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GAS COOLED LEADS

R.P. Shutt, M.L. Rehak, K.E. Hornik

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RHIC PROJECT - Magnet Division

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GAS COOLED LEADS

Part 1: Theoretical Study

R.P.Shutt, M.L.Rehak, K.E.Hornik

ABSTRACT

The intent of this paper is to cover as completely as possible and in sufficient detail the topics relevant to lead design. The first part identifies the problems associated with lead design, states the mathematical formulation, and shows the results of numerical and analytical solutions. The second part presents the results of a parametric study whose object is to determine the best choice for cooling method, material, and geometry. These findings are applied in a third part to the design of high-current leads whose end temperatures are determined from the surrounding equipment.

It is found that cooling method or improved heat transfer are not critical once good heat exchange is established. The range $5 < RRR < 100$ is found to be acceptable for the relative resistivity ratio of the material. Use of high transition temperature super conductor materials is not warranted for this application. The optimal geometry (L:length, A:cross-section) for a given current (I) follows the relation $LI/A = 2 \times 10^5$ but extends over a large range of values. Mass flow needed to prevent thermal runaway varies linearly with current above a given threshold. Below that value, the mass flow is constant with current. Transient analysis shows no evidence of hysteresis. If cooling is interrupted, the mass flow needed to restore the lead to its initially cooled state grows exponentially with the time that the lead was left without cooling.

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Nomenclature

Units are cgs, pressures are in atm, $1 \text{ atm} = 10^5 \text{ Pa}$.

A : lead copper cross-section

a : cross-section of one helium passage in a bundle

A_h : helium passage cross-section

A_g : total heat exchange area

c : copper heat capacity

c_{ph} : helium heat capacity at constant pressure

c_l : latent heat of vaporization

d : diameter of helium passage (or equivalent for rectangular passage)

D : copper lead core diameter

D_h : hydraulic diameter (4 times the hydraulic radius for a cylinder)

De : Dean's number

f_d : fin inner spacing

f_t : fin thickness

f_l : fin radial length

Gr : Grashof's number

h : heat exchange coefficient

I : current

i : current in one wire in a bundled lead

k : thermal conductivity of copper

k_h : helium thermal conductivity

L : lead length

L_o : Lorenz number

M : helium mass flow

M_{crit} : helium mass flow for which the lead is unstable

μ : helium viscosity

N : number of conductors in a bundle

Nu : Nusselt number

P : wetted perimeter of cooling passage

p : pressure in atm

P_r : Prandtl's number

ϕ : ratio of helical to straight length

Q_{in} : heat leaked into magnet at the cold end

Q_{tot} : total refrigerator load

r : radius of helium passage (or equivalent for rectangular cross-section)

R : average radius of helix core and helium passage

Ra : Rayleigh's number

Re : Reynolds number

ρ : electrical resistivity of copper

σ : copper density

σ_h : helium density

t : time

T : copper temperature

T_h : helium temperature

T_{in} : lead temperature at cold end

T_{out} : lead temperature at warm end

T_{cable} : end temperature used for cables going to the power supply

T^e : temperature of lead at exit from cryostat

T_h^e : helium temperature at exit from cryostat

T^{flag} : temperature of lead at flag

T_h^{flag} : temperature of helium at flag

X : coordinate along lead length

x : non-dimensional coordinate along lead length

1. INTRODUCTION

There is a considerable body of work on gas-cooled leads. Early papers made simplifying assumptions in order to reach analytical solutions while later papers relied on numerical methods. More recently, high transition temperature superconducting materials have been introduced in leads. Motivated by the need to design leads for the Relativistic Heavy Ion Collider (RHIC), topics relevant to lead design are presently reviewed and derived or verified.

Ideally, a lead design fulfills the following requirements:

- The lead design must minimize the total load on the refrigerator.
- Budgets for mass flow and heat conducted into the cold end of the magnet must be met.
- The operating mass flow must incorporate a given margin of safety.
- The lead temperature outside the cryostat should be above freezing temperature.
- Physical restrictions on the maximum length and outside diameter must be taken into account.
- The lead should be able to sustain the operating current for a given amount of time after an accidental interruption of helium flow.

It will be assumed at first that the end temperatures of the lead are fixed at known values in order to investigate in the most general terms the following topics:

- cooling: effectiveness of heat exchange, in particular fin geometry when coolant fluid follows a helical path
- geometry: optimal dimensions of the lead
- material: material type, in particular effect of the relative resistivity ratio (RRR)
- mass flow adjustment for varying current

Once these effects are understood, a design including adjacent equipment affecting the lead's performance is developed for high-current leads. The leads considered here are cooled by forced flow helium which is externally controlled. This differs from leads cooled

by helium boil off where the mass flow is the result of heat leaking into the liquid at the cold end.

Cooling is achieved here by means of a helical flow path machined around a copper rod. Other schemes use slotted disks, leads made of bundles and concentric tubes. Most of the results obtained here for helically cooled leads apply to those other cooling methods.

The theory presented here has led to the development of computer codes used in the design of three types of leads. One computer program models a high current lead which is made of a copper rod with a helical flow path. The second program models the CQS (corrector-quadrupole-sextupole) lead¹ made of a number of individually insulated wires (carrying currents of the order of 100 A) cooled by helium flowing in a flexible tube. The third computer program models the same wires (not insulated) held by a helical plastic core with helical cooling flow path.

2. MATHEMATICAL MODEL AND SOLUTIONS

2.1. Governing equations

2.1.1. Static analysis

The basic heat transfer equations that are used in the analysis of the helium cooled lead assemblies are well established. They express that the change in conducted heat over a section is equal to the balance between resistive heat generated and heat removed by the coolant.

$$\frac{d}{dX} \left(kA \frac{dT}{dX} \right) = \frac{-\rho I^2}{A} + \frac{hA_s}{L} (T - T_h), T(0) = T_{in}, T(L) = T_{out} \quad (2.1)$$

$$Mc_{ph} \frac{dT_h}{dX} = \frac{hA_s}{L} (T - T_h), T_h(0) = T_{in} \quad (2.2)$$

The second equation accounts for the rise in the coolant's temperature as it removes heat from the lead. To preserve the generality of this study, the end temperatures, T_{in} , T_{out}

are assumed to be fixed at 4.4 K where $X=0$ and at 293 K where $X=L$. Proper modeling of the end temperatures will be achieved in section 4 by including the equipment attached to the lead in the model.

T is the copper temperature, T_h is the helium temperature, k and ρ are the copper thermal conductivity and electrical resistivity, respectively. X is the position along the lead, L is the lead length, A is the copper cross-section and I is the current. c_{ph} is the helium heat capacity at constant pressure and M is the mass flow. h is the convection heat transfer coefficient of the cooling fluid per unit area, A_s is the total heat exchange area. The latter are explicitly stated in Appendix D.

An expression for the pressure drop can be found in Appendix F. A known pressure is assumed at the cold end. The temperature profile of the lead is affected by pressures through helium properties which are pressure dependent.

The leads considered here are cooled by forced flow. The mass flow is regulated by a valve and treated as a known value. If the lead were cooled by boil-off, the mass flow would be given by the boundary condition

$$Q_{in} = M c_l \quad (2.3)$$

where c_l is the latent heat of vaporization. Q_{in} is the heat conducted by the lead at the cold end. Appendix B compares boil-off to forced flow cooling.

2.1.2. Dynamic analysis

When dynamic effects are accounted for, Eq. (2.1) and Eq. (2.2) become:

$$\sigma A c \frac{\partial T}{\partial t} = \frac{\partial}{\partial X} \left(k A \frac{\partial T}{\partial X} \right) + \frac{\rho I^2}{A} - \frac{h A_s}{L} (T - T_h) \quad (2.4)$$

$$\sigma_h A_h c_{ph} \frac{\partial T_h}{\partial t} = -M c_{ph} \frac{\partial T_h}{\partial X} + \frac{h A_s}{L} (T - T_h) \quad (2.5)$$

σ , σ_h are the densities of copper and helium, c is the copper heat capacity, c_{ph} is the helium heat capacity at constant pressure, A_h is the cross-sectional area of the helium passage.

These are general equations which will be used to determine the lead response to current and mass flow ramp up and down. In particular a transient analysis will be needed to determine whether the lead designed for RHIC can sustain a minimum of 30 seconds without reaching the 450 K to 500 K range in the event of accidental interruption of helium flow. In this event, and neglecting conducted heat along the lead's length, Eq. (2.4) can be integrated to give an estimate for the time that a point along the lead at temperature T_1 takes to reach the temperature T_2 .

$$t = \sigma \left(\frac{A}{I} \right)^2 \int_{T_1}^{T_2} \frac{c}{\rho} dT \quad (2.6)$$

2.1.3. Numerical results

The numerical method described in Appendix C is used to solve Eq. (2.1) and Eq. (2.2). Fig. I.1 shows the lead and helium temperatures along the lead's length for different mass flows. With $L=70$ cm, $D=1.5$ cm, $A=1.76$ cm², $I=6300$ A, $T_{in}=4.4$ K, $T_{out}=293$ K, the lead can operate with a mass flow as low as 0.34 g/sec but at $M=0.33$ g/sec temperatures are already out of bounds. The figure reveals a close coupling between helium and lead temperatures indicating almost perfect heat exchange. The pressure drop is 0.5 atm when a pressure of 5 atm is assumed at the cold end. One should note that the temperature profile just before burn-out is very similar to that at the operating mass flow, giving no warning that a decrease of 0.01 g/sec will cause thermal runaway.

Fig. I.2 shows lead temperature profiles for a large range of mass flows as well as the case where there is no mass flow and no current.

Results of a dynamic analysis are shown in Fig. I.3. The initial temperature profile is that of the steady state case in Fig. I.1 with the operating mass flow of 0.41 g/sec. At time $t=0$ the mass flow is interrupted and the figure shows that the point at $X=62$ cm along the lead, initially at 70 K, would reach 440 K in 80 seconds without cooling. The time estimated from Eq. (2.6) is in good agreement at 90 seconds.

2.2. Closed form solution

Eq. (2.1) and Eq. (2.2) can be simplified to the point where it is possible to obtain a closed form solution. Assumptions and solution are given in Appendix A. It is assumed that the material's electrical resistivity is linear with temperature, its thermal conductivity is constant, and that the heat exchange with helium is perfect.

The analytical solution will be used next to confirm the existence of a critical mass flow and of a combination of values of L , I , and A which minimize the critical mass flow.

2.2.1. Comparison between closed form and numerical solutions

When compared with the numerical solution, it is found in general that agreement disappears as the purity of the material increases. An impure copper could be reasonably modeled by the analytical solution (see Fig. I.4 for an OFHC copper). But when the RRR is as high as 160, the analytical solution differs considerably from the numerical analysis (see Fig. I.5 for a copper with RRR=160). Pure copper has a high thermal conductivity at low temperatures and the assumption that k is constant is not valid. This assumption holds better for impure coppers such as OFHC (or a copper with RRR=20) whose thermal conductivity is smaller than that of pure copper by more than an order of magnitude. The analytical solution is shown for a range of mass flows in Fig. I.6. (A lead of length 45.7 cm was considered here.)

2.2.2. Critical mass flow

One of the solutions for the temperature given in Appendix A is singular when

$$M = \frac{2\sqrt{L_o}}{c_{ph}} \sqrt{I^2 - \frac{1}{L_o} \left(\frac{\pi k A}{L} \right)^2} \quad (2.7)$$

This result is called the burn-out condition in Jones².

For leads operating at a current which is large compared to the conductivity term $\frac{1}{\sqrt{L_o}} \left(\frac{\pi k A}{L} \right)$, the critical mass flow is approximately linear with current. This linear relation

between mass flow and current would apply to all materials obeying the W-F-L law:

$$M \simeq \frac{2I\sqrt{L_o}}{c_{ph}} \quad (2.8)$$

With the Lorenz number $L_o = 2.445 \times 10^{-8}$, $c_{ph} \simeq 4$, $M = 7.8 \times 10^{-5}I$. It will be seen in section 3.4 that the numerical solution, where none of the simplifying assumptions have been made, supports this result.

2.2.3. Optimal geometry

It is desirable to find the combination of L , I , A such that the total refrigerator load is minimized. To this effect $M=0$ is substituted in Eq. (2.7) which can be solved in terms of LI/A as:

$$\frac{LI}{A} = \frac{\pi k}{\sqrt{L_o}} \quad (2.9)$$

For an OFHC copper where $k \simeq 6$, one obtains $\frac{LI}{A} = 1.2 \times 10^5$. This expression is also independent of material as long as the Wiedemann-Franz law is obeyed. The existence of an optimal value for LI/A is mentioned in Güsewell and Haebel³. This is a theoretical value for the optimal LI/A but the numerical results in section 3.3 will show that this optimum is very flat and covers a wide range of values.

2.3. Normalized parameters

The combinations of parameters for which results are invariant are determined here. Eq. (2.1) and Eq. (2.2) are multiplied by L/I and the non-dimensional $x = X/L$ is used to obtain

$$\frac{A}{LI} \frac{d}{dx} \left(k \frac{dT}{dx} \right) = -\rho \frac{LI}{A} + (hA_s/I)(T - T_h) = -\rho \frac{LI}{A} + \frac{Mc_{ph}}{I} \frac{dT_h}{dx} \quad (2.10)$$

This shows that two leads made of the same material would have the same temperature profiles if (LI/A) , (hA_s/I) , (M/I) are kept identical. The numerical analysis to follow (see section 3.3) will confirm this fact. In addition it will be found that this result still holds even when (hA_s/I) is not kept constant as long as there is good heat exchange.

Moreover the definition of optimal design involves only (LI/A) and (M/I) as will be shown next. The total refrigerator load is defined as

$$Q_{tot}/I = 80M/I + Q_{in}/I \quad (2.11)$$

where 80 represents the refrigerator efficiency and

$$Q_{in} = kA \frac{dT}{dX} \Big|_{X=0} \quad (2.12)$$

is the conducted heat at the cold end. One can rewrite

$$Q_{tot}/I = 80(M/I) + k(LI/A)^{-1} \frac{dT}{dx} \Big|_{x=0} \quad (2.13)$$

. The optimal design of a lead for one current can be obtained by scaling (LI/A) and (M/I) with current. This also shows that a lead can only be optimized for one current since to keep (LI/A) constant at a different I , (L/A) would need to be modified.

3. PARAMETRIC STUDY

The cooling mode considered here consists of a helical path machined in a copper rod. Alternate cooling schemes promoting good heat exchange such as a bundle arrangement are shown to be equally feasible in Appendix D.

3.1. Parametric study on cooling

3.1.1. Helicity effect on heat transfer

When helium flows in a curved duct such as in the helical passage presently considered, heat exchange is promoted in two ways. First, the helical path along the copper rod is longer than if the path were straight along the rod length. Second, the heat transfer is enhanced in curved ducts. This is reflected in a larger Nusselt number, Nu_{heliz} (see Appendix E). The effects of improved heat transfer in a helical tube as opposed to a straight tube are illustrated in Fig. I.7. Curve A shows the temperature profile if the cooling passage were straight and $L = 45.7$ cm long. Curve B shows temperatures when a passage with the same geometry spirals around the lead's copper core. The effective length is $L_{heliz} = \phi \times L_{straight}$ (where $\phi \simeq 20$ for a typical case) and cooling is considerably improved. Curve C takes into account the improved heat transfer (which translates into a $Nu_{heliz}/Nu = 1.7$ factor in this example) due to the curvature of the helix. The difference between curves B and C is very small and indicates that effects due to the curvature of the flow path do not improve dramatically the heat exchange. It will be shown in the following that once sufficient heat exchange has been established, further improvements have little effect, since heat exchange depends on the available temperature difference ($T - T_h$).

3.1.2. Parametric study on fin geometry

The existence of a preferred set of fin dimensions which would maximize the heat transfer is investigated next. The gains of improved heat exchange are also evaluated. An explicit expression of the cooling term as a function of fin geometry is given in Appendix D (Eq. (D.15)) as

$$(hA_s)_{heliz} = GJ\phi, \quad J = \frac{(f_d + 2f_l)(f_d + f_l)^{1.8}}{(2f_d f_l)^{1.8}}, \quad \phi = \frac{L_{heliz}}{L_{straight}} = \pi \frac{D + f_l}{f_d + f_t} \quad (3.1)$$

where G is independent of fin geometry. f_l , f_d , f_t are the fin's radial length, inner spacing and thickness, respectively.

The two factors which contribute to increases in hA_s are considered separately. First, J is shown as a function of f_l or f_d in Fig. I.8 where it is seen to vary at most by a factor of 3 for the values of f_l , f_d under consideration. Next ϕ alone is increased in Fig. I.9 which shows that for values greater than 20 temperature profiles have converged. It can

be concluded that when sufficiently good heat exchange has been established, further manipulations of fin dimensions do not significantly affect results.

3.2. Parametric study on material

3.2.1. Material properties

Material properties have been obtained from tables⁴ or generated following the procedure described in Lock⁵.

Mattiessen's rule is used to determine the resistivity as follows: $\rho = \rho_T + \rho_0$, where ρ_T is the temperature dependent part which is given and ρ_0 depends on the purity and is calculated from the relative resistivity ratio. An equivalent of this rule is used for the thermal conductivity: $1/k = \rho_0/(L_o T) + P(T)$, where L_o is the Lorenz number and $P(T)$ is the temperature dependent part which is known. Variations of the electrical resistivity are measured by the relative resistivity ratio defined as $RRR = \rho(273)/\rho_0$. Material properties generated according to this procedure are consistent with available measured data in Fig. I.10. Conductivities can differ by an order of magnitude. The measured conductivity of an ETP copper (with an $RRR=100$) matches generated data for $RRR=50$. The measured conductivity of an OFHC copper (with an $RRR=106$) differs from the data generated for $RRR=100$.

3.2.2. Effects of material purity

The dependence of the lead's performance on the RRR of the material is investigated in this section using a 55 cm long lead of diameter $D=1.5$ cm. It is assumed that the refrigeration budget allows a mass flow of 0.06 g/sec/kA. At 6300 A the allowable mass flow is 0.38 g/sec. Table 3.1 shows that if the RRR of the material is less than 10, the critical mass flow is higher than the allowed value. For materials with $RRR \geq 10$, the lead can operate within budget. Values for Q_{tot}/I are minimum for the range $5 \leq RRR \leq 100$. The table shows that pure materials ($RRR > 100$) conduct large amounts of heat. It is thus not advantageous to use high purity copper. Equally undesirable for this application

are very impure materials such as brass ($RRR=2$) where resistive heat is so large that mass flows in excess of the budget are required .

Table 3.1: Effect of material

RRR material	M g/sec	Q_{in} W	$Q_{tot/I}$ W/kA	M_{crit} g/sec
2	0.800	2.2	10.5	0.80
5	0.440	3.4	6.1	0.44
10	0.380	3.7	5.4	0.36
50	0.380	4.4	5.5	0.28
100	0.380	10.4	6.5	0.26
200	0.380	23.4	8.5	0.24
500	0.380	49.1	12.6	0.23

This is further illustrated in Fig. I.11 for a mass flow of 0.45 g/sec. Temperature profiles rise significantly as the RRR decreases (i.e. the material becomes impure) thus pointing to the use of a high purity material. But the table accompanying the figure shows that large increases in conducted heat occur as the material's purity increases.

3.2.3. High transition temperature superconducting materials

A recent development in leads consists of using materials which have no electrical resistivity at low temperatures (below 80 K). These materials, usually ceramics, have in addition thermal conductivities which are smaller than those of copper by several orders of magnitude. The performance of a composite lead whose cold end is made of a ceramic material is compared to that of a conventional lead. Assumptions in favor of composite leads have been made: the electrical resistivity and thermal conductivities are equal to zero at temperatures below 80 K.

Table 3.2: Comparison between conventional and high transition temperature superconducting leads (HTSC).

lead	I A	L cm	D cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA
all copper	6300.	70	1.5	0.33	1.0	4.4
HTSC	6300	70	1.5	0.33	0.	4.2
all copper	1600	70	1.0	0.07	0.9	4.0
HTSC	1600	70	1.0	0.07	0.0	3.5

For a current of 6300 A Table 3.2 shows that when ρ , k are set equal to zero for temperatures below 80 K there is a small reduction in Q_{tot}/I (defined in Eq. (2.11)), mostly due to a reduction in conducted heat since the critical mass flow does not change significantly. A comparison between the two types of leads is shown in Fig. I.12. Replacing copper by ceramic will not reduce the resistive heat in the whole lead but is effective in reducing the conducted heat at the cold end of the lead. The conducted heat is however a small contribution to the total refrigerator load for high currents.

For a current of 1600 A, the table shows a 20% reduction in total refrigerator load when the cold end of the lead is replaced by a ceramic material. A comparison between the two types of leads is shown in Fig. I.13. At lower currents a smaller mass flow is required to cool the lead and the conducted heat becomes a larger component of the total refrigerator load. Therefore gains from the use of ceramic materials can be expected to be higher at 1600 A than at 6300 A.

In a third example which corresponds to an actual application (Shutt, Hornik, Rehak)¹² wires carrying 100 A each and 90 cm long are inserted in a pipe through which helium is flowing.

The gain on the total refrigerator load is here 22 %. The length of 90 cm was the result of an optimization where the conducted heat was minimized while keeping the resistive heat relatively low. If the length had been 45 cm, then gains with HTSC materials in total refrigerator load would have been 66 %. However it is considerably simpler to increase the length of the lead (that is optimize the design) than use a composite lead.

Table 3.3: Comparison between conventional and high transition temperature superconducting leads (HTSC).

lead	L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA
all copper	90	0.065	0.95	5.12
HTSC	90	0.050	C.	4.0
all copper	45	0.05	6.54	8.8
HTSC	45	0.045	0	3

3.3. Parametric study on geometry

The search for those parameters which minimize the total refrigerator load given in Eq. (2.11) is performed by varying one of the three parameters I , L , A at a time. Here I is fixed while L and A are varied. Fig. I.14 is a graph of Q_{tot}/I versus LI/A taken from tables (Fig. I.15) generated for $I = 5500$ A and for $I = 1600$ A which shows that all points fall on the same curve. This fact is all the more remarkable in that dimensions affecting cooling were not scaled accordingly. This confirms the predictions of the analytical solution in 2.2 that results are invariant (within a good approximation) when there is good heat exchange and when LI/A is kept constant. The graph also shows that while the optimum LI/A is approximately 2×10^5 , the range of LI/A for which Q_{tot}/I is small is considerably larger.

Tapering the cross-section to reduce resistive heat at the warm end was found to be ineffective. Gains on resistive heat are lost to increased conducted heat.

3.4. Critical mass flow at different currents

A lead optimized for a given current may be required to carry a range of currents in which case the mass flow must be adjusted accordingly. The critical mass flow is approximately a linear function of the current as can be seen in Fig. I.16 for four different materials with different RRR. The figure also shows a curve for an OFHC copper where, at low currents, the mass flow is small and practically constant. A qualitative explanation for

this behavior is provided by the closed form solution in section 2.2 (Eq. (2.7) and Eq. (2.8))

3.5. Transient behavior

The transient behavior of the lead is studied for the case where current as well as mass flow can vary in time.

It is first verified that if there is a temporary change in current or mass flow, the lead will revert in time to its cooled state. The example considered is that of a 70 cm long lead of diameter 1.5 cm which carries initially no current and is cooled by a mass flow of 0.2 g/sec. The lead is ramped up to 10 000 A at a rate of 1000 A/sec. This is followed by a ramp down at the same rate (see Fig. I.17). The mass flow remains constant throughout at 0.20 g/sec. Fig. I.18 shows temperatures along the lead length at different times. Although the current pulse lasts for 20 seconds, the lead needs 60 seconds to return to its initial state. Fig. I.19 shows temperature variations with time at four locations along the lead. Temperature maxima occur at 16 seconds, or 6 seconds after peak current goes through the lead. Peak temperatures are more pronounced closer to the warm end of the lead.

If cooling is abruptly interrupted but the current maintains its operating value, progressively high temperature profiles are obtained. It is found that an increasingly high mass flow is required to bring the lead back to a stable temperature profile. Fig. I.20 shows that a lead, initially operating at 0.36 g/sec will need 0.43 g/sec to return to its initial state if coolant has been interrupted for 120 seconds. The mass flow needed for recovery is approximately an exponential function of the interval of time between interruption and resumption of mass flow. When the recovery mass flow is used, the lead will revert in time to its cooled state prior to mass flow interruption.

The fact that there are multiple values for critical mass flow (for instance 0.36 g/sec and 0.43 g/sec) is the result of different initial temperature profiles. With an initially warm lead, the mass flow needed to operate the lead will be higher than if the lead had been originally cold. It is thus recommended that the lead be cooled prior to turning the current on.

Apparent multiple values for critical mass flow are also present in the static analysis which requires an initial guess for temperatures. The example of a lead with $L=70$ cm, $D=1.5$ cm, ends fixed at 4.4 K and 293 K, and at $I=6300$ A is taken to illustrate this. If the initial guess is a linear distribution between 4.4 K and 293 K, the critical mass flow is 0.34 g/sec. This is the mass flow needed for recovery since the lead started warm. When the solution thus obtained at $M=0.34$ g/sec is used as a new initial guess, the critical mass flow drops to 0.27 g/sec. Further iterations do not change this value. Thus, for an initially cooled lead, all initial guesses will converge to a unique temperature distribution for which there is a unique critical mass flow.

This dependence of the critical mass flow on the initial guess is less pronounced when surrounding equipment is included in the model (0.29 g/sec versus 0.28 g/sec) as will occur in the next sections. Results throughout this paper were obtained using a linear temperature distribution for initial guess in order to obtain conservative results. If the lead is initially cooled, the critical mass flow will be lower than indicated.

4. OPTIMAL LEAD DESIGN FOR RHIC

The following describes the design of two leads optimized to carry currents of 6300 A and 1600 A, respectively, in the Relativistic Heavy Ion Collider (RHIC) .

4.1. Frost-free lead

One major difficulty associated with lead design is the elimination of frost or condensation (see Appendix I). The authors have investigated⁶ the possibility of designing a lead which will not frost while meeting budget and safety requirements. The underlying principle is to increase the temperature in a short section of the lead by replacing the helical flow path with a straight flow path, thus reducing cooling. The lead is stabilized against thermal runaway by increasing the copper cross-section in the adjacent section. Such a lead consists of three parts (see Fig. I.21): a helical section (A), a section with straight internal flow (B) acting as a built-in heating unit to raise the temperature above freezing),

and a section with a large copper cross-section (C) to stabilize (by reduction of resistive heat) the lead against thermal instability. By adjusting the lengths and diameters of the various sections, it is possible to design a lead which will not frost over the range between 3000 A to 6300 A and still meet design constraints. One possible draw back of this design is that dimensions are tied to the purity of the material which must then be carefully controlled.

The main limitation of this design is that a lead which does not frost at low currents requires an excessive amount of cooling at high currents. This design is not suited to the present application where the lead must operate at little or no current as well as high currents.

An alternate design using both vacuum jacket and a variable cross-section allowing frost-free operation over a larger range of currents is discussed in reference.⁷

To illustrate the design principles developed in the first part of this paper, a lead which is a copper rod with a helical flow path over its entire length is used. The section in the atmosphere is enclosed in a vacuum jacket to avoid frost (see Fig. I.22).

4.2. Lead and equipment

The helical lead extends from the cold end which is immersed in helium, to a copper block (D) outside the cryostat where helium is removed from the system. The copper block ("flag") is attached to a set of braids (E), introduced for stress relief purposes. The braids are connected to a large copper block (F) to which cables (G) going to the power supply are connected. This equipment is cooled by air convection only. Expressions applying to air cooling used in the model are given in Appendix G. The cold end of the lead is immersed in a helium flow of 100 g/sec and it can be assumed that the cold end is heat sunk at 4.4 K. A pressure of 5 atm and room temperature of 293 K are assumed there.

The material used here is an OFHC copper. The dimensions of the fins are: fin radial length of 0.25 cm, fin inner spacing of 0.20 cm, fin thickness of 0.08 cm.

Since there are physical limits on the core diameter of the helix D ($D \simeq \sqrt{4A/\pi}$), it will appear instead of A in the tables. L denotes here the total length of the lead from the cold end to the flag.

The budget allows :

Table 4.1: Budget

M/I	Q_{in}/I	Q_{tot}/I
g/sec/kA	W/kA	W/kA
0.06	1.2	6.0

4.3. End temperatures

The boundary condition used at the warm end consists of stating that somewhere along the cables to the power supply thermal equilibrium between resistive heat and convective air cooling exists. This is formulated by : $h(T_{cable} - T_{air}) = \rho I^2 / A_{cable}$ where h combines convection cooling and cable insulation $h = (h_{air}^{-1} + h_{cable}^{-1})^{-1}$. T_{cable} is the temperature of the copper of the cable. The location at which this thermal equilibrium occurs is unknown, but a lower bound can be determined numerically by successively increasing the length of the cable in the computer model until results remain invariant. This end condition can also be obtained analytically by integrating the equation $\frac{d}{dX}(kA \frac{dT}{dX}) = \frac{-\rho I^2}{A} + (hA_s/L)(T - T_{air})$, $T(0) = T_0$, $\frac{dT}{dX}|_{X=L_{cable}} = 0$. The solution is of the form $T = c_1 e^{\beta X} + c_2 e^{-\beta X} + T_{cable}$, $T_{cable} = T_{air} + \frac{\rho I^2}{hA_s/L}$ where c_1 , c_2 are integration constants, X indicates the distance along the cable length. A length of 100 cm was found to be representative. After a very short length (between 50 to 100 cm), it is found that $T \simeq T_{cable} = 359K$. This number is very close to $T_{cable} = 363K$ obtained when using the electrical code for 10 cables spaced on a tray with an ambient temperature of 308 K.

4.4. Optimal geometry for the 6300 A lead

Values of Q_{tot}/I for an acceptable range of lengths and diameters are shown in Fig. I.23 and Fig. I.24. All tables show that there is an optimal length which will minimize Q_{tot}/I . Table 4.2 summarizes dimensions resulting in the smallest value of $Q_{tot}/I=3.6$ W/kA. The critical mass flow decreases with increasing D , as expected in a lead where resistive heat dominates conducted heat.

Table 4.2: Optimal dimensions for the 6300 A lead

L cm	50	55	85	90
D cm	1.5	1.5	2.0	2.0

As an example the length $L=72$ cm is chosen, 57 cm of which are inside the cryostat and 15 cm are between the cryostat and the flag. These 15 cm are needed to satisfy requirements for electrical insulation between current carrying parts and the cryostat wall. The fin's dimensions are: inner spacing $f_d=0.20$ cm, radial length $f_l=0.20$ cm, thickness $f_t=0.08$ cm. The lead's performance at the critical mass flow and at the operating mass flows is shown in Fig. I.25. An operating mass flow 20% above the critical mass flow is used in Table 4.3 where T^e is the lead temperature at the exit of the cryostat and T_h^e is that of the helium at that same location. Lead and helium temperatures before entering the flag are denoted by T^{flag} and T_h^{flag} . All these values are well below room temperature and the use of a vacuum jacket is recommended as a means of preventing frost.

If the stainless steel pipe enclosing the helix is exposed to the cryostat's vacuum, a study in Appendix H indicates that fifteen layers of superinsulation are sufficient to reduce the heat load due to radiation on the lead.

If the current remains at its operating value after the mass flow is turned off, Fig. I.26 shows that the lead can sustain 30 seconds without cooling.

The relation between current and critical mass flow is shown in Fig. I.27. When there is no current and no mass flow the conducted heat amounts to 34.4 W. A mass flow of 0.09 g/sec is sufficient to reduce the conducted heat to 4.3 W. The relationship is linear above

Table 4.3: Temperatures at operating and critical mass flows, $I=6300$ A, $L=72$ cm, $D=1.5$ cm

I	M	Q_{in}	Q_{tot}/I	T^e	T_h^e	T^{flag}	T_h^{flag}	Δp
A	g/sec	W	W/kA	K	K	K	K	atm
6300.	0.290	1.0	3.8	54.0	52.3	270.6	260.2	1.3
6300.	0.350	0.9	4.6	36.3	35.2	213.4	197.6	0.6

3000 A. A bilinear function of M_{crit} versus I can be used to approximate these data:

$$I < 3000 \text{ A}, M_{crit} = 0.12 \text{ g/sec}, I > 3000 \text{ A}, M_{crit} = 0.12 + 5 \times 10^{-5} (I - 3000) \quad (4.1)$$

The value $\frac{dM}{dI}$ of 5×10^{-5} compares well with the value of 7.8×10^{-5} obtained analytically in Eq. (2.8). Critical mass flow and Q_{tot}/I versus current, tabulated in Fig. I.27, are shown in Fig. I.28 and Fig. I.29.

The importance of modeling the surrounding equipment is illustrated in Table 4.4. The actual flag temperature in the complete model (and in actuality) is lower than room temperature. The critical mass flow is thus considerably smaller in the model including equipment. By fixing the warm end of the lead at 293 K, the critical mass flow is overestimated by 17%.

Table 4.4: Comparison between complete model and model assuming warm end fixed at 293 K, $I=6300$ A, $L=72$ cm, $D=1.5$ cm

case	M_{crit}	Q_{in}	Q_{tot}/I	T^e	T_h^e	T^{flag}	T_h^{flag}	Δp
A	g/sec	W	W/kA	K	K	K	K	atm
compl.model	0.290	1.0	3.8	54.0	52.3	270.6	260.2	1.3
fixed end	0.340	1.0	4.5	44.3	43.0	293.0	275.1	1.1

4.5. Optimal geometry for the 1600 A lead

As in the preceding example, optimal values for L and D are found by searching for the smallest values of Q_{tot}/I in tables shown in Fig. I.30 and Fig. I.31. Dimensions which result in $Q_{tot}/I=3.7$ W/kA are summarized in Table 4.5.

Table 4.5: Optimal dimensions for the 1600 A lead

L cm	50	55	60	65	70	80	85
D cm	0.75	0.75	0.75	0.75	0.75	1.0	1.0

For illustration purposes L=76 cm and D=1.0 cm are selected. The fin's dimensions are: inner spacing $f_d=0.20$ cm, radial length $f_l=0.25$ cm, thickness $f_t=0.08$ cm. Contrary to the case in Table 4.2, the critical mass flow increases with diameter. This can be explained by the preponderance of conducted heat over resistive heat. The cable's end temperature $T_{cable} = 318$ K is based on the assumption that there are four cables well spaced on a horizontal tray. The performance of the lead at critical and operating mass flows is shown in Fig. I.32. Temperatures at the critical and operating mass flows are compared in Table 4.6. A dynamic analysis showing evolution of temperatures profiles with time when helium flow is turned off appears in Fig. I.33. The lead can sustain the required 30 seconds without cooling.

Table 4.6: Temperatures at operating and critical mass flows, I=1600 A, L=76 cm, D=1.0 cm

I A	M g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
1600.	0.070	0.4	3.7	91.9	89.6	261.3	253.4	0.2
1600.	0.084	0.2	4.3	59.3	57.6	236.6	229.9	0.2

The relation between critical mass flow and current is tabulated in Fig. I.34. When there is no current and no mass flow the conducted heat is 15.1 W this can be reduced to

practically zero by applying an operating mass flow of 0.07 g/sec. Critical mass flow and Q_{tot}/I versus current are shown in Fig. I.28 and Fig. I.29, respectively.

4.6. Optimal geometry for a range of currents

In general, a lead optimized for high currents would conduct too much heat at lower currents where less mass flow is needed. A lead optimized for low currents would require an excessive mass flow at high current in order to control the resistive heat. Fig. I.29 shows that the 6300 A lead operating at 1600 A would consume 4 W/kA or 6.4 W more than the optimized 1600 A lead. Similarly, the 1600 A lead running at 6300 A would require 1.3 W/kA or 8.2 W more than the optimized lead for that current. If a lead is to be optimized over a range of currents, say 1600 A and 6300 A, rather than at a specific current, then its dimensions should be such that the curve of Q_{tot}/I versus I falls between the two curves in Fig. I.29. A lead whose dimensions were averaged between the two extremes would achieve this result.

5. CONCLUSIONS

The mathematical model describing lead behavior has been presented and solved both analytically and numerically. The analytical solution, which is too approximate for practical applications, was nevertheless useful in revealing the existence of a critical mass flow and of an optimal geometry. The numerical solution, free of the simplifying assumptions, was used to perform a parametric study which results in the following conclusions:

- Cooling.

Increased heat exchange due to the curvature of the helical passage has a small effect on the lead performance. There is a priori no advantage in using helically cooled leads over using other types of leads such as bundles as long as equally good heat exchange exists. Changing the fin's aspect ratio or pitch for the helically cooled lead does not bring significant gains.

- Material.

Pure materials conduct too much heat while impure materials generate too much resistive heat. An acceptable range is $5 \leq RRR \leq 100$. Composite leads with high transition temperature superconducting material at the cold end are effective in reducing conducted heat but do not significantly reduce resistive heat (which is generated almost in its entirety in the conventional part of the lead). Since conducted heat is a small contribution to the total refrigerator load in high current leads, the use of high transition temperature does not appear to be warranted in this application.

- Geometry.

The optimal aspect ratio of the lead at a given current I is found by a systematic search for the combination of length L and cross-section A which will minimize the total refrigerator load. There is an optimal value for LI/A which is independent of precise cooling details as long as good heat exchange exists. This optimum is very flat and the lead can operate efficiently from optimum. The optimal value of LI/A depends on material as well as lead end conditions and must be determined for each type of lead.

- Mass flow adjustment for variable current.

The critical mass flow is found to be approximately linear with current for a large range of materials when the current is above a minimal value. Below that threshold a constant mass flow must be used to limit conducted heat into the cold end.

- Lead design.

A lead made of a continuous copper rod with a helical flow path was chosen. This lead design is characterised by the fact that it incorporates all relevant parts of surrounding equipment into the numerical model. Instead of assuming an end temperature at the warm end of the lead, the model includes copper blocks and cables up to the point where thermal equilibrium is reached in the cables leading to the power supply. It was found that a vacuum jacket is needed to prevent frost build-up if the lead is to be used over a wide range of currents including no current.

Some of the results which have appeared in the literature (burn-out condition for the mass flow, invariance of design with LI/A) have been restated and verified here. Original work, to the best of the authors' knowledge, consists of assessing helicity effects on heat exchange and pressure drops, showing that once good heat exchange has been established further improvements bring few gains, and stressing the importance of complete modeling of the end conditions.

A. Appendix: Closed form solution

If the following assumptions are introduced in the governing equations Eq. (2.1) and Eq. (2.2), they are reducible to a form which can easily be integrated analytically:

- $\frac{dT}{dX} = \frac{dT_h}{dX}$. With good heat exchange this is a reasonable assumption.
- The material obeys the Wiedemann–Franz law $\rho k = L_o T$ where L_o is a constant. L_o does in fact vary with T but this dependence is small when compared to the other parameters (typically by a factor 2).
- k is constant. This assumption holds better for impure materials where the conductivity is small.
- $A(x)$ is constant.

The equation to be solved is :

$$\frac{d^2 T}{dX^2} + B \frac{dT}{dX} + CT = 0, \quad B = -\frac{Mc_{ph}}{kA}, \quad C = \left(\frac{I}{kA}\right)^2 L_o \quad (A.1)$$

with the boundary conditions: $T(0) = T_1$, $T(L) = T_2$.

The form of the solutions depends on the sign of the discriminant $\Delta = 4C - B^2$.

$$\Delta = 0, \quad T = (c_1 X + c_2) e^{-BX/2}, \quad c_1 = T_1, \quad c_2 = -\left(T_1 - e^{BL/2} T_2\right) / L \quad (A.2)$$

$$\Delta > 0, \quad T = \left(c_1 \sin\left(\sqrt{\Delta} X/2\right) + c_2 \cos\left(\sqrt{\Delta} X/2\right)\right) e^{-BX/2} \quad (A.3)$$

$$c_1 = -\frac{\cos\left(\sqrt{\Delta} L/2\right) T_1 - e^{BL/2} T_2}{\sin\left(\sqrt{\Delta} L/2\right)}, \quad c_2 = T_1 \quad (A.4)$$

$$\Delta < 0, \quad T = c_1 e^{(\sqrt{-\Delta}-B)X/2} + c_2 e^{-(\sqrt{-\Delta}-B)X/2} \quad (A.5)$$

$$c_1 = -\frac{e^{(-\sqrt{-\Delta}-B)L/2} T_1 - T_2}{e^{(\sqrt{-\Delta}-B)L/2} - e^{(-\sqrt{-\Delta}-B)L/2}} \quad (A.6)$$

$$c_2 = \frac{e^{(\sqrt{-\Delta}-B)L/2} T_1 - T_2}{e^{(\sqrt{-\Delta}-B)L/2} - e^{(-\sqrt{-\Delta}-B)L/2}} \quad (A.7)$$

where c_1 , c_2 are integration constants determined from the boundary conditions.

One of the solutions, Eq. (A.3), is singular when $\sin(\sqrt{\Delta} L/2) = 0$. This occurs when $\sqrt{\Delta} L/2 = n\pi$, $n = 0, 1, 2, \dots$. The case where $n=0$ is eliminated since it corresponds to $\Delta = 0$ for which the solution given in Eq. (A.2) is not singular. The values that n can assume

are limited to, say N , by the condition that $\Delta > 0$. Solving $\sqrt{\Delta}L/2 = n\pi$, $n = 1, 2, \dots, N$ for M results in

$$M = \frac{2\sqrt{L_o}}{c_{ph}} \sqrt{I^2 - \frac{1}{L_o} \left(\frac{n\pi k A}{L} \right)^2}, \quad n = 1, 2, \dots, N \quad (A.8)$$

The largest critical mass flow occurs for $n=1$. Eq. (2.11) indicates that there may be multiple values for the critical mass flow. But the physical significance of the critical mass flows for $n=2, \dots, N$ is not established.

The solution for T given by Eq. (A.3) can be differentiated with respect to X to obtain the gradient of the temperature. Of particular interest is the the temperature gradient at the warm end, $\frac{dT}{dX}|_{X=L}$, which is equal to zero when M is equal to the critical mass flow.

B. Appendix: Boil-off versus forced flow

In the case of boil-off, an additional iteration cycle is needed in the numerical method described below (Appendix C) since the mass flow is given by the conducted heat at the cold end (as indicated in Eq. (2.3)). An intial guess for the mass flow is used to determine Q_{in} which is then used to refine the guess for the mass flow.

The intial guess assumes that temperatures and pressures have a linear distribution between values at $x=0$ and at $x=L$. This guess approximates the temperature distribution due to conduction of heat between the values of the end temperatures.

A comparison of numerical results between forced flow and boil off cooling shows that boil off consumes more mass flow but is more effective in reducing Q_{in} . Q_{tot} which is given in Eq. (2.11) is thus smaller for boil-off at low currents (or for short leads) where conducted heat dominates resistive heat. At high currents (or for long leads) where resistive heat dominates, forced flow is more effective. These results are illustrated in Table B.1 for increasing currents and in Table B.2 for increasing lengths.

Table B.1: Forced flow versus boil-off cooling with increasing current, $L=55$ cm , $D=1.5$ cm

cooling	I A	M g/sec	Q_{in} W	Q_{tot}/I W/kA
forced flow	1000	0.20	40.6	42.2
boil-off	1000	0.153	3.1	15.34
forced flow	3000	0.90	25.9	11.03
boil-off	3000	0.168	3.3	5.58
forced flow	5000	.20	5.3	4.26
boil-off	5000	0.206	3.7	4.04
forced flow	7000	0.30	1.5	3.64
boil-off	7000	0.303	1.5	3.68

Table B.2: Forced flow versus boil-off cooling with increasing length, $I=6300$ A, $D=1.5$ cm

cooling	L cm	M g/sec	Q_{in} W	Q_{tot}/I W/kA
forced flow	45	0.255	6.2	4.2
boil-off	45	0.266	3.6	3.9
forced flow	55	0.265	1.8	3.65
boil-off	55	0.27	1.6	3.68
forced flow	65	0.27	1.3	3.63
boil-off	65	0.273	1.3	3.67
forced flow	75	0.275	1.2	3.68
boil-off	75	0.278	1.2	3.72

C. Appendix: Numerical method

In view of the limitations of the closed form solution, the numerical method will be taken here as the only reliable form of analysis. A simple finite difference scheme is applied to Eq. (2.1) and Eq. (2.2). For instance, $\frac{dT}{dX}$ is replaced by $\frac{T^i - T^{i-1}}{\Delta X}$. The result is a matrix formulation of the type: $[M][T] = [R]$ where $[M]$ is a matrix of the coefficients of the vector $[T]$ which contains all the unknown values for T^i and T_h^i . Since $[R]$ is a vector whose values depend on $[T]$, an initial guess has to be made and the problem has to be solved iteratively until convergence (with a tolerance on temperature of 0.01 K) has been reached. The problem is treated here as a boundary value problem where T , T_h are given at the cold end and T is given at the warm end (or determined by interaction with surrounding parts). An alternate approach, not used here, consists of fixing T , T_h and $\frac{dT}{dX}$ at the warm end and adjusting $\frac{dT}{dX}$ until the proper temperature is reached at the warm end.

D. Appendix: Heat exchange parameters with forced convection

The heat exchange parameters are now explicitly given in terms of the geometry of the cooling passage and helium properties for the case of a single straight passage, a helical passage, and the case of N parallel passages in series. Heat exchange parameters for a helical passage and for a series of parallel passages will be compared.

The heat exchange parameter h for the case where the coolant goes through an internal passage is⁸:

$$h = Nu k_h / D_h \quad (D.1)$$

where the hydraulic diameter is defined as

$$D_h = 4A_h / P \quad (D.2)$$

A_h is the cross-sectional area of the cooling passage, P is the wetted perimeter.

Nu , the Nusselt number is defined as

- for turbulent flow $Re > 2300$ (Dittus-Boelter equation),

$$Nu = 0.023 Re^{0.8} P_r^{0.4} \quad (D.3)$$

- for laminar flow $Re < 2300$,

$$Nu = 4.36 \quad (D.4)$$

Re is the Reynolds number

$$Re = \frac{4M}{\pi \mu D_h}, \quad (D.5)$$

and P_r is the Prandtl number

$$P_r = \frac{c_{p_h} \mu}{k_h} \quad (D.6)$$

k_h is the helium conductivity, μ is its viscosity, c_{p_h} is its heat capacity. These properties depend on pressure and temperature.

D.0.1. Single straight cooling passage

For a straight cooling passage of circular cross-section of diameter d , the total heat exchange area is $A_s = PL$, the wetted perimeter is $P = \pi d$, the flow area and hydraulic diameter are $A_h = \pi d^2/4$, $D_h = d$. These expressions and the turbulent Nusselt number for a straight passage are used in Eq. (D.1) to give

$$\frac{hA_s}{I} = H \left(\frac{M^{0.8}}{I} \right) \left(\frac{P}{P^{0.8}} \right) \left(\frac{L}{d} \right) \quad (D.7)$$

where $H = 0.023 k_h \left(\frac{4}{\mu} \right)^{0.8} P_r^{0.4}$ is a function of helium properties only.

D.0.2. Helical cooling passage

For the case of a helical passage with rectangular cross-section, the total heat exchange area is

$$A_s = \phi P_s L \quad (D.8)$$

with

$$P_s = 2f_l + f_d \quad (D.9)$$

ϕ is a factor which accounts for the longer length that a helix covers along the rod: if the flow is straight $\phi = 1$, if the flow is helical around the rod of diameter D ,

$$\phi = \frac{L_{heliz}}{L_{straight}} = \frac{\pi(D + d)}{p} \quad (D.10)$$

where p is the helix pitch

$$p = f_d + f_t \quad (D.11)$$

and d is the diameter of an equivalent circular passage. The wetted perimeter used in Eq. (D.2) is

$$P = 2(f_l + f_d) \quad (D.12)$$

$$d = \sqrt{A_h/\pi} = \sqrt{4f_l f_d/\pi} \quad (D.13)$$

f_l , f_d , f_t are the fins radial length, inner spacing and thickness, respectively.

The Nusselt number for a straight passage, Nu , will be used here instead of the Nusselt number for a helical passage, Nu_{heliz} , (given in Appendix E) in order to simplify the present discussion.

For this geometry and for turbulent flow, substituting into Eq. (D.1) leads to

$$\left(\frac{hA_s}{I}\right)_{heliz} \simeq H \left(\frac{M_h}{I}\right) \left(\frac{L_h}{d_h}\right) \phi \quad (D.14)$$

where subscript h refers to a helix.

The explicit expression for $(hA_s)_{heliz}$ in terms of fin geometry is

$$(hA_s)_{heliz} = G\phi \frac{(f_d + 2f_l)(f_d + f_l)^{1.8}}{(f_d f_l)^{1.8}} \quad (D.15)$$

where $G = 0.023(\frac{4M_h}{\pi\mu})^{0.8} P_r^{0.4} k_h(\frac{L}{l})$ is independent of fin geometry.

D.0.3. N straight cooling passages in parallel

If there are N passages in parallel, such as in a bundle arrangement, the conditions under which Eq. (2.1) can still be used are determined here. The model assumes that the helium flows in N identical circular passages, each with mass flow M_b . Let us consider a lead made up of a bundle of N wires with cross-section $a = A/N$ and carrying $i = I/N$ each. The governing equation equivalent to Eq. (2.1) would be , for each wire:

$$\frac{d}{dX} \left(ka \frac{dT}{dX} \right) = \frac{-\rho i^2}{a} + \frac{hA_s}{L} (T - T_h), T(0) = T_{in}, T(L) = T_{out} \quad (D.16)$$

where hA_s refers to the cooling of one individual wire. Multiplying this equation by N results in

$$\frac{d}{dX} \left(kA \frac{dT}{dX} \right) = \frac{-\rho I^2}{A} + N \frac{hA_s}{L} (T - T_h) \quad (D.17)$$

Which has the same form as Eq. (2.1) if the heat exchange parameter is multiplied by N . The numerical method developed for helical leads can thus also be used for bundles as long as proper terms are used. T and T_h are the temperatures of one rod and of a helium passage. This method has been generalized and applied to the case where different wires carry different currents in (Shutt, Hornik, Rehak)¹.

Thus one would use, with subscript b to refer to a bundle lead:

$$\left(\frac{hA_s}{I} \right)_{bundle} \simeq HN \left(\frac{M_b}{I} \right) \left(\frac{L_b}{d_b} \right) \quad (D.18)$$

D.0.4. Comparison between bundle and helix cooling

The two types of cooling for leads with same total copper cross-section and carrying same total current I are compared here. The ratio of the two expressions in Eq. (D.14) and Eq. (D.18) is

$$\frac{(hA_s/I)_{bundle}}{(hA_s/I)_{helix}} = \left(\frac{NM_b L_b}{d_b} \right) \left(\frac{d_h}{M_h L_h} \right) \quad (D.19)$$

If it is assumed that the same total mass flow is used for the two leads: $NM_b = M_h$ and that the flow is turbulent in both leads, it is found that cooling with a bundle lead is identical to cooling with a single helix when $\frac{d_h}{L_h} = \frac{d_b}{L_b}$. This indicates that, theoretically, there is no preferred cooling method since parameters can be adjusted to provide the same

cooling. A specific comparison would of course require that the proper regimes, turbulent or laminar, be used.

E. Appendix: Heat transfer in helical passage

A curved duct has increased heat exchange and friction when compared to a straight passage⁹. The centrifugal force causes a velocity gradient with high velocities at the outer wall and decreased velocities at the inner tube wall. The higher outer wall velocity increases the heat transfer as well as friction.

For a helical passage the heat exchange parameter in Eq. (D.1) becomes:

$$h = \phi Nu_{heliz} k_h / D_h \quad (E.1)$$

The Nusselt number for a helical passage, Nu_{heliz} , is given next.

Let De be the Dean number defined as :

$$De = Re \left(\frac{r}{R} \right)^{1/2} \quad (E.2)$$

where

$$R = (D/2 + d) \quad (E.3)$$

is the average radius to the center of the cooling passage and $r=d/2$. Whether the flow is turbulent or laminar is determined by the value of a critical Reynolds number:

$$Re_{heliz} = 2100 \left(1 + 12 \left(\frac{R}{r} \right)^{-0.5} \right) \quad (E.4)$$

This criterion is different than that used for straight passages.

- If $Re < Re_{heliz}$, the laminar heat transfer Nusselt number is:

$$Nu_{heliz} = \left(x_1^3 + 1.816 (De/x_4)^{1.5} \right)^{1/3} \quad (E.5)$$

$$x_4 = 1.0 + 1.15/P_r, \quad x_3 = (1.0 + 1342/(De^2 P_r))^2, \quad x_1 = 4.364 + 4.636/x_3$$

- If $Re > Re_{heliz}$, the turbulent heat transfer is:

- if $1.5 \times 10^3 < Re < 2 \times 10^4$ then

$$Nu_{heliz} = Nu (1 + 3.4r/R) \quad (E.6)$$

- if $Re > 2 \times 10^4$ then

$$Nu_{heliz} = Nu \left(1 + 3.6 (1 - r/R) (r/R)^{0.8} \right) \quad (E.7)$$

F. Appendix: Pressure drop in helical passage

The general expression for pressure drop (in atmospheres) is

$$\frac{dp}{dX} = f_{straight} \frac{10^{-6}}{D_h} \frac{M^2}{2\sigma_h A_h^2} \quad (F.1)$$

For a helical flow path the friction factor f_{heliz} replaces $f_{straight}$ in Eq. (F.1) which becomes

$$\frac{dp}{dX} = \phi f_{heliz} \frac{10^{-6}}{D_h} \frac{M^2}{2\sigma_h A_h^2} \quad (F.2)$$

where ϕ (see Eq. (D.10)) accounts for the longer flow path in a helix. The friction factors are given next.

F.0.1. Friction in a straight tube

If $Re > 2100$, the flow is turbulent and the friction factor for a straight tube is:

$$f_{straight} = 4 \left(0.0014 + \frac{0.125}{Re^{0.32}} \right) \quad (F.3)$$

This formula is preferable to the Moody diagram for numerical applications and gives the same results as the Blasius equation.

If $Re < 2100$, the flow is laminar:

$$f_{straight} = 64/Re \quad (F.4)$$

F.0.2. Friction in a helical tube

If $Re > 2100(1 + 12(R/\tau)^{-0.5})$ the flow is turbulent:

- if $0.0034 < Re(R/\tau)^{-2} < 300$ then

$$f_{heliz} = 0.00725 (\tau/R)^{0.5} + 0.076 Re^{-0.25} \quad (F.5)$$

- if $Re(\tau/\frac{d}{2}) > 300$, $f_{heliz}/f_{straight} = 1$

If $Re < 2100(1 + 12(R/\tau)^{-0.5})$ the flow is laminar:

- if $De < 30$ then $f_{heliz}/f_{straight} = 1$
- if $De < 300$ then $f_{heliz}/f_{straight} = 0.419 De^{0.275}$
- if $De > 300$ then $f_{heliz}/f_{straight} = 0.1125 De^{0.5}$

G. Appendix: Free convection

In general one has:

$$h = Nu \frac{kA}{L} \quad (G.1)$$

for a plate and

$$h = Nu \frac{kD}{L} \quad (G.2)$$

for a cylinder of diameter D. Nu is the Nusselt number which is given below for different configurations. h is then used to determine the heat exchanged per unit length, q, in $q = \frac{hA}{L} \Delta T$.

G.0.1. Vertical plate

For a the outer surfaces of a vertical plate, or for a vertically oriented cylinder whose aspect ratio satisfies

$$\frac{D}{L} \geq \frac{35}{(Gr(L))^{\frac{1}{4}}} \quad (G.3)$$

The Nusselt number is:

$$Nu(L) = \left(0.825 + \frac{0.387 (Ra(L))^{\frac{1}{6}}}{\left(1 + \left(\frac{0.492}{Pr} \right)^{\frac{9}{16}} \right)^{\frac{8}{27}}} \right)^2 \quad (G.4)$$

$Ra(L)$ is the Rayleigh number $Ra(L) = Gr(L) \times Pr$

$Gr(L)$ is the Grashof number $Gr(L) = \frac{g\beta(T-T_{air})L^3\sigma^2}{\mu^2}$

where $g = 980$ is the acceleration due to gravity

$c_{pa} = 1.01$, $\mu_a = 1.85 \times 10^{-4}$, $k_a = 2.62 \times 10^{-4}$, $\sigma_a = 1.161 \times 10^{-3}$ are the heat capacity, viscosity, thermal conductivity, density of air.

$\beta = \frac{1}{T_{av}}$ is the volumetric thermal expansion coefficient.

$$T_{av} = \frac{T + T_{air}}{2}$$

T = surface temperature of copper conductor

T_{air} = environmental air temperature

L = height of plate or cylinder

D = diameter of cylinder

All properties are evaluated at the film temperature $T_f = (T + T_{air})/2$.

For the inner surfaces between two vertical plates:

$$Nu = \frac{Ra(L_i)}{24} \frac{L_i}{L} \left(1 - e^{\frac{-35}{Ra(L_i)} \frac{L_i}{L}} \right)^{3/4} \quad (G.5)$$

L_i = inner distance between two plates

G.0.2. Horizontal plate

For the upper side of a flat plate which is warmer than the environment

If $10^4 < Ra < 10^7$ then $Nu = 0.54Ra^{1/4}$

If $10^7 Ra < 10^{11}$ then $Nu = 0.15Ra^{1/3}$

For the lower side of a flat plate which is warmer than the environment

If $10^5 Ra < 10^{10}$ then $Nu = 0.27Ra^{1/4}$

G.0.3. Horizontal cylinder

For horizontally oriented cylinders of diameter D , one uses $h = Nu \frac{kD}{L}$ with

$$Nu(D) = \left(0.60 + \frac{0.387(Ra(D))^{\frac{1}{6}}}{\left(1 + \left(\frac{0.559}{Pr} \right)^{\frac{9}{16}} \right)^{\frac{8}{27}}} \right)^2 \quad (G.6)$$

where the Grashof number is now

$$Gr(D) = \frac{g\beta(T - T_{air})D^3\sigma^2}{\mu^2} \quad (G.7)$$

H. Appendix: Superinsulation

It is assumed that the stainless steel pipe enclosing the lead inside the cryostat is directly exposed to the cryostat's vacuum. The lead will be wrapped in layers of superinsulation to shield it from radiation. Superinsulation has been treated previously¹⁰⁻¹¹. Three effects are considered when dealing with superinsulation: radiation, residual gas pressure and solid heat conduction. From this work, one relevant equation is extracted:

$$q_{sup} = \frac{1}{n} (q_c + q_{rad} + q_{cond}) A_{sup} \quad (H.1)$$

where q_c is the heat conduction through a partially evacuated space, q_{rad} is the heat leak by radiation, and q_{cond} is the heat conduction through the layers of superinsulation.

$$q_c = 1.22P \left(T_o^{\frac{1}{4}} - T_e^{\frac{1}{4}} \right) \quad (H.2)$$

$$q_{rad} = 1.29 \times 10^{-12} E \left(T_o^{\frac{17}{4}} - T_e^{\frac{17}{4}} \right) \quad (H.3)$$

$$q_{cond} = K (T_o - T_e) \quad (H.4)$$

where

P = residual gas pressure (Torr)

T_o = inner surface temperature of vacuum chamber (K)

T_e = outer surface temperature of flexible tube passing helium (very close to T_h) (K)

E = emissivity coefficient

K = equivalent solid heat conduction coefficient that can be estimated for superinsulation ($\frac{W}{cm^2 K}$). Depends on the contact between adjacent superinsulation layers.

n = number of layers of superinsulation

q_{sup} = heat passing through 1 cm length of superinsulation (W/cm)

A_{sup} = average perimeter of superinsulation wound around cooled object (cm)

For this analysis, a worst case is considered where T_o is set equal to air temperature and T_e is the helium temperature since the temperature gradient in the wall pipe enclosure is less than 1K. The heat that passes through the superinsulation is treated as another heat source warming the helium. Another case considered is where T_o is assumed to be at 80 K.

The results of these analyses with a residual gas pressure of 10^{-4} Torr can be seen in Table H.1. The number of layers is varied while keeping the residual pressure constant to see the effect that the number of layers of insulation has on the amount of heat conducted (Q_{in}^{sum} is the sum of all the heat sources at the cold end) at the cold end of the lead, before it enters the large helium volume. This analysis shows that in the worst case where $T_0 = 293$ K, fifteen layers of insulation are sufficient to keep the heat passing through the superinsulation into the lead at a minimum. For this case Q_{in}^{sum} is 1.6 W with fifteen layers of superinsulation. An increase in the number of layers of superinsulation to twenty-five

reduces Q_{in}^{sum} to 1.4 W which is practically identical to the case where no heat reaches the lead where $Q_{in}^{sum} = 1.3$ W.

Table H.1: Q_{in}^{sum} versus number of layers of superinsulation

Number of layers	0	5	10	15	20	25	30	35	∞
Q_{in}^{sum} W, $T_0 = 293$ K	6.5	1.9	1.6	1.5	1.5	1.4	1.4	1.4	1.3
Q_{in}^{sum} W, $T_0 = 80$ K	1.7	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3

I. Appendix: Condensation

Ideally, a lead should be designed so that it meets requirement concerning mass flow and heat leaked into the cold end, while at the same time exiting the cryostat at a temperature greater than the dew point temperature of the surrounding air. It is possible to design a lead which will not experience condensation over a limited range of currents (see 4.1). However in this application the lead must operate over a large range of currents, including no current. Thus the use of a vacuum jacket around the portion of the lead in the atmosphere is recommended. This will prevent frost or condensation in the sensitive area where current carrying wires are located. However the helium temperature remains low as it leaves the lead. Condensation or frost on the pipes carrying helium to the refrigerator is thus to be expected, albeit in less bothersome locations. The dew point temperature is the temperature below which condensation would begin to form on a surface. This temperature depends on other properties of the air such as its relative humidity. Fig. I.35 gives dew point temperatures for various humidity and air temperatures. If, for example, the air is at 293 K (20° C) and the relative humidity is 60%, the dew point temperature is 285 K (12° C). This means that if the surface exposed to the air is 285 K or less, condensation will form.

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Some of the theoretical studies and most of the lead designs presented here were the outcome of very fruitful discussions with A. Nicoletti.

Lead temperature at 6300 A

Figure I.1: I=6300 A, lead and helium temperatures at operating and critical mass flow

Lead temperature at 6300 A

Figure I.2:: I=6300 A, lead temperatures at different mass flows

Figure I.3: I=6300 A, lead and helium temperatures after interruption of flow

Lead temperature at 6300 A

Figure I.4: Analytical vs numerical for OFHC

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Figure I.5: Analytical vs numerical for RRR=160

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Figure I.6: Analytical solution

Figure I.7: Temperature for a straight path, for a helix treated as a straight path, for a helix treated as a curved path

Figure I.8: Cooling parameter J vs fin geometry

J vs fd for various fl

J vs fl for various fd

Lead temperature at 6300 A, $M=0.38$ g/sec for different ϕ

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NOTE. Markers are placed at every 3 data points.

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Figure I.9: 6300 A lead temperature vs $\phi = L_{helix}/L_{straight}$

Figure I.10: Electrical resistivity and conductivity of various copper types.
s. Comparison between generated and measures properties.

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Lead temperature at 6300 A

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Figure I.11: 6300 A lead temperature vs RRR

Lead temperature at 6300 A

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Figure I.12: Conventional and HTSC lead at 6300 A

Lead temperature at 1600 A

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Figure I.13: Conventional and HTSC lead at 1600 A

Figure I.15: Refrigerator load vs LI/A

Table 1: $I=5500$ A, $LI/A \times 10^{-5}$, M_{evit} (g/sec), Q_{in} (W), $Q_{tot}/I = (80M_{evit} + Q_{in})/I$ (W/kA) for various L (cm) and A (cm^2)

L	A	1.	1.2	1.5	1.8	2.	2.2	2.5
40.0	LI/A	2.2	1.83	1.47	1.22	1.10	1.00	0.88
40.0	M_{evit}	0.24	0.24	0.22	0.20	0.18	0.17	0.13
40.0	Q_{in}	1.12	1.27	1.08	18.0	37.54	45.39	101.27
40.0	Q_{tot}/I	3.89	3.72	4.49	6.16	9.44	10.72	20.30
45.0	LI/A	2.47	2.06	1.65	1.37	1.24	1.12	0.99
45.0	M_{evit}	0.24	0.24	0.23	0.21	0.20	0.19	0.16
45.0	Q_{in}	1.00	1.08	2.62	11.82	18.77	27.2	59.42
45.0	Q_{tot}/I	3.67	3.69	3.82	5.20	6.32	7.71	13.13
50.0	LI/A	2.75	2.29	1.83	1.53	1.37	1.25	1.10
50.0	M_{evit}	0.25	0.24	0.23	0.22	0.21	0.20	0.18
50.0	Q_{in}	0.98	1.01	1.65	5.53	11.29	18.34	37.78
50.0	Q_{tot}/I	3.81	3.67	3.65	4.21	5.11	6.24	9.45
55.0	LI/A	3.02	2.52	2.02	1.68	1.51	1.37	1.21
55.0	M_{evit}	0.25	0.25	0.24	0.23	0.22	0.21	0.20
55.0	Q_{in}	0.97	0.95	1.01	2.02	5.33	10.88	19.01
55.0	Q_{tot}/I	3.81	3.81	3.67	3.71	4.17	5.03	6.37
60.0	LI/A	3.30	2.75	2.20	1.83	1.65	1.50	1.32
60.0	M_{evit}	0.26	0.25	0.24	0.24	0.23	0.22	0.21
60.0	Q_{in}	0.93	0.94	0.96	1.06	2.04	5.21	11.28
60.0	Q_{tot}/I	3.95	3.81	3.67	3.68	3.72	4.15	5.11
65.0	LI/A	3.57	2.98	2.38	1.99	1.79	1.62	1.43
65.0	M_{evit}	0.26	0.25	0.24	0.23	0.23	0.23	0.22
65.0	Q_{in}	0.91	0.92	0.94	1.12	1.41	2.06	5.92
65.0	Q_{tot}/I	3.95	3.80	3.66	3.55	3.60	3.72	4.28

Table 1: $I=1600$ A, $LI/A \times 10^{-5}$, M_{evit} (g/sec), Q_{in} (W), $Q_{tot}/I = (80M_{evit} + Q_{in})/I$ (W/kA) for various L (cm) and A (cm^2)

L	A	0.317	0.381	0.474	0.572	0.634	0.697	0.793
43.59	LI/A	2.2	1.83	1.47	1.22	1.10	1.00	0.88
43.59	M_{evit}	0.07	0.07	0.07	0.06	0.06	0.05	0.04
43.59	Q_{in}	0.33	0.39	0.86	4.53	5.67	12.29	23.27
43.59	Q_{tot}/I	3.70	3.74	4.04	5.83	6.55	10.18	16.54
48.94	LI/A	2.47	2.06	1.65	1.37	1.24	1.12	0.99
48.94	M_{evit}	0.07	0.07	0.07	0.06	0.06	0.06	0.05
48.94	Q_{in}	0.31	0.32	0.47	3.87	4.38	5.33	12.44
48.94	Q_{tot}/I	3.69	3.70	3.80	5.42	5.74	6.33	10.27
54.48	LI/A	2.75	2.29	1.83	1.53	1.37	1.25	1.10
54.48	M_{evit}	0.08	0.07	0.07	0.07	0.06	0.06	0.06
54.48	Q_{in}	0.27	0.30	0.33	0.82	3.84	4.19	5.47
54.48	Q_{tot}/I	4.17	3.69	3.71	3.88	5.40	5.82	6.42
59.83	LI/A	3.02	2.52	2.02	1.68	1.51	1.37	1.21
59.83	M_{evit}	0.08	0.07	0.07	0.07	0.07	0.06	0.06
59.83	Q_{in}	0.27	0.29	0.30	0.39	0.60	3.71	4.41
59.83	Q_{tot}/I	4.17	3.68	3.68	3.75	3.88	5.32	5.70
65.38	LI/A	3.30	2.75	2.20	1.83	1.65	1.50	1.32
65.38	M_{evit}	0.08	0.07	0.07	0.07	0.07	0.07	0.06
65.38	Q_{in}	0.26	0.29	0.28	0.31	0.39	0.59	3.68
65.38	Q_{tot}/I	4.17	3.68	3.68	3.69	3.75	3.87	5.30
70.73	LI/A	3.57	2.98	2.38	1.99	1.79	1.62	1.43
70.73	M_{evit}	0.08	0.08	0.07	0.07	0.07	0.07	0.07
70.73	Q_{in}	0.26	0.25	0.28	0.28	0.31	0.40	0.73
70.73	Q_{tot}/I	4.16	4.16	3.67	3.68	3.69	3.75	3.96

Figure I.16: Critical mass flow vs current for different materials

critical mass flow vs current for ofhc

current vs time

Figure I.17: Current history

Lead temperature

Figure I.18: Temperature versus distance. Dynamic lead response to ramp up and down to 10000 A at 1000 A/sec

Temperature vs time at 4 locations of a 70cm lead

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Figure I.19: Temperature versus time. Dynamic lead response to ramp up and down to 10000 A at 1000 A/sec

Critical mass flows for lead recovery

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Figure I.20: Recovery mass flow vs time elapsed

Figure I.21: Frost free-lead design (not used here)

Figure I.22: Lead design requiring vacuum jacket

Figure I.23: Tables used to determine optimal dimensions for the 6300 A lead, D=1.0 and D=1.25 cm

Table 1: Variations of aspect ratio at I=6300 A, D=1.0 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
60	0.340	0.9	4.5	43.6	41.2	479.3	501.2	7.2
65	0.360	0.9	4.7	30.7	29.7	251.5	216.8	0.2
70	0.370	0.9	4.8	31.8	30.8	250.1	214.6	0.3
75	0.390	0.8	5.1	31.7	30.9	239.9	201.2	0.3
80	0.410	0.8	5.3	31.5	30.7	232.9	191.4	0.3

Table 2: Variations of aspect ratio at I=6300 A, D=1.25 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
30	0.270	3.0	3.9	39.5	37.8	297.3	285.6	1.1
35	0.270	2.3	3.8	58.8	55.5	377.4	376.8	3.0
40	0.280	1.1	3.7	41.1	39.5	289.4	276.3	1.1
45	0.280	1.1	3.7	44.5	42.8	308.7	298.3	1.5
50	0.280	1.2	3.7	472.9	439.4	1248.8	1261.6	9.7
55	0.290	1.0	3.8	42.9	41.4	293.9	281.0	1.2
60	0.300	1.0	4.0	40.6	39.3	273.7	257.6	0.9
65	0.300	1.0	4.0	43.7	42.2	280.8	265.7	1.0
70	0.310	1.0	4.1	41.1	39.8	268.6	251.2	0.8
75	0.320	0.9	4.2	40.5	39.2	258.2	238.9	0.7
80	0.330	0.9	4.3	39.0	37.9	251.8	230.3	0.5

Figure I.24: Tables used to determine optimal dimensions for the 6300 A lead, D=1.5 and D=2.00 cm

Table 3: Variations of aspect ratio at I=6300 A, D=1.50 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
45	0.270	1.9	3.7	58.2	56.0	289.3	281.4	1.8
50	0.270	1.3	3.6	66.8	64.1	302.9	296.2	2.2
55	0.270	1.2	3.6	86.1	81.9	342.4	339.0	4.4
60	0.280	1.0	3.7	57.3	55.3	280.5	271.7	1.6
65	0.280	1.0	3.7	62.1	59.7	286.8	278.6	1.7
70	0.290	1.0	3.8	53.5	51.8	268.0	257.8	1.3
75	0.300	1.0	4.0	50.6	49.1	255.1	243.4	1.0

Table 4: Variations of aspect ratio at I=6300 A, D=2.0 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
75	0.260	3.9	3.9	144.8	141.3	307.7	305.4	20.8
80	0.270	1.6	3.7	110.0	107.0	280.4	276.3	3.3
85	0.270	1.3	3.6	118.7	115.5	285.1	281.4	4.1
90	0.270	1.2	3.6	123.5	120.3	292.5	290.4	6.2
95	0.280	1.0	3.7	94.6	91.9	266.4	261.1	2.3

Lead temperature at 6300 A

Figure I.25: 6300 A lead at operating and critical mass flows

Lead temperature at 6300 A

Figure I.27: Response of lead optimized for 6300 A over a range of currents and at critical mass flow

I A	M g/sec	Q_{in} W	Q_{tot}/I W/kA	T^c K	T_h^c K	T^{flag} K	T_h^{flag} K	Δp atm
0.	0.000	34.4	-	194.0	4.4	268.0	4.4	0.0
0.	0.090	4.3	-	82.6	81.0	228.1	213.0	0.7
500.	0.090	4.4	23.2	84.5	82.9	229.8	214.4	0.6
1000.	0.090	4.7	11.9	90.8	89.1	241.6	184.0	0.7
1500.	0.100	3.7	7.8	84.2	82.4	229.9	230.1	0.8
2000.	0.100	4.8	6.4	96.8	94.8	243.8	226.1	0.9
2500.	0.110	4.3	5.2	101.6	99.4	247.8	246.3	1.1
3000.	0.120	4.6	4.7	124.2	121.5	268.8	261.4	1.6
3500.	0.140	3.3	4.1	135.9	132.7	285.2	282.9	2.5
4000.	0.170	1.0	3.6	93.8	90.8	268.9	264.6	2.3
4500.	0.190	0.9	3.6	100.4	96.8	288.0	284.1	3.6
5000.	0.220	0.8	3.7	71.2	68.6	268.6	262.1	2.4
5500.	0.240	0.9	3.7	76.9	73.7	291.6	285.7	3.7
6000.	0.270	1.0	3.8	60.7	58.6	278.3	269.8	1.9
6500.	0.300	1.0	3.9	53.3	51.7	274.4	263.3	1.0
7000.	0.330	1.1	3.9	48.8	47.3	273.0	260.7	0.7
7500.	0.360	1.2	4.0	45.9	44.5	275.2	261.2	0.7
8000.	0.390	1.2	4.1	43.7	42.4	279.5	263.7	0.9

Mcrit versus I for the 1600 A and 6300 A leads

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Figure I.28: Critical mass flow versus current for the 6300 A and 1600 A leads

Q_{tot}/I versus I for the 1600 A and 6300 A leads

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Figure I.29: Total refrigerator load versus current for the 6300 A and 1600 A leads

Figure I.30: Tables used to determine optimal dimensions for the 1600 A lead, D=0.50 and D=0.75 cm

Table 5: Variations of aspect ratio at I=1600 A, D=0.50 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
45	0.080	0.3	4.2	27.0	25.3	263.7	241.2	0.1
50	0.080	0.3	4.2	30.9	29.2	269.1	249.7	0.1
55	0.090	0.3	4.7	26.3	24.9	251.6	218.7	0.1
60	0.090	0.3	4.7	29.3	28.0	254.0	222.8	0.1
65	0.090	0.3	4.7	32.3	30.9	256.9	227.4	0.1

Table 6: Variations of aspect ratio at I=1600 A, D=0.75 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
40	0.070	0.7	3.9	48.4	46.7	260.0	252.0	0.1
45	0.070	0.4	3.7	49.6	47.8	262.3	254.9	0.1
50	0.070	0.3	3.7	52.4	50.6	264.5	257.5	0.1
55	0.070	0.3	3.7	53.2	51.3	266.6	259.9	0.1
60	0.070	0.3	3.7	56.5	54.5	269.0	262.8	0.1
65	0.070	0.3	3.7	60.6	58.2	271.7	265.9	0.1
70	0.070	0.3	3.7	63.7	61.0	276.2	271.2	0.1
75	0.080	0.3	4.2	45.3	43.9	248.7	237.6	0.1

Figure I.31: Tables used to determine optimal dimensions for the 1600 A lead, D=1.0 and D=1.25 cm

Table 7: Variations of aspect ratio at I=1600 A, D=1.0 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
60	0.070	1.3	4.3	80.7	78.6	255.4	250.8	0.2
65	0.070	0.9	4.1	85.8	83.5	257.8	250.8	0.2
70	0.070	0.6	3.9	87.2	84.9	259.5	253.0	0.2
75	0.070	0.4	3.8	92.0	89.7	260.4	255.7	0.2
80	0.070	0.3	3.7	92.2	89.9	262.7	252.9	0.2
85	0.070	0.3	3.7	97.1	94.7	264.2	253.8	0.2
90	0.080	0.2	4.2	66.2	64.3	242.9	236.5	0.2

Table 8: Variations of aspect ratio at I=1600 A, D=1.25 cm

L cm	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
60	0.090	0.3	4.7	29.3	28.0	254.0	222.8	0.1
65	0.090	0.3	4.7	32.3	30.9	256.9	227.4	0.1
70	0.100	0.2	5.1	29.3	28.2	246.2	206.4	0.1
75	0.100	0.2	5.1	32.1	31.0	248.2	209.7	0.1

Lead temperature at 1600 A

Figure I.33: Dynamic 1600 A lead analysis after flow interruption

Figure I.34: Response of lead optimized for 1600 A over a range of currents and at critical mass flow

I A	M_{crit} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^c K	T_h^c K	T^{flag} K	T_h^{flag} K	Δp atm
0.	0.000	15.1	-	209.2	4.4	280.7	4.4	0.0
0.	0.070	0.0	-	56.0	54.6	239.9	231.4	0.1
500.	0.070	0.0	11.3	57.5	56.0	241.5	233.7	0.1
1000.	0.070	0.1	5.7	63.5	61.7	247.1	240.0	0.1
1500.	0.070	0.3	3.9	83.3	81.1	261.0	257.5	0.2
2000.	0.090	0.3	3.8	72.4	70.0	262.0	256.4	0.3
2500.	0.110	0.4	3.7	70.1	67.3	269.9	264.3	0.4
3000.	0.140	0.5	3.9	52.2	50.2	258.6	249.1	0.4
3500.	0.170	0.5	4.0	45.7	44.2	256.5	243.5	0.5
4000.	0.210	0.6	4.3	38.1	36.9	243.9	224.1	0.4
4500.	0.240	0.6	4.4	37.1	35.9	247.4	224.7	0.5
5000.	0.280	0.7	4.6	34.4	33.3	243.5	214.8	0.4
5500.	0.320	0.7	4.8	33.2	32.2	243.8	209.1	0.3
6000.	0.360	0.8	4.9	32.9	32.0	243.1	208.0	0.3
6500.	0.410	0.8	5.2	32.0	31.1	240.4	199.5	0.3
7000.	0.470	0.9	5.5	30.8	30.0	234.0	188.5	0.4
7500.	0.530	0.9	5.8	30.4	29.7	233.0	181.2	0.5
8000.	0.590	0.9	6.0	30.5	29.9	234.7	177.1	0.6

Dew point temps for various humidities and air temps

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Figure I.35: Dew point (surface) temperatures T_d for various air humidity H and air temperatures T_a

GAS COOLED LEADS

Part 2: Frost Free Lead Optimized for a Specific Current

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ABSTRACT

This paper presents a lead design which would allow frost-free operation over a limited range of currents. This lead is made of three sections: a uniformly cooled helix, a relatively thin tube acting as a heating unit and a large copper tube at the warm end. Dimensions for all the sections were found such that a 6300 A lead and a 1600 A lead would operate within budget and not freeze. Such a lead is as efficient as a uniform lead made of a single helically cooled section and whose dimensions have been optimized. The difference lies in a more advantageous temperature profile where the section of the lead exposed to air is above freezing. The preferred temperature profile can be achieved without compromising efficiency.

While it is always possible to design a lead which will not freeze at a given current, it is much more difficult to design a lead which will not freeze over a large range of currents. The present design covers 4000 A to 8000 A only. If the lead is to operate over a range of currents, it is more advantageous to use a lead designed for high currents than one designed for low currents.

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Nomenclature

Δp : pressure drop across the lead assuming 5 atm at the cold end

M_{crit} : helium mass flow for which the lead is unstable

M_{op} : helium mass flow 20 % above M_{crit}

Q_{in} : heat conducted into magnet at the cold end

Q_{tot} : total refrigerator load

T : lead temperature

T_h : helium temperature

T^e : temperature of lead at exit from cryostat

T_h^e : helium temperature at exit from cryostat

T^{flag} : temperature of lead at flag

T_h^{flag} : temperature of helium at flag

1. INTRODUCTION

In the Relativistic Heavy Ion Collider (RHIC), there are 28 leads carrying 6300 A for the main dipoles, four 6300 A for the quadrupoles, and 28 leads operating at 1600 A for the insertion magnets DX and D0.

The cooling of the leads considered here is achieved by machining a helical path in a copper rod and enclosing the rod in a stainless steel pipe. Results in this paper are obtained with a computer model developed by the authors. Theory, parametric study and optimal design for leads with uniform cross-section and cooling are covered in reference ¹. Due to electrical creep path requirements, a 15 cm long section of the lead extends outside the cryostat and is exposed to the atmosphere. During operation, frost is expected to form on this section unless a vacuum jacket is used or heaters are activated. The specific task here is to design a lead which will perform without frost over the portion exposed to air.

The goal of the present design is to meet all of the following requirements:

- the lead must minimize the total refrigerator load and the lead temperature outside the cryostat should be above freezing temperature.
- the lead must operate at a mass flow 20 % above the critical mass flow.
- the lead should be able to sustain the design current for 30 sec after an interruption of helium flow.
- budgets for mass flow M , total refrigerator load $Q_{tot} = 80 \times M + Q_{in}$ (80 represents the efficiency of the refrigerator) and heat conducted at the cold end, Q_{in} , must be met.

Table 1.1: Budget

M/I g/sec/kA	Q_{in}/I W/kA	Q_{tot}/I W/kA
0.06	1.2	6.0

In this paper the possibility of designing a lead whose warm end does not freeze, at least over a range of currents, is investigated. Contrary to commonly used leads which

have uniform cooling and cross-section, such a lead would be made of three sections in order to achieve its goal. The performance of this non-uniform lead must be compared to that of a uniform lead whose dimensions have been optimized to put a minimum load on the refrigerator.

Gas cooled leads become unstable when the mass flow of the coolant is reduced below a critical value, M_{crit} . M_{crit} will be defined in the following as the mass flow where temperatures reach the 450 K to 500 K range. The operating mass flow, M_{op} , is defined as being 20% higher than the critical mass flow.

When frost is not a concern, an optimal design is obtained by varying systematically the length and cross-section of the lead (uniformly cooled and of constant cross-section) for a given current. The optimal design is that for which the total refrigeration load, Q_{tot} , is the smallest.

2. OPTIMAL LEAD DESIGN FOR A CURRENT OF 6300 A

The underlying principle of a frost-free lead is to increase the temperature in a short section of the lead by replacing the helical flow path with a straight flow path, thus reducing cooling. To compensate for this temperature raising section, the lead is stabilized against thermal runaway by increasing the copper cross-section in the adjacent section. Such a lead consists thus of three parts (see Fig. 4.2): a helical section (A), a section with straight internal flow (B) acting as a built-in heating unit to raise the temperature above freezing), and a section with a large copper cross-section (C) to stabilize (by reduction of resistive heat) the lead against thermal runaway. By adjusting the lengths and diameters of the various sections, it is possible to design a lead which will not frost over the range between 4000 A to 6300 A and still meet design constraints.

The following dimensions were found to provide an acceptable lead design:

- Lead inside cryostat:
 - length with helical cooling: 35.0 cm, core diameter : 1.5 cm.
 - fin radial length: 0.25 cm, fin inner distance: 0.2 cm, fin thickness : 0.08 cm.

- length with internal flow: 10.0 cm, inner diameter: 0.50 cm, outer diameter: 2.00 cm.
- The connector has a considerable amount of copper to stabilize the lead against burnout:
 - length: 15.00 cm, outer diameter: 5.00 cm, inner diameter: 0.50 cm.
- The flag's dimensions are:
 - total length: 16 cm.
 - effective length through which current goes: 10 cm (estimate from present connector center to braid connection center).
 - thickness: 2.54 cm.
- A set of four flexible Burndy B2E12 braids (see section E in Fig. 4.2) introduced for stress relief purposes connects the flag to a large copper block. The braid's parameters are:
 - length: 15.00 cm, height: 8.00 cm.
 - total cross-section: 4 pairs of braids $\times 2 \times 1.56 \text{ cm}^2 = 12.5 \text{ cm}^2$.
- The braids are connected to a large copper block (see section F in Fig. 4.2), whose exact dimensions are not critical and were approximated as follows:
 - length: 15 cm.
 - thickness: 2.54 cm.
 - height: 36 cm.
- Finally a set of 10 cables with Hypalon (Neoprene) insulation (see section G in Fig. 4.2), is attached to this block. The cables are carried in two horizontal rows of five separated by a solid plate and resting in a tray.
 - cable neoprene insulation thickness: 0.40 cm.
 - total cable copper cross-section: $10 \times 1325 \times \pi \times 0.05^2 / 4 \text{ cm}^2 = 26.0 \text{ cm}^2$.
 - outer cable diameter including neoprene: 2.282 cm.

Table 2.1: Critical mass flow and values at operating mass flow for several lead designs, $I=6300$ A.

lead case	M_{crit} g/sec	M_{op} g/sec	Q_{in} W	$Q_{tot/I}$ W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
1.existing	1.2	1.500	0.2	19.1	31.4	24.4	365.1	69.6	0.3
2.helical conn.	0.28	0.340	0.9	4.5	36.3	35.2	214.5	199.4	0.3
3.large conn.	0.27	0.320	0.9	4.2	201.8	188.6	242.6	205.3	0.2
4.three sect.	0.28	0.340	1.0	4.5	273.4	169.1	296.5	224.1	0.2
5.one sec.opt.	0.27	0.320	0.9	4.2	38.6	37.5	226.7	212.8	0.3

T^e is the lead temperature as it exits the cryostat, T^{flag} is the lead temperature where it is connected to the flag, subscript h denotes helium temperatures, Δp is the helium pressure drop across the lead.

- Case 1: existing lead, Fig. 4.4. For comparison purposes, the case of an existing lead used for magnet tests is shown in Table 2.1. This lead is helically cooled with a length of 45.7 cm and diameter 1.5 cm inside the cryostat. The connection between cryostat and flag is made by means of a thin copper pipe whose outer diameter was chosen to match the core diameter of the helix of 1.5 cm and with inner diameter 0.8 cm. The reduction in cooling is accompanied by a reduction in copper cross-section, and this takes place in the most sensitive location of the lead. All these factors contribute to raise considerably the mass flow required to prevent burnout. It must be recognized that the characteristics of this lead which was previously carefully optimized to operate at 8500 A using a formula²($LI/A=2.2 \times 10^5$, L is the length, I is the current, A is the cross-section), are completely altered when the connector is attached to its warm end. A far less efficient lead than originally intended is in its place.
- Case 2: existing lead with helix continued all the way to the flag, Fig. 4.1. This case corresponds to a uniform lead, $45.7 + 15 = 60.7$ cm long with a core

diameter of 1.5 cm. The previous design has been improved in that the helix extends all the way to the flag and the critical mass flow is below budget. However, the lead comes out of the cryostat at a very low temperature.

- Case 3: existing lead with thicker connector, Fig. 4.3. An alternate way to improve the lead in case 1 is to greatly increase the copper cross-section of the connector. Results are shown in Case 3 where a pipe with 5 cm outer diameter and 0.5 cm inner diameter has been used for the connection between lead and flag. The critical mass flow is considerably lower than case 1 and even somewhat lower than case 2. Temperatures are higher than in the preceeding case but still below freezing,.
- Case 4: proposed lead made of three sections, Fig. 4.2. The lead is made of three sections: a 35 cm long helix with core diameter 1.5 cm, followed by a 10 cm long heating unit with outer diameter 2.0 cm and inner diameter 0.5 cm, and terminated a 15 cm long connecting element outside the cryostat with outer diameter 5 cm and inner diameter 0.5 cm. These are the above given dimensions. Due to presence of the heating unit, the critical mass flow goes up, but not significantly and the temperature of the portion of the lead which is in the atmosphere is above freezing. Temperature profiles are shown in Fig. 4.5.
- Case 5: optimized lead made of one helical section, Fig. 4.1. This is the case of a uniform lead whose dimensions have been optimized to produce the lowest Q_{tot} according to the procedure briefly recalled above and described in greater detail in reference ¹. The optimal length of 55 cm (total lead length: 40 cm inside cryostat+15 cm connector) and diameter of 1.5 cm are close to the corresponding dimensions of cases one to four. The critical mass flow and conducted heat are practically the same, which results in comparable efficiency. Comparing case 4 to case 5 shows that a more desirable temperature profile can be achieved, without departing from optimal efficiency, with a lead made of three sections rather than with a uniform lead consisting of single helical section.

The proposed dimensions result in a design which achieves the three goals: the lead operates at 20 % above M_{crit} (considerably reduced from case 1 in Table 2.1), the temperature is above freezing point at the exit from the cryostat (but liquid condensation is still possible), and a dynamic analysis shows that it can sustain the current 30 seconds without helium flow.

2.1. Sensitivity study

The possibility for inaccuracies in the model increases proportionately with the number of sections. In order to refine the model and determine with greater accuracy the dimensions of the various sections, feedback from tests is needed. The errors on the parameters and assumptions can accumulate to produce two extreme cases: one where the lead would be too warm and another where it would be too cold. A parametric study has revealed that results are most sensitive to the following parameters:

- mass flow $\pm 5\%$.
- material RRR 100 ± 20 . A material with high RRR (relative resistivity ratio) will conduct more heat but generate less resistive heat and temperatures will be lower than with a material with lower RRR.
- helix length (modeling uncertainty) $35\text{ cm} \pm 2\text{ cm}$, a helix (with a relatively small cross-section) will raise temperatures by getting longer. It is assumed that the 10 cm of the heating element can be accurately controlled.
- connector length $15\text{ cm} \pm 2\text{ cm}$, a shorter connector (with a relatively large cross-section) results in raised temperatures.

The first two items reflect fluctuations in the mass flow control and variations of material properties. The error in the last two items comes from approximations and uncertainties introduced during the modeling process.

Table 2.2 shows that even for the worst case the lead will not burn (the critical mass flow is comparable to the reference case). On the other extreme the lead is expected to experience frost, and testing is required to narrow modeling uncertainties to the point where one can be assured that the lead will not freeze. The connector's cross-section was chosen to induce freezing rather than burning in worst case situations. Since measured data

are not available for materials with different RRR, these material properties are numerically generated using Mattiessen's rule for the resistivity and an equivalent of Mattiessen's rule for the conductivity³. The reference case in Table 2.2 was obtained using a RRR=100. Results differ principally in Q_{in} from case 4 shown in Table 2.1 where material properties obtained from tables for an OFHC copper were used.

Table 2.2: Sensitivity to parameters

parameter	warm	reference	cold
mass flow g/s	0.324	0.340	0.360
RRR	80	100	120
helix length cm	37	35	32
connector length cm	13	15	17
Q_{in} W	13.2	16.5	19.7
Q_{tot}/I W/kA	6.2	6.9	7.7
T_{lead}^e K	301.4	267.3	235.4
T_{he}^e K	197.1	159.6	129.8
T_{lead}^{flag} K	316.9	291.1	267.9
T_{he}^{flag} K	246.0	216.1	191.2

2.2. Behavior of the 6300 A lead over a range of currents

This lead was designed to operate at 6300 A without frost and at 20% above the critical mass flow. In general conditions which were met at that current will not be maintained at different currents. During ramp-up for instance, the lead carries currents between 0 to 6300 A for which the mass flow must be adjusted.

Table 2.3 shows that there will be frost at currents below 4000 A since the current is too low to generate enough heat in the heating unit, and mass flow is needed to reduce heat conducted at the cold end. But from 4000 A to 8500 A the lead satisfies the margin and temperature requirements. Fig. 4.6 shows the critical mass flow as a function of current. M_{crit} varies linearly with current as long as $I \geq 4000$ A, below that value M_{crit} is constant. The change in slope corresponds to the point where the lead ceases to be free of frost.

When the lead is not carrying current a certain amount of mass flow is needed to prevent excessive heat conduction. The rule of thumb consisting of using half the mass flow used for the operating current, here 6300 A, is shown below. The following shows the amount of heat conducted for three cases of mass flows.

- When $I=0$, $M=0$, $Q_{in}=56.1$ W.
- When $I=0$, $M = \frac{0.06 \times 6.3}{2} = 0.19 \text{ g/sec}$, $Q_{in}=3.9$ W.
- When $I=0$, $M=0.23$ g/sec is needed to keep Q_{in} under 1 W.

3. OPTIMAL LEAD DESIGN FOR A CURRENT OF 1600 A

When the same approach is applied to a 1600 A lead, the following dimensions are found:

- Lead inside the cryostat:
 - length with helix: 50.0 cm, core diameter : 0.8 cm.
 - length with internal flow: 13.0 cm, inner diameter: 0.50 cm, outer diameter: 1.30 cm.
 - total length inside cryostat: 63.0 cm.

Table 2.3: Temperature versus current at operating mass flow using the 6300 A lead design.

current A	M_{crit} g/sec	M_{op} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
0.	0.00	0.000	56.1	-	243.9	4.4	254.8	4.4	0.0
0.	0.08	0.100	21.0	-	208.1	140.4	227.0	166.0	0.2
500.	0.08	0.100	21.2	58.3	209.0	141.5	227.9	167.1	0.2
1000.	0.08	0.100	21.7	29.7	211.8	144.9	230.3	170.6	0.2
1500.	0.09	0.110	20.2	19.3	214.0	146.5	232.5	170.5	0.2
2000.	0.09	0.110	21.7	15.2	221.9	154.2	239.4	178.8	0.3
2500.	0.09	0.110	23.8	13.0	233.7	168.5	249.6	193.2	0.3
3000.	0.09	0.110	27.5	12.1	253.8	193.7	266.9	217.8	0.3
3500.	0.10	0.120	30.2	11.4	283.0	227.2	292.3	249.3	0.4
4000.	0.12	0.140	30.7	10.5	315.0	263.3	319.4	282.1	0.7
4500.	0.16	0.190	14.6	6.6	279.1	211.9	293.2	250.9	0.6
5000.	0.20	0.240	4.3	4.7	262.7	181.7	281.5	227.1	0.3
5500.	0.23	0.280	1.8	4.4	262.7	172.6	283.7	221.8	0.2
6000.	0.26	0.310	1.3	4.3	276.9	179.1	297.9	231.5	0.2
6500.	0.30	0.360	0.9	4.6	272.5	164.0	296.8	220.7	0.2
7000.	0.33	0.400	1.0	4.7	282.1	164.6	307.5	224.8	0.2
7500.	0.37	0.440	1.0	4.8	298.2	170.2	323.9	234.7	0.2
8000.	0.42	0.500	1.0	5.1	299.7	160.9	328.2	229.3	0.3
8500.	0.44	0.530	1.1	5.1	341.6	187.1	366.6	262.0	0.3

• connector length:15cm, inner diameter 0.5cm ,outer diameter 5 cm.

- The flag and block dimensions are unchanged from those of the previous case with the exception that there are only two braids and four cables going to the power supply.

Table 3.1: Critical mass flow and values at operating mass flow for several lead designs, $I=1600\text{ A}$

lead case	M_{crit} g/sec	M_{op} g/sec	Q_{in} W	$Q_{tot/I}$ W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
1.one sec.opt.	0.07	0.084	0.2	4.3	58.7	57.0	236.7	230.2	0.2
2.large conn.	0.064	0.077	0.4	4.1	234.8	229.1	246.6	235. 2	0.2
3.three sect.	0.067	0.080	0.3	4.2	263.7	181.0	269.6	220.0	0.1

- Case 1: optimal lead made of one helical section, Fig. 4.1. A uniform lead 80 cm long with a core diameter of 1.0 cm was found to be optimal for this current according to reference¹. The temperature of the portion of the lead exposed to the atmosphere is well below freezing.
- Case 2: optimal lead with section in atmosphere replaced by a large connector, Fig. 4.3. The portion in the atmosphere of the previous case has been replaced by a large copper connector. Temperatures are now higher but still below freezing and the total refrigerator load has decreased.
- Case 3: proposed lead made of three sections, Fig. 4.2. The dimensions proposed above have been used, temperatures in the atmosphere are now considerably higher while margin on mass flow has been maintained and the refrigerator load is comparable to the preceeding cases. Requirements concerning margin on mass flow, frost and dynamic behavior after mass flow interruption are satisfied.

Table 3.3 shows the critical mass flow, the operating mass flow and temperature and refrigerator loads at that mass flow. In order to span the range of 1600 A to 6300 A current values one could use the 1600 A design, and adjust the mass flow for higher currents. But

since the heating unit generates heat proportional to the square of the currents, mass flows well above those budgeted are needed at 6300 A. Conversely, operating the 6300 A lead at low currents conducts considerable amounts of heat. Reducing conduction requires a mass flow (exceeding budget) which induces frost below 4000 A.

Assuming that current varies with time, and that the duration of discrete current values I_j is t_j , one can use $\sum_{j=1}^{j=N} Q_{tot,j} t_j / \sum_{j=1}^{j=N} t_j$ as a measure of overall efficiency of the lead. If an equal amount of time is spent at each of the currents shown in Table 2.3 and Table 3.3, overall efficiency is measured by $\sum_{j=1}^{j=N} Q_{tot,j} / N$ where N is the number of currents considered. Table 3.2 shows that the 6300 A lead design is twice as efficient as the 1600 A design.

Table 3.2: Averaged total refrigerator load over the range of currents from 0 to 8500 A, comparison between 6300 A lead and 1600 A lead

case	6300 A lead	1600 A lead
$\sum_{j=1}^{j=N} Q_{tot,j} / N$ (W)	32.7	58.5

4. CONCLUSIONS

As a consequence of electrical creep path requirements, a distance of 15 cm must be maintained between cryostat wall and conducting cables. Leads thus extend at least 15 cm outside the cryostat wall. This section accumulates frost on leads with uniform cooling and cross-section. If a lead contains a built-in heating element, the current going through the lead is used to warm the section in the atmosphere and thus eliminate frost. To increase the lead's efficiency (or equivalently stabilize the lead against thermal runaway), the section in the atmosphere has a large copper cross-section.

The design procedure used here is that of trial and error. The solutions are therefore not unique nor optimized for minimum mass flow. However these non-uniform leads were found to be slightly more efficient than a uniform lead with optimized dimensions.

Table 3.3: Temperature versus current at operating mass flow using the 1600 A lead design

current A	M_{crit} g/sec	M_{op} g/sec	Q_{in} W	Q_{tot}/I W/kA	T^e K	T_h^e K	T^{flag} K	T_h^{flag} K	Δp atm
0.	0.00	0.00	13.1	-	279.5	4.4	282.1	4.4	0.0
0.	0.00	0.048	0.3	-	246.8	167.3	256.5	221.0	0.0
500.	0.04	0.048	0.4	8.4	250.3	174.5	259.3	226.3	0.0
1000.	0.05	0.060	0.2	5.0	254.4	173.4	262.7	220.6	0.1
1500.	0.07	0.084	0.2	4.6	258.6	162.3	266.3	204.2	0.1
2000.	0.09	0.108	0.3	4.5	273.4	169.9	278.7	206.8	0.1
2500.	0.12	0.144	0.3	4.7	265.5	166.9	273.2	216.2	0.1
3000.	0.15	0.180	0.4	4.9	285.6	182.9	290.5	237.5	0.1
3500.	0.19	0.228	0.4	5.3	309.8	199.7	310.6	255.5	0.1
4000.	0.25	0.300	0.5	6.1	314.6	189.1	314.9	249.8	0.1
4500.	0.34	0.408	0.4	7.4	291.2	151.5	296.3	215.9	0.1
5000.	0.44	0.528	0.4	8.5	286.6	139.1	292.8	204.0	0.1
5500.	0.57	0.684	0.4	10.0	245.3	100.3	259.3	161.2	0.1
6000.	0.75	0.900	0.4	12.1	186.2	56.3	211.7	107.9	0.2
6500.	0.96	1.152	0.4	14.2	156.1	39.5	188.1	83.9	0.3
7000.	1.18	1.420	0.4	16.3	138.9	32.0	175.2	71.4	0.3
7500.	1.44	1.730	0.4	18.5	124.7	26.9	164.6	61.9	0.5
8000.	1.71	2.050	0.4	20.5	114.8	23.7	157.4	55.6	0.6
8500.	1.94	2.330	0.4	22.0	109.7	22.3	154.6	52.4	0.8

If the operating mass flow is adjusted to the operating current, the 6300 A lead can carry currents as low as 4000 A without frost. Below 4000 A temperatures are well below freezing since flow is needed to limit conducted heat and there is not enough current to generate sufficient resistive heat. The 1600 A lead operates with no frost between 0 and 5000 A and one can advance the possibility of mechanically switching between the two types of leads at 4500 A. The complexity of installing two leads with switches, or of designing a lead with variable length may be greater than warranted. It is nevertheless preferable to use a lead that doesn't freeze at the operating current since that is where the magnets will be operated for a long time. As a rule, it is more advantageous to operate leads designed for high currents at lower currents than vice-versa.

The equipment connecting the lead to the power supply must be included in the lead design. Some components such as flag, braids, block and cables cooled by air convection are less critical than the connection between cryostat and flag. They must nevertheless be incorporated in the model.

The leads were designed for a specific material, here an OFHC copper with relative resistivity ratio $RRR=100$. Since results are sensitive to the RRR it is essential that some indication of the range of the RRR be known before adjusting the final dimensions.

It also should be noted that the helium leaves the system well below room temperature. The problem of frosting is not eliminated but is moved to a less sensitive location where measures can be used to prevent condensation.

One possible draw back of this design is that dimensions of the various sections of the lead are tied to the purity of the material which must then be carefully controlled. These dimensions are also closely tied to the many other assumptions made in the model. Building and testing this type of lead is necessary in order to reduce the uncertainty in the model.

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Figure 4.1: Uniform lead and surrounding equipment

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Figure 4.2: Non-uniform lead and surrounding equipment, frost-free at 6300 A

Figure 4.3: Non-uniform lead with large connector only

Figure 4.4: Existing lead used for magnet testing

Lead temperature at 6300 A

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Figure 4.5: Temperature of frost-free lead at 6300 A at $M_{op}=0.34$ g/sec

Mcrit versus I for the 6300 A lead

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Figure 4.6: Critical mass flow versus current for the 6300 A lead

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