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**THEORETICAL STUDIES OF CAPILLARITY,
PHASE CHANGE AND RELOCATION PHENOMENA
ENCOUNTERED IN A SRS ASSEMBLY MELTDOWN (U)**

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ABSTRACT

Information available in the literature on the retention and sliding of liquid drops on solid surfaces is reviewed and interpreted in terms of the behavior of liquefied fuel during a decay-heat dominated meltdown event in a Savannah River Plant (SRP) reactor. The information strongly suggests that surface tension effects dominate the fuel drainage regime and that the potential for fuel/target contact is high. The two relevant problems analyzed are: the melting of eutectic forming target alloy in contact with a fuel deposit and the sliding and freezing of target- or fuel-melt drops on the vertical assembly structure.

Note: The Savannah River Site is referred to as the Savannah River Plant (its former name) throughout this report.

1.0 INTRODUCTION

In a previous report [1] it was emphasized that process assembly meltdown behavior following a loss of cooling is strongly dependent on whether or not surface tension effects dominate the fuel-melt draining process. The liquefied fuel may appear in the form of a draining film if the underlying solid fuel-ring surface is well wetted by its molten phase. On the other hand, if the fuel melt does not wet the surface of the fuel ring, or wets only imperfectly, the fuel may appear as a collection of stationary drops that do not spread. In fact the drops may reach a size that is sufficient to result in fuel contact with the adjacent target ring.

Since our previous work [1], several references have been found that clearly demonstrate a high propensity for fuel on horizontal surfaces to ball-up into drops of considerable size relative to the dimensions of the channels between fuel and target tubes. While little is yet known about the sizes of fuel drops on the vertical surfaces of interest, rational size estimates are presented here on the basis of the known behavior of non-metallic liquid drops on inclined surfaces. These estimates suggest that fuel drop/target contacting is likely to occur during a decay-heat driven meltdown accident. While much remains to be learned about fuel melt behavior in the laboratory, we feel that the gross features are now known well enough to justify the formulation of theoretical models which will shed light on the fuel assembly meltdown process. In this spirit we have initiated studies on the melting of eutectic forming target alloy material in contact with fuel and on the sliding and freezing of molten fuel and target drops on vertical assembly structure.

2.0 MELT-DROP STATICS AND DYNAMICS ON VERTICAL SURFACES

Some of the subject matter presented here was already covered in the previous report [1]; however, it is worth repeating this material for those readers who do not have immediate access to this reference.

Historical y, core melt progression modeling has been based on the assumption that the melt material spreads completely over the relatively cold, intact fuel surface in the form of a film. The leading edge of the draining melt may make an angle with the solid. This so-called "contact angle" is measured in the liquid. If the contact angle is zero the solid is said to be completely wet and the film flow regime prevails. Frequently the contact angle is finite and the solid is incompletely wet. A finite contact angle may result in a melt drainage regime that is best described as a rivulet or droplet-type flow. An individual drop appears to stick somewhat in its downward path and the lower edge usually has a larger angle of contact (advancing contact angle) than the upper edge of the drop (receding contact angle). The theoretical treatment of the droplet flow regime is complicated by two factors: first, that the contact angles of a stationary droplet depend on the structure and composition of both the melt and the underlying surface; second, that the contact angles are rarely definite when the droplet is moving, but may have any values between two extremes, according to the speed with which the drop advances over the surface. Fortunately, the equilibrium contact angle of molten aluminum on its oxide has been measured. With this information, together with available observations on the adhesion of non-metallic liquids to inclined solid surfaces, we can estimate the static volume of fuel drops on vertical surfaces of solid-aluminum-clad fuel.

Droplets on a non-horizontal surface will not drain unless their mass exceeds a certain critical, minimum (or "static") configuration that is a strong function of the contact angles (advancing and receding). The ability of water drops to stick to mirrors and windows is a common phenomenon which

can be observed in steamy bathrooms and on window panes during a rain. Thus molten fuel-material droplets may initially remain in place on the fuel tube surface, without experiencing any relocation until they agglomerate with nearby fuel drops or are captured by the advancing fuel-tube melt zone. The droplets will travel downward only until their mass falls below the static value. On the other hand, many of the fuel drops may not relocate over the surface of the fuel tube if their critical size is large enough to bridge the channel between the fuel and target rings. Rapid freezing of fuel drops on the target material will render the drops immobile until the underlying target fails by melting and/or mechanical swelling.

Dussan and Chow [2] derived approximate solutions for the volume V_s of the largest static drop that can "stick" to an inclined solid surface and the area A_s of the solid wetted by the static drop. These are:

$$V_s \approx 2\pi\theta_A \left(\frac{2\sigma \epsilon \theta_A}{\rho g \sin\gamma} \right)^{3/2} \quad (2-1)$$

$$A_s \approx \frac{8 \epsilon \theta_A \sigma}{\rho g \sin\gamma} \quad (2-2)$$

In the above equations σ and ρ are the surface tension and density of the drop material, g is the gravitational constant, θ_A is the advancing contact angle, γ is the angle of inclination of the surface, and

$$\epsilon = 1 - \theta_R/\theta_A \quad (2-3)$$

where θ_R is the receding contact angle (see Figure 2-1). The effective thickness or radius of the drop, R , is evaluated by expressing the quotient of V divided by A_w :

$$R = \theta_A \left(\frac{\epsilon \sigma \theta_A}{2\pi \rho g} \right)^{1/2} \quad (2-4)$$

Figure 2-1 Schematic of fuel drop "sticking" to surface of solid fuel cylinder; illustrating advancing and receding contact angles ($\theta = \pi/2$).

If the area wetted by the drop A exceeds the static value A_s then, according to Dussan and Chow [2], the downward velocity of the drop U is given by

$$U = \frac{16 V \epsilon}{\pi^2 a^3 (1/\kappa_A + a/\kappa_R)} \left(\frac{\pi a^2}{A_s} - 1 \right) \quad (2-5)$$

where "a" represents the radius that a circular solid area wetted by the drop would attain if it had the same area as the actual non-circular wetted solid. Obviously, in order for $U > 0$, V and πa^2 must take on values that exceed the corresponding static values given by Equations (2-1) and (2-2). The quantities κ_A^{-1} and κ_R^{-1} are the rates of change of the contact angles with respect to drop velocity; namely,

$$\kappa_A^{-1} = \frac{d\theta_A}{dU} \quad ; \quad \kappa_R^{-1} = \frac{d\theta_R}{dU} \quad (2-6)$$

Large positive values of κ_A^{-1} and/or κ_R^{-1} result in a slowly moving drop. Equation (2-5) breaks down in the limit $\kappa_A^{-1} \rightarrow \kappa_R^{-1} \rightarrow 0$. In this extreme viscous forces created by the motion within the interior of the drop, rather than capillary forces, control the downward movement of the drop. The above equations are subject to the limitation that $\epsilon = (\theta_A - \theta_R)/\theta_A \ll 1$. While the theory that resulted in Equation (2-5) is quite elegant, it is not predictive since two parameters have been introduced, namely κ_A , κ_R , whose values are difficult to measure and are certainly unknown for aluminum/uranium fuel melts on fuel tube surfaces. In the absence of information regarding κ_A and κ_R , one option is to assume for the time being that the downward velocity of the drop is a known constant quantity. Alternatively, the influence of viscous or capillary resistance forces upon the drop motion may be ignored. As will be seen later on, this limit of a drop sliding in free fall under gravity provides useful estimates of the maximum possible distance travelled by a solidifying fuel drop on a vertical surface. Hopefully realistic drop sliding velocities will be inferred from motion picture records obtained during future laboratory studies of fuel melt drops on vertical surfaces.

An alternative relation to Equation (2-4) for the critical size drop on a vertical surface is the one proposed by Furnidge [3]; namely

$$\rho V_s g \sin \gamma = w \sigma (\cos \theta_R - \cos \theta_A) \quad (2-7)$$

where w is the width of the drop. This expression is in excellent agreement with his experiments on the maximum static size of water solution drops on solid surfaces. Equation (2-7) is valid for all values of the ratio $(\theta_A - \theta_R)/\theta_A$ and, therefore, would appear to be more general than Equation (2-4). However, since the drop width w is unknown, it cannot be applied to predict the critical drop size without invoking an assumption regarding the geometry of the drop. Assuming that the drop is near hemispherical in shape, the thickness R follows immediately from Equation (2-7) and is

$$R = \left[\frac{3\sigma (\cos \theta_R - \cos \theta_A)}{\pi \rho g \sin \gamma} \right]^{1/2} \quad (2-8)$$

If we assume that the drop takes the form of a cube with R the length of the sides, we get

$$R = \left[\frac{\sigma (\cos \theta_R - \cos \theta_A)}{\rho g \sin \gamma} \right]^{1/2} \quad (2-9)$$

The above two equations differ only by the factor $(3/\pi)^{1/2}$ and therefore give similar numerical results for the thickness of the static drop.

It is apparent from the above discussion that two conditions must be met if fuel-target contacting is to occur via a capillary mechanism. First, the advancing contact angle, which can be identified with the contact angle that the fuel melt makes on a horizontal surface (see Section 3.0), should be as large as possible. Small contact angles lead to surface wetting and film flow, which is an ineffective flow regime with regard to the desired bridging of the channels. Secondly, the difference between the two contact angles on a vertical surface, sometimes referred to in the literature as contact angle hysteresis, must be large. As far as the available evidence

goes, it appears that the dirtier and rougher the surface, the greater is the contact angle hysteresis. Contact angle measurements were performed by Neumann et al. [4] on steam condenser surfaces as a function of time of condenser operation. Indeed, while the advancing contact angle between water drops and the condenser surface remained at its initial value of $\sim 100^\circ$, the receding contact angle decreased from about 60° to essentially 0° after 14 hours of condenser operation. This decrease was attributed to the deterioration of the condenser surface with operating time.

3.0 ESTIMATE OF STATIC FUEL DROP SIZE

It is pertinent to begin this section by noting that layers of grease or oxide not more than a few molecules thick can increase the contact angle very considerably. In particular, the films of oxide so firmly held by aluminum are well known to be the cause of the difficulty in wetting this metal with liquid metals, and consequently of obtaining a satisfactory solder. Indeed, there is considerable evidence in the literature that suggests that molten aluminum (fuel) approaches the condition of near complete non-wettability on aluminum clad. Visual examinations of overheated fuel tubes that apparently experienced incipient fuel melting and clad failure reveal a fuel-drop morphology on the intact vertical aluminum cladding [7]. Koehler [5] and Dawahl and Federmann [6] measured the contact angle between molten aluminum and solid aluminum oxide over the temperature range 600 to 1100°C. The contact angle below 900°C was found to be quite large and equal to 155°. While this system is not identical to the aluminum/uranium alloy fuel melt and aluminum clad substrate pair of interest in severe accident analysis, it is felt that the contact angle for the two material pairs should be similar, owing to the presence of an oxide layer on the clad prior to melting and the small volumetric concentration of the uranium in the fuel melt.

No direct measurements on the shape of the contact line of aluminum drops on vertical surfaces have been reported. We attempt below to estimate the values of θ_A and θ_R for molten aluminum on a vertical oxide substrate from the information that is presently available. Experimental measurements indicate that the contact angle for liquid drops on a horizontal surface is approximately equal to the advancing contact angle for the liquid drop when the surface is tilted from the horizontal. For example, Adam [8] reports a contact angle for water on a horizontal wax surface of 105°, while Furnidge [3] gives a value of $\theta_A = 111^\circ$ for the advancing contact angle between water and a wax surface tilted at an angle at which the drop begins to move. The near constancy of θ_A for drops on a tilting surface before drop motion

begins was confirmed experimentally for several non-metallic liquids by Macdougall and Ockrent [9]. Thus it appears reasonable to assume that the contact angle of 155° measured for aluminum drops on horizontal oxide surfaces [5,6] is equal to the advancing contact angle θ_A for a fuel melt drop on a vertical clad surface. Obviously the receding contact angle for the fuel melt drop is difficult to estimate accurately. The photographs of overheated fuel tubes [7] reveal that the lower edges of the few refrozen fuel drops that remained on the intact cladding have much larger angles than the upper edges of the drops. Unfortunately, the quality of the photographs with regard to the details of the shape of an individual drop are not sufficient to infer accurate values of θ_R . The figures do indicate, however, that $\theta_R < 90^\circ$.

If σ for molten fuel alloy is taken comparable to the corresponding surface tension for molten aluminum, at, say 933K, we estimate $\sigma = 0.93 \text{ N m}^{-1}$. The density of molten fuel $\rho = 3400 \text{ kg m}^{-3}$. From the prior discussion we have the estimates $\theta_A = 155^\circ$ and $\theta_R = 90^\circ$. We regard the latter estimate as rather conservative. Since the submerged aluminum clad is nearly always contaminated with foreign matter, it is likely that θ_R is closer to 0° than to 90° . Substituting these physical properties into Equations (2-1), (2-2), and (2-4), we predict the following geometric properties of the critical static configuration of a fuel drop clinging to a vertical fuel tube:

$$V_s = 1.50 \times 10^{-6} \text{ m}^3 \quad (3-1)$$

$$A_s = 6.04 \times 10^{-4} \text{ m}^2 \quad (3-2)$$

$$R = 6.06 \text{ mm} \quad (3-3)$$

The critical fuel drop thickness given by Equation (3-3) exceeds the dimensions of the channels between the inner and outer fuel and target rings of 5.14 and 4.32 mm, respectively. If the receding contact angle is reduced from 90° to 72° we would predict the bridging of the largest channel between the two co-axial fuel tubes by static fuel drops. Thus as the alloy core

melts and flows through breaches in the aluminum cladding it will form drops of fuel on the surface of the cladding, as illustrated in Figure 3-1. Apparently the breaches in the cladding are formed before the fuel core achieves a molten state [7]. As the fuel is heated several hundred degrees above its normal operating temperature blisters begin to appear on the surface of the clad. By the time the fuel temperature is elevated to about 600°C localized openings that penetrate the thickness of the clad have developed. The calculations presented in the foregoing suggest that before the fuel drops can grow to their critical size and relocate to lower positions along the fuel tube they will contact the adjacent fuel or target tube. The implications of fuel-target contacting with regard to the meltdown process are discussed in the sections that follow.

As noted earlier, the Dussan and Chow [2] theory can be applied to static drops if the contact angle hysteresis, $\theta_A - \theta_R$, is not an appreciable fraction of the advancing contact angle θ_A . For the upper bound of the cases of interest here, name $\theta_A = 155^\circ$, $\theta_R = 90^\circ$, we find $(\theta_A - \theta_R)/\theta_A = 0.42$. While small, this is probably not small enough to ensure accurate predictions of the static drop size using Equation (2-4) with this θ_R . The application of Equation (2-8) results in $R = 4.92$ mm, which is a somewhat smaller thickness for the static fuel drop than that predicted with Equation (2-4). The predicted drop thicknesses are nearly equal to the fuel assembly channel diameters. Again, these estimates are based on what is regarded as an upper bound to the receding contact angle θ_R . Reference to previous experience with contact angle hysteresis on engineering surfaces would suggest a much lower θ_R value and correspondingly thicker static fuel drops relative to the dimensions of the channels. Figure 3-2 shows the predicted drop thicknesses over the range of θ_R 's from 0 to 90°.

Figure 3-1 Proposed schematic of fuel drop forming above breach in cladding and exhibiting non-wetting behavior.

Figure 3-2 Drop thickness as a function of receding contact angle, indicating the uncertainty in the predictions.

4.0 SOLIDIFICATION OF STATIC FUEL DROPS/SLUGS

Consider a fuel drop growing on the surface of a fuel tube by being continuously supplied by fuel through a localized failure site. Once the growing drop makes contact with the surface of the adjacent tube it will enlarge its area of apparent contact and display a distinct contact angle with the opposite surface. Thus the liquid drop is transformed into a liquid slug, as shown in Figure 4-1, which bridges the channel between two co-axial tubes. If the channel is one of the channels that separates a fuel tube from a target tube, fuel solidification begins as soon as the fuel drop contacts the target surface. Assuming the fuel slug remains in place during the solidification process, the amount of fuel solidified can be estimated by equating the latent heat released by the fuel with the sensible heat gained by the target; namely,

$$\delta = \frac{\rho_T c_T (T_{mp} - T_o)}{\rho h_{fs}} h \quad (4-1)$$

In the above expression δ is the final thickness of solidified fuel; c_T , ρ_T , T_o and h are the specific heat, density, initial temperature and thickness of the target, respectively; and h_{fs} , T_{mp} , and ρ are the latent heat of fusion, melting point, and the density of the liquid fuel slug. The initial target temperature is taken to be approximately the boiling point of coolant, $T_o = 373K$. The thickness of the outer target tube $h = 2.0$ mm. The remaining quantities in Equation (4-1) have the values $c_T = 1150 \text{ J kg}^{-1} \text{ K}^{-1}$, $\rho_T = 2500 \text{ kg m}^{-3}$, $T_{mp} = 919K$, $h_{fs} = 2.2 \times 10^5 \text{ J kg}^{-1}$ and $\rho = 3400 \text{ kg m}^{-3}$; so that, the thickness of the solidified fuel slug $\delta = 4.2$ mm. Thus there is sufficient heat capacity in the cold outer target ring to allow the solidification front to almost traverse the width of the channel (4.32 mm). The decay heat generation rate then results in the near uniform heating of the target and refrozen fuel to melting. The details regarding the melting of the target material is postponed until Section 5.0.

Figure 4-1 Schematic of fuel drop transformed into fuel slug upon contacting surface of target tube.

A critical question with regard to the discussion given in the foregoing is whether or not the liquid fuel slug remains in place as it solidifies. As the freezing process takes place, unsolidified fuel is continuously supplied to the melt slug through the breach in the clad of the overheated fuel tube. The unsolidified portion of the slug may exceed the critical volume required for draining and slide away from its initial point of contact with the target. Through the mechanism of coalescence other drops may now be swept from their initial sites, thereby preventing the buildup of thick frozen slugs that adhere to the target material. This possibility presents itself when only a few failure sites are present through which the fuel can flow to the outer clad surface. For example, in the extreme of only one failure opening in the clad it is easy to visualize the continuous draining of fuel over a rather small fraction of the available target surface area. Thus the initial behavior of the fuel material and the subsequent melt progression dynamics of fuel and target assemblies depends on a race between local fuel accumulation and fuel solidification. The outcome of this race depends on the number of breaches in the clad.

Because the decay heat rate Q peaks at the axial center of the assembly, we anticipate that melting of the fuel tubes will first occur at this location. Following incipient melting at the axial center, the melt zone containing a mixture or slush of the solid and liquid fuel phases will grow within the fuel tubes in the upward and downward directions. The situation is illustrated in Figure 4-2, where $\pm Z(t)$ represent the upper and lower boundaries of the fuel melt zone relative to the center of the assembly at $z = 0$. Note from Figure 4-2 that the axial distance z is taken to be positive in the upward direction. The time t is measured from the onset of fuel melting at the assembly midplane. As the melt process proceeds, fuel melt will breach the still solid, but weakened cladding and move over the outer surfaces of the fuel tubes. If the parcels of fuel melt that emerge from the tube do not relocate an appreciable distance from the breach site, an assumption whose validity is yet to be demonstrated (see Section 6.0), then the axial temperature profile is strictly a function of the assembly axial power profile, $Q(z)$.

Figure 4-2 Meltdown of a fuel cylinder; indicating instantaneous extent of fuel melt zone and axial temperature profile.

The distance dependence of the volumetric decay heating rate is usually well-represented by cosine form

$$\frac{Q(z)}{\bar{Q}} = A_N \cos\left(\frac{\pi z}{L}\right) + B_N \quad (4-2)$$

where $\bar{Q}$ is the volumetric heat generation rate averaged over the length L of the fuel tubes, and where the constants A_N and B_N are given by

$$A_N = \frac{N - 1}{1 - 2/\pi} \quad ; \quad B_N = \frac{1 - 2N/\pi}{1 - 2/\pi} \quad (4-3)$$

The constant parameter N is the ratio of the peak-to-average volumetric heat generation rate (i.e., $N = Q(0)/\bar{Q}$). In Ref. [1], expressions were derived for the instantaneous half-thickness of the melt zone, $Z(t)$, and the instantaneous rate of fuel melting $\dot{m}$ within the melt zone. These expressions are

$$\frac{Z(t)}{L} = \frac{1}{\pi} \arccos \left[\frac{1}{\frac{A_N \bar{Q} t}{\rho c (T_{mp} - T_o)} + \frac{A_N}{A_N + B_N}} - \frac{B_N}{A_N} \right] \quad (4-4)$$

$$\dot{m} = \frac{2 AL \bar{Q}}{h_{fs}} \left[\frac{A_N}{\pi} \sin\left(\frac{\pi Z}{L}\right) + B_N \frac{Z}{L} \right] \quad (4-5)$$

where A is the cross-sectional area of the fuel tube, T_o is the saturation temperature of the coolant, and c is the specific heat of solid fuel.

The table below gives the extent of the melt zone and the fuel melting rate within the outer fuel tube as a function of time for $\bar{Q} = 4.1 \times 10^7 \text{ J m}^{-3} \text{ s}^{-1}$, $L = 4 \text{ m}$, $\rho = 3600 \text{ kg m}^{-3}$, $c = 800 \text{ J kg K}^{-1}$, $T_{mp} - T_o = 546 \text{ K}$, $A_N = 0.826$, $B_N = 0.474$ (corresponding to $N = 1.3$), and $A = 5.8 \times 10^{-4} \text{ m}^2$.

Table 4-1
MELT ZONE EXTENT AND OUTER FUEL
TUBE MELTING RATE VERSUS TIME

t, s	Z, m	$\dot{m}$, kg s ⁻¹
0	0	0
0.5	0.292	0.082
1.0	0.411	0.114
2.0	0.574	0.158
3.0	0.695	0.189
5.0	0.877	0.234
10.0	1.178	0.302

Several points should be noted about Table 4-1. Since the cross-sectional area of the inner fuel tube is approximately equal to that of the outer fuel tube, the results presented in Table 4-1 also apply approximately to the inner fuel tube. The table values end at 10 s from the inception of melting at the assembly midplane. This time corresponds approximately to the time τ it takes to convert solid to melt at the midplane without increasing the fuel temperature above the solidus temperature $T_{mp} = 919\text{K}$; namely,

$$\tau = \frac{\rho h_{fs}(1 - \epsilon)}{N \dot{Q}} = 11.3 \text{ s} \quad (4-6)$$

where ϵ is the solid phase volume fraction at the solidus equilibrium condition ($\epsilon = 0.237$). Table 4-1 can be extended beyond $t = \tau$ by developing a model of fuel tube melting that tracks the location of the melt line, where the temperature first rises to the solidus temperature, and tracks the position where the solid component of the fuel slush is reduced to ϵ .

However, the information presented in Table 4-1 is all that is required for present purposes.

As mentioned previously, whether the fuel drops that form at clad failure sites run-off the target or freeze in place depends on whether there is sufficient time to accomplish appreciable fuel drop solidification before the drops accumulate enough melt to exceed the maximum possible static drop size. To derive a criterion for the immobilization of the fuel drops via freezing, we compare the time scale for drop solidification with the time it takes a surface drop continuously supplied by fuel melt through a clad failure site to grow to the maximum static size.

The solidification of fuel on target substrate is described by a diffusion wave solution between a constant temperature boundary condition at the moving phase boundary within the fuel slug and the finite region composed of the freeze layer and the target ring. The solution to this problem results in a characteristic time for fuel-slug freezing of

$$\tau_{fr} = \frac{(\delta + h)^2}{\alpha} \quad (4-7)$$

where δ is given by Equation (4-1), h is the thickness of the target ring, and α is the thermal diffusivity of the target material. The quantity δ can also be identified with the width of the channel between the fuel and target tubes, since the target heat capacity is sufficient to grow a frozen-fuel layer that traverses the channel dimension. Denote n by the concentration of failure sites in the cladding (in units of sites m^{-2}) through which the fuel flows to the outer surface of the cladding. A characteristic time τ_g for fuel slug growth can now be obtained by equating the fuel mass produced by melting during time τ_g with the mass of fuel melt contained within the maximum static slug, summed over the number of failure sites; that is,

$$\dot{m}\tau_g = n(2Z)\pi D(2V_s)\rho \quad (4-8)$$

In the above expression D is either the outer diameter of the outer fuel ring or the inner diameter of the inner fuel ring, $2Z$ is the axial extent of the melt zone, and $2V_s$ is the largest static fuel slug that can adhere to the channel walls. The fact that the line of contact between the liquid slug that bridges the channel between the co-axial fuel and target tubes is approximately twice as long as that of a drop attached to a single surface motivated our choice of $2V_s$ rather than V_s for the static fuel slug volume. By equating r_{fr} with r_g we find that clad failure site concentrations of about

$$n = \frac{\dot{m}(\delta + h)^2}{4\pi\alpha Z D V_s \rho} \quad (4-9)$$

are required to ensure that the fuel solidification process on the target surface keeps up with the outpouring of fuel from the failure sites. At 3 s into the fuel-tube melting phase of the accident, Table 4-1 indicates $Z = 0.695$ m and $\dot{m} = 0.189$ kg s⁻¹ for the outer fuel tube ($D = 8.13 \times 10^{-2}$ m). The thermal diffusivity of the target material $\alpha = 3.7 \times 10^{-5}$ m² s⁻¹, while the density of the fuel $\rho = 3400$ kg m⁻³. In addition, from Equation (3-1) and the discussion following Equation (4-1), we have $\delta = 4.2$ mm, $h = 2.0$ mm and $V_s = 1.50 \times 10^{-6}$ m³. Substituting these estimates into Equation (4-9) we predict that 53 failure sites m⁻² of cladding surface is sufficient to ensure that the fuel melt freezes in place on the target material immediately after being released from the adjacent fuel tube. This concentration of failure sites is equivalent to only 19 holes in the cladding that surrounds the instantaneous fuel melt zone. Recall that at 3 s into the melting process the melt zone extends over a distance of $2Z = 1.4$ m. Had we chosen a receding contact angle of 0° rather than 90° the minimum concentration of failure sites for fuel solidification in place on the adjacent target surface is reduced to 5. Based on these predictions we consider that it is more likely for the fuel that pours forth from the cladding to freeze in place on the target surface than to dislodge and roll off the surface and leave the target material behind. An examination of Table 4-1 reveals that the ratio $\dot{m}/Z$ is rather insensitive to time. It follows from Equation (4-9) that our prediction of the threshold failure

site density for prevention of fuel run-off is essentially independent of time.

If the number of failure sites turns out to be less than that given by Equation (4-9) then one must deal with the mechanics and thermal phenomena associated solely with sliding fuel drops and/or slugs early on in the assembly melt progression process. If n is greater than that given by Equation (4-9) then, as we shall demonstrate in the next section, the motion of both fuel and target drops is important. Certainly fuel drops that appear in the channel between the two heated fuel tubes will exceed the static drop size and slide considerable distances before coming to rest on the lower and cooler surfaces of the tubes. A model for the solidification of sliding fuel or target drops on fuel or target surfaces is developed in a subsequent section. In the next section we present a description of the heat-transfer characteristics between a frozen fuel deposit (drop) and the underlying vertical target tube. This analysis should provide a feeling for the order in which the fuel and target materials drain.

5.0 MELTING OF THE EUTECTIC FORMING LITHIUM/ALUMINUM TARGET ALLOY IN CONTACT WITH A HEAT GENERATING FUEL DEPOSIT

The problem analyzed in this section is essentially an extension of the work reported in Ref. [10], in which an integral method was employed to treat the thermal behavior of a molten fuel deposit that suddenly appears on the surface of a target ring. The high thermal conductivity of the fuel/target composite was found to result in the removal of pre-existing temperature gradients, while rapid solidification interrupted the fuel draining process before a significant fraction of the fuel melt was removed. Thus the temperature profile through the thickness of the frozen fuel drop/target composite was predicted to become uniform before the melting temperature of the composite was reached. The work reported in Ref. [10] did not include the melting or remelting of the target or fuel deposit materials. Because of alloy solution effects the temperature at which melting begins in the target is slightly less than that for the fuel. However, since the fuel deposit is self-heated and always experiences a slightly higher temperature than the target, it is not immediately evident whether or not complete, local melting of the target domain takes place before the onset of fuel melting. In this section we extend the work of Ref. [10] and study the local melting of binary, eutectic forming target material that is covered on one surface by a refrozen fuel drop.

We consider the composite solid shown in Figure 5-1. The target material occupies the region $-h < y < 0$ while the heat generating fuel deposit (drop) occupies the region $0 < y < \delta$. At time $t = 0$ both regions are at the initial temperature T_0 , which corresponds to the solidus temperature of the Li-Al target alloy. The energy generated in the fuel melt begins to raise the temperature in the fuel above T_0 . This leads to a temperature difference between the fuel and the target and, consequently, the conduction of heat out of the fuel into the cooler target (see Figure 5-1). The target material begins to melt.

Figure 5-1 Schematic of the target alloy melting problem, indicating geometry and nomenclature.

Unlike the melting of a pure material in which the process takes place isothermally at a sharp solid-liquid moving boundary, the melting of a binary, eutectic forming solution occurs over a temperature range within which a solid-liquid two-phase region exists. In the temperature range of interest here the interface temperature at $y = 0$ is higher than the solidus temperature, but lower than the liquidus temperature of the target. There is, then, only one moving boundary in the target, i.e., the solidus front, behind which a solid-liquid two-phase region grows continuously until it occupies the entire thickness of the target ring. Thereafter one must deal with binary melting in a finite domain. While there is an extensive literature on phase change in binary, eutectic forming solutions (see, e.g., Refs. [11-13]), to the author's knowledge the situation described above has not been investigated.

The objective of the analysis presented here is to determine the difference in temperature between the heat generating fuel and the melting target substrate during the transient. This temperature difference determines whether the target substrate relocates before the fuel deposit melts or whether both materials melt and relocate together. To investigate the transient behavior of the composite binary system, the following simplifying assumptions are made:

- (A1) The vertical extent of the fuel deposit is large enough so that heat transfer is virtually one-dimensional and in the tranverse y -direction (see Figure 5-1).
- (A2) The physical properties within the fuel and target materials are constant but different from each other.
- (A3) Natural convection heat transfer in the two-phase melt region can be ignored in comparison with heat conduction.
- (A4) The exposed surfaces of the target and fuel at $y = -h$ and $y = \delta$, respectively are insulated boundaries.

- (A5) Local thermodynamic equilibrium exists within the two-phase region such that the solid concentration and the temperature are fixed by the shapes of the solidus and liquidus curves of the binary eutectic phase diagram of the Li-Al target.
- (A6) Over the small temperature range of interest, the weight fractions of the solute (Li) along the solidus and liquidus curves can be regarded as linear functions of temperature.
- (A7) The mass diffusivity of the solute (Li) is much less than the thermal diffusivity of the target material so that the growth of the two-phase region is largely conduction controlled.

With the foregoing assumptions, the governing equations describing the temperature field within the fuel/target composite are

$$\frac{\partial^2 T_f}{\partial y^2} + \frac{Q}{k_f} = \frac{1}{\alpha_f} \frac{\partial T_f}{\partial t} \quad 0 \leq y \leq \delta \quad (5-1)$$

for the fuel region, and

$$\frac{\partial^2 T_T}{\partial y^2} + \frac{A}{k_T} = \frac{1}{\alpha_T} \frac{\partial T_T}{\partial t} \quad -h \leq y \leq 0 \quad (5-2)$$

for the target region. In the above equations the subscripts f and T refer to the properties of the fuel and target materials, respectively, α is the thermal diffusivity, Q is the decay heat generation rate per unit volume of fuel deposit, and the parameter A is an effective volumetric heat sink. As the two-phase melt zone continuously grows, the latent heat of melting or fusion h_{fs} is "absorbed" accordingly. As is usual in problems of this type (see Refs. [11-13]), the heat of melting can be treated as a volumetric heat sink within the melting zone, expressed as

$$A = \rho_T h_{fs} \frac{\partial f_s}{\partial t} \quad (5-3)$$

where ρ_T is the density of the melting target material and f_s is the local solid fraction within the melt zone. The initial and boundary conditions are:

$$t = 0: \quad T_T = T_f = T_o \quad (\text{solidus temperature}) \quad (5-4)$$

$$y = -h: \quad \frac{\partial T_T}{\partial y} = 0 \quad (5-5)$$

$$y = 0: \quad T_T = T_f, \quad k_T \frac{\partial T_T}{\partial y} = k_f \frac{\partial T_f}{\partial y} \quad (5-6)$$

$$y = \delta: \quad \frac{\partial T_f}{\partial y} = 0 \quad (5-7)$$

In accord with Assumption A5, the solid fraction f_s may be related to the temperature within the two-phase melt region by the so-called lever rule. This rule places the alloy composition at the "center of gravity" between the liquidus and solidus curves of the Li-Al binary eutectic phase diagram shown in Figure 5-2, and results in the expression

$$f_s = \frac{C_L(T_T) - C_o}{C_L(T_T) - C_s(T_T)} \quad (5-8)$$

for the solid fraction. The weight fractions $C_L(T_T)$ and $C_s(T_T)$ of the Li component are determined by the solidus and liquidus curves of the phase diagram given in Figure 5-2. The quantity C_o is the actual solution concentration of Li; its value for the target tubes is typically 1.91 weight percent. Reasonable linearizations of the solidus and liquidus curves are (see Assumption A6 and Figure 5-2)

$$C_s = m_s(T_{mp} - T_T) \quad (5-9)$$

$$C_L = m_L(T_{mp} - T_T) \quad (5-10)$$

Figure 5-2 Binary eutectic phase diagram for Li-Al alloy.

where $m_s = 0.0830$, $m_L = 0.239$, and T_{mp} is the melting point of pure aluminum (933K). Substituting Equations (5-9) and (5-10) into Equation (5-8) yields

$$f_s = \frac{m_L(T_{mp} - T_T) - m_s(T_{mp} - T_o)}{(m_L - m_s)(T_{mp} - T_T)} \quad (5-11)$$

Note that in deriving the above equation we made use of the linear relationship between the actual solution concentration C_o and the solidus temperature T_o inferred from Equation (5-9); namely,

$$C_o = m_s(T_{mp} - T_o) \quad (5-12)$$

This gives the value $T_o = 910K$.

We are now in a position to express the volumetric heat sink "A" in terms of temperature. This is accomplished by differentiating Equation (5-11) with respect to time and substituting the result into Equation (5-3) to obtain

$$A = - \frac{m_s \rho_T h_{fs}}{m_L - m_s} \frac{T_{mp} - T_o}{(T_{mp} - T_T)^2} \frac{\partial T_T}{\partial t} \quad (5-13)$$

Eliminating A between Equations (5-2) and (5-13) yields the sought energy equation for the target region:

$$\frac{\partial^2 T_T}{\partial y^2} = \frac{1}{a_T} \frac{\partial T_T}{\partial t} \quad (5-14)$$

where a_T is the apparent thermal diffusivity of the target material which incorporates the effects of melting the binary Li-Al alloy; it is given by

$$a_T = \alpha_T \left[1 + \frac{m_s}{m_L - m_s} \frac{h_{fs}(T_{mp} - T_o)}{c_T(T_{mp} - T_T)^2} \right]^{-1} \quad (5-15)$$

where c_T is the specific heat of the target material.

Figure 5-3 shows the relation between the ratio a_T/α_T and T_T over the range of the target temperature values of practical interest. This plot was constructed using the property value estimates $h_{fs} = 4.0 \times 10^5 \text{ J kg}^{-1}$ and $c_T = 1150 \text{ J kg}^{-1} \text{ K}^{-1}$. Note that the apparent thermal diffusivity involving phase change is much smaller than the molecular thermal diffusivity. This result has the physical meaning that the apparent heat capacity for the melting target alloy material is much larger than that for the solid alloy. The governing system of equations can be solved with the a_T versus T_T relation given by Equation (5-15), but the resulting equations are unwieldy and difficult to interpret. To simplify matters, the apparent thermal diffusivity will be represented by a suitable average value $\bar{a}_T$. The curve presented in Figure 5-3 suggests that

$$\bar{a}_T \approx 0.077 \alpha_T \quad (5-16)$$

Upon making this assumption, we have reduced the problem to one of pure heat conduction in a composite finite body containing two regions. In particular we seek a solution to Equation (5-1) and Equation (5-14) with a_T replaced by $\bar{a}_T$. A generally valid solution to this linear system can be obtained by the application of the Laplace transform technique, but the result is rather cumbersome. An alternate technique involves finding solutions valid for small and large times. The small-time solution is valid during the period in which the thickness of the thermal boundary layer in the solid fuel is less than the thickness of the solid fuel region. For our purposes the small-time solution is not too useful and we will focus our attention only on the large-time solution.

It is convenient to begin with some general considerations based on experience with conduction problems in finite regions involving internal heat generation and adiabatic boundaries or constant heat flux boundary conditions. The general solutions for the temperature fields in these problems always appear in the form of a sum of a quadratic function in the space variable y , containing a term that is linear in time, and an infinite

Figure 5-3 Normalized apparent thermal diffusivity of two-phase liquid-solid region versus temperature.

series made up of the harmonics of the Fourier series for the initial temperature. Once the thermal conduction waves reach the boundaries of the region, i.e., at the end of the small-time regime, all the harmonic terms disappear, leaving the quadratic temperature distribution whose shape remains constant in time. In other words, in the large-time regime the constant heat generation rate results in a rise in the temperature that is everywhere linear in time. Thus we seek solutions to Equations (5-1) and (5-14) that satisfy the conditions

$$\frac{\partial T_f}{\partial t} = \frac{\partial T_T}{\partial t} = \lambda = \text{const.} \quad (5-17)$$

The solutions to these equations are

$$T_f = \lambda t + f(y) \quad (5-18)$$

$$T_T = \lambda t + g(y) \quad (5-19)$$

where $f(y)$ and $g(y)$ are unknown functions of the distance variable y .

Eliminating the terms $\partial T_f / \partial t$ and $\partial T_T / \partial t$ in Equations (5-1) and (5-14) via Equation (5-17) and integrating the results yields the following temperature profiles:

$$T_f = \left(\frac{\lambda}{\alpha_f} - \frac{Q}{k_f} \right) \frac{y^2}{2} + c_1(t)y + c_2(t) \quad (5-20)$$

$$T_T = \frac{\lambda}{\bar{\alpha}_T} \frac{y^2}{2} + c_3(t)y + c_4(t) \quad (5-21)$$

where $c_1 \dots c_4$ are unknown and possibly time-dependent integration variables. From the boundary conditions at $y = 0$, $y = -h$ and $y = \delta$ [see Equations (5-5) through (5-7)] we have the following relations:

$$c_2 = c_4 \quad ; \quad k_f c_1 = k_T c_3 \quad (5-22)$$

$$c_3 = \frac{\lambda h}{\bar{a}_T} \quad (5-23)$$

$$c_1 = - \left(\frac{\lambda}{\alpha_f} - \frac{Q}{k_f} \right) \delta \quad (5-24)$$

It turns out that c_1 and c_3 are constants of integration. From Equations (5-23) and (5-24) and the second equality of Equation (5-22) we find that the temperature rise rate everywhere in the composite is

$$\lambda = \frac{Q\delta}{\frac{k_f \delta}{\alpha_f} + \frac{k_T h}{\bar{a}_T}} \quad (5-25)$$

Comparing Equation (5-18) with Equation (5-20) and Equation (5-19) with Equation (5-21), it is clear that these solutions are compatible only if the temperature profiles take the forms [see, also Equations (5-22) - (5-24)]

$$T_f = \lambda t + \left(\frac{\lambda}{\alpha_f} - \frac{Q}{k_f} \right) \frac{y^2}{2} - \left(\frac{\lambda}{\alpha_f} - \frac{Q}{k_f} \right) \delta y + c_2 \quad (5-26)$$

$$T_T = \lambda t + \frac{\lambda}{\bar{a}_T} \frac{y^2}{2} + \frac{\lambda h}{\bar{a}_T} y + c_2 \quad (5-27)$$

where c_2 is a constant of integration. Without solving for the constant c_2 we can obtain the expression for the maximum temperature difference between the fuel and target regions. By evaluating T_f at $y = \delta$ and T_T at $y = -h$ (see Figure 5-1) and subtracting the latter expression from the former, we get

$$T_f(\delta) - T_T(-h) = \Delta T = \frac{\lambda h^2}{2\bar{a}_T} \left(1 + \frac{k_T \delta}{k_f h} \right) \quad (5-28)$$

In deriving the above equation we made use of the fact that

$$\frac{\lambda}{\alpha_f} - \frac{Q}{k_f} = - \frac{\lambda k_T h}{\bar{a}_T k_f \delta} \quad (5-29)$$

as determined from Equation (5-25). To determine the constant c_2 in Equations (5-26) and (5-27) we require that the temperature distributions satisfy the global heat balance

$$Q\delta t = \frac{k_T}{\bar{a}_T} \int_{-h}^0 (T_T - T_o) dy + \frac{k_f}{\alpha_f} \int_0^{\delta} (T_f - T_o) dy \quad (5-30)$$

This condition states that the heat generated within the fuel is absorbed by both the target and fuel regions. Substitution of Equations (5-26) and (5-27) into Equation (5-30) and carrying out the indicated integrations gives

$$c_2 = T_o + \frac{\lambda^2 k_T h^3}{3Q\delta \bar{a}_T^2} \left[1 - \frac{\bar{a}_T}{\alpha_f} \left(\frac{\delta}{h} \right)^2 \right] \quad (5-31)$$

Finally, substitution of this expression into Equations (5-26) and (5-27) gives

$$T_f = T_o + \frac{\lambda^2 k_T h^3}{3Q\delta \bar{a}_T^2} \left[1 - \frac{\bar{a}_T}{\alpha_f} \left(\frac{\delta}{h} \right)^2 \right] + \lambda t + \frac{\lambda k_T h}{\bar{a}_T k_f \delta} \left(\delta y - \frac{y^2}{2} \right) \quad (5-32)$$

$$T_T = T_o + \frac{\lambda^2 k_T h^3}{3Q\delta \bar{a}_T^2} \left[1 - \frac{\bar{a}_T}{\alpha_f} \left(\frac{\delta}{h} \right)^2 \right] + \lambda t + \frac{\lambda}{\bar{a}_T} \left(\frac{y^2}{2} + hy \right) \quad (5-33)$$

These results give the temperature distribution as a function of the coordinate y and time t . These expressions are exact in the limit as $t \rightarrow \infty$. Based on experience with analogous heat conduction problems, they should predict the values of T_f and T_T to within about 10 percent for $t \approx 0.1 h^2/\bar{a}_T$. In terms of real time we find that the solutions are accurate for, say, the outer target ring beyond $t \approx 0.14$ s (see below for property value

estimates). Once the temperature distribution $T_T(y,t)$ is known, one can calculate the solid mass fraction distribution at any time $t > 0.14$ s in the two-phase zone by the application of Equation (5-11). However, as already mentioned, the main purpose of this section is to calculate the maximum temperature difference ΔT between the fuel deposit and the adjacent target substrate. Accordingly, we focus our attention on Equations (5-25) and (5-28).

Calculations are performed using the following thermo-physical properties of the target/fuel composite: $k_f = 149 \text{ W m}^{-1} \text{ K}^{-1}$, $k_T = 105 \text{ W m}^{-1} \text{ K}^{-1}$, $\alpha_f = 5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, and $\alpha_T = 3.7 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$. From Equation (5-16) the apparent thermal diffusivity of the melting target region $\bar{\alpha}_T = 2.85 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$. The fuel deposit generates heat at the volumetric rate $Q = 4.1 \times 10^7 \text{ W m}^{-3}$ and has an assumed thickness $\delta = 4 \text{ mm}$. The thickness $h = 2 \text{ mm}$, corresponding to the outer target ring. Substituting these property estimates into Equation (5-25) we predict a temperature rise rate $\lambda = 1.93^\circ \text{C s}^{-1}$. From Equation (5-28) we get $\Delta T = 3.3^\circ \text{C}$. Since the difference between the fuel solidus and the target solidus is 919K minus 910K , or 9°C , we conclude that the molten target material may break out of its cladding due to local clad failures before the fuel deposit melts. From this estimate, then, it seems unlikely that both the deposited (and solidified) fuel drop and substrate target material will become molten simultaneously and mix. However, a significant vertical separation should not develop between the fuel deposit and the target material. If the quantity of target melt that pours through the cladding is less than the capillary "stopping mass", the target material will cling to the target tube just below the fuel deposit (drop) that was responsible for heating and melting the target material. Even if the mass of the target melt exceeds the capillary stopping mass, it is not likely to slide very far from its original position. This will be demonstrated in the next section where we develop a model for the solidification of sliding fuel (or target) drops on fuel (or target) tubes.

6.0 SOLIDIFICATION MODEL OF A SLIDING DROP OR SLUG IN A VERTICAL ASSEMBLY CHANNEL

The motion and freezing of a fuel or target drop on the vertical fuel or target tube may be regarded as belonging to the class of problems on liquid solidification on moving surfaces. A drop moving down the surface at instantaneous speed U as viewed from the frame of reference at rest with respect to the drop is shown in Figure 6-1. Before being covered by the drop, the fuel or target substrate (hereafter referred to as the "wall") is at a uniform temperature T_0 which is below the freezing (solidus) point, T_{mp} , of the drop material. As the upward translating wall passes beneath the drop, a solidified layer of thickness $\delta(z)$ grows in the z -direction starting from the leading edge of the drop.

In the present treatment of the sliding, solidifying drop, the following simplifying assumptions are made:

1. The drop material begins to slide as soon as its temperature exceeds the solidus temperature of the drop material. Thus the sensible heat (i.e., superheat) within the drop is ignored in comparison with the latent heat of drop solidification h_{fs} .
2. The wall is semi-infinite in the negative y -direction (see Figure 6-1). This implies that the thermal penetration front within the heated segment of the wall does not traverse the wall thickness during the time it takes for the segment to pass under the drop. Heat transfer is assumed to occur in the y -direction only.
3. The properties of the liquid and solidified drop materials are assumed to be identical.

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Figure 6-1 Schematic of drop freezing on an upward translating wall (or fuel tube), illustrating temperature profile and flow crust wake.

4. The solidification process takes place at a constant temperature T_{mp} . The solidification front $\delta(z)$ is sharp and planar on the scale of the freeze-layer thickness; that is, the effects of the solution properties of the fuel or target drops are ignored.
5. The formulation begins by assuming that the approaching wall temperature T_o and the wall velocity U are constants. However, once the formulation is combined with the overall drop mass and momentum balances, these quantities are allowed to vary in a quasi-steady manner as the drop travels along the wall.
6. The drop is assumed to be hemispherical in shape and its instantaneous radius is denoted by the symbol R . If the melt material bridges the dimension of the channel width, it is assumed to form a slug whose lines of contact with the channel walls are squares. The instantaneous length of each side of the square is denoted by the symbol L .

With the above physical picture, the equations governing the thickness $\delta(z)$ of the solidified layer, the temperature $T_d(y,z)$ of the solidified portion of the drop, and the temperature $T_w(y,z)$ of the wall region covered by the drop are as follows (see Figure 6-1):

$$\rho_d c_d U \frac{\partial T_d}{\partial z} = k_d \frac{\partial^2 T_d}{\partial y^2} \quad (6-1)$$

for the solidified-layer region,

$$\rho_w c_w U \frac{\partial T_w}{\partial z} = k_w \frac{\partial^2 T_w}{\partial y^2} \quad (6-2)$$

for the wall region (fuel or target tube) and

$$z = 0: \quad \delta = 0, \quad T_w = T_o \quad (6-3)$$

$$y = 0: \quad T_d = T_w, \quad k_d \frac{\partial T_d}{\partial y} = k_w \frac{\partial T_w}{\partial y} \quad (6-4)$$

$$y = \delta: \quad T_d = T_{mp}, \quad \rho_d U h_{fs} \frac{d\delta}{dz} = k_d \left(\frac{\partial T_d}{\partial y} \right) \quad (6-5)$$

$$y \rightarrow \infty: \quad T_w = T_o \quad \text{or} \quad \frac{\partial T_w}{\partial y} = 0 \quad (6-6)$$

for the boundary conditions. In the above equations the quantities k , ρ , and c have their usual meanings. The subscript d refers to the physical properties and temperature of the drop while the subscript w refers to those of the wall.

Equations (6-1) through (6-6) can be solved by means of a similarity transformation. We shall present this transformation directly, omitting a detailed description of how it can be obtained. We then have

$$\eta = \frac{y}{\delta(z)}, \quad \text{where } \delta(z) = 2\lambda \left(\frac{\alpha_d z}{U} \right)^{1/2} \quad (6-7)$$

$$\theta_d(\eta) = \frac{T_d - T_o}{T_{mp} - T_o}, \quad \theta_w(\eta) = \frac{T_w - T_o}{T_{mp} - T_o} \quad (6-8)$$

where η is the independent similarity variable, λ is the so-called crust-growth constant, which must be determined as part of the solution, and θ is the dimensionless temperature. In the similarity coordinate system, the solidified layer region is given by $0 \leq \eta \leq 1$ and the wall region lies in the domain $\eta \leq 0$.

Substituting Equations (6-7) and (6-8) into Equations (6-1) through (6-6), we obtain

$$\theta_d'' + 2\lambda^2 \eta \theta_d' = 0 \quad (6-9)$$

$$\theta_w'' + 2\lambda^2 \left(\frac{\alpha_d}{\alpha_w} \right) \eta \theta_w' = 0 \quad (6-10)$$

subject to the boundary conditions

$$\theta_d'(0) = \frac{k_w}{k_d} \theta_w'(0) \quad ; \quad \theta_d(0) = \theta_w(0) \quad (6-11)$$

$$\theta_w(-\infty) = 0 \quad ; \quad \theta_d(1) = 1 \quad (6-12)$$

$$\theta_d'(1) = \frac{2 \lambda^2 h_{fs}}{c_d(T_{mp} - T_o)} \quad (6-13)$$

where the primes denote total derivatives with respect to η . The integration of Equations (6-9) and (6-10) can be completed in closed form using the appropriate boundary conditions given by Equations (6-11) and (6-12) to get

$$\theta_d = \frac{\text{erf}(\lambda\eta) + 1/\sigma}{\text{erf}(\lambda) + 1/\sigma} \quad \text{for } 0 \leq \eta \leq 1' \quad (6-14)$$

$$\theta_w = \frac{\text{erf}\left[(\alpha_d/\alpha_w)^{1/2} \lambda \eta\right] + 1}{\sigma \text{erf}(\lambda) + 1} \quad \text{for } \eta \leq 0 \quad (6-15)$$

where σ is defined as

$$\sigma = \frac{k_w \rho_w c_w}{k_d \rho_d c_d} \quad (6-16)$$

The unknown crust-growth constant λ can now be determined by substituting Equation (6-14) into Equation (6-13). The result is the following implicit relationship between λ and the parameters of the problem:

$$\pi^{1/2} \lambda e^{\lambda^2} [1/\sigma + \text{erf}(\lambda)] = \frac{c_d(T_{mp} - T_o)}{h_{fs}} \quad (6-17)$$

A typical instantaneous crust thickness distribution in the wake of the drop is shown schematically in Figure 6-2. The crust thickness is a maximum at

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Figure 6-2 Top view of hemispherical drop and cross section of crust wake.

position "a" at the rear of the drop and gradually decreases to zero at positions b and d. This is because the wall is covered by the liquid drop for the longest period of time along the drop centerline between a and c. The freeze time is reduced along straight-line segments on either side of the centerline and decreases with increasing lateral distance from the centerline. The instantaneous cross-sectional area A of the frozen wake left behind by the drop is [see Figure 6-2 and Equation (6-17)]:

$$A = \int_0^{\pi} \delta(\phi) R \sin\phi \, d\phi = \int_0^{\pi} 2\lambda \left(\frac{\alpha_d 2R \sin\phi}{U} \right)^{1/2} R \sin\phi \, d\phi$$

$$= 4\lambda \left(\frac{\alpha_d 2R}{U} \right)^{1/2} R \int_0^{\pi/R} (\sin\phi)^{3/2} \, d\phi$$

or, upon evaluating the integral*,

$$A = \frac{2\lambda \left[\Gamma\left(\frac{1}{4}\right) \right]^2}{3} \left(\frac{\alpha_d R^3}{\pi U} \right)^{1/2} \quad (6-18)$$

The volume rate of loss $\dot{V}_d$ of drop material is, then,

$$\dot{V}_d = UA = \frac{2\lambda \left[\Gamma\left(\frac{1}{4}\right) \right]^2}{3} \left(\frac{\alpha_d UR^3}{\pi} \right)^{1/2} \quad (6-19)$$

If the molten material is in the form of a slug that bridges the width of the channel, frozen wakes will be left behind on both of the channel walls. One of the walls (wall 1) may be a target tube while the other (wall 2) is a fuel tube. Alternatively, the slug may slide over the surfaces of two concentric fuel tubes. As already mentioned, the slug is assumed to be

*. $\Gamma\left(\frac{1}{4}\right) = 3.6256$

in the form of a parallelepiped of instantaneous length and width L ; its thickness is equal to the width of the channel. It is obvious that the total cross-section area A of the frozen wakes left behind on the channel walls (1 and 2) is

$$A = [\delta_1(L) + \delta_2(L)]L \quad (6-20)$$

It follows from Equation (6-7) that the volume rate of loss $\dot{V}_{sl}$ of the slug material is

$$\dot{V}_{sl} = UA = 2(\lambda_1 + \lambda_2) \left(\alpha_d UL^3 \right)^{1/2} \quad (6-21)$$

where λ_i ($i = 1, 2$) is obtained from the solution of Equation (6-17) using the physical properties of wall i .

The solutions of Equations (6-19) and (6-21) for the drop and slug melt volumes, respectively, can not be obtained unless some model or formula is provided for the velocity U of the drop or slug. As mentioned in Section 2.0, an obvious difficulty in applying the available theoretical models on the motion of sliding drops is that the parameters involved are presently difficult to estimate. We proceed here by investigating an instructive limiting case in which the frictional and capillary resistance effects are neglected and attention is focused on drops (slugs) that accelerate under the influence of gravity alone. In this limiting case the sliding velocity of the melt drop is related to the vertical distance of travel z by

$$U = (2gz)^{1/2} \quad (6-22)$$

where g is the gravitational constant. This formula will result in the prediction of the maximum possible vertical displacement of the drop or slug.

Let us first consider the case of a target melt drop formed beneath a fuel deposit and sliding away from its point of origin (or clad rupture

site). The temperature of the target tube over the distance traversed by the sliding drop is taken to be constant and equal to T_o . The volume of the drop is governed by the simple mass balance

$$\frac{dV_d}{dt} = -\dot{V}_d \quad (6-23)$$

where $\dot{V}_d$ is given by Equation (6-19). The vertical distance z from the rupture opening is

$$\frac{dz}{dt} = U \quad (6-24)$$

Actually time is of little intrinsic interest in the present problem - we are more concerned with the distance travelled by the drop when its size is reduced to the static volume V_s . Accordingly, time t can be suppressed by dividing Equation (6-23) by Equation (6-24). Combining this result with Equations (6-19) and (6-22) gives the following differential equation for the drop volume as a function of distance travelled:

$$\frac{dV_d}{dz} = -2.873 \lambda \left(\frac{\alpha_d V_d}{g^{1/2} z^{1/2}} \right)^{1/2} \quad (6-25)$$

Integrating this expression between the limits $V_d = V_{d,o}$ at $z = 0$ to $V_d = V_s$ at $z = z_t$, where $V_{d,o}$ is the initial volume of the drop and z_t is the distance travelled by the drop when its size is reduced to the static volume V_s , and solving the result for z_t gives

$$z_t = 0.421 \left(\frac{g^{1/2} V_{d,o}}{\alpha_d \lambda^2} \right)^{2/3} \left[1 - \left(\frac{V_s}{V_{d,o}} \right)^{1/2} \right]^{4/3} \quad (6-26)$$

Assuming that the temperature of the target wall is approximately equal to the boiling point of water, we estimate $T_o \approx 400K$. From this estimate and the physical property assignments $c_d = 1150 \text{ J kg}^{-1} \text{ K}^{-1}$, $h_{fs} = 4.0 \times 10^5 \text{ J kg}^{-1}$, $T_{mp} = 910K$ for the target melt, the growth constant λ for the drop

solidification process can be obtained by solving Equation (6-17). The result is $\lambda = 0.46$. Note that for a drop of molten target material sliding down a target cylinder the parameter σ defined by Equation (6-16) is unity. Suppose the drop is set in motion when its volume is equal to twice the static volume given by Equation (3-1); due, perhaps, to the coalescence of two adjacent static drops. Thus $V_{d,o} = 3.0 \times 10^{-6} \text{ m}^3$. Taking $\alpha_d = 3.7 \times 10^{-5}$, together with the above estimates, we predict from Equation (6-26) that $z_t = 10.0 \text{ cm}$. This is the maximum possible distance travelled by the drop before it is reduced to its static size. Thus the target material drop does not travel very far from the fuel deposit (drop or slug) responsible for producing the drop in the first place. Indeed, from a neutronics point of view, 10 cm is not a large separation distance.

Let us now examine the problem of a fuel slug that slides over the channel walls formed by the two coaxial fuel tubes. In the case the fuel temperature varies with axial distance z . Since the axial temperature distribution of the fuel tubes are identical, the growth constants λ_1 and λ_2 must also be identical. Thus we can rewrite Equation (6-21) in the form

$$\dot{V}_{sl} = - \frac{dV_{sl}}{dt} = 4\lambda \left(\frac{\alpha_d U V_{sl}^{3/2}}{\delta_{ch}^{3/2}} \right) \quad (6-27)$$

where δ_{ch} is the width of the channel. Again, the volume of the slug is assumed to be that of a parallelepiped of square cross section that fills the width of the channel; so that,

$$V_{sl} = L^2 \delta_{ch} \quad (6-28)$$

Equation (6-28) was used in deriving Equation (6-27).

During the early stages of assembly melting the axial temperature distribution is well represented by the formula

$$\frac{T(z) - T_o}{T_{mp} - T_o} = \frac{A_N \cos\left(\frac{\pi z}{L}\right) + B_N}{A_N + B_N} \quad (6-29)$$

where the axial distance z is measured from the midplane of the fuel tubes. This formula follows directly from Equation (4-2) and the defensible assumption that the effect of axial conduction on the temperature profile is negligible (see Ref. [1]). As the fuel slug slides downward the temperature of the channel walls "felt" by the slug varies in accord with Equation (6-29). Thus the crust growth constant that appears in Equation (6-27) is determined from Equation (6-17) with T_o replaced by $T(z)$:

$$\begin{aligned} \pi^{1/2} \lambda e^{\lambda^2} [1/\sigma + \operatorname{erf}(\lambda)] &= \frac{c_d [T_{mp} - T(z)]}{h_{fs}} \\ &= \frac{A_N}{A_N + B_N} \cdot \frac{c_d (T_{mp} - T_o)}{h_{fs}} \cdot \left(1 - \cos \frac{\pi z}{L}\right) \end{aligned} \quad (6-30)$$

The crust growth "constant" λ for solidification of fuel on the heated fuel tubes is typically small compared with unity. This is because the right-hand-side of Equation (6-30) is small. Using the first term of a series expansion of the left-hand-side of the above equation gives approximately

$$\lambda = \frac{A_N}{A_N + B_N} \cdot \frac{\sigma c_d (T_{mp} - T_o)}{\pi^{1/2} h_{fs}} \cdot \left(1 - \cos \frac{\pi z}{L}\right) \quad (6-31)$$

By employing Equations (6-22), (6-24), and (6-31), the differential equation for V_{sl} , namely Equation (6-27), can be rewritten in the form

$$\begin{aligned} V_{sl}^{-3/4} \frac{dV_{sl}}{dz} &= - \frac{4}{2^{1/4} \pi^{1/2}} \cdot \frac{A_N}{A_N + B_N} \cdot \frac{\sigma c_d (T_{mp} - T_o)}{h_{fs}} \\ &\quad \cdot \left(\frac{\alpha_d}{\delta_{ch}^{3/2} g^{1/2}} \right)^{1/2} \frac{\left[1 - \cos \frac{\pi z}{L}\right]}{z^{1/4}} \end{aligned} \quad (6-32)$$

Early in the assembly melting process the fuel slug will form in the vicinity of the midplane of the fuel tubes. Of course, the motion of the slug will begin at this location. Denoting the initial volume of the slug by $V_{sl,o}$, we have the initial condition

$$V_{sl} = V_{sl,o} \quad \text{at} \quad z = 0 \quad (6-33)$$

Moreover, assuming that the travel distance z_t of the slug is such that $z_t \ll L$, we have

$$1 - \cos \frac{\pi z}{L} \approx \frac{\pi^2}{2L^2} z^2 \quad (6-34)$$

By substituting this expression into Equation (6-32) and integrating the result we derive

$$z_t = 1.06 \left(\frac{A_N + B_N}{A_N} \right)^{4/11} \left[\frac{h_{fs}}{\sigma c_d (T_{mp} - T_o)} \right]^{4/11} \left(\frac{g^{1/2} \delta_{ch}^{3/2} L^4 v_{sl,o}^{1/2}}{\alpha_d} \right)^{2/11} \left[1 - \left(\frac{V_s}{V_{sl,o}} \right)^{1/4} \right]^{4/11} \quad (6-35)$$

for the distance the fuel slug travels before its size is reduced to the static volume V_s .

The appropriate numerical values of the parameters that appear in Equation (6-35) for a fuel slug sliding between the assembly fuel tubes are $A_N = 0.826$, $B_N = 0.474$, $h_{fs} = 2.2 \times 10^5 \text{ J kg}^{-1}$, $\sigma = 1.0$, $c_d = 800 \text{ J kg K}^{-1}$, $T_{mp} = 919\text{K}$, $T_o = 400\text{K}$, $\delta_{ch} = 6.85 \times 10^{-3} \text{ m}$, $L = 4 \text{ m}$, and $\alpha_d = 5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$. Substitution of these values into Equation (6-35) gives $z_t = 0.89 \text{ m}$ for the displacement of a fuel slug that descends from the core midplane and has an initial volume equal to twice the static value $V_s = 3 \times 10^{-6} \text{ m}^3$. Note that the maximum value of the growth constant λ experienced by the fuel slug is obtained by substituting z_t for z in Equation (6-31). For the

present example this gives $\lambda = 0.16$. Thus our assumption of small λ is valid.

We conclude this section by noting that, in contrast with the movement of target drops (or fuel drops) over target cylinders, the distance travelled by a fuel slug moving between the coaxial fuel tubes can be substantial.

7.0 DISCUSSION AND CONCLUDING REMARKS

The available experimental and theoretical evidence pertaining to the behavior of liquefied aluminum/uranium fuel alloy on an intact clad surface strongly suggests that during a Mk-22 assembly meltdown the fuel will relocate over the clad in the form of individual drops. Moreover, the evidence suggests that the size of the fuel drops may be sufficient to bridge the channel gaps that separate the fuel tubes from the target tubes. The principal facts that lead to these conclusions can be summarized as follows: (i) molten aluminum exhibits almost complete non-wetting behavior on horizontal aluminum oxide surfaces, (ii) the affinity between oxygen and aluminum (clad) is well known and results in the formation of aluminum surface oxide films at temperatures as low as room temperature, (iii) the difference between advancing and receding contact angles ("hysteresis") of liquid drops on inclined solid surfaces increases in the presence of such surface films, and (iv) the size of the drops that stick to inclined surfaces is directly proportional to the contact angle hysteresis.

The final stage of the thermal interaction between a segment of target tube in contact with a finite region of self-heated refrozen fuel was investigated theoretically. The fuel was assumed to bridge the assembly channel when it was in its molten state. This study completes an earlier analysis [10] of the initial stages of fuel/target composite heat transfer, and takes into account the binary eutectic forming properties of the lithium/aluminum alloy target. The theory indicates that complete melting of the target segment will occur before the solid fuel deposit is heated to its solidus (melting) temperature. Thus the underlying target material is likely to break through its clad and relocate as a drop to a lower position on the target cylinder before the fuel deposit responsible for melting the target begins to descend. However, additional theoretical work presented here and directed at the movement of target drops (or fuel drops) shows that the target drop will not slide very far (on the scale of the active core length) from its original position before coming to rest. Thus, while intimate

mixing of target and fuel is precluded by virtue of the different solidus temperatures for these materials, the picture that emerges of an assembly meltdown under decay heat conditions is one in which the target fuel drops descend, on a stop and go basis, just ahead of their parent fuel drops by a distance of the order of, at most, only ten drop diameters. From a neutronics point of view, such a meltdown morphology should not yield results that are too different from those based on the assumption of mixing of fuel and target at the location where the fuel first contacts the target cylinder.

As far as future work on understanding SRP assembly meltdown behavior is concerned, at this juncture the greatest single need is for a thorough experimental study designed to measure both the contact angle hysteresis of static fuel drops on vertical clad surfaces and the speed with which drops whose sizes exceed the static drop size slide over the clad surface. By placing a fuel drop of known volume on a clad surface, and measuring the angle of inclination of the surface when the drop is about to be released from its static configuration, the theory reviewed in Section 2.0 can be used to determine the contact angle hysteresis. The drop thickness and drop velocity of descent can be obtained by photographic techniques. Such an experiment would be considerably less expensive than the popular assembly meltdown tests and would provide in a timely manner crucial information concerning the potential for capillarity-induced fuel-target contacting. In addition, the necessary fundamental information required for core-melt progression modeling would be gained from the experiment. Ultimately an assembly meltdown test should be performed to determine the way in which the relocating fuel is distributed among the channels.

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