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**SENSITIVITY ANALYSIS OF THE NOBLE GAS
TRANSPORT AND FATE MODEL:
CASCADR9**

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ABSTRACT

CASCADR9 is a desert alluvial soil site-specific noble gas transport and fate model. Input parameters for CASCADR9 are: man-made source term, background concentration of radionuclides, radon half-life, soil porosity, period of barometric pressure wave, amplitude of barometric pressure wave, and effective eddy diffusivity. Using average flux, total flow, and radon concentration at the 40 day mark as output parameters, a sensitivity analysis for CASCADR9 is carried out, under a variety of scenarios. For each scenario, the parameter to which output parameters are most sensitive are identified.

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1.0 INTRODUCTION

The basic mathematics and physics for the high dry desert alluvial soil site-specific noble gas transport and fate model, CASCADER, has been discussed in detail by Lindstrom et al. (1992a and 1993a). The details of the associated FORTRAN 77 code for CASCADR8, the homogeneous porous medium version, and for CASCADR9, the layer region heterogeneous porous medium version, can be found in Cawfield et al. (1994a and b). Two realistic applications of the CASCADR8 model were made to the "usual" shallow subsurface burial of thorium low level waste (LLW). Lindstrom et al. (1992b) discussed the simulation of surface fluxes of radon-220 derived from thorium-232 LLW. Lindstrom et al. (1993b) discussed the simulation of surface fluxes of radon-222 derived from the thorium-230 "impurity," shipped to the Nevada Test Site (NTS), in the thorium-232 waste stream. An important application of CASCADR9 was using the model to help design the depth of burial of the mixed thorium-232/thorium-230 LLW placed under the usual "slab or 17 foot thick region" (Becker, 1992) of depleted uranium LLW. This detailed study can be found in Lindstrom et al. (1993c).

In all the above mentioned publications, limited sensitivity analyses have been carried out on both CASCADR8 and CASCADR9. Since CASCADR9 essentially contains CASCADR8 as a special case, a sensitivity analysis on the layered heterogeneous high dry desert alluvial soil model CASCADR9 is only considered.

In Appendix F of Cawfield et al. (1994a), a detailed verification was made of CASCADR9 by comparing the numerical model simulations to those obtained from an analytic structure modeling the same noble gas transport in dry desert alluvial soils found at NTS. Several more "verification" experiments are included below. They arise naturally in the explanation of "negative correlations" (negative sensitivity coefficients), which on the surface, seem to be counter intuitive.

2.0 GOVERNING EQUATIONS

2.1 Transport and Fate Equations

Figure 2.1.1 is a sketch of the conceptualized transport and fate processes operating in the typical "one-dimensional" representative elementary volume (REV) between x and $x+\Delta x$ on the right-hand side of the half-space ($x \geq 0$) region. The REV is assumed to lie between two textural contrasts (Figure 2.1.2). This figure illustrates a four-chain of radionuclides. The mathematical symbols are defined in Appendix A. Application of the mass balance principle in the REV leads to the following equations:

$$\frac{\partial C^1}{\partial t} + \frac{1}{\epsilon(1+R^1)} \frac{\partial (q_{air} C^1)}{\partial x} + \left(\lambda_1 + \frac{\lambda_{irr}^1 f_{irr1} (1-\epsilon)}{\epsilon(1+R^1)} \right) C^1 =$$

$$\frac{D^1}{1+R^1} \frac{\partial^2 C^1}{\partial x^2} + \frac{(1-\epsilon) \rho_p S_o(x) + (1-\epsilon) S_{sx}(x) \left(\frac{M_1}{M_0} \right)}{\epsilon(1+R^1)} \quad \left(\frac{gm}{cm^3 air} \right) \quad (2.1.1)$$

for the first daughter radionuclide and

$$\frac{\partial C^j}{\partial t} + \frac{1}{\epsilon(1+R^j)} \frac{\partial (q_{air} C^j)}{\partial x} + \left[\lambda_j + \lambda_{irr}^j f_{irrj} \frac{(1-\epsilon)}{\epsilon(1+R^j)} \right] C^j =$$

$$\frac{D^j}{1+R^j} \frac{\partial^2 C^j}{\partial x^2} + \lambda_{j-1} \left(\frac{1+R^{j-1}}{1+R^j} \right) \left(\frac{M_j}{M_{j-1}} \right) C^{j-1} \quad , \quad \left(\frac{gm}{cm^3 air} \right) \quad (2.1.2)$$

for the j th daughter radionuclide, $j = 2, 3, \dots, M$.

The detailed development of these transport and fate equations together with their associated sorption rules is given in Lindstrom et al. (1992a). In equations 2.1.1 and 2.1.2 the reversibly sorbed daughters affect the transport and fate via the use of linear equilibrium distribution coefficients and ultimately through retention coefficients such as

$$R^j = \frac{1-\epsilon}{\epsilon} f_{revj} \rho_p K_{eqj} \quad (2.1.3)$$

Figure 2.1.2 Schematic diagram of the homogeneous soil layers making up the heterogeneous vadose zone.

The irreversibly bound daughters' sorption rules are given in Lindstrom et al. (1992a). In equations 2.1.1 and 2.1.2 the effect of barometric pressure induced advection is included via the spatially and temporally varying function $q_{air}(x,t)$. Lacking values of the reversible and irreversible sorption coefficients for radon-222 on NTS soils, an error on the conservative side was selected by setting all sorption parameters to zero. The other data required to run the model, e.g., porosities, diffusivities, etc., can be found in Lindstrom et al. (1992c and 1993b).

2.2 Barometric Induced Air Pumping

The details of barometric induced air pumping in CASCADER are given in Lindstrom et al. (1992a). Briefly, the one-dimensional Darcy air flow on the homogeneous interior of each soil layer is defined as

$$q_{air} = -\frac{\kappa_{air}}{\eta_{air}} \left(\frac{\partial P_{air}}{\partial x} - \rho_{air} g \right) \quad (2.2.1)$$

(Lindstrom et al., 1992a). The assumptions are made that 1) the air can be treated as an ideal gas so that

$$\rho_{air} = \frac{M_{air}}{R_g T} P_{air} \quad , \quad (2.2.2)$$

and 2) isothermal conditions hold. Combining these two definitions (constitutive equations) with the continuity equation (mass balance) in the REV obtains

$$\frac{\partial(\epsilon \rho_{air})}{\partial t} + \frac{\partial(\rho_{air} q_{air})}{\partial x} = 0, \quad (2.2.3)$$

which provides us a way of estimating the advective velocity $V_{air}(x,t)$ since

$$q_{air} = \epsilon V_{air} \quad . \quad (2.2.4)$$

CASCADR9 makes use of all of these relationships together with initial and boundary data to estimate the advective velocity of the radionuclides.

2.3 Interfacial or Contrast Coupling

Figure 2.3.1 shows a sketch of the following Gaussian "pill box" concept at an arbitrary interface I_j , $j = 1, 2 \dots N_I$, between two soil layers with properties I_j^L and I_j^U , respectively.

Figure 2.3.1 Gaussian pill box of arbitrary cross sectional area γ (cm^2) and thickness T (cm).

The interface or coupling condition, at contrast I_j and derived in detail in Lindstrom et al. (1993c), is

$$q_{ck}^x|_{x_{ij}^-} = q_{ck}^x|_{x_{ij}^+} \quad (2.3.1)$$

or upon substitution for q_{cj}^x then,

$$\epsilon_{ij-} \left(-D_-^K \frac{\partial C^K}{\partial x} \Big|_{x_{ij}^-} + (V_{air} C^K) \Big|_{x_{ij}^-} \right) = \epsilon_{ij+} \left(-D_+^K \frac{\partial C^K}{\partial x} \Big|_{x_{ij}^+} + (V_{air} C^K) \Big|_{x_{ij}^+} \right) . \quad (2.3.2)$$

A similar argument on the barometric air pumping concept at arbitrary interface I_j leads to

$$q_{air}|_{x_{ij}^-} = q_{air}|_{x_{ij}^+} . \quad (2.3.3)$$

Since by assumption there are no sources or sinks of mass, either radionuclide or air, at the interfacial nodes, continuity in the radionuclide concentration at the interfacial node is required. That is,

$$C^K(x_{ij}^-, t) = C^K(x_{ij}^+, t) = C^K(x_{ij}, t) . \quad (2.3.4)$$

Similar reasoning on the barometric pressure field leads to

$$P_{air}(x_{ij}^-, t) = P_{air}(x_{ij}^+, t) = P_{air}(x_{ij}, t) . \quad (2.3.5)$$

Using equations 2.3.3 and 2.3.4 in equation 2.3.2 reduces 2.3.2 to the simplified form

$$-\epsilon_{ij-} \left(D_-^K \frac{\partial C^K}{\partial x} \right) \Big|_{x_{ij}^-} = -\epsilon_{ij+} \left(D_+^K \frac{\partial C^K}{\partial x} \right) \Big|_{x_{ij}^+} . \quad (2.3.6)$$

2.4 Boundary Conditions, Initial Conditions, and Source Representations

Figures 2.4.1 and 2.4.2 show the conceptual structure used at the two boundaries. Either the same boundary conditions as described in detail in CASCADR8 (Lindstrom et al., 1992a) or direct users supplied barometric pressure wave boundary conditions are used in CASCADR9. The initial pressure distribution in the barometric pumping submodel is

$$P_{air}(x, 0^+) = P_A e^{\frac{M_{air} g x}{R_g T}} . \quad (2.4.1)$$

At the soil-atmospheric boundary layer interface the boundary condition on the pressure field is either

$$P_{air}(0,t) = P_A + \sum_{i=1}^{15} \left(\Delta P_{ci} \cos\left(\frac{2\pi t}{L_i}\right) + \Delta P_{si} \sin\left(\frac{2\pi t}{L_i}\right) \right) , \quad (2.4.2)$$

or a user defined (subsequently smooth cubic splined interpolated) data table for barometric pressure $P_{air}(0,t)$.

At the water table the boundary condition is

$$q_{air}|_{x=L_x} = 0 \quad (2.4.3)$$

or

$$\left(\frac{\partial P_{air}}{\partial x} - \frac{M_{air} g}{R_g T} P_{air} \right)_{x=L_x} = 0 . \quad (2.4.4)$$

Refer again to Figures 2.4.1 and 2.4.2. They illustrate the boundary layer concepts used at the soil-atmosphere interface and the soil-water table interface respectively. The water table and the soil are coupled together at the plane $x=L_x$. Continuity in the normal component of the flux yields the partial derivative boundary condition

$$-\epsilon D^j \frac{\partial C^j}{\partial x} \Big|_{x=L_x} = -\epsilon_{in} D_{in}^j \left(\frac{C_{in}^j - C^j(L_x, t)}{\Delta x_{in}} \right) . \quad (2.4.5)$$

By similar reasoning at the soil-atmosphere interface, the boundary condition becomes

$$-\epsilon_{out} D_{out}^j \left(\frac{C^j(0,t) - C_{out}^j}{\Delta x_{out}} \right) = -\epsilon D^j \frac{\partial C^j}{\partial x} \Big|_{x=0} . \quad (2.4.6)$$

In the case of the initial conditions for the j th radionuclide chemical concentration, $C^j(x, 0^+)$ is obtained from the secular equilibrium conditions in the Bateman Chain so that

$$C^j(x, 0^+) = \frac{\lambda_{j-1} \left(\frac{1+R^{j-1}}{1+R^j} \right) \left(\frac{M_j}{M_{j-1}} \right) C^{j-1}(x, 0^+)}{\lambda_j + \lambda_{irr}^j f_{irrj} \left(\frac{1-\epsilon}{\epsilon (1+R^j)} \right)} \quad (2.4.7)$$

for $j = 2, 3, 4, \dots M$. $C^1(x, 0^+)$ is prescribed by the user of CASCADR9.

The constant in time background source function $S_o(x)$ is defined from $C_o(x)$ as

$$S_o(x) = \frac{\left(\lambda_1 + \frac{\lambda_{irr}^1 f_{irr1} (1-\epsilon)}{\epsilon (1+R^1)} \right) C_o^1(x, 0^+) \epsilon (1+R^1)}{(1-\epsilon) \rho_p} \quad (2.4.8)$$

Because the representation of the "man-made source function," $S_{sx}(x)$ is highly radionuclide specific and because the Area 5 Radioactive Waste Management Site (RWMS) at NTS seems to be a logical place to store very long lived radionuclides, such as depleted uranium (du), $S_{sx}(x)$ with du was selected for illustration.

The depth to ground water is about 240 meters, the average annual precipitation is about 120 mm/year, and the annual evapotranspiration is close to 2,000 mm/year in Area 5. Under current atmospheric driving conditions, liquid water does not seem to penetrate beyond the rooting zone of 100 cm for the open high desert alluvial soils found at NTS (Hunter, 1993). Our own vadose zone hydrologic modeling studies corroborate Hunter's experimental findings. Thus, Figure 2.4.3 shows a sketch of Pit 6 ([po6u, old to8u] Morgan, 1992). The length, width, and depth of Pit 6 are 192 m, 30.49 m, and 7.32 m, respectively. The waste packages are assumed to be the usual fashion, four high. A 0.102 m gap (filled with native soil) is left between each box. A

total of three gaps makes 0.305 m of extra space. Thus, stacking the boxes four high totals 5.18 m from top to bottom.

The volume inside Pit 6 available for waste storage is computed as follows:

$$\text{Volume} = (30.49 \text{ m}) (5.18 \text{ m}) (183 \text{ m}) = 28,903 \text{ m}^3$$

$$\text{Each box measures } 1.22 \text{ m} \times 1.22 \text{ m} \times 2.13 \text{ m} = 3.17 \text{ m}^3$$

$$\text{There are then estimated to be } (28,903 \text{ m}^3)/(3.17 \text{ m}^3/\text{box}) = 9,116 \text{ boxes}$$

$$\text{Assuming the actual storage rate at 95 percent implies } (9,116 \text{ boxes } [0.95]) = 8,661 \text{ boxes.}$$

The assumption (conservative) is made that each box contains 1×10^6 gm of pure uranium-238 at time $t=0$. Figure 2.4.4 shows the ingrowth of radium-226 the source or parent for our radon-222, the "chemical" of interest. The peak ingrowth of radium-226 occurs at about 4 million years (not shown). The peak value for radium-226 is 0.34 gm for the given 1 million gram initial mass of uranium-238. Figure 2.4.4 was generated from a table of mass ingrowth data obtained from the chemical mass form of the classical Bateman decay chain of uranium-238, called CHAIN 1 (Lindstrom et al., 1992b). Observe in Figure 2.4.4 that the ingrowth is a very slow increasing function of time. For short periods of time, say one to two years, the mass of radium-226 can be treated as a constant with the actual mass value of radium-226 obtained directly from the graph and scaled accordingly since the classical Bateman decay chain is a linear system of ordinary differential equations. For illustrative purposes the value of radium-226 at 4 million years (the peak value) was selected, which is 0.34 gm radium-226, to constitute the radium source on a per box basis. Hence,

$$(8,661 \text{ boxes}) \text{ at } (1 \times 10^6 \text{ gm U-238/box}) = 8.661 \times 10^9 \text{ gm U-238 in Pit 6.}$$

Also,

$$(8,661 \text{ boxes}) \text{ at } (0.34 \text{ gm Ra-226/box}) = 2,945 \text{ gm Ra-226 in Pit 6.}$$

The volume of the waste is estimated to be 28,903 m³ or 2.890 x 10¹⁰ (cm³). The source function is then defined by assuming that the 2,942 gm of radium-226 is instantaneously and uniformly mixed into the 2.890 x 10¹⁰ cm³ volume. No credit is taken for the slow decay (corrosion in the case of steel drums) of the waste packages. Were this known, the release rate could be modified accordingly. Therefore, estimating

$$S_{sx} = \frac{(0.693)}{(1600 \text{ yr}) \left(3.15576 \times 10^7 \frac{\text{sec}}{\text{yr}} \right)} \left(\frac{2,942 \text{ gm Ra-226}}{2.890 \times 10^{10} \text{ cm}^3} \right) =$$

$$1.39864 \times 10^{-18} \frac{\text{gm}}{\text{cm}^3 - \text{sec}} \quad (2.4.9)$$

as the man-made source function.

3.0 SOLUTION PROCEDURE

While the system of coupled parabolic pde's, together with all the initial and boundary data, is linear and can in principle be solved exactly, it is nevertheless intractable. To date, this system has not been solved. The usual methods of Laplace transforms and Fourier transforms (Lindstrom and Oberhettinger, 1975) were tried. However, the transformed system was still intractable for the general case. Therefore, a mixture of time convoluted ordinary difference equations and finite difference methods was used to approximate the solution. The advection portions of the parabolic pde's are handled using an explicit Euler-Lagrange method outlined by Lindstrom (1991 and 1992). A Lagrange cubic spline algorithm effects the interpolation of each of the m radionuclides.

3.1 Transport and Fate Equations

Since the material derivative or derivative following a particle moving along a material flow or trackline is defined for radionuclide 1 as

$$\frac{dC^1}{dt} = \frac{\partial C^1}{\partial t} + \frac{V}{1+R^1} \frac{\partial C^1}{\partial x} \quad , \quad (3.1.1)$$

the transport and fate equation for this nuclide can be rewritten as

$$\begin{aligned} \frac{dC^1}{dt} + \left(\lambda_1 + \lambda_{irr}^1 f_{irr1} \frac{(1-\epsilon)}{\epsilon(1+R^1)} + \frac{1}{1+R^1} \frac{V_{air}^2}{D_{air}} - V_{air} \frac{M_{air} g}{R_g T (1+R^1)} - \frac{1}{1+R^1} \frac{1}{P_{air}} \frac{\partial P_{air}}{\partial t} \right) C^1 = \\ \frac{D^1}{1+R^1} \frac{\partial^2 C^1}{\partial x^2} + \frac{S_o(x)}{1+R^1} \frac{(1-\epsilon)}{\epsilon} \rho_p + \frac{(1-\epsilon)}{\epsilon(1+R^1)} S_{SP}(x) \left(\frac{M_1}{M_0} \right) e^{-\lambda_o t} \quad . \end{aligned} \quad (3.1.2)$$

For radionuclides $j = 2, 3, \dots, m$,

$$\frac{dC^j}{dt} + \left(\lambda_j + \lambda_{irr}^j f_{irrj} \frac{(1-\varepsilon)}{\varepsilon(1+R^j)} + \frac{1}{1+R^j} \frac{V_{air}^2}{D_{air}} - V_{air} \frac{M_{air} g}{R_g T (1+R^j)} - \frac{1}{1+R^j} \frac{1}{P_{air}} \frac{\partial P_{air}}{\partial t} \right) C^j =$$

$$\frac{D^j}{1+R^j} \frac{\partial^2 C^j}{\partial x^2} + \lambda_{j-1} \left(\frac{1+R^{j-1}}{1+R^j} \right) \left(\frac{M_j}{M_{j-1}} \right) C^{j-1} \quad .$$

(3.1.3)

The detailed development of the approximating expressions for the irreversible sorptions (included in CASCADR9 but not exercised in this sensitivity study) can be found in Lindstrom et al. (1992a).

The split operator Euler-Lagrange method, introduced many years ago and more recently revived and reviewed by Baptista et al. (1984) and Baptista (1987), is the numerical technique used here. Basically, the operator is split into the advection portion $\frac{dC(\cdot)}{dt}$, which is solved locally in time, along a trackline (a Lagrangian point of view), and the dispersion (plus other chemical and/or biological processes) portion, which is solved locally in time in a fixed or Eulerian coordinate system. The name ELM therefore follows accordingly. This method is closely related to it's precursor, the classical method of characteristics (MOC).

On the space-time cylinder D , where

$$D = \left\{ (x,t) \text{ in } \mathbb{R}^2 \mid 0 < x < L_x \text{ and } t > 0 \right\} \quad ,$$

(3.1.4)

a lattice of N_{x+1} points called nodal points are superimposed. These points are defined as: $x_0 = 0$, $x_1 = \Delta x$, $x_2 = x_1 + \Delta x$, ... , $x_{k+1} = x_k + \Delta x$, ... x_{N_x} = soil-water table bounding plane. In time $t_0 = 0$, $t_1 = t_0 + \Delta t$, $t_2 = t_1 + \Delta t$, ... $t_n = t_{n-1} + \Delta t$, At lattice or nodal point (x_k, t_n) , $C^j(x_k^j, t_n) \doteq \chi_{k,n}$, for $j = 1, 2, 3, \dots m$ is approximated. A full space-time discretization, which is spatially centered and backward in time for equation 3.1.2 (radionuclide 1 transport and fate equation), can be written as

$$\begin{aligned}
& \frac{\chi_{k,n+1}^1 - \chi_{k(n)}^{1*}}{\Delta t} + \left(\lambda_1 + \lambda_{irr}^1 f_{irr1} \frac{(1-\varepsilon)}{\varepsilon(1+R^1)} + \frac{1}{1+R^1} \frac{V_{airk,n+1}^2}{D_{air}} \right. \\
& \left. - V_{airk,n+1} \frac{M_{air} g}{R_g T(1+R^1)} - \frac{1}{1+R^1} \frac{1}{\bar{P}_{airk,n+1}} \left(\frac{\bar{P}_{airk,n+1} - \bar{P}_{airk,n}}{\Delta t} \right) \right) \chi_{k,n+1}^1 = \\
& \frac{D^1}{(1+R^1) \Delta x^2} (\chi_{k+1,n+1}^1 - 2\chi_{k,n+1}^1 + \chi_{k-1,n+1}^1) + \frac{1-\varepsilon}{\varepsilon} \frac{\rho_p}{1+R^1} S_{ok} + \frac{(1-\varepsilon) S_{sp}(x)}{\varepsilon(1+R^1)} \left(\frac{M_1}{M_0} \right) e^{-\lambda_0 t}
\end{aligned} \tag{3.1.5}$$

(Lapidus and Pinder, 1982). Similarly, for the remaining radionuclides ($j = 2, 3, \dots m$) the fully discretized approximation to equation 3.1.2 is obtained as

$$\begin{aligned}
& \frac{\chi_{k,n+1}^j - \chi_{k(n)}^{j*}}{\Delta t} + \left(\lambda_j + \lambda_{irr}^j f_{irrj} \frac{(1-\varepsilon)}{\varepsilon(1+R^j)} + \frac{1}{1+R^j} \frac{V_{airk,n+1}^2}{D_{air}} \right. \\
& \left. - V_{airk,n+1} \frac{M_{air} g}{R_g T(1+R^j)} - \frac{1}{1+R^j} \frac{1}{\bar{P}_{airk,n+1}} \left(\frac{\bar{P}_{airk,n+1} - \bar{P}_{airk,n}}{\Delta t} \right) \right) \chi_{k,n+1}^j = \\
& \frac{D^j}{(1+R^j) \Delta x^2} (\chi_{k+1,n+1}^j - 2\chi_{k,n+1}^j + \chi_{k-1,n+1}^j) + \lambda_{j-1} \left(\frac{1+R^{j-1}}{1+R^j} \right) \left(\frac{M_j}{M_{j-1}} \right) \chi_{k,n+1}^{j-1} .
\end{aligned} \tag{3.1.6}$$

The highly detailed explanation of the "star points" and the interpolated concentrations found at the star points is given in Lindstrom et al. (1992a). Also, from Lindstrom et al. (1992a) there are the definitions

$$D_{air}(x) = \frac{\kappa_{air} \bar{P}_{air} e^{\frac{M_{air} g x}{R_g T_{air}}}}{\varepsilon \eta_{air}(T_{air})} , \tag{3.1.7}$$

the "pneumatic diffusivity" (cm²/sec),

$$K_{air} = \frac{\kappa_{air} \bar{P}_{air} g e^{\frac{M_{air} g x}{R_g T_{air}}}}{\epsilon \eta_{air}}, \quad (3.1.8)$$

the "pneumatic conductivity" (cm/sec);

$$\bar{P}_{air} = \frac{M_{air} \bar{P}_{air}}{R_g T_{air}}, \quad (\text{gm/cm}^3), \quad (3.1.9)$$

the Ideal gas law, and from Lindstrom et al. (1993a);

$$\kappa_{air} = \frac{\epsilon^4 d^2}{36(1-\epsilon)^2}, \quad (\text{cm}^2) \quad (3.1.10)$$

which empirically relates the gas permeability to the porosity and is called the Kozeny relationship in the engineering literature ($d^2 = 3.05 \times 10^{-5}$ cm for Area 5, NTS soils); and

$$D^1 = D_0 \epsilon^{3/2} \quad (3.1.11)$$

which empirically defines the radon diffusion coefficient (cm²/sec) in terms of the porosity (Marshall, 1959). $D_0 = 0.1$ cm²/sec at 20°C in free air (Nazaroff, 1992). The "pneumatic" definitions are used in the following "air-pumping" discretizations while the radon diffusion coefficient appears in equation 3.1.5. The full discretization of the surface barometric pressure change induced gas pressure field is easily written as (in time-space implicit form)

$$\frac{\bar{P}_{airk,n+1} - \bar{P}_{airk,n}}{\Delta t} + \frac{\partial K_{air}}{\partial x} \bigg|_{x_k} \bar{P}_{airk,n} = D_{air}(x_k) \left\{ \frac{\frac{\bar{P}_{airk+1,n+1} - \bar{P}_{airk,n+1}}{\Delta x_k} - \frac{\bar{P}_{airk,n+1} - \bar{P}_{airk-1,n+1}}{\Delta x_{k-1}}}{\left(\frac{\Delta x_{k-1} + \Delta x_k}{2} \right)} \right\} \quad (3.1.12)$$

where $\tilde{P}_{air k, n} \equiv P_{air}(x_k, t_n)$, $k = 1, 2, 3 \dots N_x$. Recasting this linear tridiagonal implicit system into its usual unconditionally stable form obtains

$$\begin{aligned}
 & -\frac{2 D_{air k} \Delta t}{\Delta x_{k-1} (\Delta x_{k-1} + \Delta x_k)} \tilde{P}_{air k-1, n+1} \\
 & + \left(1 + \frac{2 D_{air k} \Delta t}{\Delta x_{k-1} + \Delta x_k} \left(\frac{1}{\Delta x_{k-1}} + \frac{1}{\Delta x_k} \right) + \Delta t \left. \frac{\partial K_{air}}{\partial x} \right|_{x_k} \right) \tilde{P}_{air k, n+1} \\
 & - \frac{2 D_{air k} \Delta t}{\Delta x_k (\Delta x_{k-1} + \Delta x_k)} \tilde{P}_{air k+1, n+1} = \tilde{P}_{air k, n} \quad .
 \end{aligned} \tag{3.1.13}$$

3.2 Approximations to Interfacial Coupling, Boundary, and Initial Conditions

CASCADR9 uses the split operator Euler-Lagrange finite difference numerical method, detailed by Lindstrom et al. (1992), in solving for the radionuclide concentration distribution over space and time for prescribed initial and boundary conditions. Thus, in the air pumping submodel the interfacial coupling conditions of 2.3.3 and 2.3.5 are present. Using the same one-sided first difference approximation for $\frac{\partial P_{air}}{\partial x}$ as appears in CASCADR8 (Lindstrom et al., 1992a),

$$\begin{aligned}
 & -K_{air l_j^-} \left(\frac{P_{air l_j} - P_{air l_j-1}}{\Delta x_{l_j-1}} - \frac{M_{air} g}{R_g T_{air}} P_{air l_j} \right) = \\
 & -K_{air l_j^+} \left(\frac{P_{air l_j+1} - P_{air l_j}}{\Delta x_{l_j}} - \frac{M_{air} g}{R_g T_{air}} P_{air l_j} \right)
 \end{aligned} \tag{3.2.1}$$

as the interfacial coupling condition on the barometric pressure field and similarly for radionuclide k condition 2.3.6 becomes

$$-\epsilon_{ij}^- D_-^K \left(\frac{C_{ij}^K - C_{ij-1}^K}{\Delta x_{ij-1}} \right) = -\epsilon_{ij}^+ D_+^K \left(\frac{C_{ij+1}^K - C_{ij}^K}{\Delta x_{ij}} \right) . \quad (3.2.2)$$

At all other interior points of each soil layer the split operator Euler-Lagrange method is used. The smooth cubic Lagrange spline method of interpolating the advected radionuclide chemical concentration as per detailed in Lindstrom et al. (1992) is retained in CASCADR9. The thickness of each soil layer as well as the nodal point spacing (either uniform or nonuniform spacing) is the same in CASCADR9 as in CASCADR8.

The boundary condition on the barometric air pressure field at the surface $x=0$, defined by equation 2.4.2, remains unchanged in that the user chooses to either define the pressure at the surface as

$$\bar{P}_{air0,n+1} = P_A + \sum_{i=1}^{15} \left(\Delta P_{ci} \cos \left(\frac{2\pi t_{n+1}}{L_i} \right) + \Delta P_{si} \sin \left(\frac{2\pi t_{n+1}}{L_i} \right) \right) . \quad (3.2.3)$$

or to define the pressure at the surface via a smooth cubic spline (fit several points ahead in time in CASCADR9) to real barometric pressure data (15 minute type usually).

In the equation shown in Figure 2.4.2, $\left. \frac{\partial P_{air}}{\partial x} \right|_{x=L_x}$ is approximated via the one-sided two-point difference approximation

$$\left. \frac{\partial P_{air}}{\partial x} \right|_{x=L_x} \doteq \frac{P_{air}(X_{Nx}) - P_{air}(X_{Nx-1})}{\Delta x_{Nx}} . \quad (3.2.4)$$

Substitution into the no flux bottom boundary condition, with subsequent rearrangement, leads to the simple form

$$-P_{airNx-1} + \left(1 - \frac{M_{air} g}{R_g T} \Delta x_{Nx} \right) P_{airNx} = 0 . \quad (3.2.5)$$

For the j th radionuclide at the soil-water table interface, the "gradient component" is approximated via the one sided difference approximation

$$\left. \frac{\partial C^j}{\partial x} \right|_{x=L_x} \doteq \frac{C^j(x_{Nx}, t) - C^j(x_{Nx} - \Delta x_{Nx}, t)}{\Delta x_{Nx}} \quad (3.2.6)$$

to obtain

$$-\epsilon D^j \left(\frac{C^j(x_{Nx}, t) - C^j(x_{Nx-1}, t)}{\Delta x_{Nx}} \right) = -\epsilon_{in} D_{in}^j \left(\frac{C_{in}^j - C^j(L_x, t)}{\Delta x_{in}} \right) \quad (3.2.7)$$

Boundary condition (3.2.7) is used since it retains the important physical concept of flux balance at the water table. A choice of erring on the conservative side was made in setting $C_{in}^j = 0$ and using $D_{in}^j = 10D^j$, for each j , in view of the lack of knowledge of these parameters values at the water table.

For the j th radionuclide at the air-soil interface, the "gradient component," is approximated via the one-sided approximation

$$\left. \frac{\partial C^j}{\partial x} \right|_{x=0} \doteq \frac{C^j(x_1, t) - C^j(0, t)}{\Delta x_0} \quad (3.2.8)$$

to obtain

$$-\epsilon D^j \left(\frac{C^j(x_1, t) - C^j(0, t)}{\Delta x_0} \right) = -\epsilon_{out} D_{out}^j \left(\frac{C^j(0, t) - C_{out}^j}{\Delta x_{out}} \right) \quad (3.2.9)$$

Boundary condition (3.2.9) is used since it also retains the important physical concept of flux balance at the soil-atmosphere interface. Well-known historical data on radon-222 near-surface air concentrations at the RWMS in Area 5 of NTS clearly show that the radon-222 air concentration one to two meters above the land surface is below detection limits. Thus, C_{out}^j was set to zero for all j .

The initial condition on the gas pressure field becomes

$$\bar{P}_{\text{air},k,0} = P_{\text{air}}(0) e^{\frac{M_{\text{air}} g x_k}{R_g T}}, \quad (3.2.10)$$

$$k = 0, 1, 2, 3 \dots, N_{Lx}.$$

The initial distribution for the j th radionuclide is obtained by using the secular equilibrium distribution equation (2.4.7) where the user must supply the initial distribution of radionuclide $C^j(x,0^+)$. In virtually all cases discussed below, $C^j(x,0^+)$ is simply set at a uniformly equal concentration distribution. This means that the first ten or so simulated days consists of "transient data" and must therefore be treated as such.

3.3 Two-Time Level Time Marching Algorithm

The two-time level, time marching algorithm used in CASCADR8 is preserved in CASCADR9. The only difference between the two numerical methods comes in the definitions of the tridiagonal system coefficients at each soil layer interface or contrast nodal point. The interface conditions are used directly to define the linear system coefficients. Both the air pumping submodel and the radionuclide transport and fate portions are treated in this way at the contrasts. Because of the low order of accuracy associated with the two-point one-sided derivatives, the nodal point spacing in the neighborhood of the contrast nodes is kept quite small and gradually expands in spacing size as distance increases from the interfaces. Also, the Lagrangian treatment of the advection terms (particle back tracking) is carried out only on the interior nodal points of each homogeneous region between the contrasts. Nodal spacing close to the leading and lagging edges of the man-made waste package region (i.e., source region where $S_{SX}(x) > 0$), are again kept small, but on an expanding scale as distance increases away from the edges. This scheme tends to preserve the accuracy of the approximate solution.

Between any two-time levels, t_n and t_{n+1} , the air pumping portion is solved first. Then the new pressure field is used to estimate the "latest" advection velocity distribution. This information is fed into the "particle back-tracker." The particle starting points or "star points" are then estimated. The smooth cubic Lagrange spline routine is called to estimate the radionuclide concentration at each "star point." The remaining procedure is to serially use the classical tridiagonal system solver (Thomas algorithm [Press et al., 1987]) to effect the updated radionuclide concentration field at time level t_{n+1} . After all radionuclide concentration fields have been updated by serially solving equations 3.1.5 and 3.1.6 the entire procedure is repeated. This is basically a recursive procedure. Additional checks on the mass balance, surface barometric pressure wave curvature, mass account, etc., are imbedded in the various subroutines.

4.0 Sensitivity Analysis

Sensitivity can be defined as the degree to which a change in an input parameter, relative to its uncertainty, impacts the output. The method used here (slightly modified from, for example, Iman and Helton, 1988) was:

- 1. Standardize each input parameter by subtracting off its central value and dividing by one-half its range. Calculate all first-order interaction terms (or higher, if need be) and all quadratic (or higher, if need be) terms. Linearly transform all interaction and quadratic terms, so they are on the same scale (matching maxima and minima) as the transformed input parameters.**
- 2. Standardize the output parameter(s) by subtracting off the mean and dividing by the sample standard deviation. 'Mean' and 'standard deviation' are used here as purely descriptive statistics.**
- 3. Model the standardized output parameter (s) as a function which is linear in the standardized input parameters, their standardized first-order interactions, and the standardized quadratic terms, using least squares or some other form of regression.**
- 4. Assess the goodness-of-fit of the linear approximation.**
- 5. If the linear approximation is "adequate" (an imprecisely defined term), the sensitivity of the output parameter to each input parameter is measured by the absolute value of the corresponding linear coefficient.**

Comments on each step follow:

- 1. Clearly, sensitivity cannot depend on the units of the input parameters. In the case at hand, input parameters cross many orders of magnitude. Standardizing renders the input parameters unitless and (except in pathological**

cases) of the same order of magnitude. With the logarithmic spacing used here, this results in the coding: low value -0.8, medium-low value -0.4404, mid-value 0.0, medium-high value 0.5394, high value 1.20. Interaction and quadratic terms are transformed for the same reason the input parameters are: to facilitate comparison. These parameters are transformed linearly, so the smallest value codes into -0.8 and the greatest value codes into 1.20.

2. Standardizing makes the output unitless and of a controlled order of magnitude.
3. Comparing the outputs of least-squares regression, L^1 regression, and various robust regressions is desirable. If, as in this case, all give approximately the same answers, the least-squares estimate is used. If answers differ, the reason for this difference should be investigated.
4. Data are deterministic. As no randomness exists, formal goodness-of-fit tests are meaningless. Informal tests can be conducted via examination of residuals and various descriptive statistics. Adequacy of the planar approximation can be assessed by fitting quadratic terms. If the coefficients associated with these terms are 'small' in absolute value, where 'smallness' is subjective, the linear approximation is judged adequate.
5. Sensitivity is assessed via absolute values of coefficients, with input parameters to which the output parameters are 'sensitive' having coefficients with large absolute values.

Other methods of sensitivity analysis exist; each implicitly defines 'sensitivity' differently. It is often desirable to repeat the analysis with several methods. If all agree as to which input parameters the output is most sensitive, as occurred here, the analysis is complete. If not, the reason for the lack of agreement should be determined.

4.1 Effect of Depth of Water Table Numerical Experiment

As was previously stated, the depth to the water table in Area 5 of NTS is about 240 meters. In Yucca Flat (Area 3) where another LLW radionuclide site exists the depth to the water table is about 450 meters. The accuracy of the numerical model is directly related to the largest spacing between nodal points. For a fixed simulation time the actual computer run time per simulation is directly related to the number of nodal points used across the flow domain. Hence, since there is a need to make a large number of computer runs for the sensitivity analysis, it is advantageous to first determine if there is the need to go all the way to the real world water table, or possibly "shorten up" the flow domain significantly in the mathematical sense without sacrificing much of the real effect of the bottom reflection boundary (on the gas pressure field) and the absorbing effect on the radionuclides. Therefore, a series of "bottom boundary effect" computer runs using CASCADR9 was done first. Table 4.1.1 summarizes the "best guess" at the dry desert alluvial soil gas flow properties for the RWMS facility in Area 5 at NTS. The column labeled "mid value" represent the mean values or best guess values from the three pilot wells recently drilled around the RWMS (Sully, 1993). It was decided to use the "logarithmic spacing" about the central point in the experimental design space since most studies done to date indicate a multiplicative error structure rather than an additive error structure. The factor 1.5 was chosen to cause the extreme values to vary up and down by 50 percent of the "mean value." The two "extra points" will allow for estimating nonlinear effects other than simply kth order interactions. The mid value of ϵ , the porosity, yields $K_{air} = 4.0 \times 10^{-8} \text{ cm}^2$ (4 Darcys) and $D^1 = 0.0225 \text{ cm}^2/\text{sec}$.

Figures 4.1.1 through Figures 4.1.5 summarize this numerical experiment. The "response variable," chosen to illustrate the effect of depth of the water table, is the radionuclide concentration at selected depths as a function of time. The depths chosen are given in Table 4.1.2.

Table 4.1.1 Parameter Values Used in the Sensitivity Analysis for CASCADR9

Parameter Name	Symbol	Units	Low Value	Mid Low	Mid Value	Mid High	High Value
1. Background Source Uniform Concentration	S_0	gm/cm ³ -sec Reg AL	9.20×10^{-19} 9.20×10^{-29}	1.13×10^{-18} 1.13×10^{-28}	1.38×10^{-18} 1.38×10^{-28}	1.69×10^{-18} 1.69×10^{-28}	2.07×10^{-18} 2.07×10^{-28}
2. Man-Made Source Concentration $Z_{Top} = 3 \text{ m}$ $Z_{Bot} = 8 \text{ m}$ } 5 m thick	S_{sx}	gm/cm ³ -sec Reg AL	9.33×10^{-19} 9.33×10^{-29}	1.14×10^{-18} 1.14×10^{-28}	1.4×10^{-18} 1.4×10^{-28}	1.71×10^{-18} 1.71×10^{-28}	2.10×10^{-18} 2.10×10^{-28}
3. Radon Half-life	$T_{1/2}$	sec	2.22×10^5	2.72×10^5	3.327×10^5	4.07×10^5	4.99×10^5
4. Porosity	ϵ	cm ³ /cm ³	0.25	0.30	0.37	0.45	0.56
5. Period of Barometric Wave	L	sec	5.76×10^4	7.05×10^4	8.64×10^4	1.06×10^5	1.30×10^5
6. Amplitude of Barometric Wave	Δpos_1	mb	3.33	4.08	5.0	6.12	7.5
7. Effective "Eddy" Diffusivity Across Thin Boundary Layer	D_{out}	cm/cm ² -sec Low High	0.07 1.33	0.08 1.63	0.1 2.0	0.12 2.45	0.15 3.00
		Multiplier	1/1.5	$1/\sqrt{1.5}$	1	$\sqrt{1.5}$	1.5

Radon diffusion coefficient is estimated from: $D^1 = (0.1) \epsilon^{3/2} (\text{cm}^2 \text{sec})$.

Gas permeability K_{air} is estimated from $K_{air} = \frac{\epsilon^4 d^2}{36(1-\epsilon)^2}$, $d^2 = 3.05 \times 10^{-5} \text{ cm}^2$.

Pneumatic diffusivity = $D_{air} = \frac{K_{air} \bar{p}}{\epsilon \eta}$
(used in air pumping part)

Figure 4.1.1 Gas-phase concentrations (pCi/cm³) at selected horizons. $Z_{wt} = 240$ m.

Figure 4.1.2 Gas-phase concentrations (pCi/cm³) at selected horizons. $Z_{wt} = 120$ m.

Figure 4.1.3 Gas-phase concentrations (pCi/cm³) at selected horizons. $Z_{wt} = 60$ m.

Figure 4.1.4 Gas-phase concentrations (pCi/cm³) at selected horizons. $Z_{wt} = 30$ m.

Figure 4.1.5 Gas-phase concentrations (pCi/cm³) at selected horizons. $Z_{wt} = 50$ m.

Table 4.1.2 Printout Nodal Positions

Figure #	Nodal Positions (depth below land surface, m)								
	1	2	3	4	5	6	7	8	9
4.1.1	0	1.0	3.0	5.5	8.0	25.0	50.0	100.0	240.0
4.1.2	0	1.0	3.0	5.5	8.0	25.0	50.0	100.0	120.0
4.1.3	0	1.0	3.0	5.5	8.0	25.0	60.0	-	-
4.1.4	0	1.0	3.0	5.5	8.0	25.0	30.0	-	-
4.1.5	0	1.0	3.0	5.5	8.0	25.0	30.0	-	-

Four other response variables sampled over time obtained at the same nodal positions are given in Appendix B. These variables are surface flux density of radon-222 ($\text{pCi}/\text{cm}^2\text{-sec}$), surface cumulative radon-222 flux (pCi), gas pressure distribution (mb), and induced advective velocity (cm/sec).

It is clear from studying these figures that it is possible to move the bottom boundary up to about 50 meters without sacrificing anything substantive. However, the reader should realize that these runs were made assuming a gas permeability of 4 Darcys. An increase in the gas permeability causes a corresponding increase in both the pneumatic diffusivity and pneumatic conductivity with a corresponding square root increase in the "skin depth." "Too large" of an increase in the gas permeability may void the decision to place the water table bottom at 50 meters below the land surface.

Figure 4.1.6 shows a plot of the SUN SPARC 2.0 run time as a function of the depth to the water table. Clearly, for a depth of 50 meters it takes about five minutes of SUN SPARC 2.0 time to simulate 40 days of field time. The five min/run is attractive since it now allows the rather large number of computer runs for the full sensitivity analysis to be made in a reasonable amount of real time.

Figure 4.1.6 Computer run time in minutes, SUN SPARC 2.0[®], as a function of depth to water table.

4.2 Sensitivity Analysis

Cumulative flow (10^4cm^2 area used), average flux density ($\text{pCi}/\text{cm}^2\text{-sec}$), and radon concentration at the air-soil interface point, are the output pCi parameters of interest. As sensitivity of a nonlinear model is a local concept, six scenarios, labeled Scenario One through Scenario Six, are considered:

Scenario Number	Central Value of Background Concentration	Central Value of Man-Made Concentration	Central Value of Effective Eddy Diffusivity
1	1.38E-18	1.40E-18	0.10
2	1.38E-18	1.40E-18	2.00
3	1.38E-18	1.40E-28	0.10
4	1.38E-18	1.40E-28	2.00
5	1.38E-28	1.40E-18	0.10
6	1.38E-28	1.40E-18	2.00

Two other scenarios, in which both background and man-made concentrations are at their low value, might have been considered. These are omitted, as such small sources would result in a flux, flow, and radon concentration of almost zero.

4.2.1 Description of Scenarios

Some input parameters maintain constant central values and half-ranges through all scenarios. These are summarized in Table 4.2.1.1.

Table 4.2.1.1

Parameter	Central Value	Half-Range
Half-Life	332,700 (sec)	138,625 (sec)
Porosity	0.37 (cm^3/cm^3)	0.1547667 (cm^3/cm^3)
Period of Pressure Wave	86,400 (sec)	36,000 (sec)
Amplitude of Pressure Wave	5.0 (mb)	2.08333 (mb)

Other input parameters have central values and half-ranges that vary, between "high" and "low" settings, with scenario. These are summarized in Table 4.2.1.2.

Table 4.2.1.2

Input Parameter	Central Value (high)	Half-Range (high)	Central Value (low)	Half-range (low)
Background Concentration	1.38E-18 $\frac{\text{gm}}{\text{cm}^3}$	5.75E-19 $\frac{\text{gm}}{\text{cm}^3}$	1.38E-28 $\frac{\text{gm}}{\text{cm}^3}$	5.75E-29 $\frac{\text{gm}}{\text{cm}^3}$
Man-made Source	1.40E-18 $\frac{\text{gm}}{\text{cm}^3\text{-sec}}$	5.83E-19 $\frac{\text{gm}}{\text{cm}^3\text{-sec}}$	1.40E-28 $\frac{\text{gm}}{\text{cm}^3\text{-sec}}$	4.1667E-2 $\frac{\text{gm}}{\text{cm}^3\text{-sec}}$
Effective Eddy Diffusivity	2.00 $\frac{\text{cm}^2}{\text{sec}}$	8.3335E-1 $\frac{\text{cm}^2}{\text{sec}}$	1.01E-1 $\frac{\text{cm}^2}{\text{sec}}$	4.1667E-2 $\frac{\text{cm}^2}{\text{sec}}$

Means and standard deviations of the output parameters, for each scenario, are summarized in Table 4.2.1.3.

Table 4.2.1.3

Scenario	Flux, Mean $\frac{\text{pCi}}{\text{cm}^2\text{-sec}}$	Flux, Standard Deviation $\frac{\text{pCi}}{\text{cm}^2\text{-sec}}$	Flow, Mean pCi	Flow, Standard Deviation pCi	Concentration, Mean $\frac{\text{gm}}{\text{cm}^3}$	Concentration Standard Deviation $\frac{\text{gm}}{\text{cm}^3}$
One	1.249	1.375	3.02E10	3.32E10	6.26E-15	6.86E-15
Two	1.147	1.183	2.83E10	2.94E10	3.89E-16	4.45E-16
Three	2.51E-5	2.08E-5	884,607	719,494	1.26E-19	1.00E-19
Four	2.37E-5	1.88E-5	841,562	657,836	8.01E-29	7.36E-29
Five	1.249	1.375	3.02E10	3.32E10	6.26E-15	6.85E-15
Six	1.147	1.183	2.83E10	2.94E10	3.89E-16	4.50E-16

4.2.2 Curve-Fitting and Conclusions

For all scenarios, all input and output parameters were standardized, using methods described previously. Least-squares curves were fit, including coefficients for all first-order terms, all first-order interactions, and all quadratic terms. Coefficients of these

fitted curves appear, in tabular form, in Appendix D. Five consequences follow from a study of these tables:

1. The dominant input parameters for determining flux and flow are porosity, half-life, and man-made concentration in Scenarios One, Two, Five, and Six (concentration of man-made radionuclides high). The dominant input parameters for determining flux and flow are porosity, half-life, and background concentrations in Scenarios Three and Four (concentrations of man-made radionuclides low). The dominant input parameter for determining radon concentration, in all scenarios, are the same as the input parameters for determining flux and flow, with the addition of effective eddy diffusivity. Amplitude and frequency of pressure wave are, in all scenarios, of secondary importance for determining output parameters.
2. The coefficients of flux and flow are very similar. This holds because, for each scenario, each is almost a linear transform of the other. This is demonstrated in Figure 4.2.2.1, in which a scatterplot of flux versus flow, for Scenario One (background concentration high, man-made concentration high, effective eddy diffusivity low) appears. Scatterplots for other scenarios are similar.
3. The coefficients for Scenario One (background concentration high, man-made concentration high, effective eddy diffusivity low) and Scenario Five (background concentration low, man-made concentration high, effective eddy diffusivity low) are almost identical. The coefficients for Scenario Two (background concentrations high, man-made concentration high, effective eddy diffusivity high) and Scenario Six (eddy diffusivity high) are almost identical. The matched scenarios differ only in level of background concentration. This indicates that, if the concentration of man-made radionuclides is near its "high" value, the effects of background concentrations will be washed out.

Figure 4.2.2.1 Scatterplot of flow versus flux, Scenario One. Other scenarios produce similar scatterplots.

4. Interaction terms are only large (where "large" is, admittedly, subjective) if the linear terms are large. Hence, one need only worry about interactions of those input parameters identified as "key parameters."
5. Quadratic terms can be used to assess the adequacy of the planar approximation. For the output parameter "radon concentration", in all scenarios, the quadratic term associated with effective eddy diffusivity is, if not "large", perhaps non-negligible. Similarly, for the output parameters "flux" and "flow", the quadratic term associated with porosity, for Scenarios Four and Five are perhaps, non-negligible. These terms are not so large that they invalidate the sensitivity analysis. However, they do show that, for the appropriate scenario, the model is highly nonlinear in these input parameters.

Other regression (L^1 , Tukey's biweight) gave approximately the same coefficients. Hence, least-squares coefficients were used.

Adequacy of approximation was assessed by studying the residuals and by calculation of r^2 , used here as a purely descriptive statistic. Although residual plots indicated regularity not captured by the fitted curves, the absolute values of the residuals tended to be small, compared to the output parameters. Hence, the fitted curves can be viewed as adequate, if imperfect, representations. For the three output parameters and the six scenarios, the values of r^2 ranged from 0.968 to 0.997. This range of values for r^2 indicate that, in all cases, the fitted models explain most of the (non-random) variability.

Classic sensitivity analysis (as appearing in, for example, Iman and Helton, 1988) uses only the first-degree terms, and omits the interaction and quadratic terms. In view of the extreme nonlinearity of the model, retaining at least some of these terms was thought advisable. The decision to include all interactions and quadratic terms was made to avoid any controversy over which ones should be included. Had the analysis

not included any interactions or quadratic terms, the list of input parameters identified as those to which the model is most sensitive would not have changed.

4.2.3 Competing Definitions of Sensitivity

The methods used here are not the only ones in the literature. Gardner et al. (1981) recommended using simple correlation coefficients between inputs and outputs. IAEA has advocated using Spearman's (IAEA, 1989). The use of informal graphical methods (see, for example, Helton et al. 1993) have been proposed. All these methods yielded the same results as the standardized regression method used here, and hence are not described in detail. This suggests that the conclusions reached here depend little on the definition of sensitivity adopted.

5.0 Conclusions

For each scenario, a few input parameters essentially determined the output parameters, with the other variables contributing little. Typically, these input parameters are the only ones whose interaction terms have a non-negligible impact on the output parameter. The key input parameters, in order of importance as indicated by the absolute value of the linear term, are:

<u>Parameters to Which the Model is Sensitive</u>			
Scenario	Flux	Flow	Radon Concentration
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made (4) Effective Eddy Diffusivity
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made (4) Effective Eddy Diffusivity
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Background (3) Half-life	(1) Porosity (2) Background (3) Half-life	(1) Porosity (2) Background (3) Effective Eddy Diffusivity (4) Half-life
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Half-life (3) Background	(1) Porosity (2) Half-life (3) Background	(1) Porosity (2) Background (3) Effective Eddy Diffusivity (4) Half-life
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made (4) Effective Eddy Diffusivity
Background high Man-made high Eddy diffusive low	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made	(1) Porosity (2) Half-life (3) Man-made (4) Effective Eddy Diffusivity

For all scenarios, porosity and half-life are primary drivers for all output parameters, concentration of man-made radionuclides is a crucial input parameter for determining all output parameters, unless the man-made concentration is low and background is high, in which case background concentration takes it place. Effective eddy diffusivity is important in determining radon concentration, but not flux or flow. The physical reasoning behind this counter intuitive result is given in detail in Appendix C. It is analogous to the concept of the rate limiting step in a Bateman cascade chain of radionuclides. The slowest process becomes the rate limiting step with virtually all other radionuclides down the chain being governed by this slowest step. Diffusion of radon gas in the dry soil appears to be the rate limiting transport process with the movement away from the air-soil surface being a much faster process even at the smallest value of D_{out}^1 considered here. Amplitude and frequency of the pressure wave are of relatively little consequence, at least for the output parameters studied here.

APPENDIX A

NOMENCLATURE

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	total cross-sectional area at source region-soil interface	(cm ²)
V_o	total volume of source region	(cm ³)
ϵ_{in}	porosity of water table region	(cm ³ /cm ³)
V_{in}	average flow velocity in the water table region	(cm/sec)
D_{in}^j	average diffusion coefficient of jth radionuclide in the water table region	(cm ² /sec)
ϵ	porosity of soil region (homogeneous)	(cm ³ /cm ³)
V_{air}	average gas flow velocity (seepage velocity) at (x,t) in the soil region	(cm/sec)
V_{out}	average gas flow velocity in the boundary layer region exterior to the soil-atmosphere boundary $X = 0$	(cm/sec)
D_j	average diffusion coefficient of jth radionuclide in soil region	(cm ² /sec)
Δx_{in}	thickness of water table coupling region	(cm)
Δx_{out}	thickness of boundary layer	(cm)
Δx	spacing between nodal points	(cm)
D_{out}^j	effective eddy diffusivity of jth radionuclide in the boundary layer region	(cm ² /sec)
C^j	concentration of jth radionuclide in the soil subregion	(gm/cm ³)
R^j	linear equilibrium retardation coefficient of jth radionuclide in the soil subregion	(dimensionless)

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
SA_j	specific activity of the radionuclide j	(pCi/gm)
M_s	initial mass of radionuclide 1 source material in the source region	(gm)
λ_j	decay coefficient for jth radionuclide	(1/sec)
λ_{irr}^j	rate coefficient for irreversible binding of jth radionuclide	(1/sec)
ρ_p	porous medium particle density	(gm/cm ³)
f_{revj}	fraction of porous medium solids available for reversible binding of jth radionuclide	(dimensionless)
f_{irrj}	fraction of porous medium solids available for irreversible binding of jth radionuclide	(dimensionless)
K_{eqj}	linear equilibrium reversible sorption coefficient for radionuclide j onto the solid particles	($\frac{\text{cm}^3 \text{ air}}{\text{gm particles}}$)
q_{air}	specific discharge of air in the porous medium	(cm ³ /cm ² -sec)
$S_o(x)$	background source rate function constant in time, distributed in space for first radionuclide in the transport and fate region	($\frac{\text{gm radionuclide}}{\text{gm soil-sec}}$)
$S_s(x)$	man-made source rate function, constant constant in time, distributed in space, for first radionuclide in the transport and fate region	($\frac{\text{gm radionuclide}}{\text{cm}^3\text{-sec}}$)
κ_{air}	air permeability	(cm ²)
K_{air}	pneumatic conductivity	(cm/sec)
η_{air}	air viscosity	(dyne-sec/cm ²)

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
ρ_{air}	air density	(gm/cm ³)
g	gravitational acceleration	(cm/sec ²)
ΔP_A	amplitude of barometric pressure change	(dyne/cm ²)
P_{air}	barometric air pressure	(dyne/cm ²)
$\bar{P}_{air}$	average barometric pressure	(dyne/cm ²)
$\bar{\rho}_{air}$	average air density	(gm/cm ³)
M_{air}	molecular mass of air	(gm/mole)
M_j	atomic mass of radionuclide j	(gm/mole)
R_g	gas constant	($\frac{erg}{mole \cdot ^\circ K}$)
T	kelvin temperature	(°k)
D_{air}	pneumatic diffusivity	(cm ² /sec)
N_x	number of equal length subintervals into which $[0, L_x]$ is subdivided	(dimensionless)
Δt	time increment between adjacent time levels	(sec)
$C_{um, n+1}^j$	cumulative mass loss of the jth radionuclide from the surface $x = 0$	(gm or pCi)
$q_{cj, n+1}$	flux of radionuclide j at time t_{n+1} at the atmosphere-soil interface	(gm/cm ² -sec)

APPENDIX B

Four more response variables used in assessing the effect of depth to the water table are included below. The four response variables are : 1) radon-222 surface flux density, pCi/cm²-sec, 2) cumulative radon-222 surface flux (pCi [10⁴cm² area used]), 3) gas pressure at selected horizons (m bar), and 4) induced advective gas velocity at selected horizons (cm/sec).

The data input file C9402400.Dat summarized all the input data for the first depth to water table numerical experiment.

There are five sets of output and data files; corresponding to the depths: 240 m, 120 m, 60 m, 30 m, and 50 m, respectively.

Figure B.1 Effect of depth to the water table from a depth of 240 meters.
Surface flux density verses time in days.

Figure B.1A Effect of depth to the water table from a depth of 240 meters.
Cumulative surface flow through 1m² of surface area verses time.

Figure B.1B Effect of depth to the water table from a depth of 240 meters.
Barometric pressure at selected horizons (Table 4.1.2).

Figure B.1C Effect of depth to the water table from a depth of 240 meters. Induced bulk soil gas velocity verses time at selected horizons (Table 4.1.2).

Run Datafile is (C9402400.DAT) Transport of (Radon gas) chain.

This is the "baseline" run for the sensitivity runs. The idea is a homogeneous column, with a buried source and a very weak background. Use single sine wave for baro.

We'll start with this, then raise the floor (water table) till we start to see an effect. What we want to do is to use a shallow column for the sens tests, but not *too* shallow.

Note -- Comment lines now end with "\$" in column 1
\$

SET OF FLAGS: NFLAG(I): 01 02 03 04 05 06 07 08 09 10
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NLAYER NODES NUCS NOUT
1 271 1 600

Spacing is 'A' for Absolute, 'R' for Relative
'R'

ZOFF, then NM1 spacings for DZZ
0.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

1*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

3*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

231*1.0D2

Spacing for layers: 'A' for absolute, 'R' for relative
'A'

BOTTOM DEPTH FOR N-1 LAYERS; LAST ONE IS DEPTH OF WATER TABLE, NOT LISTED
NDEP; then 'N' for nodes or 'Z' for actual depths
09 'Z'

PRINT AT THESE NODAL POSITIONS

0.000D0 1.000D2 3.000D2 5.500D2 8.000D2

2.500D3 5.000D3 1.000D4 2.400D4

INITIAL TIME: T0

0.0D0

FINAL TIME: TIME

3.456D+6

NUMBER OF PRINT TIMES: NPRT

360

DT

60.0D0

PARTDN EPS KAPPA ONE ROW FOR EACH LAYER

2.6D0 0.37D0 4.00D-8

```

EPSIN, EPSOT
0.37D0, 1.0D0
DSO1 DSO2, DSO3, DSOn
0.10D0
DSP1 DSP2 DSP3 DSPn ONE ROW FOR EACH LAYER
0.022D0
DSOT1, DSOT2, DSOT3, DSOTn
3.60D2
FREV1 FREV2 FREV3 FREVn ONE ROW FOR EACH LAYER
0.10D0
FIRR1 FIRR2 FIRR3 FIRRn ONE ROW FOR EACH LAYER
0.10D0
KEQ1 KEQ2 KEQ3 KEQn ONE ROW FOR EACH LAYER
0.0D0
THALF[0..MNUCS]
4.990D+10, 3.327D+5, 1.800D+2, 1.560D+3, 1.140D+3
AM[0..MNUCS]
226.D0, 222.D0, 218.D0, 214.D0, 214.D0
SA[0..MNUCS]
0.989D12, 1.5377D17, 2.768D20, 3.28D19, 4.41D19
LTIRR1 LTIRR2 LTIRR3 LTIRRn; ONE ROW FOR EACH LAYER
0.0D0
AREA
1.0D+4
COT1, COT2, COT3, COTn
0.0D0
CIN1, CIN2, CIN3, CINn
1.30D-28
DXIN
100.0D0
DXOUT
100.0D0
TAIR, RG, MASSA, GRAV, RHOWAT
2.93D+2, 8.3143D+7, 29.0D0, 9.8D+2, 1.0D0
PAIRO AND NPRES
910.0D0, 1
POC'S FROM 1 TO 15
+0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0
+0.0000D0, -0.0000D0, +0.0000D0, -0.0000D0, 0.0000D0
0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
POS'S FROM 1 TO 15
-5.0000D0, +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0
-0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0, 0.0000D0
0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
PERIODS OF EACH COMPONENT IN SECS.
8.6400D+4, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
Number of Source Regions; 'N' for nodes, 'Z' for depths.
01 'Z'
INITIAL DISTRIBUTION AND SOURCES
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01 3.00D2 8.00D2 1.30D-28, 1.40D-18

```

Figure B.2 Effect of depth to the water table from a depth of 120 meters.
Surface flux density verses time in days.

Figure B.2A Effect of depth to the water table from a depth of 120 meters.
Cumulative surface flow through 1m² of surface area verses time.

Figure B.2B Effect of depth to the water table from a depth of 120 meters.
Barometric pressure at selected horizons (Table 4.1.2).

Figure B.2C Effect of depth to the water table from a depth of 120 meters. Induced bulk soil gas velocity verses time at selected horizons (Table 4.1.2).

Run Datafile is (C9402401.DAT) Transport of (Radon gas) chain.

This is the "baseline" run for the sensitivity runs. The idea is a homogeneous column, with a buried source and a very weak background. Use single sine wave for baro.

We'll start with this, then raise the floor (water table) till we start to see an effect. What we want to do is to use a shallow column for the sens tests, but not *too* shallow.

This run raises the wt from 240 to 120 m . . .

Note -- Comment lines now end with "\$" in column 1

\$

SET OF FLAGS: NFLAG(I): 01 02 03 04 05 06 07 08 09 10
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NLAYER NODES NUCS NOUT
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Spacing is 'A' for Absolute, 'R' for Relative
'R'

ZOFF, then NM1 spacings for DZZ
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1.0D1 2.0D1 6.0D1

1*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

3*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

112*1.0D2

Spacing for layers: 'A' for absolute, 'R' for relative
'A'

BOTTOM DEPTH FOR N-1 LAYERS; LAST ONE IS DEPTH OF WATER TABLE, NOT LISTED
NDEP; then 'N' for nodes or 'Z' for actual depths
09 'Z'

PRINT AT THESE NODAL POSITIONS

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2.500D3 5.000D3 1.000D4 1.200D4

INITIAL TIME: T0

0.0D0

FINAL TIME: TIME

3.456D+6

NUMBER OF PRINT TIMES: NPRT

360

DT

60.0D0

PARTDN EPS KAPPA ONE ROW FOR EACH LAYER
 2.6D0 0.37D0 4.00D-8
 EPSIN, EPSOT
 0.37D0, 1.0D0
 DSO1 DSO2, DSO3, DSO_n
 0.10D0
 DSP1 DSP2 DSP3 DSP_n ONE ROW FOR EACH LAYER
 0.022D0
 DSOT1, DSOT2, DSOT3, DSOT_n
 3.60D2
 FREV1 FREV2 FREV3 FREV_n ONE ROW FOR EACH LAYER
 0.10D0
 FIRR1 FIRR2 FIRR3 FIRR_n ONE ROW FOR EACH LAYER
 0.10D0
 KEQ1 KEQ2 KEQ3 KEQ_n ONE ROW FOR EACH LAYER
 0.0D0
 THALF[0..MNUCS]
 4.990D+10, 3.327D+5, 1.800D+2, 1.560D+3, 1.140D+3
 AM[0..MNUCS]
 226.D0, 222.D0, 218.D0, 214.D0, 214.D0
 SA[0..MNUCS]
 0.989D12, 1.5377D17, 2.768D20, 3.28D19, 4.41D19
 LTIRR1 LTIRR2 LTIRR3 LTIRR_n; ONE ROW FOR EACH LAYER
 0.0D0
 AREA
 1.0D+4
 COT1, COT2, COT3, COT_n
 0.0D0
 CIN1, CIN2, CIN3, CIN_n
 1.30D-28
 DXIN
 100.0D0
 DXOUT
 100.0D0
 TAIR, RG, MASSA, GRAV, RHOWAT
 2.93D+2, 8.3143D+7, 29.0D0, 9.8D+2, 1.0D0
 PAIRO AND NPRES
 910.0D0, 1
 POC'S FROM 1 TO 15
 +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0
 +0.0000D0, -0.0000D0, +0.0000D0, -0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 POS'S FROM 1 TO 15
 -5.0000D0, +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0
 -0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 PERIODS OF EACH COMPONENT IN SECS.
 8.6400D+4, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 Number of Source Regions; 'N' for nodes, 'Z' for depths.
 01 'Z'
 INITIAL DISTRIBUTION AND SOURCES
 00 0.00D0 2.40D4 1.30D-28, 0.00D-00
 01 3.00D2 8.00D2 1.30D-28, 1.40D-18

Figure B.3 Effect of depth to the water table from a depth of 60 meters.
Surface flux density verses time in days.

Figure B.3A Effect of depth to the water table from a depth of 60 meters. Cumulative surface flow through 1 m^2 of surface area versus time.

Figure B.3B Effect of depth to the water table from a depth of 60 meters.
Barometric pressure at selected horizons (Table 4.1.2).

Figure B.3C Effect of depth to the water table from a depth of 120 meters. Induced bulk soil gas velocity versus time at selected horizons (Table 4.1.2).

Run Datafile is (C9402402.DAT) Transport of (Radon gas) chain.

This is the "baseline" run for the sensitivity runs. The idea is a homogeneous column, with a buried source and a very weak background. Use single sine wave for baro.

We'll start with this, then raise the floor (water table) till we start to see an effect. What we want to do is to use a shallow column for the sens tests, but not *too* shallow.

This run raises the wt from 120 to 060 m . . .

Note --- Comment lines now end with "\$" in column 1

\$

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Spacing is 'A' for Absolute, 'R' for Relative
'R'

ZOFF, then NM1 spacings for DZZ
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1.0D1 2.0D1 6.0D1

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4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

3*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

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1.0D1 2.0D1 6.0D1

051*1.0D2

Spacing for layers: 'A' for absolute, 'R' for relative
'A'

BOTTOM DEPTH FOR N-1 LAYERS; LAST ONE IS DEPTH OF WATER TABLE, NOT LISTED
NDEP; then 'N' for nodes or 'Z' for actual depths
07 'Z'

PRINT AT THESE NODAL POSITIONS

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2.500D3 6.000D3

INITIAL TIME: T0

0.0D0

FINAL TIME: TIME

3.456D+6

NUMBER OF PRINT TIMES: NPRT
 360
 DT
 60.0D0
 PARTDN EPS KAPPA ONE ROW FOR EACH LAYER
 2.6D0 0.37D0 4.00D-8
 EPSIN, EPSOT
 0.37D0, 1.0D0
 DSO1 DSO2, DSO3, DSO_n
 0.10D0
 DSP1 DSP2 DSP3 DSP_n ONE ROW FOR EACH LAYER
 0.022D0
 DSOT1, DSOT2, DSOT3, DSOT_n
 3.60D2
 FREV1 FREV2 FREV3 FREV_n ONE ROW FOR EACH LAYER
 0.10D0
 FIRRI FIRRI2 FIRRI3 FIRRI_n ONE ROW FOR EACH LAYER
 0.10D0
 KEQ1 KEQ2 KEQ3 KEQ_n ONE ROW FOR EACH LAYER
 0.0D0
 THALF[0..MNUCS]
 4.990D+10, 3.327D+5, 1.800D+2, 1.560D+3, 1.140D+3
 AM[0..MNUCS]
 226.D0, 222.D0, 218.D0, 214.D0, 214.D0
 SA[0..MNUCS]
 0.989D12, 1.5377D17, 2.768D20, 3.28D19, 4.41D19
 LTIRRI LTIRRI2 LTIRRI3 LTIRRI_n; ONE ROW FOR EACH LAYER
 0.0D0
 AREA
 1.0D+4
 COT1, COT2, COT3, COT_n
 0.0D0
 CIN1, CIN2, CIN3, CIN_n
 1.30D-28
 DXIN
 100.0D0
 DXOUT
 100.0D0
 TAIR, RG, MASSA, GRAV, RHOWAT
 2.93D+2, 8.3143D+7, 29.0D0, 9.8D+2, 1.0D0
 PAIRO AND NPRES
 910.0D0, 1
 POC'S FROM 1 TO 15
 +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0
 +0.0000D0, -0.0000D0, +0.0000D0, -0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 POS'S FROM 1 TO 15
 -5.0000D0, +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0
 -0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 PERIODS OF EACH COMPONENT IN SECS.
 8.6400D+4, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 Number of Source Regions; 'N' for nodes, 'Z' for depths.
 01 'Z'
 INITIAL DISTRIBUTION AND SOURCES
 00 0.00D0 2.40D4 1.30D-28, 0.00D-00
 01 3.00D2 8.00D2 1.30D-28, 1.40D-18

Figure B.4 Effect of depth to the water table from a depth of 30 meters.
Surface flux density verses time in days.

Figure B.4A Effect of depth to the water table from a depth of 30 meters.
Cumulative surface flow through 1m² of surface area verses time.

Figure B.4B Effect of depth to the water table from a depth of 30 meters.
Barometric pressure at selected horizons (Table 4.1.2).

Figure B.4C Effect of depth to the water table from a depth of 30 meters. Induced bulk soil gas velocity verses time at selected horizons (Table 4.1.2).

Run Datafile is (C9402403.DAT) Transport of (Radon gas) chain.

This is the "baseline" run for the sensitivity runs. The idea is a homogeneous column, with a buried source and a very weak background. Use single sine wave for baro.

We'll start with this, then raise the floor (water table) till we start to see an effect. What we want to do is to use a shallow column for the sens tests, but not *too* shallow.

This run raises the wt from 060 to 030 m . . .

Note -- Comment lines now end with "\$" in column 1

\$

SET OF FLAGS: NFLAG(I): 01 02 03 04 05 06 07 08 09 10
0, 0, 0, 0, 0, 0, 0, 0, 0, 0

NLAYER NODES NUCS NOUT
1 061 1 600

Spacing is 'A' for Absolute, 'R' for Relative
'R'

ZOFF, then NM1 spacings for DZZ
0.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

1*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

3*1.0D2

6.0D1 2.0D1 1.0D1
4.0D0 3.0D0 2.0D0 1.0D0

1.0D0 2.0D0 3.0D0 4.0D0
1.0D1 2.0D1 6.0D1

021*1.0D2

Spacing for layers: 'A' for absolute, 'R' for relative
'A'

BOTTOM DEPTH FOR N-1 LAYERS; LAST ONE IS DEPTH OF WATER TABLE, NOT LISTED
NDEP; then 'N' for nodes or 'Z' for actual depths
07 'Z'

PRINT AT THESE NODAL POSITIONS

0.000D0 1.000D2 3.000D2 5.500D2 8.000D2

2.500D3 3.000D3

INITIAL TIME: T0

0.0D0

FINAL TIME: TIME

3.456D+6

NUMBER OF PRINT TIMES: NPRT

360

DT
 60.0D0
 PARTDN EPS KAPPA ONE ROW FOR EACH LAYER
 2.6D0 0.37D0 4.00D-8
 EPSIN, EPSOT
 0.37D0, 1.0D0
 DSO1 DSO2, DSO3, DSO_n
 0.10D0
 DSP1 DSP2 DSP3 DSP_n ONE ROW FOR EACH LAYER
 0.022D0
 DSOT1, DSOT2, DSOT3, DSOT_n
 3.60D2
 FREV1 FREV2 FREV3 FREV_n ONE ROW FOR EACH LAYER
 0.10D0
 FIRRI FIRRI2 FIRRI3 FIRRI_n ONE ROW FOR EACH LAYER
 0.10D0
 KEQ1 KEQ2 KEQ3 KEQ_n ONE ROW FOR EACH LAYER
 0.0D0
 THALF[0..MNUCS]
 4.990D+10, 3.327D+5, 1.800D+2, 1.560D+3, 1.140D+3
 AM[0..MNUCS]
 226.D0, 222.D0, 218.D0, 214.D0, 214.D0
 SA[0..MNUCS]
 0.989D12, 1.5377D17, 2.768D20, 3.28D19, 4.41D19
 LTIRR1 LTIRR2 LTIRR3 LTIRR_n; ONE ROW FOR EACH LAYER
 0.0D0
 AREA
 1.0D+4
 COT1, COT2, COT3, COT_n
 0.0D0
 CIN1, CIN2, CIN3, CIN_n
 1.30D-28
 DXIN
 100.0D0
 DXOUT
 100.0D0
 TAIR, RG, MASSA, GRAV, RHOWAT
 2.93D+2, 8.3143D+7, 29.0D0, 9.8D+2, 1.0D0
 PAIRO AND NPRES
 910.0D0, 1
 POC'S FROM 1 TO 15
 +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0
 +0.0000D0, -0.0000D0, +0.0000D0, -0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 POS'S FROM 1 TO 15
 -5.0000D0, +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0
 -0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 PERIODS OF EACH COMPONENT IN SECS.
 8.6400D+4, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 Number of Source Regions; 'N' for nodes, 'Z' for depths.
 01 'Z'
 INITIAL DISTRIBUTION AND SOURCES
 00 0.00D0 2.40D4 1.30D-28, 0.00D-00
 01 3.00D2 8.00D2 1.30D-28, 1.40D-18

Figure B.5 Effect of depth to the water table from a depth of 50 meters.
Surface flux density verses time in days.

Figure B.5A Effect of depth to the water table from a depth of 50 meters.
Cumulative surface flow through 1m² of surface area verses time.

Figure B.5B Effect of depth to the water table from a depth of 50 meters.
Barometric pressure at selected horizons (Table 4.1.2).

Figure B.5C Effect of depth to the water table from a depth of 50 meters. Induced bulk soil gas velocity verses time at selected horizons (Table 4.1.2).

Run Datafile is (C9402404.DAT) Transport of (Radon gas) chain.

This is the "baseline" run for the sensitivity runs. The idea is a homogeneous column, with a buried source and a very weak background. Use single sine wave for baro.

We'll start with this, then raise the floor (water table) till we start to see an effect. What we want to do is to use a shallow column for the sens tests, but not *too* shallow.

This run raises the wt from 030 to 050 m . . .
That is, 50 m seems to be the choice . . .

Note -- Comment lines now end with "\$" in column 1

\$

SET OF FLAGS: NFLAG(I): 01 02 03 04 05 06 07 08 09 10
0, 0, 0, 0, 0, 0, 0, 0, 0, 0

NLAYER	NODES	NUCS	NOUT
1	081	1	600

Spacing is 'A' for Absolute, 'R' for Relative
'R'

ZOFF, then NM1 spacings for DZZ
0.0D0

1.0D0	2.0D0	3.0D0	4.0D0
	1.0D1	2.0D1	6.0D1

1*1.0D2

6.0D1	2.0D1	1.0D1	
4.0D0	3.0D0	2.0D0	1.0D0

1.0D0	2.0D0	3.0D0	4.0D0
	1.0D1	2.0D1	6.0D1

3*1.0D2

6.0D1	2.0D1	1.0D1	
4.0D0	3.0D0	2.0D0	1.0D0

1.0D0	2.0D0	3.0D0	4.0D0
	1.0D1	2.0D1	6.0D1

041*1.0D2

Spacing for layers: 'A' for absolute, 'R' for relative
'A'

BOTTOM DEPTH FOR N-1 LAYERS; LAST ONE IS DEPTH OF WATER TABLE, NOT LISTED
NDEP; then 'N' for nodes or 'Z' for actual depths

07 'Z'

PRINT AT THESE NODAL POSITIONS

0.000D0 1.000D2 3.000D2 5.500D2 8.000D2

2.500D3 5.000D3

INITIAL TIME: T0

0.0D0

FINAL TIME: TIME

3.456D+6

NUMBER OF PRINT TIMES: NPRT

360

DT
 60.0D0
 PARTDN EPS KAPPA ONE ROW FOR EACH LAYER
 2.6D0 0.37D0 4.00D-8
 EPSIN, EPSOT
 0.37D0, 1.0D0
 DSO1 DSO2, DSO3, DSO_n
 0.10D0
 DSP1 DSP2 DSP3 DSP_n ONE ROW FOR EACH LAYER
 0.022D0
 DSOT1, DSOT2, DSOT3, DSOT_n
 3.60D2
 FREV1 FREV2 FREV3 FREV_n ONE ROW FOR EACH LAYER
 0.10D0
 FIRR1 FIRR2 FIRR3 FIRR_n ONE ROW FOR EACH LAYER
 0.10D0
 KEQ1 KEQ2 KEQ3 KEQ_n ONE ROW FOR EACH LAYER
 0.0D0
 THALF[0..MNUCS]
 4.990D+10, 3.327D+5, 1.800D+2, 1.560D+3, 1.140D+3
 AM[0..MNUCS]
 226.D0, 222.D0, 218.D0, 214.D0, 214.D0
 SA[0..MNUCS]
 0.989D12, 1.5377D17, 2.768D20, 3.28D19, 4.41D19
 LTIRR1 LTIRR2 LTIRR3 LTIRR_n; ONE ROW FOR EACH LAYER
 0.0D0
 AREA
 1.0D+4
 COT1, COT2, COT3, COT_n
 0.0D0
 CIN1, CIN2, CIN3, CIN_n
 1.30D-28
 DXIN
 100.0D0
 DXOUT
 100.0D0
 TAIR, RG, MASSA, GRAV, RHOWAT
 2.93D+2, 8.3143D+7, 29.0D0, 9.8D+2, 1.0D0
 PAIRO AND NPRES
 910.0D0, 1
 POC'S FROM 1 TO 15
 +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0
 +0.0000D0, -0.0000D0, +0.0000D0, -0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 POS'S FROM 1 TO 15
 -5.0000D0, +0.0000D0, +0.0000D0, +0.0000D0, -0.0000D0
 -0.0000D0, +0.0000D0, -0.0000D0, +0.0000D0, 0.0000D0
 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0, 0.0000D0
 PERIODS OF EACH COMPONENT IN SECS.
 8.6400D+4, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0, 0.0000D+0
 Number of Source Regions; 'N' for nodes, 'Z' for depths.
 01 'Z'
 INITIAL DISTRIBUTION AND SOURCES
 00 0.00D0 2.40D4 1.30D-28, 0.00D-00
 01 3.00D2 8.00D2 1.30D-28, 1.40D-18

APPENDIX C

VERIFICATION MODEL

Several special case solutions of the underlying model for CASCADR9 are found below. The highly abridged solution method has been adapted from the more detailed procedures given in Appendix F of Cawlfeld et al. (1994a).

In the absence of any barometric pressure changes at the surface, there is no induced gas advection. Furthermore, suppose that there is no irreversible first order binding present in the soil. Radon-222 is chosen as the model radionuclide of interest, and only radon-222 is considered below. Let the porous medium be homogeneous and very dry. Equation 2.1.1 assumes the form

$$\frac{\partial C^1}{\partial t} + \lambda_1 C^1 = \frac{D^1}{1+R^1} \frac{\partial^2 C^1}{\partial X^2} + \lambda_1 C_{BG} + \frac{1-\epsilon}{\epsilon(1+R^1)} \frac{M_1}{M_0} S_{SX}(x) \quad (C.1)$$

There are no interfacial couplings since there are no contrasts. Let the transport domain be

$$D = \left\{ (x,t) \text{ in } \mathbb{R}^2 \mid t > 0 \text{ and } 0 < x < \infty \right\} \quad (C.2)$$

where D is a semi-infinite space-time cylinder for the linear parabolic equation (C.1).

Assume that the constant in time background concentration C_{BG} is uniform over $\{0, \infty\}$. Let $S_{SX}(x)$ be a constant in time, rectangular in space, man-made source function defined on $\{0, \infty\}$ as

$$S_{SX}(x) = \begin{cases} 0, & x < x_T \\ S_0, & x_T < x < x_B \\ 0, & x > x_B \end{cases} \quad (C.3)$$

The initial condition is assumed to be

$$C^1(x, 0^+) = C_{BG}, \quad 0 < x < \infty \quad (C.4)$$

The boundary conditions are:

$$-\varepsilon D^1 \frac{\partial C_1}{\partial x} \Big|_{x=0^+} = -\frac{D_{out}}{\Delta x_{out}} (C_1(0^-, t) - C_{out}) \quad (C.5)$$

and

$$\lim_{x \rightarrow \infty} \frac{\partial C_1}{\partial x} = 0 \quad \text{or} \quad \lim_{x \rightarrow \infty} C_1 < \infty. \quad (C.6)$$

The solution method used is outlined in Cawlfeld et al. (1994a). Only the barest of details are given below.

First of all, separate the above problem into three problems, one for each of the three regions: $0 < x < x_T$, $x_T < x < x_B$, and $x > x_B$. The first two regions are coupled at $x = x_T$ and the latter two regions are coupled at $x = x_B$. At these "coupling points" we assume continuity in concentration and normal component of the gradient. Thus, for the region $0 < x < x_T$ the following problem exists.

$$\frac{\partial C_1}{\partial t} + \lambda_1 C_1 = \frac{D^1}{1+R^1} \frac{\partial^2 C_1}{\partial x^2} + \lambda_1 C_{BG} \quad (C.7)$$

$$\text{I.C.} \quad C_1(x, 0^+) = C_{BG}, \quad 0 \leq x < x_T \quad (C.8)$$

$$\text{B.C.} \quad \text{i)} \quad -\varepsilon D^1 \frac{\partial C_1}{\partial x} \Big|_{x=0^+} = -\frac{D_{out}}{\Delta x_{out}} (C_1(0^-, t) - C_{out}) \quad (C.9)$$

and

$$\text{ii)} \quad C_1(x_T^-, t) = C_1(x_T^+, t) \quad (C.10)$$

$$-\frac{\partial C_1}{\partial x}(x_T^-, t) = -\frac{\partial C_1}{\partial x}(x_T^+, t). \quad (C.11)$$

For the region $x_T < x < x_B$ the following problem exists.

$$\frac{\partial C_1}{\partial t} + \lambda_1 C_1 = \frac{D^1}{1+R^1} \frac{\partial^2 C_1}{\partial x^2} + \lambda_1 C_{BG} + \frac{1-\varepsilon}{\varepsilon(1+R^1)} \frac{M_1}{M_0} S_0 \quad (C.12)$$

$$\text{I.C.} \quad C_1(x, 0^+) = C_{BG}, \quad x_T \leq x \leq x_B \quad (C.13)$$

$$\text{B.C.} \quad \text{i) } C_1(x_T^-, t) = C_1(x_T^+, t) \quad (C.14)$$

$$- \frac{\partial C_1}{\partial x}(x_T^-, t) = - \frac{\partial C_1}{\partial x}(x_T^+, t) \quad (C.15)$$

and

$$\text{ii) } C_1(x_B^-, t) = C_1(x_B^+, t) \quad (C.16)$$

$$- \frac{\partial C_1}{\partial x}(x_B^-, t) = - \frac{\partial C_1}{\partial x}(x_B^+, t) . \quad (C.17)$$

For the region $x > x_T$ the following problem exist.

$$\frac{\partial C_1}{\partial t} + \lambda_1 C_1 = \frac{D^1}{1+R^1} \frac{\partial^2 C_1}{\partial x^2} + \lambda_1 C_{BG} \quad (C.18)$$

$$\text{I.C.} \quad C_1(x, 0^+) = C_{BG}, \quad x \geq x_T, \quad (C.19)$$

$$\text{B.C.} \quad \text{i) } C_1(x_B^-, t) = C_1(x_B^+, t) \quad (C.20)$$

$$- \frac{\partial C_1}{\partial x}(x_B^-, t) = - \frac{\partial C_1}{\partial x}(x_B^+, t) . \quad (C.21)$$

and

$$\lim_{x \rightarrow \infty} \frac{\partial C_1}{\partial x} = 0 . \quad (C.22)$$

Following Cawfield et al. (1994a), the above system of transport and fate equations are Laplace transformed equations. The resulting second order linear ordinary differential equations are easily solved. In the transformed variable it is found that:

For $0 \leq x \leq x_T$,

$$\hat{C}_1(x,s) = \alpha_0 e^{-x\gamma} + \alpha_1 e^{x\gamma} + \frac{C_{BG}}{s} . \quad (C.23)$$

For $x_T \leq x \leq x_B$,

$$\hat{C}_1(x,s) = \beta_0 e^{-x\gamma} + \beta_1 e^{x\gamma} + \frac{C_{BG}}{s} + \frac{1-\epsilon}{\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} . \quad (C.24)$$

For $x \geq x_B$,

$$\hat{C}_1(x,s) = \delta_0 e^{-x\gamma} + \delta_1 e^{x\gamma} + \frac{C_{BG}}{s} \quad (C.25)$$

In the above three "solutions," γ is defined as

$$\gamma = \sqrt{\frac{(1+R^1)}{D^1}(s+\lambda_1)} , \quad (C.26)$$

and the six "constants" α_0 , α_1 , β_0 , β_1 , δ_0 , and δ_1 are determined by the two boundary conditions and the four coupling conditions.

Application of the Laplace transformed bottom boundary condition to distribution (C.25) immediately leads to the requirement that $\delta_1 \equiv 0$. Application of the Laplace transformed top boundary condition to distribution (C.23) yields

$$\alpha_0 = \left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \alpha_1 - \frac{\frac{D_{out}^1}{\Delta x_{out}} \frac{1}{s} (C_{BG} - C_{out})}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \quad (C.27)$$

Clearly then, back substitution for α_0 in (C.23) obtains

$$\hat{C}_1(x,s) = \left(\left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) e^{-x\gamma} + e^{x\gamma} \right) \alpha_1 - \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{BG} - C_{out}}{s} \right) e^{-x\gamma} + \frac{C_{BG}}{s} . \quad (C.28)$$

Since $\delta_1 \equiv 0$, in the region $x \geq x_B$, the transformed distribution is

$$\hat{C}_1(x,s) = \delta_0 e^{-x\gamma} + \frac{C_{BG}}{s} . \quad (C.29)$$

Application of the remaining four "interior coupling conditions obtains the following system of four equations in four unknowns.

$$\begin{aligned} & \left(\left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) e^{-x_T \gamma} + e^{x_T \gamma} \right) \alpha_1 - \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{BG} - C_{out}}{s} \right) e^{-x_T \gamma} \\ &= \beta_0 e^{-x_T \gamma} + \beta_1 e^{x_T \gamma} + \frac{1-\epsilon}{\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \end{aligned} \quad (C.30)$$

$$\left(- \left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) e^{-x_T \gamma} + e^{x_T \gamma} \right) \alpha_1 + \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{BG} - C_{out}}{s} \right) e^{-x_T \gamma}$$

$$= -\beta_0 e^{-x_T \gamma} + \beta_1 e^{x_T \gamma} \quad (C.31)$$

$$\beta_0 e^{-x_B \gamma} + \beta_1 e^{x_B \gamma} + \frac{1-\epsilon}{\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} = S_1 e^{-x_B \gamma} \quad (C.32)$$

and

$$-\beta_0 e^{-x_B \gamma} + \beta_1 e^{x_B \gamma} = -S_1 e^{-x_B \gamma} \quad (C.33)$$

78 After considerable amount of algebraic manipulation the four unknown "constants" α_1 , β_0 , β_1 , and δ_0 are found to be

$$\alpha_1 = \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} (+ e^{-x_T \gamma} - e^{-x_B \gamma}) \quad (C.34)$$

$$\beta_0 = \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) (e^{-x_T \gamma} - e^{x_B \gamma}) - e^{x_T \gamma} \right) - \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{BG} - C_{out}}{s} \right) e^{-x_T \gamma}$$

$$(C.35)$$

$$\beta_1 = -\frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} e^{-x_B \gamma} \quad (C.36)$$

and

$$\delta_1 = \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) (e^{-x_T \gamma} - e^{x_B \gamma}) - e^{x_T \gamma} \right) - \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{BG} - C_{out}}{s} \right) e^{-x_T \gamma} \\ + \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} e^{x_B \gamma} . \quad (C.37)$$

Back substitution for these "constants" into transformed distribution (C.28) for $0 \leq x < x_T$, (C.29) for $x > x_B$, and (C.24)

for $x_T \leq x \leq x_B$, obtains

for $0 \leq x \leq x_T$,

$$\hat{C}_1(x, s) = \frac{C_{BG}}{s} + \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{out} - C_{BG}}{s} \right) e^{-x \gamma} \\ + \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) (e^{-(x+x_T) \gamma} - e^{-(x+x_B) \gamma}) + (e^{-(x_T-x) \gamma} - e^{-(x_B-x) \gamma}) \right) , \quad (C.38)$$

for $x_T \leq x \leq x_B$,

$$\begin{aligned} \hat{C}_1(x,s) = & \frac{C_{BG}}{s} + \frac{1-\varepsilon}{\varepsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \\ & + \frac{1-\varepsilon}{2\varepsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\left(\frac{\varepsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\varepsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(e^{-(x_T+x)\gamma} - e^{-(x_B+x)\gamma} \right) - \left(e^{-(x-x_T)\gamma} - e^{-(x_B-x)\gamma} \right) \right) \\ & + \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\varepsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{out} - C_{BG}}{s} \right) e^{-(x_T+x)\gamma}, \end{aligned} \quad (C.39)$$

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and for $x > x_B$,

$$\begin{aligned} \hat{C}_1(x,s) = & \frac{C_{BG}}{s} + \frac{1-\varepsilon}{2\varepsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\left(\frac{\varepsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\varepsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(e^{-(x+x_T)\gamma} - e^{-(x+x_B)\gamma} \right) - e^{-(x-x_T)\gamma} + e^{-(x-x_B)\gamma} \right) \\ & + \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\varepsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{out} - C_{BG}}{s} \right) e^{-(x_T+x)\gamma}. \end{aligned}$$

(C.40)

At this point it is convenient to let $D_{out}^1 \rightarrow \infty$ in each of the transformed distributions. Doing this yields the special case of infinitely tight coupling to the atmospheric concentration at the soil-atmosphere interface. In Appendix F of Cawfield et al. (1994a), these limiting form arose following the natural course of their solution procedure. If in each transformed distribution both C_{out} and C_{BG} are to zero and take the above indicated limit as D_{out}^1 tends large, transformed distributions (C.38) - (C.40) agree exactly with distributions (F.37 - F.39) in Cawfield et al. (1994a). It is also possible, in principle, to apply the Laplace inversion theorem to transformed distributions (C.38) - (C.40) as Cawfield et al. (1994a) did, and obtain the analytical representation for each sub region and hence for all x in $[0, \infty)$ over all time $t \geq 0$. However, steady-state (long time) distributions are the only interest here. These long time distributions are easily obtained directly from the residue theorem in complex function theory applied to the simple first order pole at $s = 0$ in each sub distribution. The long time distribution is then defined to be:

for $0 \leq x < x_T$,

$$C^1(x, \infty) = C_{BG} + \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) (C_{out} - C_{BG}) e^{-x_1 \sqrt{\frac{\lambda_1 (1+R^1)}{D^1}}} + \frac{1-\epsilon}{2\epsilon (1+R^1)} \left(\frac{M_1}{M_0} \right) \left(\frac{S_0}{\lambda_1} \right) \left(\left(\frac{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(e^{-\frac{(x+x_T) \sqrt{\lambda_1 (1+R^1)}}{D^1}} - e^{-\frac{(x+x_B) \sqrt{\lambda_1 (1+R^1)}}{D^1}} \right) + e^{-\frac{(x_T-x) \sqrt{\lambda_1 (1+R^1)}}{D^1}} - e^{-\frac{(x_B-x) \sqrt{\lambda_1 (1+R^1)}}{D^1}} \right), \quad (C.41)$$

for $x_T \leq x < x_B$,

$$C^1(x, \infty) = C_{BG} + \left(\frac{\frac{D'_{out}}{\Delta x_{out}}}{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D'_{out}}{\Delta x_{out}}} \right) (C_{out} - C_{BG}) e^{-\frac{(x+x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}}$$

$$+ \frac{1-\varepsilon}{2\varepsilon(1+R^1)} \left(\frac{M_1}{M_0} \right) \left(\frac{S_0}{\lambda_1} \right) \left(2 + \frac{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 - \frac{D'_{out}}{\Delta x_{out}}}{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D'_{out}}{\Delta x_{out}}} \left(e^{-\frac{(x+x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}} - e^{-\frac{(x+x_B) \sqrt{\lambda_1(1+R^1)}}{D^1}} \right) - e^{-\frac{(x-x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}} - e^{-\frac{(x_B-x) \sqrt{\lambda_1(1+R^1)}}{D^1}} \right),$$

(C.42)

and for $x > x_B$

$$C^1(x, \infty) = C_{BG} + \left(\frac{\frac{D'_{out}}{\Delta x_{out}}}{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D'_{out}}{\Delta x_{out}}} \right) (C_{out} - C_{BG}) e^{-\frac{(x+x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}}$$

$$+ \frac{1-\varepsilon}{2\varepsilon(1+R^1)} \left(\frac{M_1}{M_0} \right) \left(\frac{S_0}{\lambda_1} \right) \left(\frac{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 - \frac{D'_{out}}{\Delta x_{out}}}{\varepsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D'_{out}}{\Delta x_{out}}} \left(e^{-\frac{(x+x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}} - e^{-\frac{(x+x_B) \sqrt{\lambda_1(1+R^1)}}{D^1}} \right) - e^{-\frac{(x-x_T) \sqrt{\lambda_1(1+R^1)}}{D^1}} + e^{-\frac{(x_B-x) \sqrt{\lambda_1(1+R^1)}}{D^1}} \right).$$

(C.43)

The transformed surface flux density of radon-222 is defined as

$$\begin{aligned}\hat{F}(s) &= -\epsilon D^1 \frac{\partial \hat{C}_1}{\partial x} \Big|_{x=0} \\ &= +\epsilon D^1 \gamma \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(\frac{C_{out} - C_{BG}}{s} \right)\end{aligned}\quad (C.44)$$

$$-\epsilon D^1 \gamma \frac{1-\epsilon}{2\epsilon(1+R^1)} \frac{M_1}{M_0} \frac{S_0}{s(s+\lambda_1)} \left(\frac{\epsilon D^1 \gamma - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon D^1 \gamma + \frac{D_{out}^1}{\Delta x_{out}}} \left(-e^{-x_T \gamma} + e^{-x_B \gamma} \right) + e^{-x_T \gamma} - e^{-x_B \gamma} \right).$$

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The long time or steady-state surface flux density is immediately written down as

$$\begin{aligned}F(\infty) &= \epsilon \sqrt{D^1(1-R^1)} \lambda_1 \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) (C_{out} - C_{BG}) \\ &\quad - \epsilon \sqrt{\frac{D^1}{\gamma_1(1+R^1)}} \frac{1-\epsilon}{2\epsilon} \left(\frac{M_1}{M_0} \right) S_0 \left(\frac{\epsilon \sqrt{D^1(1+R^1)} \lambda_1 - \frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1(1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(-e^{-x_T \sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} + e^{-x_B \sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} \right) + e^{-x_T \sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} - e^{-x_B \sqrt{\frac{\lambda_1(1+R^1)}{D^1}}}.\end{aligned}\quad (C.46)$$

To the best of our knowledge, these distributions have never before been published.

In almost all real world environmental surveillance data at one and two meters screen height, C_{out} is below the limits of detectability. Therefore, in the following verification analysis set $C_{out} = 0$.

Scenario 1, no man-made source only the background source.

In this case, $S_0 = 0$. Since $C_{out} = 0$, the steady-state surface concentration $C^1(0, \infty)$ is easily seen to be

$$C^1(0, \infty) = C_{BG} \left(1 - \frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \quad (C.47)$$

or

$$C^1(0, \infty) = C_{BG} \frac{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1}{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \quad (C.48)$$

which clearly indicates a positive correlation with ϵ , D^1 , R^1 , and λ_1 and a negative correlation with D_{out}^1 . For the steady-state surface flux density under these conditions

$$F(\infty) = -C_{BG} \epsilon \sqrt{D^1 (1+R^1)} \lambda_1 \left(\frac{\frac{D_{out}^1}{\Delta x_{out}}}{\epsilon \sqrt{D^1 (1+R^1)} \lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \quad (C.49)$$

which indicates (ignoring the overall minus sign, which itself indicates a loss from the surface) a positive correlation with ϵ , D^1 , R^1 , and λ_1 , and D_{out}^1 . It also clearly shows the "saturation" (asymptotic) of the surface so that as D_{out}^1 increases $F(\infty)$ asymptotically approaches a finite limiting form governed by the diffusion process in the soil pores.

Table C.1 summarizes the effect of varying D_{out}^1 on the surface concentration and surface flux density using formulas C.48 and C.49 for the analytic model and the formulas found in CASCADR9. In CASCADR9 the amplitude of the single one day pressure wave at the surface (varying about a mean pressure of 910 mb) has been set to zero thereby causing the induced advection to vanish. In all cases $R^1 = 0$ (no reversible sorption).

Table C.1 Comparison of the numeric and analytic indicators of performance as a function of the effective eddy diffusivity, D_{out}^1 . $\Delta x_{out} = 100.0$ cm, $\epsilon = 0.37$, $D^1 = 0.0225$ cm²/sec, $\lambda_1 = 2.0833 \times 10^{-6}$ 1/sec, $C_{BG} = 0.2$ pCi/cm³. Both indicator variables are 40th day averages in the numerical model.

D_{out}^1 (cm ² /sec)	Surface Flux Density (pCi/cm ² -sec)		Surface Gas-Phase Concentration (pCi/cm ³)	
	Analytic	Numeric	Analytic	Numeric
3.6×10^{-3}	4.9675×10^{-6}	5.3083×10^{-6}	0.13800	0.14745
3.6×10^{-2}	1.3105×10^{-5}	1.4169×10^{-5}	0.03640	0.03936
3.6×10^{-1}	1.5673×10^{-5}	1.7008×10^{-5}	0.00435	0.00472
3.6	1.5986×10^{-5}	1.7356×10^{-5}	0.00044	0.00048
3.6×10^1	1.6018×10^{-5}	1.7391×10^{-5}	4.45×10^{-5}	4.83×10^{-5}
3.6×10^2	1.6021×10^{-5}	1.7395×10^{-5}	4.45×10^{-6}	4.83×10^{-6}
3.6×10^3	1.6021×10^{-5}	1.7395×10^{-5}	4.45×10^{-7}	4.83×10^{-7}

Scenario 2, No background source only the man-made source.

It is assumed that the man-made source consist of depleted uranium (DU), which has decayed through the usual Bateman Chain (Lindstrom et al., 1992a, Appendix B), to the its maximum value for radium-226. This actually takes about 4 million years. However, for the sake of illustration assume the peak value and the LLW pit loading rate discussed in Section 2. Thus, $S_0 = 1.39 \times 10^{-18}$ gm/cm³-sec and is constant in time. $M_1 = 222$ while $M_0 = 226$. Let $x_T = 300$ cm and $x_B = 800$ cm. These numbers are peculiar to the current depth of burial practice for DU in the RWMS in Area 5 of

the NTS. The remaining system parameters are the same as those chosen for scenario 1 above. Hence, with $C_{out} = 0$ and $C_{BG} = 0$ we have

$$C^1(0, \infty) = \frac{1-\varepsilon}{2\varepsilon(1+R^1)} \left(\frac{M_1}{M_0} \right) \left(\frac{S_0}{\lambda_1} \right) \left(\frac{2\varepsilon\sqrt{D^1(1+R^1)}\lambda_1}{\varepsilon\sqrt{D^1(1+R^1)}\lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(e^{-x_T\sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} - e^{-x_B\sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} \right) \quad (C.50)$$

for the long time radon-222 concentration at the surface. Furthermore, using (C.46) yields

$$F(\infty) = -\sqrt{\frac{D^1}{\gamma_1(1+R^1)}} \left(\frac{1-\varepsilon}{2} \right) \left(\frac{M_1}{M_0} \right) S_0 \left(\frac{2\frac{D_{out}^1}{\Delta x_{out}}}{\varepsilon\sqrt{D^1(1+R^1)}\lambda_1 + \frac{D_{out}^1}{\Delta x_{out}}} \right) \left(e^{-x_T\sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} - e^{-x_B\sqrt{\frac{\lambda_1(1+R^1)}{D^1}}} \right) \quad (C.51)$$

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for the long time radon-222 surface flux density.

Table C.2 summarizes the effect of varying D_{out}^1 on the surface concentration and the surface flux density of radon-222 given that only the man-made buried source of radon gas is operating.

Table C.2 Comparison of the numerical and analytical indicators of performance as a function of the effective eddy diffusivity, D_{out}^1 .
 $\Delta x_{out} = 100$ cm, $\varepsilon = 0.37$, $D^1 = 0.0225$ cm²/sec, $\lambda_1 = 2.0833 \times 10^{-6}$ 1/sec, $C_{out} = C_{BG} = 0$, $S_0 = 0.69 \times 10^{-18}$ gm/cm³-sec, SA radon-222 is 1.55377×10^{17} pCi/gm, $M_1 = 222$ gm, $M_0 = 226$ gm, $x_T = 300$ cm, $x_B = 800$ cm. Both indicator variables are 40th day averages in the numerical model.

D_{out}^1 (cm ² /sec)	Surface Flux Density (pCi/cm ² -sec)		Surface Gas-Phase Concentration (pCi/cm ³)	
	Analytic	Numeric	Analytic	Numeric
0.01	0.21165	0.22737	2131.83	2279.71
0.1	0.35292	0.38277	355.48	383.88
1.0	0.37816	0.41085	38.09	41.09
10.0	0.38088	0.41388	3.84	4.14
100.0	0.38116	0.41419	0.38	0.41

The agreement between the analytic and numeric methods is considered very good in view of the fact that one sided approximations to the gradients are used at both the atmosphere-soil interface and at the water table. Nonuniform spacing is also used internally in CASCADR9. The formulas for the flux density are those shown in Lindstrom et al. (1992a) as numbers 3.5.1 and 3.5.2. They are

$$q_c = -\epsilon D^1 \left(\frac{C_{1,n+1}^1 - C_{0,n+1}^1}{\Delta x_1} \right) + \epsilon V_{air 0,n+1} C_{0,n+1}^1 \quad (C.52)$$

if $V_{air 0} \leq$ and

$$q_c = -D_{out}^1 \left(\frac{C_{0,n+1}^1 - C_{out}^1}{\Delta x_{out}} \right) + V_{out} C_{out}^1 \quad (C.53)$$

If $V_{air 0} > 0$.

In the case of no air pumping, such as in Scenarios 1 and 2 above, $V_{air} = 0$ and only the gradient approximation contributes to the surface flux density. Therefore, the use of equations C.52 and C.53 helps us understand what appears to be a counter intuitive flux density effect.

Still operating under the assumptions of Scenario 2, except now another sequence of five runs was made using an amplitude of 5 mb in the one day sinusoidal barometric

pressure wave. CASCADR9 results are shown graphically in Figure C.1 and tabularly in Table C.3. In addition to the 40th day average flux density columns for the cumulative surface flux (Area $A = 10^4 \text{cm}^2$), the 40th day average surface concentration, and the 40th day average first interior nodal point concentration. At first glance intuition would have us believe that under air pumping, by increasing D_{out}^1 the long time average surface flux density should increase. However, the phenomenon shown is clearly understood by referring to the two concentration columns as a function of D_{out}^1 and flux density formulas C.52 and C.53. Under no air pumping $V_{\text{air}} = 0$ and only the gradient term contributes. Study of Table C.3 reveals that as D_{out}^1 increases with both air pumping and no air pumping the gradient in surface concentration increases, $\Delta x_1 = 0.1 \text{ cm}$ in all cases. Hence, this term increases asymptotically as D_{out}^1 increases despite the decreasing C_1^0 and C_1^1 . However, when air pumping is operating, in addition to an increasing gradient there is a large bulk chemical movement due to the product of V_{air} and C_1^0 when D_{out}^1 is small. In fact if D_{out}^1 were to approach zero, a physical impossibility since it can go no lower than the binary diffusion coefficient of radon-222 in still dry air, the gradient would vanish yet formula C.52 indicates considerable radon-222 surface loss due to the air pumping. Note that as D_{out}^1 increases, both the surface and first interior nodal point concentrations decrease rapidly. This does not affect the gradient, however, it does cause the bulk transport contribution to the surface flux density to rapidly decrease. This then provides the explanation for what appears on the surfaces to be counter-intuitive. Namely, the negative correlation of the 40th day average surface flux density under air pumping with increasing D_{out}^1 , but a positive correlation of the 40th day average surface flux density under no air pumping with increasing D_{out}^1 .

Table C.3 40th day average surface flux density, cumulative flux, and surface and first interior nodal point chemical concentration.

No Air Pump:

DSOT	Avg Flux (pCi/cm²-sec)	Cum Flux (pCi)	Avg Con Surf (gm/cm³)	Avg Con node 2 (gm/cm³)
1.000000E-02	0.227316	5.227795E+09	1.478284E-14	1.496036E-14
0.100000	0.382776	9.142739E+09	2.489274E-15	2.788203E-15
1.00000	0.410864	9.879766E+09	2.671821E-16	5.880328E-16
10.0000	0.43881	9.960006E+09	2.691557E-17	3.501363E-16
100.000	0.414187	9.968097E+09	2.693546E-18	3.261532E-16

With Air Pump:

DSOT	Avg Flux	Cum Flux	Avg Con Surf	Avg Con node 2
1.000000E-02	0.550873	1.275514E+10	1.458037E-14	1.475546E-14
0.100000	0.425958	1.025473E+10	2.351414E-15	2.633788E-15
1.00000	0.418460	1.014229E+10	2.666568E-16	5.868767E-16
10.0000	0.418565	1.015220E+10	2.716330E-16	3.533590E-16
100.000	0.418588	1.015349E+10	2.721599E-18	3.295500E-16

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Figure C.1 40-day average flux density (m-222) with and without air pumping.

Figure C.2 40th day average concentration at the surface and first interior nodal points.

APPENDIX COEFFICIENTS OF FIT

For the six scenarios, the coefficients for the first interactions, and all quadratic terms were calculated. The constant terms, which have no impact on sensitivity, are not shown.

Scenario One: Background concentration high, man-made concentration low, eddy diffusivity low

Input Parameter	Coefficient (flux)	Coef (flow)
background concentration	-0.00078	-0.00078
man-made concentration	0.28098	0.28098
half-life	0.42956	0.39356
porosity	0.55708	0.58508
period of pressure wave	-0.04214	-0.04414
amplitude of pressure wave	0.05306	0.06006
effective eddy diffusivity	-0.03184	-0.02784
background x man-made	0.00011	0.00011
background x half-life	-0.00014	-0.00004
background x porosity	0.00001	-0.00001
background x period	-0.00014	-0.00024
background x amplitude	0.00015	0.00017
background x eddy diff.	-0.00009	-0.00009
man-made x half-life	0.25355	0.22938

man-made x porosity	0.31372	0.33341	0.26668
man-made x period	-0.03166	-0.02833	0.00551
man-made x amplitude	0.03240	0.03758	-0.02008
man-made x eddy diff.	-0.02243	-0.01882	-0.15257
half-life x porosity	0.40975	0.40056	0.33577
half-life x period	-0.04379	-0.03484	0.00815
half-life x amplitude	0.03704	0.03984	-0.03787
half-life x eddy diff.	-0.03932	-0.03128	-0.23761
porosity x period	-0.07355	-0.07383	0.02101
porosity x amplitude	0.08563	0.09466	-0.03922
porosity x eddy diff.	-0.05810	-0.04969	-0.22721
period x amplitude	-0.01879	-0.01609	0.01101
period x eddy diff.	0.03237	0.03656	-0.00046
amplitude x eddy diff.	-0.03252	-0.03148	0.01978
background ²	0.00144	0.00090	-0.00090
man-made ²	-0.00069	-0.00125	-0.00313
half-life ²	0.03164	0.00562	0.03182
porosity ²	0.01701	0.03544	-0.03927
period ²	-0.00572	0.00590	-0.01590
amplitude ²	-0.00210	-0.00252	-0.00045
eddy diff ²	0.00652	0.00528	0.06643

Scenario Two: Background concentration high, man-made concentration high, effective eddy diffusivity

Input Parameter	Coefficient (flux)	Coefficient (flow)	Coefficient (radon concentration)
background concentration	0.00125	0.00128	-0.00052
man-made concentration	0.29703	0.29291	0.29337
half-life	0.43983	0.39754	0.43629
porosity	0.56561	0.59499	0.54928
period of pressure wave	0.00303	0.00544	0.00642
amplitude of pressure wave	0.01688	0.02498	0.00455
effective eddy diffusivity	0.00114	0.00140	-0.27415
background x man-made	0.00001	0.000020	0.00020
background x half-life	-0.00035	-0.00021	-0.00000
background x porosity	-0.00017	-0.00020	0.00021
background x period	-0.00001	-0.00006	0.00001
background x amplitude	0.00003	0.00005	0.00000
background x eddy diff.	0.00000	0.00000	0.00003
man-made x half-life	0.26049	0.23292	0.23468
man-made x porosity	0.31760	0.33927	0.28141
man-made x period	-0.00436	0.00325	0.00061
man-made x amplitude	0.01196	0.01793	0.00491

man-made x eddy diff.	-0.00001	0.00015	-0.15893
half-life x porosity	0.38920	0.38455	0.34362
half-life x period	-0.00466	0.00621	0.00189
half-life x amplitude	0.00576	0.01244	-0.00265
half-life x eddy diff.	-0.00090	-0.00056	-0.22981
porosity x period	-0.00375	0.00543	0.00821
porosity x amplitude	0.03727	0.04698	0.01894
porosity x eddy diff.	-0.00027	0.00004	-0.27148
period x amplitude	-0.00106	0.00477	0.00411
period x eddy diff.	0.00155	0.00106	-0.00256
amplitude x eddy diff.	-0.00016	-0.00016	-0.00048
background ²	-0.00210	-0.00217	0.00081
man-made ²	-0.00219	-0.00257	-0.00319
half-life ²	0.03327	0.00466	0.02482
porosity ²	0.00064	0.02166	-0.00855
period ²	-0.01855	-0.00267	-0.01372
amplitude ²	-0.00025	-0.00027	0.00272
eddy diff ²	-0.00209	-0.00221	0.07152

Scenario Three: Background concentration high, man-made concentration low, effective eddy diffusivity low

Input Parameter	Coefficient (flux)	Coefficient (flow)	Coefficient (radon concentration)
background concentration	0.44920	0.45457	0.50592
man-made concentration	-0.00336	-0.00256	-0.00149
half-life	-0.21442	-0.18626	-0.23798
porosity	0.75157	0.75371	0.69445
period of pressure wave	-0.06616	-0.06917	0.02303
amplitude of pressure wave	0.07405	0.07862	-0.06598
effective eddy diffusivity	-0.04036	-0.03529	-0.43590
background x man-made	0.00042	0.00042	0.00045
background x half-life	-0.11096	-0.09594	-0.11085
background x porosity	0.38579	0.38905	0.32270
background x period	-0.03679	-0.03605	0.01271
background x amplitude	0.03975	0.04214	-0.03248
background x eddy diff.	-0.02166	-0.01889	-0.20155
man-made x half-life	-0.00009	-0.00008	-0.00009
man-made x porosity	0.00035	0.00031	0.00041
man-made x period	-0.00022	-0.00027	0.00009
man-made x amplitude	0.00022	0.00022	-0.00011

man-made x eddy diff.	-0.00011	-0.00010	-0.00013
half-life x porosity	-0.18267	-0.15586	-0.14408
half-life x period	0.01801	0.01633	-0.00654
half-life x amplitude	-0.02820	-0.02485	0.00735
half-life x eddy diff.	-0.00387	-0.00565	0.08374
porosity x period	-0.08392	-0.09154	0.04150
porosity x amplitude	0.09996	0.10200	-0.07033
porosity x eddy diff.	-0.05800	-0.05159	-0.25664
period x amplitude	-0.01944	-0.01967	0.01968
period x eddy diff.	0.03808	0.04226	-0.00604
amplitude x eddy diff.	-0.03570	-0.03521	0.03541
background ²	-0.00255	0.00448	-0.00646
man-made ²	0.00591	-0.00388	0.00252
half-life ²	0.04500	0.04016	0.04635
porosity ²	0.09840	0.09887	0.03399
period ²	0.00197	0.01203	-0.01311
amplitude ²	0.00063	-0.00077	0.00374
eddy diff ²	0.01215	0.01007	0.10182

Scenario Four: Background concentration high, man-made concentration low, effective eddy diffusivity high

Input Parameter	Coefficient (flux)	Coefficient (flow)	Coefficient (radon concentration)
background concentration	0.046537	0.47073	0.44059
man-made concentration	-0.00212	-0.00143	-0.00346
half-life	-0.25000	-0.21955	-0.23807
porosity	0.74542	0.74998	0.69806
period of pressure wave	-0.00126	-0.00004	0.00662
amplitude of pressure wave	0.02277	0.02669	0.00406
effective eddy diffusivity	-0.00186	-0.00111	-0.42115
background x man-made	0.00035	0.00035	0.00055
background x half-life	-0.13006	-0.11390	-0.11311
background x porosity	0.38308	0.38755	0.32908
background x period	-0.00294	0.00125	0.00333
background x amplitude	0.01545	0.01745	0.00599
background x eddy diff.	0.00052	0.00052	-0.19997
man-made x half-life	-0.00011	-0.00010	-0.00023
man-made x porosity	0.00023	0.00022	0.00063
man-made x period	-0.00002	-0.00006	0.00002
man-made x amplitude	0.00006	0.00006	0.00000

man-made x eddy diff.	-0.00000	-0.00001	-0.00027
half-life x porosity	-0.21567	-0.18796	-0.18409
half-life x period	0.00316	0.00264	-0.00070
half-life x amplitude	-0.01834	-0.01666	-0.01172
half-life x eddy diff.	-0.00103	-0.00101	0.10954
porosity x period	0.00124	0.00077	0.01604
porosity x amplitude	0.04330	0.04397	0.01905
porosity x eddy diff.	0.00052	0.00048	-0.31558
period x amplitude	-0.00049	0.00144	0.00636
period x eddy diff.	0.00147	0.00136	-0.00586
amplitude x eddy diff.	-0.00035	-0.00048	-0.00038
background ²	-0.00336	-0.00468	-0.00510
man-made ²	0.00368	0.00244	0.00604
half-life ²	0.05128	0.04588	0.04797
porosity ²	0.08444	0.08414	0.05807
period ²	-0.01104	0.00207	-0.00706
amplitude ²	0.00536	0.00412	0.00803
eddy diff ²	0.00366	0.00240	0.10282

Scenario Five: Background concentration low, man-made concentration high, effective eddy diffusivity low

Input Parameter	Coefficient (flux)	Coefficient (flow)	Coefficient (radon concentration)
background concentration	-0.00078	-0.00048	0.00047
man-made concentration	0.28098	0.28045	0.31127
half-life	0.42956	0.39329	0.48539
porosity	0.55706	0.58526	0.53910
period of pressure wave	-0.04214	-0.04417	0.01256
amplitude of pressure wave	0.05306	0.06075	-0.03223
effective eddy diffusivity	-0.03184	-0.02760	-0.26857
background x man-made	0.00011	0.00011	0.00011
background x half-life	-0.00014	-0.00004	-0.00012
background x porosity	0.00001	-0.00005	0.00012
background x period	-0.00014	-0.00020	0.00004
background x amplitude	0.00015	0.00017	-0.00003
background x eddy diff.	-0.00009	-0.00008	0.00008
man-made x half-life	0.25356	0.22938	0.25698
man-made x porosity	0.31372	0.33341	0.26668
man-made x period	-0.03166	-0.02833	0.00541
man-made x amplitude	0.03249	0.03758	-0.02008

man-made x eddy diff.	-0.02243	-0.01882	-0.15257
half-life x porosity	0.40976	0.40057	0.33577
half-life x period	-0.04379	-0.03484	0.00815
half-life x amplitude	0.03704	0.03984	-0.03787
half-life x eddy diff.	-0.03932	-0.03128	-0.23761
porosity x period	-0.07355	-0.07383	0.02101
porosity x amplitude	0.08563	0.09466	-0.03921
porosity x eddy diff.	-0.05810	-0.04969	-0.22721
period x amplitude	-0.01879	-0.01609	0.01101
period x eddy diff.	0.03237	0.03656	-0.00046
amplitude x eddy diff.	-0.03252	-0.03148	0.01978
background ²	0.00144	0.00090	-0.00090
man-made ²	-0.00069	-0.00125	-0.00313
half-life ²	0.03164	0.00562	0.03182
porosity ²	0.01701	0.03544	-0.03927
period ²	-0.00572	0.00590	-0.01593
amplitude ²	-0.00210	-0.00252	-0.000405
eddy diff ²	0.00652	0.00528	0.06643

Scenario Six: Background concentration low, man-made concentration high, effective eddy diffusivity high

Input Parameter	Coefficient (flux)	Coefficient (flow)	Coefficient (radon concentration)
background concentration	0.00124	0.00127	-0.00052
man-made concentration	0.29701	0.29290	0.29337
half-life	0.43984	0.39755	0.43632
porosity	0.56560	0.59498	0.54930
period of pressure wave	0.00303	0.00544	0.00562
amplitude of pressure wave	0.01688	0.02498	0.00455
effective eddy diffusivity	0.00115	0.00140	-0.27415
background x man-made	0.00001	0.00002	0.00020
background x half-life	-0.00035	-0.00021	-0.00000
background x porosity	-0.00018	-0.00021	0.00020
background x period	-0.00001	-0.00006	0.00001
background x amplitude	0.00003	0.00005	0.00000
background x eddy diff.	0.00000	0.00000	0.00003
man-made x half-life	0.26048	0.23291	0.23469
man-made x porosity	0.31759	0.33926	0.28142
man-made x period	-0.00436	0.00325	0.00061
man-made x amplitude	0.01195	0.01793	0.00491

man-made x eddy diff.	-0.00001	0.00015	-0.15893
half-life x porosity	0.38921	0.38455	0.34364
half-life x period	-0.00466	0.00621	0.00189
half-life x amplitude	0.00577	0.01244	-0.00265
half-life x eddy diff.	-0.00090	-0.00056	-0.22983
porosity x period	-0.00375	0.00543	0.00821
porosity x amplitude	0.03727	0.04697	0.01894
porosity x eddy diff.	-0.00027	0.00004	-0.27148
period x amplitude	-0.00106	0.00477	0.00411
period x eddy diff.	0.00155	0.00106	-0.00256
amplitude x eddy diff.	-0.000106	-0.000106	-0.00048
background ²	-0.00210	-0.00217	0.00081
man-made ²	-0.00219	-0.00257	-0.00319
half-life ²	0.03326	0.00446	0.02482
porosity ²	0.00064	0.02166	-0.00856
period ²	-0.01855	-0.00267	-0.01372
amplitude ²	-0.00025	-0.00027	0.00272
eddy diff ²	-0.00209	-0.00221	0.07152

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