1 of 2

Methodology for Reliability Based Condition Assessment

Application to Concrete Structures in Nuclear Plants

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Abstract

Structures in nuclear power plants may be exposed to aggressive environmental effects that cause their strength to decrease over an extended period of service. A major concern in evaluating the continued service of such structures is to ensure that in their current condition they are able to withstand future extreme load events during the intended service life with a level of reliability sufficient for public safety. This report describes a methodology to facilitate quantitative assessments of current and future structural reliability and performance of structures in nuclear power plants. This methodology takes into account the nature of past and future loads, and randomness in strength and in degradation resulting from environmental factors. An adaptive Monte Carlo simulation procedure is used to evaluate time-dependent system reliability. The methodology is applied to concrete structures for illustrative purposes. The time-dependent reliability is sensitive to the time-varying load characteristics and to the choice of initial strength and strength degradation models but not to correlation in component strengths within a system. Identifying the critical components in the system before performing the system reliability analysis enables the size of the analysis to be reduced.

The evaluation of time-dependent reliability and degradation of concrete structures provides a basis for selecting appropriate periods for continued service and/or for determining optimum intervals and extent of inspection and maintenance. Inspection/maintenance strategies are identified that minimize the expected future cost of keeping the failure probability of a structure at or below an established target failure probability during its anticipated service period.

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Glossary

C	=	damage growth rate;
C_f	=	expected losses due to failure of a structure given failure occurs;
C_{ins}	=	cost of inspection;
C_{rep}	=	cost of repair;
C_T	=	total cost of a structure;
D	=	dead load;
D_n	=	nominal dead load;
$d(x)$	=	detectability function;
E	=	earthquake load;
E_{ss}	=	safe shut-down earthquake load;
$F(t)$	=	cumulative distribution function (cdf) of time to failure, T ;
$F_c(t)$	=	compressive strength of concrete at time t ;
$F_S(x)$	=	cdf of load intensity;
$F_X(x)$	=	cdf of X ;
F'_X	=	prior cdf of damage intensity;
F''_X	=	posterior cdf of damage intensity;
f'_c	=	specified compressive strength of concrete at 28 or 90 days;
$f_{R_0}(r)$	=	probability density function (pdf) of initial structural capacity;
f_t	=	tensile strength of concrete;
f_u	=	tensile strength of steel tendon;
$f_X(x)$	=	pdf of X ;
f'_X	=	prior pdf of damage intensity;
f''_X	=	posterior pdf of damage intensity;
f_y	=	specified yield strength of reinforcing bar;
$G(t)$	=	time-dependent degradation function (random function);
$g(t)$	=	time-dependent degradation function (deterministic function);
$g_i(t)$	=	degradation function of i^{th} component;
$h(t)$	=	hazard function;

h_V	= importance sampling function;
L	= live load;
$L(t)$	= reliability function of a component;
L_n	= nominal live load;
$L_S(t)$	= reliability function of a system;
m	= number of inspection/repair operations during the life time of a structure;
$N(t)$	= number of load events that occur within the interval $(0, t)$;
$P[\bullet]$	= probability of event in brackets;
P	= accidental pressure load;
P_a	= nominal accidental pressure load;
P_f	= failure probability;
P'_f	= established target failure probability;
q	= load combination;
$R(t)$	= structural capacity at time t ;
R_0	= initial structural capacity;
R_n	= nominal initial structural capacity;
R_c	= corrosion rate;
S_j	= overall structural action due to the j^{th} load event;
T	= random time to failure;
T	= nominal accidental temperature;
T_I	= damage initiation time (random variable);
T_i	= time at which i^{th} load event occurs;
t_L	= service life of a structure;
t_{P_i}	= time at which i^{th} partial inspection/repair is carried out;
t_{R_i}	= time at which i^{th} full inspection/repair is carried out;
V_X	= coefficient of variation (c.o.v.) of X ;
W_I	= random variable uniformly distributed in $(0, t_L)$;
$X(t)$	= damage intensity at time t ;
$\hat{X}$	= estimator of X ;
x_{cr}	= critical value of damage for partial inspection;
x_{th}	= minimum detectable value of damage;
λ	= mean rate of occurrence of load events;

- ρ = correlation coefficient between initial structural capacities in a system
(Chapter 5);
- ρ = reinforcement ratio (Chapter 6);
- μ_X = mean of X ;
- $\nu(w)$ = damage initiation rate;
- $\Phi(\bullet)$ = standard normal distribution function;
- σ_X = standard deviation of X ;
- τ = mean duration of load events.

Chapter 1

Introduction

1.1 Background

Engineered systems are subject to a phenomenon known as aging, referring to time-dependent changes that may impact the ability of the system to withstand various demands from operation, the environment, and accidents. Vora, et al, (1991) define aging as cumulative changes in a system over time from natural internal processes, external factors, service wear, excessive testing, and improper installation and use. In reinforced concrete structures, for example, the engineering properties of concrete change over an extended service life from the specified properties used as the basis for structural design calculations. Some of these changes may be beneficial to performance, e.g., concrete strength may increase by a factor of two or more over a period of 25 years if it is protected from adverse environmental effects (Washa and Wendt, 1975; Washa, et al, 1989). There are, however, many environmental factors that can cause deleterious changes in construction materials (Tuutti, 1982; Clifton and Knab, 1989). In reinforced concrete structures, the high density of carbon dioxide in industrial areas penetrates the concrete cover on the reinforcement and may lead to the initiation of corrosion, resulting in concrete cracking and a reduction of structural capacity. Sulfates in certain soils attack concrete in contact with the soil and result in cracking and, in severe cases, concrete disintegration. Thus, the structural resistance is actually time-dependent and may decrease with time, causing the risk of structural failure to accelerate.

Facility life extension is assuming greater importance in many areas due to the current economic climate (CERF, 1991; ASME, 1992). In particular, during the next 15 years, the operating licenses for a number of nuclear power plants (NPPs) in the United States will expire. Faced with the prospect of having to replace the lost generating capacity from other sources and substantial shutdown and decommissioning cost, many utilities are expected to seek renewals of their operating licenses. Continued service of a 1000-MW plant for an additional 20 years may result in a present worth saving of perhaps \$1 billion per plant (Jaske and Shah, 1990). NRC's Structural Aging Program is being conducted to evaluate the reliability of concrete structures in NPPs for continued service

(Naus, et al, 1989).

As a part of the license renewal process, as presently constituted (July 1993), a NPP owner/operator will, in general, provide evidence that the safety-related systems in the plant have not degraded during its previous service history to the point where public health and safety might be endangered by significant environmental or abnormal operating challenges during the license extension period. This evidence would include data on initial NPP design and construction parameters, operating history, an assessment of current condition, and projected performance during the license extension period requested. While qualitative decision guidelines related to these factors are available for some engineered systems, quantitative methodologies to support license extension applications, their evaluation and regulatory rule-making are currently in a rudimentary stage of development (Naus, 1986; Ellingwood and Mori, 1992). Thus, there is some urgency in developing the methodology needed to support the evaluation of license extension applications.

A major concern in evaluating NPP structures for continued service is in ensuring that their capacity to withstand extreme events has not degraded due to structural aging during their previous service history. Although properly designed and maintained structures normally perform well over an extended period of time, the design margins of safety are assumed to remain relatively constant for the entire life of a structure (Somerville, 1986; Vesikari, 1988), typically 50 to 100 years or more. Structural performance in service is ensured by the inherently conservative nature of design. There is no presumption in code development that periodic inspection, maintenance and repair are carried out.

Much of the focus in research on the impact of aging on NPP structures has been on mechanical/electrical components. Unlike structures, they are easily replaced with a minimum disruption of facility operation if they wear out. Test data are available for modeling their degradation. Structures, in contrast, are passive. Their degradation may not be noticeable until failure is impending. Moreover, structural components are practically impossible to replace without taking the facility out of service for an extended time. Despite the importance of structures to plant safety, only limited data are available to evaluate their performance in service.

Current codes of practice for civil structures (e.g., ACI Standard 318, ACI Standard 349) take into account the uncertain nature of resistance and loads only in a subjective way. However, the safety problem in structural engineering can be treated more rationally with probabilistic methods. These methods provide basic tools for evaluating structural safety quantitatively (Melchers, 1987). Uncertainties in loads, material properties, and construction practice, which have been traditionally dealt with by empirical safety factors, can be taken into account explicitly and consistently in probabilistic safety assessment. Such methods have been used to develop a first-generation of limit state design codes, in which the safety checks are associated with a specified limit state probability or reliability (Ellingwood, et al, 1982; Galambos, et al, 1982; Hwang, et al, 1987). However, neither existing nor proposed limit states design codes explicitly address strength degradation due to aging.

Time-dependent aging of a structure must be taken into account to evaluate reliability during its service life. Few reliability analyses have been performed of concrete structures

in which the time-dependent change in resistance is taken into account. Siemes, et al (1985) and Vesikari (1988) presented some examples to evaluate the failure probability of concrete structures with deterioration caused by corrosion of reinforcement. In these studies, the limit state of the structure was defined as either the spalling of the concrete cover or the appearance of surficial cracks. However, the probability of structural failure under a random sequence of loads was not considered.

Reliability-based methods for assessing the current condition and future safety of concrete structures in NPPs should take into account the stochastic nature of past and future loads, randomness in strength, and uncertainty in material changes due to aging and possible degradation, mechanistic models used to predict time-dependent structural behavior, and inspection and nondestructive evaluation techniques. Recently, methods have been developed to perform these assessments (Ellingwood and Mori, 1992; Mori, 1992). These methods can be used to answer the following questions or at least to frame the questions in a logical way:

1. Which aging factors are particularly significant for concrete structures in terms of their future reliability?
2. To what extent has the original strength of the structure degraded over time? What would be the service life of a concrete structure in an aggressive environment?
3. What is the residual life or strength of the structure, and how would it respond to a design-basis event?
4. Which nondestructive evaluation techniques or in situ strength measurements are most useful for demonstrating reliability of an existing structure?
5. What sort of inspection procedures and sampling plans should be implemented to provide the needed assurance of reliability?
6. What repair techniques are most effective in extending useable life and enhancing reliability?

Costs of inspection and repair may be a significant part of the overall lifetime cost of a structure. These costs should be balanced by the benefits to be gained, both in economic and reliability terms. There are tradeoffs between the extent and accuracy of inspection, required level of reliability, and cost. One could envision two alternatives: relatively frequent but cursory inspections or infrequent, comprehensive inspections. Reliability-based condition assessment and life prediction methodologies should provide guidance to schedule the routine inspection/maintenance, if necessary, to keep the failure probability of the structure at or below an established target probability during its service life.

1.2 The Structural Aging Program

The structural aging program (SAG) is being conducted under the sponsorship of the USNRC by Oak Ridge National Laboratory. The principal objectives of the Structural

Aging Program (SAG) are to (1) develop criteria to ensure that degradation of plant components and structures does not endanger public health and safety, and (2) provide a technical basis for criteria and guidelines for license review and extension. In support of these objectives, the following issues are being addressed:

- Which components and structures are susceptible to detrimental aging effects? Which of these could be replaced and which are irreplaceable?
- What are the significant degradation mechanisms? Can they be modeled mathematically and how?
- What criteria and supporting evidence are necessary for license extension evaluations?
- What sort of inspection/maintenance programs will be necessary to ensure acceptable in-service performance?

The main tasks being conducted as part of the SAG are summarized in Figure 1.1[†].

1.3 Research Objectives and Scope

This research is in support of Task S.4 of the Structural Aging Program. Task S.4 is summarized in Figure 1.2. The goal of this task is to develop a set of probability-based tools to facilitate quantitative assessments of current and future structural reliability and performance of concrete structures. This goal is supported by the following objectives:

1. Identify predictive models that enable the change in strength of concrete structures over time to be evaluated in terms of initial conditions, service load history, and aggressive environmental factors.
2. Formulate probabilistic methods to predict the structural reliability of existing concrete structures during their service lives, taking into account initial conditions of the structure, service history, and structural aging.
3. Develop methods to identify optimum inspection/maintenance strategies which minimize the expected lifetime cost of the structure and ensure that the structure has a sufficient level of reliability for public safety during a prescribed service life.

In previous work (Ellingwood and Mori, 1992), degradation mechanisms affecting the strength of reinforced concrete structures in NPPs were identified and their potential for impacting time-dependent strength was described. Basic statistical data on structural loads and strengths were compiled. Concepts of time-dependent reliability analysis were

[†]Tables and Figures for each chapter are collected at the end of the chapter of this report.

introduced and illustrated with simple components. Finally, the role played by in-service inspection in maintaining reliability at or above a desired level was described. This report completes the development of time-dependent reliability methods for structural systems, performs an analysis to indicate sensitivity of reliability to data quality and various modeling assumptions, and describes a procedure for determining optimal inspection/repair policies for reliability assurance.

1.4 Outline of Study

This report is organized into seven chapters with tables and figures appearing at the end of each chapter. Two addenda are attached to Chapter 5 providing details of derivations.

In Chapter 2, common degradation mechanisms affecting concrete and reinforcement are reviewed, with particular attention to mathematical models that could be used in predicting stochastic deterioration in structural strength. Mechanisms are identified that may cause strength degradation in a concrete member that may not be apparent from a cursory visual inspection.

Chapter 3 reviews the data that are available to describe statistically loads acting on concrete structures and components and the structural resistance in common limit states. Many of these data were determined in previous research to develop probability-based design requirements for ordinary building structures and for concrete structures in nuclear power plants (Ellingwood, et al, 1982; Galambos, et al, 1982; Hwang, et al, 1987).

Chapter 4 presents the basic reliability analysis concepts and methods for time-dependent reliability assessment. The basic (undegraded) strength models presented in Chapter 3 are combined with the degradation relationships in Chapter 2 to determine the probability distribution of strength as a function of time. This time-dependent strength model is convolved with the stochastic load models to determine limit state probabilities and hazard functions (conditional failure rates) for a period of 60 years. Time-dependent reliability of a component subjected to more than one load process is also addressed. Illustrations of the time-dependent reliability analysis are presented using idealized strength degradation models. Statistical data describing loads and resistance for concrete structures in nuclear power plants are used for illustrative purposes in this chapter as well as in the subsequent chapters.

In Chapter 5, the methods for evaluating the time-dependent reliability of single components are extended to systems of components in order to evaluate the reliability of a structure. Because of the complexity of the problem, efficient Monte Carlo methods are developed to perform the multivariate reliability analyses.

The role of in-service inspection (ISI) and maintenance in maintaining reliability is addressed in Chapter 6. The strength reduction of a component is evaluated taking the effectiveness of inspection/maintenance into account. The probability of detecting dam-

age of a given size or extent is considered and models of damage initiation and intensity are introduced to take into account the effect of damages overlooked at an inspection or detected but not repaired. The effects of in-service inspection and maintenance are incorporated in the time-dependent reliability analysis, and are illustrated using idealized damage detection and repair functions. Inspection/maintenance strategies are identified that minimize the expected total future cost, while keeping the failure probability at or below an established target failure probability during the service life of a structure. The method is illustrated using an one-way slab subjected to corrosion initiated by carbonation of concrete.

Chapter 7 summarizes the main conclusions and makes recommendations for future study.

Figure 1.1: Overview of Structural Aging Program

Figure 1.2: Task S4: Quantitative Methodology for Service Life Prediction

Chapter 2

Time-Dependent Strength of Concrete Structures in NPPs

2.1 Introduction

The performance of a structure in service is assessed by comparing its state of behavior to one of several limit states. When a structure or structural component becomes unfit for its intended purpose, it is said to have reached a limit state. In the reliability-based condition assessment methodology developed subsequently, structural performance in the presence of uncertainty is measured in terms of a limit state probability. Ultimate limit states relate to safety under extreme conditions. A concrete structure is considered to have failed when a load effect (e.g., axial force, moment, shear or some combination of these effects) to which it is subjected exceeds its available capacity at that time. Such an event should have a low probability of occurrence. Serviceability limit states relate to disruption of function under conditions of normal usage. Excessive deflections or crack widths under normal service loads are examples of serviceability limit states in concrete structures.

The service life of reinforced or prestressed concrete structural components and systems is dependent on how the material properties, and thus structural resistance, change over time. Such changes are brought about by naturally occurring chemical and physical processes, external environmental factors, excessive loads during testing, environmental or accidental events, wear during service operation, and improper use and maintenance. Aging and durability issues introduce the time factor into limit states design. One cannot deal with durability issues rationally without introducing the notion of design or service life (Somerville, 1986). The limit state probability must be determined with respect to a service life in order to be useful as a decision variable. The service life is defined as that period during which the structure is able to withstand all loads safely. A design service life (or maintenance interval) is that period during which the probability of the structure performing its intended functions is acceptable.

Concrete that has been properly designed, mixed, placed and cured increases in strength well beyond its specified compression strength at 28 or 90-days, f'_c , used as a basis for design, provided that water is available for continuous hydration (Washa and Wendt, 1975). It is not uncommon for concrete strength to increase by a factor of 2 or more over 50 years if it is protected from aggressive environments. The increase in concrete strength as a function of time, $f_c(t)$, can be modeled as,

$$f_c(t) = \frac{t}{\alpha + \beta \cdot t} \cdot f'_c \quad (2.1)$$

in which t is in days, and α and β are constants. For Type II cement, $\alpha = 8$ and $\beta = 0.67$ (ACI SP-27, 1971). This model is based on experimental data up to 1 year.

Experiments conducted over a period of 50 years (Washa and Wendt, 1975; Washa, et al, 1989) show that the increase in compressive strength is roughly proportional to the logarithm of the age during the first 10 to 50 years and the strength changes very little thereafter. Thus, $f_c(t)$ can be modeled as,

$$f_c(t)/f_{c_{28}} = \begin{cases} \alpha + \beta \ln t & t < t_M \\ \alpha + \beta \ln t_M & t \geq t_M \end{cases} \quad (2.2)$$

in which $f_{c_{28}}$ is the 28-day compressive strength, t is in days, and age to maturity t_M depends on the chemical composition of concrete. For concrete made with relatively low C_2S , which contributes the long-term strength of concrete, $t_M \approx 10$ years, and $f_c(t_M)/f_{c_{28}} = 1.67$ yielding $\alpha = 0.541$ and $\beta = 0.138$. The coefficient of variation (c.o.v.) for the 50-year compressive strength for a given set of variables is low, varying between 5 and 10 %. High quality concrete with low permeability may maintain its integrity and have a service life of several hundred years if the conditions to which it is exposed are not severe (Somerville, 1986). However, environmentally aggressive conditions may limit the above increases or cause $f_c(t)$ to decrease over a service period (Ellingwood and Mori, 1992).

2.2 Degradation Mechanisms

Time-dependent reliability analysis and service life predictions for reinforced concrete structures require time-dependent stochastic models of the structural strength. In concept, stochastic strength models can be derived from: (1) mathematical models describing the effects of the aging process resulting from service and environmental factors on steel and concrete materials and component geometry; (2) accelerated life testing; or (3) a combination of the two. At the current state of the art, these effects are often known qualitatively; however, quantitative models that describe material degradation

processes often are empirical in nature (Clifton and Knab, 1989; Clifton, 1991). The service life determinations often require that these models be extrapolated outside the range of experimental data. Deterioration for several common degradation mechanisms is illustrated qualitatively by the models in Figure 2.1.

Degradation mechanisms for concrete buildings, bridges, and structures in nuclear power plants (NPPs) have been summarized elsewhere (ACI, 1985; Naus, 1986; Shah and MacDonald, 1988; Hookham, 1991; Ellingwood and Mori, 1992). In prestressed concrete structures, the prestressing system appears to be most vulnerable to degradation, while in ordinary reinforced concrete structures, corrosion of the reinforcement appears most important. Environmental factors and degradation mechanisms that are most important in determining the service life of concrete structures are summarized in Table 2.1. Degradation mechanisms may occur in combination. However, the necessary data to consider such dependencies do not exist (Clifton and Knab, 1989; Clifton, 1991). Thus, at the current state of the art, it is necessary to consider their effects independently.

Concrete Deterioration

Attack of concrete by sulfate and acids, alkali-aggregate reactions, leaching and salt recrystallization, and other modes of chemical, physical or environmental attack may result in surficial cracking, expansive internal reactions, or both, that lead to internal disintegration of the concrete and loss of load-carrying capacity. Simple relations must be used to model the effect on concrete of environmental factors in time (Clifton, 1991). In general, the depth of attack or deterioration, $x(t)$, can be modeled by,

$$x(t) = \begin{cases} 0 & ; t < t_I \\ K(t - t_I)^\alpha & ; t \geq t_I \end{cases} \quad (2.3)$$

in which t_I is the induction time required to initiate degradation, t is elapsed time, K is a rate parameter dependent on the aggressiveness of the environment and characteristics of the concrete in situ, and α depends on the basic nature of the attack. This kinetic model is basically empirical in nature, with constants that must be determined by the nature of the degradation process. If the process is strictly diffusion-controlled, $\alpha = 1/2$; if the process is strictly rate-controlled, $\alpha = 1$.

Sulfate attack:

$$x(t) = Kt^\alpha; \quad \alpha \geq 1 \quad (2.4)$$

Leaching of calcium hydroxide:

$$x(t) = Kt \quad (2.5)$$

Reinforcement Corrosion

Protection of reinforcement against corrosion involves a physical barrier and a chemical barrier. The physical barrier is related to the amount of concrete cover and the presence of cracks. The chemical barrier comes from the alkalinity of portland cement concrete, which passivates the reinforcement surface and provides a natural barrier against corrosion. Tensile forces that develop in the concrete from volume expansion of the corroding steel cause cracking and subsequent destruction of the concrete cover and a decrease in structural capacity dependent on the area of reinforcement lost due to corrosion. Corrosion products also can cause deterioration in the bond between reinforcement and concrete.

Penetration of the concrete by atmospheric carbon dioxide reduces the natural alkalinity of the concrete. When the penetration reaches the reinforcement, it destroys the passive oxide layer protecting the reinforcement and allows active corrosion to initiate. Carbonation is basically a diffusion-controlled process. The depth of carbonation, $x(t)$, is given as,

$$x(t) = K\sqrt{t} \quad (2.6)$$

in which t = elapsed time and K = constant dependent on the composition of the concrete and the concentration of carbon dioxide in the environment. If concrete is sheltered from rain, K is estimated by (Tuutti, 1982)

$$K = 26(W/C - 0.3)^2 + 1.6 \quad (\text{mm/year}) \quad (2.7)$$

where W/C is the water-cement ratio of the concrete, by weight.

Cracking in concrete components and structures may have little effect on their strength. However, the carbon dioxide in the air penetrates the concrete more easily at a crack (Vesikari, 1988), and the initiation period of carbonation decreases dramatically. In this case, the depth of carbonation at a crack is obtained from,

$$x(t) = 50\sqrt{w} \sqrt[4]{t} \quad (\text{mm}) \quad (2.8)$$

where w is the crack width (mm).

Chloride ions may be present as a result of deicing salts, natural constituents of certain aggregates, and certain chemical admixtures. The diffusion of chloride ions to the depth of the reinforcement can be predicted by (Vesikari, 1988; Clifton, 1991),

$$\frac{C}{C_s} = 1 - \text{erf} \left(\frac{x}{2\sqrt{D \cdot t}} \right) \quad (2.9)$$

in which C = chloride concentration for initiation of corrosion, C_s = surface chloride content, D = diffusion coefficient for chlorides, and $erf(\bullet)$ = error function. Setting C equal to the critical concentration for initiation, this equation may be solved for the initiation time, t_I , required to depassivate the surface of the reinforcement. Alternatively and more simply,

$$x(t) = K\sqrt{t} \quad (2.10)$$

in which $x(t)$ = depth of chloride ion penetration at time t and K = constant dependent on the permeability and moisture content of the concrete and concentration of chloride ions.

Active corrosion begins once the level of carbonation or aggressive ion penetration has reached the reinforcement. Once active corrosion has initiated, the corrosion rate, r_c , is dependent on the corrosion current, area of anodes, relative humidity and temperature (Tuutti, 1982). The depth of penetration of active corrosion, $x(t)$, is given as,

$$x(t) = R_c \cdot t \quad (2.11)$$

If the propagation time is evaluated as the time required for spalling of the concrete cover, the period of active corrosion, t_P , is given as,

$$t_P = k_a \cdot c \quad (2.12)$$

in which c = reinforcement cover and k_a = constant. The total service life, t_L , would then be (Vesikari, 1988),

$$\begin{aligned} t_L &= t_I + t_P \\ &= K \cdot c^2 + k_a \cdot c \end{aligned} \quad (2.13)$$

in which t_I = time required to initiate active corrosion. In most instances, t_P is substantially less than t_I .

2.3 Summary

The relations presented in this chapter are used to determine the dependence of structural strength on time in the reliability analysis. The decrease in component strength as a function of time is computed directly from $x(t)$. The random nature of the rate

parameter, K , for each mechanism is considered in modeling the degradation of structural components later in this study.

Table 2.1: Mechanisms Affecting the Performance of Concrete Structures

Concrete
Aggressive chemical attack from sulfates and acids
Alkali-aggregate reactions
Leaching
Freeze-thaw cycling
Elevated temperature effects
Creep and shrinkage
Deformed Bar Reinforcement
Corrosion
Elevated temperature
Prestressing Reinforcement
Corrosion from carbonation, chloride attack
Elevated temperature
Detensioning

Figure 2.1: Material Degradation (Clifton and Knab, 1999)

Chapter 3

Statistical Data on Loads and Resistance

3.1 Introduction

The condition assessment and time-dependent reliability analysis require probabilistic descriptions of loads and structural strengths. Some of the statistical data needed to support this methodology have been collected as part of previous research to develop probability-based design requirements for ordinary building construction and for nuclear power plant structures (Ellingwood, et al, 1982; Galambos, et al, 1982; Hwang, et al, 1987). Load and resistance models and data have been analyzed previously in detail (Ellingwood and Mori, 1992) and are summarized in this chapter.

3.2 Structural Loads

The occurrence and variation in intensity of a structural load, $X(t)$, in space and in time is modeled as a stochastic process. It is convenient to classify these loads in a general sense according to the way in which they vary in time, as illustrated by the typical sample functions in Figure 3.1. Two sets of probability density functions (pdf) are also shown in Figure 3.1; $f_X(x)$ describes the intensity of the load at any time, while $f_Z(x)$ describes the maximum load within the interval of time $(0, t)$. Permanent loads (Figure 3.1(a)) are random in intensity but do not vary significantly in time. The probability density function of the load at any point in time, $f_X(x)$, is sufficient to describe the statistical characteristics of the load process, and is identical to $f_Z(x)$. Loads with significant variation in time may act essentially continuously (Figure 3.1(b)) or intermittently (Figure 3.1(c)). The probability density function at any point in time is insufficient to describe these load processes statistically or to determine $f_Z(x)$; information concerning

the time-variation of the load process is required as well.

A relatively simple model of the overall temporal variation in structural loads is provided by assuming that the changes in the load magnitude occur in time according to a Poisson process. Under this assumption, the number of load changes, $N(t)$, in the interval $(0, t]$ are described by the probability function,

$$P[N(t) = n] = \frac{(\lambda t)^n \cdot \exp(-\lambda t)}{n!} ; \quad n = 0, 1, 2, \dots \quad (3.1)$$

in which $P[\bullet]$ = probability of the event in the brackets and the parameter λ = mean rate of occurrence of the events. The duration of one load event, T_i , also is a random variable. If the events occur according to a Poisson process with parameter λ and the load is a continuous variable load (Figure 3.1(b)), the duration has an exponential distribution, with mean value $E[T_i] = \tau = 1/\lambda$. If the load is intermittent ($X(t) = 0$ for significant periods of time (Figure 3.1(c)), and $E[T_i] = \tau$, the mean occurrence rate is $\lambda = p/\tau$, in which p = probability that the load is nonzero at any point in time.

If the load intensity varies negligibly or slowly during each load event, its effect on a structure is essentially static. For reliability analysis purposes, then, the load intensity can be treated as constant during any event. The actual time-varying load thus is replaced with the sequence of load pulses illustrated by the dotted lines in Figures 3.1(b) and 3.1(c). It is customary to assume that the pulse magnitudes are identically distributed and statistically independent random variables, each with cumulative distribution function (cdf), $F_X(x)$. Such a load process is referred to as a Poisson pulse process (Larrabee and Cornell, 1981; Pearce and Wen, 1984). The parameters λ , τ , p and $F_X(x)$ provide a complete description of the macro-variation of the load in time.

Load statistics used in the subsequent time-dependent reliability analysis are summarized in Table 3.1. These statistics are believed to be reasonable and conservative. Only limited data on NPP loads are available; more detailed descriptions are given elsewhere (Ellingwood and Mori, 1992; Mori, 1992).

3.3 Structural Resistance

The strength, R , of a reinforced concrete component is described by (Ellingwood and Hwang, 1985),

$$R = B \cdot R_m(X_1, X_2, \dots, X_m) \quad (3.2)$$

in which $X_1, X_2, \dots$ are basic random variables that describe yield strength of reinforcement, compressive or tensile strength of concrete, and structural component dimensions or section properties. The function $R_m(\bullet)$ describes the strength based on principles of

structural mechanics. Modeling assumptions invariably must be made in deriving $R_m(\bullet)$, and the factor B describes errors introduced by modeling and scaling effects. The probability distribution of B describes bias and uncertainty that are not explained by the model $R_m(\bullet)$ when values of all variables X_i are known. The probability distribution of B can be assumed to be normal (MacGregor, et al, 1983). A more accurate behavioral model will lead to a decrease in the variability in B and thus in R .

Probability models for R usually must be determined from the statistics of the basic variables, X_i , since it seldom is feasible to test a sufficient sample of structural components to determine the cumulative distribution function (cdf) of R directly. Typical statistical data on material strengths and dimensions of reinforced concrete structural elements are summarized in Table 3.2 (MacGregor, et al, 1983; Ellingwood and Hwang, 1985; Ellingwood and Mori, 1992). The material strengths presented are based on "static" rates of load. Long-term strength changes in the concrete or steel due to maturity of concrete, environmental stressors, and possible corrosion of reinforcement are not reflected in these statistics. When data on the strength of concrete or steel can be obtained by limited in situ sampling as part of a condition assessment of a specific structure, that data should be utilized.

Typical means and c.o.v. in strength for reinforced concrete structural elements are summarized in Table 3.3. These estimates were obtained by Monte Carlo simulation using the strength and dimensional variabilities in Table 3.2. The modeling uncertainty was described by a normal distribution, with a mean $\mu_B = 1.05$ and c.o.v. $V_B = 0.06$ for flexure and compression and with $\mu_B = 1.15$ and $V_B = 0.15$ for shear. In all cases, the nominal flexural strength for concrete structural members, M_u , the nominal shear strength, V_u and the nominal axial strength R_u were computed according to ACI Standards 318 and 349.

Table 3.1: Statistical Properties of NPP Loads

Load	Rate of Occurrence	Duration	Mean ⁺	C.o.v.	Pdf
Dead	n/a	n/a	$1.0D_n$	0.07, 0.10	Normal
Live	0.5 yr	3 mo	$0.4L_n$	0.50	Type I
Earthquake	0.11 yr	30 sec	$0.078E_{ss}$	0.85 [*]	Type II [*]
Equipment	n/a	40 yr	$1.0Q_n$	0.04	Normal
Acc. Pressure	1/10000 yr	20 min	$0.8P_a$	0.20	Type I
Acc. Temp.	1/10000 yr	20 min	$0.9T_a$	0.08	Type I

⁺ D_n , L_n , Q_n , P_a and T_a are nominal loads;

E_{ss} is safe-shutdown earthquake load.

^{*}For the pdf of annual maximum values, $F_{E_{ann}}(x)$

Table 3.2: Statistical Data for Reinforced Concrete

Parameter	Mean	C.o.v.	Pdf
Material Strength			
f_c	$960 + 0.8f'_c$	0.12	Normal
f_t	$6.4 \sqrt{f'_c}$	0.18	Normal
f_y (ASTM A615/Gr.60)	67 ksi	0.11	Lognormal
f_u (ASTM A416/A421)	270 ksi	0.04	Lognormal
Dimensions			
Overall dimensions (in)	Nominal, h	$0.4/h$	Normal
Placement of reinf.(in)	Nominal, d	$0.6/d$	Normal

Table 3.3: Probabilistic Descriptions of Resistance of Concrete Components

Limit state	Mean*	C.o.v.	Pdf
Beam, flexure	$1.11M_n$	0.13	Lognormal
Beam, shear	$1.22V_n$	0.18	Lognormal
Wall, shear	$1.35V_n$	0.18	Lognormal
Short wall, shear	$1.70V_n$	0.18	Lognormal
Slabs, flexure	$1.12M_n$	0.14	Lognormal
Short column, compression	$1.13P_n$	0.14	Lognormal
Short column, tension	$1.12P_n$	0.13	Lognormal

*The nominal values M_n , V_n , and P_n are the capacities that would be computed from the ACI Standards 318 or 349 (ACI 318, 1989; ACI 349, 1990)

3.1(a) Permanent Loads

3.1(b) Continuous Variable Loads

3.1(c) Intermittent Variable Loads

Figure 3.1: Typical Sample Functions of Structural Loads

Chapter 4

Time-Dependent Structural Reliability Analysis

4.1 Introduction

The ability to evaluate structural behavior in service is affected by uncertainties in the structural load environment, material strengths, dimensions, and models used to predict response. As a result, it is not possible to ensure acceptable performance in an absolute sense. Instead, engineers, code-writers, and regulatory officials can only provide design requirements that result in structures and components with limit state probabilities that are “acceptably” low.

The conceptual basis for the analysis of risk and uncertainty in structural performance is provided by classical structural reliability theory (e.g., Melchers, 1987). The uncertainty in structural behavior is taken into account by modeling the loads on the structure and the strength as random variables, as described in Section 3. In the simplest model of performance of a component with an overall resistance, R , and a dimensionally consistent structural action due to the applied loads, S , the limit state probability or probability of failure can be determined as,

$$P_f = P[R < S] = \int_0^{\infty} F_R(s) f_S(s) ds \quad (4.1)$$

where $F_R(s)$ = cumulative distribution function (cdf) of R , defined as $F_R(s) = P[R < s]$ and $f_S(s)$ = probability density function (pdf) of S , obtained by differentiating its cdf, $F_S(s)$. The reliability is defined as $L = 1 - P_f$. The limit state probability, P_f , provides a quantitative measure of safety of the component or system that reflects the uncertainties in R and S explicitly through the probability laws of R and S .

4.2 Time-dependent Reliability Analysis

Structural loads generally vary randomly in time, as illustrated in Chapter 3. Thus, the probability that a structure or component continues to operate successfully during time interval $(0, t]$ is dependent on t .

4.2.1 Single Components Subjected to One Load Process

If the strength is time-invariant but the load is a function of time, the structure survives if it survives the maximum load to occur within the time interval $(0, t]$. The probability of this event can be calculated in a straightforward fashion if the load history is modeled as a sequence of identically distributed and statistically independent random variables, each with cdf $F_S(s)$. If n loads act within the interval $(0, t]$, the cdf of the maximum load in that interval is,

$$F_{S_{max}}(s, t) = \{F_S(s)\}^n \quad (4.2)$$

If the number of loads is a Poisson process, with mean occurrence parameter λ , then,

$$F_{S_{max}}(s, t) \approx \exp[-\lambda t\{1 - F_S(s)\}] \quad (4.3)$$

In either case, the reliability, $L(t)$, can be obtained as,

$$L(t) = \int_0^\infty F_{S_{max}}(r, t) f_R(r) dr \quad (4.4)$$

in which $f_R(r)$ = pdf of R . $L(t)$ is a monotonically decreasing function of t (Ellingwood and Mori, 1992).

When the strength is a stochastic function of time as well, a different approach is required. Let T = the random time to "failure," failure being defined as loss of load-carrying capacity, excessive deformation, or any other limit state of interest. If both load and resistance are functions of time, the structure fails if $R(t) < S(t)$ for at least one of the load events in $(0, t)$. The probability of failure in the time interval $(0, t]$, $P_f(t)$ is given by,

$$P_f(t) = P[R(t) < S(t) \text{ in } (0, t]] \quad (4.5)$$

Since this event is identical to the event that $T \leq t$, this equation also defines the

cumulative distribution function of T ;

$$P_f(t) = F(t) = P[T \leq t] \quad (4.6)$$

The reliability function is defined as,

$$L(t) = P[T > t] = 1 - F(t) \quad (4.7)$$

The probability density function of T is given as,

$$f(t) = \frac{dF(t)}{dt} = \frac{-dL(t)}{dt} \quad (4.8)$$

$$f(t)dt = P[t < T \leq t + dt] \quad (4.9)$$

in which dt = a small increment of time. $f(t)$ can be thought of as an unconditional failure rate.

A hazard or conditional failure rate function, $h(t)$, is defined as the conditional probability that failure occurs in the interval $(t, t + dt]$, given that the structure has survived up to t ;

$$h(t)dt = P[t < T \leq t + dt | T > t] \quad (4.10)$$

$$= \frac{f(t)dt}{1 - F(t)} \quad (4.11)$$

Using Eqn.(4.8),

$$L(t) = \exp\left[-\int_0^t h(\tau)d\tau\right] \quad (4.12)$$

Typical functions $F(t)$, $L(t)$, $f(t)$ and $h(t)$ are illustrated in Figure 4.1. If failures occur purely at random during $(0, t]$, $h(t) = \alpha$ (constant), and the reliability function is simply,

$$L(t) = \exp(-\alpha t) \quad ; t \geq 0 \quad (4.13)$$

When failure is due to structural aging, however, $h(t)$ increases with time. Clearly, the management objective in proper design, inspection and maintenance is to keep $F(t)$ below some acceptable threshold probability during the service life of the structure.

The duration of significant load events generally is very short, and thus such events occupy only a small fraction of the total life of a component when viewed on the timescale of the service life of a concrete structure, say 40 years or more. Such structural load events can be modeled as an intermittent Poisson process with mean occurrence rate λ , mean duration τ , and load intensities, S_j , assumed to consist of identically distributed and statistically independent random variables described with distribution function $F_S(x)$. Sample functions of resistance and load are illustrated in Figure 4.2, in which $\mu_R(t)$ and $\mu_S(t)$ are time-dependent mean strength and load. It is assumed that the event duration is sufficiently short that any change to the state of the structure occurs only during the application of the load (i.e., cumulative damage during a load event does not occur).

Assume temporarily that the initial strength of the structural component, R_0 , is deterministic and equal to r . The strength of the component deteriorates with time due to environmental factors according to,

$$r(t) = r \cdot g(t) \quad (4.14)$$

in which $r(t)$ is the strength at time t , and $g(t)$ is the degradation function. It is assumed that $g(t)$ is independent of the load history and is deterministic. It has been found that variability in the degradation function is of minor importance when compared to mean degradation and load process characteristics (Ellingwood and Mori, 1992; Mori, 1992; Mori and Ellingwood, 1993). This will be illustrated subsequently. If $N_S(t_L) = n$ events occur within time interval $(0, t_L]$ at deterministic times $t_j, j = 1, \dots, n$, the reliability function is represented as follows:

$$L(t_L) = P[r \cdot g_1 > S_1 \cap \dots \cap r \cdot g_n > S_n] \quad (4.15)$$

$$= \prod_{j=1}^n F_S(r \cdot g_j) \quad (4.16)$$

in which $g_j = g(t_j)$.

In general, the loads occur randomly at times $\underline{T} = \{T_1, \dots, T_n\}$ described by the joint probability density function $f_{\underline{T}}(\underline{t})$. Removing the conditioning on random occurrence time $\underline{T}$, on the number of load events, $N_S(t_L)$, within time interval $(0, t_L]$, and on the initial strength, R_0 , the reliability function becomes (Ellingwood and Mori, 1992; Mori, 1992):

$$L(t_L) = \int_0^\infty \exp \left[-\lambda t_L \cdot \left[1 - \frac{1}{t_L} \int_0^{t_L} F_S\{r \cdot g(t)\} dt \right] \right] f_{R_0}(r) dr \quad (4.17)$$

in which $f_{R_0}(r)$ is the probability density function of the initial strength, R_0 .

4.2.2 Load Combinations

A structure is often subjected to more than one load process. If a component is subjected to two statistically independent load processes with intensities S_1 and S_2 , respectively, but only S_1 varies in time, then from Eqn.(4.17) and by the theorem of total probability the reliability function becomes

$$L(t_L) = \int_0^\infty \int_0^\infty \exp \left[-\lambda_{S_1} t_L \left[1 - \frac{1}{t_L} \int_0^{t_L} F_{S_1} \{r \cdot g(t) - s_2\} dt \right] \right] f_{S_2}(s_2) f_{R_0}(r) ds_2 dr \quad (4.18)$$

in which λ_{S_1} is the mean occurrence rate of S_1 , F_{S_1} and f_{S_2} are the cdf of S_1 and the pdf of S_2 , respectively.

When several time-dependent load processes act on a structure, the probability distribution of the structural action due to the combined loads can be obtained approximately as a function of the statistical descriptors of the individual loads (Wen, 1977; Hwang, et al, 1987). At any instant in time, the structure is subjected to one and only one combination of loads. Thus, the failure probability of the structure during its service life can be estimated from the sum of the failure probabilities of these mutually exclusive load combinations. For example, if a component is subjected to two time-dependent load processes with intensity S_1 and S_2 , respectively, then the failure probability under the following three mutually exclusive non-zero load combinations must be evaluated: only S_1 is non-zero (S_1); only S_2 is non-zero (S_2); or both S_1 and S_2 are non-zero ($S_1 + S_2$). The failure probability of the component, P_f , can be expressed as,

$$P_f = P_f^{(S_1)} + P_f^{(S_2)} + P_f^{(S_1+S_2)} \quad (4.19)$$

in which $P_f^{(q)}$ is the limit state probability under the non-zero load combination q . This probability can be computed approximately by Eqns.(4.7) and (4.17) with $F_S(s) = F_q(s)$ and $\lambda = \lambda_q$, in which F_q is the cdf of the intensity of the load combination q and λ_q is the incidence rate of the non-zero load combination. For $q = (S_1 + S_2)$, the cdf can be evaluated by the convolution integral as,

$$F_{S_1+S_2}(s) = \int_0^\infty F_{S_1}(s - s_2) f_{S_2}(s_2) ds_2 \quad (4.20)$$

The incidence rate of combination $S_1 + S_2$ is evaluated approximately as,

$$\lambda_{S_1+S_2} \approx \lambda_{S_1} \lambda_{S_2} (\tau_{S_1} + \tau_{S_2}) \quad (4.21)$$

in which λ_{S_i} and τ_{S_i} are the mean occurrence rate and the mean duration of S_i . Eqns.(4.19) - (4.21) can be generalized if a component is subjected to more than two statistically independent load processes (Wen, 1977).

4.2.3 Illustration of Component Reliability Analysis

The effect of degradation in component strength on the component reliability function is illustrated using the methods developed in Sections 4.2.1 and 4.2.2 and several simple parametric representations of time-dependent load and strength. A more complete parametric sensitivity study is available elsewhere (Ellingwood and Mori, 1992; Mori and Ellingwood, 1993); here some of the main results are summarized. The range of load and resistance parameters used in this parametric sensitivity study are selected to be representative of the existing statistical data presented in Tables 3.1 and 3.3, but no particular structural component is envisioned in the illustrations. In each analysis, the reference period, t_L , is assumed to be 60 years, the sum of the initial operating period of 40 years plus a tentative license extension of 20 years.

The degradation models considered in this parametric study are illustrated in Figure 4.3 and summarized in Table 4.1. Degradation in strength due to corrosion of reinforcement and sulfate attack are modeled in an approximate manner by the linear, and parabolic functions, respectively. The square root function represents the effect of degradation mechanisms in which the basic process is diffusion-controlled and the rate decreases in time. The degradation rate parameter is given with reference to the strength at 40 years. For example, setting $g(40) = 0.8$ means that the strength at 40 years is 80 % of the initial strength (cf. Eqn.(4.14)). Thus a linear model with $g(40) = 0.8$ in Table 4.1, for example, might correspond to a case where the initial diameter of the reinforcement, D , is 32 mm and the corrosion rate, r_c is constant and approximately equal to 0.04 mm/year.

Let first us consider the combination of live and dead loads ($L + D$). The probabilistic models of load events are presented in Table 3.1. The mean value of initial strength is evaluated based on one of the current design requirements for concrete structures (ACI 318, 1989)

$$0.9M_n = 1.4D_n + 1.7L_n \quad (4.22)$$

in which M_n = nominal flexural capacity, and D_n and L_n = nominal dead and live loads, respectively. The mean values of R , D and L are $\mu_R = 1.15M_n$, $\mu_D = 1.0D_n$, $\mu_L = 0.40L_n$ and it is assumed that $L_n = D_n$. The c.o.v. of the initial strength, V_R , is assumed to equal 0.15. The effect of variability in dead load on $F(t)$ was found to be negligible compared with the effect of other time-dependent factors (Mori, 1992) and $V_D = 0.0$.

The effect of variability in degradation function, $G(t)$, on the limit state probability

is presented in Figures 4.4(a) - 4.4(c) using the linear degradation model,

$$G(t) = 1 - At \quad (4.23)$$

where A is a random variable normally distributed with mean value, $\mu_A =$ (a) 0.0075, (b) 0.005 and (c) 0.0025, corresponding to $E[G(40)] =$ (a) 0.7, (b) 0.8, and (c) 0.9, respectively, and with coefficient of variation, $V_A = 0.1, 0.3$ or 0.5 (only for case (c)). The corresponding c.o.v.'s in the degradation functions are also presented in the figures. The failure probability is evaluated by Monte Carlo simulation. The variability in $G(t)$ has a second-order effect on $F(t)$ compared with other time-dependent factors considered subsequently, if the $V_{G(60)}$ is less than about $0.1 - 0.15$. Accordingly, the variability in the degradation function is not considered in subsequent illustrations.

The effect of the shape of the degradation function on the limit state probability, $F(t)$, is presented in Figures 4.5(a) and (b) for $\lambda = 0.5$. (Note the change in scale of ordinate.) The shape of the degradation functions has a greater influence as $g(40)$ decreases. For lower residual strength at time t , the failure probability is sensitive to small changes in the strength; this effect accumulates with time. Accordingly, the appropriate modeling of $g(t)$ is especially important when the residual strength at t_L may be relatively low.

The effect of the load occurrence rate, λ , on the probability of failure is shown in Figure 4.6(a) using the linear strength degradation model with $g(40) = 0.8$. In Figure 4.6(b), the failure probability associated with nondegrading component strength, i.e. $g(40) = 1.0$, is presented for comparison. The failure probability increases with the mean occurrence rate of the load events, as would be expected. However, order of magnitude changes in λ have less than order of magnitude effects on $F(40)$.

The combinations of accidental pressure and dead loads ($D + P$), and live, earthquake and dead loads ($D + L + E$) demonstrate further the effect of the parameters describing the occurrence and intensity of the load events presented in Table 3.1. The mean value of initial strength is evaluated based on the following current design requirements for concrete structures in NPPs (ACI 349, 1990),

$$0.9M_n = -0.9D_n + 1.5P_a \quad \text{for } D + P \quad (4.24)$$

$$0.9M_n = D_n + L_n + E_{ss} \quad \text{for } D + L + E \quad (4.25)$$

in which P_a = accidental pressure load and E_{ss} = safe shut-down earthquake loads. Structural actions due to P_a and E_{ss} are assumed to equal to $3.0D_n$ and $2.0D_n$, respectively. The mean occurrence rates for the combination of $L + E$ is $\lambda_{L+E} = 0.014$ from Eqn.(4.21). To evaluate the failure probability under $D + L + E$, the convolution integral in Eqn.(4.20) is computed by Gauss-Legendre quadrature.

The effect of the overall degradation in 40 years on the failure probability is presented in Figures 4.7 and 4.8 for $D + P$ and $D + L + E$, respectively, assuming linear strength degradation. The shape of the degradation function also affects the limit state probability, as shown in Figures 4.9(a), (b), and (c) for $D + P$, and in Figures 4.10(a), (b), and (c) for $D + L + E$. The degradation models affect the failure probability under either $D + P$ or $D + L + E$ in a manner similar to that under $D + L$ (see Figure 4.5).

However, the failure probability under $D + P$ is smaller by four orders of magnitude than that under $D + L$ because of the small mean occurrence rate of the loads. Thus, in an absolute sense, the failure probability under $D + P$ is less sensitive to the degradation model than is P_f under $D + L$.

The failure probability under $D + L + E$ is also less sensitive to the degradation rate and the shape of the degradation function. With the linear degradation model and $g(40) = 0.7$, the failure probability under $D + L + E$ increases by a factor of five above that without degradation, while the failure probability under $D + L$ increases by more than two orders of magnitude above that without degradation (Ellingwood and Mori, 1992). When the degradation rate is small ($g(40) \geq 0.9$), the effect of the shape of the degradation function is negligible up to 60 years. This insensitivity of the failure probability under $D + L + E$ to the degradation model is due to the small mean occurrence rate and large variability in load E ; once a load event occurs, the intensity of the load may be large enough to cause failure of the component regardless of whether or not the component has degraded.

The hazard functions, $h(t)$, associated with the strength degradation models presented in Figures 4.5(b) and 4.5(c) ($g(40) = 0.8, 0.9$ and $\lambda = 0.5$) are illustrated in Figures 4.11(a) and 4.11(b). The hazard function associated with the square root model is nearly linear in time for $g(40) = 0.9$. However, $h(t)$ clearly is nonlinear for the linear and parabolic degradation models, and its curvature increases as $g(40)$ decreases. Therefore, linear hazard functions might be reasonable if the degradation mechanism can be described by a diffusion type model with small rate; such models, however, generally are not flexible enough to analyze aging in concrete structural components.

In the above illustrations in Figures 4.4 through 4.11, only the active stage of component strength degradation is considered. In previous work (Ellingwood and Mori, 1992), it was shown that ignoring the initiation phase may lead to an overly pessimistic appraisal of reliability. The effect of variability in the initiation period, T_I , on the failure probability of a component is illustrated in Figures 4.12 - 4.14, using square root, linear, and parabolic strength degradation models expressed as,

$$G(t; T_I) = \begin{cases} 1 & ; t < T_I \\ 1 - a(t - T_I)^\alpha & ; t \geq T_I \end{cases} \quad (4.26)$$

in which the degradation rate, a is determined from the condition $G(40|T_I = 0) = 0.8$. It is assumed that T_I is lognormally distributed with mean $\mu_{T_I} = 10, 20$, or 30 years and with c.o.v. $V_{T_I} = 0.0$ (deterministic initiation time), 0.3 or 0.5. The load combination of $D + L$ is considered. When α in Eqn.(4.26) is small (i.e., square root or linear), the variability in T_I has negligible effect when $\mu_{T_I} = 10$ and 20 years. On the other hand, when α is large (i.e., parabolic model), the effect of the variability in T_I increases. When $\mu_{T_I} = 20$ or 30 years, $F(60)$ with $V_{T_I} = 0.5$ is more than twice as large as $F(60)$ without variability in T_I . The strength with the parabolic degradation model decreases mainly in later stages of exposure, and whether or not this late stage is included during the service-life is highly dependent upon the initiation period.

4.3 Summary

Time-dependent reliability analysis of single components indicates that the reliability is sensitive to the choice of initial component strength and strength degradation models, including initiation period, the mean occurrence rate of load events, and coefficient of variation of the intensity of time varying load. However, it is not sensitive to the variability in initiation period when the strength degradation of a component can be modeled by a linear or a square root function. Appropriate degradation characteristics and load process statistics must be identified to utilize the above methodology, especially in realistic condition assessments. In the next chapter, the reliability of structural systems is considered.

Table 4.1: Degradation Model

Shape of the Degradation Function	Degradation Rate $g(40)$	Corresponding Degradation Mechanism
Linear: $g(t) = 1 - at$	0.7, 0.8, 0.9	Corrosion
Parabolic: $g(t) = 1 - at^2$	0.7, 0.8, 0.9	Sulfate Attack
Square root: $g(t) = 1 - a\sqrt{t}$	0.7, 0.8, 0.9	Diffusion Controlled Degradation

Figure 4.1: Time-dependent Reliability Functions

Figure 4.2: Schematic Representation of Load Process and Degradation of Resistance

Figure 4.3: Typical Degradation Functions

4.4(a) $g(40)=0.7$ 4.4(b) $g(40)=0.8$ Figure 4.4: Dependence of Failure Probability on $V_G(t)$

4.4(c) $g(40)=0.9$

4.5(a) $g(40)=0.8$ 4.5(b) $g(40)=0.9$

Figure 4.5: Dependence of Single Component Failure Probability on Degradation Model: D+L

4.6(a) Linear Strength Degradation: $g(40)=0.8$

4.6(b) Nondegrading Strength

Figure 4.6: Dependence of Single Component Failure Probability on the Mean Occurrence Rate of Load Events

Figure 4.7: Dependence of Single Component Failure Probability on Degradation Rate: D+P

Figure 4.8: Dependence of Single Component Failure Probability on Degradation Rate: D+L+E

4.9(a) $g(40) = 0.7$ 4.9(b) $g(40) = 0.8$

Figure 4.9: Dependence of Single Component Failure Probability on Degradation Model: D+P

$$4.9(c) \quad g(40) = 0.9$$

4.10(a) $g(40)=0.7$ 4.10(b) $g(40)=0.8$

Figure 4.10: Dependence of Single Component Failure Probability on Degradation Model: D+L+E

4.11(a) $g(40)=0.8$ 4.11(b) $g(40)=0.9$

Figure 4.11: Hazard Function of Single Component

Figure 4.12: Dependence of Failure Probability on the Variability in Initiation Period: Square Root Strength Degradation

Figure 4.13: Dependence of Failure Probability on the Variability in Initiation Period: Linear Strength Degradation

Figure 4.14: Dependence of Failure Probability on the Variability in Initiation Period: Parabolic Strength Degradation

Chapter 5

Time-Dependent System Reliability Analysis

5.1 Introduction

A structural system composed of beams, columns, slabs, and walls can be modeled by a combinations of two fundamental subsystems: series systems and parallel systems (Melchers, 1987). A series system fails if any of its components fail, while a strictly parallel system fails only if all its components fail. A statically indeterminate structure generally fails only if failure occurs in more than one member or at more than one point, and thus more resembles a parallel system. However, the reliability analysis of a structure modeled as a series system of components provides a lower bound estimate of the actual system reliability. If F_i denotes the event that component i fails within time interval $(0, t_L]$, the reliability function of a series system, $L_S(t_L)$, can be expressed by

$$L_S(t_L) = 1 - P[F_1 \cup F_2 \cup \dots \cup F_m] \quad (5.1)$$

$$= P[\overline{F}_1 \cap \overline{F}_2 \cap \dots \cap \overline{F}_m] \quad (5.2)$$

in which $\overline{F}_i$ is the complement of event F_i and m is the number of components.

Stochastic dependence between component failures makes an exact calculation of the probability of the union of failure events difficult even for small systems. The exact failure probability of a system can be calculated simply with single variate normal distribution functions under the assumptions that resistances are normally distributed and equally correlated, that the load is deterministic and constant in time and that all elements have the same failure probability (Grigoriu and Turkstra, 1979). In order to relax the assumption that component resistances are equally correlated, Thoft-Christensen and Sorensen (1982) showed that using the average of the correlation coefficients in the above formulation provides a good approximation of system reliability. Bounds on the system reliability generally are easier to determine (Ditlevsen, 1979; Ramachandran, 1984). Cornell (1967) introduced first-order bounds; however, these often are too wide to be

of practical use in structural reliability calculations. Ditlevsen's second-order bounds make use of individual and pair-wise failure event probabilities $P[F_i]$ and $P[F_i \cap F_j]$. These bounds can be quite narrow and insensitive to the correlation between failure events provided that the correlation coefficient is less than about 0.7 (Ditlevsen, 1979; Thoft-Christensen and Sorensen, 1982). These bounding and approximating methods have been developed assuming that the resistance is independent of time.

Monte Carlo simulation can be used to deal with the complexities of time-dependent reliability analysis. Rapid advances in computer technologies in recent years have made this method attractive. However, simulation methods may be time-consuming for structural reliability problems in which small failure probabilities are to be evaluated. In this chapter, efficient methods of Monte Carlo simulation for time-dependent system reliability analysis are developed. These methods are used subsequently for parametric analyses of time-dependent reliability of degrading concrete structural systems in NPPs.

5.2 Formulation of Time-dependent System Reliability

Let us assume that n discrete load events occur at deterministic times t_j , $j = 1, \dots, n$. Load S_j acting on the structure induces a structural action (e.g., moment, axial force, shear, etc.) $c_i S_j$ in member i . For the present, assume that the initial strength of the structural components, $R_0 = \{R_1, \dots, R_m\}$, is deterministic and equal to $r = \{r_1, \dots, r_m\}$. The change in strength of each component in time is described by a degradation function, g_{ij} , which is the fraction of the initial strength of member i remaining at time t_j , i.e., $g_{ij} = g_i(t_j)$. Following Eqn.(5.2), the time-dependent reliability function for a series system, $L_S(t_L)$, is represented as,

$$\begin{aligned}
 L_S(t_L) &= P \left(\begin{array}{ccccccc} r_1 g_{11} > c_1 S_1 & \cap & \dots & \cap & r_1 g_{1n} > c_1 S_n \\ \cap & r_2 g_{21} > c_2 S_1 & \cap & \dots & \cap & r_2 g_{2n} > c_2 S_n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \cap & r_m g_{m1} > c_m S_1 & \cap & \dots & \cap & r_m g_{mn} > c_m S_n \end{array} \right) \\
 &= P \left[\left\{ \bigcap_{i=1}^m \frac{r_i \cdot g_{i1}}{c_i} > S_1 \right\} \cap \dots \cap \left\{ \bigcap_{i=1}^m \frac{r_i \cdot g_{in}}{c_i} > S_n \right\} \right] \\
 &= P \left[\min_{i=1}^m \frac{r_i \cdot g_{i1}}{c_i} > S_1 \cap \dots \cap \min_{i=1}^m \frac{r_i \cdot g_{in}}{c_i} > S_n \right] \tag{5.3}
 \end{aligned}$$

Assuming that loads S_j are identically distributed and statistically independent random variables,

$$L_S(t_L) = \prod_{j=1}^n F_S \left(\min_{i=1}^m \frac{r_i \cdot g_{ij}}{c_i} \right) \tag{5.4}$$

The conditioning on the number of load events, $N = n$, the times at which they occur, $\underline{T} = \underline{t}$, and the initial strength, $\underline{R}_0 = \underline{r}$ must be removed to take into account the randomness in occurrence of load events and in strength. Applying a procedure similar to that used to analyze component reliability (Eqns.(4.16) - (4.17)) the failure probability for the system is evaluated as,

$$\begin{aligned} F(t_L) &= 1 - L_S(t_L) \\ &= \underbrace{\int_0^\infty \cdots \int_0^\infty}_{m\text{-fold}} F(t_L | \underline{R}_0 = \underline{r}) \cdot f_{\underline{R}_0}(\underline{r}) d\underline{r} \end{aligned} \quad (5.5)$$

in which

$$\begin{aligned} F(t_L | \underline{R}_0 = \underline{r}) &= 1 - L_S(t_L | \underline{R}_0 = \underline{r}) \\ &= 1 - \exp \left[-\lambda t_L \cdot \left\{ 1 - \frac{1}{t_L} \int_0^{t_L} F_S \left(\min_{i=1}^m \frac{r_i \cdot g_i(t)}{c_i} \right) dt \right\} \right] \end{aligned} \quad (5.6)$$

and $f_{\underline{R}_0}(\underline{r})$ is the joint probability density function (pdf) of the initial component strengths. Eqn.(5.5) cannot be integrated in closed form because of the $(m+1)$ -fold integration. The analysis of even a 2-component system requires lengthy computations.

Monte Carlo simulation is one of the most powerful and commonly used techniques for analyzing complex physical problems. Although Monte Carlo simulation is sometimes criticized as being crude and time-consuming, several variance reduction techniques have been developed to reduce the number of samples required to estimate the probability of failure; these techniques include stratified sampling, Latin hypercube sampling, directional simulation, and importance sampling (McGrath and Irving, 1975; McKay, et al, 1979; Rubinstein, 1981; Verma, et al, 1989). In such procedures, the sampling is altered so that the rare events can be observed more frequently. Among these, importance sampling has been shown in recent studies (e.g., Harbitz, 1986; Bucher, 1988; Karamchandani, et al, 1989; Melchers, 1989) to have the potential for increasing the efficiency of structural reliability analysis.

The limit state functions for the analysis of time-dependent system reliability are complicated. Since a limit state function has to be considered for each load event, the number of limit state functions is random and the functions change in time. Besides, a large number of random variables are associated with this problem; intensity of loads, the times of load occurrences, and initial strengths of components. Therefore, performing the m -fold integration in Eqn.(5.5) by Monte Carlo simulation while evaluating Eqn.(5.6) numerically in closed form is more feasible than a simulation in which all time-dependent random variables are generated. Note that the time-dependent aspects of the problem are handled outside the Monte Carlo simulation.

5.3 Efficient Monte Carlo Methods

5.3.1 Importance Sampling

Suppose $g(\underline{X})$ is a random function, where $\underline{X}$ is a random vector described by a joint pdf $f_{\underline{X}}(\underline{x})$. The expected value of $g(\underline{X})$ is denoted by the integral,

$$\mu = E[g(\underline{X})] = \int \dots \int_{\Omega} g(\underline{x}) f_{\underline{X}}(\underline{x}) d\underline{x} \quad (5.7)$$

where Ω is the basic space of $\underline{X}$. A basic Monte Carlo simulation would obtain an estimate of μ by generating a sequence of n random vectors $\underline{X}_k, k = 1, 2, \dots, n$, from $f_{\underline{X}}(\underline{x})$ and computing an estimate of the sample mean by

$$\hat{\mu} = \frac{1}{n} \sum_{k=1}^n g(\underline{X}_k) \quad (5.8)$$

The law of large numbers (Benjamin and Cornell, 1970) ensures the convergence of $\hat{\mu}$ to μ for large n .

Clearly, $\hat{\mu}$ is also a random variable with mean μ and variance σ^2/n for independent samples, where σ^2 is the variance of $g(\underline{X})$. σ^2 is estimated by sample variance, S^2 , as,

$$S^2 = \frac{1}{n-1} \sum_{k=1}^n [g(\underline{X}_k) - \hat{\mu}]^2 \quad (5.9)$$

The variance of $\hat{\mu}$, $S_{\hat{\mu}}^2$, is estimated by

$$S_{\hat{\mu}}^2 = S^2/n \quad (5.10)$$

The estimator, $S_{\hat{\mu}}$, or the estimator of the coefficient of variation (c.o.v.) of $\hat{\mu}$, $S_{\hat{\mu}}/\hat{\mu}$, is often used as a measure of the accuracy of the simulation (McGrath and Irving, 1975; Karamchandani, et al, 1989), and the sampling plan must be designed so that $S_{\hat{\mu}}$ is small. Eqn.(5.10) suggests that there are two ways to improve the accuracy of an estimator: increase the number of samples, n , which is costly, or decrease the sample variance, S , by modifying the basic sampling procedure.

In importance sampling, the basic sampling procedure is modified by replacing the joint pdf, $f_{\underline{X}}(\underline{x})$, with a so-called importance sampling density function, $h_V(\underline{x})$. The

function $h_V(\underline{x})$ focuses the sampling on more important regions in Eqn.(5.7), and accordingly, the sample variance is reduced. From Eqn.(5.7) μ is expressed as,

$$\mu = \int \dots \int_{\Omega} \frac{g(\underline{x})f_{\underline{X}}(\underline{x})}{h_V(\underline{x})} h_V(\underline{x}) d\underline{x} \quad (5.11)$$

Provided that $h_V(\underline{x}) > 0$ for values of $\underline{x}$ in which $g(\underline{x})f_{\underline{X}}(\underline{x}) \neq 0$, the estimator for μ is given by

$$\hat{\mu} = \frac{1}{n} \sum_{k=1}^n \frac{g(\underline{X}_k)f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)} \quad (5.12)$$

and the estimator for the variance of μ is given by

$$S_{\hat{\mu}}^2 = \frac{1}{n(n-1)} \sum_{k=1}^n \left[\frac{g(\underline{X}_k)f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)} - \hat{\mu} \right]^2 \quad (5.13)$$

Eqn.(5.13) shows that if $g(\underline{x}) > 0$ and if

$$h_V(\underline{x}) = h_{V_{opt}}(\underline{x}) \equiv \frac{g(\underline{x})f_{\underline{X}}(\underline{x})}{\mu} \quad (5.14)$$

where $h_{V_{opt}}(\underline{x})$ is the optimal importance sampling function, then $S_{\hat{\mu}}^2 = 0$, and the exact solution is obtained with one sample. Although this implies that the optimal importance sampling cannot be performed without knowing μ (which is the quantity sought and is unknown), a large improvement can still be expected if a function close to the form $g(\underline{x})f_{\underline{X}}(\underline{x})/\mu$ is selected for $h_V(\underline{x})$. Figure 5.1 shows an example of a reasonable importance sampling function, which tends to emphasize the region of $g(\underline{x})f_{\underline{X}}(\underline{x})/\mu$ that is most important. However, selecting an importance sampling function, especially its location, must be done with care. If an importance sampling function is selected inappropriately, the results may err by orders of magnitude (Karamchandani, et al, 1989; Ibrahim, 1991). A number of recent studies have developed techniques for selecting "good" importance sampling functions with reasonable effort (e.g., Harbitz, 1986; Bucher, 1988; Karamchandani, et al, 1989; Melchers, 1990). It should be noted that spending a lot of time to select an importance sampling function may reduce or eliminate any advantage of this method over a basic Monte Carlo simulation.

5.3.2 Application of Importance Sampling to Structural Reliability Analysis

The failure probability of a structural system, P_f , can be formulated as (Melchers, 1987),

$$P_f = \int \dots \int_D f_{\underline{X}}(\underline{x}) d\underline{x} \quad (5.15)$$

where $f_{\underline{X}}(\underline{x})$ is the joint pdf of random variables $\underline{X}$. The failure domain, D , is described by a set of limit state functions, $G_i(\cdot)$:

$$G_i(\underline{x}) \begin{cases} < 0, & \text{for } \underline{x} \text{ in failure domain} \\ = 0, & \text{for } \underline{x} \text{ on limit state surface} \\ > 0, & \text{for } \underline{x} \text{ in safe domain} \end{cases} \quad (5.16)$$

Eqn.(5.15) may be rewritten in the form of Eqn.(5.7) using the indicator function, $I[\bullet] = 1$ if $\bullet$ is true and zero otherwise,

$$P_f = \int \dots \int_{\Omega} I \left[\bigcup_{i=1}^m G_i(\underline{x}) < 0 \right] f_{\underline{X}}(\underline{x}) d\underline{x} \quad (5.17)$$

Introducing the importance sampling function, $h_V(\underline{x})$, Eqn.(5.17) can be rewritten as,

$$P_f = \int \dots \int_{\Omega} \frac{I \left[\bigcup_{i=1}^m G_i(\underline{x}) < 0 \right] f_{\underline{X}}(\underline{x})}{h_V(\underline{x})} h_V(\underline{x}) d\underline{x} \quad (5.18)$$

Eqn.(5.17) also can be rewritten in terms of conditional probabilities as (Karamchandani and Cornell, 1991),

$$\begin{aligned} P_f &= \int \dots \int_{\tilde{\Omega}} \left\{ \int \dots \int_{\tilde{\Omega}} I \left[\bigcup_{i=1}^m G_i(\tilde{\underline{x}}, \underline{\tilde{x}}) < 0 \right] f_{\underline{X}|\underline{\tilde{X}}}(\underline{\tilde{x}}|\tilde{\underline{x}}) d\underline{\tilde{x}} \right\} f_{\underline{\tilde{X}}}(\tilde{\underline{x}}) d\tilde{\underline{x}} \\ &= \int \dots \int_{\tilde{\Omega}} P_{f|\underline{\tilde{X}}=\tilde{\underline{x}}} \cdot f_{\underline{\tilde{X}}}(\tilde{\underline{x}}) d\tilde{\underline{x}} \end{aligned} \quad (5.19)$$

where $\tilde{X}$ and $\check{X}$ are subsets of random vector X , $f_{\tilde{X}|\check{X}}(\tilde{x}|\check{x})$ is the conditional pdf of $\tilde{X}$ given $\check{X}$, $\tilde{\Omega}$ and $\check{\Omega}$ are the sample space of $\tilde{X}$ and $\check{X}$, respectively, and $\tilde{\Omega} \cup \check{\Omega} = \Omega$ and $\tilde{\Omega} \cap \check{\Omega} = \phi$. Observe that Eqn.(5.19) has the same form as Eqn.(5.5). Introducing the importance sampling function, $h_V(\tilde{x})$, Eqn.(5.19) can be rewritten as,

$$P_f = \int \dots \int_{\tilde{\Omega}} \frac{P_{f|\check{X}=\check{x}} \cdot f_{\tilde{X}}(\tilde{x})}{h_V(\tilde{x})} h_V(\tilde{x}) d\tilde{x} \quad (5.20)$$

In most previous studies, the importance sampling is applied to structural reliability analysis expressed by Eqn.(5.18) rather than Eqn.(5.20).

To apply importance sampling effectively, it is necessary to have some information about the "region of interest." Harbitz (1986) proposed using $f_{\check{X}}$ for h_V , but with its mean value point, μ_V , shifted to the checking point, the point on the limit state surface with minimum distance from the origin in the reduced space, which can be obtained from first-order (FO) theory (Melchers, 1987). Schueller and Stix (1987) also suggested that μ_V be located at the checking point, but that an independent multivariate normal pdf be used for h_V because it is simple to manipulate and the quality of the estimator is less sensitive to the choice of density type than to its parameters. Because of the nature of FO analysis, a limit state function is required to apply this approach, preferably one that is differentiable. Otherwise, identification of the checking point requires lengthy computations and may be very difficult. Melchers (1989) recommended the use of an independent multivariate normal pdf for h_V but with μ_V at the point $\tilde{x}^*$ on the limit state surface for which $f_{\check{X}}$ is a maximum (i.e. the point of maximum likelihood). If this point can not be obtained in closed form, it can be approximately obtained by simulation. This approximate point can be updated in the course of simulation to estimate P_f . In these approaches, when a reliability problem involves multi-limit states, a multi-modal h_V is used with a mode corresponding to each location $\tilde{x}_i^*$, the checking point or the point of maximum likelihood for the i^{th} limit state.

Bucher (1988) proposed that μ_V be located at the mean, $\mu_{V_{opt}}$, of the optimal importance sampling function and that a single-mode h_V be used for problems involving either a single limit state or multi-limit states. This point can be interpreted as the centroid of the integrand in Eqn.(5.17). The mass of the integrand is expected to be concentrated around this point, and accordingly, this is the region of interest. This approach may not work well for problems in which the integrand in Eqn.(5.17) takes on large values far away from $\mu_{V_{opt}}$; in that case, multi-modal importance sampling might be more appropriate. It would be expected that using an importance sampling function for each limit state of interest would be less efficient than using only one importance sampling function when the "checking" points lie reasonably together (Schueller, et al, 1989). Moreover, considerable computations would be required to identify the μ_V for each limit state function. Melchers (1990) observed that the estimate of P_f is not sensitive to the precise choice of "checking point." With the same analogy, this observation would apply for numerical integration expressed in the form of Eqn.(5.20) if one importance sampling function is sufficient and if the integrand is a smooth function.

Karamchandani and Cornell (1991) suggested using $f_{\underline{X}}$ for h_V but with $\underline{\mu}_V$ at the point of maximum $P_f|_{\tilde{X}=\tilde{x}} \cdot f_{\tilde{X}}(\tilde{x})$ in Eqn.(5.20). This scheme was developed with the idea that there is only one region of $\tilde{X}$ in which the integrand is relatively large. However, the scheme must be modified for problems in which there is more than one such region. Identification of all such regions is not a trivial task.

In the time-dependent reliability analysis described in the preceding section, Eqn.(5.6) shows that (i) $F(t_L|\underline{R}_0 = \underline{r})$ is a non-increasing function in terms of each random variable, R_i , (ii) the contours of $F(t_L|\underline{R}_0 = \underline{r})$ are convex with respect to the origin (see Figure 5.2), and (iii) there is more than one region in which $F(t_L|\underline{R}_0 = \underline{r})f_{\underline{R}_0}(\underline{r})$ is relatively large. Two possible cases are illustrated in Figure 5.3. Either way one can expect that the important region is not spread over the domain of $\underline{R}$ in this problem, and using one importance sampling function with $\underline{\mu}_V$ at $\underline{\mu}_{V_{opt}}$ seems feasible for time-dependent reliability analysis. However, $\underline{\mu}_{V_{opt}}$ is also an unknown parameter and must be located adaptively as part of the simulation.

5.3.3 Adaptive Sampling

The parameters of the importance sampling function can be improved to reduce the variance of the estimated value of P_f by using an adaptive sampling algorithm (e.g., Bucher, 1988; Melchers, 1990; Ibrahim, 1991; Karamchandani and Cornell, 1991). Through adaptive sampling, one can get more information, and it would be wasteful if this information were not utilized in estimating P_f .

It was shown previously that there exists an optimal importance function with which the sample variance becomes zero, as expressed by Eqn.(5.14). Then element, i , of the mean vector of the optimal importance variable, $\underline{\mu}_{V_{opt}}$, is expressed as,

$$\mu_{V_{opt}i} = \underbrace{\int_0^\infty \cdots \int_0^\infty}_{m\text{-fold}} x_i h_{V_{opt}}(\underline{x}) d\underline{x} \quad (5.21)$$

$$= \underbrace{\int_0^\infty \cdots \int_0^\infty}_{m\text{-fold}} x_i \frac{g(\underline{x}) f_{\underline{X}}(\underline{x})}{P_f} d\underline{x} \quad (5.22)$$

Although it is not possible to obtain $\underline{\mu}_{V_{opt}}$ before sampling, it can be estimated in the course of the Monte Carlo simulation to determine P_f . From Eqns.(5.12) and (5.22),

$\underline{\mu}_{V_{opt}}$ is estimated by

$$\hat{\mu}_{V_{opt}i} = \frac{\sum_{k=1}^n X_{k_i} \frac{g(\underline{X}_k) f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)}}{\sum_{k=1}^n \frac{g(\underline{X}_k) f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)}} \quad (5.23)$$

$$= \frac{\sum_{k=1}^n X_{k_i} \frac{g(\underline{X}_k) f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)}}{n \cdot \hat{P}_f} \quad (5.24)$$

in which $\hat{P}_f$ represents the estimate of P_f . $\hat{\mu}_{V_{opt}}$ is not an unbiased estimator of $\underline{\mu}_{V_{opt}}$; however, it is asymptotically unbiased as $n \rightarrow \infty$ (see Addendum 5A at the end of this chapter). If a problem is symmetric in terms of random variables X_i , $i = m_1, m_1 + 1, \dots, m_2$, the mean values of these random variables described by the optimal importance sampling distribution function are identical to one another. Therefore, the mean values can be estimated by

$$\hat{\mu}_{V_{opt}i} = \frac{1}{n \cdot \hat{P}_f} \sum_{j=m_1}^{m_2} \left[\sum_{k=1}^n X_{k_j} \frac{g(\underline{X}_k) f_{\underline{X}}(\underline{X}_k)}{h_V(\underline{X}_k)} \right], \quad i = m_1, \dots, m_2 \quad (5.25)$$

This mean operation accelerates convergence to $\mu_{V_{opt}i}$, but cannot be applied when the maximum integrand point method is used for $\underline{\mu}_V$. It is clear from Eqn.(5.24) that little additional effort is required to estimate $\underline{\mu}_{V_{opt}}$ in addition to P_f . Thus, $\underline{\mu}_{V_{opt}}$ can be evaluated while estimating P_f , and the location of $h_V(\underline{x})$ can be updated during the sampling process. This updating can be continued until no significant improvement in accuracy is obtained. Since the mean vector and covariance matrix uniquely define a multivariate normal (or multivariate lognormal) probability density function, such a pdf is suggested for the iterative procedure.

When the simulation is carried out with only one importance sampling function located at the most important checking point (the checking point closest to the origin in the reduced space) for a problem with multiple checking points, the variance of the importance sampling variables, σ_V , must be large enough so as not to neglect the influence of the density of probability in the regions of the other checking points (Schueller and Stix, 1987). Karamchandani, et al (1989) observed that if the variances of the importance sampling variables are too small, the resulting estimates of the failure probability are significantly lower than the correct value even though the estimated c.o.v., $S_{\hat{P}_f}/\hat{P}_f$, may be small. With the same analogy, a larger σ_V must be used for a single-mode importance sampling function if $\underline{\mu}_V$ is located far from the optimal location. As shown schematically in Figure 5.3, the integrand in Eqn.(5.5) is relatively large over a large

area. Therefore, the variance of the importance sampling variables must be large enough during the preliminary analysis to include the influence of those "important" points. This is especially true for the first set of samples, for which the mean of the importance sampling variables may be some distance from $\underline{\mu}_{V_{opt}}$. The variance of the importance sampling variables can be reduced as the preliminary sampling is repeated and as the mean vector of the importance sampling variables becomes closer to $\underline{\mu}_{V_{opt}}$.

On the other hand, too large a variance in the importance sampling variables leads to a large statistical error in the estimator (Ouypraprasert, 1988). These observations suggest that there exists an optimal ratio of the standard deviation of an importance sampling variable to that of the corresponding original variable, $k_{opt} = \sigma_{V_i}/\sigma_{R_i}$, which gives an estimator of P_f close to the exact value and minimizes the estimate of c.o.v. of $\hat{P}_f$.

The optimal common ratio, k_{opt} , for the importance sampling function with $\underline{\mu}_V = \underline{\mu}_{V_{opt}}$ is estimated iteratively, after a stable estimate of $\underline{\mu}_{V_{opt}}$ is obtained. Here, the common series of sample vectors from a standard joint normal distribution function are used (correlated sampling technique, Rubinstein, 1981) so that relative accuracy can be compared with a smaller number of samples. In the iteration the common ratio, $\sigma_{V_i}/\sigma_{R_i}$, $i = 1, \dots, m$ is reduced as long as the c.o.v. of $\hat{P}_f$ decreases and significant decrease in $\hat{P}_f$ is not observed, or the ratio is increased until the c.o.v. of $\hat{P}_f$ increases and no significant increase in $\hat{P}_f$ is observed. Underestimation of P_f due to too small a ratio, $\sigma_{V_i}/\sigma_{R_i}$, is avoided by this procedure. Then, the main sampling for estimating P_f is carried out.

The information obtained through the preliminary sampling should be utilized in estimating P_f to maximize the efficiency of the simulation. The estimators of P_f obtained through preliminary analysis, $\hat{P}_{f_{pre}}$, and its estimated variance, S_{pre}^2 , are evaluated by

$$\hat{P}_{f_{pre}} = \frac{1}{n_{pre}} \sum_{j=1}^{n_{adp}} \left[\sum_{k=1}^{n_{sub}} \frac{g(\underline{X}_{kj}) f_{\underline{X}}(\underline{X}_{kj})}{h_{V_j}(\underline{X}_{kj})} \right] \quad (5.26)$$

$$S_{pre}^2 = \frac{1}{n_{pre}(n_{pre} - 1)} \sum_{j=1}^{n_{adp}} \left[\sum_{k=1}^{n_{sub}} \left\{ \frac{g(\underline{X}_{kj}) f_{\underline{X}}(\underline{X}_{kj})}{h_{V_j}(\underline{X}_{kj})} - \hat{P}_{f_{pre}} \right\}^2 \right] \quad (5.27)$$

in which n_{adp} is the number of adaptations to evaluate $\underline{\mu}_{V_{opt}}$, n_{sub} is the number of samples generated in each subset for updating $\underline{\mu}_{V_{opt}}$, and $n_{pre} = n_{adp} \cdot n_{sub}$ is the total number of samples used to estimate $\underline{\mu}_{V_{opt}}$. The estimators of P_f obtained through the main analysis, $P_{f_{main}}$, and its estimated variance, S_{main}^2 , are evaluated by Eqn.(5.12) and Eqn.(5.13), respectively.

Since it is expected that $\hat{P}_{f_{main}}$ is more accurate than $\hat{P}_{f_{pre}}$, it is reasonable to give larger weight to $\hat{P}_{f_{main}}$ than $\hat{P}_{f_{pre}}$ in estimating P_f . Let σ_{main}^2 and σ_{pre}^2 be the variance

of $\hat{P}_{f_{\text{main}}}$ and $\hat{P}_{f_{\text{pre}}}$, respectively. Let $\hat{P}_f$ be the new estimator expressed as,

$$\hat{P}_f = (1 - w)\hat{P}_{f_{\text{main}}} + w\hat{P}_{f_{\text{pre}}} \quad (5.28)$$

The variance of the revised estimator

$$\sigma^2 = (1 - w)^2 \sigma_{\text{main}}^2 + w^2 \sigma_{\text{pre}}^2 \quad (5.29)$$

is minimized when

$$w = \frac{\sigma_{\text{main}}^2}{\sigma_{\text{main}}^2 + \sigma_{\text{pre}}^2} \quad (5.30)$$

Thus, the estimator expressed as

$$\hat{w} = \frac{S_{\text{main}}^2}{S_{\text{main}}^2 + S_{\text{pre}}^2} \quad (5.31)$$

is used as a weight for combining $\hat{P}_{f_{\text{main}}}$ and $\hat{P}_{f_{\text{pre}}}$. The resulting estimator

$$\hat{P}_f = (1 - \hat{w})\hat{P}_{f_{\text{main}}} + \hat{w}\hat{P}_{f_{\text{pre}}} \quad (5.32)$$

is no longer unbiased; however, it is asymptotically unbiased as $n_{\text{main}}, n_{\text{pre}} \rightarrow \infty$, where n_{main} and n_{pre} are the number of samples used to evaluate $\hat{P}_{f_{\text{main}}}$ and $\hat{P}_{f_{\text{pre}}}$, respectively (see Addendum 5B at the end of this chapter).

5.4 Examples of Adaptive Sampling

The efficiency of the importance sampling scheme described above is demonstrated through an analysis of two- and five-component systems with degrading component strength. As the correlation of the initial strength of components becomes higher, the joint pdf is oriented more strongly in one direction, and thus the region over which the integrand in Eqn.(5.5) takes on large values (area of interest) is narrower. Therefore, the most severe test of importance sampling with a single importance sampling function involves components with independent initial strength.

The probabilistic models of load events used in the illustration are summarized in Table 3.1. Only the combination of time varying live load, L , and time-invariant dead load, D , is considered for illustrative purposes. The variation of dead load is ignored because its effect on the structural reliability is negligible compared with the effect of other parameters such as strength degradation models and coefficient of variation of the intensity of time varying load, as discussed in Chapter 4. The mean value of initial strength of a component is evaluated based on one of the current design requirements for concrete structures (ACI 318, 1989),

$$0.9M_n = 1.4D_n + 1.7L_n \quad (5.33)$$

in which M_n = nominal capacity, and D_n and L_n = nominal dead and live loads, respectively. Eqn.(5.33) would govern design for flexure, for example. The initial strength, R_0 is described by a lognormal distribution with mean, $\mu_R = 1.15M_n$, and c.o.v., $V_R = 0.15$. Assuming that $\mu_D = 1.00D_n$, $\mu_L = 0.40L_n$, and $D_n = L_n$, the central factor of safety, $\mu_R/(\mu_D + \mu_L)$, is 2.83. The systems analyzed are summarized in Table 5.1. $g_i(40) = 0.8$ means that 80 % of the initial strength of component i remains at 40 years. Strength degradation is assumed to occur linearly, representative, e.g., of loss in flexural strength of a reinforced element due to corrosion of reinforcement.

For each system, three separate simulations are carried out using both adaptive sampling and basic Monte Carlo sampling with 1000 samples to illustrate the reproducibility of the results. (Recall that Monte Carlo simulation is used to remove the conditioning on R_0 in Eqn.(5.5), not to evaluate Eqn.(5.6).) A multivariate lognormal pdf is selected for the importance sampling function. The preliminary analyses in the adaptive sampling for locating $\mu_{V_{opt}}$ is carried out with 50 samples for each subset ($n_{sub} = 50$), and k_{opt} is sought with 20 samples per adaptation. The number of samples used to select k_{opt} are not counted in the total number of samples.

In the preliminary analysis, the ratio of the standard deviation of an importance sampling variable to that of the corresponding original variable, $k = \sigma_{V_i}/\sigma_{R_i}$ for all i , is set equal to 3.0 for the first subset of samples, 2.2 for the second and the third subsets, and 1.6 for the rest of the subsets. These values of k are obtained empirically; Melchers (1990) observed that good results can be obtained with k in the range 1 - 2, while Ouyyornprasert (1988) suggested that any value of k between 2 - 5 can be used. The failure probability is estimated for periods of 20, 40 and 60 years. The results are summarized in Tables 5.2 - 5.5; the "exact" values presented were obtained from Eqns.(5.5) and (5.6) by basic Monte Carlo simulation with $n = 400,000$ for examples 1 - 3 and $n = 1,000,000$ for example 4.

In the first example (Table 5.2), it is assumed that the degradation rates of all five components are identical and that $g_i(40) = 0.8$. The adaptive sampling method is more accurate than basic Monte Carlo simulation if t_L is small and $F(t_L)$ is small. As t_L increases and $F(t_L)$ approaches the level of 10^{-2} , the method becomes less advantageous.

In general, the reliability function of a series system increases as the number of components decreases (Thoft-Christensen and Sorensen, 1982). In the second example

(Table 5.3), the number of components is decreased from five to two. The series system effect on failure probability is reduced and the failure probability decreases by a factor of 2. Correspondingly, the accuracy of basic Monte Carlo simulation decreases. The accuracy of importance sampling, however, does not change significantly relative to the first example.

In the third example (Table 5.4), the degradation of four components out of five is increased to $g(40) = 0.95$, rendering their effect on the system failure probability less critical as time increases. As it is assumed that the strengths of components linearly degrade with time, all components maintain close to their initial strength at the early stage of service life, and accordingly, the noncritical components still contribute to failure probability (cf. the columns for $t_L = 20$ in Tables 5.2 and 5.4). On the other hand, the contribution of noncritical components decreases with time (cf. the columns for $t_L = 60$ in Tables 5.2 and 5.4). The adaptive sampling deals with this change, and continues to provide good accuracy.

In the last example (Table 5.5), the central factor of safety for all components is increased by 50% from the first example, leading to a lower failure probability. The estimates obtained from basic Monte Carlo simulation are unstable and the accuracy decreases substantially in some cases. On the other hand, adaptive sampling still provides a good estimate, although the accuracy is slightly decreased.

The optimal common factor, k_{opt} , is in the range of 1 - 2 in these examples, which agrees with the observation by Melchers (1990). This value tends to decrease slightly as time increases (i.e., as failure probability increases). As the failure probability decreases, the importance region is shifted toward smaller values in the lower tail of the pdf for resistances. Since this tail is spread over a wider region, a larger variance for the importance sampling variable is required to cover this region.

5.5 Illustration of Time-dependent System Reliability

The effect of degradation in component strength on the system reliability function is illustrated using several simple parametric representations of time-dependent strength. Those parameters which should receive special attention in subsequent data acquisition are identified. The probabilistic models of load events used in the illustration of system reliability are summarized in Table 3.1. Only the combination of time-varying live load, L , and time-invariant dead load, D , is considered for illustrative purposes. The initial strength, R_0 , of each component is described by a lognormal distribution with mean, $\mu_R = 1.15M_n$, and c.o.v., $V_R = 0.15$. The components are assumed to be designed using Eqn.(5.33) with the assumption of $L_n = D_n$. Only linear degradation models are considered for all components. Without loss of generality, the c_i 's in Eqn.(5.6) are assumed to be unity for all components. In each analysis, a two-component system ($m = 2$) and five-component system ($m = 5$) are considered.

The strengths of the components in a system are correlated because of common

construction materials, fabrication and construction practices. The sensitivity of the reliability function to the number of components and to the stochastic dependence between their strengths is investigated by assuming that the strengths of the components are identically distributed and equally correlated pair-wise, with correlation coefficient, ρ . A multivariate lognormal pdf with the same correlation coefficient matrix as that of the original pdf is selected for the importance sampling function.

The failure probability, $F(t_L)$, is shown in Figures 5.4(a) with $V_L = 0.5$ and 5.4(b) with $V_L = 0.8$, assuming that the degradation rate equals 0.8 at 40 years for all components. The failure probability at a given time t_L increases as the number of components increases and as the correlation coefficient decreases. However, the effect of ρ is relatively small, particularly when $\rho < 0.8$, and decreases as the coefficient of variation of load intensity increases. The reduction in the sensitivity of the reliability function to ρ is due to the fact that dependence among component failure events increases from common random loads with large variabilities. This result is consistent with the results of a previous study (Thoft-Christensen and Sorensen, 1982), which showed that the effect of strength correlation on the system failure probability is significant for the limiting case in which the loading is deterministic.

It is unreasonable to expect that perfect correlation exists among the strengths of components. In light of the relative insensitivity to ρ , it seems appropriate to model systems such as those considered herein as being composed of components with independent strengths, particularly when the coefficient of variation of load intensity is large relative to that of resistance. Such a model would be conservative, of course. However, the reliability is substantially less affected by this assumption than it is by the choice of strength degradation model.

In general, a system consists of components with different statistical characteristics of strength and because of this, some components contribute more to the system failure probability than others. Such "critical" components arise from basically two sources: (a) Difference in initial safety factor or statistics of initial strength, and (b) Difference in degradation mechanism and associated degradation rate and initiation time. The combination of these cases also may cause a change in which component is critical at a given time.

The effect of the statistics of initial strength of components on system reliability are shown in Figures 5.5(a) and 5.5(b). The degradation rate is assumed such that $g(40) = 0.8$ for all components, while the mean initial strengths of all components except one (the critical component) are increased by 5%, 10% or 20%. The c.o.v. of the initial strength for all components is equal to 0.15. It is assumed that the initial strengths of the components are statistically independent of one another. As the mean strength of noncritical components increases, the failure probability of the system approaches to that of a single critical component. If the mean strengths of noncritical components are increased by more than 20%, the contribution of these components to system reliability is negligible and the size of the system for reliability analysis effectively can be reduced by treating those components as if their strengths are deterministic and equal to their respective mean values.

The effect of degradation rates of components on system reliability are shown in

Figures 5.6(a) - 5.6(f). The degradation rates are set so that $g(40) = 0.8$ for one component (critical component) while $g(40) = 0.9, 0.95$ or 0.98 for the other component(s) (noncritical component(s)). The initial strengths of all the components are assumed to be identically and lognormally distributed and equally correlated pair-wise with $\rho = 0.0$ in Figures 5.6(a) and 5.6(b), $\rho = 0.5$ in Figures 5.6(c) and 5.6(d), and $\rho = 0.8$ in Figures 5.6(e) and 5.6(f). These system failure probabilities are compared with the failure probability of the single critical component in the system and the failure probability of a system consisting of only critical components. At the early stage of the service life when all components remain close to their initial strength, the system failure probability is close to that of the system consisting of only critical components. At this stage, noncritical component(s) still contribute to the system failure. As time increases, however, the contribution of noncritical component(s) decreases and the system failure probability approaches that of the single critical component. For service lives of 40 - 60 years, the effect of noncritical component(s) is negligible and the system reliability can be determined approximately as that of the single critical component. Therefore, identification of critical components is important in a time-dependent reliability analysis of a complex system in order to keep the system model at a manageable size. The effect of strength correlation on system failure probability is small, as was also shown in Figures 5.4(a) and 5.4(b).

5.6 Summary

The adaptive Monte Carlo simulation is feasible for time-dependent system reliability analysis of structures of the type discussed here. Unlike systems evaluated by basic Monte Carlo simulation, the accuracy of the failure probability evaluated by adaptive importance sampling is relatively insensitive to the order of the probability. Accordingly, the advantage of using adaptive importance sampling increases as the failure probability decreases.

The reliability of the system is sensitive to the choice of initial component strength, strength degradation rate, and coefficient of variation of the intensity of time-varying load. Less sensitivity was shown to dependence in component strengths within a system because of the large variability in time varying load. The system reliability is also insensitive to the existence of statistically "noncritical" components. Therefore, it is important to identify the noncritical components before analysis so that their contribution to the reliability analysis can be excluded and the size of the system reliability analysis can be reduced. Such components in NPP can be identified using a ranking methodology developed in Task S.3 of the Structural Aging Program (Hookham, 1991).

Addendum 5A: Biased estimator of $\underline{\mu}_{V_{opt}}$

Let

$$\hat{\theta}_{1,i} = \frac{1}{n} \sum_{k=1}^n X_{k,i} \cdot g(\underline{X}_k) \quad (5.34)$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{k=1}^n g(\underline{X}_k) \quad (5.35)$$

in which $g(\underline{X})$ is a random function and $\underline{X}_k$ is a random vector described by a joint pdf $f_{\underline{X}}(\underline{x})$. Then

$$E[\hat{\theta}_{1,i}] = E[X_i \cdot g(\underline{X})] \quad (5.36)$$

$$E[\hat{\theta}_2] = E[g(\underline{X})] = P_f \quad (5.37)$$

From Eqn.(5.22),

$$E[X_i \cdot g(\underline{X})] = P_f \cdot \mu_{V_{opt},i} \quad (5.38)$$

Thus, the law of large numbers (Benjamin and Cornell, 1970) ensures that $\hat{\theta}_{1,i}$ and $\hat{\theta}_2$ converge to $P_f \cdot \mu_{V_{opt},i}$ and P_f , respectively, when $n \rightarrow \infty$. This also means that

$$\lim_{n \rightarrow \infty} \{E[\hat{\theta}_{1,i}/\hat{\theta}_2]\} = \lim_{n \rightarrow \infty} \{E[\hat{\mu}_{V_{opt},i}]\} \rightarrow \mu_{V_{opt},i} \quad (5.39)$$

Therefore, $\hat{\mu}_{V_{opt}}$ is an asymptotically unbiased and consistent estimator of $\underline{\mu}_{V_{opt}}$.

Addendum 5B: Biased estimator of P_f

Substitution Eqn.(5.31) into Eqn.(5.32) yields,

$$\hat{P}_f = \frac{S_2^2 \cdot \hat{P}_{f1} + S_1^2 \cdot \hat{P}_{f2}}{S_1^2 + S_2^2} \quad (5.40)$$

Since $\hat{P}_{f1}$ and $\hat{P}_{f2}$ are the unbiased estimator of P_f ,

$$E[\hat{P}_{f1}] = E[\hat{P}_{f2}] = P_f \quad (5.41)$$

Also, since S_1^2 and S_2^2 are the unbiased estimator of σ_1^2 and σ_2^2 , respectively,

$$E[S_1^2] = \sigma_1^2 \quad (5.42)$$

$$E[S_2^2] = \sigma_2^2 \quad (5.43)$$

The law of large numbers ensures that $\hat{P}_{f1}$ and S_1^2 converge to P_f and σ_1^2 , respectively, when the number of samples for the estimator $\hat{P}_{f1}$, $n_1 \rightarrow \infty$. Similarly, $\hat{P}_{f2}$ and S_2^2 converge to P_f and σ_2^2 , respectively, when the number of samples for the estimator $\hat{P}_{f2}$, $n_2 \rightarrow \infty$. Thus,

$$\lim_{n_1, n_2 \rightarrow \infty} E[S_2^2 \cdot \hat{P}_{f1} + S_1^2 \cdot \hat{P}_{f2}] \rightarrow (\sigma_1^2 + \sigma_2^2) \cdot P_f \quad (5.44)$$

Also,

$$E[S_1^2 + S_2^2] = E[S_1^2] + E[S_2^2] = \sigma_1^2 + \sigma_2^2 \quad (5.45)$$

From Eqns.(5.40), Eqn.(5.44) and (5.45),

$$\lim_{n_1, n_2 \rightarrow \infty} E \left[\frac{S_2^2 \cdot \hat{P}_{f1} + S_1^2 \cdot \hat{P}_{f2}}{S_1^2 + S_2^2} \right] = \lim_{n_1, n_2 \rightarrow \infty} \hat{P}_f \rightarrow P_f \quad (5.46)$$

Therefore, $\hat{P}_f$ is an asymptotically unbiased and consistent estimator of P_f .

Table 5.1: Models Analyzed in Examples

Example	Number of components	$\mu_{R_i}/(\mu_{D_i} + \mu_{L_i})$	$g_i(40)$
1	5	2.83	0.8 for all i
2	2	2.83	0.8 for all i
3	5	2.83	0.8 for $i = 1$, 0.95 for $i = 2, \dots, 5$
4	5	4.25	0.8 for all i

Table 5.2: Comparison of results for Example 1 (Five Components)

Method	Run No.	$t_L = 20$			$t_L = 40$			$t_L = 60$		
		p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}
Adaptive sampling	1	5.91×10^{-4}	0.04	1.4	3.86×10^{-3}	0.04	1.4	2.19×10^{-2}	0.03	1.3
	2	6.12×10^{-4}	0.04	1.5	3.97×10^{-3}	0.04	1.4	2.24×10^{-2}	0.03	1.3
	3	6.21×10^{-4}	0.04	1.5	4.07×10^{-3}	0.04	1.4	2.30×10^{-2}	0.03	1.3
Simple Monte Carlo simulation	1	4.60×10^{-4}	0.09	-	3.25×10^{-3}	0.08	-	2.00×10^{-2}	0.06	-
	2	5.73×10^{-4}	0.11	-	3.89×10^{-3}	0.09	-	2.27×10^{-2}	0.06	-
	3	8.20×10^{-4}	0.24	-	4.83×10^{-3}	0.14	-	2.54×10^{-2}	0.07	-
Exact	-	6.22×10^{-4}	-	-	4.06×10^{-3}	0.03	-	2.29×10^{-2}	-	-

Table 5.3: Comparison of results for Example 2 (Two Components)

Method	Run No.	$t_L = 20$			$t_L = 40$			$t_L = 60$		
		p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}
Adaptive sampling	1	2.67×10^{-4}	0.03	1.6	1.79×10^{-3}	0.03	1.6	1.05×10^{-2}	0.03	1.6
	2	2.54×10^{-4}	0.03	1.6	1.71×10^{-3}	0.02	1.6	1.00×10^{-2}	0.03	1.6
	3	2.67×10^{-4}	0.03	1.6	1.78×10^{-3}	0.03	1.5	1.05×10^{-2}	0.03	1.4
Simple Monte Carlo simulation	1	2.02×10^{-4}	0.14	-	1.46×10^{-3}	0.12	-	0.93×10^{-2}	0.09	-
	2	2.73×10^{-4}	0.15	-	1.89×10^{-3}	0.12	-	1.14×10^{-2}	0.09	-
	3	4.68×10^{-4}	0.40	-	2.55×10^{-3}	0.26	-	1.25×10^{-2}	0.12	-
Exact	-	2.64×10^{-4}	-	-	1.78×10^{-3}	-	-	1.05×10^{-2}	-	-

Table 5.4: Comparison of results for Example 3

Method	Run No.	$t_L = 20$			$t_L = 40$			$t_L = 60$		
		p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}
Adaptive sampling	1	3.45×10^{-4}	0.06	1.6	1.35×10^{-3}	0.05	1.6	6.18×10^{-3}	0.05	1.5
	2	3.70×10^{-4}	0.05	1.6	1.56×10^{-3}	0.05	1.5	6.86×10^{-3}	0.04	1.4
	3	3.65×10^{-4}	0.05	1.5	1.56×10^{-3}	0.05	1.4	6.71×10^{-3}	0.04	1.3
Simple Monte Carlo simulation	1	2.58×10^{-4}	0.10	-	1.12×10^{-3}	0.11	-	5.59×10^{-3}	0.10	-
	2	3.38×10^{-4}	0.12	-	1.48×10^{-3}	0.12	-	7.08×10^{-3}	0.11	-
	3	4.64×10^{-4}	0.25	-	1.70×10^{-3}	0.17	-	7.15×10^{-3}	0.11	-
Exact	-	3.61×10^{-4}	-	-	1.46×10^{-3}	-	-	6.52×10^{-3}	-	-

Table 5.5: Comparison of results for Example 4

Method	Run No.	$t_L = 20$			$t_L = 40$			$t_L = 60$		
		p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}	p_f	c.o.v.	k_{opt}
Adaptive sampling	1	1.28×10^{-6}	0.06	1.6	1.34×10^{-5}	0.06	1.5	1.36×10^{-4}	0.05	1.3
	2	1.34×10^{-6}	0.06	1.5	1.38×10^{-5}	0.06	1.5	1.42×10^{-4}	0.05	1.4
	3	1.31×10^{-6}	0.06	1.6	1.37×10^{-5}	0.05	1.5	1.43×10^{-4}	0.04	1.4
Simple Monte Carlo simulation	1	0.65×10^{-6}	0.15	-	0.81×10^{-5}	0.13	-	0.96×10^{-4}	0.10	-
	2	0.99×10^{-6}	0.21	-	1.13×10^{-5}	0.17	-	1.25×10^{-4}	0.13	-
	3	2.65×10^{-6}	0.59	-	2.37×10^{-5}	0.47	-	2.06×10^{-4}	0.34	-
Exact	-	1.37×10^{-6}	-	-	1.42×10^{-5}	-	-	1.43×10^{-4}	-	-

Figure 5.1: Illustration of Importance Sampling Concept

Figure 5.2: Illustration of $F(t_L | R_0 = r)$ and $f_{R_0}(r)$ in Two Dimensions

5.3(a) Two Components Equally Important

5.3(b) Two Components, but Only One Important

Figure 5.3: Contours of Probability for $F(t_L | R_0 = r)$, $f_{R_0}(r)$ and $F(t_L | R_0 = r) f_{R_0}(r)$
(X: Centroid of the integrand, $F(t_L | R_0 = r) f_{R_0}(r)$)

5.4(a) $V_R=0.15, V_L=0.5$ 5.4(b) $V_R=0.15, V_L=0.8$ Figure 5.4: Dependence of System Failure Probability on ρ

5.5(a) Two Component Systems

5.5(b) Five Component Systems

Figure 5.5: Dependence of System Failure Probability on Initial Strength

5.6(a) Two Component Systems with $\rho_{ij}=0$ 5.6(b) Five Component Systems with $\rho_{ij}=0$

Figure 5.6: Dependence of System Failure Probability on Degradation Rate

5.6(c) Two Component Systems with $\rho_{ij}=0.5$ 5.6(d) Five Component Systems with $\rho_{ij}=0.5$

5.6(e) Two Component Systems with $\rho_{ij}=0.8$ 5.6(f) Five Component Systems with $\rho_{ij}=0.8$

Chapter 6

The Role of Inspection/Repair in Maintaining Reliability of Concrete Structures

6.1 Introduction

The analysis of reliability of components and structures as functions of time enables the analyst to determine the service period beyond which the desired reliability of the structure cannot be ensured. To extend service period, the structure should be inspected and maintained. The intervals and extent of inspection and maintenance that are required to ensure the performance during its intended service life can be determined using the time-dependent reliability analysis presented in Chapters 4 and 5. In a general sense, when an inspection is performed, we learn something about the condition of the structure that may allow us to revise the probabilistic strength models and degradation models used in the time-dependent reliability analysis (e.g., Turkstra, et al, 1988; Ciampoli, 1989; Bartlett and Sexsmith, 1991; Ellingwood and Mori, 1992). A conceptual illustration of the effect of this process on the hazard function, $h(t)$, is presented in Figure 6.1. The effect of inspection/repair is to remove larger defects from the structure and upgrade its strength, thus reducing its conditional failure rate (see Eqn.(4.10)). As the structure ages, the hazard function increases until another inspection/repair operation occurs. Inspection/maintenance strategies should be designed so that the integrated effect of $h(t)$ in Figure 6.1 must remain below an established target probability during its required service period.

In order to design an inspection/maintenance strategy, it is necessary to evaluate the effectiveness of periodic inspection and maintenance on the failure probability of a component. In studies to evaluate the failure probability of metallic structures subjected to fatigue, it has been assumed that a fatigued component is replaced if the intensity of detected damage exceeds a critical value (Madsen, 1987; Thoft-Christensen and Sorensen,

1987; Fujita, et al, 1989). However, a component may not even be replaced or repaired during maintenance unless damage is detected and is larger than a certain threshold. In this case, the effect of damage overlooked at an inspection or detected but not repaired also should be considered. To accomplish this task, it is necessary to consider the probability of detecting damage of a given size or extent and to introduce models of damage initiation and intensity. The number of damage zones and the damage intensity before and after maintenance are random variables. Tang and Halim (1988) developed a procedure for updating the distribution function of damage intensity immediately following maintenance through such modeling. However, the distribution function of damage intensity at some time after maintenance is also necessary to evaluate time-dependent reliability because new damages may initiate and undetected or unrepaired damages may grow after maintenance.

No nondestructive evaluation (NDE) method can detect small defects with certainty. The imperfect nature of NDE methods is random and can be described in statistical terms. This randomness affects the calculated reliability of the component. Figure 6.2 illustrates conceptually the probability, $d(x)$, of detecting a defect of size x . Such a relation exists, at least conceptually, for each in-service inspection technology. The NDE method should be able to distinguish between different defects with a high level of reliability so that needless repairs can be avoided. Inspections for the purpose of reliability-based condition assessment and evaluation of suitability for continued service of concrete structures and components may require a greater detection reliability and a more quantitative measure of defects and accumulated damage than traditional surficial in-service inspection of concrete structures (House, 1987). The massive size of NPP structures poses some special problems in NDE, since many of the techniques have limited applicability because of section thicknesses and reinforcing bar sizes and patterns.

The strength of structural components in an aggressive environment may degrade with time according to $R(t) = R_0 \cdot G(t)$, in which R_0 = component capacity in the undegraded (original) state and $G(t)$ = time dependent degradation function defining the fraction of initial strength remaining at time t . The component strength degradation must be related to the damage intensities to evaluate the effect of periodic maintenance from a reliability point of view. The estimated failure probability for some future period then can be revised assuming a prescribed inspection/maintenance history.

In this chapter, methods for performing this evaluation that were introduced previously (Ellingwood and Mori, 1992) are developed and their application to reinforced concrete structural components is illustrated. As noted in Chapter 5, a system reliability analysis often can be reduced to a single component reliability analysis if the critical component can be identified. If this cannot be done, the method still can be applied to the system reliability using the adaptive Monte Carlo simulation technique discussed in Chapter 5. Since inspection and maintenance are costly, there are tradeoffs between the extent and accuracy of inspection, required level of reliability, and cost. The application of the method to evaluate the failure probability of a component with periodic inspection/maintenance and to design an optimum inspection/maintenance strategy which minimizes the future expected cost is demonstrated later in this chapter.

6.2 Role of Inspection on Time-Dependent Reliability

6.2.1 Degradation Function for a Component Based on Individual Damage Intensities

The damage intensity is modeled as a state variable taking a value within the interval $[0,1]$; the values 0 and 1 indicate no damage and no residual strength, respectively. Examples of this state variable describing damage would include the ratio of area of reinforcement lost due to corrosion to the original area, the ratio of strength loss of concrete due to sulfate attack to the strength of undegraded concrete, and the ratio of the depth of crack in a member to the member thickness. The following assumptions are made:

- (a) Initiation of damages in a component is described by a Poisson process in which the expected number of damages in time interval $(t, t + \Delta t]$ ($t > 0$) is $\int_t^{t+\Delta t} \nu(\tau) d\tau$. The damage initiation rate $\nu(t)$ is dependent on the surface area or volume of the component.
- (b) Damages initiate homogeneously over the surface area or volume of the component.
- (c) Once a damage initiates at location j , it grows according to

$$X_j(t) = \begin{cases} 0 & ; 0 \leq t < T_{I_j} \\ C_j(t - T_{I_j})^\alpha & ; t \geq T_{I_j} \end{cases} \quad (6.1)$$

in which $X_j(t)$ $j = 1, 2, \dots$ is the intensity of damage at time t , T_{I_j} $j = 1, 2, \dots$ are the random initiation times of damage, C_j 's are damage growth rates which are identically distributed and statistically independent random variables described by a cumulative distribution function (cdf) $F_C(c)$, and α is an deterministic parameter. The assumption of independent C_j 's provides a conservative estimate of failure probability. Parameters C and α depend on the degradation mechanism (e.g., Clifton and Knab, 1989; Clifton, 1991). If damage grows linearly, $\alpha = 1$.

- (d) The degradation function $G(t)$ for a component is defined in terms of damage intensities as

$$G(t) = 1 - \max_{all j} \{X_j(t)\} \quad (6.2)$$

Consider damages which initiate within interval $(t_1, t]$. Assume first that the number of these damages, $N_I(t_1, t)$, is n . From assumption (c), the cdf of $X_j(t)$, $F_{X_j}(x; t_1, t)$, is expressed as,

$$F_{X_j}(x; t_1, t) = P[C_j(t - T_{I_j})^\alpha < x] \quad (6.3)$$

$$= \int_{t_1}^t F_C \left(\frac{x}{(t-\tau)^\alpha} \right) f_{T_{I_j}}(\tau) d\tau \quad (6.4)$$

where $f_{T_{I_j}}(\tau)$ is the probability density function (pdf) of T_{I_j} . Given $N_I(t_1, t) = n$, the rank-ordered initiation times, $T_{I_1}, T_{I_2}, \dots, T_{I_n}$ are n order statistics of random variables $W_{I_1}, W_{I_2}, \dots, W_{I_n}$ which are statistically independent and identically distributed with pdf expressed as (Taylor and Karlin, 1984),

$$f_{W_I}(w) = \begin{cases} \frac{\nu(w)}{\int_{t_1}^t \nu(\tau) d\tau} & ; t_1 \leq w \leq t \\ 0 & ; \text{Otherwise} \end{cases} \quad (6.5)$$

Therefore, the cdf of the intensity of an arbitrary damage, $X(t)$, which initiates within $(t_1, t]$ is,

$$F_X(x; t_1, t) = \int_{t_1}^t F_C \left(\frac{x}{(t-\tau)^\alpha} \right) f_{W_I}(\tau) d\tau \quad (6.6)$$

Note that $f_{W_I}(w)$ is an unconditional pdf. Since the C_j 's and W_{I_j} 's are statistically independent of one another, the $X_j(t)$'s are also statistically independent. Accordingly, the cdf of $X_{max}(t_1, t) = \max\{X_j(t) \text{ which initiated within } (t_1, t]\}$ is,

$$F_{X_{max}}(x; t_1, t | N_I(t_1, t) = n) = [F_X(x; t_1, t)]^n \quad (6.7)$$

Removing the condition that $N_I(t_1, t) = n$,

$$\begin{aligned} F_{X_{max}}(x; t_1, t) &= \sum_{n=0}^{\infty} [F_X(x; t_1, t)]^n \cdot \frac{\left(\int_{t_1}^t \nu(\tau) d\tau \right)^n \cdot \exp \left(- \int_{t_1}^t \nu(\tau) d\tau \right)}{n!} \\ &= \exp \left[- \int_{t_1}^t \nu(\tau) d\tau \{ 1 - F_X(x; t_1, t) \} \right] \end{aligned} \quad (6.8)$$

From assumption (d), the mean and variance of the degradation function are evaluated by

$$\begin{aligned} E[G(t)] &= E[1 - X_{max}(0; t)] \\ &= 1 - \int_0^1 [1 - F_{X_{max}}(x; 0, t)] dx \end{aligned} \quad (6.9)$$

and

$$\begin{aligned} Var[G(t)] &= Var[X_{max}(0; t)] \\ &= \int_0^1 2x[1 - F_{X_{max}}(x; 0, t)]dx - \{E[G(t)]\}^2 \end{aligned} \quad (6.10)$$

6.2.2 Degradation Function After Repair

Assume that during inspection/maintenance at time t_R the entire component is inspected (full inspection) and all detected damages are repaired immediately and completely. The repaired parts of the component are restored to their initial strength levels. (If the repair is not fully effective, an additional variable is added to the analysis. This can be handled, but at the expense of significant additional computational effort.) Then the effect of inspection/maintenance depends on the detectability function, $d(x)$, associated with a NDE method. The inspection with higher $d(x)$ makes repair more likely and, accordingly, leads to higher values of the mean degradation function, $g(t)$. In the limit, if an inspection is perfect, i.e., $d(x) = 1$ for $x > 0$, then the component is restored to its original condition by the repair, i.e., $g(t_R) = 1$.

Step Detectability Function

Assume that the detectability function of an inspection, $d(x)$, is a step function defined as

$$d(x) = \begin{cases} 0 & ; \quad 0 \leq x < x_{th} \\ 1 & ; \quad x_{th} \leq x \leq 1 \end{cases} \quad (6.11)$$

where x_{th} is the minimum detectable value of damage. In other words, damage with intensity greater than x_{th} is detected with probability one, while damage with intensity less than x_{th} is overlooked with probability one. Subsequently, more general detectability functions will be considered.

Following an inspection at t_R , some of the damages will be repaired and the cdf describing $X(t)$ and $X_{max}(t)$ will change. Under the assumption that damage initiation is described by a Poisson process, the intensities of damages that initiate after repair at time t_R are independent of repair, and only the pdf of the intensities of damages initiating before t_R is updated. Let us consider damages which exist at time t and initiate within (A) $(0, t_R]$ and (B) $(t_R, t]$ where $t > t_R$ (as illustrated in Figure 6.3). Then the updated cdf of $X_{max}(t)$ after inspection/repair, $F''_{X_{max}}(x; t_R, t)$, is,

$$F''_{X_{max}}(x; t_R, t) = F_{X_{max(A)}}(x; t_R, t) \cdot F_{X_{max(B)}}(x; t_R, t) \quad (6.12)$$

in which $F_{X_{\max(A)}}(x; t_R, t)$ and $F_{X_{\max(B)}}(x; t_R, t)$ are the cdf of maximum intensity of damages in group (A), $X_{\max(A)}(t)$, and maximum intensity of damages in group (B), $X_{\max(B)}(t)$, respectively.

The cdf of the intensity of damage in group (A) at time t , $F_{X(A)}(x; t_R, t)$, is expressed as,

$$\begin{aligned}
 F_{X(A)}(x; t_R, t) &= P[X(t) < x | X(t_R) < x_{th}] \\
 &= P[C(t - W)^\alpha < x | C(t_R - W)^\alpha < x_{th}] \\
 &= \frac{\int_0^{t_R} F_C \left(\min \left\{ \frac{x}{(t - \tau)^\alpha}, \frac{x_{th}}{(t_R - \tau)^\alpha} \right\} \right) f_{W_I}(\tau) d\tau}{\int_0^{t_R} F_C \left(\frac{x_{th}}{(t_R - \tau)^\alpha} \right) f_{W_I}(\tau) d\tau} \quad (6.13)
 \end{aligned}$$

If $N_I(0, t_R) = n$ and the number of damages remaining after repair, $N_A(t_R) = n_A$ ($0 \leq n_A \leq n$), the cdf of $X_{\max(A)}(x)$ is,

$$F_{X_{\max(A)}}(x; t_R, t | N_I(0, t_R) = n \cap N_A(t_R) = n_A) = [F_{X(A)}(x; t_R, t)]^{n_A} \quad (6.14)$$

Removing the conditions that $N_I(t_R) = n$ and $N_A(t_R) = n_A$,

$$\begin{aligned}
 F_{X_{\max(A)}}(x; t_R, t) &= \sum_{n=0}^{\infty} \sum_{n_A=0}^n [F_{X(A)}(x; t_R, t)]^{n_A} \binom{n}{n_A} p^{n_A} (1-p)^{(n-n_A)} \\
 &\quad \cdot \frac{\left(\int_0^t \nu(\tau) d\tau \right)^n \cdot \exp \left(- \int_0^t \nu(\tau) d\tau \right)}{n!} \\
 &= \exp \left[-p \int_0^{t_R} \nu(\tau) d\tau \{1 - F_{X(A)}(x; t_R, t)\} \right] \quad (6.15)
 \end{aligned}$$

in which

$$\begin{aligned}
 p &= P[\text{A damage is not repaired at } t_R] \\
 &= F_X(x_{th}; 0, t_R) \quad (6.16)
 \end{aligned}$$

Under the assumption that damage initiation is described by a Poisson process,

$F_{X_{max(B)}}(x; t)$ is given by

$$F_{X_{max(B)}}(x; t) = F_{X_{max}}(x; t_R, t) \quad (6.17)$$

in which $\nu(t)$ in Eqn.(6.8) is replaced by the nonstationary damage initiation rate after repair, $\nu^*(t)$, defined by

$$\nu^*(t) = \begin{cases} \nu(t - t_R) & ; \text{ for part repaired} \\ \nu(t) & ; \text{ for part not repaired} \end{cases} \quad (6.18)$$

Substitution of Eqns.(6.8), (6.17) and (6.15) into Eqn.(6.12) leads to

$$F''_{X_{max}}(x; t_R, t) = \exp[-p \int_0^{t_R} \nu(\tau) d\tau \cdot \{1 - F_{X(A)}(x; t_R, t)\} - \int_{t_R}^t \nu^*(\tau) d\tau \cdot \{1 - F_X(x; t_R, t)\}] \quad (6.19)$$

The mean and variance of the degradation function are evaluated by Eqns.(6.9) and (6.10), respectively.

In some cases, only a portion of a component is inspected and possibly repaired. If a % (in area, volume, etc.) of a component is inspected, the cdf of the degradation function is evaluated by the law of total probability as,

$$F_{X_{max}}(x; t_R, t) = \frac{a}{100} \cdot F_{X_{max}}(x|\text{repaired}) + (1 - \frac{a}{100}) \cdot F_{X_{max}}(x|\text{not repaired}) \quad (6.20)$$

where $F_{X_{max}}(x|\text{repaired})$ and $F_{X_{max}}(x|\text{not repaired})$ are evaluated by Eqn.(6.19) and Eqn.(6.8), respectively.

General Detectability Function

In general, the detectability function, $d(x)$, is not a step function but rather a non-decreasing function of damage intensity (see Figure 6.2). Noting that a detectability function has the same property as a cdf, i.e., non-decreasing and taking a value within $[0, 1]$, $d(x)$ can be interpreted as describing an inspection with a step detectability function in which the threshold value, X_{th} , is a random variable with cdf, $F_{X_{th}}(x) = d(x)$.

In short,

$$\begin{aligned}
 d(x) &= P[\text{Detect damage with intensity as small as } x] \\
 &= P[X_{th} \leq x] \\
 &= F_{X_{th}}(x)
 \end{aligned} \tag{6.21}$$

$F_{X_{th}}(x)$ can be interpreted as the distribution function of the smallest detectable defect size.

The cdf of damage intensity in group (A) after inspection is evaluated using Bayes theorem and the law of total probability (cf. Eqn.(6.13)) as,

$$\begin{aligned}
 F_{X_{(A)}}(x; t_R, t) dx &= P[X(t) \leq x | \text{Not repaired at } t_R] \\
 &= P[X(t) \leq x | X(t_R) < X_{th}] \\
 &= k \cdot \int_0^1 P[X(t) \leq x \cap X(t_R) < x_{th}] \cdot f_{X_{th}}(x_{th}) dx_{th}
 \end{aligned} \tag{6.22}$$

in which k is a normalizing constant so that $F_{X_{(A)}}(x; t_R, t)$ is a legitimate cdf. Therefore,

$$F_{X_{(A)}}(x; t_R, t) = k \cdot \int_0^1 F_{X_{step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) \cdot f_{X_{th}}(x_{th}) dx_{th} \tag{6.23}$$

where $F_{X_{step}}(x; x_{th})$ is the cdf of damage intensity inspected with step detectability function with a threshold value x_{th} , evaluated by Eqn.(6.15), and

$$f_{X_{th}}(x) = \frac{d}{dx} d(x) \tag{6.24}$$

The cdf of damage intensity in group (B) after inspection remains unchanged. The mean and variance of the degradation function are evaluated by Eqns.(6.9) and (6.10), respectively, after substituting Eqns.(6.23) into (6.19). Eqn.(6.20) is still applicable if only a portion of the component is inspected.

Partial Repair

Sometimes only damages with intensity greater than a critical value, x_{cr} , are repaired. If we assume that the true damage intensity is known, if detected, the cdf of the intensity

of the damages in group (A) is,

$$\begin{aligned}
F_{X(A)}(x; t_R, t) &= P[X(t) < x | X(t_R) < X_{th} \cup X(t_R) < x_{cr}] \\
&= k \int_0^1 P[X(t) < x \cap X(t_R) < \max\{x_{th}, x_{cr}\}] \cdot f_{X_{th}}(x_{th}) dx_{th} \\
&= k \cdot [F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{cr}; 0, t_R) \cdot d(x_{cr}) \\
&\quad + \int_{x_{cr}}^1 F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) \cdot f_{X_{th}}(x_{th}) dx_{th}] \quad (6.25)
\end{aligned}$$

Therefore,

$$F_{X(A)}(x; t_R, t) = k \int_0^1 F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) \cdot f_{X_{eq}}(x_{th}) dx_{th} \quad (6.26)$$

where $f_{X_{eq}}(x)$ is the derivative of an “equivalent” detectability function, $d_{eq}(x)$, defined as

$$d_{eq}(x) = \begin{cases} 0 & ; \quad 0 \leq x < x_{cr} \\ d(x) & ; \quad x_{cr} \leq x \leq 1 \end{cases} \quad (6.27)$$

In general, the true damage intensity is not known even if damage is detected. Instead, the intensity detected would be described by a conditional distribution function, $F_{X_I|X(t_R)}(x_I|x)$, where X_I is the damage intensity indicated by an inspection. This conditional cdf can be formulated from experimental data. Then the cdf of the intensity of the damage in group (A) is,

$$\begin{aligned}
F_{X(A)}(x; t_R, t) &= P[X(t) < x | X(t_R) < X_{th} \cup X_I < x_{cr}] \\
&= k \cdot \{P[X(t) < x \cap X(t_R) < X_{th}] + P[X(t) < x \cap X_I < x_{cr}]\} \quad (6.28)
\end{aligned}$$

where

$$\begin{aligned}
P[X(t) < x \cap X(t_R) < X_{th}] &= \int_0^1 F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) \cdot f_{X_{th}}(x_{th}) dx_{th} \quad (6.29)
\end{aligned}$$

and

$$P[X(t) < x \cap X_I < x_{cr}]$$

$$\begin{aligned}
&= \int_0^1 P[X(t) < x \cap X_I < x_{cr} | X(t_R) = x_{th}] \cdot f_X(x_{th}, t_R) dx_{th} \\
&= \int_0^1 P[X(t) < x \cap X(t_R) = x_{th}] P[X_I < x_{cr} | X(t_R) = x_{th}] dx_{th} \\
&= \int_0^1 \frac{\partial}{\partial x_{th}} \left\{ F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) dx \right\} \\
&\quad \cdot \frac{1}{f_X(x_{th}; t_R)} \cdot \frac{\partial}{\partial x_{th}} F_{X_I X(t_R)}(x_{cr}, x_{th}) dx_{th} \\
&= - \int_0^1 [F_{X_{Step}}(x; x_{th}) \cdot F_X(x_{th}; 0, t_R) dx \\
&\quad \cdot \frac{\partial}{\partial x_{th}} \left\{ \frac{1}{f_X(x_{th}; t_R)} \frac{\partial}{\partial x_{th}} F_{X_I X(t_R)}(x_{cr}, x_{th}) \right\}] dx_{th} \tag{6.30}
\end{aligned}$$

Therefore, $F_{X_{(A)}}(x; t_R, t)$ is evaluated by the same expression as Eqn.(6.26), in which the equivalent detectability function now is defined as

$$d_{eq}(x) = d(x) - \frac{1}{f_X(x_{th}; t_R)} \frac{\partial}{\partial x_{th}} F_{X_I X(t_R)}(x_{cr}, x_{th}) \tag{6.31}$$

6.2.3 Degradation with Initiation Period

As discussed in Chapter 2, the service life of a concrete structure is defined as the sum of an initiation period and a propagation period for some degradation mechanisms such as corrosion (Tuutti, 1982; Vesikari, 1988; Clifton, 1991). In such cases damages initiate only after the initiation period, during which little or no degradation occurs. Let us define the initiation period as ending at time t_{I_1} when the first damage initiates and assume that the damage initiation after t_{I_1} is described by a Poisson process with mean initiation rate, $\nu(t)$. Assumptions (b), (c) and (d) made in Section 6.2.1 are also made here. Then the same approach can be followed to determine the cdf of maximum damage intensity before and after inspection/repair at time t_R , except that there is one extra damage which initiates at time t_{I_1} .

The cdf, $F_{X_1}(x; t)$, of the intensity of the first damage, $X_1(t)$, before inspection/repair is,

$$\begin{aligned}
F_{X_1}(x; t) &= P[C(t - t_{I_1})^\alpha < x] \\
&= F_C\left(\frac{x}{(t - t_{I_1})^\alpha}\right) \tag{6.32}
\end{aligned}$$

With assumption (c), the cdf of maximum damage intensity at time t before inspec-

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tion/repair is,

$$F_{X_{1max}}(x; t) = F_{X_1}(x; t) \cdot F_{X_{max}}(x; t_{I_1}, t) \quad (6.33)$$

in which $F_{X_{max}}(x; t_{I_1}, t)$ is given by Eqn.(6.8).

If the structure is inspected by an NDE method described by a step detectability function and all detected damages are repaired completely, the cdf, $F''_{X_1}(x; t)$, of the intensity of the first damage after inspection/repair at t_R , if it is not repaired, is,

$$\begin{aligned} F''_{X_1}(x; t) &= P[C(t - t_{I_1})^\alpha < x | C(t_R - t_{I_1})^\alpha < x_{th}] \\ &= \frac{F_C\left(\min\left\{\frac{x}{(t - t_{I_1})^\alpha}, \frac{x_{th}}{(t_R - t_{I_1})^\alpha}\right\}\right)}{F_C\left(\frac{x_{th}}{(t - t_{I_1})^\alpha}\right)} \end{aligned} \quad (6.34)$$

With assumption (c), the cdf of maximum damage intensity at time t before an inspection/repair is expressed as

$$F''_{X_{1max}}(x; t) = F''_{X_1}(x; t) \cdot F''_{X_{max}}(x; t_R, t) \quad (6.35)$$

in which $F''_{X_{max}}(x; t_R, t)$ is given by Eqns.(6.8), (6.13), (6.16) and (6.19) in which 0 is replaced by t_{I_1} . Ignoring the initiation phase may lead to an overly pessimistic appraisal of reliability (Ellingwood and Mori, 1992). Therefore the appropriate evaluation of initiation period is important to the reliability assessment.

6.2.4 Illustration of the Effect of a Single Inspection/Repair Operation on Reliability

Step detectability function

In order to study a simple case, let us assume that:

- (a) Every part of the structure is inspected (full inspection).
- (b) The initiation of damages is described by a stationary Poisson process with parameter ν which is dependent on structure surface area or volume.
- (c) Damage grows linearly as a function of time as described by Eqn.(6.1) with $\alpha = 1$.
- (d) The degradation rate, C , is lognormally distributed with two possible mean values, $\mu_C = 0.00125$ or 0.0025 , which corresponds to $E[X(40)|T_I = 0] = 0.05$ or 0.10 , respectively, and with coefficient of variation, $V_C = 0.5$.

(e) The detectability function is the step function described by Eqn.(6.11).

The mean and c.o.v. of $G(t)$ evaluated by Eqns.(6.9) and (6.10) are shown in Figures 6.4(a) and 6.4(b), respectively, corresponding to linear damage growth rate $E[X(40)|T_I = 0] = 0.05$ and 0.10 . The mean of the degradation function, $g(t)$, decreases as ν increases, as would be expected. The low values of the c.o.v. of $G(t)$ for $g(t) > 0.6$ suggest that it is reasonable to evaluate the time-dependent reliability of a component considering only the first-order statistics (mean, median or mode) of its degradation function.

The effect of repairing all detected damages on $E[G(t)] = g(t)$ is illustrated using several simple parametric representations of damage initiation, growth and detectability. It is assumed that the expected rate of occurrence of new damage indicators, ν , is 5/year. Inspection/repair is carried out at $t_R=20$ years. Figures 6.5(a) and 6.5(b) show the mean degradation function for no repair and for several detection thresholds x_{th} greater than 0.01. As expected, the mean degradation function decreases as the threshold of damage detection increases. It can also be noted that an inspection method with a specified threshold of detection is relatively more effective for the case in which $E[X(40)|T_I = 0] = 0.10$ and degradation occurs more rapidly.

Failure probabilities evaluated with the degradation functions illustrated in Figures 6.5(a) and 6.5(b) are presented in Figures 6.6(a) and 6.6(b) for the reinforced concrete component designed for flexure as before, using the design load combination for dead plus live load in ACI Standard 349 (1990),

$$0.9M_n = 1.4D_n + 1.7L_n \quad (6.36)$$

in which M_n = nominal strength determined from specified steel and concrete strengths and component dimensions and D_n and L_n are specified nominal dead and live loads. The probabilistic models of the loads are presented in Table 3.1. The initial resistance, R_0 , is described by a lognormal distribution, with mean of $1.15M_n$ and c.o.v. of 0.15 and it is assumed for illustrative purposes that $L_n = D_n$. The slope of $F(t)$ changes at the time of repair. An inspection with a lower detection threshold flattens the slope and, accordingly, leads to a lower time-dependent failure probability subsequent to the inspection.

General detectability function

The effect on the mean degradation function of inspection/repair described by several general detectability functions is illustrated in Figures 6.7(a) and 6.7(b). All assumptions in the previous section are made except for assumption (e). Two detectability functions are considered; in the first, X_{th} is uniformly distributed (i.e., $d(x)$ is linear between x_{min} and x_{max} , where $d(x_{min}) = 0$ and $d(x_{max}) = 1$), while in the second, X_{th} is lognormally distributed with mean, $\mu_{X_{th}}$, equal to 0.03 or 0.05, and coefficient of variation, $V_{X_{th}}$, equal to 0.3 or 0.5. The mean of the degradation function now must be evaluated by Monte Carlo simulation. Linear damage growth models with $E[X(40)|T_I = 0] = 0.05$

or 0.10 are considered and it is assumed that the rate of damage initiation, ν , is 5/year and inspection/repair is carried out at $t_R = 20$ years. The mean degradation function decreases as $V_{X_{th}}$ increases (which would result in lower reliability); however, the effect of uncertainty in detectability, measured by the general shape of $d(x)$ (Figure 6.2), is not significant and decreases with elapsed time since inspection. This insensitivity of the mean degradation to the choice of detectability function suggests that a general detectability function might be approximated for practical purposes by a step function with $x_{th} = \mu_{X_{th}}$. This would be advantageous because information on $\mu_{X_{th}}$ may be more readily available based on current experience with reinforced concrete structures than information on $d(x)$. Indeed, for certain structures, only visual inspection of exposed surfaces may be feasible, making information other than $\mu_{X_{th}}$ nearly impossible to estimate.

6.3 Degradation Function after m^{th} Inspection/ Repair

During the service life of a structure, more than one inspection/repair might be carried out. In such a case, the degradation function of a component/structure after, say, the m^{th} inspection/repair must be estimated taking the effects of the preceding inspections/repairs into account. Assume that a component is inspected/repared at times $t_R = \{t_{R_1}, t_{R_2}, \dots, t_{R_m}\}$. Each inspection is carried out using an NDE method described by a step detectability function with threshold value x_{th_j} , $j = 1, \dots, m$, and all detected damages are repaired. Then the cdf of the intensity of the damage which initiates within the interval of $(t_{R_{j-1}}, t_{R_j}]$ ($t_{R_0} = 0$) and remains after m^{th} inspection/repair, $F_X^{(xj)}(x; t, t_R, x_{th})$ (which will be expressed as $F_X^{(xj)}(x; t, m)$ for simplicity in the following), is,

$$F_X^{(xj)}(x; t, m) = P[X(t) < x | X(t_{R_j}) < x_{th_j} \cap \dots \cap X(t_{R_m}) < x_{th_m}] \quad (6.37)$$

$$= \frac{P \left[C(t - T_I)^\alpha < x \cap \left(\bigcap_{k=j}^m C(t_{R_k} - T_I)^\alpha < x_{th_k} \right) \right]}{P \left[\bigcap_{k=j}^m C(t_{R_k} - T_I)^\alpha < x_{th_k} \right]} \quad (6.38)$$

Assume for the moment that the number of damages which initiate within the interval of $(t_{R_{j-1}}, t_{R_j}]$, $N_I(t_{R_{j-1}}, t_{R_j})$, is n ; $F_X^{(*)j}(x; t, m)$ is expressed as (cf. Eqns.(6.13)),

$$\begin{aligned}
 F_X^{(*)j}(x; t, m) &= \frac{\int_{t_{R_{j-1}}}^{t_{R_j}} P \left[C < \frac{x}{(t-\tau)^\alpha} \cap \left(\bigcap_{k=j}^m C < \frac{x_{th_k}}{(t_{R_k} - \tau)^\alpha} \right) \right] f_{W_I}(\tau) d\tau}{\int_{t_{R_{j-1}}}^{t_{R_j}} P \left[\bigcap_{k=j}^m C < \frac{x_{th_k}}{(t_{R_k} - \tau)^\alpha} \right] f_{W_I}(\tau) d\tau} \\
 &= \frac{\int_{t_{R_{j-1}}}^{t_{R_j}} F_C \left(\min_{k=j}^m \left\{ \frac{x}{(t-\tau)^\alpha}, \frac{x_{th_k}}{(t_{R_k} - \tau)^\alpha} \right\} \right) f_{W_I}(\tau) d\tau}{\int_{t_{R_{j-1}}}^{t_{R_j}} F_C \left(\min_{k=j}^m \left\{ \frac{x_{th_k}}{(t_{R_k} - \tau)^\alpha} \right\} \right) f_{W_I}(\tau) d\tau} \quad (6.39)
 \end{aligned}$$

If the number of damages remaining after t_{R_m} , N_m , is n_m , then the cdf of the maximum intensity of the damages which initiated within the interval of $(t_{R_{j-1}}, t_{R_j}]$ and remain after t_m , $X_{max}^{(*)j}(t; t_m)$, is,

$$F_{X_{max}}^{(*)j}(x; t) = [F_X^{(*)j}(x; t, m)]^{n_m} \quad (6.40)$$

Removing the conditionings of $N_I(t_{R_{j-1}}, t_{R_j}) = n$ and $N_m = n_m$, $F_{X_{max}}^{(*)j}(x; t)$ is expressed as,

$$F_{X_{max}}^{(*)j}(x; t) = \exp[-p_j \nu(t_{R_j} - t_{R_{j-1}}) \{1 - F_X^{(*)j}(x; t, m)\}] \quad (6.41)$$

in which

$$\begin{aligned}
 p_j &= P[\text{a damage remains without being repaired after } t_{R_m}] \\
 &= F_X^{(*)j}(x_{th_m}; t_{R_m}, m-1) \quad (6.42)
 \end{aligned}$$

Finally, the cdf, $F_{X_{max}}''(x; t)$ of the maximum intensity of damages existing at time t after the m^{th} inspection/repair is,

$$\begin{aligned}
 F_{X_{max}}''(x; t) &= P[\max\{X_{max}(t_m; t), \max_{j=1}^m \{X_{max}^{(*)j}(t_m; t)\}\} < x] \\
 &= F_{X_{max}}(x; t_{R_m}, t) \cdot \prod_{j=1}^m F_{X_{max}}^{(*)j}(x; t) \quad (6.43)
 \end{aligned}$$

in which $F_{Xmax}(x; t_{R_m}, t)$ and $F_{Xmax}^{(*)j}(x; t)$ are evaluated by Eqn.(6.8) and Eqn. (6.41), respectively.

The mean degradation function can be evaluated by Eqn.(6.9). An extensive computational effort is required to evaluate the mean degradation function because the integrand in Eqn.(6.39) is discontinuous. However, if

$$t_{R_j}/t_{R_i} \geq x_{th_j}/x_{th_i} \quad (6.44)$$

for $j > i$, Eqn.(6.39) can be simplified as,

$$F_X^{(*)j}(x; t, m) = \frac{\int_{t_{R_{j-1}}}^{t_{R_j}} F_C \left(\min \left\{ \frac{x}{(t - \tau)^\alpha}, \frac{x_{th_m}}{(t_{R_m} - \tau)^\alpha} \right\} \right) d\tau}{\int_{t_{R_{j-1}}}^{t_{R_j}} F_C \left(\frac{x_{th_j}}{(t_{R_j} - \tau)^\alpha} \right) d\tau} \quad (6.45)$$

The condition of Eqn.(6.44) is achieved if the same detection threshold values are assumed for all inspections.

Illustration of Full Inspection Strategies

The effect of repairs on the mean degradation function is illustrated in Figures 6.8(a) - 6.8(b) making assumptions (a) - (e) from the previous illustration. It is assumed that the mean initiation rate of new damages, ν , is 5/year and a damage grows linearly after its initiation ($\alpha = 1$). Inspections/repairs are carried out at 20, 30, 40 and 50 years with (a) $x_{th_j} = x_{th} = 0.05$ when $E[X(40)|T_I = 0] = 0.05$ and (b) $x_{th_j} = x_{th} = 0.08$ when $E[X(40)|T_I = 0] = 0.10$. For comparison, the mean degradation functions for a component without repair and for a component with one inspection/repair at 30 years with $x_{th} = 0.01$ are also presented in the figures. With multiple repairs, the mean degradation function can be kept within a narrow range during the service life of a structure; the width and the location of the range can be controlled by the frequency and the threshold value of the inspections.

Failure probabilities evaluated with the degradation functions illustrated in Figures 6.8(a) - 6.8(b) are presented in Figures 6.9(a) - 6.9(b). The component is designed for flexure by Eqn.(6.36), and the load and initial strength statistics are same as before. As illustrated in Figures 6.5(a) and 6.5(b), the slope of $F(t)$ changes at the time of repair. Although the change of slope due to repair with $x_{th} =$ (a) 0.05 or (b) 0.08 is not as large as that with $x_{th} = 0.01$, the combined effect of multiple repairs with $x_{th} =$ (a) 0.05 or (b) 0.08 leads to a failure probability during 60 years that is compatible with that of the single intensive repair. This suggests the existence of an optimum inspection/maintenance strategy with which the failure probability of the component is

kept below an established target probability during its service life and the total expected cost, defined as the sum of the cost of inspections/repairs and the expected loss due to failure, is minimized. Optimum inspection strategies are discussed in detail later in this chapter.

6.4 Partial Inspection Strategies

In many instances only a small part of a structure or a component is inspected to assess its condition because of the cost of inspection. Inherent in this procedure is the assumption that any part of the component is representative of the component as a whole, and the rest of the component is amenable to a statistical representation (Chapman, 1983). At each time when a partial inspection is carried out we gain some information about the strength and degradation of the component and make a decision whether or not to repair the component, depending on the result of the inspection. Assume that each inspection is carried out by a procedure described by a step detectability function. A decision policy at each inspection can be established as follows:

1. Inspect $a\%$ of the component (Stage I).
2. If the maximum intensity of damages detected at Stage I, $X_{maxI}(t_R)$, is less than the predetermined critical value, x_{cr} , then perform no further inspection until the next scheduled inspection.
3. If $X_{maxI}(t_R) \geq x_{cr}$, then inspect another $a\%$ of the component (Stage II).
4. If the maximum intensity of damages detected at Stage II, $X_{maxII}(t_R)$, is less than x_{cr} , then perform no further inspection until the next scheduled inspection.
5. If $X_{maxII}(t_R) \geq x_{cr}$, then inspect the entire component and repair all detected damages (Stage III).
6. If a Stage III inspection is carried out at t_R , then during the rest of its service life the component will be fully inspected and all detected damages will be repaired.
7. The parameters of the degradation model are updated after each partial inspection (Stage I and Stage II) but the component is not repaired. Since the portion of a component partially inspected is generally small, the impact of repairing the detected damages on the degradation function and thus on the failure probability of the component is small.

This decision policy is summarized in Figure 6.10. A series of partial inspections with this policy leads to a decision tree as shown in Figure 6.11. A similar tree was used by Thoft-Christensen and Sorensen (1987) with a simpler policy. Although other policies can be envisioned, this policy is consistent, in a general sense, with acceptance criteria currently in use for weldments and certain other critical structures. The total number of branches after the m^{th} inspection is $2^m + 2^{m-1} + 1$.

Besides the assumptions (a) - (d) made in Section 6.2.1, assume that the mean damage initiation rate, ν , and the mean of the damage growth rate, $\bar{C}$, are random variables jointly distributed with pdf, $f_{\nu\bar{C}}(\zeta, \eta)$. Furthermore, assume that the cdf of the maximum intensity of damages existing at time t without repair can be evaluated approximately by Eqns.(6.5), (6.6) and (6.8) by substituting $\nu(\tau) = \mu_\nu$ and $\bar{C} = \mu_C$, where μ_ν and μ_C are the means of μ and $\bar{C}$, respectively.

After each inspection μ_ν and μ_C are updated depending on the decision made at the inspection, i.e., the path to be taken in Figure 6.11. Based on Bayes theorem (Benjamin and Cornell, 1970), the posterior joint density function of ν and $\bar{C}$, $f''_{\nu\bar{C}|I}(\zeta, \eta)$, is,

$$f''_{\nu\bar{C}|I}(\zeta, \eta) = kL(\zeta, \eta|I)f'_{\nu\bar{C}}(\zeta, \eta) \quad (6.46)$$

in which k is a normalizing constant, $L(\zeta, \eta|I)$ is the likelihood function of taking a path given $\nu = \zeta$ and $\bar{C} = \eta$, and $f'_{\nu\bar{C}}(\zeta, \eta)$ is the prior joint density of ν and $\bar{C}$. Then the parameters of the degradation model are updated as,

$$\mu''_\nu = \int_0^\infty \int_0^\infty \zeta f''_{\nu\bar{C}|I}(\zeta, \eta) d\eta d\zeta \quad (6.47)$$

$$\mu''_C = \int_0^\infty \int_0^\infty \eta f''_{\nu\bar{C}|I}(\zeta, \eta) d\eta d\zeta \quad (6.48)$$

At an inspection, one of three decisions is made. The probabilities of making each decision are,

$$P[\text{Stage I}] = F_{X_{max}}(x_{cr}; 0, t_R) \quad (6.49)$$

$$P[\text{Stage II}] = [1 - F_{X_{max}}(x_{cr}; 0, t_R)] F_{X_{max}}(x_{cr}; 0, t_R) \quad (6.50)$$

$$P[\text{Stage III}] = [1 - F_{X_{max}}(x_{cr}; 0, t_R)]^2 \quad (6.51)$$

in which $F_{X_{max}}(x; 0, t)$ is evaluated by Eqn.(6.8) substituting $\nu = a\mu''_\nu/100$ and $\mu_C = \mu''_C$. Therefore, the likelihood function of taking any path is,

$$\begin{aligned} L(\zeta, \eta|I) &= \prod_i P[\text{Stage I at } t_{R_i} | \nu = \zeta, \bar{C} = \eta] \\ &\quad \cdot \prod_j P[\text{Stage II at } t_{R_j} | \nu = \zeta, \bar{C} = \eta] \\ &\quad \cdot P[\text{Result of the } k^{\text{th}} \text{ inspection} | \nu = \zeta, \bar{C} = \eta] \end{aligned} \quad (6.52)$$

Illustration of Partial Inspection Strategies

In order to study a simple case, let us assume that prior to the first inspection ν and $\bar{C}$ are statistically independent and uniformly distributed within the intervals of $\pm 50\%$ from their mean values. Then the prior joint pdf of ν and $\bar{C}$ is given by

$$f'_{\nu\bar{C}}(\zeta, \eta) = \frac{1}{\mu_\nu \cdot \mu_C} \quad (6.53)$$

The effect of variabilities in ν and $\bar{C}$ on the mean and c.o.v. of a degradation function without inspection/repair is illustrated in Figure 6.12 assuming that $\mu_\nu = 5/\text{year}$ and damages grow linearly. Mean $g(t)$ and $V_{G(t)}$ with deterministic ν and $\bar{C}$ are also shown in Figures 6.12(a) and 6.12(b). The difference in $g(t)$ between these two cases is negligible and the difference in $V_{G(t)}$ is small. Since the variabilities in ν and $\bar{C}$ are decreased by updating the joint pdf at each path taken after a partial inspection (cf. Eqn.(6.46)), $V_{G(t)}$ with partial inspections will be smaller than that without partial inspections. Thus, the result of partial inspection strategies can be compared with those of full inspection strategies.

The effect of partial or possible full inspection/repair on the failure probability of a component is illustrated in Figures 6.13(a) and 6.13(b). The component is designed for flexure by Eqn.(6.36). The statistics of the initial resistance and of the loads are the same as those used in two previous examples. Partial inspections are scheduled at 20, 30, 40 and 50 years. 5 % of the component is inspected at each partial inspection. As an additional constraint, if a Stage III inspection is carried out at 20 years, the component also is fully inspected/repared at 40 years. However, if a Stage III inspection is carried out at 30 years or later, the component is not inspected/repared during the rest of its service life. The critical value for decision at partial inspection, x_{cr} , and the threshold value of damage detection for full inspection are same at all inspections and equal (a) 0.03 and (b) 0.05. For the purpose of comparison, the result of the full inspection strategy described in Figure 6.9 is also shown in the Figure 6.13. These partial inspection strategies are compatible with the full inspection strategies in terms of failure probability for a lifetime of 60 years. Thus, the choice of a strategy should be based on the expected total cost associated with each inspection strategy.

The application of the above methodology of condition assessment and life prediction of concrete structures to determine the optimum inspection/repair strategies is demonstrated in the next section.

6.5 Optimum Inspection/Repair Strategy

Costs of inspection and repair may be a significant part of the overall lifetime cost of a concrete structure. These costs should be balanced by the benefits to be gained, both in economic and reliability terms. There are tradeoffs between the extent and accuracy of inspection, required level of reliability, and cost. One can perform this tradeoff by defining an objective function that takes into account the cost of failure, cost of inspection, and costs of repair. The failure probability of a component during its service life, $F(t)$, provides one of several constraints on the optimization.

To design an optimum inspection/maintenance program, the following optimization problem must be solved:

$$\begin{array}{ll} \text{Minimize} & C_T \\ \text{Subject to} & F(t_L) \leq P_{f_T} \end{array} \quad (6.54)$$

in which P_{f_T} is an established target failure probability during the service life, t_L , of a component/structure and C_T is the total cost of inspection/repair plus expected losses due to failure of a structure. Other possible constraints might include the minimum and maximum time intervals between inspections, and the minimum threshold value of detection of the NDE method which is available.

Some studies have been done to determine optimal inspection/maintenance strategies for metallic structures subjected to fatigue, assuming that a component is replaced if the intensity of detected damage exceeds a critical value (e.g., Thoft-Christensen and Sorensen, 1987). The cost of repair (i.e., cost of replacement) was assumed to be constant in these studies. However, a component may not be replaced; instead, only detected damages might be repaired. In this case, the cost of repair would be a function of damage intensities to be repaired. This aspect also should be considered in designing an optimum strategy.

Optimum inspection/repair strategies are illustrated using several parametric representations of relative cost of inspection, repair and losses due to structural failure. In order to study a simple case, let us assume that:

- (a) Damage grows linearly as a function of time as described by Eqn.(6.1) with $\alpha = 1$.
- (b) The degradation rate, C , is lognormally distributed with a mean value that is allowed to vary (see Table 6.1) and a c.o.v., $V_C = 0.5$.
- (c) The detectability function is the step function described by Eqn.(6.11),
- (d) All detected damages are repaired.

Let us assume that the total cost can be expressed as

$$C_T = C_{ins} + C_{rep} + C_f \cdot F(t_L) \quad (6.55)$$

in which C_{ins} is expected inspection cost, C_{rep} is expected repair cost, and C_f is the loss due to structural failure. Assume further that C_f is constant, and C_{ins} and C_{rep} are evaluated by the following formulas:

$$\begin{aligned} C_{ins} &= \alpha_{ins} \cdot \sum_{j=1}^m E[A_{ins_j}] (1 - x_{th_j})^b \\ &= \alpha_{ins} (1 - x_{th})^b \sum_{j=1}^m E[A_{ins_j}] \end{aligned} \quad (6.56)$$

$$C_{rep} = \alpha_{rep} \cdot \sum_{j=1}^m E[A_{rep_j}] E[X_{max}(t_{R_j})]^2 \quad (6.57)$$

in which α_{ins} and α_{rep} are constants, A_{ins_j} and A_{rep_j} are the normalized area (or volume) of a component to be inspected and repaired at the j^{th} inspection, respectively, and $X_{max}(t_{R_j})$ is the maximum damage intensity prior to the repair at t_{R_j} . In Eqns.(6.56) and (6.57), it is assumed that C_{ins} depends on the quality and extent of inspection, while C_{rep} is a linear function of the area to be repaired and a nonlinear function of maximum damage intensity to be repaired. The parameter, b , in Eqn.(6.56) is set equal to 20 so that $C_{ins}|_{x_{th}=0.1}/C_{ins}|_{x_{th}=0.01} \approx 0.15$. For strategies involving full inspections,

$$E[A_{ins_j}] = E[A_{rep_j}] = 1 \quad \text{for } j = 1, \dots, m \quad (6.58)$$

The component is designed for flexure by Eqn.(6.36), assuming that $L_n = D_n$. The initial resistance, R_0 , is described by a lognormal distribution, with mean of $1.15M_n$ and c.o.v. of 0.15. The probabilistic models of load events are presented in Table 3.1. No data could be located to define the costs of inspection, repair and failure in absolute terms. Thus, several cases were considered in which the relative costs were varied to determine the sensitivity of the optimal strategy to these relative costs. These relative costs of inspection, repair and losses due to failure determined by α_{ins} , α_{rep} , and C_f in Eqns.(6.55) - (6.57) are summarized in Table 6.1. P_{f_T} is assumed to equal 4×10^{-4} in 60 years.

The gradient-based nonlinear optimization program MINOS (Murtagh and Saunders, 1987) was employed to perform the minimum cost optimization. Since the mathematical programming problem expressed by Eqn.(6.54) is nonconvex, the solution depends on the starting point and might be locally rather than globally optimum. To test the validity of a solution, several different starting points were selected. In the following, "optimum solution" means a locally optimum solution.

The future inspection and repair costs have not been discounted to present worth in the interest of simplicity.

Full Inspection Strategies

The decision variables are the times at which inspections/repairs are carried out, $t_R = \{t_{R_1}, \dots, t_{R_m}\}$, and the threshold value of detection for inspections, $x_{th_1} = \dots = x_{th_m} = x_{th}$. The number of inspections, m , during the service life of a structure is considered to be given. The influence of m can be taken into account by solving the optimization problem for a number of different values of m and comparing the optimum total costs C_T . The value, m , with the smallest C_T is the optimal value for m . With this approach, the more complicated integer programming problem can be avoided (Thoft-Christensen and Sorensen, 1987).

Besides the constraint on failure probability in Eqn.(6.54), the following constraints are introduced by assuming that operational characteristics of the structure preclude inspection more often than every 4 years and that x_{th} of the best available inspection procedure, $x_{th_{min}}$, is either 0.05 or 0.005:

$$\Delta t_{R_j} \geq 4 \text{ years}, \quad j = 1, \dots, m+1 \quad (6.59)$$

$$x_{th} \geq x_{th_{min}} \quad (6.60)$$

in which $\Delta t_{R_j} = t_{R_j} - t_{R_{j-1}}$, $t_{R_0} = 0$ and $t_{R_{m+1}} = t_L$.

The optimum solutions for full inspection/repair strategies with given values of m are shown in Tables 6.2 - 6.9 and the expected cost associated with these solutions and the mean degradation function associated with the optimum policy are presented in Figures 6.14 - 6.21. In the first three examples, the same structural component is considered but the ratios among α_{ins} , α_{rep} , and C_f are different. When the repair is relatively expensive (Table 6.2 and Figures 6.14(a) and 6.14(b)), t_{R_i} tends to shift toward the early period of the service life. With this strategy, the component can be repaired when it is lightly damaged, and accordingly, the cost of repair can be reduced. If the service life of a component is extended beyond $t_L = 60$ years, a large repair cost may be incurred at that time. Because there is some time between the last scheduled inspection/repair and t_L , the component would be degraded relatively heavily at t_L (cf. Figure 6.14(b)). If it is considered possible at the time of the reliability analysis that the service life might be extended, another constraint, such as a lower bound of $g(t)$, would be recommended. Since the inspection is relatively inexpensive, x_{th} has little effect on the total cost, and accordingly, the optimal x_{th} equals its lower bound (Eqn.(6.60)). Moreover, since only the minimum extent of repair is performed to meet the constraint in terms of failure probability, the failure probability equals its upper bound (Eqn.(6.54)). The optimum number of inspections/repairs is three; however, the total cost function is insensitive to m if $2 \leq m \leq 5$ (cf. Figure 6.14(a)). In this case, then, 2 inspections might be an acceptable alternative policy.

When the total expected cost is dominated by the expected losses due to structural failure (Table 6.3 for $m = 1, \dots, 5$ and Figures 6.15(a) and 6.15(b)), the failure probability associated with the optimal strategy does not reach its upper bound; inspection/repair should be carried out with the best available method of damage detection at nearly uniform intervals during the lifetime of the component. With this strategy, the maximum effect of repair on reducing failure probability can be obtained. Based on this observation, the optimum full inspection policy is to perform inspections/repairs at uniform intervals during the service life with the best available method of damage detection. The expected total cost of these strategies is also presented in Figure 6.15(a). Since the failure probability can be reduced by increasing the number of inspections/repairs, the optimum number of inspections, m_{opt} , reaches its upper bound (cf. Eqn.(6.59)) of 14. With this strategy, the strength of the component can be kept near its original state throughout its service life (cf. Figure 6.15(b)). The reduction of C_T by increasing m is small for m greater than about 7.

In the third example (Table 6.4 and Figures 6.16(a) and 6.16(b)), it is assumed that the repair cost is constant and that losses due to structural failure are small and can be excluded from the optimization problem. In this case, the objective function can be rewritten as,

$$\text{Minimize } C_{ins} \quad (6.61)$$

Given that the number of inspections is m , the first inspection is carried out later than in Examples 1 and 2. A component needs to be damaged relatively heavily for a relatively cursory NDE method to have much impact on the reliability of the component. The later inspections (for $m > 1$) are scheduled approximately uniformly through the rest of the service life. This result is consistent with the results of a previous study (Thoft-Christensen and Sorensen, 1987), which showed that the first inspection interval of the optimum inspection strategy for a metallic component subjected to fatigue is larger than subsequent intervals between inspections, assuming that only the inspection cost depends on the decision variables. Therefore, it is important to estimate the cost of inspection for each NDE method available, the cost of repair given the level of damage, and the losses due to structural failure before determining the optimum inspection/repair strategy.

In the fourth example (Table 6.5 and Figures 6.17(a) and 6.17(b)), the damage growth rate and the relative cost are the same as that of Example 1 but $x_{th_{min}}$ is increased to 0.05. The optimization problem is solved for $m \geq 3$ because the constraint on failure probability cannot be met when $m \leq 2$. The times of inspection shift toward the later stage of service life because an NDE method with relatively large detection threshold cannot work effectively if the damage intensity is small. As a result, the intensities of detected damages are relatively large, which leads to expensive repair. The optimum number of inspections/repairs is three; however, C_T is insensitive to m for $3 \leq m \leq 5$.

In the fifth and sixth examples, $x_{th_{min}}$ and the relative cost are the same as those in Example 1 (cf. Table 6.2 and Figures 6.14(a) and 6.14(b)), but the damage growth rate is different. For the case that $E[X(40)|T_I = 0] = 0.03$ (Table 6.6 and Figures 6.18(a) and 6.18(b)), the optimization problem is solved for $m \leq 3$ because reductions in C_T are not expected with a larger number of inspections; for the case that $E[X(40)|T_I = 0] = 0.10$

(Table 6.7 and Figures 6.19(a) and 6.19(b)), it is solved for $3 \leq m \leq 9$ because the constraint on failure probability cannot be met when $m \leq 2$. Optimum inspection/repair times, t_R , shift toward the later period of the service life as the mean degradation rate increases. The number of inspection/repair operations required at the optimum strategy increases with the mean degradation rate ($m_{opt} = 1$ for $E[X(40)|T_I = 0] = 0.03$, $m_{opt} = 3$ for $E[X(40)|T_I = 0] = 0.05$ and $m_{opt} = 8$ for $E[X(40)|T_I = 0] = 0.10$). Thus, knowledge of the mean degradation rate is also important for designing the optimum inspection strategy.

In the seventh and eighth examples, it is assumed that damages grow nonlinearly, but that the other parameters are the same as those in Example 1. The optimum solutions for square root damage growth are presented in Table 6.8 and Figures 6.20(a) and 6.20(b), while those for parabolic damage growth are presented in Table 6.9 and Figures 6.21(a) and 6.21(b). As observed in Example 1, t_R shifts toward the early period of the service life when relative repair cost is high. However, since the component degrades relatively rapidly in the early stage of its service life for the case of square root damage growth ($\alpha = 0.5$ in Eqn.(6.1)), more frequent maintenance is required than for the case when damages grow linearly. When damages grow parabolically ($\alpha = 2$ in Eqn.(6.1)), relatively infrequent inspections can keep the strength of the component at near its original state so that the stage of rapid damage growth can be avoided during the intended service life. Thus, the additional constraint of maximum damage level at t_L is more important than in the case of linear damage growth if the service life might be extended. The optimum number of inspections/repairs increases as α in Eqn.(6.1) decreases.

The intervals between inspections/repairs are nearly uniform in these examples regardless of damage accumulation. It is assumed that damage initiation is stationary and the threshold value of damage detection is the same in all cases. Since most rapidly growing damages which are overlooked at an inspection initiate after the previous inspection, the damage intensity remaining immediately following an inspection is nearly identically distributed after the second inspection/repair. If damage initiation is nonstationary, the intervals decrease as the damage initiation rate increases as seen in Examples 1, 5 and 6.

Partial Inspection Strategies

The decision policy for partial inspection strategies described in the preceding section is also employed here. In the optimization of a partial inspection strategy, there exists a number of decision variables, such as the number of partial inspections to be scheduled, m_P , critical value of decision at the i^{th} partial inspection, x_{cr_i} , times at which the i^{th} partial inspection is carried out, t_{P_i} , the number of full inspections after a Stage III inspection is performed at t_{P_i} (Branch i), m_i , the threshold value of the j^{th} inspection/repair in Branch i , $x_{th_{ij}}$, and the time at which full inspection/repair is carried out in Branch i , $t_{R_{ij}}$. Because extensive computational effort is required to optimize all decision variables, the following relations among variables are assumed:

$$x_{cr_i} = x_{cr} \quad (6.62)$$

$$x_{thij} = x_{th} \quad (6.63)$$

$$x_{th} = x_{cr} \quad (6.64)$$

$$t_{P_i} = t_{P_1} + (i - 1)\Delta t_P \quad (6.65)$$

$$t_{R_{ij}} = t_{R_{i1}} + (j - 1)\Delta t_{R_i} \quad (6.66)$$

$$\Delta t_{R_i} = \alpha_R \cdot t_{P_i} \quad (6.67)$$

$$m_i = m_P - i + 1 \quad (6.68)$$

; for $j = 1, \dots, m_i$ and $i = 1, \dots, m_P$

in which Δt_P and Δt_{R_i} are the intervals between partial inspections and those between full inspections/repairs after a Stage III inspection is carried out, respectively. Furthermore, it was observed in the analysis of optimum full inspection/repair strategies that x_{th} tends to take the minimum value when inspection is inexpensive relative to repair and failure costs. Since this usually is the case in practice, it can be assumed that $x_{th} = x_{th_{min}}$. Thus, the problem is reduced to one having three decision variables, t_{P_1} , Δt_P , and α_R .

The optimum partial inspection strategies for Example 1 (see Table 6.1) are presented in Table 6.10. Because of the large number of branches, extensive computational effort is required to obtain the optimum solution by MINOS for $m_P = 4$. Alternatively, the optimum solutions were obtained by solving with several combinations of $(t_{P_1}, \Delta t_P, \alpha_R)$. The optimum full inspection/repair strategy is also presented in Table 6.10 for comparison. The optimum partial inspection strategy ($m_P = 4$) is slightly more cost-efficient ($C_T = 11.4$) than the optimum full inspection/repair strategy ($C_T = 11.5$). The benefit of partial inspection/repair would increase if all decision variables were optimized.

On the other hand, a partial inspection/repair strategy is less feasible than the optimum full inspection/repair strategy when the total expected cost is dominated by losses due to structural failure (cf. Table 6.3 and Figure 6.15). In such a case, the component should be repaired frequently so that the hazard function and failure probability are kept small.

6.6 Illustration with One-way Slab

Up to this point, the methodology for condition assessment and life prediction of concrete structures has been illustrated using simple conceptual models. In order to provide a more practical illustration, the methodology is applied to the evaluation of a simply supported one-way slab (see Figure 6.22(a)) subjected to corrosion of reinforcement initiated following carbonation of the concrete cover. Let us assume that,

- (a) The slab is subjected to time invariant dead load, D , and intermittent live load L , both of which are distributed uniformly, on the slab.
- (b) The slab fails only due to flexure,

- (c) The slab is under-reinforced, and fails if all the reinforcing bars yield,
- (d) The internal moment within the midspan range $\pm 0.225\ell$ due to the dead and live load is nearly uniform and equals $(D + L)\ell^2/8$ (see Figure 6.22(b) and 6.22(c)). This assumption simplifies the reliability analysis and provides a conservative estimate of the failure probability of the slab,
- (e) Yielding of reinforcing bars occurs only within the midspan range $\pm 0.225\ell$; the probability that any reinforcing bar yields outside this range is negligible,
- (f) Compressive strength of the concrete changes with time due to continuous hydration of cement, while the section area of the reinforcement decreases with time due to corrosion. Other engineering properties of the slab are time-invariant.

6.6.1 Degradation Function of One-Way Slab

As reinforcing bars are generally ductile, the one-way slab can be modeled as a strictly parallel system composed of ductile components. The reinforcing bars are assumed to be supplied by the same manufacturer, and thus the yield strengths and the section areas of reinforcing bars are assumed to be perfectly correlated. Because a one-way slab can be modeled as a beam, its initial flexural strength, R_0 , is expressed as (ACI 318, 1989),

$$R_0 = \alpha_0(1 - 0.59\alpha_0)bd^2F_{c28} \quad (6.69)$$

in which

$$\alpha_0 = \frac{\rho F_y}{F_{c28}} \quad (6.70)$$

$$\rho_0 = \frac{n_t A_t}{bd} = \text{reinforcement ratio} \quad (6.71)$$

F_{c28} is the compressive strength of concrete at 28 days, n_t is the number of reinforcing bars, and F_y and A_t are yield strength and section area of a reinforcing bar, respectively. Similarly, the flexural strength at time t , $R(t)$, is expressed as,

$$R(t) = \alpha(t)(1 - 0.59\alpha(t))bd^2F_c(t) \quad (6.72)$$

in which

$$\alpha(t) = \frac{\rho(t)F_y}{F_c(t)} \quad (6.73)$$

$$\rho(t) = \frac{\sum_{i=1}^{n_t} A_{t,i}(t)}{bd} \quad (6.74)$$

$F_c(t)$ is the compressive strength of concrete at time t and

$$A_{t,i}(t) = \min_{all j} \{A_{t,ij}(t)\} \quad (6.75)$$

in which $A_{t,ij}(t)$ is the residual section area at location j of the i^{th} reinforcing bar at time t . Then the degradation function of the slab is,

$$G(t) \equiv \frac{R(t)}{R_0} \quad (6.76)$$

$$\begin{aligned} &= \frac{\rho(t)F_y \left[1 - 0.59 \left(\frac{\rho(t)F_y}{F_c(t)} \right) \right]}{\rho_0 F_y \left[1 - 0.59 \left(\frac{\rho_0 F_y}{F_{c28}} \right) \right]} \\ &= \frac{\sum_{i=1}^{n_t} A_{t,i}(t) \left[1 - 0.59 \left(\sum_{i=1}^{n_t} \frac{A_{t,i}(t)F_y}{bdF_c(t)} \right) \right]}{n_t A_t \left[1 - 0.59 n_t A_t \left(\frac{F_y}{bdF_{c28}} \right) \right]} \\ &= \frac{\frac{1}{n_t} \sum_{i=1}^{n_t} \frac{A_{t,i}(t)}{A_t} \left[1 - 0.59 n_t A_t \left(\frac{1}{n_t} \sum_{i=1}^{n_t} \frac{A_{t,i}(t)}{A_t} \frac{F_y}{bdF_c(t)} \right) \right]}{\left[1 - 0.59 n_t A_t \left(\frac{F_y}{bdF_{c28}} \right) \right]} \end{aligned} \quad (6.77)$$

Now let us define the intensity of damage at location j of the i^{th} reinforcing bar at time t as,

$$X_{ij}(t) = 1 - \frac{A_{t,ij}(t)}{A_t} \quad (6.78)$$

Then the degradation function of the i^{th} reinforcing bar is,

$$G_i(t) = 1 - \max_{all j} \{X_{ij}(t)\} \quad (6.79)$$

$$= \frac{\min_{\text{all } j} \{A_{t,j}(t)\}}{A_t} \quad (6.80)$$

$$= \frac{A_{t,i}(t)}{A_t} \quad (6.81)$$

Since the expected value of $G_i(t)$ can be approximated by

$$E[G_i(t)] \approx \frac{1}{n_t} \sum_{k=1}^{n_t} \frac{A_{t,k}(t)}{A_t} \quad (6.82)$$

the degradation function of the one-way slab can be expressed as,

$$G(t) \approx E[G_i(t)] \frac{\left[1 - 0.59n_t A_t \left(E[G_i(t)] \frac{F_y}{bdF_c(t)} \right) \right]}{\left[1 - 0.59n_t A_t \left(\frac{F_y}{bdF_{c28}} \right) \right]} \quad (6.83)$$

The mean degradation function of the one-way slab can be evaluated approximately by,

$$E[G(t)] \approx E[G_i(t)] \frac{\left[1 - 0.59n_t E[A_t] \left(E[G_i(t)] \frac{E[F_y]}{bdE[F_c(t)]} \right) \right]}{\left[1 - 0.59n_t E[A_t] \left(\frac{E[F_y]}{bdE[F_{c28}]} \right) \right]} \quad (6.84)$$

in which $E[G_i(t)]$ can be evaluated by Eqns.(6.9) and (6.19).

6.6.2 Description of Slab Analyzed

The dimensions of the one-way slab and the nominal material strength are shown in Table 6.11. The reinforcement ratio of the slab is

$$\rho = \frac{n_t a_t}{bd} \quad (6.85)$$

$$= \frac{\frac{30}{4/12} \frac{1}{4} \pi \left(\frac{0.75}{12} \right)^2}{30 \frac{10}{12}} \quad (6.86)$$

The nominal flexural strength of the one-way slab, M_n is given by

$$M_n = bd^2 f'_c \alpha_0 (1 - 0.59 \alpha_0) \quad (6.87)$$

where

$$\alpha_0 = \frac{\rho_0 f_y}{f'_c} \quad (6.88)$$

$$= \frac{0.0110 \cdot 60}{4} = 0.165 \quad (6.89)$$

Thus,

$$\begin{aligned} M_n &= 30(10/12)^2 (4 \cdot 12^2) 0.165 (1 - 0.59 \cdot 0.165) \\ &= 1787 \text{ (kip-ft)} \end{aligned} \quad (6.90)$$

It is assumed that this slab is designed based on the current design requirement of concrete structures expressed by Eqn.(6.36) and that $L_n = D_n$. The nominal combined load effect of dead and live load is 1039 kip-ft, or the nominal load effects of dead and live load are both 519.5 kip-ft. The corresponding nominal uniform load, Q_n , is related to nominal load effect as

$$D_n + L_n = \frac{1}{8} Q_n b \ell^2 \quad (6.91)$$

Therefore, $Q_n = 0.308$ kips/ft². The self weight of the one-way slab is 0.141 kips/ft².

The initial resistance of the one-way slab, R_0 , is described by a lognormal distribution with mean value, $\mu_R = 1.12M_n$, and the coefficient of variation 0.14 (cf. Table 3.3). The live load effect is described by a Type I largest value distribution with mean value, $\mu_L = 0.4L_n$, and the coefficient of variation 0.5. Since the effect of variation of dead load on the failure probability of a component is small as illustrated in Chapter 4, the intensity of dead load is assumed to be deterministic and equal to μ_D .

For ordinary concrete sheltered from rain and without cracks, the depth of carbonation is expressed by Eqns.(2.6) and (2.7). If the water-cement ratio equals 0.5 and if the concrete cover of the reinforcing bars is 1 inch (25 mm), the induction period, t_I , is

$$t_I = \left[\frac{25}{26(0.5 - 0.3)^2 + 1.6} \right] = 90 \text{ years} \quad (6.92)$$

Therefore, the corrosion of reinforcement due to carbonation would not be a problem for an ordinary concrete structure. However, concrete structures are susceptible to cracking due to shrinkage, freeze-thaw cycling, excessive tensile stresses, etc. In general, these cracks have little effect on the compressive strength of concrete. However, the carbon dioxide of the air penetrates the concrete more easily at a crack than at an unbroken concrete surface (Vesikari, 1988), and the induction period of carbonation may decrease. The depth of carbonation at a crack is obtained from Eqn.(2.8). If the thickness of the concrete cover is 1 inch (25 mm) and crack width is 4×10^{-3} inch (0.1 mm), the induction period is

$$t_I = (25/50)^4 \cdot (1/0.1)^2 = 6.3 \text{ years} \quad (6.93)$$

Since the initiation time of a crack and its width is random, let us assume that the initiation of corrosion is described by Poisson process with parameter $\nu(t)$ expressed as,

$$\nu(t) = \begin{cases} 0 & , \text{ for } t < t^* \\ \nu & , \text{ for } t \geq t^* \end{cases} \quad (6.94)$$

For illustration, let us assume that

- t^* is deterministic and equal to 10 years. As shown in Chapter 4, the variability in the initiation period is negligible.
- The mean initiation rate of corrosion within the critical area of a bar, ν_{bar} is 0.05/bar/year (susceptible area only); i.e. the mean initiation rate of corrosion within the critical area of the slab is 4.5/year,
- The intensity of a damage due to corrosion is,

$$X_{ij}(t) \approx \frac{4R_c}{D}(t - T_I) \quad (6.95)$$

in which R_c is the corrosion rate and T_I is the initiation time of the corrosion; Eqn.(6.95) is valid if the depth of corrosion is small relative to the diameter of the reinforcing bar.

- R_c is a time-invariant random variable described by a lognormal distribution with mean value, μ_{R_c} , of 1.2×10^{-3} inch/year (30 $\mu\text{m}/\text{year}$) and coefficient of variation, V_{R_c} , of 0.5 (Vesikari, 1988). Then the damage growth rate, C , in Eqn.(6.1),

$$C = \frac{4R_c}{D} \quad (6.96)$$

- is also a time-invariant random variable described by a lognormal distribution with $V_C = 0.5$. The mean of the damage growth rate, μ_C , equals 6.40×10^{-3} /year for $D = 3/4$ inch (19 mm),
- The 28-day specified compressive strength of concrete equals 4000 psi (27.6 MPa). The mean compressive strength at 28 days is evaluated by the equation in Table 3.2,
 - Concrete matures with time; the compressive strength increases during the first 10 years and does not change thereafter. The mean compressive strength at time t is evaluated by Eqn.(2.2) with $\alpha = 0.541$, and $\beta = 0.138$. Substituting $E[F_c(28 \text{ days})] = 4160$ psi into Eqn.(2.2) (in psi units) leads to

$$E[F_c(t)] = \begin{cases} 2250 + 573 \ln t, & t < 10 \text{ years} \\ 6950, & t \geq 10 \text{ years} \end{cases} \quad (6.97)$$

in which t is in days. The uncertainty in $F_c(t)$ due to maturing is neglected because of the small variation in strength as discussed in Chapter 2.

6.6.3 Illustration

The mean degradation functions of the one-way slab subjected to corrosion are presented in Figure 6.23. Three cases are illustrated; the slab is fully inspected/ repaired at $t_R = 30$ years with $x_{th} = 0.01$; at $t_R = 20, 30, 40$, and 50 years with $x_{th} = 0.08$; or not inspected/repaired. For comparison, the mean degradation function of a one-way slab with continuous hydration of cement but without corrosion is also presented. Without corrosion, the strength of the slab increases about 5 %, while it decreases 17.5 % at 60 years due to corrosion without repair.

The failure probabilities of the one-way slab with mean strength degradation illustrated in Figure 6.23 are presented in Figure 6.24 along with the failure probability with $g(t) = 1$. Since the latter can be considered as the intended design failure probability, (e.g., Galambos, et al, 1982; Hwang, et al, 1987) this value would be the target failure probability for inspection/maintenance strategy if degradation occurs. Since $F(40)$ without repair is smaller than $F(40)$ with $g(t) = 1$, inspection/repair would not be necessary if the intended service life is 40 years or less. If the intended service life is 60 years, some inspections/repairs are required. Both strategies described in Figure 6.24 are feasible and compatible in terms of failure probability. The selection of the appropriate strategy depends of the total cost associated with each.

The optimum inspection/repair strategies with given value of m for this slab for $t_L = 60$ years are presented in Table 6.12 and Figures 6.25(a) and 6.25(b) for full inspection/repair strategies and in Table 6.13 for partial inspection strategies. It is assumed that the total cost is evaluated by Eqns.(6.55) - (6.57) with $\alpha_{ins} : \alpha_{rep} : C_f = 1:5000:10^4$. The optimum solution for partial inspection strategies for $m_P = 4$ is obtained by solving several combinations of $(t_{P1}, \Delta t_P, \alpha_R)$. The optimum partial inspection is performed

when $m_P = 3$; however, the associated expected total cost is larger than that associated with the optimum full inspection/repair strategy. Thus, the optimum strategy is to perform two inspection/repair operations at 17.7 and 25.0 years with $x_{th} = 0.005$.

6.7 Summary

The degradation in strength of a component with random damage depends on the damage mechanism, frequency of in-service inspection/maintenance, quality of NDE technology used in inspection, and quality of repair. The variance in the component strength degradation is small even if the variance of individual damage intensities is large. This result suggests that it is reasonable to consider the degradation function to be deterministic, in first approximation. The degradation function is sensitive to the mean damage growth rate after initiation and to the threshold of detection of the detectability function, $d(x)$. However, it is not sensitive to the precise shape of $d(x)$.

Optimum strategies for inspection and maintenance can be devised so as to minimize the expected total cost while maintaining the limit state probability of structures and components below a desired acceptable threshold. These strategies are sensitive to the relative cost of inspection, repair and failure as well as to the shape and rate of the damage growth model. Near-uniform inspection intervals appear to be optimal for a wide range of NDE techniques and inspection, repair and failure costs.

Table 6.1: Models Analyzed in Examples

Example	$x_{th_{min}}$	$E[X(40) T_I = 0]$	Relative cost $\alpha_{ins} : \alpha_{rep} : C_f$	Damage growth model (cf. Eqn.(6.1))
1	0.005	0.05	1:5000:10 ⁴	Linear ($\alpha = 1$)
2	0.005	0.05	1:1000:10 ⁷	Linear ($\alpha = 1$)
3	0.005	0.05	1:0:0	Linear ($\alpha = 1$)
4	0.05	0.05	1:5000:10 ⁴	Linear ($\alpha = 1$)
5	0.005	0.03	1:5000:10 ⁴	Linear ($\alpha = 1$)
6	0.005	0.10	1:5000:10 ⁴	Linear ($\alpha = 1$)
7	0.005	0.05	1:5000:10 ⁴	Square root ($\alpha = 0.5$)
8	0.005	0.05	1:5000:10 ⁴	Parabolic ($\alpha = 2$)

Table 6.2: Optimum Solutions for Full Inspection Strategies for Example 1 (Table 6.1)

m	$F(60)$ (10 ⁻⁴)	x_{th}	t_{R_i}				
			1	2	3	4	5
1	4.00	0.005	21.8	—	—	—	—
2	4.00	0.005	10.8	20.6	—	—	—
3	4.00	0.005	7.9	13.8	20.3	—	—
4	4.00	0.005	5.8	10.3	15.0	20.3	—
5	4.00	0.005	4.1	8.1	12.1	16.1	22.2

Table 6.3: Optimum Solutions for Full Inspection Strategies for Example 2 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}				
			1	2	3	4	5
1	3.43	0.005	30.7	—	—	—	—
2	2.52	0.005	20.3	39.6	—	—	—
3	2.21	0.005	15.2	29.2	44.0	—	—
4	2.05	0.005	12.5	23.7	35.3	47.3	—
5	1.97	0.005	10.0	18.6	27.5	37.4	48.2

Table 6.4: Optimum Solutions for Full Inspection Strategies for Example 3 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}				
			1	2	3	4	5
1	4.00	0.023	33.1	—	—	—	—
2	4.00	0.046	26.1	45.3	—	—	—
3	4.00	0.055	26.9	37.3	47.9	—	—
4	4.00	0.059	25.5	32.0	40.1	50.5	—
5	4.00	0.062	23.7	30.4	37.3	44.8	52.3

Table 6.5: Optimum Solutions for Full Inspection Strategies for Example 4 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}				
			1	2	3	4	5
3	4.00	0.05	22.0	31.1	41.0	—	—
4	4.00	0.05	4.0	22.3	31.6	40.9	—
5	4.00	0.05	4.0	8.0	22.1	31.1	41.0

Table 6.6: Optimum Solutions for Full Inspection Strategies for Example 5 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}		
			1	2	3
1	3.95	0.005	6.5	—	—
2	3.60	0.005	5.3	9.3	—
3	3.03	0.005	6.4	10.6	15.0

Table 6.7: Optimum Solutions for Full Inspection Strategies for Example 6 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}								
			1	2	3	4	5	6	7	8	9
3	4.00	0.005	12.2	23.5	35.7	—	—	—	—	—	—
4	4.00	0.005	9.8	16.9	24.5	34.9	—	—	—	—	—
5	4.00	0.005	6.9	14.2	20.2	27.0	34.5	—	—	—	—
6	4.00	0.005	5.9	10.8	15.7	21.3	27.4	34.3	—	—	—
7	4.00	0.005	4.7	8.7	12.9	17.7	22.5	27.4	34.3	—	—
8	4.00	0.005	4.0	8.0	12.2	16.2	20.6	25.2	29.6	34.1	—
9	3.47	0.005	4.0	8.0	12.0	16.0	20.0	24.0	28.0	32.0	36.0

Table 6.8: Optimum Solutions for Full Inspection Strategies for Example 7 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}							
			1	2	3	4	5	6	7	8
2	4.00	0.005	16.3	33.1	—	—	—	—	—	—
3	4.00	0.005	9.4	19.2	29.6	—	—	—	—	—
4	4.00	0.005	6.5	13.4	20.7	28.6	—	—	—	—
5	4.00	0.005	5.0	10.3	15.8	21.9	28.1	—	—	—
6	4.00	0.005	4.0	8.0	12.2	16.8	21.9	27.8	—	—
7	3.93	0.005	4.0	8.0	12.0	16.0	20.0	24.0	28.0	—
8	3.39	0.005	4.0	8.0	12.0	16.0	20.0	24.0	28.0	32.0

Table 6.9: Optimum Solutions for Full Inspection Strategies for Example 8 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}			
			1	2	3	4
1	4.00	0.005	21.8	—	—	—
2	3.88	0.005	14.5	22.1	—	—
3	3.25	0.005	12.8	18.4	24.7	—
4	2.91	0.005	11.8	16.3	21.2	26.5

Table 6.10: Optimum Solutions for Partial Inspection Strategies for Example 1 (Table 6.1)

m	$F(60)$ (10^{-4})	x_{cr}	t_{P_1}	Δt_P	α_R	C_T	C_{ins}	C_{rep}	$C_f F(60)$
2	4.00	0.005	13.2	4.0	0.7	13.20	1.26	7.94	4.00
3	4.00	0.005	9.1	4.0	0.6	11.48	1.94	5.54	4.00
4*	4.00	0.005	5.8	4.0	0.5	11.36	2.18	5.18	4.00
Full	4.00	—	—	—	—	11.45	2.71	4.74	4.00

* Optimum partial inspection policy

Table 6.11: Engineering Properties of One-Way Slab

Width, b	30 ft
Span, ℓ	30 ft
d	10 in
Height, h	11 in
Concrete cover,	1 in
Diameter of reinforcing bar, D	3/4 in
Distance between reinforcing bars, s	4 in
f'_c	4000 psi
f_y	60 ksi (Grade 60)

Table 6.12: Optimum Solutions for Full Inspection Strategies for One-Way Slab

m	$F(60)$ (10^{-4})	x_{th}	t_{R_i}			
			1	2	3	4
1	1.50	0.005	25.5	—	—	—
2	1.50	0.005	17.7	25.0	—	—
3	1.43	0.007	15.5	20.5	26.0	—
4	1.15	0.007	15.1	19.3	24.2	29.9

Table 6.13: Optimum Solutions for Partial Inspection Strategies for One-Way Slab

m	$F(60)$ (10^{-4})	x_{cr}	t_{P_1}	Δt_P	α_R	C_T	C_{ins}	C_{rep}	$C_f F(60)$
2	1.50	0.005	19.2	4.0	0.84	4.35	1.47	1.36	1.50
3*	1.50	0.005	15.5	4.0	0.77	4.29	1.96	0.83	1.50
4	1.49	0.005	12.0	4.0	0.70	4.44	2.10	0.84	1.49
Full	1.50	—	—	—	—	4.09	1.81	0.78	1.50

* Optimum partial inspection policy

Figure 6.1: Role of Inspection in Controlling Hazard Function

Figure 6.2: Defect Detectability Function

Figure 6.3: Schematic Representation of Damage Growth

6.4(a) $E[X(40)|T_I=0]=0.05$ 6.4(b) $E[X(40)|T_I=0]=0.10$

Figure 6.4: Mean and C.o.v. of Degradation Function without Inspection/repair

$$6.5(a) \quad E[X(40)|T_1=0] = 0.05$$

$$6.5(b) \quad E[X(40)|T_1=0] = 0.10$$

Figure 6.5: Mean Degradation Function with Repair at $t_R=20$ years

6.6(a) $E[X(40)|T_1=0]=0.05$ 6.6(b) $E[X(40)|T_1=0]=0.10$ Figure 6.6: Failure Probability of a Component Repaired at $t_R = 20$ years

$$6.7(a) \quad E[X(40)|T_1=0] = 0.05$$

$$6.7(b) \quad E[X(40)|T_1=0] = 0.10$$

Figure 6.7: Mean Degradation Function with Repair Using Several General Detectability Functions

$$6.8(a) \quad E[X(40)|T_1=0] = 0.05$$

$$6.8(b) \quad E[X(40)|T_1=0] = 0.10$$

Figure 6.8: Mean Degradation Function of a Component with Full Inspection

6.9(a) $E[X(40)|T_1=0]=0.05$ 6.9(b) $E[X(40)|T_1=0]=0.10$

Figure 6.9: Failure Probability of a Component With Full Inspection

Figure 6.10: Decision Policy for Partial Inspection

Figure 6.11: Partial Inspection/Repair Strategies

6.12(a) $E[X(40)|T_I=0]=0.05$ 6.12(b) $E[X(40)|T_I=0]=0.10$

Figure 6.12: Mean and C.o.v. of Degradation Function without Inspection/repair

6.13(a) $E[X(40)|T_1=0]=0.05$ 6.13(b) $E[X(40)|T_1=0]=0.10$

Figure 6.13: Failure Probability of a Component with Partial Inspection

6.14(a) Cost Function at Optimum Solution

6.14(b) Mean Degradation Function with Optimum Policy

Figure 6.14: Optimum Solutions for Full Inspection Strategies for Example 1 (Table 6.1)

6.15(a) Cost Function at Optimum Solution

6.15(b) Mean Degradation Function with Optimum Policy

Figure 6.15: Optimum Solutions for Full Inspection Strategies for Example 2 (Table 6.1)

6.16(a) Cost Function at Optimum Solution

6.16(b) Mean Degradation Function with Optimum Policy

Figure 6.16: Optimum Solutions for Full Inspection Strategies for Example 3 (Table 6.1)

6.17(a) Cost Function at Optimum Solution

6.17(b) Mean Degradation Function with Optimum Policy

Figure 6.17: Optimum Solutions for Full Inspection Strategies for Example 4 (Table 6.1)

6.18(a) Cost Function at Optimum Solution

6.18(b) Mean Degradation Function with Optimum Policy

Figure 6.18: Optimum Solutions for Full Inspection Strategies for Example 5 (Table 6.1)

6.19(a) Cost Function at Optimum Solution

6.19(b) Mean Degradation Function with Optimum Policy

Figure 6.19: Optimum Solutions for Full Inspection Strategies for Example 6 (Table 6.1)

6.20(a) Cost Function at Optimum Solution

6.20(b) Mean Degradation Function with Optimum Policy

Figure 6.20: Optimum Solutions for Full Inspection Strategies for Example 7 (Table 6.1)

6.21(a) Cost Function at Optimum Solution

6.21(b) Mean Degradation Function with Optimum Policy

Figure 6.21: Optimum Solutions for Full Inspection Strategies for Example 8 (Table 6.1)

6.22(a) Simply Supported One-Way Slab

6.22(c) Moment Diagram of One-Way Slab Used in Reliability Analysis

Figure 6.22: One-Way Slab and Its Moment Diagram

Figure 6.23: Mean Degradation Function of One-way Slab

Figure 6.24: Failure Probability of One-way Slab

6.25(a) Cost Function at Optimum Solution

6.25(b) Mean Degradation Function with Optimum Policy

Figure 6.25: Optimum Solution for Full Inspection Strategies for One-Way Slab

Chapter 7

Conclusions and Recommendations

This report has described a methodology to facilitate quantitative assessments of current and future structural reliability and performance of concrete structures in NPPs, taking into account those aging effects that might diminish their ability to withstand future extreme events.

Chapter 2 reviewed common degradation mechanisms affecting concrete and reinforcement. Environmental factors that may have a significant effect on the time-dependent reliability of concrete structures and should be considered in the reliability analysis were identified.

Chapter 3 summarized data gathered in previous work (Ellingwood and Mori, 1992) to describe statistically loads acting on concrete structures and the structural resistance in common limit states.

Chapter 4 presented the basic concepts for time-dependent reliability assessment of a single component. Time-dependent strength models were convolved with the stochastic load models to determine limit state probabilities and hazard functions (conditional failure rates) for a period of 60 years. The sensitivity of the reliability to various parameters describing load occurrence and strength degradation was illustrated using simple examples. The modeling of loads that vary in time and the mean value of the degradation function describing the change in strength in time have a significant impact on time-dependent reliability. The variability in dead load and in degradation function have a less significant effect and can be neglected for practical purposes. The limit state probability and hazard functions both show a relatively sharp increase in magnitude shortly after 40 years. This suggests that in an aggressive environment, evidence of past acceptable performance may not be sufficient to ensure future acceptable performance. Periodic inspection may be required to ensure acceptable performance in the future.

In Chapter 5, the evaluation of time-dependent reliability of single components was

extended to a system of components. An adaptive Monte Carlo simulation procedure combined with conditional expectation was developed. Parametric analyses showed that the time-dependent system reliability is sensitive to the choice of mean initial component strength, mean strength degradation rate, and general stochastic characteristics of the time varying load. Less sensitivity was shown to dependence in component strengths within a system. The presence of "critical" components within the system controls the system reliability, and the dimension of the reliability analysis can be reduced if such components can be identified prior to the analysis. Similar results have been obtained in a different context in seismic PRA and margin studies.

The role of in-service inspection and maintenance on time-dependent reliability was addressed in Chapter 6. The quality of an inspection is described by a detectability function, $d(x)$. The distribution function of damage intensity after an inspection and repair is updated using a Bayesian analysis. With this posterior distribution, the time-dependent strength after the inspection and repair can be also evaluated. Using this information combined with time-dependent reliability analysis developed in Chapters 4 and 5, changes in the failure probability of the component can be evaluated for arbitrary inspection intervals and NDE detection capabilities.

The effects of in-service inspection and maintenance on time-dependent reliability analysis were illustrated using idealized damage detection and repair functions. Quantitative information about damage growth such as the mean rate of corrosion, the mean rate of reduction of strength due to sulfate attack, etc. is required in order to implement the method fully. The degradation function is sensitive to the threshold of detection of the detectability function, $d(x)$, but not to its precise shape. In other words, a step function describing the ability of an NDE method to detect damage of particular size may be sufficient to model the effect of NDE on strength degradation. General detectability functions may not be available for use in concrete structures because of a shortage of experimental data. However, it may be possible to model a step detectability function for each NDE technology based on current knowledge, since all that is required is a detection threshold.

Optimum strategies for inspection and maintenance were devised that minimize the future cost maintaining the limit state probability of structures and components below a desired acceptable threshold. These strategies are sensitive to the relative cost of inspection, repair and failure as well as to the rate of degradation. In order to design an optimum inspection strategy, additional information about the cost of inspection, repair and failure is required. However, it was found that when the cost of failure is higher by order(s) of magnitude than the cost of inspection or repair, as it is in NPP structures, a near-optimal policy is to inspect at near uniform intervals of time throughout the projected service life or service life extension.

Several areas for future work were identified. They include:

1. Supporting Data - Time-dependent reliability analysis provides a quantitative framework for condition assessment of concrete structures. The methodology is data-intensive in comparison to the subjective rating procedures that often are used in the in-service evaluation of ordinary civil structures. Implementation and extension of the method to realistic condition assessment is difficult due to a lack

of supporting quantitative data on strength degradation models, including initiation and rate of damage growth, the mean occurrence rate of load events and c.d.f of the intensity of time-varying loads. The threshold value of defect detection associated with each NDE method must be identified to evaluate the effect of inspection/repair on structural failure probability. Information about relative cost of inspection, repair and losses due to failure is required to design an optimum inspection strategy. It is hoped that the availability of this methodology will motivate the development of the necessary supporting data.

2. Interactions between Degradation Mechanisms - A structural component may degrade due to more than one environmental factor. In such a case, a damage model reflecting the interactions between degradation mechanisms must be developed. With such a model combined with time-dependent reliability analysis developed in this study, the failure probability of a component can be evaluated.
3. Verification - The reliability models established for condition assessment should be validated through application to laboratory and prototypical structures. In laboratory tests, the effects of environmental factors can be evaluated under controlled conditions. Selecting an existing structure with available design documentation as a prototype, the analysis should be run forward to the present time and the current predicted condition should be compared with the estimate of actual present condition. The results can be utilized to modify the theoretical models.
4. User-Oriented Guideline - The condition assessment and life prediction methodology may not be suitable for routine use in practice without simplification. User-oriented requirements and guidelines for in-service evaluation of concrete structures should be developed.

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