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XENON CALCULATIONS FOR I AND E FUEL ELEMENTS

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XENON CALCULATIONS FOR I AND E FUEL ELEMENTS

INTRODUCTION

This document describes in detail the calculations that are basic to the xenon problem. While much of this work appears elsewhere, herein an attempt has been made to bring it together and make it as understandable as possible. The principal purpose of this document is to calculate the equilibrium xenon poisoning for the I and E geometry fuel element and compare the results to the equilibrium xenon poisoning for the solid geometry fuel element. Further objectives are to provide an understandable reference for those with interest in the basic xenon formulation, and to serve as a training guide for new personnel.

SUMMARY OF RESULTS

Figures 1, 2, 3, and 4 compare the variation in equilibrium poison values for both xenon and potential iodine poisoning with power level 1, for both solid and I and E natural uranium in non-flattened piles.

The present xenon tables used by Area Physicists are for reactors with solid fuel elements; however, a simple correction can be made that will allow them to be used for I and E loadings. At 1500 MW the 2004 tube reactor loaded with I and E fuel elements has about 0.4 per cent more xenon and 7.5 per cent more potential iodine poisoning than

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SUMMARY OF RESULTS (Continued)

a reactor loaded with solid fuel elements and similarly for a 3220 tube reactor operating at 3000 MW the corrections are 0.9 per cent for xenon and 8.2 per cent for the iodine.

The complete poison expression for xenon can be written as follows:

$$\text{Poison (mk)} = \frac{AW}{1 + BW} \quad \begin{array}{l} \text{A and B are constants} \\ \text{W is power in MW} \\ \text{Poison = Reactivity loss in } \underline{\text{milli k}} \end{array}$$

J. A. Wheeler (2) reduced the poison formula to this simplified form. The method of calculating the constants is shown later in this report. For an unflattened Hanford Reactor, the values of the constants are:

	<u>2004-tube Reactor</u>	<u>3220-tube Reactor</u>
Solid Loading	A = 0.10663 B = 0.003366	A = 0.05722 B = 0.001822
I and E Loading	A = 0.11454 B = 0.003651	A = 0.06237 B = 0.001976
I and E, E	A = 0.1003 B = 0.002884	A = 0.04695 B = 0.001491

Poison is in terms of milli k; for green natural uranium

$$1 \text{ milli k} = 41.0 \text{ inhours.}$$

J. O. Erkmann's method of calculating the xenon poisoning as a function of radial flattening is based upon tube power and is described in Appendix C. Using this method the saturated xenon poisoning, L_s , is:

$$L_s = \frac{A'W}{1 + B'W}$$

The values for a 2004-tube pile are:

	<u>I and E, Natural U</u>	<u>Solid, Natural U</u>	<u>I and E, Enriched U</u>
A'	$\frac{160.73 \Sigma F_1^3 N}{N \Sigma F_1^2}$	$\frac{149.75 \Sigma F_1^3 N}{N \Sigma F_1^2}$	$\frac{134.18 \Sigma F_1^3 N}{N \Sigma F_1^2}$
B'	$\frac{5.123 \Sigma F_1^3 N}{N \Sigma F_1^2}$	$\frac{4.728 \Sigma F_1^3 N}{N \Sigma F_1^2}$	$\frac{3.860 \Sigma F_1^3 N}{N \Sigma F_1^2}$

For a 3220-tube reactor:

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SUMMARY OF RESULTS (Continued)

	<u>I and E, Natural U</u>	<u>Solid, Natural U</u>	<u>I and E, Enriched U</u>
A'	$\frac{137.25 \Sigma F_1^3}{N \Sigma F_1^2}$	$\frac{128.01 \Sigma F_1^3}{N \Sigma F_1^2}$	$\frac{105.36 \Sigma F_1^3}{N \Sigma F_1^2}$
B'	$\frac{4.374 \Sigma F_1^3}{N \Sigma F_1^2}$	$\frac{4.042 \Sigma F_1^3}{N \Sigma F_1^2}$	$\frac{3.299 \Sigma F_1^3}{N \Sigma F_1^2}$

Potential iodine poisoning may be expressed as

$$\text{Poison (mk)} = A_2 W$$

Where $A_2 = \text{Constant}$

Poison = Reactivity in milli k

W = Power level in megawatts

Values for an unflattened 2004-tube pile are:

	<u>I and E, Natural U</u>	<u>Solid, Natural U</u>	<u>I and E, Enriched U</u>
A_2	0.0827	0.0759	0.0653

For a 3220-tube reactor:

A_2	0.0430	0.0398	0.0461
-------	--------	--------	--------

Using Erkman's iodine poisoning expression:

$$\text{Poison (mk)} = (C) A' p$$

	<u>I and E, Natural U</u>	<u>Solid, Natural U</u>	<u>I and E, Enriched U</u>
Values of (C) for 2004-tube pile	110.79	103.02	94.81
Values of (C) for 3220-tube pile	95.25	88.06	81.32

It can be seen that by using the two previous sets of constants, the xenon poisoning and potential iodine poison differ slightly. Since the methods for obtaining the constants differ, it is not unreasonable that they do not give exactly the same results.

APPLICATION

To make the xenon and iodine corrections for the I and E fuel element simply use Figures 1, 2, 3, and 4 to obtain the corrections depending upon reactor and power level, and apply the correction to the present xenon tables by multiplying each value of xenon and iodine by the appropriate correction factor.

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APPLICATION (Continued)

Example

A 2004-tube reactor operating at 1500 MW and fueled with I and E fuel elements. $W^*/W = .82$.

If the reactor were fueled with solid fuel elements the current xenon tables show that

$$\begin{aligned} \text{Iodine} &= 3434 \\ \text{Xenon} &= 963 \end{aligned}$$

Applying the correction factor for the I and E fuel

$$\begin{aligned} \text{Iodine} &= 3434 (1.075) = 3692 \\ \text{Xenon} &= 963 (1.004) = 967 \end{aligned}$$

The same method can be used for transient values of xenon and iodine.

OUTLINE OF CALCULATIONS

The discussion of the xenon calculations proceeds through the following steps:

1. The local xenon reactivity effect is calculated, considering the neutron absorption in xenon as a change in the thermal utilization of the lattice.
2. The lattice power per unit length (local power) is calculated from the known flux traverse through the reactor.
3. The local reactivity effect in terms of local power is reduced to the expression:

$$\frac{dk}{k} = - \frac{f \gamma_y (1+\delta)}{\gamma} \frac{w}{w+w^*}$$

4. The local reactivity effect is averaged over the pile to obtain the complete pile effect. This is done by weighting the local reactivity effect by the square of the local power or flux. (Appendix B)
5. The xenon poisoning expression is then reduced to:

$$L_s = \frac{AW}{1+BW} \quad L_s = \text{Saturated xenon poison}$$

A & B = Constants

W = Power level in MW

A and B are evaluated for the case of an unflattened pile.

6. Erkman's method of calculating xenon poisoning for flattened piles on the basis of tube factors is described in Appendix C - (3), (9), (10).
7. Potential iodine poisoning constants are calculated for 2004 and 3220-tube piles. The general solution is very similar to the xenon poisoning calculation.

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DISCUSSION

NOTATION

ϕ = Thermal neutron flux

f = Thermal utilization

γ = Neutrons per fission - 2.5

N_{25} = Density of U-235 atoms = 0.03396×10^{22} per cm^3 for Natural Uranium
= 0.0452×10^{22} per cm^3 for E Metal (0.946 per cent enriched)

σ_f = Fission cross section of U-235

σ_x = Capture cross section of Xe-135

Y = Total fission yield Xe-135 and I-135 = 6.4 per cent (5)

m = Atomic density of I-135

x = Atomic density of Xe-135, ground state

x^* = Atomic density of Xe-135, excited isomer

η = Neutrons produced in thermal fission per neutron abs. in U = 1.334 (6)

λ = Decay constant of I-135 = 2.882×10^{-5} Sec^{-1}

λ^* = Decay constant of Xe-135 (excited isomer) = 77.02×10^{-5} Sec^{-1}

λ_x = Decay constant of Xe-135 (ground state) = 2.091×10^{-5} Sec^{-1}

β = Fast fission per thermal fission (7)

K = 3.222×10^{-11} watt sec per fission

The following values of quantities fundamental to the calculations were used:

	<u>2004-Tube Reactor</u>	<u>3220-Tube Reactor</u>
f , solid natural uranium fuel element	.871	.871
f , I and E enriched fuel element	.873	.876
f , I and E natural uranium fuel element	.870	.875
ϕ , solid natural uranium	.065	.065
ϕ , I and E natural uranium	.056	.056
ϕ , I and E enriched uranium	.056	.056

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DISCUSSION (Continued)

NOTATION (Cont'd)

$$\phi_u (\text{Solid}) \propto \left[I_0(\gamma r) + \frac{r^2}{x^4 - r^4} \right]^{(4)}$$

$$\phi_u (I \text{ and } E) \propto \left[CI_0(\gamma r) + DK_0(\gamma r) \right]$$

2004-Tube Reactor

3220-Tube Reactor

$$x^4 = 22.06941$$

Same

$$\gamma = 0.7135$$

r = radius in cm

$$C = 0.4695$$

Same

$$D = 0.3803$$

$$\gamma = 1.04518$$

r = Radius in cm

ϕ_u expressions are fit to experimental traverses through the uranium.

	<u>Old</u>	<u>K</u>
L Effective Pile Length (including augmentation distance)	812 cm	950 cm
R The equivalent bare pile radius of the reactor. It includes an extrapolation distance and is usually 50-60 cm larger than R_0 .	27.62 lattice units	34.63 lattice units
R_0 The radius of a cylinder having the same cross section as the active face of the reactor	25.26 lattice units	32.01 lattice units

XENON FORMATION

The net growth of xenon can be found by using the following differential equations:

For iodine-135

$$\frac{dm}{dt} = Y (1-s) \phi N \sigma_f (1+\delta) - \lambda_m$$

For xenon-135 (excited isomer)

$$\frac{ds}{dt} = (1-r) \lambda_m - \lambda_s$$

For the xenon ground state

$$\frac{dx}{dt} = S Y \phi \Sigma_f (1+\delta) + r \lambda_m + \lambda_s - (\phi \sigma_x + \lambda) x$$

DISCUSSION (Continued)

XENON FORMATION (Cont'd)

The equilibrium or steady state conditions are found by setting the rate of change equal to 0 and solving as follows:

$$\frac{dm}{dt} = 0 = Y(1-s) \phi N \sigma_f (1+\delta) - \lambda m$$

$$m_0 = \frac{Y(1-s) \phi N \sigma_f (1+\delta)}{\lambda}$$

$$\frac{dz}{dt} = 0 \quad \text{then}$$

$$z_0 = \frac{Y(1-r)(1-s) \phi N \sigma_f (1+\delta)}{\lambda}$$

$$\frac{dx}{dt} = 0$$

$$x_0 = \frac{sY \phi \Sigma_f (1+\delta) + r [Y(1-s) \phi \Sigma_f (1+\delta)] + Y(1-r)(1-s) \phi \Sigma_f (1+\delta)}{\phi \sigma_x + \lambda}$$

$$x_0 = \frac{Y \phi \Sigma_f (1+\delta)}{\phi \sigma_x + \lambda}$$

LOCAL REACTIVITY EFFECT

The poisoning due to xenon may be regarded as a change in thermal utilization of the lattice.

$$f = \frac{\text{Thermal neutrons absorbed in fuel}}{\text{Thermal neutrons absorbed}}$$

$$\text{So } \frac{1}{f} = \frac{\Sigma_a \phi V_{\text{cell}}}{\Sigma_a \phi V_{\text{fuel}}}$$

Taking the derivative to determine the change in thermal utilization due to xenon, we have,

$$\frac{d}{d\lambda} \left(\frac{1}{f} \right) = \frac{\Sigma_a \phi V_{\text{cell}}}{\Sigma_a \phi V_{\text{fuel}}} \cdot \frac{d}{d\lambda} \left(\frac{1}{f} \right) = \frac{\int_u x \sigma_x \phi dv}{N_u \sigma_u \int_u \phi dv}$$

Using the substitution $\frac{\lambda}{\gamma} = \frac{N_{25} \sigma_{f25}}{N_{25} \sigma_{a25} + N_{28} \sigma_{a28}} = \frac{N_{25} \sigma_{f25}}{\Sigma_a^u} ; \Sigma_a^u = \frac{N_{25} \sigma_{f25}}{\gamma}$

$$\frac{d}{d\lambda} \left(\frac{1}{f} \right) = \frac{\gamma \int_u x \sigma_x \phi dv}{N_{25} \sigma_{f25} \int_u \phi dv} \quad \frac{d}{d\lambda} \left(\frac{1}{f} \right) = -\frac{f \gamma \int_u x \sigma_x \phi dv}{N_{25} \sigma_{f25} \int_u \phi dv}$$

DISCUSSION (Continued)

LOCAL REACTIVITY EFFECT (Cont'd)

Now substituting the steady state solution of the differential equation for ground state xenon,

$$x = \frac{Y \phi \Sigma_f (1+\delta)}{\phi \sigma_x + X}$$

We now have:

$$\frac{df}{f} = \frac{dk}{k} = \frac{-f \int \frac{Y \phi \Sigma_f (1+\delta) \sigma_x \phi dv}{\phi \sigma_x + X}}{Y N_{25} \sigma_{f25} \int \phi dv}$$

Or

$$\frac{dk}{k} = \frac{-f \int Y (1+\delta) \int \frac{\phi \sigma_x dv}{X + \phi \sigma_x}}{\int \phi dv}$$

For the I and E natural uranium fuel element

$$\phi = a [C I_0(\mu r) + D K_0(\mu r)]^*$$

For simplification let $[C I_0(\mu r) + D K_0(\mu r)] = [S]$

Thus:

$$\frac{dk}{k} = \frac{-f \int Y (1+\delta)}{\int \phi dv} 2\pi a \frac{\int_{r_1}^{r_0} \frac{[S]^2}{\frac{X}{\sigma_x a} + [S]} r dr}{\int_{r_1}^{r_0} a [S] 2\pi r dr}$$

* The expression $\phi = a [C I_0(\mu r) + D K_0(\mu r)]$ is a very close approximation to the experimental flux (4) through the I and E fuel element.

$$r_0 \text{ outside slug radius} = 1.74 \text{ cm}$$

$$r_1 \text{ inside slug radius} = 0.6109 \text{ cm}$$

$$\mu = 1.04518$$

$$C = 0.4695$$

$$D = 0.3803$$

DISCUSSION (Continued)LOCAL REACTIVITY EFFECT (Cont'd)

This expression can be approximated as follows:

In a low flux the term $\frac{X}{\sigma_x a} + [S]$ approaches $\frac{X}{\sigma_x a}$ and

$$\lim_{a \rightarrow 0} \frac{dk}{k} = - \frac{f \gamma Y (1+\delta)}{\gamma} \frac{2\pi a \int_{r_1}^{r_0} \frac{[S]^2 r dr}{\frac{X}{\sigma_x a}}}{\int_{r_1}^{r_0} a [S] 2\pi r dr}$$

$$\lim_{a \rightarrow 0} \frac{dk}{k} = - \frac{f \gamma Y (1+\delta)}{\gamma} \frac{\sigma_x a}{X} \frac{\int_{r_1}^{r_0} [S]^2 r dr}{\int_{r_1}^{r_0} [S] r dr}$$

The solution of
$$\frac{\int_{r_1}^{r_0} [S]^2 r dr}{\int_{r_1}^{r_0} [S] r dr}$$

is in Appendix A and is numerically equal to 0.8420.

$$\lim_{a \rightarrow 0} \frac{dk}{k} = \left(- \frac{f \gamma Y (1+\delta)}{\gamma} \frac{a \sigma_x}{X} \right) (0.8420)$$

In a high flux $\frac{X}{\sigma_x a}$ becomes negligible. Thus:

$$\lim_{a \rightarrow \infty} \frac{dk}{k} = - \frac{f \gamma Y (1+\delta)}{\gamma}$$

$$\lim_{a \rightarrow \infty} \frac{dk}{k} = - \frac{.87 (1.334) (6.4) (1 + 0.056)}{2.5} = 31.375 \text{ mk}$$

From Appendix D, 1 mk = 41.0323 inhours

$$\lim_{a \rightarrow \infty} \frac{dk}{k} = 1287.38 \text{ inhours}$$

DISCUSSION (Continued)

LOCAL REACTIVITY EFFECT (Cont'd)

As in HW-17171, pp 93-94, a good approximation for

$$\frac{\int_a \frac{[S]^2}{\sigma_x a} 2\pi r dr}{\int_a [S] 2\pi r dr} \approx \frac{0.8420}{\frac{X}{a\sigma_x} + .8420}$$

It may be seen that this value is exact for $a = 0$ and $a = \infty$. Making this substitution

$$\frac{dk}{k} = -f \frac{\eta}{\gamma} Y (1+\delta) \frac{0.8420}{\frac{X}{a\sigma_x} + 0.8420}$$

The constant a , which defines the absolute flux in the fuel is related to local power in the following manner:

The power (in watts) per unit length of fuel element is,

$$w = K \sum_f (1+\delta) \int_{r_i}^{r_o} \phi dv = K \sum_f \epsilon \int [S] dv$$

$$\int_{r_i}^{r_o} \phi dv = 6.9693 a \quad (\text{Appendix A})$$

$$w = (3.222 \times 10^{-11})(.03396 \times 10^{22})(550 \times 10^{-24})(1.056)(6.9693 a) \int \phi dv$$

$$w = 0.4429 \times 10^{-10} a$$

$$a = 2.2578 \times 10^{10} w.$$

Then for equilibrium xenon,

$$\frac{dk}{k} = -f \frac{\eta}{\gamma} Y (1+\delta) \frac{0.8420}{\frac{X}{2.2578 \times 10^{10} w \sigma_x} + .8420}$$

rearranging

$$\frac{dk}{k} = -f \frac{\eta Y (1+\delta)}{\gamma} \frac{w}{w+w^*}$$

where

$$w^* = \frac{6.9693 K \sum_f (1+\delta) X}{.8420 \sigma_x} = 1.9998 \frac{\sigma_f}{\sigma_x}$$

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DISCUSSION (Continued)

LOCAL REACTIVITY EFFECT (Cont'd)

The local reactivity change

$$\frac{dk}{k} = - \frac{f \gamma Y (1+\delta)}{\sqrt{\quad}} \frac{w}{w+w^*}$$

must be averaged over the entire reactor. This is done by weighting the change according to the square of the local flux. In a uniform loading the local power is proportional to the local flux. Thus

$$\text{Poison} = \frac{f \gamma Y (1+\delta)}{\sqrt{\quad}} \frac{\int \frac{w^3}{w+w^*} dv}{\int w^2 dv}$$

The integrals are approximated in Appendix B assuming a non-flattened flux distribution over the reactor. In an actual case where tube powers are available Erkman's method (3), (9), (10), Appendix C, is more appropriate.

In Appendix B it is shown that for a 2004-tube pile (natural uranium)

$$\frac{\int \frac{w^3}{w+w^*} dv}{\int w^2 dv} = \frac{0.6137 \frac{w_1}{w^*}}{1 + 0.6137 \frac{w_1}{w^*}}$$

Where $w_1 = \frac{W}{0.4934}$ where W is power level in M.W.

rearranging

$$\frac{\int \frac{w^3}{w+w^*} dv}{\int w^2 dv} = \frac{1.2438 \frac{W}{w^*}}{1 + 1.2438 \frac{W}{w^*}}$$

and the complete poison expression

$$\text{Poison} = \frac{.870 (1.334)(1.056) \gamma}{2.5} \frac{1.2438 \frac{W}{w^*}}{1 + 1.2438 \frac{W}{w^*}}$$

$$\text{Poison (mk)} = 10 (.4902) \gamma \frac{1.2438 \frac{W}{w^*}}{1 + 1.2438 \frac{W}{w^*}}$$

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DISCUSSION (Continued)

LOCAL REACTIVITY EFFECT (Cont'd)

As previously noted,

$$w^* = 1.9998 \frac{\sigma_f}{\sigma_x}$$

The ratio $\frac{\sigma_f}{\sigma_x}$ is taken to be a constant, equal to the average value for the reactor.

This assumption and its significance are discussed thoroughly in HW-23729. The value used is $\frac{\sigma_f}{\sigma_x} = 1.627 \times 10^{-4}$ as reported in HW-23729. Making this substitution the

xenon poison effect may be reduced to the expression

$$\text{Poison (mk)} = \frac{A W}{1 + B W}$$

Where

$$\begin{aligned} A &= \frac{10 (0.4902) 6.4\% 1.2438}{w^*} \\ &= \frac{4.902 \cdot 6.4 \cdot 1.2438}{1.9998 \times 10^2 \cdot 1.627} = 0.11993 \\ B &= \frac{1.2439}{w^*} \\ &= \frac{1.2438}{1.9998 \times 1.627 \times 10^2} = 0.003823 \end{aligned}$$

Thus:

$$\text{Poison (mk)} = \frac{0.11993 W}{1 + 0.003823 W} \quad \checkmark$$

for an unflattened 2004-tube pile loaded with I and E natural uranium fuel elements. The constants A and B for solid natural uranium, I and E natural uranium, and I and E enriched metal for both 2004 and 3220-tube piles reported in the summary.

POTENTIAL IODINE POISONING

The I-135 contained in a reactor is potential poison since it decays to Xe-135. It is assumed in the potential poison calculation that all of the iodine is suddenly converted into xenon.

The solution of this problem is much simpler than the xenon reactivity effect since the amount of iodine is simply flux dependent.

$$m_s = \frac{Y (1-s) \phi N \sigma_f (1+\delta)}{I}$$

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DISCUSSION (Continued)

POTENTIAL IODINE POISONING (Cont'd)

As before in the xenon calculation

$$\frac{dk}{k} = - \frac{f \gamma \int_u \phi m_s \sigma_x dv}{\sqrt{\sigma_f} N_{25} \int_u \phi dv}$$

The solution being

$$\frac{dk}{k} = - \frac{f \gamma \sigma_x (1-s) Y 0.8420 W}{\sqrt{I} 6.9693 K N \sigma_f}$$

for a 2004-tube pile loaded with I and E natural.

Reduced:

$$\frac{dk}{k} = - \frac{f \gamma \sigma_x (1-s) Y 0.8420 \quad 0.6139 W}{\sqrt{I} 6.9693 K N_{25} \sigma_f \quad 0.4934}$$

$$\text{Poison (mk)} = -0.08269 W$$

Erkman uses tube factors in this manner.

Looking back

$$\frac{dk}{k} = - \frac{f \gamma \sigma_x (1-s) Y 0.8420 w}{\sqrt{I} 6.9693 K N \sigma_f}$$

$$w = \frac{\int_0^{\pi/2} \int_0^R w_1^3 \sin^3 \frac{\pi x}{L} z^3(r) 2\pi r dr dx}{\int_0^{\pi/2} \int_0^R w_1^2 \sin^2 \frac{\pi x}{L} z^2(r) 2\pi r dr dx}$$

Integrating over x then converting to tube power

$$w = 1.667 \times 10^3 \text{ AP for 2004-tube pile}$$

$$w = 1.4249 \times 10^3 \text{ AP for 3220-tube pile}$$

$$\begin{aligned} \frac{dk}{k} &= - \frac{(0.87)(1.334)(6146)(0.95)(0.064)(0.8420)(1.667 \times 10^3) \text{ AP}}{(2.5)(2.882 \times 10^{-5})(6.9693)(3.222 \times 10^{-11})(0.03396 \times 10^{22})} \\ &= \text{poison (mk)} = -110.79 \text{ AP} \end{aligned}$$

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DISCUSSION (Continued)

XENON TRANSIENT CALCULATIONS

Xenon transient problems are explained very thoroughly in Volume I, "Pile Physics Manual of Standard Practices."

The calculations I have made might aid an area physicist who has xenon tables for a pile loaded with solid fuel elements, but must make predictions for one loaded with I and E fuel elements.

Calculation of Xenon and Iodine for Mixed Fuel Loadings

Since the constants in the xenon and iodine equations differ among fuel element types, the iodine and xenon concentrations cannot be calculated directly for mixed fuel loadings. However, the concentration for each fuel type can be calculated separately and the total poison or potential poison contribution is the sum of the parts.

The total power output of all tubes of fuel type m may be written

$$w_m = \sum P_i$$

where the P_i are the individual tube powers and the summation is over all tubes of type m. Then the power of a full pile loading of type m fuel with the existing flux distribution would be:

$$\frac{w_m \sum_{i=1}^n F_i}{\sum_m F_i} = \frac{\sum P_i \sum_{i=1}^n F_i}{\sum_m F_i}$$

Since F_i is proportional to P_i for a given fuel type.

$$\frac{\sum_m P_i}{\sum_m F_i} = \left(\frac{P}{F} \right)_m$$

Thus the saturated xenon strength of a full pile loading of fuel m would be:

$$\text{Full Pile Xenon}_{(m)} = \frac{A'_m \left(\frac{P}{F} \right)_m \sum_{i=1}^n F_i}{1 + B'_m \left(\frac{P}{F} \right)_m \sum_{i=1}^n F_i}$$

The contribution of the partial load is the full pile contribution times the fraction of the integrated flux squared in the region occupied by fuel m; or

$$L_{S(m)} = \left[\frac{A'_m \left(\frac{P}{F} \right)_m \sum_{i=1}^n F_i}{1 + B'_m \left(\frac{P}{F} \right)_m \sum_{i=1}^n F_i} \right] \left[\frac{\sum F_i^2}{\sum_{i=1}^n F_i^2} \right]$$

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DISCUSSION (Continued)

XENON TRANSIENT CALCULATIONS (Cont'd)

The total xenon strength is the sum of the contributions of each fuel type, or

$$L_s = \sum_{\text{All } m} L_s(m)$$

The potential iodine poisoning may be found similarly by making the same substitution for full pile power and taking the flux-squared fraction for each fuel type.

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FIGURE 1

POWER M.W.

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FIGURE II

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FIGURE III

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milli h

120
110
100
90
80
70
60
50
40
30
20
10

Power M.W.

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APPENDIX A

Solution of $\frac{\int \phi^2 dv}{\int \phi dv}$

Solving first the denominator integral, it is found that the exact solution of $\int \phi dv$ is:

$$\int \phi dv = \int_{R_1}^{R_0} a [C I_0(\gamma r) + D K_0(\gamma r)] 2\pi r dr \quad \begin{matrix} x = \gamma r \\ dx = \gamma dr \end{matrix}$$

$$= \frac{2\pi a}{\gamma^2} \int_{R_1 \gamma}^{R_0 \gamma} [C I_0(x) + D K_0(x)] x dx$$

Integrating by parts

$$\frac{2\pi a}{\gamma^2} \left\{ \int_{R_1 \gamma}^{R_0 \gamma} C I_0(x) x dx + \int_{R_1 \gamma}^{R_0 \gamma} D K_0(x) x dx \right\}$$

$$\int_{R_1 \gamma}^{R_0 \gamma} C I_0(x) x dx = C [x I_1(x)]_{R_1 \gamma}^{R_0 \gamma} = 1.0444$$

$$\int_{R_1 \gamma}^{R_0 \gamma} D K_0(x) x dx = D [x K_1(x)]_{R_1 \gamma}^{R_0 \gamma} = 0.1673$$

This is not correct

$$\frac{2\pi a}{\gamma^2} \{1.2117\} = 6.9693 a \quad \leftarrow \text{not equal}$$

$$\frac{6.9693}{1.2117} = 5.7517$$

$$\frac{5.7517}{2.17} = 2.6505 \neq \frac{1}{2.17}$$

Similar integration for solid natural and enriched I and E fuel elements

6.9693 a for I and E Natural Uranium

11.5654 a for Solid Natural Uranium

6.772 a for I and E Enriched Uranium

Using Weddle's rule an approximation is made for $\int \phi^2 dv$ where $\phi = a [A I_0(\gamma r) + B K_0(\gamma r)]$.

$$6.9693 = 2\pi \cdot 1.1072$$

See next pg.

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APPENDIX A (Continued)

The following example is for I and E natural uranium fuel element:

	<u>r</u>	<u>H</u>	<u>C, I₀(Mr)</u>	<u>D K₀(Mr)</u>	<u>[R]</u>	<u>C</u>	<u>[S] rc</u>	<u>[S]² rc</u>
1.	.6109	1.04518	.5186	.2774	.7960	1	.4863	.3749
2.	.7050	"	.5354	.2370	.7724	5	2.7225	2.1030
3.	.7991	"	.5550	.2038	.7588	1	.6064	.4601
4.	.8932	"	.5775	.1762	.7537	6	4.0392	3.0444
5.	.9873	"	.6030	.1530	.7560	1	.7464	.5642
6.	1.0814	"	.6319	.1333	.7652	5	4.1375	3.1660
7.	1.1755	"	.6641	.1165	.7806	2	1.8352	1.4326
8.	1.2695	"	.7000	.1021	.8021	5	5.0915	4.0835
9.	1.3636	"	.7399	.0896	.8295	1	1.1311	.9383
10.	1.4577	"	.7841	.0789	.8630	6	7.5480	6.5136
11.	1.5518	"	.8329	.0695	.9024	1	1.4003	1.2637
12.	1.6495	"	.8866	.0613	.9479	5	7.8005	7.3945
13.	1.74	"	.9457	.0542	.999	1	1.7398	1.7396

$$\Sigma, (S)^2 rc = 39.2847$$

$$\int_{R_1}^{R_0} (S)^2 r dr = \frac{.09409}{10} (\text{increment in } r) \Sigma_1 (S)^2 rc$$

$$= \frac{3}{10} .09409 \cdot 39.2897 = .9337$$

$$\frac{\int \phi^2 dv}{\int \phi dv} = \frac{\int 2\pi a^2 \int [C I_0(Mr) + D K_0(Mr)]^2 r dr}{\int 2\pi a \int [C I_0(Mr) + D K_0(Mr)] r dr} = \frac{.9337}{1.1092} a$$

must be $\frac{1}{2} = .8420 a$

for Solid Slug 1.2978 a

APPENDIX B

INTEGRATION OF EFFECT OVER REACTOR

To find the total effect upon a pile, the change in reactivity must be averaged over the entire pile. This is done by weighting the reactivity change according to the square of the local power or flux.

$$\frac{dk}{k} = - \frac{f \gamma (1+\delta)}{\gamma} \frac{\int \frac{w^3 dv}{w + w^*}}{\int w^2 dv}$$

The integral

$$\int \frac{w^3 dv}{w + w^*}$$

can be evaluated over an equivalent bare reactor whose dimensions are those of the active zone plus the augmentation distance due to the reflector savings.

$$w = w_1 \sin \frac{\pi x}{L} \zeta(r)$$

w_1 = power of the central cm slab of center tube in watts

x = variable coordinate parallel to the tube

L = effective pile length (includes augmentation length)

$\zeta(r)$ = function giving power distribution in a radial direction, and it will depend upon flattening and non-flattened distribution.

$$\zeta(r) = J_0 \left(2.405 \frac{r}{R} \right)$$

Where r is the radial distance from the center of the pile
 R is the radius of the pile.

$$\begin{aligned} \int \frac{w^3 dv}{w + w^*} &= \int \frac{\left[w_1 \sin \left(\frac{\pi x}{L} \right) \zeta(r) \right]^3 dv}{\left[w_1 \sin \left(\frac{\pi x}{L} \right) \zeta(r) \right] + w^*} \\ &= 2 \int_0^{\pi/2} \int_0^R \frac{w_1^3 \sin^3 \left(\frac{\pi x}{L} \right) \zeta^3(r)}{w_1 \sin \left(\frac{\pi x}{L} \right) \zeta(r) + w^*} 2\pi r dr dx \end{aligned}$$

Substituting $y = \frac{\pi x}{L}$

$$= 2w_1 \int_0^{\pi/2} \int_0^R \frac{\sin^3 y \zeta^3(r) 2\pi r dr}{w_1 \sin y \zeta(r) + w^*} \frac{dy L}{\pi}$$

$y = \frac{\pi x}{L}$
 $dy = \frac{\pi}{L} dx$

APPENDIX B (Continued)

Note that the integrals are taken over the entire reactor volume including augmentation distances. This is necessary since the reflector causes the pile size to be effectively larger than it really is.

$$\begin{aligned} \int \frac{w^3 dy}{w + w^*} &= 2 w_1^2 \int_0^{\pi/2} \int_0^R \frac{\sin^3 y z^2(r) 2 L r dr dy}{\sin y + \frac{w^*}{w_1 z(r)}} \\ &= 4 w_1^2 L \int_0^R z^2(r) r \int_0^{\pi/2} \frac{\sin^3 y dy dr}{\sin y + \frac{w^*}{w_1 z(r)}} \end{aligned}$$

The integral

$$\int_0^{\pi/2} \frac{\sin^3 y dy dr}{\sin y + \frac{w^*}{w_1 z(r)}}$$

is approximated in HW-23729 like this.

Treating $\frac{w^*}{w_1 z(r)}$ as a parameter we see that for large values of the parameter

$$\text{when } \frac{w^*}{w_1 z(r)} \rightarrow \infty \quad \int_0^{\pi/2} \frac{\sin^3 y dy}{\sin y + \frac{w^*}{w_1 z(r)}} = \frac{w_1 z(r)}{w^*} \int_0^{\pi/2} \sin^3 y dy = \frac{2}{3} \left[\frac{w_1 z(r)}{w^*} \right]$$

$$\text{and when } \frac{w^*}{w_1 z(r)} \rightarrow 0 \quad \int_0^{\pi/2} \frac{\sin^3 y dy}{\sin y + \frac{w^*}{w_1 z(r)}} = \int_0^{\pi/2} \sin^2 y dy = \frac{\pi}{4}$$

The limiting values suggest the approximation

$$\int_0^{\pi/2} \frac{\sin^3 y dy}{\sin y + \frac{w^*}{w_1 z(r)}} \approx \frac{\pi/4}{1 + \frac{3\pi w^*}{8w_1 z(r)}}$$

and as shown in HW-23729, the approximation is quite good.

APPENDIX B (Continued)

Substituting the approximation

$$\begin{aligned} \int \frac{w^3 dv}{w + w^*} &= 4 w_1^2 L \int_0^R \frac{\frac{\pi r}{4} z^2(r)}{1 + \frac{3\pi w^*}{8w_1} z(r)} dr \\ &= 4w_1^2 L \pi/4 \int_0^R \frac{r z^3(r)}{z(r) + \frac{3\pi w^*}{8w_1}} dr \end{aligned}$$

The remaining integral can be approximated much the same as the previous one, letting $\frac{3\pi w^*}{8w_1}$ be a parameter.

As

$\frac{3\pi w^*}{8w_1} \rightarrow \infty$ then

$$\int_0^R \frac{z^3(r) r dr}{z(r) + \frac{3\pi w^*}{8w_1}} = \frac{8w_1}{3\pi w^*} \int_0^R z^3(r) r dr$$

and as

$$\frac{3\pi w^*}{8w_1} \rightarrow 0 \quad \int_0^R \frac{z^3(r) r dr}{z(r) + \frac{3\pi w^*}{8w_1}} = \int_0^R z^2(r) r dr$$

which leads to the approximation

$$\int \frac{w^3 dv}{w + w^*} \approx \frac{w_1^2 L \pi \int_0^R z^2(r) r dr}{1 + \frac{3\pi w^*}{8w_1} \frac{\int_0^R z^2(r) r dr}{\int_0^R z^3(r) r dr}}$$

Now dividing by $\int w^2 dv$

$$\begin{aligned} \frac{\int \frac{w^3 dv}{w + w^*}}{\int w^2 dv} &\approx \frac{\frac{w_1^2 L \pi \int_0^R z^2(r) r dr}{1 + \frac{3\pi w^*}{8w_1} \frac{\int_0^R z^2(r) r dr}{\int_0^R z^3(r) r dr}}}{\frac{w_1^2 L \pi \int_0^R z^2(r) r dr}{1 + \frac{3\pi w^*}{8w_1} \frac{\int_0^R z^2(r) r dr}{\int_0^R z^3(r) r dr}}} \\ &= \frac{1}{1 + \frac{3\pi w^*}{8w_1} \frac{\int_0^R z^2(r) r dr}{\int_0^R z^3(r) r dr}} \end{aligned}$$

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APPENDIX B (Continued)

Since $\frac{\int z^3(r) r dr}{\int z^2(r) r dr} = 0.7232$

$$\frac{\int \frac{w^3 dv}{w + w^*}}{\int w^2 dv} = \frac{1}{1 + \frac{3\pi w^* 1.3832}{8w_1}} = \frac{0.6137 \frac{w_1}{w^*}}{1 + 0.6137 \frac{w_1}{w^*}}$$

TOTAL PILE POWER

The pile power W (in megawatts) is given by

$$W = \int w dv$$

$$w = w_1 \sin \frac{\pi x}{L} Z(r)$$

and $Z(r) = J_0 \left[\frac{2.405 r}{R} \right]$ for an unflattened pile.

Using the assigned flux distribution over the reactor the integral is solved:

$$W = w_1 4\pi \int_{\frac{L-L_1}{2}}^{L/2} \sin \frac{\pi x}{L} dx \int_0^{R_0} z(r) r dr$$

$$\int_{\frac{L-L_1}{2}}^{L/2} \sin \frac{\pi x}{L} dx = \int_{10^\circ}^{90^\circ} \sin y \frac{dy L}{\pi} = \frac{.9848}{\pi}$$

$$\int_0^{R_0} z(r) r dr = \int_0^{(25.27)(.0891)} J_0(x) \frac{x}{\mathcal{M}} \frac{dx}{\mathcal{M}} = \frac{1}{\mathcal{M}^2} \int_0^{2.2003} x J_0(x) dx$$

$$= \frac{1}{.00758} [2.2003 (0.55592)]$$

$$W = w_1 4 (812) (0.9848) \frac{1}{.00758} [2.2003 (0.55592)]$$

$$W = 0.5166 w_1 \text{ M.W. for 2004-tube pile.}$$

$$W = 0.9539 w_1 \text{ M.W. for 3220-tube pile.}$$

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APPENDIX C

ERKMAN METHOD

Appendix B is followed until

$$\frac{\int \frac{w^3 dv}{w + w^*}}{\int w^2 dv} = 1 + \frac{3\pi}{8} \frac{w^*}{w_1} \frac{\int z^2 r dr}{\int z^3 r dr}$$

Then the total reactivity change is

$$\frac{dk}{k} = -f \frac{\gamma Y}{\gamma} (1 + \delta) \frac{1}{1 + \frac{3\pi}{8} \frac{w^*}{w_1} \frac{\int z^2 r dr}{\int z^3 r dr}}$$

It is convenient to use central tube power, where a cosine distribution along the length of the tube is assumed.

p = Central tube power in M.W.

$$p = 2 (10^{-6}) \int_{\frac{L-L_0}{2}}^{L/2} w_1 \sin \frac{\pi x}{L} dx$$

$$= 2 (10^{-6}) \frac{L}{\pi} w_1 \int_{9.53^\circ}^{90^\circ} \sin y dy$$

$$= 2 (10^{-6}) w_1 \frac{L}{\pi} .9863 \text{ M.W.}$$

$$w_1 = \frac{p \pi}{1.9726 L \times 10^{-6}} \text{ M.W.}$$

Now

$$L_B = - \frac{f \gamma Y (1 + \delta)}{\gamma} \left[\frac{1}{1 + \frac{3\pi \cdot 1.9726 L \times 10^{-6} w^*}{p \pi} \frac{\int z^2 r dr}{\int z^3 r dr}} \right]$$

When

$$A = \frac{\int z^3 r dr}{\int z^2 r dr}$$

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APPENDIX C (Continued)

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$$L_s = \frac{.87 (1.334)(.064)(1.056)}{2.5}$$

$$\frac{A_p}{w^* 1.9726 L \frac{3}{8} + A_p}$$

$$L_s = \frac{31.375 A_p}{A_p + .7397 w^* L \times 10^{-6}}$$

Evaluation of A is accomplished by using tube fractions defined as

W = Total power

$$F_i = p_i \frac{N}{W}$$

N = Number of tubes in pile

p_i = Ratio of power generations in i^{th} tube to average tube power.

The quantities $\sum_{i=1}^N F_i$, $\sum_{i=1}^N F_i^2$, and $\sum_{i=1}^N F_i^3$ are used often and are calculated by an IBM machine.

A_p is defined thus

$$A_p = \frac{w}{N} \frac{\sum F_i^3}{\sum F_i^2} = \frac{\int p^3 z^3 r dr}{\int p^2 z^2 r dr} = \frac{\sum p_i^3}{\sum p_i^2}$$

Then as before

$$L_s = \frac{31.375 A_p}{A_p + 0.1952}$$

when as before w^* in an I and E fuel element is 325.37 watts.

~~APPENDIX C~~
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Using excess reactivity expression from HW-53441-RD

$$\rho = \frac{\beta}{\lambda + \tau} + \frac{\tau}{\lambda + \tau} \sum_j \sum_i \frac{\beta_{ij} t_{ij}}{\tau + t_{ij}}$$

ρ = Excess reactivity in units of k

τ = Stable pile period

β_{ij} = Fraction of the total fission neutrons liberated which are in the i^{th} group produced by the j^{th} fissionable isotope.

t_{ij} = Mean life of delayed neutron precursors of the i^{th} group produced by the j^{th} fissionable isotope.

A period of 3600 seconds has $\rho = .02437$ mk, which is equal to one inhour, for a pile fueled with green natural uranium.

Then 1 mk = 41.0 inhours.

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