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EFFECTS OF BOND QUALITY AND DIMENSIONAL STABILITY ON SIDE RUPTURE RESISTANCE OF NON-BUMPER FUEL ELEMENTS

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EFFECTS OF BOND QUALITY AND DIMENSIONAL
STABILITY ON SIDE RUPTURE RESISTANCE OF
NON-BUMPER FUEL ELEMENTS

by

R. R. Bloomstrand
Advance Fuel Engineering
Fuels Preparation Department

January 5, 1962

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Verified By DK Hansen, 3-29-94

SYNOPSIS

Based on a study of reactor performance of non-bumper I&E fuel elements canned over the period from August 3, 1959 through April 19, 1961, involving a total of 95 side failures, it is concluded that resistance to side failure is strongly influenced by bond quality, as measured by total external bond count, and dimensional stability, as measured by tube filling capacity.

A mathematical model describing the relationship that existed between performance level and the above quality attributes was developed and is included in the report.

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INTRODUCTION

Over the past several years, there have been many occasions where poor rupture performance has been qualitatively related to bond quality or dimensional stability. During this same general period, samples of production fuel elements have been routinely measured and irradiated under the Quality Certification Program (PT-216).

Acting with the knowledge that the current rupture model has been proven to be capable of quantifying fuel performance and with the belief that the Quality Certification measurements of total bond count and tube filling capacity (a measure of dimensional stability) physically portray "quality", a study was undertaken to attempt to relate these measured quality parameters with observed reactor performance in terms of side failures.

SUMMARY AND CONCLUSIONS

This study covered the reactor performance of non-bumper I&E fuel elements which were canned during the period from August 3, 1959, through April 19, 1961. Ninety-five side failures, consisting of 77 side hot-spot and 18 side-other failures, make up the "rupture data" for the study. The statistics used were average log total external bond count (LBC), average tube filling capacity (TFC), and the metal performance level parameter for side failures (q_{side}). Methods of analysis are discussed in subsequent sections of this report.

The principal conclusion resulting from the analysis of these data is that resistance to side failures is strongly influenced by bond quality, as measured by total external bond count, and dimensional stability, as measured by tube filling capacity. A mathematical model describing the relationship that existed between metal performance level and the above-noted quality attributes was developed and is as follows:

$$q_{side} = 0.9758 - 1.2928(LBC) - 0.3630(TFC) + 0.4868(LBC)(TFC)$$

where q_{side} = metal performance level parameter for side failures,
multiplied by 10^{39} .

LBC = average logarithm of total external bond count.

TFC = average tube filling capacity.

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RECOMMENDATIONS

The apparent strong effect of "bond quality" on metal performance level provides incentive for additional work as outlined in the following recommended action program.

1. Make the production bond testers more accurate and more reproducible, at sensitivity levels high enough to measure product variability.
2. Investigate the feasibility of using total bond count as an acceptance criterion for production fuel elements and as an on-line quality index for process control.
3. Determine the mechanism(s) by which "unbondedness" causes fuel failure.
4. Determine the "types" of unbondedness that control the reactor failure rates and, for example, explain the observed interaction of total bond count and dimensional stability on the metal performance level.

Admittedly, this is an ambitious program. Some of the problems may prove to be difficult or, within reasonable limits of time, manpower and money, impossible to solve. Nevertheless, this study has shown a significant relationship between fuel performance and two measurable quality parameters. Failure to follow up this finding with a strong effort to determine the reasons behind the relationship and to find a practical set of standards by which the bond count parameter can be used as a powerful reject criterion is inconsistent with our goals of improving and understanding fuel performance.

DISCUSSION

I. General

Over the past several years there have been many occasions where poor rupture performance has been qualitatively related to bond quality. For example, the initial irradiation experience of X-8001 clad fuel elements in CY-1958 was relatively poor, and bonding difficulties were believed causal. Poor performance of multiple side failure lots has been traced to "more than normal" porosity in two instances.* Dimensional stability has likewise been believed to contribute to fuel performance, and a consistent relationship between warp and hot-spots has been observed in non-bumper fuel elements.⁽¹⁾

* By chance, several unirradiated pieces from lots which had incurred multiple side failures were available for destructive testing.

(1) HW-67120, "Analysis of Quality Certification Data - Effect of Warp on the Incidence of Hot-Spots", LT Hagie and JL Jaech, 10/14/60 (Unclassified).

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Samples of production fuel elements have been routinely measured and irradiated under the Quality Certification Program (PT-216) since mid-1959. The measurements are believed to physically portray "quality". The capability for quantifying fuel performance exists using the current rupture model.* This study was undertaken to attempt to relate measured quality parameters with observed reactor performance.

II. Data and Method of Analysis

The statistics used in this study consisted of average log total external bond count (LBC), average tube filling capacity (TFC), and the metal performance level parameter for side failures (q_{side}). Groups of fuel elements were defined by canning date periods, and each of the above statistics was estimated for the various groups. Canning dates covered the period from August 3, 1959 through April 19, 1961. This over-all period was broken down into 17 sub-periods under the arbitrary criteria that each sub-period be at least 25 days long and include at least four side failures. A total of 95 side failures, consisting of 77 side hot-spot and 18 side-other failures, were included in the analysis. Second failures from multiple failure lots were excluded. The data are summarized in Table I.

Tube filling capacity (TFC) was selected as the measure of dimensional stability, rather than warp. Theoretically, the former may more accurately represent the degree of process tube annulus restriction. Flow restrictions induced thereby, over and above those caused by fuel cocking, could be expected to play some part in the formation and relative severity of hot spots and hot areas. Clearly, this is an important phase in the mechanism producing side failures. The method of calculating tube filling capacity and the details of the square root transformation have been documented.^(2,3) Values of average TFC in this study ranged from 2.92 to 4.00 ($\sqrt{\text{mils}}$).

Total external bond count potentially measures any unbondedness on the external surface of the fuel element. Unbondedness includes "porosity", "brittle" or "broken" bonds, "non-wets" of various forms, etc. The degree and extent of these defects probably varies from time to time. To some

* The current rupture model was originally derived from side failure experience for solid fuel elements. Its adequacy for I&E fuel has been repeatedly tested and demonstrated over the past three years.

- (2) HW-70643, "Describing Fuel Element Distortion Using Orthogonal Polynomials", JL Jaech, 8/4/61 (Unclassified).
- (3) HW-67530, "Analysis of Quality Certification Data - Warp, TFC, and ΔOD_2 As A Function of Reactor Operating Conditions", LT Hagie and JL Jaech, 11/23/60, (Secret), Appendix B.

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TABLE I

FUEL PERFORMANCE AND QUALITY DATA

<u>Canning Dates</u>	<u>Average LBC</u>	<u>Average TFC</u>	<u>q_{side}</u>	<u>Number of Side Failures</u>
1959 - Aug 3 to Aug 27	1.98	2.97	0.233	6
Aug 28 to Sep 21	2.06	2.91	0.161	4
Sep 22 to Oct 16	1.89	3.33	0.357	9
Oct 17 to Nov 11	1.10	3.89	0.285	7
Nov 12 to Dec 6	1.05	3.71	0.226	6
Dec 7 to Dec 31	1.01	3.27	0.159	5
1960 - Jan 1 to Jan 25	1.19	3.32	0.160	6
Jan 26 to Feb 19	1.05	3.19	0.099	4
Feb 20 to Apr 9	1.15	3.04	0.058	5
Apr 10 to May 29	0.79	4.00	0.076	8
May 30 to Jun 23	0.85	3.90	0.071	4
Jun 24 to Aug 12	0.89	3.81	0.050	6
Aug 13 to Sep 6	1.03	3.40	0.072	5
Sep 7 to Oct 1	0.95	3.89	0.054	4
Oct 2 to Nov 20	0.90	3.59	0.038	6
Nov 21 to Dec 15	0.83	3.41	0.067	5
Dec 16 to 1961 Apr 19	0.84	2.92	0.013	5

extent, any or all of these types of defects are reflected in the total count data used in this study.* Total bond count data vary because of "true" product variation, and, for a constant product, will also vary because of tester variation.**

Figure 1 shows average log bond count data for Quality Certification charges plotted versus canning date. Normally, each point is the average of 32-38 natural uranium fuel elements from a single lot. Log bond count is routinely calculated in the PDQ program (computer program for Quality Certification). The basis for the logarithmic transformation is documented.⁽⁴⁾ These data show that short-term variation (within-a-day and day-to-day) is quite large. An estimate of how much of the variation is due to the product and how much is due to the testers cannot be made as the necessary data were not available for the subject time. Nonetheless, these data do indicate a clear response of total bond count to changes in the canning process. For example, on March 25, 1960, vibration frequency was markedly increased--the plot shows a corresponding reduction in bond count. An effect of pressure quenching is indicated by the downward trend in bond count during the conversion period (late May 1961 through late August 1961) and an apparent continuation at the reduced level. This leads us to believe that while we cannot separate tester and product variability on the short run for these historical data, the average bond count for many fuel elements, i.e., over a long period of time under the sampling plan that was followed, probably is largely a measure of product quality. Values of average LBC used in this study ranged from 0.79 to 2.06.

Over the period of time examined it is our understanding that there have been two major modifications to the bond testers, i.e., major from the standpoint of data analysis.

1. Prior to about August, 1960, the sound frequency was 20 megacycles; this frequency was then increased to 25 megacycles, the transition period for all testers being about four months. Most of the data of Figure 1 during this transition period were probably from 25 megacycle testers. Theoretically, tester sensitivity would be increased somewhat by this step. From about January, 1961, on, sound frequency has been maintained at 25 megacycles.

* Bond count data since August, 1961, are possibly different in this respect. Because of changes in tester set-up procedure, counts attributable to "brittle" bonds may have been reduced. See p. 9 for further discussion of this point.

** Over-all tester variability is made up of short-term variation within a tester, termed reproducibility, plus long-term variation within a given tester plus variation between testers. The latter two can be termed relative bias or relative accuracy.

- (4) HW-61394, "Basis for Logarithmic Transformation of Total Count Data", JL Jaech, 8/4/59 (Unclassified).

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Figure 1

AVERAGE LOG TOTAL BOND COUNT (EXTERNAL)
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2. Beginning about August, 1961, tester set-up procedure was modified so as to deliberately reduce tester sensitivity to counts "emanating" from so-called "brittle" bonds. There are no data, at this time, from which to judge the effect this may have had on the total count of the Quality Certification sample. This is of concern mainly from a performance prediction standpoint, since data subsequent to August, 1961, were not included in this study.

Reactor performance of the fuel elements is measured by the metal performance level parameter (q) for side failures. This parameter is estimated by dividing the observed number of side failures (R) by the change in rupture index (ΔRI).⁽⁵⁾

Failures classified as side hot-spot and side-other were combined. With the existing rupture classification system these two categories are frequently interchanged. Except for "classic" examples at the ends of the classification spectrum, past experience has shown a considerable overlap in what is called a hot-spot failure and what is not. This occurs primarily because there are many observers at many examination sites. Rupture classifications are, in general, more descriptive of the physical appearance of the piece than of the cause of failure, and viewing facilities are far from optimum. These factors make rupture classifications quite subjective. Some unknown fraction of side-other failures is truly caused by something other than accelerated hot-spot corrosion. The cause(s) of failure might be inclusions, groove or general corrosion, or end closure defects, to name a few. The fact remains, however, that a large proportion of non-ruptured fuel elements exhibit hot spots or hot areas (which themselves are difficult to differentiate). It follows, then, that a failure by any mechanism has a reasonable chance of being incorrectly classified, in the sense of using classification to imply failure mechanism, as a hot-spot rupture.

Further evidence of a common denominator within the side failure grouping is the fact that over the years the proportion of side failures classified as side hot-spots has been remarkably constant at about three out of four. Furthermore, this proportion has been independent of the performance level. In the author's opinion there is more to be gained than lost by grouping all side failures into one category for purposes of evaluating fuel performance. Values of q_{side} used in this study ranged from 0.013×10^{-39} to 0.357×10^{-39} .

Rupture index on a canning date basis is not directly computed by existing programs. For purposes of this study, this measure of severity of reactor service was approximated. Any error introduced by this approximation would have negligible effect upon the results since the largest error component in the estimate of q_{side} is the random error associated with the observed number of ruptures. The procedure used in approximating rupture index by canning date is discussed in Appendix A.

The quality and performance data summarized for this study were analyzed and correlated using the generalized regression analysis program, "George", for the IBM 7090 developed by Barbara B. Vinson of the Operations Research and Synthesis Operation, HLO.

(5) HW-65550, "Use of Control Charts for Analysis of Plant Rupture Experience", RR Bloomstrand, 6/9/60 (Secret)

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III. Results

It is concluded, from this study, that bond quality and dimensional stability strongly influence the side failure performance of non-bumper fuel elements. The following model was developed, from the data of this report, describing the relationship that existed:

$$q_{\text{side}} = 0.9758 - 1.2928(\text{LBC}) - 0.3630(\text{TFC}) + 0.4868(\text{LBC})(\text{TFC})$$

where q_{side} = metal performance level parameter for side failures, multiplied by 10^{39} .

LBC = average logarithm of total external bond count.

TFC = average tube filling capacity.

The relative effects of changes in LBC and TFC are illustrated in Table II, below.

TABLE II

<u>LBC</u>	<u>TFC</u>	<u>q_{side}</u>
2.0	2.75	.069
2.0	4.00	.832
0.9	2.75	.019
0.9	4.00	.112

When LBC is 2.0 (corresponds roughly to an average arithmetic total bond count of 175) q_{side} changes twelve-fold between TFC = 2.75 and TFC = 4.00. When LBC is 0.9 (an average arithmetic total bond count of 12) the same change in TFC produces a six-fold increase in q_{side} . Likewise for low TFC values, q_{side} is less sensitive to changes in the bond count level than at the higher TFC values. This results from the interaction between LBC and TFC.

The multiple correlation coefficient was high, 0.876, indicating that about 76 per cent of the observed variation in q is accounted for by the model. The model yields negative q values when LBC and/or TFC values are quite small. This is of little consequence, however, since the limits within which q would be expected to fall embrace positive values.* It is important to stress that the primary purpose of the analysis was to identify factor(s) affecting fuel performance, and this was certainly accomplished. The fact that LBC and TFC interact to affect fuel performance is, of itself, an important finding.

* See Appendix B for discussion on the choice of the model, confidence limits of the parameter estimates, comparison of observed and predicted values, etc.

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An effect of "bond quality" on rupture resistance can be inferred without the discipline of the statistical analysis. This is illustrated in Figure 2. On this chart q_{side} is plotted versus TFC for three periods of time that are clearly defined by known process changes or gross differences in measured attributes, i.e., changes that are so large and consistent that it is virtually inconceivable that they are not, in fact, real. Referring to the data of Table I, page 6, the time periods are: 1) August 3, 1959 through October 16, 1959; 2) October 17, 1959 through April 9, 1960; and 3) April 10, 1960 through April 19, 1961. Periods (1) and (2) are distinguishable on the basis of a gross difference in bond count. Period (3) begins, essentially, with a process change.* Notably, significant interaction between LBC and TFC is evident from inspection of the chart.

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* Vibration frequency increased markedly on March 25, 1960.

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APPENDIX A

ESTIMATION OF SEVERITY OF SERVICE

Adjustment of Rupture Index Values

Rupture index (ΔRI), a measure of severity of reactor service, is routinely computed on a monthly basis. The same formula is used for all process tubes. It can be shown,* for constant fuel quality, that the expected number of ruptures per unit ΔRI (q) will differ for various fuel types and loading configurations. These differences in q theoretically arise from differences in heat flux and surface temperature due to differences in charge length, fuel and tube geometry, R values, and axial flux distribution. The observed relationships between q 's for various fuel types agree with those predicted theoretically. This indicates that quality, over the short run, is constant between fuel types.

It is possible, then, to monitor quality changes more efficiently by adjusting or normalizing calculated rupture index (ΔRI) values for all fuel types to a common basis, and using total plant rupture experience.

For this study the following factors were used to adjust rupture index values to a common base:

<u>Type Fuel and Charge Length</u>	<u>ΔRI Adjustment* Factor</u>
O-Natural, 34 pcs.	1.0 (Base)
O-Natural, 32 pcs.	1.7
O-Enriched, 34 pcs.	6.3
O-Enriched, 43 pcs.	1.7
C-Natural, 32 pcs.	3.4
C-Enriched, 34 pcs.	15.1
K-Natural, 38 pcs.	0.41
K-Enriched, 46 pcs.	1.17

* Based essentially on observed rupture experience over the period from April 1960 through May 1961.

*Calculations not included in this report.

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Approximating Δ RI for Canning Date Periods

As mentioned above, rupture index is computed monthly. These values largely reflect the contributions of material discharged at goal exposure, because of the strong influence of exposure on the probability of incurring a rupture. Residence time to reach goal exposure is not constant for all tubes, therefore, the Δ RI data of a given month include material from a wide span of canning dates. The distribution of canning date-within-monthly Δ RI cannot be accurately determined. It can, however, be roughly approximated from the distribution of time between canning date and failure date of rupture because Δ RI is directly proportional to number of ruptures for constant quality. For this analysis, the distribution of canning date-within-monthly Δ RI was estimated from the time between canning and failure of about 130 ruptures over the period January 1960 through September 1961, as follows:

TABLE I

Time Between Canning Date and Failure Date (Months)		Frequency (%)
<u>Greater Than</u>	<u>Equal to or Less Than</u>	
1	2	5
2	3	23
3	4	38
4	5	20
5	6	10
6		4

This distribution represents the "average" condition over the entire period. During CY-1960 only, the distribution was skewed more toward the shorter residence times; during CY-1961 it was skewed more toward the higher residence times. Actually, considerable departure from the above distribution is tolerable before the results of this study would be significantly affected. This is clear from inspection of the monthly adjusted Δ RI data in Table II where it can be seen that the sliding cumulative summation of Δ RI, say for three month periods, does not change abruptly.

The distribution in Table I, and the data of Table II were used to approximate the Δ RI experienced in material canned during each of the 17 time periods used for this analysis. The results are shown in Table III.

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TABLE II

HISTORICAL Δ RI DATA
(Non-Bumper Fuel Only)

<u>Year and Month</u>		<u>Adjusted ΔRI</u>
1959	Oct	27
	Nov	29
	Dec	25
1960	Jan	25
	Feb	24
	Mar	21
	Apr	32
	May	42
	Jun	34
	Jul	43
	Aug	48
	Sep	61
	Oct	51
	Nov	58
	Dec	65
1961	Jan	95
	Feb	74
	Mar	61
	Apr	100
	May	72
	Jun	62
	Jul	102
	Aug	71
	Sep	62

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TABLE III

HISTORICAL ARI DATA BY CANNING DATES

	<u>Period</u>	<u>Approximate Adjusted ARI</u>
1959	Aug 3 to Aug 27	25.8
	Aug 28 to Sep 21	24.9
	Sep 22 to Oct 16	25.2
	Oct 17 to Nov 11	24.6
	Nov 12 to Dec 6	26.6
	Dec 7 to Dec 31	31.4
1960	Jan 1 to Jan 25	37.4
	Jan 26 to Feb 19	40.4
	Feb 20 to Apr 9	86.2
	Apr 10 to May 29	101.7
	May 30 to Jun 23	56.3
	Jun 24 to Aug 12	120.6
	Aug 13 to Sep 6	69.5
	Sep 7 to Oct 1	74.0
	Oct 2 to Nov 20	156.7
	Nov 21 to Dec 15	74.6
1961	Dec 16 to Apr 19	400.3

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APPENDIX BDEVELOPMENT OF THE MODEL

A series of models relating q , LBC, and TFC, were developed from the data of this report to attempt to quantify the influence of bond quality and dimensional stability on side rupture resistance. When more than one independent variable (x) is believed to influence some observed value (y), and there is no previous experience or physical law that can be used to determine the form of the model that should be used, the technique of multiple regression can be used to fit various models to the data, and the "best" model selected. The "best" model is not necessarily one that fits the data perfectly. For example, a quadratic equation can fit three data points exactly--perfect correlation. The equation, however, does not necessarily describe the underlying true relationship between the variables. Simply fitting a curve is, of itself, an incomplete analysis. Acceptance at face value can be entirely misleading because random errors or even the fact that there is a paucity of data, as in this example, is ignored.

From inspection of the data of this report, it seemed likely that one of the four following models would describe the relationship between q , LBC, and TFC.

Let $y = q$

$x_1 = \text{LBC}$

$x_2 = \text{TFC}$

- (1) $y = a_0 + a_1x_1 + a_2x_2$ (linear effects only)
- (2) $y = a_0 + a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_2^2$ (linear and quadratic effects)
- (3) $y = a_0 + a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_2^2 + a_5x_1x_2$ (linear and quadratic effects plus an interaction)
- (4) $y = a_0 + a_1x_1 + a_2x_2 + a_3x_1x_2$ (linear plus interaction)

Model (3), which tested the data for virtually everything, was rejected because most of the parameter estimates (the a 's in the model) were found to be not different from zero. The interaction parameter (a_5 in the model) was significant. As a matter of interest, the multiple correlation coefficient was high, >0.85 .

Model (2) was rejected on the same basis as Model (3); only the linear effect of LBC (a_1x_1) was significant.

Model (1), the simplest of the four, described the data quite well. All the parameter estimates were significant; a_1 was highly significant. The multiple correlation coefficient, however, was not particularly high, 0.71, considering there were only two independent variables.

Model (4) was the logical choice as being "best." Adding the interaction term to Model (1) raised the multiple correlation coefficient to 0.876, the highest of the group. The linear effects of LBC and TFC, and the interaction are all highly significant, i.e., the values of a_1 , a_2 , and a_3 have a small probability (<1%) of being zero.

The parameter estimates for Model (4) and their standard deviations are as follows:

$$a_0 = 0.9758 ; \sigma = 0.3913$$

$$a_1 = -1.2928 ; \sigma = 0.3674$$

$$a_2 = -0.3630 ; \sigma = 0.1215$$

$$a_3 = 0.4868 ; \sigma = 0.1184$$

Comparison of the observed and expected (those calculated by the model) values is shown in Table I.

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TABLE I

Point No.	x_1 x_2	Observed y	Expected y	Difference (O-E)	σ of Predicted y
1	1.98	0.233	0.200	0.033	0.029
	2.97				
2	2.06	0.161	0.174	-0.013	0.036
	2.91				
3	1.89	0.357	0.387	-0.030	0.038
	3.33				
4	1.10	0.285	0.224	-0.061	0.024
	3.89				
5	1.05	0.226	0.168	0.058	0.016
	3.71				
6	1.01	0.159	0.091	0.068	0.015
	3.27				
7	1.19	0.160	0.155	0.005	0.014
	3.32				
8	1.05	0.099	0.091	0.008	0.017
	3.19				
9	1.15	0.058	0.087	-0.029	0.019
	3.04				
10	0.79	0.076	0.041	0.035	0.028
	4.00				
11	0.85	0.071	0.075	-0.004	0.021
	3.90				
12	0.89	0.050	0.093	-0.043	0.017
	3.81				
13	1.03	0.072	0.114	-0.042	0.013
	3.40				
14	0.95	0.054	0.134	-0.080	0.018
	3.89				
15	0.90	0.038	0.081	-0.044	0.014
	3.59				
16	0.83	0.067	0.043	0.024	0.017
	3.41				
17	0.84	0.016*	0.024	-0.008	0.030
	2.92				

* Erroneously used in the machine computations. Value should have been 0.013.