression $\nabla^2 \phi + B\phi = 0$ for the geometry of interest.
- (2) Divide the system into an appropriate number of regions.
- (3) Apply the proper conditions to the fluxes and slopes of the fluxes at the region boundaries. Combine the resulting equations properly to obtain the "critical," or "continuity" equations.

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II. DISCUSSION (Continued)

- C. (4) Substitute the known bucklings into the critical equations, and solve for the unknown buckling and the flux distribution.

The term "buckling" calculation covers a wide variety of techniques. The exact method used depends upon the situation and accuracy desired. Various numbers of neutron energy groups may be used to describe the neutron energy spectrum. Several different models have been developed to describe the "wandering" or "diffusion" of a thermal neutron prior to its capture. The simplest possible approximation, one-group diffusion theory, is used throughout this report. Better approximations will undoubtedly be adopted in the future. However, since the Hanford reactors are large and loaded with reasonably uniform material, the one-group diffusion theory approximation provides fairly accurate results.

D. Estimating Bucklings of Uniformly Perturbed Lattices

Non-standard materials such as control rods, poison columns, and enriched charges, are commonly located in the lattices of reactors to control local flux and total power. These materials produce local perturbations in the lattice K_{00} . It is often necessary to assign an "effective" material buckling to a lattice containing a uniform arrangement of non-standard material. The effect of a single, centrally located foreign charge on the reactivity, or K_{eff} , of a reactor is usually determined during startup tests. If not, it can be computed theoretically (to a reasonable accuracy). For the purposes of this report, it will be assumed that the reactivity effects of single centrally located foreign charges are known.

The effect of a uniform arrangement of non-standard charges on the lattice material buckling can be computed from the single column reactivity values. The method outlined below is based upon applying the flux squared weighting theorem to the single column reactivity values. Hence the accuracy of the computed buckling effects is subject to the limitations of this theorem. If the flux perturbations caused by the single columns at time of testing were large, more elegant techniques should be used. However, most of the single columns used did not produce large perturbations in the total flux distribution during the tests.

Derivation

1. Estimating the Buckling Effect of Non-Uranium Columns

Suppose a uniform cylindrical lattice is loaded until a super-critical state is reached. If a poisonous charge is placed in the central lattice cell, the K_{eff} of the reactor will be reduced. The change in K_{eff} is equivalent to a known uniform reduction in the lattice material buckling. The reduction in buckling associated with a uniform array of these columns may be computed as follows:

Suppose we wish to compute the effect of n such columns spread uniformly throughout the reactor. The "effective" area of control

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II. DISCUSSION (Continued)

D. Derivation (Cont'd)

of a single column is $\frac{\text{Area of loaded lattice}}{n}$.

(This effective area is obviously not exactly the same as the region of control; however, it is beyond the scope of this document to give a detailed discussion of the microscopic flux distribution in the vicinity of a control element.) The flux distribution in the uniformly loaded cylinder is:

$$\phi = J_0 \left(\frac{2.405 r}{R_p} \right)$$

R_p = Effective pile radius

If the cylinder is large, and the flux perturbation caused by the column is small, $\phi(r)$ after the column is introduced is still:

$$\phi(r) = J_0 \left(\frac{2.405 r}{R_p} \right)$$

According to the flux squared weighting theorem:

$$\oint_{B_c} \phi^2 dA \approx \oint_{B_p} \phi^2 dA$$

Rearranging:

$$\oint_{B_c} \phi^2 dA = \frac{\oint_{B_p} \phi^2 dA}{\oint_c \phi^2 dA}$$

Substituting the expression for the flux:

$$\oint_{B_c} \phi^2 dA \approx \frac{\oint_{B_p} \phi^2 dA}{\oint_c \phi^2 dA} = \frac{\int_0^{R_p} 2\pi r J_0^2 \left(\frac{2.405 r}{R_p} \right) dr}{\int_0^{R_c} 2\pi r J_0^2 \left(\frac{2.405 r}{R_p} \right) dr}$$

$$\begin{aligned} R_c &= \text{"Effective radius of control"} \\ &= \left(\frac{\text{Effective area of control}}{\pi} \right)^{1/2} \\ &= \left(\frac{\text{Area of loaded lattice}}{\pi n} \right)^{1/2} \end{aligned}$$

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II. DISCUSSION (Continued)

D. Derivation (Cont'd)

The integration is performed as follows:

$$\int x \left[J_0^2(\alpha x) \right] dx = \frac{x^2}{2} \left\{ \left[J_0^2(\alpha x) \right] - \left[-J_1^2(\alpha x) \right] \right\}$$

$$\int x \left[J_0^2(\alpha x) \right] dx = \frac{x^2}{2} \left\{ \left[J_0^2(\alpha x) \right] + \left[J_1^2(\alpha x) \right] \right\}$$

Substituting this expression for the integrals:

$$\delta B_c \approx \delta B_p \frac{\left[\frac{r^2}{2} \left(J_0^2\left(\frac{2.405 r}{R_p}\right) + J_1^2\left(\frac{2.405 r}{R_p}\right) \right) \right]_0^{R_p}}{\left[\frac{r^2}{2} \left(J_0^2\left(\frac{2.405 r}{R_p}\right) + J_1^2\left(\frac{2.405 r}{R_p}\right) \right) \right]_0^{R_c}}$$

Inserting the limits:

$$\delta B_c \approx \delta B_p \frac{R_p^2 J_1^2(2.405)}{R_c^2 \left[J_0^2\left(\frac{2.405 R_c}{R_p}\right) + J_1^2\left(\frac{2.405 R_c}{R_p}\right) \right]}$$

δB_c is the reduction in lattice material buckling caused by columns charged in the density specified by R_c . Note that regardless of the density used (subject to the restriction that the pile must be near critical) the overall flux distribution in a reactor uniformly loaded with the columns will be given by $J_0\left(\frac{2.405 r}{R_p}\right)$. Further, this is the

reduction in lattice material buckling within the column region associated with columns in the density specified by R_c regardless of the geometrical arrangement of the poisoned region.

2. Estimating the Buckling Effect of Control Rods

Experimental data on the reactivity effects of the horizontal control rods is not very plentiful. It is difficult to use the data which are available because the rod systems are not symmetrical about the center of the rod plane, the rod plane is rectangular, and the reactor loadings were usually non-uniform during the tests. For these reasons the buckling effects of the control rods were calculated theoretically.

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II. DISCUSSION (Continued)

D. Derivation (Cont'd)

The following discussion presents the results of the theoretical calculations, and outlines a method for using these results to estimate the buckling effect of the HCR's in the radial (temperature map) plane.

The reduction in buckling (δB_c) within the control area was calculated using the methods outlined by D. E. Simpson in reference 1. The results were:

Reactor	δB_c
B, D, DR, F, H	59.5 μb
C	72.5 μb^*
K	59.0 μb

* Value for the new half-rods. It is assumed that this value also applies to the old cadmium rods.

The reactivity worth of a single centered HCR (or a symmetrical pattern of rods) may be estimated by transposing to an equivalent cylinder in the HCR plane and using standard "flux matching" techniques and the buckling effects listed above.

The number of HCR's (16.5) required to hold the green, fully loaded K Reactors critical was measured during the K pile startup tests. These data provide an indication of the accuracy of the calculated rod effects. The radial buckling required to support 16.5 HCR's was calculated using $\delta B_c = 59 \mu b$. The results were:

Theoretical Radial Buckling (Assuming $\delta B_c = 59 \mu b$)	Observed Radial Buckling (From Wet Critical Size)
53.2 μb	55.2 μb

The reactivity worth of a single centered HCR was calculated for the various reactors using the δB_c values listed above. The rods are assumed to extend completely through the reactor side-to-side.

Reactor	δB_p^{**}
B, D, DR, F, H	4.7 μb
C	5.4 μb
K	3.4 μb

** δB_p is the equivalent uniform reduction in the material buckling of the pile caused by a single centered HCR.

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II. DISCUSSION (Continued)D. Derivation (Cont'd)

Since we wish to approximate the reactor by an equivalent right cylinder (axis in the front-to-rear direction), we must estimate the buckling effects in the radial (front face) plane. The ϕ_{B_p} values listed above may be used to estimate the radial ϕ_{B_c} associated with various control patterns as follows:

- 1) Estimate the number of effective central (front-to-rear) HCR's in the area of interest using perturbation theory.

$$N = \frac{\sum_{i=1}^n \int_{\text{control}} \phi^2 dx}{\int_{\text{centered rod}} \phi^2 dx}$$

N = Effective centered (front-to-rear) rods

n = Number of HCR's passing through the region of interest

$$\phi = \sin \frac{\pi x}{L}$$

- 2) Transpose the reactor to an equivalent right cylinder (axis in the front-to-rear direction). Estimate ϕ_{B_c} within the control region of interest by using the method outlined for non-standard charges, the proper ϕ_{B_p} from the values listed above and the R_c which corresponds to the N computed above. This is equivalent to assuming that the rods travel along the axis of the cylinder.

Values of the integrals needed for computing N are:

Reactor	Rod Bank	$\frac{\int_{\text{rod}} \phi^2 dx}{\int_{\text{center rod}} \phi^2 dx}$	Remarks
B,C,D,DR,F,H	Upstream	.54	Assumes rod system displaced 14 inches to the front
	Center	.98	
	Downstream	.77	
K	Inboard	.94	
K	Outboard	.57	

E. Multi-Region Cylindrical Coordinate Critical Equations

For simplicity of treatment, the Hanford reactors may be approximated by an equivalent right cylinder. (Better geometric approximations should be adopted as time permits. The so-called "variational technique"⁷ appears to be the only reasonable candidate.) The critical conditions are described by:

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II. DISCUSSION (Continued)

E. (1) $\nabla^2 \phi - \sum_a \phi + K \sum_a \phi = \frac{\partial \phi}{\partial T} = 0$

$$\nabla^2 \phi + \frac{(K-1) \sum_a \phi}{D} = 0$$

$$\nabla^2 \phi + B \phi = 0$$

In cylindrical coordinates this expression becomes:

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} + B \phi = 0$$

This has the solutions:

(1) $B = 0$:

$$\phi = A + C \ln r.$$

(2) $B > 0$:

$$\phi = A J_0(b r) + C Y_0(b r)$$

$$b = \sqrt{|B|}$$

(3) $B < 0$:

$$\phi = A I_0(b r) + C K_0(b r)$$

$$b = \sqrt{|B|}$$

These expressions may be used to derive the critical equations for an arbitrary number of regions of uniform bucklings. The problem may be presented pictorially as follows:

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II. DISCUSSION (Continued)

E. Equations may be generated which will permit solutions to the following types of problems: (Note that the difference is artificial.)

- (1) Given the n bucklings, find the uniform change in buckling required to produce the critical state. Also find the flux distribution.
- (2) Given $n-1$ of the bucklings, and the fact that a critical state exists, find the unknown buckling. Also find the critical flux distribution.

The equations are generated by applying the following boundary conditions:

$$\begin{aligned}\phi_1 \Big|_{R_1} &= \phi_{1+1} \Big|_{R_1} \\ \frac{d\phi_1}{dr} \Big|_{R_1} &= \frac{d\phi_{1+1}}{dr} \Big|_{R_1} \quad ? \quad \left(\text{not } D_i \frac{d\phi_i}{dr} \Big|_{R_i} = D_{i+1} \frac{d\phi_{i+1}}{dr} \Big|_{R_i} \right) \\ \phi_n \Big|_{R_n} &= 0\end{aligned}$$

For example, suppose we are interested in solving a three-region problem. Further, suppose the B_i 's are all positive:

$$(1) \quad \phi_1 = A J_0(b_1 r) + C Y_0(b_1 r)$$

Since $Y_0 \rightarrow \infty$ as $r \rightarrow 0$

$$\phi_1 = A J_0(b_1 r)$$

Since one of the constants is arbitrary, set $A = 1$

$$\phi_1 = J_0(b_1 r)$$

$$(2) \quad \phi_2 = A_1 J_0(b_2 r) + A_2 Y_0(b_2 r)$$

$$(3) \quad \phi_3 = A_3 J_0(b_3 r) + A_4 Y_0(b_3 r)$$

Applying the boundary conditions:

$$(1) \quad J_0(b_1 R_1) = A_1 J_0(b_2 R_1) + A_2 Y_0(b_2 R_1)$$

$$(2) \quad b_1 J_1(b_1 R_1) = A_1 b_2 J_1(b_2 R_1) + A_2 b_2 Y_1(b_2 R_1)$$

$$(3) \quad A_1 J_0(b_2 R_2) + A_2 Y_0(b_2 R_2) = A_3 J_0(b_3 R_2) + A_4 Y_0(b_3 R_2)$$

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II. DISCUSSION (Continued)

$$E \quad (4) \quad A_1 b_2 J_1(b_2 R_2) + A_2 b_2 Y_1(b_2 R_2) = A_3 b_3 J_1(b_3 R_2) + A_4 b_3 Y_1(b_3 R_2)$$

$$(5) \quad A_3 J_0(b_3 R_p) + A_4 Y_0(b_3 R_p) = 0$$

These equations are most easily solved by renaming the bessel functions and working in symbolics as follows:

$J_0(b_1 R_1) = X_1$	$J_0(b_3 R_2) = X_9$
$J_0(b_2 R_1) = X_2$	$Y_0(b_3 R_2) = X_{10}$
$Y_0(b_2 R_1) = X_3$	$b_2 J_1(b_2 R_2) = X_{11}$
$b_1 J_1(b_1 R_1) = X_4$	$b_2 Y_1(b_2 R_2) = X_{12}$
$b_2 J_1(b_2 R_1) = X_5$	$b_3 J_1(b_3 R_2) = X_{13}$
$b_2 Y_1(b_2 R_1) = X_6$	$b_3 Y_1(b_3 R_2) = X_{14}$
$J_0(b_2 R_2) = X_7$	$J_0(b_3 R_p) = X_{15}$
$Y_0(b_2 R_2) = X_8$	$Y_0(b_3 R_p) = X_{16}$

Equations 1-5 may then be rewritten:

$$\begin{aligned} (1) \quad X_1 &= A_1 X_2 + A_2 X_3 \\ (2) \quad X_4 &= A_1 X_5 + A_2 X_6 \\ (3) \quad A_1 X_7 + A_2 X_8 &= A_3 X_9 + A_4 X_{10} \\ (4) \quad A_1 X_{11} + A_2 X_{12} &= A_3 X_{13} + A_4 X_{14} \\ (5) \quad A_3 X_{15} + A_4 X_{16} &= 0 \end{aligned}$$

After appropriate algebraic manipulation, the resulting solution is:

$$\frac{A_2 X_7 + A_3 X_8}{A_2 X_{11} + A_3 X_{12}} = \frac{X_9 - D X_{10}}{X_{13} - D X_{14}}$$

$$A_2 = \frac{X_3 X_{14} - X_1 X_6}{X_3 X_5 - X_2 X_6}$$

$$A_3 = \frac{X_2 X_{14} - X_1 X_5}{X_2 X_6 - X_3 X_5}$$

$$D = \frac{X_{15}}{X_{16}}$$

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II. DISCUSSION (Continued)

- E. Similar equations may be derived for any combination of signs of B_1 and number of regions.

Routine manual solutions to problems for up to three regions are probably practical (although three-region problems are not particularly pleasant). Machine computations are required for routine solutions to problems containing four or more regions as the complexity and time required for solution rapidly becomes limiting. It seems advisable at this time to generate a versatile program for a computer which will solve these problems for an arbitrary number of regions, and an arbitrary arrangement of signs on B_1 .

F. Listing of Cold Wet Hanford Lattice Material Bucklings and Other Reactor Constants

Note: The values for solid slugs were obtained from startup test results. I and E and "E" slug values were obtained from exponential and test pile data available at the time the analysis was performed. The solid slug values are listed in references 4, 5, and 6, and summarized in reference 10.

<u>Lattice Type</u>	<u>Fuel Type</u>	<u>B_m (ub) Green</u>
8-3/8 inch B,D,IR,F,H	Solid Natural	71.8
	I and E Natural (0 II)	60.1
	I and E E (.94 0 II)	270

$$mk/\mu b = -.618 \text{ (best value)}$$

$$\text{Effective radius of loaded pile} = 537.3 \text{ cm}$$

$$\text{Radial augmentation length} \approx 63 \text{ cm}$$

$$\text{Effective pile radius} \approx 600 \text{ cm}$$

$$\text{Longitudinal augmentation length} \approx 43 \text{ cm}$$

$$\text{Radial geometric buckling} \approx \left(\frac{2.405}{600} \right)^2 \approx 16.1 \mu b$$

$$\text{Longitudinal geometric buckling} \approx (\pi/L)^2$$

$$32 \text{ slugs} \approx 15.1 \mu b$$

$$34 \text{ slugs} \approx 13.5 \mu b$$

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II. DISCUSSION (Continued)

<u>F.</u>	<u>Lattice Type</u>	<u>Fuel Type</u>	<u>B_m (μb) (Green)</u>
	8-3/8 inch C	Solid Natural	66.2
		I and E Natural C II	47.2*
		"C" Metal (Full Charge)	711
		Solid E ~.94 Enriched	280
		I and E E ~.94 Enriched C II	265

Other reactor constants are the same as for 8-3/8 inch B, D, DR, F, H.

*From a very rough extrapolation of dry lattice values. C data fits quite well if this value is used.

<u>Lattice Type</u>	<u>Fuel Type</u>	<u>B_m (μb) (Green)</u>
7-1/2 inch K	Solid Natural	61.9
	I and E Natural K II	51.9
	Solid E (.94)	280
	I and E E (.94) K II	270
	"C" (Partial Column loading used)	860

$mk/\mu b \approx -.535$ (best value)

Effective radius of loaded pile ≈ 610 cm

Radial augmentation length ≈ 52 cm

Effective pile radius ≈ 662 cm

Longitudinal augmentation length ≈ 31 cm

Radial geometric buckling $\approx \left(\frac{2.405}{662}\right)^2 \approx 13.2 \mu b$

Longitudinal geometric buckling (38 slugs) $= 11.6 \mu b$.

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II. DISCUSSION (Continued)

F. The following assumptions were made for all calculations:

- (1) Material bucklings and other reactor constants listed above are correct.
- (2) Effects of poisonous materials distributed throughout lattice calculated by method outlined in Section D. Effects of single columns taken from startup test results. See reference 10 for listing.
- (3) Cold pile B_m varies with exposure as indicated in reference 9. The results listed in this reference were used for all lattices containing natural fuel elements.

G. Results of Cold Reactivity Calculations

The as-loaded reactivities of the cold Hanford reactors can be computed by using the material bucklings and methods developed in the preceding sections. Cold reactivities were computed for each of the lattice types, and for both the solid and I and E fuel elements. Three-region cylindrical coordinate equations were used. The results are compared to the observed pile reactivities in the following summary. The observed pile reactivity is defined as:

Pile Reactivity, R , = Pile Excess Plus Reactivity Held in Xenon

Most of the outages listed were essentially rods-out minimums.

Reactor	Outage Date	Natural Metal Type	Radial Enr. Type and Amount	Flattening Columns (Equivalent)	R Calculated 1h	R Observed 1h
B	5-22-58	Solid	Solid E - ~30 Cols.	~31 H ₂ O Cols.	714	758
B	12-22-58	I and E	I and E E ~50 Cols.	~35 H ₂ O Cols.	570	605
D	1-30-57	Solid	None	~27 H ₂ O Cols.	709	706
DR	3-20-58	Solid	14 Solid E	~28 H ₂ O Cols.	716	737
H	3-19-58	Solid	Solid E ~260 Cols.	~32 H ₂ O Cols.	1023	1025
H	1-24-59	I and E	I and E E ~230 Cols.	~40 H ₂ O Cols.	797	757
Average * Now 9-Rod Pile		I and E	I and E E ~60 Cols.	~28 H ₂ O Cols.	630	(300 MWD/Ton Exposure)

* Recent information indicates that D Reactor lattice material buckling may be ~3-5 μ b less than the value listed for I and E fuel elements. The validity of the data must be checked before any definite conclusions can be drawn. This discrepancy may be due to balls remaining in the stack from past ball drops.

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II. DISCUSSION (Continued)

G. (Cont'd)

<u>Reactor</u>	<u>Outage Date</u>	<u>Natural Metal Type</u>	<u>Radial Enr. Type and Amount</u>	<u>Flattening Columns (Equivalent)</u>	<u>R Calculated ih</u>	<u>R Observed ih</u>
C	1-8-56	Solid (Low Goal)	C ~97 Cols.	~24 H ₂ O Cols.	810	782
C	1-12-59	I and E	E ~260 Cols.	~7 H ₂ O Cols.	804	825
K	Early 1957	Solid	C ~70 Cols.	0	920	~ 930
K	Late '58 Early '59	I and E	E ~250 Cols.	~5 H ₂ O Cols. ~50 Spike E	960	~ 980-1020

The accuracy indicated by the results listed above is very satisfactory. Since the outages were chosen at random, these results indicate the accuracy which can be expected. The results are encouraging from another point of view. Since the lattice material bucklings and methods used may be assumed to be correct, such calculations provide a rough check on the accuracy of the xenon equations. If one assumes that the methods and bucklings are correct, the results indicate that the xenon equations must also be valid.

The enrichment and flattening in some of the reactors is not uniform (heavy near-side enrichment concentrations to support HCR's). The multi-region cylindrical coordinate calculation assumes a uniform loading. The effects of this non-uniformity can be investigated using multi-region slab calculations. One case was investigated for the K Reactors. The effect of the non-uniformity was found to be insignificant ($< 2 \mu b$). However, the effect should be investigated more thoroughly for all of the reactors, and the various possible loading patterns.

Of particular interest is the agreement between the calculated and observed reactivities for K, C, H Reactors. By the time of the transition from the solid load to I and E's the "cold clean unenriched" reactivities of these reactors as calculated by the standard techniques were 200-350 ih less than the values observed during pile startup tests. This reactivity loss was commonly referred to as the "ghost loss" because it could not be explained. The buckling calculations point out that the lattice reactivities have not changed since startup. The "ghost" loss was caused by improper evaluation of the enrichment reactivity worth.

H. Hot Reactivity Calculations

The reactivity status and flux distributions of the operating reactors are of vital economic concern. In general, the radial flux distribution of the operating Hanford reactors may be characterized by a flat zone containing HCR's, some flattening columns and perhaps a few spike enrichment columns. The flat zone is usually surrounded by a ring of "radial" enrichment (up to two to three lu thick). The loading outside the enrichment ring usually

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II. DISCUSSION (Continued)

- H. consists of natural uranium elements plus a few empty, damaged, channels and perhaps a "fringe poison plus compensating enrichment" load. The reactor may then be characterized by three major regions with perhaps a few subdivisions within the flat zone and/or fringe. At any rate the radial plane of the reactors may be represented pictorially as follows:

B_1 = Hot material buckling within the flat zone = B (hot) + δB_c
 + $\delta B_p \approx 0$. B_1 must be reduced to essentially zero ($K_\infty \approx 1$)
 within the flat zone to produce a "flat" flux distribution.
 δB_c = Buckling effect of control rods. δB_p = Buckling
 effect of poisonous charges.

B_2 = Hot material buckling of enriched ring $\approx$ cold effective buckling
 + δB_T .
 δB_T = Change in buckling resulting from bringing the cold,
 xenon-free pile to a hot condition. δB_T is assumed to be
 independent of ϕ for the purpose of this discussion.

B_3 = Hot material buckling of the fringe $\approx$ cold effective
 buckling + δB_T .

It is immediately apparent that the flat zone may be treated as a separate and isolated unit because its hot B_m must be ≈ 0 . The number of radial enrichment tubes, etc., required to support the edge leakage and hence supply a critical condition has no effect on the flat zone. Therefore, the hot reactivity problem can be broken into two parts: (1) computing the poison and/or rod densities within the flat zone required to reduce the hot pile B_m to zero and produce the "flat" condition, (2) computing the radial enrichment density and arrangement necessary to supply the critical condition. This computation must be performed as a "buckling" calculation with the appropriate boundary conditions.

(The discussions of this section are, of course, idealized. B is never exactly zero everywhere within the flat zone. Also, the radial enrichment arrangement is never such that the pile would be exactly critical if $B = 0$ everywhere within the flat zone.)

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II. DISCUSSION (Continued)

H. (Cont'd)

Flat Zone Considerations

As was indicated in the preceding discussion the flat zone may be treated as an isolated unit which has a hot $B = 0$ (radial component of B). Stated mathematically:

$$B \approx B_{\text{cold}} + \delta B_T + \delta B_p + \delta B_c = 0$$

$$\delta B_T = \delta B \text{ from xenon} + \delta B \text{ from coefficients}$$

For the purposes of this discussion the following assumptions will be made:

- (1) B cold is known
- (2) δB from xenon is known
- (3) δB from coefficients is known.

In actual practice the δB from coefficients is not very well known unless data from some previous operating experience is available.

If these assumptions are made, then the problem reduces to one of determining the rod pattern, flattening density, and spike enrichment loading which will provide the δB needed to force B to zero. In actual practice a rod pattern is selected, and a flattening (including spike enrichment) density which will permit this rod pattern must be found. Minor errors in this estimated density are accommodated by varying the rod pattern as the equilibrium condition is approached. The buckling effects of the rod and flattening arrangements may be computed by the methods outlined in Section D.

As was indicated previously, the δB associated with the temperature coefficients is usually not known for a particular slug unless some operating data is available. However, the effects can be estimated roughly from experience with other slugs. A flattening pattern which will allow as much uncertainty as possible to be accommodated by the rod system should be selected. In actual practice the transition from one load to another is never a one-step process. This permits determining the optimum equilibrium arrangement before the load is completed.

Radial Enrichment Considerations

The effective buckling of the fringe (region exterior to the flat zone) needed to supply a critical condition must be calculated by "flux" matching techniques. The hot material buckling of the natural uranium must be assumed known. (Again this may be estimated from $B = B_{\text{cold}} + \delta B_T$). If this assumption is made, the problem reduces to one of finding that enrichment arrangement which will satisfy the critical condition.

The situation may be pictured as follows for three-regions:

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II. DISCUSSION (Continued)

H. (Cont'd)

$$B_1 = 0$$

$$B_2 = ?$$

$$B_3 = B_m \text{ cold} + \delta B_T$$

This problem may be attacked in two ways:

- (1) Assume that all tubes in region two are charged with enrichment and find R_2 (hence the number of tubes) which satisfies the critical condition.
- (2) Assume that the value of R_2 is known (hence the enrichment will all be located within a particular region) and find the B_2 needed to supply a critical condition. The number of enrichment tubes can then be found by simply computing the E density required to supply this B.

Method (2) will be selected for the example which follows because it leads to more useful solutions.

Derivation

In this particular situation it is obvious that fringe B's are usually >0 . Therefore, the fringe fluxes are usually given by:

$$\phi_1 = A_1 J_0(b_1 r) + C_1 Y_0(b_1 r)$$

Applying the appropriate boundary conditions:

$$(1) \phi_1 = A = 1 \text{ The flux within Region 1 is constant. } \phi_1 \Big|_{R_1} = \phi_2 \Big|_{R_2}$$

$$1 = A_2 J_0(b_2 R_1) + C_2 Y_0(b_2 R_1)$$

$$(2) \frac{d\phi_1}{dr} \Big|_{R_1} = \frac{d\phi_2}{dr} \Big|_{R_1} = 0$$

$$0 = -A_2 b_2 J_1(b_2 R_1) - C_2 b_2 Y_1(b_2 R_1)$$

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II. DISCUSSION (Continued)

H. Derivation (Cont'd)

$$(3) \quad \phi_2 \Big|_{R_2} = \phi_3 \Big|_{R_2}$$

$$A_2 J_0(b_2 R_2) + C_2 Y_0(b_2 R_2) = A_3 J_0(b_3 R_2) + C_3 Y_0(b_3 R_2)$$

$$(4) \quad \frac{d\phi_2}{dr} \Big|_{R_2} = \frac{d\phi_3}{dr} \Big|_{R_2}$$

$$A_2 b_2 J_1(b_2 R_2) + C_2 b_2 Y_1(b_2 R_2) = A_3 b_3 J_1(b_3 R_2) + C_3 b_3 Y_1(b_3 R_2)$$

$$(5) \quad \phi_3 \Big|_{R_3} = 0$$

$$A_3 J_0(b_3 R_3) + C_3 Y_0(b_3 R_3) = 0.$$

Using symbolics to simplify the notation:

$$X_1 = J_0(b_2 R_1)$$

$$X_8 = Y_0(b_3 R_2)$$

$$X_2 = Y_0(b_2 R_1)$$

$$X_9 = b_2 J_1(b_2 R_2)$$

$$X_3 = b_2 J_1(b_2 R_1)$$

$$X_{10} = b_2 Y_1(b_2 R_2)$$

$$X_4 = b_2 Y_1(b_2 R_1)$$

$$X_{11} = b_3 J_1(b_3 R_2)$$

$$X_5 = J_0(b_2 R_2)$$

$$X_{12} = b_3 Y_1(b_3 R_2)$$

$$X_6 = Y_0(b_2 R_2)$$

$$X_{13} = J_0(b_3 R_3)$$

$$X_7 = J_0(b_3 R_2)$$

$$X_{14} = Y_0(b_3 R_3)$$

Using this notation the equations become:

$$(1) \quad 1 = A_2 X_1 + C_2 X_2$$

$$(2) \quad 0 = A_2 X_3 + C_2 X_4$$

$$(3) \quad A_2 X_5 + C_2 X_6 = A_3 X_7 + C_3 X_8 \quad A_2 = -C_2 X_4/X_3$$

$$(4) \quad A_2 X_9 + C_2 X_{10} = A_3 X_{11} + C_3 X_{12}$$

$$(5) \quad A_3 X_{13} + C_3 X_{14} = 0 \quad A_3 = -C_3 X_{14}/X_{13}$$

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II. DISCUSSION (Continued)

H. Derivation (Cont'd)

Eliminating the A_1 's in the usual fashion:

$$\frac{(X_6 - X_5 X_4)}{X_3} = \frac{(X_8 - X_7 X_{14})}{X_{13}}$$

$$\frac{(X_{10} - X_9 X_4)}{X_3} = \frac{(X_{12} - X_{11} X_{14})}{X_{13}}$$

This is the "critical" equation. Once the b 's which satisfy the critical condition are found the fluxes may be found by solving for the constants in equations 1-5. The value of b_2 found specifies the amount of enrichment needed to satisfy the critical condition (support the fringe leakage).

This set of equations (or similar equations) may be used to define the amount of radial enrichment required to support various flat zone sizes (and hence various flattening efficiencies) and natural fringe bucklings. The corresponding flattening efficiency may be evaluated as follows:

$$\text{Efficiency} = \frac{\int_0^{R_1} \phi_1 dA + \int_{R_1}^{R_2} \phi_2 dA + \int_{R_2}^{R-D} \phi_3 dA}{\int_0^{R-D} \phi_1 dA}$$

D = augmentation distance

Since the expressions for the fluxes are combinations of J_0 and Y_0 , and the constants are pure numerics, the efficiency expression is mathematically integrable. The flux is not perfectly flat within region 1 due to non-uniform poisoning, etc. In general, this "non-flatness" effect is ~5-10 per cent. Hence, a more representative expression for efficiency is:

$$\text{Efficiency} \approx \frac{.90 \int_0^{R_1} \phi_1 dA + \int_{R_1}^{R_2} \phi_2 dA + \int_{R_2}^{R-D} \phi_3 dA}{\int_0^{R-D} \phi_1 dA}$$

$$\text{ECT} \approx (\text{Efficiency})(N)$$

N = Number of power generating lattice cells in the active zone.

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II. DISCUSSION (Continued)

H. Derivation (Cont'd)

The preceding derivations assume that the radial enrichment density is uniform. However, since the HCR's enter from the near side of the reactor an increased near-side density is often needed to support the HCR's. This may be accommodated in one of the two following ways:

- (1) Compute the $\oint B$ associated with the HCR pattern of interest assuming that the HCR's are spread uniformly through the fringe. Compute the E density required to supply the critical condition with the resulting fringe buckling, hence the required number of enrichment columns. Then compute the number of E columns required if there were no HCR's passing through the fringe. The difference is the number of E columns required to support the HCR's, and these E columns should all be located on the near side. The remainder should be located uniformly. The efficiency resulting from the flux distribution calculated for the condition resulting with the HCR's should very nearly describe the actual efficiency. Or,
- (2) Compute the $\oint B$ associated with the HCR's in the "effective region" of the HCR's. Compute the enrichment density needed to supply a critical condition for a reactor with the resulting fringe buckling. This defines the required enrichment loading within the "effective" region of control. The density in the rest of the ring is computed by assuming that no HCR's pass through the fringe. This method is probably superior to method (1).

Preliminary Fringe Buckling Calculations

A program was written for the 650 (the actual program writing was done by D. E. Goins) to compute enrichment requirements for various flattening efficiencies. The techniques used were:

- (1) Compute the material buckling required to support various flat zone sizes assuming that the fringe loading is uniform (fringe completely loaded with one kind of fuel).
- (2) Compute the flattening efficiencies which result if these conditions obtain.
- (3) Assume that the flux distribution and "effective" fringe bucklings obtained for a given efficiency are characteristic of this efficiency.
- (4) Radial enrichment must be changed if the natural load buckling is less than the required "effective" fringe buckling. The amount of enrichment required for various flattening efficiencies and natural load bucklings was calculated from the relationship:

$$B_{\text{eff}} = B_n \int_n \phi^2 dA + B_E \int_E \phi^2 dA.$$

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II. DISCUSSION (Continued)

- H. The results were useful for preliminary studies. They led to a more comprehensive study of the use of buckling calculation techniques for reactivity accounting. However, since the results are limited by the accuracy of the flux squared weighting theorem, and the assumptions regarding the flux distributions, they will not be included in this report.

I. Use of Buckling Calculations for Routine Reactivity Calculations and Accounting

Since pile reactivities and flux distributions must be calculated by "flux matching" or "buckling" techniques, it seems obvious that these methods should be adopted for routine reactivity calculating and accounting. The primary obstacle which must be overcome before these techniques can be adopted for routine use is the cumbersome and time-consuming calculations required for adequate description of the reactor loadings. A rough glance at the variation in loadings from reactor to reactor, and during a reactor operating period will give an appreciation of the problem. These techniques cannot be easily adopted unless most of the computational work is done by machine before the results are needed. The following is a sketchy outline of a scheme which seems feasible to the author:

- (1) Write a comprehensive and versatile program for a 709, or analogue computer. This program must be made versatile enough to compute virtually all conceivable reactor conditions.
- (2) Publish the results of the machine computations in the form of readily useable charts and/or curves. These data must cover the entire range of normal operating experience.
- (3) Revise the normal reactivity calculating and accounting schemes to be consistent with the requirements of the proposed system.

As indicated under heading (2), the data would be available to the pile physicists in the form of charts and/or curves. The physicist would select that data which most nearly fit the condition of interest. The cold startup prediction, equilibrium rod pattern, enrichment loading, etc., would be selected based upon an appropriate use of the data.

The data and method requirements for the various types of operation are:

(1) Cold Startup Conditions

Charts or curves of cold pile reactivities computed by the methods outlined under section E must be available. These data must be versatile enough to accommodate enrichment changes, central zone loading variations (including temporary poison loadings), exposure variations, control rod effects, etc. Xenon effects can be calculated from the existing tables.

(2) Transient Operation

Non-equilibrium transient conditions can be evaluated by the methods discussed previously. However, since these conditions are so varied, it will be very difficult to cover all conceivable situations.

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II. DISCUSSION (Continued)

- I. (2) Therefore, the data required to describe these situations must be generated after cold startup and equilibrium data. In this sense it must be considered of secondary importance. It will obviously be necessary to carry on with the current mode of evaluation until the necessary data are generated.

(3) Equilibrium Operation

The data and methods needed to describe the equilibrium condition are adequately described in Section H. Again it will be necessary to generate data regarding the enrichments needed to support various flattening efficiencies and natural fringe bucklings.

In a sense the hot and cold reactivity calculations are separate and distinct problems. They are related only insofar as one is interested in the change in buckling resulting from bringing the pile from a cold to an operating condition. However, it can easily be seen that the hot material bucklings can be evaluated from operating data even if no cold buckling data are available. In general one is interested in the change in the buckling resulting from bringing the pile to an operating condition because this aids in defining coefficient effects, etc. However, cold startup reactivity observations should be regarded as verifications, or if you like establishments, of cold material bucklings. It is the opinion of the author that much more detailed attention should be devoted to studies of the hot material bucklings than has been the case in the past. In short, more detailed study of the hot reactor, and less worry about the cold reactor seems appropriate.

J. Sample Calculation

Problem - Compute the uniform reduction in material buckling required to obtain the critical state for H Reactor as loaded on 3-19-58. Compare the computed uniform reduction to the observed value.

Data - From Reference 11

ΣR_N^* in poisonous materials within the flat zone ≈ 621.2 ih.

ΣR_N in enriched charges within the flat zone ≈ 40.6 ih.

Number of E columns in radial enrichment ≈ 256 (charge make up: 20 centered 6.54" solid E fuel elements, nine upstream 8.9" solid natural, eight downstream 8.9" solid natural).

1238 tubes located inside the enrichment ring.

Enrichment ring two lattice units thick.

* R_N is defined as the reactivity effect of a charge located at the geometric center of the uniformly loaded reactor.

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II. DISCUSSION (Continued)

J. Example Calculation (Cont'd)

Calculation

(1) Effective pile radius ≈ 600 cm.

(2) Inner radius of enrichment ring:

$$\frac{\pi R^2}{[(2.54)(8.375)]^2} = 1238 \text{ tubes}$$

$$R \approx 422.3 \text{ cm}$$

(3) Outer radius of enrichment ring:

$$422.3 + ((2)(2.54)(8.375)) = 464.8 \text{ cm}$$

(4) Number of tubes in enrichment ring:

$$\frac{(\pi) [(464.8)^2 - (422.3)^2]}{[(2.54)(8.375)]^2} = 262.3 \text{ tubes}$$

(5) Buckling effect of non-standard material within flat zone:

$$\text{Effective water columns} \approx \frac{621.2 - 40.6}{18} \approx 32.3$$

Assuming that the water columns are distributed uniformly within the flat zone,

$$\text{Area of flat zone} = \pi R^2 = 17.83 \times 10^4 \pi$$

$$\text{Area per water column} = \frac{17.83 \times 10^4 \pi}{32.3} = .552 \times 10^4 \pi$$

$$R_c = \text{Effective radius of a single water column} = 74.3 \text{ cm.}$$

$\delta B_p \approx .455 \text{ mk} \approx .75 \mu\text{b}$ for a single water column charged in the center of the green, fully loaded reactor.

$$\delta B_c \approx \frac{\delta B_p R_p^2 J_1^2(2.405)}{R_c^2 \left[J_0^2 \left(\frac{2.405 R_c}{R_p} \right) + J_1^2 \left(\frac{2.405 R_c}{R_p} \right) \right]}$$

$$\delta B_c \approx \frac{(.75)(600^2)(.27)}{(.552 \times 10^4) \left[J_0^2 \left(\frac{2.405 \times 74.3}{600} \right) + J_1^2 \left(\frac{2.405 \times 74.3}{600} \right) \right]}$$

$$\delta B_c \approx 13.7 \mu\text{b}$$

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II. DISCUSSION (Continued)

J. Calculation (Cont'd)

(6) Effective material buckling of radial E charges:

$$B_m \text{ column} \approx \frac{B_m E \int_E \phi^2 dA + B_m \text{Nat.} \int_{\text{nat}} \phi^2 dA}{\int_{\text{column}} \phi^2 dA}$$

$$\approx (280)(.72) + (71)(.28) \approx 221 \mu\text{b.}$$

$$\text{Number of effective full E charges in E ring} = \left(\frac{221}{280}\right) 256 = 202$$

(7) Bucklings of the various regions:

Flat zone exposure $\approx 250 \text{ MWD/T}$

Fringe (outside E ring) exposure $\approx 400 \text{ MWD/T}$

1 = Flat Zone
2 = Enrichment Ring
3 = Fringe

$R_1 = 422.3 \text{ cm}$
 $R_2 = 464.8 \text{ cm}$
 $R_3 = 600.0 \text{ cm}$

δB_1 due to exposure $\approx -1 \mu\text{b}$

δB_3 due to exposure $\approx +3 \mu\text{b}$

B_2 does not change with exposure

$$B_g \text{ axial} = \left(\frac{\pi}{809}\right)^2 \approx 15.1 \mu\text{b}$$

$$B_{m1} = 71.8^* - 1 - 13.7 = 57.1 \mu\text{b}$$

$$B_{r1} = \text{Radial buckling} = 57.1 - 15.1 = 42.0 \mu\text{b}$$

*Note: B_m solid regular = $71.8 \mu\text{b}$

B_m solid E = $280 \mu\text{b}$

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II. DISCUSSION (Continued)

J. Calculation (Cont'd)

$$B_{m2} = \left(\frac{202}{262.3} \right) (280)* + \left(\frac{60.3}{262.3} \right) 71.8* = 232.1 \mu b$$

$$B_{r2} = 232.1 - 15.1 = 217.0 \mu b$$

$$B_{m3} = 71.8* + 3 = 74.8 \mu b$$

$$B_{r3} = 74.8 - 15.1 = 59.7 \mu b$$

*Note: B_m solid regular = 71.8 μb

B_m solid E = 280 μb

(8) Calculating the uniform δB required for criticality:

The critical equations are: (See Section E)

$$\frac{A_2 X_7 + A_3 X_8}{A_2 X_{11} + A_3 X_{12}} = \frac{X_9 - D X_{10}}{X_{13} - D X_{14}}$$

Where:

$$A_2 = \frac{X_3 X_4 - X_1 X_6}{X_3 X_5 - X_2 X_6}$$

$$A_3 = \frac{X_2 X_4 - X_1 X_5}{X_2 X_6 - X_3 X_5}$$

$$D = \frac{X_{15}}{X_{16}}$$

and the X_1 's are defined in the following table. Note that the value of each X_1 is listed for $\delta B = 43 \mu b$.

<u>Name</u>	<u>Function</u>	<u>Value</u> for $\delta B = 43.0$
X_1	$J_0(b_1 R_1)$ or $I_0(b_1 R_1)$	1.045
X_2	$J_0(b_2 R_1)$	.017
X_3	$Y_0(b_2 R_1)$	-.337
X_4	$b_1 J_1(b_1 R_1)$ or $-b_1 I_1(b_1 R_1)$	$-.215 \times 10^{-3}$
X_5	$b_2 J_1(b_2 R_1)$	-4.445×10^{-3}

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II. DISCUSSION (Continued)

J. Calculation (Cont'd)

<u>Name</u>	<u>Function</u>	<u>Value</u> for $\delta B = 43.0$
X ₆	= $b_2 Y_1(b_2 R_1)$	$-.620 \times 10^{-3}$
X ₇	= $J_0(b_2 R_2)$	.185
X ₈	= $Y_0(b_2 R_2)$	-.263
X ₉	= $J_0(b_3 R_2)$	.282
X ₁₀	= $Y_0(b_3 R_2)$	.497
X ₁₁	= $b_2 J_1(b_2 R_2)$	-3.284×10^{-3}
X ₁₂	= $b_2 Y_1(b_2 R_2)$	-2.730×10^{-3}
X ₁₃	= $b_3 J_1(b_3 R_2)$	2.376×10^{-3}
X ₁₄	= $b_3 Y_1(b_3 R_2)$	$-.672 \times 10^{-3}$
X ₁₅	= $J_0(b_3 R_3)$	-.023
X ₁₆	= $Y_0(b_3 R_3)$	.505

The problem is to find the value of δB which satisfies the critical equations. Making a fortuitous choice for δB (it helps to have solved the problem previously):

$$\text{Try } \delta B = 43.0 \text{ } \mu\text{b}$$

$$B_1 = 42.0 - 43.0 = -1 \text{ } \mu\text{b} = -1 \times 10^{-6}$$

$$b_1 = \sqrt{|B_1|} = 1 \times 10^{-3}, b_1 R_1 = .42$$

$$B_2 = 217.0 - 43.0 = 174.0 \text{ } \mu\text{b} = 174 \times 10^{-6}$$

$$b_2 = 13.19 \times 10^{-3}, b_2 R_1 = 5.57 \quad b_2 R_2 = 6.13$$

$$B_3 = (59.7 - 43.0) \times 10^{-6} = 16.7 \times 10^{-6}$$

$$b_3 = 4.09 \times 10^{-3}, b_3 R_2 = 1.90, b_3 R_3 = 2.45$$

Thumbing through J_0 , Y_0 , etc. tables, one obtains the numbers which are listed above opposite the functions under the heading "value for $\delta B = 43.0$." Substituting these numbers into the appropriate equations:

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II. DISCUSSION (Continued)

J. Calculation (Cont'd)

$$A_2 = \frac{(-.337)(-.215 \times 10^{-3}) - (1.045)(-.620 \times 10^{-3})}{(-.337)(-4.445 \times 10^{-3}) - (.017)(-.620 \times 10^{-3})} = \frac{.720}{1.509} = +.477$$

$$A_3 = \frac{(.017)(-.215 \times 10^{-3}) - (1.045)(-4.445 \times 10^{-3})}{-1.509 \times 10^{-3}} = \frac{+4.641}{-1.509} = -3.076$$

$$D = -.046$$

$$\frac{(.477)(.185) + (-3.076)(-.263)}{(.477)(-3.284 \times 10^{-3}) + (-3.076)(-2.730 \times 10^{-3})} =$$

$$\frac{.282 - (-.046)(.497)}{2.376 \times 10^{-3} - (-.046)(-.672 \times 10^{-3})}$$

$$\frac{.897}{6.831} = \frac{.305}{2.345}$$

$$.131 = .130$$

The answer, to the nearest .5 μ b, is $\delta B = 43.0 \mu$ b. 43 μ b was indeed a fortuitous choice for a first try. One could hardly expect to be so lucky on an unbiased calculation. In general, three or more trial δB 's are necessary before the proper δB is found (the critical equations are satisfied).

- (9) Compare the calculated δB to the observed δB : (Data from Reference 11.)

Equilibrium power = 14 MW

$$\Sigma F^3 = 230$$

$$\Sigma F^2 = 2088$$

$$\text{Discharge} = 7.7 \text{ per cent } F^3$$

Time down to first indication = 23.7 hours

Reactivity at first indication = 5 ih (in period)

Uniform δB in xenon at first indication = 1021 ih =

Total uniform δB observed at critical = $\frac{1026 \text{ ih}}{23.7 \text{ ih}/\mu\text{b}^*} = 43.3 \mu\text{b}$

Predicted uniform $\delta B = 43 \mu\text{b}$

*The xenon tables used 1 mk = 38.3 ih. Hence, $\text{ih}/\mu\text{b} = (38.3)(.618) = 23.7$.

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