AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

12B.1 SENSITIVITY ANALYSIS OF NUMERICALLY-SIMULATED CONVECTIVE STORMS USING DIRECT AND ADJOINT METHODS

Seon Ki Park and Kelvin K. Droegemeier

Center for Analysis and Prediction of Storms and School of Meteorology, University of Oklahoma
Norman, Oklahoma 73019

and

Christian Bischof and Tim Knauff

Mathematics and Computer Science Division, Argonne National Laboratory
Argonne, IL 60439**1. BACKGROUND AND GOALS**

The goal of this project is to evaluate the sensitivity of numerically modeled convective storms to control parameters such as the initial conditions, boundary conditions, environment, and various physical and computational parameters. In other words, we seek the gradient of the solution vector with respect to specified parameters. One can use two approaches to accomplish this task. In the first or so-called "brute force" method, one uses a fully nonlinear model to generate a control forecast starting from a specified initial state. Then, a number of other forecasts are made in which chosen parameters (e.g., initial conditions) are systematically varied (e.g., Droegemeier and Levit, 1993). The obvious drawback is that a large number of full model predictions are needed to examine the effects of only a single parameter.

A better approach, thinking backward in time, is to specify a particular measure of the solution at the end of the forecast period, e.g., the surface pressure at a particular location at 2 hours, and then integrate backward in time to determine what parameters, say at the initial time, influence its value. This technique, which involves the adjoint of the forward model, has been used successfully to evaluate solution sensitivity (e.g., Errico and Vukicevic, 1992). Unfortunately, the formulation of an adjoint for even a modestly complex model is a tedious, highly time-intensive process that is prone to error and requires changes to the adjoint code every time the forward model code is modified. In addition, the solution sensitivity so determined is limited to linear variations about a specified forecast, and thus the adjoint method is unable to deal with regime transitions (e.g., between multicell and supercell storm types).

We describe herein an alternative, essentially automated method (ADIFOR, or Automatic Differentiation of FORtran; Bischof et al., 1992) for obtaining the solution gradient that bypasses the adjoint altogether yet provides even more information about the gradient. (ADIFOR, like the adjoint technique, is constrained by the linearity assumption.) Applied to a 1-D moist cloud model, we assess the utility of ADIFOR relative to the brute force approach and evaluate the validity of the tangent linear approximation in the context of deep convection.

2. THE NUMERICAL MODEL

We use for our experiments a 1-D warm-rain Eulerian cloud model similar to that described by Ogura and Takahashi (1971, OT71 hereafter), using a forward-upstream numerical scheme. The model domain extends 10 km vertically with a

250 m grid spacing. The model prognoses vertical velocity (w), temperature (T), water vapor mixing ratio (Q_v), cloud water mixing ratio (Q_c) and rain water mixing ratio (Q_r). The dynamical processes include advection, buoyancy, liquid water drag, lateral eddy exchange and dynamical entrainment, and the physical processes include condensation, evaporation of cloud and raindrops, and conversion from cloud to rain. The conversion rate of the latter, C_0 , is set to 0.005 inverse seconds and will be the focal point of our sensitivity study.

The base state environment is dry adiabatic from the ground (25 C) up to 1 km and, thereafter, nearly moist adiabatic. The surface relative humidity of 94.5%, increases at a rate of 5%/250 m up to 1 km, and then decreases at the same rate to the model top. Convection is initiated by imposing a parabolic updraft perturbation at low levels with a maximum of 1 m/s. Rigid lower and upper boundary conditions are used.

3. RESULTS AND DISCUSSION*Control run and validity of the tangent linear model*

The control simulation made using the fully nonlinear model (NLM) depicts the typical three-stage life cycle of a single-cell storm similar to that described by OT71. We developed from the nonlinear model code an associated first-order tangent linear model (TLM) which predicts the evolution of perturbations about a state provided by the NLM. To test the TLM, we conducted 12 experiments in which 5 to 60% errors were added to the initial updraft profile. For perturbations less than 15%, the TLM correctly describes storm evolution up to 70 min as compared to its NLM counterpart likewise perturbed. However, for larger perturbations, the NLM storm intensifies abruptly between 15-20 min and then oscillates, causing substantial disagreement with the TLM solution near cloud top where the strongest gradients of all variables exist. When even larger errors are introduced, the region of strong gradient near the cloud top moves upward during the mature stage in the NLM, causing even greater disagreement with the TLM.

Results in the vicinity of regime change

The conversion rate from cloud to rain, C_0 , plays a major role in determining the life cycle of the modeled cloud (OT71). When $C_0 \leq 4.096 \times 10^{-3}$, the cloud exhibits nearly steady behavior, while for $C_0 \geq 4.097 \times 10^{-3}$, the cloud is

unsteady, with increasing C_0 reducing the lifetime of the cloud.

For reasonably small changes in C_0 , the TLM solution agrees well with that of the NLM, provided that one is not in the vicinity of a regime change. We have examined NLM experiments for three base values of C_0 , onto each of which 10 perturbations are added ranging from 10^{-7} to 10^{-4} (i.e., 30 experiments). For the steady regime ($C_0 = 0.003$), the NLC (nonlinearity coefficient, i.e., the ratio of the higher-order nonlinear terms to the first-order linear term) is well below unity throughout the simulation, indicating that the TLM is a good approximation to the NLM solutions. For the non-steady regime ($C_0 = 0.005$), the NLC abruptly rises above unity at about 15 min for errors larger than 4×10^{-6} (Fig. 1a). For C_0 in the steady regime near the boundary of the regime change ($C_0 = 0.00409$), the NLC shows a very sharp increase for errors larger than 7×10^{-6} (Fig. 1b). The larger error leads to an earlier occurrence of the sharp increase. We also observe that the NLC is larger than unity for very small perturbations at early times for all cases. This is due to machine round-off error introduced by perturbations having a small magnitude. It is evident that there exists a specific range of perturbations for which the TLM is valid. This experiment implies that we need to exercise caution in using gradient information from the TLM/ADJM (adjoint model) system in the optimization process of data assimilation, especially when atmospheric conditions are in the vicinity of a regime change.

Fig. 1 Nonlinearity coefficients of w field for 10 perturbation runs for a) $C_0 = 0.00409$ and b) $C_0 = 0.005$.

Sensitivity of the model states to the control parameters

We explore the effects of three control parameters on cloud behavior: the cloud to rain conversion rate, C_0 , a non-dimensional proportionality constant, α , in the lateral eddy exchange, and the radius of the cloud, R . Values of these parameters in the control run are 0.005 s^{-1} , 0.2 , and 3 km , respectively. Table 1 compares the forward sensitivity coefficient (FSC) and the relative sensitivity coefficient (RSC) at $t = 60 \text{ min}$ for the total liquid water content, surface rainfall intensity and accumulated precipitation. Here, FSC is defined as the change in a dependent variable at any intermediate time with respect to a unit change in an independent variable at the initial time, while RSC represents the percentage change of the former for a 1% change in the latter.

For a unit increase in each of the three parameters, all dependent variables show the least sensitivity with respect to

Table 1. Forward vs. relative sensitivities at $t = 60 \text{ min}$ for various dependent variables

Sensitivity	Physical Control Parameter	Dependent Variables		
		Total Liquid Water Content	Rainfall Intensity at Ground	Accumulated Ground Rainfall
FSC	C_0	802.43	4145.9	1439.5
	α	-25.103	-6.2670	-1.8474
	R	0.0016	0.0004	0.0001
RSC	C_0	0.0658	2.3290	1.8230
	α	-0.0824	-0.1408	-0.0936
	R	0.0824	0.1408	0.0936

R . However, the percentage change of the total liquid water content for a 1% change in R is larger than that for a 1% change in C_0 . Furthermore, the percentage change in each dependent variable shows an identical rate of change for α and R , though in the opposite direction. Both α and R appear in the lateral eddy exchange term, which is proportional to α/R . Therefore, this behavior found was to be expected.

The FSC and local sensitivity coefficient (LSC, defined as any sensitivity normalized by the domain maximum sensitivity) of Q_r for a unit change in C_0 is presented in Fig. 2.

2. The zero contour in the decaying stage matches the axis of

Fig. 2 a) Forward sensitivity vs. b) local sensitivity of Q_r with respect to C_0 for $C_0 = 0.005$.

the maximum rainwater (AMQ_r) in control run very well, where strong gradients are observed. The FSC shows that an increase in C_0 will result in an earlier onset and termination of rainfall, which is physically plausible. However, little information is available locally for FSCs at earlier time due to their small magnitude. From the LSC analysis, we can examine the local importance of the sensitivity at any simulation time in a relative sense. In Fig. 2b, we see three distinct stages in the behavior of sensitivity, which are related to the stages in the cloud's life cycle. During the developing stage (up to 15 min), large sensitivities are observed at low levels with strong gradients both at the cloud top and bottom, which are mainly related to the local maxima of Q_r observed at those regions in control run. In the mature stage, the largest sensitivities are observed at the cloud top or in the layer just below it with strong gradient at the cloud top only. During the decaying stage, local maxima of sensitivities appear at lower layers and the strongest gradient is observed in all layers along AMQ_r . As demonstrated, the LSC provides very useful information on the local characteristics of the sensitivities.

Results from the ADIFOR run

The ADIFOR-generated code provides FSCs of all dependent variables with respect to all independent variables with a single integration. Furthermore, we can obtain correct adjoint sensitivity coefficients (ASC) by computing $\partial X(t)/\partial X(t_1)$ from reinitialized ADIFOR runs, and then accumulating them in reverse order, from the verification time to the time of interest. The total number of the dependent variables is 290, and that of independent variables is 661. Fig. 3 shows FSCs and ASCs computed from the ADIFOR-generated code for a verification time of $t = 90$ min. The labels on each axis show the height where the variables of interest are located.

Fig. 3 Results from ADIFOR run. Forward sensitivity of Q_r with respect to w for a) $t = 10$ min and b) $t = 80$ min (upper panel) and adjoint sensitivity of w with respect to Q_r for c) $t = -80$ min and d) $t = -10$ min (lower panel).

For FSCs, we discuss the sensitivity of Q_r at any model time with respect to w at the initial time. In the developing stage ($t = 10$ min), the change in Q_r is bounded at lower levels below 2 km and is mostly affected by changes in the initial state of w located in the same region. Note that the initial wind maximum is located at 1 km. The increase in w below that level induces the increase in Q_r inside the cloud layer, while an increase of the former above that level induces the increase of the latter only at the upper layer of the cloud. Throughout the mature stage, the largest sensitivity of Q_r is observed at the cloud top for errors inserted at that height in w at the initial time (not shown). The latter tendency continues at early decaying stage ($t = 80$ min). However, the area of sensitivity extends downward, which is apparently related to the downward tilt of AMQ_r . Noteworthy sensitivities are observed through all the cloud layer due to the change in the initial wind at levels below 1 km.

The lower panel of Fig. 3 represents ASCs which explain the changes in w at verification time due to the errors inserted in Q_r at any intermediate time. For changes occurred in Q_r at the lowest level (250 m) during the developing stage ($t = -80$ min), w at verification time decreases in the layer 1-4 km and increases above that layer. w is not sensitive to changes in Q_r at other levels. This implies that, in operational meteorology, we need good observations or retrieval of moisture at low levels in the developing stage to predict well the subsequent storm behavior. In the mature stage, w in all columns of the cloud is highly sensitive to changes in Q_r at 5-7 km and relatively less sensitive to changes at lower levels (not shown). The largest sensitivity is observed above the cloud top, which implies that mis-fitting of Q_r above the cloud top in the model may lead to a poor prediction of the location of the cloud top during the decaying stage. In the decaying stage ($t = -10$ min), w is most sensitive to errors in Q_r introduced at the cloud top. Those errors result in a decrease in w at 2-4.5 km and increase over the region where the cloud top is observed at verification time. Overall, the increase in Q_r on or above the cloud top results in an increase in the height of cloud top at the verification time which means longer duration of the cloud system.

This kind of global analysis of sensitivities of model states to their earlier stages is almost impossible with the traditional TLM/ADJM system. For large numbers of independent variables, the parallel processing can profitably be employed to decrease turnaround time. For an increase in parallel processors from 1 to 60, the averaging execution times showed a hyperbolic decrease. Our ultimate goal is to apply ADIFOR to the full 3-D ARPS model being developed by CAPS.

4. ACKNOWLEDGMENTS

This research was supported by NSF grant ATM92-22576 to the second author, and by grant ATM88-09862 to the Center for Analysis and Prediction of Storms. Additional support was provided to Argonne National Laboratory from the Office of Scientific Computing, U.S. Department of Energy under Contract W-31-109-Eng-38, and by the National Aerospace Agency under Purchase Order L25935D. We are grateful for Sue Weygandt of CAPS for her assistance with graphics and paste-up, and to Alan Carle of Rice University for his essential role in the ADIFOR project.

5. REFERENCES

- Bischof, C., A. Carle, G. Corliss, A. Griewank, and P. Hovland, 1992: ADIFOR - Generating derivative codes from Fortran programs. *Scientific Programming*, 1, 11-29.
- Droegemeier, K. K., and J. J. Levit, 1993: The sensitivity of numerically-simulated storm evolution to initial conditions. Preprints, 17th Conf. on Severe Local Storms, St. Louis, Missouri, Amer. Meteor. Soc., 431-435.
- Errico, R. M., and T. Vukicevic, 1992: Sensitivity analysis using an adjoint of the PSU-NCAR mesoscale model. *Mon. Wea. Rev.*, 120, 1644-1660.
- Ogura, Y., and T. Takahashi, 1971: Numerical simulation of the life cycle of a thunderstorm cell. *Mon. Wea. Rev.*, 99, 895-911.

The submitted manuscript has been authored by a contractor of the U. S. Government under contract No. W-31-109-ENG-38. Accordingly, the U. S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U. S. Government purposes.

**DATE
FILMED**

8 / 11 / 94

END