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Low Beta Equilibrium and Stability for Anisotropic Pressure Closed Field Line Plasma Confinement Systems

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Abstract

General formalism is developed to analyze the equilibrium and stability of low beta anisotropic pressure plasmas confined in closed field line magnetic systems. The formalism allows to consider rather general magnetic systems with nonuniform axis curvature and longitudinal profiles of toroidal and multipole poloidal field. It also allows having a strong pressure anisotropy corresponding to enhanced plasma pressure in mirror cells of the system. As an example of such the system we consider the recently proposed linked mirror neutron source (LMNS). Application of the above formalism to the LMNS analysis confirms most of the preliminary results, however, we obtain a considerable reduction of mirror cell axis curvature and an appreciable ellipticity of plasma cross-section in the mirror cell midplane. We have also optimized the longitudinal pressure and magnetic field distribution.

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I Introduction

Closed magnetic field line plasma confinement traps periodically attract an attention as possible alternative approaches to the magnetic fusion.¹⁻⁶ The main advantages of such the traps are steady state operation and relative simplicity of their magnetic systems in comparison with the conventional stellarators. The main disadvantage of the above magnetic traps is the absence of a rotational transform which is able to provide the nested magnetic surfaces and to prevent from rough perturbations of the field lines caused by imperfections of real magnetic systems. However, an elimination or a considerable reduction of such the rough perturbations by external field corrections seems to be possible. When residual imperfections are small, a self-consistent radial electric field appears to provide closed drift surfaces of plasma particles and to prevent from a disruption of the plasma equilibrium.

Recently⁷ a linked mirror system (LMS) with closed field lines has been proposed as a highly effective neutron source for testing materials in a fusion environment. Such the system has all the advantages inherent in previous mirror-based neutron source concepts,⁸⁻¹⁰ however, the toroidal linkage of the system allows to exclude the longitudinal energy losses and correspondingly to enhance electron temperature and power efficiency of the source. The advantages of the LMS-based neutron source are primarily achieved through two parameters. The first is the ratio p_h/p_t supposing to be large ($p_h/p_t > 5$), where p_h is the pressure of the mirror confined hot ions and p_t is the pressure of the toroidally confined warm target plasma. The second parameter is a high ellipticity ($E > 10$) of the plasma cross-section in the toroidal linkage cells which appears as a result of high ellipticity of the mirror end fans.

The previous consideration⁷ has revealed the main properties of plasma equilibrium and magnetohydrodynamic (MHD) stability in the LMS. However, some simplified assumptions have been used under the consideration. Thus, an advanced self-consistent analysis is needed to enhance the reliability of the LMS neutron source concept. The previous formalism⁴

developed for isotropic low-beta plasma equilibrium and MHD-stability analysis of closed field line systems is not applicable to LMS due to the strong plasma pressure anisotropy. Here we develop a generalized formalism which allows to analyze the equilibrium and stability of low-beta plasma with arbitrary pressure profile along field lines in a wide class of closed field line plasma confinement systems. We have obtained that the nonuniform longitudinal pressure profile results in some new qualitative features of plasma equilibrium and stability in the above systems. The formalism developed is also applied to the detailed analysis of the plasma equilibrium and stability in LMS and to optimization of LMS parameters.

II Plasma Equilibrium

An anisotropic plasma equilibrium is determined by the following equations:

$$\nabla \cdot \mathbf{P} = \mathbf{j} \times \mathbf{B} , \quad (1)$$

$$\nabla \times \mathbf{B} = \mathbf{j} , \quad (2)$$

where $\mathbf{j}$ is the plasma current and $\mathbf{B}$ the magnetic field. The pressure tensor is of the form:

$$\mathbf{P} = p_{\perp} \mathbf{I} + (p_{\parallel} - p_{\perp}) \mathbf{b} \mathbf{b} , \quad (3)$$

$\mathbf{b} = \mathbf{B}/B$ and $\mathbf{I}$ is the unit dynamic. The longitudinal component of Eq. (1) is automatically satisfied, if $p_{\parallel}$ and $p_{\perp}$ are calculated using a distribution functions of plasma particles which depends on the integrals of motion. From the transverse component of Eq. (1) it follows, using standard techniques.¹⁰ that:

$$\mathbf{j}_{\perp} = \left[(p_{\parallel} - p_{\perp})(\mathbf{b} \times \boldsymbol{\kappa}) + \mathbf{b} \times \nabla p_{\perp} \right] / B , \quad (4)$$

$$\boldsymbol{\kappa} = (\mathbf{b} \cdot \nabla) \cdot \mathbf{b} \equiv -\mathbf{b} \times (\nabla \times \mathbf{b}) .$$

The longitudinal current $j_{\parallel}$ is found using $\nabla \cdot \mathbf{j} = 0$ that leads to the relation

$$\mathbf{b} \cdot \nabla \left[\frac{1}{B^3} j_{\parallel} (B^2 + p_{\perp} - p_{\parallel}) \right] = - \frac{\mathbf{b} \times \boldsymbol{\kappa}}{B^2} \cdot \nabla (p_{\parallel} + p_{\perp}) . \quad (5)$$

Equation (5) must satisfy the following solvability condition:

$$\oint \frac{d\ell}{B^2} (\mathbf{b} \times \boldsymbol{\kappa}) \cdot \nabla(p_{\parallel} + p_{\perp}) = 0 , \quad (6)$$

where the integration is along a closed field line. Equation (6) is the most important condition of the equilibrium. Physically it corresponds to a closure of Pfirsch-Schlüter currents within the system considered.

Following the conventional procedure⁴ we assume that β is small ($\beta = 2p/B^2 \ll 1$) and invoke a rather reasonable paraxial (near axis) approximation which is valid if the cross-field pressure gradient is much greater than the field line curvature. These assumptions allow to develop a completely self-consistent formalism. Under the low- β assumption we may substitute the vacuum (nonperturbed) magnetic field to the equation (6). In the case of closed field lines it is possible to introduce Clebsh coordinates Ψ, Φ, χ as follows:

$$\mathbf{B} = \nabla\chi = \nabla\Psi \times \nabla\Phi , \quad (7)$$

to describe the vacuum magnetic field. If the plasma pressure is isotropic ($p_{\perp} = p_{\parallel}$), we can always choose Ψ to provide $p = p(\Psi)$ due to relation $\mathbf{B} \cdot \nabla p = 0$. In the general case of an anisotropic plasma the effective pressure depends generally on all three coordinates: $P \equiv p_{\perp} + p_{\parallel} = P(\Psi, \Phi, \chi)$. Nevertheless, we can again define Ψ to provide $P = 2p(\Psi)$ in the regions where the plasma column has the isotropic pressure. Such the definition of Ψ is possible, because the isotropic pressure component has to be constant along field lines. Under the above definition of Ψ we could not expect an appreciable dependence of P on Φ , because such the dependence appears to be an FLR effect. It has to be determined from the kinetic consideration taking into account a self-consistent radial electric field which appears to unify the drift trajectories of different plasma particles⁷ and to reduce correspondingly the Φ -dependence of P . In this paper we neglect FLR effects, therefore, below we shall consider $P = P(\Psi, \chi)$. Under the above assumptions we can rewrite Eq. (6) in the following simple

form:

$$\oint \frac{\partial P}{\partial \Psi} \frac{\partial B}{\partial \Phi} \frac{d\chi}{B^3} = -\frac{1}{2} \oint \frac{\partial P}{\partial \Psi} \frac{\partial}{\partial \Phi} \left(\frac{d\chi}{B^2} \right) = 0. \quad (8)$$

The Clebsh coordinates depend on parameters of the system considered. To determine these coordinates we suppose that the plasma column is located in a vicinity of a closed field line whose physical characteristics (magnetic field, curvature, etc.) are given. We name this field line as a "magnetic axis." In the case of isotropic plasma pressure equilibrium it is reasonable to define the magnetic axis as the center of closed nested surfaces $\oint dl/B = \text{const}$ which appears to be the axis of the plasma column.⁴ In the general case of anisotropic pressure, as it will be shown below, the position of the plasma column axis depends on longitudinal pressure distribution, so the plasma column is not attached to the surfaces $\oint dl/B = \text{const}$. Thus we have a lot of freedom to concretize the magnetic axis. As a rule, a spatial symmetry inherent in the closed field line magnetic systems¹⁻⁶ allows us to choose the magnetic axis to be a rather simple planar curve without a torsion.

Further, we introduce the local cartesian coordinates $\{s, x, y\}$ where s is the distance along the magnetic axis, x is the distance measured from the axis along its external normal, and y is chosen to complete the right triad.

Under the paraxial approximation it is sufficient to consider an expansion of χ of up to the 3-d order of distance from the magnetic axis.⁴ The corresponding expression takes the form:

$$\chi = \int_0^s B(s)ds - \frac{\eta' B}{4} (x^2 - y^2) - \frac{B'}{4} (x^2 + y^2) - B_3(x^3 - 3xy^2) - \phi_k(x^3 + xy^2) + \dots, \quad (9)$$

where $B(s)$ is a toroidal field at the axis, $B\eta'(s)$ and $B_3(s)$ characterize quadrupole and hexapole poloidal field components correspondingly, prime denotes the derivative with respect to s , ϕ_k has the form:

$$\phi_k = \frac{5}{16} B'k + \frac{1}{8} Bk' + \frac{1}{16} Bk\eta', \quad (10)$$

and $k(s)$ is the axis curvature. Using Eqs. (7) and (9) we calculate the magnetic field:

$$\begin{aligned} B_x &= -\frac{x}{2} (\mathcal{B}' + \eta' \mathcal{B}) - x^2(3B_3 + 3\phi_k) + y^2(3B_3 - \phi_k) + \dots, \\ B_y &= -\frac{y}{2} (\mathcal{B}' - \eta' \mathcal{B}) + 2xy(3B_3 - \phi_k) + \dots, \\ B_s &= \frac{1}{1 - kx} \left[\mathcal{B} - \frac{x^2}{4} (\mathcal{B}'' + (\eta' \mathcal{B})') - \frac{y^2}{4} (\mathcal{B}'' - (\eta' \mathcal{B})') + \dots \right]. \end{aligned} \quad (11)$$

To obtain the appropriate expressions for Ψ and Φ we have to solve the field line equations:

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{ds}{B_s}. \quad (12)$$

Using renormalized coordinates:

$$X = x\mathcal{B}^{1/2}e^{\eta/2}; \quad Y = y\mathcal{B}^{1/2}e^{-\eta/2}, \quad (13)$$

we can rewrite Eq. (12) as follows:

$$\begin{aligned} \frac{dX}{ds} &= X^2 \frac{e^{-\eta/2}}{\mathcal{B}^{1/2}} \left(\frac{3B_3 + 3\phi_k}{\mathcal{B}} + k \frac{\mathcal{B}'}{\mathcal{B}} + k\eta' \right) + Y^2 \frac{e^{3\eta/2}}{\mathcal{B}^{1/2}} \left(\frac{3B_3 - \phi_k}{\mathcal{B}} \right) + \dots, \\ \frac{dY}{ds} &= XY \frac{e^{-\eta/2}}{\mathcal{B}^{1/2}} \left(2 \frac{3B_3 - \phi_k}{\mathcal{B}} + k \frac{\mathcal{B}'}{\mathcal{B}} - k\eta' \right) + \dots. \end{aligned} \quad (14)$$

The right sides of Eq. (14) consist of second order terms only. Thus, the solution of Eq. (14) with the accuracy of up to the second order of the distance from the magnetic axis takes the form:

$$X = X_0 + X_0^2 u(s) + Y_0^2 v(s), \quad (15)$$

$$Y = Y_0 + X_0 Y_0 w(s),$$

where $X_0 = X|_{s=0}$; $Y_0 = Y|_{s=0}$, $s = 0$ is an arbitrary point at the axis, and

$$u = \int_0^s \left(k\eta' + k \frac{\mathcal{B}'}{\mathcal{B}} - 3 \frac{B_3 + \phi_k}{\mathcal{B}} \right) \frac{e^{-\eta/2}}{\sqrt{\mathcal{B}}} ds,$$

$$v = \int_0^S \frac{3B_3 - \phi_k}{B} \frac{e^{3\eta/2}}{\sqrt{B}} ds, \quad (16)$$

$$w = \int_0^S \left(-k\eta' + k\frac{B'}{B} + 2\frac{3B_3 - \phi_k}{B} \right) \frac{e^{-\eta/2}}{\sqrt{B}} ds.$$

The above expressions allow to determine the Clebsh variables Ψ and Φ . At the main order of the expansion which corresponds to the paraxial approximation a transversal cross-section of the surface $\Psi = \text{const}$ has to be closed second order curve (ellipse). Therefore, we can write the expressions for Ψ and Φ in the following general form:

$$\begin{aligned} \Psi &= (X_0 - \Delta)^2 + Y_0^2, \\ \Phi &= \arctan \left(\frac{Y_0}{X_0 - \Delta} \right), \end{aligned} \quad (17)$$

where Δ is a renormalized displacement of a plasma axis with respect to the magnetic axis, and Ψ is taken to be the magnetic flux. It is not difficult verify that expressions (17) satisfy Eq. (7). Using Eqs. (7), (11), (13), (15), and (17) we obtain by means of algebraic transformations the following expression:

$$\begin{aligned} \frac{d\chi}{B^2} &= \frac{d\chi}{B^2(\chi)} \left\{ 1 - 2k\frac{e^{-\eta/2}}{B^{1/2}}\Delta + \Delta^2 \left[-2k\frac{e^{-\eta/2}}{B^{1/2}}u + \left[k^2 - \frac{1}{4}\left(\frac{B'}{B} + \eta'\right)^2 + \frac{1}{2}\left(\frac{B'}{B} + \eta'\right)' \right] \frac{e^{-\eta}}{B} \right] + \right. \\ &+ \sqrt{\Psi} \cos \Phi \left[-2k\frac{e^{-\eta/2}}{B^{1/2}} + \Delta \left[-4k\frac{e^{-\eta/2}}{B^{1/2}}u + 2\frac{e^{-\eta}}{B} \left(k^2 - \frac{1}{4}\left(\frac{B'}{B} + \eta'\right)^2 + \frac{1}{2}\left(\frac{B'}{B} + \eta'\right)' \right) \right] \right] + \\ &+ \Psi \left[-k\frac{e^{-\eta/2}}{B^{1/2}}(u+v) + \frac{e^{-\eta}}{2B} \left[k^2 - \frac{1}{4}\left(\frac{B'}{B} + \eta'\right)^2 + \frac{1}{2}\left(\frac{B'}{B} + \eta'\right)' \right] + \right. \\ &+ \frac{e^\eta}{2B} \left[-\frac{1}{4}\left(\frac{B'}{B} - \eta'\right)^2 + \frac{1}{2}\left(\frac{B'}{B} - \eta'\right)' \right] \left. \right] + \\ &+ \Psi \cos 2\Phi \left[-k\frac{e^{-\eta/2}}{B^{1/2}}(u-v) + \frac{e^{-\eta}}{2B} \left[k^2 - \frac{1}{4}\left(\frac{B'}{B} + \eta'\right)^2 + \frac{1}{2}\left(\frac{B'}{B} + \eta'\right)' \right] \right] - \end{aligned} \quad (18)$$

$$- \frac{e^\eta}{2B} \left[-\frac{1}{4} \left(\frac{B'}{B} - \eta' \right)^2 + \frac{1}{2} \left(\frac{B'}{B} - \eta' \right)' \right] \Bigg\} ,$$

which has an appropriate form to be substituted to the equilibrium condition (8).

For simplicity we assume a conventional parabolic transversal pressure distribution:

$$P(\chi, \Psi) = \mathcal{P}(\chi) \left(1 - \frac{\Psi}{\Psi_m} \right) , \quad (19)$$

where Ψ_m labels the plasma boundary surface. Substituting Eqs. (18) and (19) into Eq. (8) we suppose that s is independent of Ψ and Φ , because $\partial s / \partial \Phi$, $\partial s / \partial \Psi$ could only add terms of the order of $\Psi^{3/2}$ and higher to the equilibrium condition (8). Changing the integration along field lines by the integration along the axis we obtain the following equilibrium condition:

$$\begin{aligned} \sqrt{\Psi} \sin \Phi \oint \mathcal{P}(s) \left\{ -k \frac{e^{-\eta/2}}{B^{3/2}} + \Delta \left[-2k \frac{e^{-\eta/2}}{B^{3/2}} u + \frac{e^{-\eta}}{B^2} \left(k^2 - \frac{1}{4} \left(\frac{B'}{B} + \eta' \right)^2 + \frac{1}{2} \left(\frac{B'}{B} + \eta' \right)' \right) \right] \right\} ds + \\ + \Psi \sin 2\Phi \oint \mathcal{P}(s) \left\{ -2k \frac{e^{-\eta/2}}{B^{3/2}} (u - v) + \frac{e^{-\eta}}{B^2} \left[k^2 - \frac{1}{4} \left(\frac{B'}{B} + \eta' \right)^2 \right. \right. \\ \left. \left. + \frac{1}{2} \left(\frac{B'}{B} + \eta' \right)' \right] - \frac{e^\eta}{B^2} \left[-\frac{1}{4} \left(\frac{B'}{B} - \eta' \right)^2 + \frac{1}{2} \left(\frac{B'}{B} - \eta' \right)' \right] \right\} ds = 0 . \end{aligned} \quad (20)$$

Both the terms in Eq. (20) have to equal zero independently. The first and the second terms describe the closure of the dipole and the quadrupole Pfirsch-Schlüter currents correspondingly.

Introducing the useful notations

$$\begin{aligned} J_1 &= - \oint ds \frac{\mathcal{P}(s)}{B^2} \left\{ e^{-\tilde{\eta}} \left[k^2 - \frac{1}{4} \left(\frac{B'}{B} + \tilde{\eta}' \right)^2 + \frac{1}{2} \left(\frac{B'}{B} + \tilde{\eta}' \right)' \right] - 2ke^{-\tilde{\eta}/2} B^{1/2} \tilde{u} \right\} ds , \\ J_2 &= - \oint ds \frac{\mathcal{P}(s)}{B^2} \left\{ e^{\tilde{\eta}} \left[-\frac{1}{4} \left(\frac{B'}{B} - \tilde{\eta}' \right)^2 + \frac{1}{2} \left(\frac{B'}{B} - \tilde{\eta}' \right)' \right] - 2ke^{-\tilde{\eta}/2} B^{1/2} \tilde{v} \right\} ds , \end{aligned} \quad (21)$$

where $\tilde{\eta} = \eta - \eta_0$, $\eta_0 = \eta|_{s=0}$, and

$$\begin{aligned}\tilde{u} &= \int_0^s \left[k \left(\tilde{\eta}' + \frac{B'}{B} \right) - 3 \frac{B_3 + \phi_k}{B} \right] \frac{e^{-\tilde{\eta}/2}}{B^{1/2}} ds, \\ \tilde{v} &= \int_0^s \left(\frac{3B_3 - \phi_k}{B} \right) \frac{e^{3\tilde{\eta}/2}}{B^{1/2}} ds,\end{aligned}\tag{22}$$

we rewrite the equilibrium conditions in the following form:

$$\delta_0 = - \oint \mathcal{P} \frac{k}{B^{3/2}} e^{-\tilde{\eta}/2} \frac{ds}{B_0^{1/2}} J_1, \tag{23}$$

$$E_0^2 \equiv e^{2\eta_0} = \frac{J_1}{J_2}, \tag{24}$$

where $\delta_0 = \Delta e^{-\eta_0/2} / B_0^{1/2}$ is the displacement of plasma axis from the magnetic axis, and E_0 is the ellipticity of plasma cross-section at $s = 0$.

Expressions (13), (15), (17), (23), and (24) completely determine the Clebsh coordinates Ψ and Φ in the main order of paraxial expansion, and, as a result, they completely determine the shape of equilibrium plasma column. In the general anisotropic pressure case the displacement δ_0 and the ellipticity E_0 are not the parameters inherent in the magnetic system only, so far as they also depend on longitudinal pressure profile. Nevertheless, an appropriate choice of the axis curvature in accordance with Eq. (23) allows us to design the magnetic system which provides $\delta_0 = 0$ for the basic regimes of the system operation with desirable longitudinal pressure profile. Integrals J_1 , J_2 can change their signs due to variations of the pressure profile and magnetic system parameters. From Eq. (24) one can see that the plasma equilibrium exists, if J_1 and J_2 have the same signs (both are positive, or both are negative). The change of sign of $J_{1,2}$ corresponds to a loss of equilibrium in terms of paraxial approximation considered. Finally we should emphasize that the above procedure allows to be extended up to the arbitrary order of expansion in powers of the distance from the axis. For example, if the system considered has a strong external hexapole magnetic field component, we should expect an appreciable triangularity of the plasma cross-section. In this case

we have to consider an expression for χ and Eq. (8) of up to the 3-d order of the distance from the axis. The corresponding parameter of triangularity appears to be calculated from the hexapole Pfirsch-Schlüter current closure condition (vanishing of the 3-d order term in Eq. (8)).

III Low- β MHD Stability

The MHD-stability problem for low- β plasmas confined in CFLS is reduced to the analysis of flute mode stability. Under paraxial approximation, as it follows from the energy principle, the flute-like mode stability criterion corresponds to positive definiteness of quadratic form:

$$W = -\frac{1}{2} \in \frac{(\xi_{\perp} \nabla \mathcal{P})(\xi_{\perp} \nabla B)}{B} dV > 0, \quad (25)$$

where $\xi_{\perp}$ is the flute cross-field plasma displacement. Using Clebsh coordinates and taking into account the equilibrium condition (8), we can rewrite Eq. (25) as follows:

$$W = \frac{1}{4} \int (\xi_{\perp} \nabla \Psi)^2 d\Psi d\Phi \oint \frac{\partial \mathcal{P}}{\partial \Psi} \frac{\partial}{\partial \Psi} \left(\frac{d\chi}{B^2} \right) > 0. \quad (26)$$

Here $(\xi_{\perp} \nabla \Psi)^2$ is an arbitrary positively defined function of Ψ and Φ . Therefore, the stability criterion is to be written in the following local form:

$$\oint \frac{\partial \mathcal{P}}{\partial \Psi} \frac{\partial}{\partial \Psi} \left(\frac{d\chi}{B^2} \right) > 0, \quad (27)$$

which is similar to the equilibrium condition (8). Substituting Eqs. (18) and (19) into Eq. (27) and taking into account notations (21) and (22) and equilibrium conditions (23) and (24) we obtain the following stability criterion:

$$G = J_1 e^{-\eta_0} + J_2 e^{\eta_0} > 0. \quad (28)$$

According to Eqs. (24) and (28) the MHD equilibrium of a low- β plasma column in CFLS is stable with respect to flute modes, if both the integrals J_1 and J_2 are positive. If

both the integrals $J_{1,2}$ are negative, the equilibrium also exists, however, it is unstable. It is obvious that when we are approaching to the flute modes stability margin (at least one of the integrals $J_{1,2}$ tends to be equal to zero) the equilibrium condition (described in terms of paraxial approximation) is broken as well. The similar correlation between MHD equilibrium and stability conditions was pointed earlier¹² for a tandem mirror geometry. This geometry can be considered as a specific limit of CFLS with $k(s) = 0$ in mirror cells and $\mathcal{P}(s) = 0$ in a toroidal linkage. In general, J_1 and J_2 do not change their signs simultaneously for the different pressure profiles and magnetic system parameters. Therefore, the stable and unstable regions in a parametric space of CFLS are separated by a zone where the plasma equilibrium is absent ($J_1 \cdot J_2 \leq 0$). Such the feature of the equilibrium is not typical for conventional plasma confinement systems. We shall discuss the above mentioned features of the equilibrium in details considering the linked mirror system in the next section.

IV Equilibrium and Stability in a Toroidally Linked Mirror System

Now we apply the results obtained in the previous sections to the analysis of low- β plasma equilibrium and stability in the Linked Mirror Neutron Source (LMNS).⁷ The magnetic configuration to be discussed is shown in Fig. 1. It consists of four individual or, equivalently, two parallel pairs of minimum- B mirror cells (midplane cross-sections 2, 6, 8, and 12) linked by two semitori with highly elliptical cross-sections (4,10). The LMNS plasma consists of two components. The first is a toroidally confined target plasma with isotropic pressure p_t constant along a field line. The second is a hot ion component which has an anisotropic pressure and it is localized near the mirror cell midplanes (2, 6, 8, 12).

The LMNS magnetic system exhibits the reflection symmetry relative to the plane 1-7 and to the plane 4-10 (see Fig. 1). Therefore, it is sufficient to consider one quarter of the system and to perform all the integrations between cross-sections 1 and 4. We choose

also $s = 0$ at the mirror midplane (cross-section 2) and assume the following magnetic axis curvature: $k(s) = k_m$ in the mirror cell (1-3) and $k(s) = -k_t$ in the toroidal link cell (3-4). The toroidal and quadrupole magnetic field components in the mirror cell will be modeled by the reasonable analytical expressions⁷:

$$\mathcal{B}(s) = \mathcal{B}_0 \left(1 + (R - 1) \sin^2 \pi \frac{s}{L} \right) ; \quad \mathcal{B}\eta' \equiv q(s) = q_0 \cos \pi \frac{s}{L} ; \quad (29)$$

where R is mirror ratio, L is total length of mirror cell axis. The magnetic field in the toroidal cell has only toroidal component $\mathcal{B}(s) = \mathcal{B}_t = \mathcal{B}_0 R$, so $\eta'(s) = 0$ in this cell. The hexapole component is $B_3(s) = 0$ everywhere. Contrary to the previous LMNS study⁷ we do not assume the circular plasma cross-section in the mirror cell midplane. The midplane ellipticity E_0 will be calculated self-consistently according to Eq. (24). Therefore, the toroidal cell ellipticity E_t depends on E_0 as follows: $E_t = E_0 E$, where relative ellipticity E is determined by the mirror cell magnetic field distribution (29):

$$E \equiv \exp \left\{ \eta \left(\frac{L}{2} \right) - \eta_0 \right\} = \exp \left\{ \int_0^{L/2} \frac{q}{\mathcal{B}} ds \right\} . \quad (30)$$

To calculate the longitudinal pressure profile we shall assume the following distribution function of hot ions:

$$f_h = f(|\mathbf{v}|) \left(1 - \frac{\cos^2 \theta_0}{\cos^2 \theta_h} \right) , \quad |\theta_0| \leq \theta_h , \quad (31)$$

which is smoother and more realistic than that which was used in the previous study.⁷ Here $\mathbf{v}$ and θ_0 are correspondingly the velocity and the pitch angle of a plasma particle at the mirror midplane. Using (29) and taking into account the magnetic moment conservation $\sin^2 \theta / \sin^2 \theta_0 = R(s) \equiv \mathcal{B}(s) / \mathcal{B}_0$ we come to the following hot ion pressure distribution:

$$\mathcal{P}_h(s) = \frac{p_{h0}}{R(s)} \left(\frac{R_h - R(s)}{R_h - 1} \right)^{3/2} \frac{6R_h - R(s)}{6R_h - 1} , \quad (32)$$

where $\mathcal{P}_h(s) = p_{h\parallel} + p_{h\perp}$, $R_h = R(l_h/2) = \sin^{-2} \theta_h$ and l_h is a hot ion component length. The total pressure distribution has the form $\mathcal{P} = \mathcal{P}_h(s) + 2p_t$. We shall also use an effective

hot component length l_h^* defined as follows:

$$l_h^* = \frac{2}{p_{h0}} \int_0^{l_h/2} \frac{B_0}{B(s)} \mathcal{P}_h(s) ds, \quad (33)$$

to compare the results of this paper with the previous results based on a step-like pressure distribution.

Even after the above simplifications and parameterization of the profiles the LMNS equilibrium and stability depend on six dimensionless parameters which determine the magnetic system and pressure distribution. They are: relative toroidal and mirror cell curvatures $K_t = k_t L$, $K_m = k_m L$; mirror ratio R ; relative ellipticity E ; target/hot plasma pressure ratio $\alpha = p_t/p_{h0}$; and relative hot component length $\lambda = l_h/L$. To analyze such the multiparametric problem we should define some optimization criterions to reduce a number of free parameters.

First of all we shall assume that K_m is chosen from Eq. (23) to provide $\delta_0 = 0$ for any variations of the other parameters. Thus, K_m becomes a dependent parameter. Then we calculate dimensionless integrals $J_{1,2}$ (normalized by a factor $p_{h0}/B_0^2 L$) and plot them as functions of $\alpha = p_t/p_{h0}$ in Fig. 2. The other parameters are fixed and chosen in accordance with the LMNS proposal⁷ as follows: $K_t = 2$, $R = 2$, $E = 12$, $\lambda = 0.3$, ($\lambda^* \equiv l_h^*/L = 0.16$). Figure 2 shows that stable equilibria ($J_{1,2} > 0$) exist, if $\alpha < 0.1$. This restriction does not allow to get $\alpha = 0.15$ which was presented in initial proposal⁷ as a desirable value. More of that, in reality α has to be appreciably less than $\alpha = 0.1$, because the toroidal ellipticity E_t goes to infinity due to vanishing of J_2 , when α approaches to the critical value. Such the behavior of E_t corresponds to the loss of equilibrium discussed in the previous section. According to Eq. (24) plasma equilibria exist also when $\alpha > 0.15$ ($J_2 < J_1 < 0$) including the case of isotropic plasma pressure ($p_{h0} = 0$), however, such the equilibria are unstable.

Singularity of E_t does not allow to consider equation $J_2 = 0$ as a marginal stability condition. Considering a real system we should keep in mind that the plasma is not allowed

to touch the material walls, so E_t has to be restricted. Therefore, a reasonable procedure of LMNS magnetic system optimization has to maximize $\alpha = p_t/p_{h0}$ for a fixed (desirable) value of E_t . When $E_t = \text{const}$ the parameter α has a maximum as a function of the parameters λ , R , E , and α increases monotonically with a decrease of the toroidal curvature K_t . Thus, we are able to optimize the parameters λ , R , E to provide the maximum of α . However, the low- β equilibrium and stability theory does not provide an algorithm for optimization of K_t . Therefore, we consider $K_t = 2$, as it has been chosen in the initial proposal.⁷ We shall also assume a fixed value $\lambda = 0.3$, because α depends only slightly on λ in the vicinity of $\lambda = 0.3$, which is suitable for the high neutron flux production.⁷

A family of the curves $\alpha(E)$ for a set of different values E_t and $R = 2$ is shown in Fig. 3. The parameters α appears to be very sensitive to variations of E . According to the optimization procedure suggested above we choose $E = E_{\text{opt}}$ which corresponds to the maximum of $\alpha(E)$. Analogously we optimize the parameter R to provide $\max \alpha(R)$ when $R = R_{\text{opt}}$. The optimized values of E and R as functions of E_t are presented in Fig. 4. Figure 5 presents the maximum values of α for a given E_t and corresponding “stability factor” S_f considered as a functions of E_t . The factor S_f is defined as the ratio of the local potential energy G defined by Eq. (28) to a value $G = G_0$ calculated for the same magnetic system parameters assuming, however, that $p_t = 0$. The stability factor S_f characterizes a rigidity of the unstable mode anchoring by the hot ion component. The factor S_f seems to be important for the ballooning mode stability.

We can see from Fig. 4 that contrary to the previous study assumption the mirror cell midplane ellipticity $E_0 = E_t/E$ appreciably differs from the unity: $E_0 \approx 1.7$. This result means that quadrupole Pfirsch-Schlüter current generated in the toroidal cell is not negligible. Figs. 4 and 5 also show that the values of α and R to be appropriate for the LMNS could be provided if $E_t \geq 15$. Finally we present in Table 1 the dimensionless MHD parameters of LMNS modified in accordance with the self-consistent MHD theory developed in this paper.

Table I Parameters of LMNS magnetic system. The first column presents the parameters of initial proposal, the second column shows the optimized results of present study.

Toroidal cell curvature $k_t L$	2	2
Effective hot component length l_h/L	0.15	0.16
Mirror ratio R	2	2
Mirror cell axis curvature $k_m L$	0.44	0.09
Toroidal cell ellipticity E_t	12	17
Mirror midplane ellipticity E_0	1	1.6
Tangent/hot plasma pressure ratio p_t/p_{h0}	0.15	0.08
Stability factor	-	0.192

V Conclusion

The general formalism is developed to analyze the MHD equilibrium and stability of low- β anisotropic pressure plasmas contained in the closed field line magnetic confinement systems. The formalism is shown to be completely self-consistent under paraxial approximation. It allows to calculate the principal parameters of the plasma column. All the calculations can be extended up to next orders of the expansion in the powers of the distance from the magnetic axis. In a general case the plasma equilibrium exhibits a nonconventional feature which is that the stable and unstable equilibrium regions in a parametric space appear to be separated by a zone where an equilibrium is absent.

An application of the formalism developed to the linked mirror neutron source analysis confirms the results of the preliminary study in general, however, we have found some new features of plasma equilibrium that has led to the necessity to modify the LMNS parameters. Simultaneously, we have got the way of how to optimize these parameters to increase the stability of the system considered. All of that allows to consider the above formalism as fruitful and useful one.

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Figure Captions

1. Structure of magnetic field lines in the toroidally linked mirror system. Cross-sections 2, 6, 8, 12 correspond to minimum- B mirror cell midplanes.
2. Dependence of J_1 and J_2 on $\alpha = p_t/p_{h0}$ for the LMNS magnetic system parameters.
3. The family of the curves $\alpha(E)$ for $R = 2$ and $E_t = \text{const.}$ The curves 1, 2, 3, 4 correspond to the following values of E_t : 12, 15, 18, 21.
4. The optimal values of E and R as the functions of E_t .
5. The optimal value of α and "stability factor" S_f as the functions of E_t .

Figure 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

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