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TITLE: DISPLACEMENT FORCES ON A NEUTRAL CURRENT DETECTOR FROM A
BUOYANT AND FLEXIBLE CABLE UNDER TENSION

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MASTER

DISPLACEMENT FORCES ON A NEUTRAL CURRENT DETECTOR FROM A BUOYANT AND FLEXIBLE CABLE UNDER TENSION

Sean Walston
May 1994

As a means for maintaining order in the 112 NCD readout cables, it is proposed to put the cables under tension in such a way that they lie as straight as possible and rest along the top surface of the acrylic D₂O vessel. However, one aim is to avoid using top anchor points to secure the NCDs in the D₂O vessel, but rather to just let the counter's inherent buoyancy maintain the necessary vertical orientation. As a result, applying a tension to the readout cable may pull the counters too far off of vertical. Whether or not the counter is pulled too far off of vertical depends in large part on the characteristics of the cable, i.e. modulus of elasticity and initial radius of curvature.

Given that the applied tension in equilibrium is just equal to the restoring force on the counter when pulled off of vertical by some arbitrarily small amount, and given that the final radius of curvature of the readout cable is 6 meters (the radius of the D₂O vessel), what then must the modulus of elasticity be for a given initial radius of curvature of the cable? This assumes that the cables will be wrapped around a spool or somehow coiled for shipment. This also assumes that upon delivery of the cables, some effort can be made to straighten them out somewhat. For this approximation, the buoyancy of the cables is assumed to be negligible. The small length of cable that does not rest against the inner surface of the D₂O vessel is also neglected.

The restoring force on a counter pulled off of vertical by an angle θ is

$$F_r = \frac{B_{NCD} \sin \theta}{2}$$

$$B_{NCD} = gV_{NCD}(\rho_{D,C} - \rho_{NCD})$$

where V_{NCD} is the volume of the detector and ρ_{NCD} is the density of the detector.

The tension required to pull the counter off of vertical is

$$T = \frac{F_r}{\cos(\theta + \xi)}$$

where ξ is the angle with respect to horizontal of the neck of the vessel from the top of the counter, or the angle along which the tension vector is acting.

The straightening force on the wire due to the tension is

$$dF_s = T \sin \phi \, d\phi$$

$$F_s = \int_0^{\frac{L}{2R}} T \sin \phi \, d\phi$$

$$F_s = T - T \cos\left(\frac{L}{2R}\right)$$

where $\phi = 0$ at the center of the wire, L is the length of the wire, and R is the radius of curvature of the wire.

The restoring force on a bent wire is equal to the straightening force on the wire when the wire is in equilibrium. So given the final radius of curvature and the diameter of the wire, the modulus of elasticity can be calculated for a given value for the initial radius of curvature.

The deflection of the wire y , as the wire goes from its initial radius of curvature R_0 to the final radius of curvature R is

$$y = \sqrt{2R_0^2 \left[1 - \cos\left(\frac{L}{2R_0}\right) \right] + 2R^2 \left[1 - \cos\left(\frac{L}{2R}\right) \right] - 2R_0R \cos\left(\frac{L}{4R} - \frac{L}{4R_0}\right) \sqrt{2 \left[1 - \cos\left(\frac{L}{2R_0}\right) \right] \left[1 - \cos\left(\frac{L}{2R}\right) \right]}}$$

The cross sectional moment of inertia for a given diameter d is

$$I = \frac{\pi}{64} d^4$$

The modulus of elasticity for a beam fixed at one end and loaded at the other is then

$$E = \frac{F_s \left(\frac{L}{2}\right)^3}{3Iy}$$

The graph below shows the maximum allowable modulus of elasticity of the cable as a function of the initial radius of curvature, given that the top of the counter deviates from vertical by 10 cm.

Length of Counter	4 m
Density of Counter	0.25 g/cm ³
Position in Equatorial Plane	(5.5 m, 1.5 m)
Angle of Deviation from Vertical	0.025 Radians
Diameter of Cable	1/4 in = 0.635 cm
Final Radius of Curvature	6 m

EXPERIMENTAL VERIFICATION

A test was conducted to verify that these equations were turning out the right results. First, the modulus of elasticity of a stainless steel welding rod was determined. One end of the rod was held fixed in a vise with most of the length of the rod sticking out horizontally, unsupported. The deflection of the end of the rod was measured and the modulus of elasticity calculated.

The modulus of elasticity for a uniformly loaded beam fixed at one end is

$$E = \frac{WL^3}{8Iy}$$

Length of Unsupported Rod	23.988 in
Diameter of Rod	0.094 in
Linear Density of Rod	1.9935×10^{-3} lb/in
Cross Sectional Moment of Inertia	3.8325×10^{-6} in ⁴
Deflection of End of Rod	0.734375 in
Modulus of Elasticity	29,329,000 psi

The value for the modulus of elasticity for steel given in the Machinery's Handbook is 30 million psi.

The rod was then bent into a near half circle, a tension applied, and the modulus of elasticity calculated again.

In general, by measuring the tip to tip chord S , the radius of curvature R can be calculated by solving for θ in the following equation:

$$\frac{2}{\theta} \sin\left(\frac{\theta}{2}\right) - \frac{S}{L} = 0$$

R can be easily calculated from θ by

$$R = \frac{L}{\theta}$$

Once the initial radius of curvature R_0 of the rod and the final radius of curvature R of the rod under tension are known, the modulus of elasticity can be calculated by the equations derived earlier.

A tension weight was connected by a string to each end of the bent rod, or hoop, and then the weights were allowed to hang over the sides of a table. The experiment was done two different ways. First, the weights were allowed to sink to equilibrium, expanding the wire hoop. Alternately, the weights were pulled down farther than equilibrium and allowed to rise to equilibrium as the wire hoop contracted. By tapping the table gently, the resulting vibrations caused the system to move towards a more stable

equilibrium. When the weights stopped moving and the wire hoop reached equilibrium, the chord length measurement was taken.

Constants for All Experiments

Length of Rod L	36.060 in
Diameter of Rod d	0.094 in
Tension Applied to Each End T	0.277 lbs
Cross Sectional Moment of Inertia I	$3.8325 \times 10^{-6} \text{ in}^4$

Expanding Wire Hoop:

Initial Chord S_0	26.475 in
Final Chord S	29.310 in
Initial Radius of Curvature R_0	13.668 in
Final Radius of Curvature R	16.513 in
Displacement y	1.965 in
Straightening Force F_s	0.149 lbs
Modulus of Elasticity E	38,769,000 psi

Initial Chord S_0	26.540 in
Final Chord S	29.605 in
Initial Radius of Curvature R_0	13.719 in
Final Radius of Curvature R	16.909 in
Displacement y	2.145 in
Straightening Force F_s	0.143 lbs
Modulus of Elasticity E	34,018,000 psi

Initial Chord S_0	26.505 in
Final Chord S	29.625 in
Initial Radius of Curvature R_0	13.692 in
Final Radius of Curvature R	16.937 in
Displacement y	2.183 in
Straightening Force F_s	0.143 lbs
Modulus of Elasticity E	33,322,000 psi

Initial Chord S_0	26.595 in
Final Chord S	29.556 in
Initial Radius of Curvature R_0	13.763 in
Final Radius of Curvature R	16.842 in
Displacement y	2.073 in
Straightening Force F_s	0.144 lbs
Modulus of Elasticity E	35,470,000 psi

Average Modulus of Elasticity	35,394,800 psi
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Contracting Wire Hoop:

Initial Chord S_0	26.480 in
Final Chord S	30.110 in
Initial Radius of Curvature R_0	13.672 in
Final Radius of Curvature R	17.654 in
Displacement y	2.576 in
Straightening Force F_s	0.132 lbs
Modulus of Elasticity E	26,203,000 psi

Initial Chord S_0	26.590 in
Final Chord S	30.330 in
Initial Radius of Curvature R_0	13.759 in
Final Radius of Curvature R	18.008 in
Displacement y	2.680 in
Straightening Force F_s	0.128 lbs
Modulus of Elasticity E	24,302,000 psi

Initial Chord S_0	26.604 in
Final Chord S	30.264 in
Initial Radius of Curvature R_0	13.770 in
Final Radius of Curvature R	17.900 in
Displacement y	2.618 in
Straightening Force F_s	0.106 lbs
Modulus of Elasticity E	20,635,000 psi

Average Modulus of Elasticity	23,719,000 psi
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Average Modulus of Elasticity for All Experiments:	29,557,000 psi
Percent Error Between Straight Wire Determination and Bent Wire Determination:	0.7 percent

TESTING ACTUAL ELECTRICAL CABLES

The moduli of elasticity for two garden variety electrical cables were determined using the bent wire method, as described above. One of the cables was a piece of Intercomp RG 58 C/U Mil Type, and the other a piece of Belden E 83242 M17/158-00001 Mil-C-1755494 ---- E34972 1C18 Shielded 200C (UL) CL2P ---- Made with Dupont Teflon, which was brown in color. Hence forth, the first cable will be referred to as simply RG 58, and the second cable as "The Brown Cable".

RG 58 Cable

Length of Wire L	15.215 in
Diameter of Wire d	0.196 in
Tension Applied to Each End T	0.0523 lbs
Cross Sectional Moment of Inertia I	$7.244 \times 10^{-5} \text{ in}^4$

Expanding Wire Hoop:

Initial Chord S_0	11.250 in
Final Chord S	14.516 in
Initial Radius of Curvature R_0	5.830 in
Final Radius of Curvature R	14.389 in
Displacement y	2.848 in
Straightening Force F_s	$7.14 \times 10^{-3} \text{ lbs}$
Modulus of Elasticity E	5,075 psi

Initial Chord S_0	11.375 in
Final Chord S	14.570 in
Initial Radius of Curvature R_0	5.933 in
Final Radius of Curvature R	14.987 in
Displacement y	2.846 in
Straightening Force F_s	$6.59 \times 10^{-3} \text{ lbs}$
Modulus of Elasticity E	4,690 psi

Initial Chord S_0	11.220 in
Final Chord S	14.600 in
Initial Radius of Curvature R_0	5.806 in
Final Radius of Curvature R	15.353 in
Displacement y	2.988 in
Straightening Force F_s	$6.29 \times 10^{-3} \text{ lbs}$
Modulus of Elasticity E	4,261 psi

Initial Chord S_0	11.320 in
Final Chord S	14.690 in
Initial Radius of Curvature R_0	5.887 in
Final Radius of Curvature R	16.632 in
Displacement y	3.064 in
Straightening Force F_s	$5.37 \times 10^{-3} \text{ lbs}$
Modulus of Elasticity E	3,552 psi

Average Modulus of Elasticity	4,394 psi
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Contracting Wire Hoop:

Initial Chord S_0	11.690 in
Final Chord S	14.860 in
Initial Radius of Curvature R_0	6.215 in
Final Radius of Curvature R	20.261 in
Displacement y	3.124 in
Straightening Force F_s	3.64×10^{-3} lbs
Modulus of Elasticity E	2,361 psi
Initial Chord S_0	12.208 in
Final Chord S	14.859 in
Initial Radius of Curvature R_0	6.769 in
Final Radius of Curvature R	20.232 in
Displacement y	2.768 in
Straightening Force F_s	3.65×10^{-3} lbs
Modulus of Elasticity E	2,671 psi
Initial Chord S_0	12.205 in
Final Chord S	14.960 in
Initial Radius of Curvature R_0	6.765 in
Final Radius of Curvature R	23.930 in
Displacement y	2.983 in
Straightening Force F_s	2.62×10^{-3} lbs
Modulus of Elasticity E	1,778 psi
Initial Chord S_0	12.475 in
Final Chord S	14.860 in
Initial Radius of Curvature R_0	7.112 in
Final Radius of Curvature R	20.261 in
Displacement y	2.576 in
Straightening Force F_s	3.64×10^{-3} lbs
Modulus of Elasticity E	2,862 psi
Average Modulus of Elasticity	2,418 psi

Average Modulus of Elasticity for All Experiments:

3,406 psi

Brown Cable

Length of Wire L	20.280 in
Diameter of Wire d	0.193 in
Tension Applied to Each End T	0.0523 lbs
Cross Sectional Moment of Inertia I	$6.811 \times 10^{-5} \text{ in}^4$

Expanding Wire Hoop:

Initial Chord S_0	14.925 in
Final Chord S	16.850 in
Initial Radius of Curvature R_0	7.715 in
Final Radius of Curvature R	9.800 in
Displacement y	1.362 in
Straightening Force F_s	0.0256 lbs
Modulus of Elasticity E	95,784 psi

Initial Chord S_0	15.062 in
Final Chord S	17.000 in
Initial Radius of Curvature R_0	7.824 in
Final Radius of Curvature R	10.034 in
Displacement y	1.391 in
Straightening Force F_s	0.0245 lbs
Modulus of Elasticity E	89,821 psi

Initial Chord S_0	14.988 in
Final Chord S	17.060 in
Initial Radius of Curvature R_0	7.765 in
Final Radius of Curvature R	10.132 in
Displacement y	1.487 in
Straightening Force F_s	0.0241 lbs
Modulus of Elasticity E	82,535 psi

Average Modulus of Elasticity	89,380 psi
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Contracting Wire Hoop:

Initial Chord S_0	14.925 in
Final Chord S	17.275 in
Initial Radius of Curvature R_0	7.715 in
Final Radius of Curvature R	10.506 in
Displacement y	1.703 in
Straightening Force F_s	0.0225 lbs
Modulus of Elasticity E	67,427 psi

Initial Chord S_0	15.062 in
Final Chord S	17.364 in
Initial Radius of Curvature R_0	7.825 in
Final Radius of Curvature R	10.673 in
Displacement y	1.689 in
Straightening Force F_s	0.0219 lbs
Modulus of Elasticity E	66,067 psi

Initial Chord S_0	14.988 in
Final Chord S	17.375 in
Initial Radius of Curvature R_0	7.764 in
Final Radius of Curvature R	10.694 in
Displacement y	1.746 in
Straightening Force F_s	0.0218 lbs
Modulus of Elasticity E	63,665 psi

Average Modulus of Elasticity **65,720 psi**

Average Modulus of Elasticity for All Experiments: **77,550 psi**

BUOYANT FORCES vs. BENDING FORCES

When the readout cable is straightened out, there is a restoring force, or bending force on the cable which tries to return it to its initial radius of curvature. If the modulus of elasticity is sufficiently low, and the cable sufficiently long, how does the buoyant force compare to the bending force? The two forces were calculated for a cable with the following characteristics:

Length of Cable	3 m = 118.11 in
Initial Radius of Curvature	1 m = 39.37 in
Final Radius of Curvature	6 m = 236.22 in
Diameter of Cable	0.00635 m = 0.25 in
Density of Cable	0.75 g/cm ³
Modulus of Elasticity	3.447×10 ⁷ Pa = 5,000 psi

$$B_{Cable} = \pi l \left(\frac{d}{2} \right)^2 g (\rho_{D_0} - \rho_{Cable})$$

$$F_{Bending} = \frac{3Ely}{l^3}$$

The 3 meter length was chosen so that at the initial radius of curvature of 1 meter, the cable does not quite form a half circle. That the actual cables are likely to be longer than 3 meters is not important since this is just a rough comparison. Also, since the length of the cable is a cubic term in the denominator, a longer cable will only serve to make the bending force smaller.

Buoyant Force	$2.33 \times 10^{-1} \text{ N} = 5.23 \times 10^{-2} \text{ lbs}$
Bending Force	$2.72 \times 10^{-4} \text{ N} = 6.10 \times 10^{-5} \text{ lbs}$

The buoyant force of the cable is approximately three orders of magnitude larger than the bending force in this case. If it is possible to select readout cable wire with a reasonable amount of buoyancy and a modulus of elasticity on the order of 5,000 psi, then the cable's buoyant force should provide more than enough tension to straighten the readout cable.

CABLE SHAPE

Since the bending forces on the cable are so small, the shape of the cable can be modeled as though it were infinitely flexible without sacrificing an unreasonable amount of accuracy. It can therefore be assumed that the readout cable will attain a shape very close to that of a catenary where the cable is not in contact with the surface of the vessel. The shape of the cable that is in contact with the vessel will of course be that of a circle.

In general, the equation of a catenary is

$$y = \cosh x$$

Since the buoyant forces point up instead of down, the equation becomes

$$y = -\cosh x$$

In order to move the vertex of the catenary to the point (a, b), the equation then becomes

$$y = b + 1 - \cosh(x - a)$$

To vary the shape of the catenary, the equation should be

$$y = b + 1 - \cosh[c(x - a)]$$

For values of c larger than 1, the catenary gets thinner as c increases. If c is less than 1 but greater than 0, the catenary gets fatter as c gets closer to 0. At $c=0$, the catenary is a straight line at $y=b$.

In general, the equation for a circle of radius R centered at the point (u, v) is

$$R^2 = (x-u)^2 + (y-v)^2$$

$$y = v \pm \sqrt{R^2 - (x-u)^2}$$

All of the parameters of the circle are exactly known.

The derivative of the catenary equation is

$$\frac{dy}{dx} = -c \sinh[c(x-a)]$$

And the derivative of the circle equation is

$$\frac{dy}{dx} = -\frac{(x-u)}{\sqrt{R^2 - (x-u)^2}}$$

There are two boundries. One is where the cable comes out of the top of the counter, and the other is where the cable comes into contact with the surface of the vessel. If the top of the counter is taken to be the origin of the coordinate system, then that boundry condition can be simplified because x and y can both be set to zero.

Boundry at top of counter:

Location: $y = 0 = b + 1 - \cosh(ca)$

Slope: $\frac{dy}{dx} = \frac{L_{NCD}}{d_{NCD}} = -c \sinh(ca)$

Boundry where the cable meets the surface of the vessel:

Location: $y = b + 1 - \cosh[c(x_b - a)] = v + \sqrt{R^2 - (x_b - u)^2}$

Slope: $\frac{dy}{dx} = -c \sinh[c(x_b - a)] = -\frac{(x_b - u)}{\sqrt{R^2 - (x_b - u)^2}}$

From the boundary conditions at the origin, several important things can be found.

$$\begin{aligned}
 0 &= b + 1 - \cosh(ca) \\
 b + 1 &= \cosh(ca) \\
 c &= \frac{\cosh^{-1}(b + 1)}{a} \\
 \frac{L_{NCD}}{d_{NCD}} &= \frac{\cosh^{-1}(b + 1) \sinh[\cosh^{-1}(b + 1)]}{a} \\
 a &= \frac{d_{NCD} \cosh(b + 1) \sinh[\cosh^{-1}(b + 1)]}{L_{NCD}} \\
 c &= \frac{L_{NCD}}{d_{NCD} \sinh[\cosh^{-1}(b + 1)]}
 \end{aligned}$$

The reader may notice that a minus sign has been dropped. This is legal and moreover necessary due to the nature of the arc hyperbolic cosine function. If there was a minus sign in the equation for a, then the vertex of the catenary would be to the left of the origin. However, the coordinate system has been defined in such a way that this is not possible.

In order to find x_s and b, the following two equations must be solved.

$$\begin{aligned}
 0 &= b + 1 - \cosh[c(x_s - a)] - v - \sqrt{R^2 - (x_s - u)^2} \\
 0 &= c \sinh[c(x_s - a)] - \frac{(x_s - u)}{\sqrt{R^2 - (x_s - u)^2}}
 \end{aligned}$$

Unfortunately, there does not appear to be an analytic solution for these two equations. So in order to solve for both x_s and b, a numerical approach is the only way to go. There are several different methods to do this, the simplest of which is Newton-Raphson. However, Newton-Raphson will only converge to a root if the initial guess is pretty close to the money. Therefore, a more globally convergent method would be better. One of the best such methods is Broyden's method.

The NCDs for which these calculations were performed were the eight farthest from the center of the vessel (circled tubes on the schematic below). These tubes are approximately 4 meters in length. The top of the tube was tipped towards the center of the vessel by an amount $d_{NCD} = 10 \text{ cm}$ from where it would be if it were exactly vertical. This corresponds to an angle of deviation of 0.025 radians. Since the top of the tube is on the origin of the coordinate system, the center of the vessel is at position (u, v) which is known to be (5.6, -2). The radius of the vessel is $R = 6$ meters.

PLAN VIEW OF EQUATORIAL PLANE OF VESSEL

The following parameters were given:

$$L_{NCD} = 4 \text{ meters}$$

$$d_{NCD} = 0.1 \text{ meters}$$

$$R = 6 \text{ meters}$$

$$u = 5.6 \text{ meters}$$

$$v = -2.0 \text{ meters}$$

The parameters of the catenary are therefore:

$$b = 0.172152 \text{ meters}$$

$$a = 8.84644 \times 10^{-3} \text{ meters}$$

$$c = 65.4123$$

The readout cable meets the surface of the vessel at:

$$x_b = 0.8 \text{ cm}$$

$$y_b = 17.1 \text{ cm}$$

SHAPE OF THE CABLE

By looking at the point where the cable meets the surface of the vessel closer and closer, it is possible to get a better idea of the smoothness of the shape of the cable.

SIZE $\times 10$

SIZE $\times 100$

SIZE $\times 1,000$

In general, the arclength S over the interval (a, b) is

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

The arclength of the catenary part of the readout cable is therefore

$$S_{\text{catenary}} = \int_0^{x_1} \sqrt{1 + \{c \sinh[c(x-a)]\}^2} dx$$

and the arclength of the readout cable which is in contact with the vessel is

$$S_{\text{circle}} = \int_{x_1}^{x_2} \sqrt{1 + \left[\frac{(x-u)}{\sqrt{R^2 - (x-u)^2}} \right]^2} dx$$

It is obvious to the most casual observer that the total arclength of the readout cable is

$$S_{cable} = S_{catenary} + S_{circle} = \int_0^{x_1} \sqrt{1 + \{c \sinh[c(x-a)]\}^2} dx + \int_{x_1}^{x_1 + \pi R - a} \sqrt{1 + \left[\frac{(x-u)}{\sqrt{R^2 - (x-u)^2}} \right]^2} dx$$

Integrating this thing numerically yields the following:

$$S_{catenary} = 0.172 \text{ m}$$

$$S_{circle} = 6.346 \text{ m}$$

$$S_{cable} = 6.518 \text{ m}$$

By taking the arclength of the cable which is in contact with the surface of the vessel, the lateral force on the counter can be found. From this, a new value for the deflection of the counter can be found, which can then be put back into the equations for the shape of the cable, and so on. The value for the deflection of the NCD should converge very quickly due to the fact that the arclength of the cable in contact with the vessel will not change by much.

CONCLUSIONS

The upshot of all of this is that the proportional counters are not likely to require an attachment scheme on the top end to keep them in a vertical orientation, but rather that their inherent buoyancy will be sufficient to ensure a very nearly vertical orientation despite forces to the contrary. Any tension required to further straighten the readout cables beyond what the buoyancy already does will most likely be very small. This is assuming that readout cable with the following characteristics can be selected: Modulus of elasticity on the order of 5,000 psi; Specific gravity on the order of 0.75 g/cm³; Initial radius of curvature on the order of 0.5 meters to 1 meter. For safety's sake, the readout cable should be wound on as large a spool as is reasonably possible.

DATE

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7/27/94

END