1 of 2

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

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*Owen C. Jones, Professor and Principal Investigator
Center for Multiphase Research
Department of Nuclear Engineering and Engineering Physics
Rensselaer Polytechnic Institute
Troy, New York 12180-3590 U.S.A.*

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1. MAJOR CONTRIBUTORS

The following personnel have been major continuing contributors to this research effort during the past year:

- Dr. O.C. Jones, Professor of Nuclear Engineering and Engineering Physics
- Dr. J.T. Lin, Assistant Research Professor
- Ms. Yingchao He, graduate student in Nuclear Engineering
- Mr. L. Ovacik, graduate student in Electric Power Engineering
- Mr. Hongjun Shu, graduate student in Engineering Physics
- Mr. B. Dunsmoor, undergraduate student in Mechanical Engineering
- Mr. R. Parker, undergraduate student in Engineering Physics

2. CONTRIBUTORS

The following personnel have been principally involved in the various stages of this project:

- Dr. O.C. Jones, Professor of Nuclear Engineering and Engineering Physics, principal investigator and overall project coordinator.
- Dr. J.C. Newell, Professor of Biomedical Engineering and leader of the Rensselaer Biomedical Imaging Team, provided valuable assistance and information in the initial organization of the effort and in the early stages of the research.
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- Dr. J. Goble, Post Doctoral Research Associate provided assistance in initial image treatment especially using ImageTool.
- Dr. J.T. Lin, first as Post-Doctoral Research Associate and then as Assistant Research Professor has been the major lead contributor in the day-to-day operation of the program, in the development of the finite element model, the block decomposition method, and the overall numerical effort in the program.
- Dr. K. Kiymik, Visiting Scientist, was responsible for the frequency multiplexing work.
- Ms. Yingchao He, graduate student in Nuclear Engineering, is the lead student responsible for developing the back projection imaging methods.
- Mr. L. Ovacik, graduate student in Electric Power Engineering, is the lead student researcher in the finite element method and numerical imaging methods development effort.
- Mr. Hongjun Shu, graduate student in Engineering Physics, was responsible for designing and constructing the high accuracy imaging electronic system and is the lead student responsible for design and construction of the PACIFIC-1 high speed system.
- Mr. H. Al-Saidi, graduate student in Engineering Physics, was responsible for the capacitive imaging effort.
- Mr. Y. Danon, graduate student in Engineering Physics, was instrumental in assisting the set-up of our overall computational system network.
- Mr. B. Dunsmoor, undergraduate student in Mechanical Engineering, is the lead person responsible for design and construction of the three-dimensional phantoms.
- Mr. R. Parker, undergraduate student in Engineering Physics, is the person responsible for design and construction of the natural convection test.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer: Cooperative Research Program

A. Theoretical Development	A.1
1. basic theory— — finite element method	A.1
1.1. Newton— Raphson Method (Yorkey's Method)	A.1
1.2. A New Algorithm to Calculate the Jacobian Matrix ()	A.3
1.2.1 Storage and Computational Improvements	A.6
2. exponential block decomposition	A.7
2.1. Exponential Scheme (FEM)	A.7
2.1.1. Background	A.7
2.1.2. Analytical Development	A.8
3. image enhancement— — finite element method	A.12
3.1. Errors in Imaging	A.12
3.1.1. Error in modelling the forward problem	A.12
3.1.2. Error in the Jacobian Matrix calculations	A.13
3.1.3. Error in placement of nodal element boundaries	A.13
3.1.4. Error in experimental data	A.13
3.1.5. Computer truncation error	A.14
3.2. 2—Dimensional Error Correction Method for Image Enhancement	A.14
3.3. Three—Dimensional Error Correction Methods for Image Enhancement	A.16
3.3.1. Explanation of 3—D error (space distortion)	A.16
3.3.2. Experimental methods for eliminating the 3—D space distortion	A.18
3.3.3. Numerical methods for eliminating the 3—D space distortion	A.20
3.4. Nomenclature	A.21
4. basic theory— — back projection	A.24
4.1. Introduction	A.24
4.2. Theoretical Background	A.24
4.3. Transformation into u,v Space	A.25
4.4. Solution of Simplified Poisson Equation (15)	A.27
4.5. Backprojection	A.30
4.6. Conclusions	A.31
5. quasi—three—dimensional back projection	A.31
5.1. BASIC THEORY	A.31
5.2. ASSUMPTION	A.34
6. basic theory— — frequency multiplexing	A.35
6.1. Frequency Multiplexing for Fast Excitation and Acquisition of Signals	A.35
6.1.1. Conceptual Description	A.35
6.1.2. Fourier Decomposition	A.35

6.1.3. First Testing of the Method	A.37
6.1.4. Second tests of the method	A.38
6.1.5. Square–Wave Excitation	A.42
6.1.6. Summary	A.42
7. basic concept—neural networks	A.44
7.1. A Simulated Neural Network Approach for Electrical Impedance Imaging	A.44
7.1.1. Artificial Neural Networks	A.44
7.1.2. Application to Imaging	A.44
7.1.3. Results	A.45
B. Numerical Results	B.47
1. Basic Finite Element Method	B.47
2. Effect of Noise	B.47
3. Block Reconstruction	B.50
3.1. Results and Discussion	B.50
4. Incomplete Excitation Sets	B.52
C. Experimental Design—Mechanical	C.56
1. Design of 2–Dimensional Phantom	C.56
1.1. Two–Dimensional Phantom Concept	C.56
1.1.1 Phantom Geometry and Electrodes	C.56
1.1.2. Overall Concept and Justification	C.58
2. Design of 3–Dimensional Phantoms	C.58
2.1. Rectangular Geometry	C.58
2.2. Cylindrical Phantoms	C.59
2.2.1. Resistive Phantom—First Design	C.59
2.2.2. Resistive Phantom—Second Design	C.60
2.2.3. Capacitive Phantom—First Design	C.60
2.2.4. Capacitive Phantom—Second Design	C.60
2.3. Back Projection Geometry	C.65
3. Natural Circulation Test	C.66
3.1 Tank Design	C.66
3.2. Heater Control Design	C.67
3.3. Experience	C.68
D. Experimental Design—Electronic	D.69
1. High Accuracy System	D.69
1.1 Static System for High Accuracy Imaging	D.69
1.1.1 Experimental System Design	D.69
1.1.2 Detailed Circuit Diagrams and PC Board Layouts	D.70
1.1.4 Computer Interfacing	D.79
1.1.5 Phase–sensitive measurement	D.80
1.1.6 Accuracy	D.80
1.1.7 Software	D.80

2. PACIFIC-1 System	D.82
2.1. PACIFIC-I System Overview	D.83
2.2. Preliminary Specifications of PACIFIC-I	D.83
3. Design of parallel, applied current imaging system with fast integrated components - PACIFIC-1	D.85
3.1. Introduction	D.85
3.2. Preliminary Specifications Overview of PACIFIC-I	D.86
3.3. Implementation of a Digital Step Function Generator	D.87
3.4. The TMS320C25 DSP	D.88
3.5. The Analog-to-Digital Converter	D.88
3.6. Digital-to-Analog Converter	D.92
3.7. High precision voltage reference	D.92
3.8. Signal conditioning and Voltage-to-current converter Circuit	D.93
3.9. Negative impedance compensation	D.95
3.10. Optoisolation	D.96
3.11. Wiring and Cable	D.97
3.11.1. Coax cable	D.97
3.11.2. Triax cable	D.97
3.12. Shield Driver	D.97
E. Experimental Results	E.100
1. Basic 2-dimensional resistive imaging using the finite element model: point electrodes	E.100
1.1. Comparing Reconstructed Images of Different Number of Excitations	E.100
1.2. Image Improvement	E.103
1.3. Comparing Reconstructed Images of Three Different Types of Excitation Patterns	E.105
2. Basic 2-dimensional resistive imaging using the finite element model: point-plate electrode comparisons	E.111
2.1. Introduction	E.111
2.2. Modeling the Phantom	E.112
2.3. Numerical Sensitivity	E.112
2.4. Modeling of Plate Electrodes	E.116
3. basic 3-dimensional imaging--fem	E.121
3.1. 3-D Imaging by 2-D Scanning Technique	E.121
3.1.1. The Experimental Results and Conclusions	E.121
4. quasi 3-dimensional imaging--back projection	E.127
F. Summary and Recommendations	F.129
1. Summary	F.129
2. Recommendations	F.133
G. Bibliography	G.135
H. Appendix 4: Publications	H.148

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Annual Technical Report for Program Renewal

A. THEORETICAL DEVELOPMENT

This part of the report summarizes the analytical methods utilized and developed to obtain impedance images from boundary-excited fields of variable conductivity. Section 1 summarizes the basic theory of the finite element method. First, the Newton-Raphson finite element method (FEM) developed by Yorkey is described in Sect. 1.1 followed by a significant new development which eliminates the need to compute the Jacobian in order to obtain the Hessian matrix (Sect. 1.2). Section 2 then outlines several innovations which, together, have both increased the accuracy of the FEM used (exponential shape functions) and reduced the time required to undertake computations by more than three orders (block decomposition). Section 3 describes errors involved in impedance imaging and methods developed to improve the basic images obtained from the FEM exponential block decomposition method. In Sect. 3.1, various sources of error are identified and discussed. Methods of error correction are developed and discussed including that for 2-D imaging (Sect. 3.2), image enhancement for 3-D imaging correcting for both voltage and current distortions (Sect. 3.3). Backprojection theory is outlined in Sects. 4 and 5 including both two-dimensional methods and quasi-three-dimensional applications. The use of multiplexing as a possible method to speed the data excitation and acquisition process is considered and basic theoretical methods are developed in Sect. 6. Finally, Sect. 7 describes the use of neural nets as an alternative approach to obtaining images from impedance data.

1. BASIC THEORY--FINITE ELEMENT METHOD

1.1. Newton-Raphson Method (Yorkey's Method)

This form of the Newton-Raphson algorithm is an iterative reconstruction technique specifically developed for inverse non-linear problems.¹⁵⁷ This algorithm has been recognized as one of the most theoretically accurate reconstruction algorithm used in EIT.

Consider the following governing equations from Maxwell's set:

$$\nabla \cdot \left(\sigma \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t} \right) = 0 \quad (1)$$

with σ the conductivity, $\mathbf{E}$ is the electric field, t is the time, and where we let

$$\mathbf{E} = \mathbf{E}_0 e^{j\omega t} \quad (2)$$

with ω the rotational frequency. We now note that

$$\partial \mathbf{E} / \partial t = j\omega \epsilon \mathbf{E} \quad (3)$$

and for a conservative field

$$\mathbf{E} = -\nabla \Phi \quad (4)$$

so that we obtain Poisson's equation for the field potential Φ as

$$\nabla \cdot (\sigma + j\omega \epsilon) \nabla \Phi = 0. \quad (5)$$

The equivalent network of field equations obtained using Kirchoff's law is

$$\mathbf{Y}_{N \times N} \mathbf{V}_{N \times P} = \mathbf{C}_{N \times P} \quad (6)$$

where $\mathbf{Y}$ is the admittance matrix, $\mathbf{V}$ is the voltage matrix, and $\mathbf{C}$ is the current matrix. Also, N is the number of total nodes, M is the number of total elements, P is the number of total current excitations, and E is the number of total measured electrodes. Following Yorkey's algorithm, we define an error function ϕ

$$\phi(\sigma) = (1/2) [\mathbf{f}(\sigma) - \mathbf{V}_0]^T [\mathbf{f}(\sigma) - \mathbf{V}_0] \quad (7)$$

with $\mathbf{V}_0$ and $\mathbf{f}(\sigma)$ the measured and the estimated (computed) electrode voltages respectively for a resistivity distribution σ , and T denotes transpose of a matrix. To find a σ -distribution which minimizes $\phi(\sigma)$, we let the derivative of the error function vanish. Thus,

$$\phi'(\sigma) = [\mathbf{f}'(\sigma)]^T [\mathbf{f}(\sigma) - \mathbf{V}_0] = 0 \quad (8)$$

where

$$\sigma_{1 \times M} = [\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_m, \dots, \sigma_M]. \quad (9)$$

The differential of the computed boundary electrode voltages is defined as

$$\mathbf{f}'(\sigma) = \frac{d\mathbf{f}(\sigma)}{d\sigma} \quad (10)$$

where $\mathbf{f}'$ is called the Jacobian matrix ($\mathbf{J}$).

The second derivative

$$\phi''(\sigma) = [\mathbf{f}'(\sigma)]^T \mathbf{f}''(\sigma). \quad (11)$$

We take the Taylor series expansion of $\phi(\sigma)$ around a point σ^k and keep the linear terms

$$\phi'(\sigma) \approx \phi'(\sigma^k) + \phi''(\sigma^k)\nabla(\sigma^k) = 0 \quad (12)$$

(where ϕ'' is called the Hessian matrix) and find the updating equation for σ

$$\Delta\sigma^k = - [f'(\sigma^k)^T f'(\sigma^k)]^{-1} f'(\sigma^k)^T [f(\sigma^k) - V_o]. \quad (13)$$

We have now derived an iterative procedure which estimates σ^k . At iteration k we have an estimate of σ^k and updated σ^k so that

$$\sigma^{k+1} = \sigma^k + \Delta\sigma^k \quad (14)$$

The correction to σ^k can be obtained after every iteration, until the convergence criteria is met. The fact that σ minimizes is guaranteed by the positive definiteness of the approximate Hessian matrix. This method, in fact, forms the basis for the reconstruction methods developed in this project.

This method forms the basis for the imaging system. The overall system uses a forward computation of potential distribution based on assumed conductivity distribution in each element and the specified boundary conditions, and a reverse computation of changes in conductivity for each element based on Eq. (14) which reduces the error between the measured and computed boundary response.

Direct application of these methods requires lengthy computation and significant resources in terms of memory, so much so that imaging of anything greater than a 16 x 16 array of elements

1.2. A New Algorithm to Calculate the Jacobian Matrix ($J^T \cdot J$)

The majority of time required in the computations is associated with computing the elements of the Jacobian matrix after each iteration. A substantial reduction in computational effort results from a new method developed which eliminates computation of the Jacobian matrix itself.

Defining matrices **A** as the square of the Jacobian

$$A \equiv [J_{M \times EP}^T * J_{M \times EP}]_{M \times M} \quad (15)$$

and **B** as

$$B \equiv -J_{M \times EP}^T [f(\sigma^k)_{EP \times 1} - V_{o_{EP \times 1}}] \quad (16)$$

Then Eq. (13) will become

$$A_{M \times M} d\sigma_{M \times 1}^k = B_{M \times 1}. \quad (17)$$

The correction term for the conductivity/permittivity of each element for k^{th} iteration can be solved by

$$d\sigma_{M \times 1}^k = A_{M \times M}^{-1} \cdot B_{M \times 1} \quad (18)$$

if the matrices A and B can be calculated Equation (18) shows that the Jacobian itself is not needed for the inverse calculation, just its square. If the square of the Jacobian, A , can be computed directly, considerable time and storage can be saved in the calculations.

Recall that the Jacobian matrix is defined by

$$J^k \equiv \frac{\partial f^k}{\partial \sigma} \quad (19)$$

with the value for the m^{th} conductivity element,

$$[J_{EP \times M}]_m = \frac{\partial f_{EP \times 1}}{\partial \sigma} \quad (20)$$

Since Eq. (19) is a derivative of one matrix with respect to another, the Jacobian matrix consists of M vectors given by

$$J_{EP \times M} = [J_{1_{EP \times 1}}, J_{2_{EP \times 1}}, \dots, J_{m_{EP \times 1}}, \dots, J_{M_{EP \times 1}}] = \quad (21)$$

$$\left\{ Tran_{NP \rightarrow EP} * \left[vec \left(- Y_{NxN}^{-1} \frac{\partial Y_{NxN}}{\partial \sigma_m} V_{NxP} \right) \right]_{NxP} \right\}_{NP \times 1}_{EP \times 1}$$

where $Tran_{NP \rightarrow EP}$ is the transition matrix which transforms from the matrix of size $N \times P$ size of all nodes, N , for all current excitations, P , into the matrix of size ExP having all measured electrodes for all current excitations.

$$\left[Y_{NxN}^{-1} \frac{\partial Y_{NxN}}{\partial \sigma_m} V_o \right] = \begin{bmatrix} y'_{m_{1,1}} V_{1,j} + y'_{m_{1,2}} V_{2,j} + \dots + y'_{m_{1,N}} V_{N,j} \\ y'_{m_{2,1}} V_{1,j} + y'_{m_{2,2}} V_{2,j} + \dots + y'_{m_{2,N}} V_{N,j} \\ \vdots \\ \sum_{\beta=1}^N y'_{nl(i)\beta} V_{\beta,j} \\ \vdots \\ y'_{m_{N,1}} V_{1,j} + y'_{m_{N,2}} V_{2,j} + \dots + y'_{m_{N,N}} V_{N,j} \end{bmatrix} \quad (22)$$

or in more compact form

$$\left[Y_{NxN}^{-1} \frac{\partial Y_{NxN}}{\partial \sigma_m} V_o \right] = \begin{bmatrix} \sum_{\alpha=1}^N y_{1,\alpha}^{-1} \sum_{\beta=1}^N y'_{m_{\alpha\beta}} V_{\beta,j} \\ \sum_{\alpha=1}^N y_{2,\alpha}^{-1} \sum_{\beta=1}^N y'_{m_{\alpha\beta}} V_{\beta,j} \\ \vdots \\ \sum_{\alpha=1}^N y_{N,\alpha}^{-1} \sum_{\beta=1}^N y'_{m_{\alpha\beta}} V_{\beta,j} \end{bmatrix}. \quad (23)$$

The element of the Jacobian matrix $\mathbf{J}$ is to pick up node electrodes from the matrix of $[Y^{-1}(dY/d\sigma_m)V]_{NP \times 1}]_{NP \times M}$ so that

$$\mathbf{A} \equiv [\mathbf{J}_{M \times EP}^T * \mathbf{J}_{EP \times M}]_{M \times M} = \mathbf{J}^2. \quad (24)$$

Therefore, element $-i, j$ of the square of the Jacobian is

$$(\mathbf{J}^2)_{(i,j)} = \sum_{e=1}^{EP} \mathbf{J}_{e,i}^* \mathbf{J}_{e,j} \quad (25)$$

where e is the index for excitation and electrodes given by $(e_a - 1) * P + e_b$ where e_a is the electrode index, from 1 to E and e_b is the excitation index, from 1 to P . Equation (25) may be written as

$$(\mathbf{J}^2)_{(i,j)} = \sum_{e=1}^{EP} \left(Y^{-1} \frac{\partial Y}{\partial \sigma_i} V \right)_e \left(Y^{-1} \frac{\partial Y}{\partial \sigma_j} V \right)_e \quad (26)$$

and, after further expansion

$$(\mathbf{J}^2)_{(i,j)} = \sum_{e=1}^{EP} \left[\sum_{\alpha=1}^N y_{nl(e),\alpha}^{-1} \sum_{\beta=1}^N y'_{i_{\alpha\beta}} V_{\beta,e_i} \right]_e \left[\sum_{\alpha=1}^N y_{nl(e),\alpha}^{-1} \sum_{\beta=1}^N y'_{j_{\alpha\beta}} V_{\beta,e_j} \right]_e. \quad (27)$$

Here, $nl(e_a)$ is the node number of electrode index e_a , $ni(\alpha)$ and $ni(\beta)$ are node numbers of the neighborhood of element i , where α and β are dummy variables from 1 to 4, $nj(\alpha)$ and $nj(\beta)$ are node numbers of the neighborhood of element j , where α and β are also dummy variables from 1 to 4. Note too that $V_{ni(\beta),e_b}$ is the voltage on the node of $ni(\beta)$ for excitation e_b , $V_{nj(\beta),e_b}$ is the voltage on the node of $nj(\beta)$ for the excitation e_b , $y'_{j_{ni(\alpha),ni(\beta)}}$ is the derivative of the admittance from node $ni(\alpha)$ to node $ni(\beta)$ with respect to the conductivity of element i , $y'_{j_{nj(\alpha),nj(\beta)}}$ is the derivative of the admittance from node $nj(\alpha)$ to node $nj(\beta)$ with respect to the conductivity of element j , $y'_{j_{nl(e_a),ni(\alpha)}}$ is the element having the coordinates [node of the electrode e_a , node on $ni(\alpha)$] in the matrix Y^{-1} .

Combining and rearranging results in

$$(J^2)_{(i,j)} = \sum_{e=1}^{EP} \left\{ \left[\sum_{a=1}^4 y_{nl(e_a),ni(\alpha)}^{-1} \sum_{\beta=1}^4 y'_{i_{n(\alpha),n(\beta)}} V_{ni(\beta),e_b} \right] \cdots \right. \\ \left. \cdots \left[\sum_{a=1}^4 y_{nl(e_a),nj(\alpha)}^{-1} \sum_{\beta=1}^4 y'_{j_{n(\alpha),n(\beta)}} V_{nj(\beta),e_b} \right] \right\}_e \quad (28)$$

where

$$B_{M \times 1} \equiv -J_{M \times EP}^T [f(\sigma^k)_{EP \times 1} - V_{o_{EP \times 1}}] = (B_j)_{M \times 1} \quad (29)$$

and

$$B_j = \sum_{e=1}^{EP} (j_e - V_e) \left[\sum_{a=1}^4 y_{nl(e_a),nj(\alpha)}^{-1} \sum_{\beta=1}^4 y'_{j_{n(\alpha),n(\beta)}} V_{nj(\beta),e_b} \right]_e \quad (30)$$

Note too that

$$y'_{m(i,j)} \equiv \text{element of} \left[\left(\frac{\partial Y_{N \times N}}{\partial \sigma_m} \right)_{N \times N} \right]_{(i,j)} \quad (31)$$

with

$$J_{e,i} \equiv \left(Y^{-1} \frac{\partial Y}{\partial \sigma_i} V \right)_e \quad (32)$$

1.2.1 Storage and Computational Improvements

From Eq. (25)–(30) the matrix $A_{M \times M}$ and matrix $B_{M \times 1}$ can be constructed directly without storing the Jacobian matrix, which is an array of size $EP \times M$. For an 8 x 8 finite element structure with 50 excitations, the Jacobian matrix will be 32 x 50 x 64 and consume 1.64MB computer memory. This is 25 times larger than A which is only a 64 x 64 array. Alternatively, for a 16x16 array with 50 excitations, $J_{EP \times M}$ will occupy 14.8 MB of computer memory which is 11 times larger than that required for A which has a size of 289 x 289, occupying 1.34 MB. (Note that each computer word at double precision occupies 16 bytes.) When high noise level exists, more than four current excitations are desirable and so memory requirements will be even larger. The computational effort for constructing the symmetric matrix A is much smaller than for J^2 and computer resources also much less than for direct calculation of the Jacobian.

For the best, or optimum, current excitations, the computational effort for the two methods is of the same order. However, the new algorithm requires significantly (order of magnitude) less computer storage and shows no degradation in the convergence relative to the direct computation utilizing the Jacobian.

2. EXPONENTIAL BLOCK DECOMPOSITION

As was stated in the previous section, application to real problems of the Newton–Raphson method applied so successfully by Yorkey would require prohibitive computational resources to obtain reasonable resolution of images. In this program, however, we have completed a major new development which circumvents the majority of the problem. This method we term “exponential block decomposition” or EBD.

Exponential block decomposition is really a combination of two separate new developments:

11. Block decomposition which undertakes outer and inner iterations in a way which reduces computational effort by approximately three orders of magnitude;
12. Exponential shape functions for the potential and conductivity fields to allow the conductivity and its first spatial derivative to vary continuously between elements.

In this way, the accuracy and resolution of fine-mesh reconstruction could be obtained with a relatively coarse, rapidly converging element structure.

2.1. Exponential Scheme (FEM)

2.1.1. Background

The exponential or exact numerical scheme developed by Patankar et. al.¹¹⁷ used for forward solutions is adapted for the solution of this inverse problem in finite element form. The method is known for use in obtaining solutions which eliminate effects of false diffusion in finite element computations. The result is used to describe real physical solutions in reconstructing the images of electrical impedance computed tomography.

The problem of solving the electrical impedance equation is governed by differential equations of the form

$$\nabla \cdot \sigma \nabla \phi = 0 \quad (33)$$

where ϕ represents the electrical potential field and σ represents the complex conductivity distribution. To solve this equation in the forward direction given the conductivity distribution and the boundary values, it is frequently necessary to obtain a numerical simulation of ϕ by finite-differencing or using finite-elements. Numerical solutions of the resulting set of algebraic equations are difficult to obtain because of a numerical instability that occurs when the first derivative term in the equation, $\nabla \phi$, dominates the transport of ϕ by diffusion. The problem shows up as unrealistic oscillatory solutions whenever the mesh size exceeds a critical value. Overcoming the problem is often computationally expensive.

Researchers have used both upwind and hybrid schemes to overcome problems of instability. However, such schemes suffer from severe false diffusion, especially when properties vary in a direction at an angle to the grid lines.

Numerical prediction of practical, false-diffusion phenomena started receiving widespread attention of research workers in the 1960s. Early investigations in this area employed finite-difference methods or finite element methods based on the central difference scheme or bilinear shape function; therefore, they were limited to pure diffusion problems. With a strong property variation or strong convective transport, a special numerical scheme is necessary. Patankar and his coworkers^{4,117,123} present the exponential or exact numerical scheme for both finite element and finite difference methods.

Finite difference schemes are suited to computational meshes produced by grid lines that are parallel to Cartesian coordinate axes or to some other regular coordinate system. Therefore, they are limited to regular-shaped domains. It is possible that extensions of these schemes to rotational curvilinear coordinates would allow their application to irregularly-shaped domains, but such extensions get rather complicated when highly irregular and multiply connected domains are encountered. Domains involving such complexities are best handled by finite-element methods as undertaken herein.

Conventional Galerkin, finite-element methods for first derivative-dominant problems experience difficulties similar to those that afflict central difference schemes. To overcome these problems, a streamline upwind scheme was used by Hughes.⁷¹ A streamline exponential scheme was also proposed by Raithby,¹²³ and subsequently extended by Baliga and Patankar,⁴ Ramadhyani and Patankar,¹²⁴ and by Hookey et. al.⁵⁸

In the method developed herein, termed "block decomposition," exponential shape functions are utilized consistent with the requirement for an exact, separable solution to Poisson's equation. This method allows internal, exponentially-shaped conductivity variations to be determined in each cell, significantly reducing the total number of cells required to depict the overall conductivity field.

2.1.2. Analytical Development

Beginning with Poisson's equation for a variable-conductivity field,

$$\nabla \cdot (\sigma \nabla \phi) = 0 \quad (34)$$

we multiply Eq. (34) with an arbitrary but continuous function W and integrate over the domain of element-e, Ω_e , to obtain the following equation:

$$\int_{V(\Omega_e)} W \nabla \cdot (\sigma \nabla \phi) dV = 0 \quad (35)$$

In the weighted-residual methods, the parameters of the approximation are determined such that equation (35) is satisfied for every independent choice of the function W . By partial integration over the element-e, a weak form of (35) becomes

$$\int_{V(\Omega_e)} W \nabla \cdot (\sigma \nabla \phi) dV = \int_{V(\Omega_e)} \nabla \cdot (W \sigma \nabla \phi) dV - \int_{V(\Omega_e)} \nabla W \cdot (\sigma \nabla \phi) dV \quad (36)$$

which vanishes due to the vanishing of the original Eq. (34). Using Gauss' theorem,

$$\int_{V(\Omega_e)} W \nabla \cdot (\sigma \nabla \phi) dV = \int_{S(\Omega_e)} (W \sigma \nabla \phi) dA - \int_{V(\Omega_e)} \nabla W \cdot (\sigma \nabla \phi) dV = 0. \quad (37)$$

Since Eq. (37) vanishes identically,

$$\int_{V(\Omega_e)} \nabla W \cdot (\sigma \nabla \phi) dV = - \int_{S(\Omega_e)} W q^e \cdot dA \quad (38)$$

where the current flux on the boundary is defined as $q^e \equiv -\sigma \nabla \phi$. The potential $\phi(x, y)$ inside the element "e" can be represented by the voltages V_1, V_2, V_3, V_4 on nodes at the corner of the rectangular element by

$$\phi(x, y) = f_1 V_1 + f_2 V_2 + f_3 V_3 + f_4 V_4 = \sum_{m=1}^{m=4} f_m V_m \quad (39)$$

where f 's are the interpolation functions. In the Rayleigh-Ritz method, the weight function W_l is determined as required to minimize the numerical error in closure of Eq. (34). In this case, $W_l = f_l$. Since $dV = dx dy * 1$,

$$\int_{V(\Omega_e)} \nabla W_l \cdot (\sigma \nabla \phi) dV = \int_{V(\Omega_e)} \left(\frac{\partial W_l}{\partial x} i + \frac{\partial W_l}{\partial y} j \right) \cdot \left(\sigma \frac{\partial \phi}{\partial x} i + \sigma \frac{\partial \phi}{\partial y} j \right) dx dy \quad (40)$$

which is

$$\int_{V(\Omega_e)} \nabla W_l \cdot (\sigma \nabla \phi) dV = \int_{V(\Omega_e)} \left(\frac{\partial W_l}{\partial x} \sigma \frac{\partial \phi}{\partial x} + \frac{\partial W_l}{\partial y} \sigma \frac{\partial \phi}{\partial y} \right) dx dy. \quad (41)$$

Writing the potential $\phi(x, y)$ in summation form yields

$$\int_{V(\Omega_e)} \nabla W_l \cdot (\sigma \nabla \phi) dV = \int_{V(\Omega_e)} \left[\sigma \frac{\partial W_l}{\partial x} \sum_{m=1}^{m=4} \left(\frac{\partial f_m}{\partial x} V_m \right) + \sigma \frac{\partial W_l}{\partial y} \sum_{m=1}^{m=4} \left(\frac{\partial f_m}{\partial y} V_m \right) \right] dx dy. \quad (42)$$

Defining an admittance matrix, K , then allows (42) to be written as

$$\int_{V(\Omega_e)} \nabla W_l \cdot (\sigma \nabla \phi) dV = \sum_{m=1}^{m=4} (K_{lm} V_m) \quad (43)$$

where

$$K_{lm} \equiv \int_{V(\Omega_e)} \left[\sigma \frac{\partial W_l}{\partial x} \left(\frac{\partial f_m}{\partial x} \right) + \sigma \frac{\partial W_l}{\partial y} \left(\frac{\partial f_m}{\partial y} \right) \right] dx dy. \quad (44)$$

If we choose W_l as f_l then

$$K_{lm} \equiv \int_{V(\Omega_e)} \left[\sigma \frac{\partial f_l}{\partial x} \left(\frac{\partial f_m}{\partial x} \right) + \sigma \frac{\partial f_l}{\partial y} \left(\frac{\partial f_m}{\partial y} \right) \right] dx dy \quad (45)$$

and the result is

$$K_{lm} V_m = I_l \quad (46)$$

Note that

$$I_l \equiv - \int_{A_e} (W_l q^e) \cdot d\vec{A} \quad (47)$$

where K_{lm} is an element of the stiffness matrix. This element also satisfies Kirchhoff's law with the voltages V_m of the nodes at the corner of the element-e and the net current I_l at the nodes into the element-e.

Since the exact solution for the two-dimension case of Eq. (34) is very complicated for irregular geometry, the two dimensional equation can be made one-dimensional. In one dimensional form, the governing equation, Eq. (1), becomes

$$\frac{d^2 \phi}{dx^2} + \frac{\sigma_x}{\sigma} \frac{d\phi}{dx} = 0 \quad (48)$$

which is one equation obtained for the separable problem. That is, if the conductivity distribution is assumed to be exponential in form, than Eq. (34) is separable with the previous result. The solution of equation (48) is

$$\phi(x) = D_0 + D_1 \int e^{-\int \frac{\sigma_x}{\sigma} dx} dx \quad (49)$$

where D_0 and D_1 are integration constants. For the exponentially-fitted conductivity distribution,

$$-\frac{\sigma_x}{\sigma} \equiv g_x \quad (50)$$

which is a constant inside the element. Therefore, the shape function for the one-dimensional solution is

$$\phi(x) = C_0 + C_1 e^{g_x x} \quad (51)$$

The approximation solution for two-dimension element according to Patankar et. al.^{4,117,123} is

$$\phi(x, y) = C_0 + C_1 e^{g_x x} + C_2 y + C_3 e^{(g_x x + g_y y)} \quad (52)$$

where r is the direction of the maximum directional derivative σ_r and n is the direction which is normal to the direction r , and both x and y are Cartesian coordinates.

We can now solve for the four unknowns C_1, C_2, C_3, C_4 to match the four given positions at node 1, node 2, node 3, node 4 of the element and given voltage V_1, V_2, V_3, V_4 . Therefore

$$\phi(x, y) = f_1 V_1 + f_2 V_2 + f_3 V_3 + f_4 V_4 = \sum_{m=1}^{m=4} f_m V_m \quad (53)$$

where f_1, f_2, f_3, f_4 can be solved in terms of x, y, r, n .

The solutions in the Cartesian coordinates obtained are:

$$1. \quad f_1 = \left(1 - \frac{\hat{\xi}}{\hat{a}}\right) \cdot \left(1 - \frac{\hat{\eta}}{\hat{b}}\right) \quad (54)$$

$$2. \quad f_2 = \frac{\hat{\xi}}{\hat{a}} \cdot \left(1 - \frac{\hat{\eta}}{\hat{b}}\right) \quad (55)$$

$$3. \quad f_3 = \frac{\hat{\xi}}{\hat{a}} \cdot \frac{\hat{\eta}}{\hat{b}} \quad (56)$$

$$4. \quad f_4 = \left(1 - \frac{\hat{\xi}}{\hat{a}}\right) \cdot \frac{\hat{\eta}}{\hat{b}} \quad (57)$$

where

$$\hat{x} = e^{g_x x} \quad (58)$$

and

$$\hat{a} = \hat{x}_2 - \hat{x}_1 \quad (59)$$

and

$$\hat{\xi} = \hat{x} - \hat{x}_1 \quad (60)$$

and

$$\hat{y} = e^{g_y y} \quad (61)$$

and

$$\hat{b} = \hat{y} - \hat{y}_1 \quad (62)$$

and

$$\hat{\eta} = \hat{y} - \hat{y} \quad (63)$$

and where

$$x \in [x_1, x_2] \quad (64)$$

and

$$y \in [y_1, y_2]. \quad (65)$$

From Eq. (45) we can calculate admittance matrix, Y_{lm} which is identical to the stiffness matrix K_{lm} , and a set of algebraic equations can be obtained for

$$Y V = I \quad (66)$$

From this point, the process is identical to that described previously.

3. IMAGE ENHANCEMENT--FINITE ELEMENT METHOD

In EIT imaging we compare the measured boundary voltages to the computed boundary voltages and minimize the difference of these voltage values by changing the discrete element conductivities of elements. When the squared sum of the differences (the method of least squares minimization) reaches a minimum then the image is obtained from the computed conductivities. In the ideal case, the minimum of this sum is zero. In practice, however, the global error does not reach to zero because of the noise in both experiments and computations.

3.1. Errors in Imaging

Basically, we can categorize the source of error into at least the following groups: error in modelling the forward problem; error in the Jacobian Matrix calculations; error in placement of nodal element boundaries; error in experimental setup and measurement; computer truncation error.

3.1.1. Error in modelling the forward problem

This error arises when we approach the forward problem by numerical methods (FEM in our case). More accurate solutions are obtained using higher order shape functions as proposed and implemented in Sect. 2. Another way to increase the accuracy is to increase the number of elements. However, an increase in the number of elements also increases the number of unknown conductivities, causing large storage requirements for the inverse solution step (storage of the Jacobian Matrix) and drastically increasing CPU time in the computations. Larger number of elements also increases the numerical errors involved.

Also, proper modelling of the electrode geometries has a significant effect on the accuracy of the forward solution. The computational model utilized herein is completely based on a point-voltage-

current model which can be approximated in experiments by small-area electrodes. Best sensitivity, however, is obtained by plate electrodes which are, however, difficult to model in the finite element method. While progress is being made in the latter area, no results have yet been forthcoming and so the method is not delineated in this report.

3.1.2. Error in the Jacobian Matrix calculations

Considering Eq. (21), the Jacobian Matrix is calculated using the elements of the inverse of the global admittance matrix, Y^{-1} , the potentials of the nodes, V , and the derivative of the global admittance matrix $\partial Y / \partial \sigma$. Therefore, the error in the Jacobian matrix basically comes from two sources. One part of the error is introduced by the inaccurate modelling the forward solution (error in the admittance network). The other part comes from the numerical error when inverting the large-sized admittance matrix.

The first part of the error may be corrected by the proposed methods to improve the forward problem modelling above. The second part can only be corrected by using robust matrix inversion algorithms and using high precision floating numbers.

3.1.3. Error in placement of nodal element boundaries

At best, the finite element result is an approximation to the real field distribution of conductivities. Obviously, discontinuities in the conductivity would result in perturbations in the electrical field and have an effect on the boundary response to the excitation. With constant-conductivity elements, exact coincidence of the element boundaries with the field discontinuities would yield significantly improved results over the case where the two did not coincide. This, in fact, was the basis for the high degree of accuracy of Yorkey's numerical experiments. Lack of this coincidence would distort the computed field relative to the actual field and increase the errors.

Use of an exponentially-varying conductivity field can significantly improve these types of errors by allowing the exponential shape function automatically approximate the discontinuous behavior. Thus, if a field boundary is located within an element of the model, the exponential shape would try to rapidly change the conductivity. The difficulty with this is that under certain circumstances, overflows or under-flows can occur leading to non-convergence of the result. While this latter difficulty has not been a problem in modeling of square geometries using square elements, it has proven a source of difficulty in modeling non-square geometries with quadrilateral elements.

3.1.4. Error in experimental data

This error is related to a number of factors including the digital measurement equipment and measurement techniques used, and the accuracy with which the experimental apparatus is constructed and modeled. The number of digits of measurement and the internal impedance of the voltmeter has a significant role on the measurement accuracy. Placement of electrodes on the boundary must also be done with care so that the numerical model and the experimental model coincide with a reasonable degree of accuracy.

Another source of error is electromagnetic interference which leads to noise on the signal. Shielding the experimental setup and the data acquisition system may prevent or reduce such radio interference noise in the data.

3.1.5. Computer truncation error

Computer truncation error can be reduced by using high precision floating numbers. In our reconstruction programs we use double precision floating numbers (16 bytes for complex and 8 bytes for real numbers). We have also developed and used algorithms specifically to reduce some of this error.

3.2. 2-Dimensional Error Correction Method for Image Enhancement

In this section, a numerical method correcting the error in the 2-dimensional forward solution is described. As derived from Maxwell's equations, the governing equation for the electrical potential ϕ is

$$\nabla \cdot \sigma \nabla \phi = 0 \quad (67)$$

for the various conductivity/permittivity σ distribution over the measured domain. The straight forward solution of Eq. (67) in finite elements is based on Kirchhoff's Law of electrical currents. As presented in Ref. 84, 85, 87, 100, 102 and 103. Kirchhoff's Law can be described in finite element form

$$Yf = C \quad (68)$$

where Y is the FEM admittance matrix as modelled by finite element method, f is the FEM voltage vector which is predicted by the finite element model, and C is the measured excitation current matrix. The experimental case, on the other hand, is described as

$$\Gamma V = C \quad (69)$$

where Γ is the real admittance matrix and V is the measured voltage vector. Then, since the current matrices are identical in each case

$$Yf = \Gamma V \quad (70)$$

Now at this point, we have Y , the computed FE admittance matrix, we have f , the computed FE voltage vector, but we don't have V , the voltage vector which, like f , includes the internal nodes as well as the boundary values. Therefore, we can not utilize (70) directly for determination of Γ , the real admittance matrix. For this, we need to perform additional steps and make one major assumption.

Pre-multiplication of (70) by Y^{-1} yields

$$f = (Y^{-1}\Gamma)V = AV \quad (71)$$

If the computation of Y is reasonably accurate, the matrix A should be close to a diagonal matrix so that $A \approx \{a_{ii}\}$ where $\{a_{ii}\}$ are the eigenvalues of A . If Y were exact, it would be identical to the actual admittance matrix and the matrix A would be an identity matrix. Now, we know neither Γ nor V since the latter includes both boundary and internal nodal voltages. However, we can say that the result of carrying out the products in (71) is

$$f_i \approx a_{ii}V_i \quad (72)$$

which provides a mapping between the measured and computed voltage domain.

Since A should be close to an identity matrix, the off-diagonal terms should be orders of magnitude smaller than unity—i.e., A is expected to be a very stiff matrix. The same should be true in either the homogeneous case or the target case. It can thus be assumed that $A_{homog} = A_{target}$. This is a major assumption in that it allows us to use the computed values of A as determined from the homogeneous case when computing target conductivity distributions.

Consider now the balance of the process, and keep in mind that only the boundary voltages are known; i.e., V is an incomplete vector. We now write

$$\Gamma V = (YA)V = C. \quad (73)$$

However, we can simply combine the A with the V and resolve the problem. Thus,

$$Y(AV) = C \quad (74)$$

or

$$YV' = C \quad (75)$$

where V' is the corrected voltage vector based on the matrix A . Note that while the corrections apply to the entire vector, boundary and internal values, only to the boundary values are known. This, now, presents a new problem to be solved

$$Yf' = C \quad (76)$$

where Y and f' are new admittance matrix and voltage vector which solves the minimization problem with respect to V' . Thus, (76) is solved for the admittance and voltage distributions such that

$$\phi^2 = (f' - V_o')^T (f' - V_o') \quad (77)$$

is minimized, where V_o' is the "corrected boundary voltage based on Eq. (74); i.e.,

$$V_o' = AV_o \quad (78)$$

Locally analytic solutions in the finite element direct solver result in "Block Decomposition" of the large mesh. These solutions solve Maxwell's equations to provide the local shape function. Both

numerical and experimental cases have been successfully undertaken. As shown in Ref. 79, the method improved computational speed by more than three orders of magnitude relative to conventional finite element solver.

For the present time, the matrix A is determined from the homogeneous case, and then applied to the target cases without change. However, for future improvement, it seems reasonable that the matrix A can be computed from many different target cases and then a method found to apply the variations of this matrix with conductivity distribution in a way which will let us apply the method directly to the unknown distribution problem. Training by neural network theory seems a likely candidate for this process.

3.3. Three-Dimensional Error Correction Methods for Image Enhancement

In the case of a 3-D electrodes and target, the governing Laplace's equation in Eq.(67) has to be solved in 3-D. This requires a more complicated formulation in the FEM, and the problem of computational resources is compounded. an alternative is to attempt to use two-dimensional methods but corrected for three-dimensional effects, and then reconstruct the third dimension by piecing together a sequence of resultant images. If the plain 2-D FEM solver is used in reconstructions, very poor results are obtained. However, the 3-D data may be mapped to an equivalent 2-D result providing the necessary correction. The following section describes how to provide the necessary corrections.

3.3.1. Explanation of 3-D error (space distortion)

We consider each slice of our tomograph as a 2-D imaging problem. To make use of the available 2-D finite element (FE) forward solver for computation of boundary potentials, we assume that the conductivity does not change along the axis of the phantom. This assumption of course causes loss of resolution along the axis of the target but makes the forward solution step much simpler.

When 3-D electrodes comprised, say, of short-length sections of rods aligned axially on the surface of the body being imaged (phantom), are used to collect data on the boundary, the variation of potentials on the electrodes is distorted due to high gradients at both ends of the electrodes. In fact, the solution of the forward problem is not a 2-D problem anymore.

Consider the geometry shown in Fig. 20. The left lower corner and right upper corner electrodes (1 and 9) are excited. The comparison of potential variation curves on the electrodes of 2-D and 3-D phantoms in the homogeneous case is shown in Fig. 21. The two potential curves intersect at electrodes 1 and 9 since the same excitation voltages are applied in the both cases, and also they intersect at 5 and 13 because of the symmetry.

Examining Fig. 21, the leakage of the current into the third dimension causes the potential curve to flatten between the excited electrodes, and this effect is magnified as the electrode length is decreased. (Note that the 2-D phantom has rod-electrodes running the entire depth of the pool, whereas the 3-D phantom has portions of these rods insulated so that only a short length actually contacts the water.) This flattening potential variation between the excited electrodes introduces 3-D space distortion (or 3-D error) in the data. The 3-D distortion also brings in loss of sensitivity to the target size.

Figure 20. Electrode numbering in the square phantom.

Figure 21. Voltage distribution on the electrodes when electrode pair 1-9 is excited.

X = 3-D; ● = 2-D homogenous phantom.

It is possible to experimentally determine the 3-D voltage error on the electrodes for the phantom and excitation patterns. This error does not change unless the geometry of the phantom, electrodes and excitation patterns are changed.

Let the number of excitations be p and the number of electrodes be e . Then, we can define the electrode voltage vector for the given excitations as $\{V\}_{ep \times 1}$ where $ep = e \cdot p$. The 3-D error vector for the homogeneous case is defined by

Figure 22. Three-dimensional error in voltage data for the homogenous case.

$$\{E_h\}_{ep \times 1} = \{V_h\}_{ep \times 1}^{3D} - \{V_h\}_{ep \times 1}^{2D} \quad (79)$$

where the superscripts 2-D and 3-D correspond to the two different situations.

The variation of 3-D error for the case of 5-cm long 3-D electrodes with eight single-pair excitations is shown in Fig. 22. The maximum error is approximately half a volt out of 7.07 V applied to the electrodes, and the error takes its maximum near the excited electrodes. This effect, which is accurately repeatable, is believed to be due to the high gradients at the corners of the electrodes causing more voltage drop than in the 2-D case.

3.3.2. Experimental methods for eliminating the 3-D space distortion

Since the conductivity variation is unknown in the nonhomogeneous inverse imaging problem, the error vector cannot be calculated. It is possible, however, that the homogeneous error is similar to that of the nonhomogeneous case. Therefore, if we use the homogeneous error vector in any conductivity distribution, we may eliminate the 3-D distortion to some degree.

The error vector may thus be calculated for the homogeneous case by Eq. (79) and then subtracted from the 3-D non-homogeneous data to give

Figure 23. 3-D phantom and 2"x2"x4" target located at (3",3").

$$\{V_{nh}\}^{3D} = \{V_{nh}\}^{3D} - \{E_h\} \quad (80)$$

where subscript *nh* corresponds to non-homogeneous data, and *corr* and *ucorr* correspond to corrected and uncorrected data respectively. Therefore, the 3-D data are approximately transformed to 2-D data by eliminating the 3-D leakage distortion.

The method suggested by Eq. (80) was investigated. The center of a 5 x 5 x 10-cm target was located at (7.5,7.5) in a 20 x 20-cm square phantom (see Fig. 23). The comparison of 3-D error curves is shown in Fig. 24. The error curves are produced by the same expression given in Eq. (79). The 2-D non-homogeneous data were collected using 2-D-electrodes and a 2-D target of the same cross section as that used in the 3-D tests. Fig. 24 shows the error variation of the first excitation for conductive and resistive targets when located at the same position. The error for homogeneous case lies between two extreme cases (perfectly conductive and perfectly resistive). Thus, if we use the homogeneous correction vector on 3-D data, our imaging problem becomes a 2-D equivalent problem in the electrode plane.

This method explained above can also be considered as a calibration of the 3-D data to the 2-D data. In Fig. 25 the data corresponding to the first excitation is shown before and after correction which significantly improves the 3-D target data.

The correction must also be made in excitation current data. The excitation currents for 8 excitations are shown in Fig. 26. The currents for 3-D case are smaller than that of 2-D case because of smaller 3-D electrode length. Also, a nonlinear relationship is found between 2-D and 3-D currents.

An alternative to these experimental methods is to correct **voltage/current** ratio. Instead of correcting the voltage and current data separately, currents are normalized and the electrode voltages in ev-

Figure 24. Variation of 3-D error in the electrode voltages for the first excitation (electrode pair 1-9 is excited). Targets: × homogenous; ▲ insulator; + conductor.

ery excitation are divided by the corresponding excitation currents. Thus, the constant voltage source excitation is converted to the equivalent constant current source excitation. Then, the same procedure is applied to correct the voltage/current data. Since the current is constant in all the excitations of both 2-D and 3-D problems it is enough to correct only the *voltage/current* ratio.

3.3.3. Numerical methods for eliminating the 3-D space distortion

In the experimental method described above, we eliminated the error related to the 3-D space distortion. In fact, the error has also a second component due to the 2-D finite element modelling of the phantom. Experimental and numerical errors are included in terms of *voltage/current* $\{V/c\}$ where the error is

$$\{E_h\} = \{E_h\}_{\text{experimental}}^{3D} + \{E_h\}_{\text{numerical}}^{2D} \quad (81)$$

where

$$\{E_h\}_{\text{experimental}}^{3D} = \left\{ \frac{V_h}{c} \right\}_{\text{experimental}}^{3D} - \left\{ \frac{V_h}{c} \right\}_{\text{experimental}}^{2D} \quad (82)$$

Figure 25. The variation of electrode voltage before and after 3-D correction (electrode pair 1-9 is excited). Symbols: × actual 2-D phantom and target; ▲ 3-D phantom and target before 3-D error correction; + 3-D phantom and target after 3-D error correction.

and where

$$\{E_h\}_{numerical}^{2D} = \left\{ \frac{V_h}{c} \right\}_{experimental}^{2D} - \left\{ \frac{V_h}{c} \right\}_{numerical}^{2D} \quad (83)$$

so that the total error becomes

$$\{E_h\} = \left\{ \frac{V_h}{c} \right\}_{experimental}^{3D} - \left\{ \frac{V_h}{c} \right\}_{numerical}^{2D} \quad (84)$$

Therefore, the error vector obtained above includes the error due to the 3-D distortion and the error from the 2-D FE modelling.

3.4. Nomenclature

- N = the number of total nodes
- M = the number of total elements
- P = the number of total current excitations

Figure 26. The variation of excitation currents before and after 3-D error correction.

- E = the number of total measured electrodes
- Y = admittance matrix
- V = the matrix of voltages on the nodes
- C = the matrix of currents on the nodes.
- e index for excitation and electrodes; given by $(e_a - 1) * P + e_b$
- e_b excitation index, from 1 to P
- e_a electrode index, from 1 to E
- $nl(e_a)$ node number of electrode index e_a
- $ni(\alpha), ni(\beta)$ node numbers of the neighborhood of element i , where α and β are dummy variables from 1 to 4
- $nj(\alpha), nj(\beta)$ node numbers of the neighborhood of element j , where α and β are dummy variables from 1 to 4
- $V_{ni(\beta), e_b}$ voltage on the node of $ni(\beta)$ for excitation e_b
- $V_{nj(\beta), e_b}$ voltage on the node of $nj(\beta)$ for the excitation e_b

- $y'_{i_{ni(\alpha),ni(\beta)}}$ derivative of the admittance from node $ni(\alpha)$ to node $ni(\beta)$ with respect to the conductivity of element i
- $y'_{j_{nj(\alpha),nj(\beta)}}$ derivative of the admittance from node $nj(\alpha)$ to node $nj(\beta)$ with respect to the conductivity of element j
- $y_{ni(e_a),ni(\alpha)}^{-1}$ the element having the coordinates [node of the electrode e_a , node of $ni(\alpha)$] in the matrix Y^{-1}

4. BASIC THEORY--BACK PROJECTION

4.1. Introduction

The relationship between applied voltage (or current) and observed voltage is a non linear function of conductivity. This means that in general the reconstruction algorithm must be iterative such as described previously for the finite element method. If the conductivity distribution changes only by a small amount from a known distribution then the reconstruction problem may be linearized.

Backprojection (Applied Potential Tomography) methods, however, are simple, linear, noniterative reconstruction processes. They do not need the solution of the forward problem during image reconstruction. Since the computation of the equipotential lines is done before the image reconstruction, this method is very fast and does not need a large storage area.

4.2. Theoretical Background

The method of backprojection along equipotential lines was developed by researchers in the University of Sheffield¹¹. A mathematical analysis of this method is reviewed here. In electrical impedance tomography, the forward problem is the first step and involves solving a Poisson partial differential equation¹

$$\sigma \nabla^2 \phi + \nabla \sigma \cdot \nabla \phi = 0 \quad (85)$$

with Neumann boundary conditions

$$\sigma \mathbf{n} \cdot \nabla \phi = |J| \quad (86)$$

where ϕ is the scalar potential, σ is the conductivity of the medium, and J is the applied surface current density.

Define

$$R = - \ln \sigma \quad (87)$$

$$\sigma = e^{-R} \quad (88)$$

so that

$$e^{-R} \nabla^2 \phi + \nabla e^{-R} \cdot \nabla \phi = 0 \quad (89)$$

or

$$e^{-R} \nabla^2 \phi - e^{-R} \nabla R \cdot \nabla \phi = 0 \quad (90)$$

1. Unless otherwise identified, nomenclature in this section is identical to that given in the previous section.

Now Poisson's equation can be written as

$$\nabla^2 \phi = \nabla R \cdot \nabla \phi \quad (91)$$

Let

$$\phi = \psi + \phi_p \quad (92)$$

where ψ is the homogenous scalar potential solution, ϕ_p is the particular scalar potential solution due to perturbations from the homogeneous field. Then,

$$\nabla^2(\psi + \phi_p) = \nabla R \cdot \nabla(\psi + \phi_p) \quad (93)$$

or

$$\nabla^2 \psi + \nabla^2 \phi_p = \nabla R \cdot \nabla \psi + \nabla R \cdot \nabla \phi_p \quad (94)$$

Since in the homogenous medium

$$\nabla^2 \psi = 0 \quad (95)$$

and

$$R(x, y) = R_0 = \text{const.} \quad (96)$$

then for the inhomogeneous field

$$\nabla^2 \phi_p = \nabla R \cdot \nabla \psi + \nabla R \cdot \nabla \phi_p \quad (97)$$

and

$$R(x, y) = R_0 + R_r(x, y). \quad (98)$$

Choose $R_p(x, y) \ll R_0$. Then $\nabla \phi_p \ll \nabla \psi$ and we may ignore this term. Thus, using subscripts i to indicate the potentials ϕ_p and ψ which depend on the i^{th} applied current density function, we show

$$\nabla^2 \phi_p = \nabla R \cdot \nabla \psi \quad (99)$$

and

$$\nabla^2 \phi_{pi} = \nabla R \cdot \nabla \psi_i. \quad (100)$$

4.3. Transformation into u,v Space

Assuming that the distance between two adjacent driving electrodes (2τ) is very small and considering the drive pair of electrodes as a dipole, a conformal transformation can transform circles in the (x,y) plane into rectangles in the (u,v) plane shown in Fig. 27. The appropriate transformation is

Figure 27. Transformation from x,y into u,v space

$$x = \frac{u}{u^2 + v^2} \quad \text{and} \quad y = \frac{v}{u^2 + v^2} \quad (101)$$

and the inverse

$$u = \frac{x}{x^2 + y^2} \quad \text{and} \quad v = \frac{y}{x^2 + y^2} \quad (102)$$

For any points on cycle I in the (x,y) plane, the following equations are satisfied:

$$I : (x - 0)^2 + (y - r)^2 = r^2 \quad (103)$$

$$I : (x^2 + y^2) = 2ry; \quad (104)$$

which when transferred to (u,v) coordinates yields

$$v = \frac{y}{x^2 + y^2} = \frac{1}{2r} \quad (105)$$

Thus, the circle in the (x,y) plane becomes a straight line on the (u,v) plane.

Consider now which points will go to $+\infty$ and $-\infty$. Assuming that δ is a very small but positive number, for point $D^+(\delta, 0)$, $D^-(-\delta, 0)$ on (x,y) see Fig. 27, then

$$D^+ : x = +\delta, \quad y = 0$$

then

$$u = \frac{x}{x^2 + y^2} = \frac{1}{\delta} > > 0 \quad \text{as } u \rightarrow \infty \quad (106)$$

and for

$$D^- : x = -\delta, \quad y = 0$$

then

$$u = \frac{x}{x^2 + y^2} = -\frac{1}{\delta} < 0 \text{ as } u \rightarrow -\infty \quad (107)$$

Under this transformation, it can be seen that circle I in the (x,y) plane is transformed into straight line ($v=E'$) parallel to the u -axis in the (u,v) plane and two end points of the current dipole $D^+(\delta, 0)$ and $D^-(-\delta, 0)$ are transformed into points at $+\infty$ and $-\infty$ of the u -axis. The straight lines parallel to the u -axis become field lines and are transformed into circles tangential to the x -axis at the point D in the (x,y) plane. Similarly, equipotential lines which are transformed into the circular lines passing through the point D in (x,y) plane. If the dipole strength is m , then the equipotential lines are the lines passing through points at which

$$V = mu = \frac{mx}{x^2 + y^2} \quad (108)$$

is constant. This method is called the dipole approach and results in significant error close to the electrodes when the gap between the drive electrodes is not sufficiently small.

4.4. Solution of Simplified Poisson Equation (15)

From the transformation given in Eqs. (101) and (102)

$$\frac{\partial u}{\partial x} = \frac{1}{x^2 + y^2} - \frac{2x^2}{(x^2 + y^2)^2} \quad (109)$$

and

$$\frac{\partial u}{\partial y} = -\frac{2xy}{(x^2 + y^2)^2} \quad (110)$$

Also

$$\frac{\partial v}{\partial x} = -2uv \quad \text{and} \quad \frac{\partial v}{\partial y} = u^2 - v^2. \quad (111)$$

Assuming that the applied current density produces a uniform electrical field with components in the homogeneous medium, we obtain

$$\frac{\partial \psi}{\partial u} = E \quad \text{and} \quad \frac{\partial \psi}{\partial v} = 0 \quad \text{and} \quad \frac{\partial \psi}{\partial x} = 0 \quad (112)$$

where u and v are orthogonal axes rotated by an angle θ from the x -axis. On any observation plane normal to the z -axis, lines of constant u represent equipotentials of ψ and lines of constant v represent current streamlines.

Thus, the x -direction differential in the Eq. (99) Laplacian may be written as

$$\frac{\partial^2 \phi_p}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial \phi_p}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial \phi_p}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial \phi_p}{\partial v} \frac{\partial v}{\partial x} \right) \quad (113)$$

so that

$$\frac{\partial^2 \phi_p}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial \phi_p}{\partial u} (v^2 - u^2) - \frac{\partial \phi_p}{\partial v} (2uv) \right) \quad (114)$$

or

$$\frac{\partial^2 \phi_p}{\partial x^2} = \left(\frac{\partial}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial}{\partial v} \frac{\partial v}{\partial x} \right) \left(\frac{\partial \phi_p}{\partial u} (v^2 - u^2) - \frac{\partial \phi_p}{\partial v} (2uv) \right). \quad (115)$$

Performing the indicated differentiations results in

$$\frac{\partial^2 \phi_p}{\partial x^2} = \frac{\partial^2 \phi_p}{\partial u^2} (v^2 - u^2)^2 - \frac{\partial^2 \phi_p}{\partial u \partial v} (v^2 - u^2)(4uv) + \frac{\partial^2 \phi_p}{\partial v^2} (4u^2 v^2). \quad (116)$$

In a directly analogous manner, the y-direction differential term in the Laplacian of Eq. (99) is

$$\frac{\partial^2 \phi_p}{\partial y^2} = \frac{\partial^2 \phi_p}{\partial u^2} (4u^2 v^2) - \frac{\partial^2 \phi_p}{\partial u \partial v} (u^2 - v^2)(4uv) - \frac{\partial^2 \phi_p}{\partial v^2} (u^2 - v^2)^2. \quad (117)$$

Adding Eqs. (116) and (117) and rearranging yields

$$\frac{\partial^2 \phi_p}{\partial x^2} + \frac{\partial^2 \phi_p}{\partial y^2} = \left(\frac{\partial^2 \phi_p}{\partial u^2} + \frac{\partial^2 \phi_p}{\partial v^2} \right) (u^2 + v^2)^2. \quad (118)$$

Similarly, the divergence product term in the same equation may be treated as

$$\nabla R \cdot \nabla \psi \equiv \frac{\partial R}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial R}{\partial y} \frac{\partial \psi}{\partial y} \quad (119)$$

which may be expanded in terms of u and v also using chain rule differentiation to obtain

$$\nabla R \cdot \nabla \psi = \left(\frac{\partial R \partial \psi}{\partial u^2} + \frac{\partial R \partial \psi}{\partial v^2} \right) (u^2 + v^2)^2. \quad (120)$$

Substitution of Eqs. (118) and (120) into Eq. (99) and using the results of (112) yields

$$\nabla^2 \phi_p = E \cdot \frac{\partial R}{\partial u} \quad (121)$$

We may now take the Fourier transform of Eq. (121). The transfer pair is

$$\phi_p(u, v) \Rightarrow \mathcal{F}\{\phi_p\}(F_u, F_v) \quad (122)$$

and

$$R(u, v) \Rightarrow \mathcal{F}\{R\}(F_u, F_v) \quad (123)$$

and

$$- (F_u^2 + F_v^2) \mathcal{F}\{\phi_p\} \Rightarrow jEF_u \mathcal{F}\{R\} \quad (124)$$

so that

$$\mathcal{F}\{\phi_p\} \Rightarrow -j \frac{EF_u \mathcal{F}\{R\}}{(F_u^2 + F_v^2)} \quad (125)$$

The Fourier transform of the gradient of v in the u direction can be obtained by multiplying both sides of Eq. (125) by jF_u to obtain

$$g_p(F_u, F_v) = jF_u \mathcal{F}\{\phi_p\} = \frac{F_u^2 E \mathcal{F}\{R\}}{F_u^2 + F_v^2}. \quad (126)$$

This gives a simple relationship between the value of the voltage gradient and the distribution of log resistance with the object. Of primary interest is the value of the gradient at the boundary of the object i.e. at $V=V_b$. Assuming the origin in (u, v) space is shifted to lie on the line $V=V_b$, the Fourier transform of the boundary gradient is given by

$$b_p(F_u) = \frac{\partial \phi_p}{\partial u} = E \int_{-\infty}^{\infty} \frac{F_u^2 \mathcal{F}\{R\}}{F_u^2 + F_v^2} dF_u \quad (127)$$

Now consider a point object at depth $q = V - V_b$ in the region. The Fourier transform of this object is given mirror symmetry about the origin. Thus,

$$\mathcal{F}\{R\} = \cos(F_u, q) \quad (128)$$

$$b_p(F_u) = E \int_0^{\infty} \frac{F_u^2 \cos(F_u, q)}{F_u^2 + F_v^2} dF_u \quad (129)$$

$$b_p(F_u) = 0.5E \cdot |F_u| \cdot \pi \cdot e^{(-q|F_u|)} \quad (130)$$

This is the Fourier transform of the boundary profile produced from a point object with a distance q from the boundary. This function can be interpreted in two ways. Firstly the complete Fourier transform of this function is given by

$$G(u) = \frac{E(q^2 - u^2)}{(u^2 + q^2)^2} \quad (131)$$

and this may easily be shown to be the field gradient produced by a small current dipole of strength E at depth q . This is a physical interpretation of Eq (130). An image reconstruction interpretation is as

follows. Eq. (130) can be divided into two terms. The exponential term can be thought of as a term representing the Fourier transform of a point response function (PRF) measured at boundary for a point object at depth q . Its transform into real space is

$$S(u) = \frac{q}{u^2 + q^2} \quad (132)$$

which means that the point response is positive and increases in width with depth into the region.

The second term is $|F_u|$ which is a ramp filter. These results can be interpreted as meaning that the boundary profile is already ramp filtered; i.e., the backprojection of boundary data can proceed without further ramp filtering.

Dividing both side of Eq. (131) by E gives an expression for $G(u)/E$. This is invariant under the transformation of Eqs. (101) and (102) yielding

$$\frac{G(u)}{E} = \frac{g_m(\theta)}{g_u(\theta)} \quad (133)$$

Back-projecting $G(u)/E$ in (u, v) space along the v axis and transforming into x, y space produces the same backprojection as back-projecting $g_m(\theta)/g_u(\theta)$ in (x, y) space along the equipotentials.

4.5. Backprojection

This method is called backprojection between equipotential lines since the method back-projects the potential difference between two equipotential lines on the surface to the resistivity value in the inside area between those two equipotential lines.¹⁴² One idea for reducing the total computational effort was to use a fast approximate method to obtain an improved first estimate of the conductivity distribution for use by the finite element code, thereby reducing the overall number of iterations required for solution. Back projection is one method utilized in the biomedical field for approximate reconstruction of medical images.

This concept is shown pictorially in Fig. 28. With current applied at one electrode and removed from another, a certain field is applied to the phantom. This potential field will be everywhere normal to the flux lines. We assume that the resistivity is uniform and calculate the expected voltages between each pair of electrodes.

For the nonhomogeneous case, the boundary voltages will depart from those expected for the homogeneous field. If the voltage on a real object was higher by some factor than for the homogenous case, the model resistivity will be multiplied by that factor everywhere in the shaded region corresponding to the area between the projection lines.²⁸ The impedance in a single element within the backprojection area will then be expressed by¹⁴²

$$\rho = \rho_o \frac{(V_i - V_j)_{\text{measured}}}{(V_i - V_j)_{\text{expected}}} \quad (134)$$

Figure 28. Methodology for back projection.

4.6. Conclusions

The backprojection method (APT) is a simple and quick method to obtain a coarse image. This simple linear reconstruction process clearly may not reconstruct with quantitative accuracy the distribution of resistivity in the area being imaged. However, a simple reconstruction method of this form does have the advantage of speed over iterative methods.

It must also be emphasized that data collection can be made very rapid and that dynamic imaging of moving structures should be possible. It is therefore important that processing times be kept to a minimum.⁹

5. QUASI-THREE-DIMENSIONAL BACK PROJECTION

5.1. BASIC THEORY

We shall repeat the simplified Poisson equation as follows:

$$\nabla^2 \phi_{pi} = \nabla R \cdot \nabla \psi_i \quad (135)$$

To illustrate how backprojection may be used in this manner, we again assume that the applied voltage produces in the homogeneous medium a uniform electrical field with components⁴⁹:

$$\frac{\partial \psi}{\partial u} = E, \quad \frac{\partial \psi}{\partial v} = 0, \quad \frac{\partial \psi}{\partial z} = 0 \quad (136)$$

where u and v are orthogonal axes rotated by an angle θ from the x -axis. On any observation plane normal to the z -axis, lines of constant u represent equipotentials of ψ and lines of constant v represent current streamlines. The perturbation potential for an infinite cylinder of radius r and log conductivity R_n , located on the z -axis is

$$\phi_{p\theta}(u, v) = E \cdot r^2 \cdot (R_n - R_o) \cdot \frac{u}{(u^2 + v^2)} \quad (137)$$

To simulate a Dirac impulse object let radius $r \rightarrow 0$, while $R_n \rightarrow \infty$; i.e., $\sigma_n \rightarrow \infty$. Then

$$r^2 \cdot (R_n - R_o) \rightarrow 1 \quad (138)$$

and

$$\phi_{p\theta}(u, v) = E \cdot \frac{u}{(u^2 + v^2)} \quad (139)$$

and

$$g_\theta(u, v) = \frac{\partial \phi_{p\theta}}{\partial u} = E \cdot \frac{(u^2 + v^2 - 2u^2)}{(u^2 + v^2)^2} \quad (140)$$

Dividing by E and considering this function along a current streamline $v = \varepsilon$ yields

$$\frac{g_\theta(u, v)}{E} = \frac{(\varepsilon^2 - u^2)}{(\varepsilon^2 + u^2)^2} \quad (141)$$

The 1-D Fourier transform of Eq. (141) is

$$\frac{G_\theta(\omega_u, \varepsilon)}{E} = |\omega_u| \cdot e^{(-\varepsilon \cdot |\omega_u|)} \quad (142)$$

For small ε

$$\frac{G_\theta(\omega_u, \varepsilon)}{E} = |\omega_u| \quad (143)$$

where $|\omega_u|$ is the "ideal" backprojection filter.

Now the $G_\theta(\omega_u, \varepsilon)/E$ for $\theta \leq 0 \leq \pi$, are assigned to the proper terms of the image's 2-D Fourier domain $F(\omega_x, \omega_y)$. A Dirac pulse results when the inverse transform $f(x, y)$ is computed. The image can then be an accurate reproduction of the original object. For large ε , the exponential term $\exp(\varepsilon|\omega_u|)$ attenuates the higher spatial frequencies in $F(\omega_x, \omega_y)$, causing blurring and distortion of the impulse object's image. In the spatial domain every point (x, y) in the image plane becomes

$$u = x \cdot \cos \theta + y \cdot \sin \theta \quad (144)$$

If the normalized gradient $g_\theta(u, v)/E$ contains contributions from a sufficient number of angles, the object's image $f(x, y)$ is obtained. For each angle θ the values of $g_\theta(u, v)/E$ must smeared back over all points along equipotential regions, i.e. $u = \text{constant}$.

The point response function (PRF) of this backprojection imaging method is not space-invariant. Further extension to continuous distributions could be approached by superposition, based on the

hypothesis of linearity underlying $\nabla^2 \phi_{pi} = \nabla R \cdot \nabla \psi_i$ and considering the reconstructed image as the sum of responses to individual impulse objects with amplitude R .

The requirement that applied current density functions produce a uniform field serves only to simplify the mathematical analysis. In practice, current may be applied by electrodes simulating point sources and sinks. Equipotentials and streamlines of ψ are then lines parallel to the u and v axes, respectively, in a space defined by a conformal mapping. Because the mapping function depends on the drive configuration, equipotential paths in the image space must be described differently.

The Neumann boundary conditions $\sigma n \cdot \nabla \phi = |J|$ forces the boundary of the volume conductor to be a streamline of ψ . Note that $\partial \psi / \partial u$ sampled at a number of points on this boundary ($v = \epsilon$) is not usually a constant. Therefore, E in the previous equations must be replaced by values of $\partial \psi / \partial u$ measured on the boundary of the homogeneous medium. In order to alleviate technical requirements on the current source, boundary potential gradients are replaced by transfer impedances.

Transfer impedances are determined by measuring both the boundary potential differences and the actual current injected through the medium. The discrete form of the equipotentials backprojection function given by ⁴⁹

$$R(x, y) \approx \frac{1}{M} \sum_{i=1}^M \sum_{k=1}^N T_{i,k}(x, y) \left[\frac{Z_{I,ik} - Z_{H,ik}}{Z_{H,ik}} \right] \quad (145)$$

where $k = 1, N$ represent a pair of potential recording electrodes, $j = 1, M$ represent a pair of current drive electrodes, $Z_{I,ik}$ is impedance in the inhomogeneous medium, k represents the k^{th} pair of recording electrodes; current applied by the i^{th} pair of drive electrodes, $Z_{H,ik}$ is the impedance in the homogeneous medium under the same situation with $Z_{I,ik}$. $T_{ik}(x, y) = 1$ if the point (x, y) belongs to the equipotential region ending at electrodes of k^{th} pair when excited by the i^{th} electrode pair or $T_{ik}(x, y) = 0$ otherwise. Also,

$$D_{ij}^2 = [U_{h,ij} - U_{nh,n}]^2 - [U_{h,ij} - U_{nh,(n+1)}]^2 \quad (146)$$

where $U_{h,ij}$ is the expected homogeneous-field potential in element i, j as calculated from infinite field theory for a dipole current pair located at the source-sink electrode locations. $U_{nh,n}$ and $U_{nh,(n+1)}$ are the measured boundary voltage at adjacent electrodes n and $n+1$ for the nonhomogeneous case.¹¹ This quantity is used to make a determination whether a given element i, j is within the boundaries of the field lines shown in the shaded area in Fig. 28. If D_{ij}^2 goes to the minimum value calculated for the nonhomogeneous case it is within the area bounded by the two potential lines. Otherwise it is not.

It is found that this judgement not always correct. Sometimes (146) will give an erroneous estimate of a cell's membership in the bounded area. For example it may give the wrong result when two adjacent boundary voltage measurements differ by a large fraction of the applied voltage. Therefore, the following judgement has been added to supplement the estimation given by Eq. (146) and provides better estimation regarding whether a cell is within the bounded region or not.

The judgement of which field point belongs to which equipotential region has been improved and is given by

$$U_{nh,n} < U_{h,ij} < U_{nh,(n+1)} \quad (147)$$

If this Eq. (147) is true then $U_{h,ij}$ definitely belongs in the region between $U_{nh,n}$ equipotential line and $U_{nh,(n+1)}$ equipotential line.

5.2. ASSUMPTION

Use an infinite medium approximation. Calculate potential field using a simple equipotential model and applying a source/sink electrode pair in the infinite medium. The potential field distribution is given by⁴⁹

$$U(x, y) = 2\pi I \sigma \left[\frac{1}{\sqrt{(x - x_{so})^2 + (y - y_{so})^2}} - \frac{1}{\sqrt{(x - x_{si})^2 + (y - y_{si})^2}} \right] \quad (148)$$

where σ is the conductivity, I is the current measured, and where (x, y) represent the coordinate locations for the point electrodes in the pair, (x_{so}, y_{so}) represent source points coordinate locations and (x_{si}, y_{si}) represent sink points coordinate locations. Get impedances Z_{ij} from both the boundary potential differences and the actual current injected into and through the medium. Finally, calculate the potential at each point current recording electrode location using Eq. (148) where the coordinates (x, y) now represent the location of the recording electrode; i.e., evaluate $U_a(x_a, y_a)$ and $U_b(x_b, y_b)$ where the subscripts a and b refer to the point of each pair of recording electrodes respectively. While there are some remaining problems, particularly is how to get sensitive observed voltage measure from the boundary, the method does provide a reasonable first image of a three-dimensional object.

6. BASIC THEORY--FREQUENCY MULTIPLEXING

6.1. Frequency Multiplexing for Fast Excitation and Acquisition of Signals

Conventionally, in the transverse plane, some preselected pattern of impedance imaging electrical signals are applied to the electrodes located circularly around the phantom, usually at uniform angles. For each excitation the signal amplitude and phase are measured at the remaining electrodes. These measurements are then used to compute an impedance cross section map of the phantom.

This generic technique is not sufficiently fast to map rapidly varying impedance cross sections, since all of the electrodes need to be excited sequentially while amplitude and phase measurements are made at each one of the other electrodes. Recent studies on EIT shows that the simultaneous current excitation method has the best image with the least noise [Hua et al.⁶⁴; Cheng et al.²²].

Particularly in multiphase flow imaging, among other applications, a full cycle of amplitude and phase measurements needs to be taken before any significant change in the impedance cross section occurs. In many cases this is not feasible due to fast flow and resultant rapid variations in internal impedance distribution relative to the possible data acquisition rate.

In an attempt to improve this situation, multiple simultaneous excitations are examined as an alternative to single sequential excitations. Preliminary studies indicate, using this technique, as long as the measurements due to each individual excitation signal can be separated, simultaneous measurements can be done reducing the data acquisition time by a factor of about the square of the number of electrodes.

6.1.1. Conceptual Description

The model presented depends on simultaneous excitation of all electrodes of the phantom with different frequencies on each electrode. As shown in Fig. 29, N equally spaced electrode currents at different frequencies flow through $(N-1)$ electrodes simultaneously and are collected by the one remaining electrode, N , which is grounded. Voltages across each of the electrodes 1 through $(N-1)$ are measured with respect to first reference electrode. Each of the measured voltage contains $(N-1)$ mixed frequencies which give information related to the impedance distribution of the phantom. For an N -electrodes system a total of $(N-1)^2$ data points are obtained (not all of which are independent).

6.1.2. Fourier Decomposition

In an N -electrode, symmetric, circular phantom, either with a homogeneous or inhomogeneous medium, the eigenvectors for the impedance matrix, Z , which relates voltages to currents as

$$V(\theta) = Z(\varrho)I(\theta) \quad (149)$$

Figure 29. Frequency multiplexing data acquisition system

are trigonometric functions. These eigenvectors are $N/2$ spatial cosine functions $\{ \cos(n\theta) \}$ and $\{(N/2)-1\}$ spatial sine functions $\{ \sin(n\theta) \}$. By applying these eigencurrents directly to the phantom, the corresponding eigenvalues, which are called characteristic impedances, can be obtained experimentally. The resulting voltage pattern, V , on the surface may be obtained by applying the matrix Z to the current pattern I [Kuo, et al, 1991] so that

$$V = Z \cdot I = \sum_{k=1}^{N-1} \langle I, T^k \rangle Z T^k \quad (150)$$

where T^k is the k th eigencurrent. Since the matrix Z is self adjoint, the characteristic impedances are doubly degenerate. Therefore, equation (150) can be expressed as

$$V = Z \cdot I = \sum_{k=1}^{N/2} \langle I, T_c^k \rangle Z_k T_c^k + \sum_{k=1}^{N/2-1} \langle I, T_s^k \rangle Z_k T_s^k \quad (151)$$

where the coefficients I, T_c^k and I, T_s^k are the Fourier coefficients of the cosine and sine terms for the current I . Thus any voltage pattern can be reconstructed with the eigencurrents and characteristic impedance. Measured voltage vectors produced in response to different frequency currents applied to electrodes can be inverted, using a Fourier decomposition method to obtain the charac-

teristic impedances for trigonometric eigencurrents. The voltage and current vectors can be represented in terms of the discrete Fourier transform components as

$$V(\theta_i) = \sum_{n=1}^{N/2} (V_{n1}^c \cos n\theta_i + V_{n2}^s \sin n\theta_i) \quad i = 1, \dots, N \quad (152)$$

and

$$I(\theta_i) = \sum_{n=1}^{N/2} (I_{n1}^c \cos n\theta_i + I_{n2}^s \sin n\theta_i) \quad i = 1, \dots, N \quad (153)$$

where the angles $\theta_i = 2\pi i/(N+1)$ for $1 < i < N/2$. Also V_{n1} and V_{n2} are the n^{th} Fourier voltage coefficients of cosine and sine, respectively, and I_{n1} and I_{n2} are the Fourier current coefficients. Equation (152) can be written in terms of characteristic impedances as

$$V(\theta_i) = \sum_{n=1}^{N/2} (Z_n I_{n1}^c \cos n\theta_i + Z_n I_{n2}^s \sin n\theta_i). \quad (154)$$

If the point $\sin \{(N/2)\theta\}$ is used as a reference point, the characteristic impedance can be found for each part of the summation of Eq. (154) as

$$Z_{n1} = V_{n1}^c / I_{n1}^c \quad n = 1, \dots, N/2 \quad (155)$$

and

$$Z_{n2} = V_{n2}^s / I_{n2}^s \quad n = 1, \dots, N/2 - 1 \quad (156)$$

6.1.3. First Testing of the Method

As a test of this methodology, different frequency independent currents $I_1, I_2, \dots, I_{(n-1)}$ were injected into a phantom and their voltage responses measured so that the Fourier spectrum (amplitude and phase) could be obtained from the data and directly related to the conductivity distribution. The major difficulty here is to generate and maintain a stable sinusoidal current and then measure and process the data accurately.

The sinusoidal function generator designed for use in these experiments depends on phase-stable sine and cosine oscillations as shown in Fig. 30. In this figure, B is a binary counter, F is a filter, z is an input for trigger pulses, and x and y are complementary outputs of the counters. In the circuit model shown in Fig. 30, the binary counters $B1$ and $B2$ produce Rademacher functions (Harmuth⁵³), from which the filters extract the fundamental sinusoidal functions. The first harmonic has three times the frequency of the fundamental oscillation.

The system model was assumed as a multiple input and a single output during each data collection period. In order to deduce the response of a single-electrode signal, power spectral density of the

Figure 30. Generator for phase locked sine and cosine functions.

mixed sinusoidal data recorded were computed using fast Fourier transform techniques. Figures 31 and 32 show respectively simulation results of multiplexed sinusoids with noise on one electrode of an eight-electrode phantom, and the equivalent power spectrum of this signal.

Experimentally, two different frequency currents which had the same amplitude were simultaneously applied to the four-electrode phantom. Measured voltage response was acquired with an HP 3258A digital voltmeter (DVM) for each of two separate targets and the results were processed by the HP-9816S computer to obtain the FFT spectrum shown in Figs. 33 and 34. From these figures we observed that changing the location of the target in the phantom changes the spectrum amplitude relative to that obtained with homogeneous conductivity distribution. For this experiment the two signal sources were constructed using an XR-2206 chip and ranged from approximately 100Hz to 150KHz. From these results, it is seen that the use of signal multiplexing produces measurable changes which may be useful to imaging of two-phase flows.

6.1.4. Second tests of the method

Figure 29 shows the general principles of the testing method. The major difficulty here is to generate and maintain a stable sinusoidal current and then measure and process the data accurately. For this second experiment eight signal sources were constructed using an XR-2206 chip ranging from approximately 1 Hz to 15KHz. In order to get accurate data, a very low-noise signal source was designed and realized. Figure 35 shows block diagram of this circuit. The most important

Figure 31. Sum of mixed sinusoidal signals.

Figure 32. Power spectral density of the mixed signal.

part of the circuit is multiple feedback bandpass (MFPB) filter, which has a narrow bandwidth to prevent noise in the signal sources and provides low cross correlation between the signals. This circuit features a minimum number of components and low sensitivity to component tolerance.

Figure 33. FFT spectrum of two mixed signals from real measurements with noise.

Figure 34. FFT spectrum of two mixed signals from real measurements with noise with a different target in the phantom from that in Fig. 33.

In this application, a 6th order MFBP filter was constructed and, according to desired bandwidth (selected to be 20 Hz at 3dB), values of Q change from 17 to 22. Each signal source has a different frequency range of 100 Hz to 640 Hz with frequency interval at 80 Hz.⁵

In order to avoid the ionization in the water, seven 5 V peak-to-peak voltage with the low frequency A.C. signal sources, which each of them has a different frequency range of 100 Hz to 560 Hz with frequency interval at 80 Hz, has been applied for excitation of the phantom. The tested

Figure 35. Block diagram of the excitation sources constructed for the second test.

Figure 36. FFT spectrum of the response signal measured from the seventh electrode of the two-dimensional phantom.

results of electrical resistivity of pure water is $4,850 \Omega\text{m}$ compared to the real value of $5,000$ at room temperature, indicating that our instrumentation and methods are reasonably accurate.

Figure 36 shows the equivalent FFT spectrum of this signal. Experimentally, seven different frequency currents which had the same amplitude were simultaneously applied to the eight-electrode phantom. Measured voltage response was acquired with an HP 3258A digital multimeter (DMM) for each of two separate targets and the results were processed by the HP-9816S computer to obtain the FFT spectrum shown in Figs. 36 and 37. From these figures we observed that changing the location of the target in the phantom changes the spectrum amplitude relative to that obtained with homogeneous conductivity distribution as previously seen.

Figure 37. FFT spectrum of the response after changing the location of the target relative to the previous measurements shown in Fig. 36.

6.1.5. Square-Wave Excitation

Another different approach related to the frequency multiplexing method is to apply current sources which sum have the harmonic distribution required to produce a square wave as the measured output voltage. The equivalent formula is

$$V(t) = \sum_{k=1,3,\dots}^{N-1} \frac{\sin kt}{k}. \quad (157)$$

Computer simulation results are shown in three dimensional representations in Figs. 38 and 39. If the conductivity of the phantom is homogenous, the Fourier decomposition of the measured output is a square wave. For the nonhomogeneous case, the shape will depart significantly from the square wave shape such as shown in Fig. 38. In these figures, only one output is shown. If all the (N-1) output voltage are recorded, information related to the entire phantom conductivity distribution could be obtained.

6.1.6. Summary

The model described, which is a multi input-single output model required that parallel time frequency analysis related with linear dynamic model of the EIT for real time processing. The state vector in this model is the discrete Fourier transform of the output signal with phase reference of the current time. This model was generalized by Bimead, at al.⁵ This is a special case of the model

Figure 38. Three-dimensional view of simulated voltage distribution in phantom with nonhomogeneous medium.

Figure 39. Fourier transform of simulated conductivity distribution of inhomogeneous medium in the phantom.

for the Kalman filter, which is the optimal linear estimator for instantaneous values of the state of a signal in the frequency domain, given measurements of the signal in time domain. Our study related to multiplexed excitation is related to Kalman filtering which itself is related to parallel time frequency analysis of measured signals.

7. BASIC CONCEPT--NEURAL NETWORKS

7.1. A Simulated Neural Network Approach for Electrical Impedance Imaging

In impedance tomography we used iterative numerical methods to calculate unknown conductivity distribution from the measured voltage data on the boundary of the problem region. In general, the reconstruction algorithms compare the measured boundary data to computed boundary data from numerical or analytical solutions of the governing partial differential equation using the given experimental excitation patterns. We then seek for a distribution of conductivity to minimize the squared sum of differences between measured and computed boundary data. This approach is basically the same for all other reconstruction algorithms used today. The disadvantage of these methods is the increasing computational burden due to the demand of high resolution images, and the current lack of adequate definition of resulting images.

When the boundary of the problem region is excited by a current source, the solution of potential variation on the boundary is unique unless the problem is ill-conditioned. This being the case, it appeared possible that the images could be obtained by using artificial neural networks. This section of the report describes briefly a small effort to investigate this possibility.

7.1.1. Artificial Neural Networks

Neural computation refers to a calculational procedure where the models of elementary brain functions are thought to be mimicked by applying neural operators as building blocks for a network. A neural network generally consists of several layers of interconnected networks. For the so called feed-forward network three types of layers are distinguished: the input layer (the first layer), the output layer (the last layer) and the hidden layers (the layers between the input and the output layers).

In feedforward networks the three main stages are distinguished: the training phase, the testing phase and the user phase. In the first phase the output is calculated for the given input and compared with the specified target pattern. The weights are dynamically adjusted according to the specific learning rule, then the weight matrix is calculated. During the testing phase the known examples are checked if the network properly generalizes. During the user phase the network is utilized for its intended purpose.

7.1.2. Application to Imaging

As an example of a neural network approach we considered a very simple imaging problem. The problem region, element and electrode numbering is shown in the Fig. 40. In this case, each element is considered as a pixel having a binary density, black or white. Therefore, we do not only simplify the problem to the limited number of pixels but also to the binary variation of the conductivity variation in the region.

In our case, there are two possible ways to generate the training patterns. The first is experimentally changing the targets and measuring the boundary voltages. This method must, in fact, be utilized in real applications. For purposes of this text, however, we simulated the phantom and target conditions on the computer and calculated the boundary voltages by either the FEM or other numerical solution method to solve Laplace's equation for the given boundary values.

Figure 40. Numbering scheme for testing of artificial neural network in imaging.

Numerical methods were thus used to generate a set of random examples; i.e. random conductivity distributions in the 4x4 mesh. Boundary voltages were computed numerically and the input patterns thus prepared as required to train the neural network program. Once the network was trained with enough number of examples, the program calculated a set of weighting factors for the network and for a given voltage pattern. The distribution can be calculated simultaneously. If this approach becomes applicable, the only computational time required is for training the network.

Using computer-generated synthetic data for the randomly chosen set of conductivity patterns, if we do not restrict the number of black and white pixels, the number of all possible patterns for a 16-pixel matrix is $2^{16} = 65,536$. Even for this simple problem there are many number of combinations. We cannot train the network with all the possible combinations. The question here is that we do not know yet how many of these combinations are enough to train the network well. To simplify the problem more, we used 500 random patterns which had a maximum of 3 black pixels, out of $16 \times 15 \times 14 = 3,360$ possible patterns. The reason for this was to simplify the problem to something the neural network could learn relatively easily with a small number of training cases.

To calculate electrode voltages, current I is injected into the electrodes 2, 3, 4, 5, 6, 7, 8 (see Fig. 40) and the current $7I$ is taken out from the electrode-1. The potential function is computed by solving Laplace's equation using the FEM. Also for the second excitation the current I is injected to the electrodes 1, 2, 3, 4, 6, 7, 8 and the current $7I$ is taken from the electrode 5. Therefore, we obtained 16 electrode voltages for a given conductivity pattern. Here we quantize the black pixels with a conductivity of 0.9 (ohm m)^{-1} and the white pixels with 0.1 (ohm m)^{-1} . This was done for all 500 random patterns. The electrode voltages are normalized by dividing the maximum electrode voltage of 500 cases. The normalization found to be necessary to have a faster convergence in computations of weights.

7.1.3. Results

The computations are performed on a 386 PC using the neural network program called META-NEURAL. The 300 patterns out of the 500 random cases were used for training the network and

200 were used to test the network. The computations for the training phase took approximately 30 hours to reach to an error of one part in 10. In the test phase the network recognized 90% of the patterns used for the test phase, consistent with the training error.

The method we presented above is yet too simple for real applications. If the number of pixels increased to have more detailed images, say 256x256, more excitations are required to collect enough information. This will cause a tremendous increase in CPU time estimated to take several hundred hours on a super computer. An additional difficulty is the exponentially increasing number of possible combinations of the patterns. In the above example we presented, we chose approximately 10% of the all combinations for training but the network still could not recognize the randomly chosen patterns 100% accurately. If we double the number of pixels (8x8) then we have to choose 25,000 training patterns to have the network trained to recognize the patterns with 3 black pixels. At this point in time, it appears that neural networks do not present an attractive choice for image reconstruction of phase distributions in gas-liquid flows.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

B. NUMERICAL RESULTS

In this part of the report, we summarize the results obtained through the development of numerical methods. This includes the implementation of the basic finite element method (Sect. 1), considerations of the effects of noise (Sect. 2), the use of block reconstruction methods (Sect. 3) including both linear and bilinear exponential shape functions for the conductivity in any given element. While the use of block decomposition methods has resulted in major improvements in computational resources required for imaging, the use of incomplete excitation sets (Sect. 4) has also had a significant effect in time savings during image computations.

1. BASIC FINITE ELEMENT METHOD

Fig. 41 shows the convergence sequence for a 64-element body with real-conductivity elements of 3:1 contrast ratio distributed in a relatively arbitrary pattern. The quantitative resistivity pattern definition is shown at the center of Fig. 42. The original pattern is chosen as uniformly of high conductivity. Gauss elimination was used to perform matrix inversions. The small square patterns are used to visually represent the degree of variation in resistivity. The dense pattern represents material with high conductivity and low resistivity. The less dense the fill pattern, the lower the conductivity as might be the case with liquid and gas mixtures.

It is seen that there is a relatively rapid convergence for elements near the boundary even though the change is from one extreme to the other. On the other hand, changes in the central region require significantly more computation for convergence due to the extreme lack of sensitivity of regions farthest from the boundaries.

Fig. 42 shows two additional patterns with the number of iterations required for convergence. The ring pattern converges much more rapidly than the annular pattern because the zones requiring the greatest changes are nearer the boundary. Furthermore, the central region required no change whereas for the annular geometry, the central zones required maximum change.

2. EFFECT OF NOISE

Global error for the three cases (arbitrary, ring, and annular) is shown in Fig. 43 indicating that the more complex the pattern, the larger the number of iterations required for convergence. (Note that

Figure 41. Typical convergence sequence for a 64 element array of quadrilateral elements with zero permittivity. Contrast ratio: 3:1.

Figure 42. Converged patterns and resistivity index for both annular and ring geometries with zero permittivity. Contrast ratio: 3:1.

Figure 43. Global error for the three patterns tested.

for a 25-element geometry, convergence within 10^{-6} is generally obtained for contrast ratios up to 100:1 within 15 iterations and convergence was obtained with all real contrast ratios tested up to $10^5:1$.)

Using constant grid size^{100,102} noise was shown to limit the accuracy of a reconstruction. This is shown in Fig. 44 for the three cases shown above in Figs. 41 and 42.

Noise can come from many sources as described in Part B, Sect. 3.1. Included in the noise is experimental noise in measurements; computer-generated noise due to machine errors; reconstruction noise due to misalignment between the field properties and the reconstruction grid. The first cannot be avoided to some extent, and in fact the accuracy of the measurements determines the accuracy and resolution of possible reconstruction. The computer-generated noise can be mitigated to a large extent by judicious and careful programming. The noise due to random placement of property boundaries relative to the grid placement can not be avoided in all cases and will also lead to limitations in reconstruction accuracy. Of course, this is the exact problem which must be faced if accurate reconstructions are to be obtained in future work.

Figure 44. Effect of noise on convergence for the annular pattern.

Global error, however, is not the entire story since the global error can be quite low and local error still significant. Figure 45 shows the variation in the local error for each of the four ring layers in the geometry. It is seen that the farther an element is from the boundary where measurements are taken the larger the error. In the case of 1% Gaussian noise, the local error in the inner elements is above 30%. Even in this case, however, the noise has little effect on the visual recognition of the pattern (Fig. 46).

3. BLOCK RECONSTRUCTION

3.1. Results and Discussion

For the first numerical simulation, a linear conductivity distribution is assumed. The reconstruction of the assumed distribution for 5x5 square meshes is simulated to ensure numerical stability. The results shown in Fig. 47 presents the convergence of the block decomposition scheme using five excitations, a factor of five less than the full excitation set. This convergence is typical of that obtained when conductivity variations between 10^{-8} and 10^8 across a given cell were tested. It is noted that

Figure 45. Local error for the annular-flow-like geometry.
 ♦ Inner Cells; • Inner Middle; + Outer Middle; x Outer Cells.

Figure 46. Converged annular pattern with 1% Gaussian noise.

even with a partial set of excitations, the convergence here is approximately one order for each three iterations.

The total computation time on the IBM-3090-200S for 20 iterations was approximately seven seconds including compilation, or about 0.3 seconds per iteration. When compared with the standard Newton-Raphson method without decomposition, the time for a given number of mesh elements is, perhaps, double that of the original method with the same number of excitations. However, now the size of the array can be reduced by a factor of seven or eight. The computational time can therefore be

Figure 47. The convergence of iterations.

decreased by approximately $8^{3.3}$ or about three orders. In addition, a fewer number of excitations are expected to be required for convergence with a coarser mesh further reducing the computation time. This method, when fully developed, may make rapid reconstruction practical.

The iteration sequence for the spatial conductivity distribution is shown in Fig. 48. At the start of the iteration sequence, the original conductivity distribution was assumed to be uniform in the entire field. In this figure, the central conductivity is shown for each element including the defined distribution (target). The conductivity for iterations 2, 5 and 10 are also shown. Note that the two lines show the target and converged conductivity distribution, but offset from one another due to the offset nature of the chart. The final values at iteration #10 agreed with the target values within three parts in 10^5 .

4. INCOMPLETE EXCITATION SETS

The methods utilized to date are based on a least-squares minimization of the error function by variations in the local element conductivity. Since this is not an exact solution of N -equations with N -unknowns, but rather is a minimization approximation, it seems natural to enquire about the use of an incomplete excitation set for the iteration sequence. Note that each excitation deleted from the iteration scheme decreases the computation time linearly.

Consider that the number of 2-pole excitations for an N -electrode system is $M=N(N-1)/2$. Thus, for 16 electrodes, there are 120 independent excitations and for a 32-electrode system, there are 496

Figure 48. Linear conductivity distribution for different iterations. Note that the two lines are identical but offset due to the offset nature of the bar graph.

possible excitations. Clearly, if the problem was an exact solution of M -mesh-element conductivities, there would require M linearly independent equations. However, it may be possible that the minimization problem does not require this number of excitations to obtain a stable solution. If a carefully-chosen subset of M , say $M/10$, $M/20$, or $M/50$ could be used, there could be a 10-, 20-, or 50-times reduction in the computational time. The question is the penalty which might arise in slower convergence, or even nonconvergence of the solution.

A field with small variations typical of that to be expected for bubbly flows was tested on a 16×16 quadrilateral mesh. Results of the image sequence are shown in Fig. 49. This is not a converged solution, but one which has obtained a value of 10^{-7} in the global error. Convergence is very slow at this point, requiring approximately 5 iterations for each order reduction in the global error.

Another concept along the same lines was also tested. This used one set of excitations to initiate the problem and carry the reconstruction until the convergence slowed down. Then, another set of five excitations was utilized to continue the reconstruction. This was found to improve the convergence of the image. Such a situation is shown in Fig. 50 for a case looking like wavy-annular flow. The image converges almost to the original conductivity distribution after 40 iterations, with global error of approximately 10^{-7} based on the original excitation set. After this iteration, a new set of five current excitation patterns was employed. The images improved as a result of the sensitivity introduced by the new pattern excitations with a two-order reduction in global error over ten iterations.

The conclusion we can draw regarding these tests is that a complete set of excitations is not necessarily needed for iterative convergence. Rather, one or more "well chosen" sets can be used in sequence

Figure 49. Numerical simulation for 16 x 16 element having variations similar to bubbly flow.

through the reconstruction. However, the price paid is certainly slower convergence in comparison with earlier results. Furthermore, we have found some cases which will not converge to a solution at all. However, this method shows considerable potential as the 5-excitations utilized herein represented a computational reduction by a factor of 50 over the complete set reconstruction. It is thought that using this method in combination with others has considerable potential. The use of partial excitation sets for experimental data will be described later in the report.

Figure 50. Numerical simulation for 16 x 16 element with supplemental current excitation.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

C. EXPERIMENTAL DESIGN--MECHANICAL

This chapter of the report summarizes the various mechanical equipment designed and constructed as part of the program. First described is the two-dimensional phantom (Sect. 1) and several three-dimensional phantoms including one of rectangular geometry (Sect. 2.1), and several of cylindrical geometry for both resistive (Sects. 2.2.1 and 2.2.2) and capacitive measurements (Sects. 2.2.3 and 2.2.4). In addition to phantoms designed for use with finite element modeling, a phantom was specially designed for three-dimensional back projection analysis (Sect. 2.3). Finally, a natural circulation geometry was designed and constructed to test the ability of the methods developed to image isotherms as they affect the local conductivity (Sect. 3).

1. DESIGN OF 2-DIMENSIONAL PHANTOM

1.1. Two-Dimensional Phantom Concept

A step-by-step numerical and experimental calibration has been configured for image reconstruction. In this manner we will approach the real problem through a sequence of planned stages, each of which can be checked against a known behavior. Firstly, we make sure there is no three-dimension error introduced by using a two-dimension reconstruction algorithm. A square phantom is used in the form of a tank 200 x 200-square, 380-mm deep. Stainless steel rods, 1.59-mm O.D. approximately 400-mm long, are used as vertical electrodes located on the inner surface of the tank to eliminate three-dimension behavior. Fixed insulator calibrators are used made of materials such as nylon, delrin and UH&M, and conductors are also used. Furthermore, empty plastic cylinders are also utilized. As shown in Fig. 51, each 380-mm long calibrator of a given cross section is placed vertically at a known location in the tank which is then filled with water when measurements are taken.

1.1.1 Phantom Geometry and Electrodes

In this section, we will introduce the two-dimensional square phantom from which we obtained data sets for Yorkey's algorithm to reconstruct images. The phantom shown in Fig. 52 is a glass tank 200 mm x 200 mm and 335 mm deep. We chose the depth of the phantom approximately 1.5 times the size of one side in order to eliminate three-dimensional effects of the field.

Figure 51. Point-electrode, two-dimensional phantom used in CMR.

Viewing Direction for subsequent photographs

Figure 52. Two-dimensional square phantom and targets used for experiments.

The electrodes used are 1/16" and 1/8" in diameter stainless steel rod electrodes. The reason for using rod electrodes is to represent point electrode model in Yorkey's method. The electrodes are vertically positioned at equal intervals around the inner surface of the phantom tangential to the inner wall of

the phantom. This phantom is designed so that we can vary the number of electrodes to make measurements for different electrode numbers. The maximum number of electrodes in this phantom can be increased up to 64.

1.1.2. Overall Concept and Justification

As report in Geddes et al. the contact impedance between stainless and saline water is a function of current density and frequency. For higher current density, the interface resistance between saline water and stainless electrodes will be lower. As far as is known, a chemical reaction between stainless steel and saline water is responsible for this influence.

The dielectric constant of pure water, as reported by Franks,³⁴ is invariant with frequency from D.C. up to beyond 10^8 Hz. Considering contact impedance around $100\ \Omega$ reported by Yorkey,¹⁵⁷ if pure water is used, the contact resistance should be negligible relative to the resistance through the bath itself.

Issacson⁷⁵ proved that a larger area electrode provides a high distinguishability. However, Hua⁶² et al. indicated larger electrodes will introduce high geometrical error into the conventional finite element model which is not equipped to handle anything but a point electrode. To simulate a point electrode in the two-dimensional sense, 1.59-mm O.D. by approximately 400-mm long stainless steel rods are used as electrodes.

In order to avoid the ionization of the pure water, a 10V peak-peak voltage with a low frequency of 60–200Hz A.C. current has been applied for excitation of the phantom. The tested results of electrical resistivity of pure water is $4,850\ \Omega\text{m}$ compared to the real value of $5,000\ \Omega\text{m}$ reported by Duffin³¹ at room temperature, indicating that our instrumentation and methods are reasonably accurate.

By making the phantom an essentially two-dimensional geometry, we have been able to eliminate errors due to three-dimensional effects. The voltage variations in the axial direction were measured and determined to be on the order of 0.1 % relative to the lateral applied voltages due to excitation.

Experiments using two 200 x 400-mm parallel plates indicated the amplitude-response of electrical resistivity is flat. That is, no variation in electrical resistivity could be determined. The reactive component at the frequency 60 Hz compared with the resistive component for the parallel tested stainless plate is around a factor of 10^{-8} . Thus, the phantom used has been determined to operate in a purely two-dimensional, resistive mode, allowing exact solutions to be obtained to compare with our reconstruction algorithms.

2. DESIGN OF 3-DIMENSIONAL PHANTOMS

2.1. Rectangular Geometry

The 2-Dimensional phantom has modified to 3-Dimensional phantom by changing the 2-dimensional rod electrodes to 3-dimensional rod electrodes. The 3-dimensional rod electrodes are designed 1/8" in diameter and 2 inches high (see Fig. 53).

Figure 53. 3-D phantom and electrodes.

The electrode height can be reduced to get more detailed 3-dimensional information for conductivity information. However, the 3-dimensional distortion becomes dominant causing insensitivity to the size of the targets.

2.2. Cylindrical Phantoms

2.2.1. Resistive Phantom--First Design

This phantom was the first design to be tested and has been described in previous reports. The design called for the phantom to have a inside diameter of 57.15 mm (2.25"), with 32 electrodes, around the circumference of this diameter. The electrodes were to be cut out of 3.175-mm thick (1/8") copper sheet and then milled to be 4.76-mm square (3/16" x 3/16"). Triaxial cable was then soldered to each electrode through a hole drilled in the center of the electrode face. The electrode-cable assemblies were then to be epoxied into a plastic ring in which 32 radial slots had been milled in one face. The back face of the electrodes were designed to be placed 0.76 mm (0.030") offset from the diameter of 57.15 mm. The assembly would then be placed in the lathe and the center diameter would be bored out to the diameter of 57.15 mm, leaving a 0.76-mm thick ring of electrodes.

To determine the feasibility of construction and testing of this phantom, a mock-up was constructed using only four electrode-cable assemblies, evenly spaced about the circumference. Difficulty in boring out the center diameter was encountered due to two electrodes catching on the cutting bit, and pulling out of the ring. This was attributed to the inability to accurately offset the electrodes, as well as ensuring good epoxy coverage over the back face of the electrodes. While this phantom has been used for subsequent development, this design concept was not utilized for further work with resistive media.

2.2.2. Resistive Phantom--Second Design

The second design for the resistive phantom (Fig. 54) used chemical etching to construct the electrodes from copper sheet electrode deposited to a thickness of 0.038 mm on a mylar backing. The (32) electrodes are 4.76-mm (3/16") high, but are 4.595-mm (0.1809") wide, with 1.02-mm (0.04") spacing between electrodes. The electrode strip is placed on the inside diameter of ring that has 32 radial slots on the top face. The inside diameter of the ring is 57.15 mm (2.25"), with a recessed slot around the circumference for the strip of electrodes. The electrodes have coaxial cable soldered to them from the back of the strip, so that the electrodes face inward. The cable-electrode strip is then epoxied into the ring, with a solid ring placed on the top face, so as to sandwich the cables into the ring. The ring assembly then has a tube with inside diameter of 57.15 mm (2.25"), attached to the top and bottom faces by flanges. This tube assembly is then mounted on a metal base for stability. A photograph of the resistive phantom is shown in Fig. 55.

2.2.3. Capacitive Phantom--First Design

This first capacitive phantom is shown in Fig. 56. The inside diameter of the tube was 57.15 mm (2.25"), with 32 electrodes on the outside of the pipe at a diameter of 66 mm (2.60"). These electrodes were constructed by chemically etching them from copper sheet on mylar backing. The electrodes were 4.76-mm square (3/16" x 3/16") with a 6.35-mm (1/4") wide tab connected to the top of each. This tab was folded back along the top of the clamping ring and used for connections to the appropriate cabling. The clamping ring was used to hold the strip of electrodes in place, and to act as support for the cabling. This phantom was constructed and tested but during testing was found to have too much parasitic capacitance between electrodes (at the cable-electrode connections) due to the connection geometry.

2.2.4. Capacitive Phantom--Second Design

This phantom used the same ideas for construction of the electrodes as the second-design resistive phantom, and was expected to eliminate the parasitic capacitance between electrodes. As shown in Fig. 57, the 32 electrodes were also chemically etched on 0.038-mm thick copper-mylar sheet. The pads are 4.76-mm (3/16") high, but were 5.22-mm (0.2054") wide leaving 1.02-mm (0.040") spacing between each electrode. The strip was placed inside a plastic ring, that had 32 radial grooves milled in the top face. The inside diameter of the ring was 62.48 mm (2.46"), with a recessed slot (ID=63.5 mm) around the circumference for the strip of electrodes. The electrodes were soldered to coaxial cable from the backside of the mylar strip (the electrodes are on the inside diameter of the strip). Parasitic capacitance was thus minimized with this design. The electrode-cable strip was epoxied into the ring with a solid ring placed on top, to sandwich the cables down into the radial grooves. The ring was cleaned of excess epoxy and placed over a tube which had been turned on a lathe to an inside diameter of 57.15 mm (2.25") and an outside diameter of 62.23 mm (2.45").

The capacitive phantom (Fig. 58) was completed in August, 1992. When tested, it was found that the small capacitance between opposite electrodes (order of 10^2 fF) could not be determined with suffi-

Figure 54. Drawing of second resistive phantom design.

Figure 55. Front and top views of the resistive phantom.

Figure 56. Drawing of first capacitive phantom design.

Figure 57. Drawing of second capacitive phantom design.

Figure 58. Capacitive Phantom

cient accuracy. Since these electrodes were deemed the largest acceptable for sufficient resolution of gas-liquid structure, it was decided to concentrate on resistive imaging.

2.3. Back Projection Geometry

The 3-D back projection phantom consists of a Plexiglass cylinder, 150-mm in diameter and 600-mm high as shown in Fig. 59. Thirty two (32) electrodes were utilized for excitation and measurement, arranged in four levels of eight electrodes each. For these experiments, the "A" level was used as a source & measurement level (any electrode on level A can be a source). Level "B" was a sink level. thus, levels A and B provided the excitation pairs. Electrodes consisted of circular washers, 25-mm in diameter, affixed to the inside of the Plexiglass test assembly.

The first set of electrodes tested were point-electrodes consisting of 32 bolts approximately 3-mm in diameter inserted through the wall of the phantom. It was found, however, that these point electrodes provided insufficient current to obtain a good image with distilled water. No testing was undertaken with salt dissolved to lower the resistivity because with lower water impedance because of potential errors associated with electrode contact impedance. Electrodes were enlarged to 25-mm diameter centered at the same locations. Successful imaging was undertaken with this final design.

Level D

Level C Figure 59. 3-D Cylindrical

Level B Phantom, 150-mm diameter,

Level A 600-mm high, Plexiglass.

3. NATURAL CIRCULATION TEST

The purpose of this part of the project was to design and manufacture a small scale water bath to be used in subsequent natural convection tests with hopes of replicating conditions of natural convection in larger scale applications. Of interest was possible ability to image the thermal patterns in the tank based on the changing electrical resistivity with temperature. This particular experiment has been designed to model a radial section of a boiling water reactor (BWR) as shown in Fig. 60.

3.1 Tank Design

The tank is a 15 degree section of the suppression and outer pools of a BWR. The heater approximates heating from the core and the aluminum wall presents a means of heat transfer to the outer pool and coolant system.

Figure 60. Overall drawing of natural convection apparatus for thermal imaging tests.

Upon consideration of geometries suggested by Hitachi, a design was chosen with a emphasis on vertical length to provide optimal heat transfer. A vertical aluminum wall was placed in the tank, separating it into two pools, between which heat transfer could occur. Aluminum was chosen for this purpose because of its high coefficient of heat transfer, low cost, and ease of machining.

A 2.0 kilowatt electric heater was selected as a heat source. The heater, installed vertically, was placed on the opposite side of the tank from the aluminum wall to maximize heat transfer by convection.

As a means of taking data in this experiment, thin, rod-electrodes oriented perpendicular with the inner surface of the apparatus, were placed evenly around the inner surface of the tank walls. Leads for the electrodes run directly through the wall to the outside of the tank using standard, bulkhead-type BNC fittings and coaxial cable connected to the imaging electronics.

3.2. Heater Control Design

Due to the highly sensitive means of collecting data in this experiment, special care was necessary when selecting electronic components for the heater control. The most important of these was the

Figure 61. Schematic of temperature control system for natural convection system.

need to minimize radio frequency interference (RFI), as any source of electronic “noise” near the convection tank would adversely affect measurements taken from the electrodes.

The overall schematic of the system is shown in Fig. 61. A solid state, silicone-controlled rectifier (SCR), relay (Omega model SSR240AC25) was chosen combined with a three-mode controller using zero-phase-firing, to avoid this problem. In an SSR, on-off switching takes place at the zero voltage crossover point of the alternating current cycle. As a result, no appreciable electrical noise is generated during switching.

This experiment is based on determining conduction currents via recorded changes in the resistivity of the water. Since the resistivity of pure water changes approximately 0.5% for every 2 degrees Celsius, it is important to be able to maintain constant control over the temperature of the water. The temperature controller (Omega model CN-2011K) features a Ramp and Soak function which provides the capability to maintain either a constant temperature or a constant rate of change over a pre-determined time interval.

3.3. Experience

It was found that the point electrodes, when used with our imaging electronics, had insufficient area when used with distilled water to provide the necessary current for imaging. Furthermore, when replaced with rod-electrodes oriented vertically along the inner surface of the tank, no images were obtained. This appears to be due to the still immature nature of our quadrilateral-element, exponential, block decomposition, finite element model (Q_uE-EBDeFEM). The quadrilateral mesh generator, when added to the square-element, EBDeFEM, yields exponentials of integrals which can have very large exponents. Differences are then obtained between these large numbers. The combination of this problem with exponential overflow or underflow in the computers has effectively prevented us from obtaining usable images for this model.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

D. EXPERIMENTAL DESIGN--ELECTRONIC

In this chapter, we summarize the electronic design results undertaken to complete the imaging measurement. This includes the high accuracy system (Sect. 1) including detailed circuit board design and layout, noise reduction methods, computer interfacing, phase measurement, accuracy considerations, and software. Initial design considerations for the PACIFIC-1 high speed system is also discussed in Sect. 2.

1. HIGH ACCURACY SYSTEM

1.1 Static System for High Accuracy Imaging

Electrical impedance tomography reconstructs a resistivity distribution by making electrical measurements on the objects periphery.⁸¹ This part of the report describes a high accuracy switching matrix measurement system designed for testing the reconstruction of electrical impedance imaging. It is the purpose of this system to determine the limits of accuracy and resolution of impedance imaging in low-resistivity, gas-liquid systems, and to aid in numerical and analytical methods development in this area. All switches are shielded relays, which are controlled by TTL-level signals from the IBM personal computer. Using this switching system, we can excite any two electrodes and measure the responses of the other two electrodes in all possible combinations.

1.1.1 Experimental System Design

Static experiments can measure the magnitude and phase of the response signal to an applied sinusoidal excitation of voltage, with the voltage being measured and utilized in the imaging computations (Fig. 62). A Zenith 386/25 personal computer is utilized to control and acquire data from several instruments through General Purpose Interface Bus (GPIB). The signal generator is utilized to excite the test section with the desired sinusoidal signal. Test signal amplitude data are obtained using a Hewlett Packard HP 3458A 8-digit integrating digital voltmeter to an accuracy of 10^{-6} with AC (Range 10V). The phase between the applied signal and the measured response is obtained by using a HP 54510A digital oscilloscope with time base resolution of 20 ps. This computer will also be used for controlling matrix switching through parallel digital I/O interface (PIO).

Figure 62. Photograph of the test assembly mock-up.

1.1.2 Detailed Circuit Diagrams and PC Board Layouts

The original concept for the electronic system was to have been an extension of methods used in the biomedical area (Fig. 63). Figure 64 shows the system block diagram.

The matrix switching system is comprised of 160 shielded switches for 32 channels: 32 each for switching excitation high signals; 32 each for low signals to any or all of 32 electrodes on the phantom test device; 32x2 each for switching high side voltage measurement to either side of the current-measuring resistors; and 32 for switching the low-side of the digital voltmeter or oscilloscope to any electrodes. All switches are CP Clare DSS3 relays, which are controlled by a TTL-level signal from the IBM personal computer through a Keithley Metrabyte PIO-96(96-channel digital input/output control board). Using this switching system, we can excite any two electrodes and measure the responses of the other two electrodes in all possible combinations. A schematic diagram of the matrix switching system is shown in Fig. 65.

In order to obtain high signal to noise ratio, the layout of the printed-circuit-board (PCB) is designed carefully. All considerations of noise reduction are applied in the PCB design. Figure 66 shows PCB configuration while Fig. 67 and Fig. 68 show the component-side and solder-side layout of PCB. Figures 69 and 70 show photographs of the component- and solder-sides of the completed circuit

Figure 63. Schematic of excitation and measurement system.

board as tested in a slightly different configuration. The board configuration was later modified to allow assembly into an RF-shielded module of a rack.

1.1.3 Noise Reduction

Our object is to design a high accuracy, static switching matrix system. High sensitivity measurements are subject to common mode noise (CMN). Normally, CMN is avoided by breaking the ground loop and employing other methods like grounding, guarding, shielding, decoupling, driven guarding, etc.. The reader is directed to Ref. 108 for an excellent discussion of these factors summarized below.

Sources of noise. The measurement of low level signals in a test system environment can best be accomplished with careful attention to the details of grounding and shielding. The following are major sources of noise coupling into these systems: conductively-coupled noise; coupling through a common impedance; electric- and magnetic-field coupled noise.

The following common noise reduction techniques apply to single point systems and multipoint systems: grounding; guarding; shielding; decoupling; driven guard. There are discussed in turn.

GROUNDING

The design objectives of a grounding system are:

- To minimize the noise voltage generated by currents from two or more circuits flowing through a common ground impedance.
- To avoid creating ground loops that are susceptible to magnetic fields and differences in ground potential.

Single Point Grounding To Avoid Creating Ground Loops. Lower frequency, high sensitivity measurements are subject to common mode noise. This noise can be avoided by breaking the ground loop with a floating input. The parallel single point ground is the most desirable configuration for

Figure 64. Block diagram of the matrix switching system.

frequencies in the audio range. This configuration eliminates cross coupling between ground currents through different circuits, avoids creating ground loops, and provides good low-frequency grounding when $f < 1$ Mhz.

Separate Ground Returns. For practical reasons, most systems require three separate ground returns. Signal grounds should be separated from hardware grounds and from noisy grounds such as those emanating from digital circuitry. Separate ground returns, carefully designed to avoid ground loops, can reduce mutual interface.

Wide Ground System. The ground distribution system is designed to be as wide as possible everywhere. In addition, each signal line is separated by a ground line.

SHIELDING

Figure 65. Schematic diagram of the matrix switching system.

Shielding against noise in systems involves shielding against both capacitive (electric) and inductive (magnetic) coupling in the system.

Surrounding the Signal Conductor with a Ground shield. The coupling noise from one conductor to the other is a result of the capacitance between the conductors and expressed as

$$V_n = j\omega R C V_s \quad (158)$$

where V_n is the noise voltage, V_s the signal voltage, ω the frequency, and RC the coupling time constant between the two conductors.

If we could reduce the capacitance, C , then the noise voltage coupled into the conductor would be reduced. Surrounding the signal conductor with a grounded shield and a uniform electric field re-

Figure 67. PC board layout of component side of the high accuracy circuit as modified to be compatible with cabinets ordered for the system.

Figure 68. PC board layout of solder-side of the high accuracy circuit as modified to be compatible with cabinets ordered for the system.

Figure 69. Photograph of component side of high accuracy board as tested.

Figure 70. Photograph of solder side of high accuracy board as tested.

duces the noise voltage on the signal conductor. Lines can be isolated by running ground conductors parallel to critical or noise-generating lines. These ground lines then act as a shield.

Since inductive noise coupling is proportional to the loop area cutting the magnetic flux of the noise, receiver circuits can be best protected against magnetic fields by decreasing the area of the receiver loop. For maximum protection against induced noise at low frequencies, the shield should not be one of the signal conductors. So, one end of the circuit must be isolated from the ground.

Creating a Ground Plane to Achieve a Shielding Effect. The copper surface will be largely given over to the ground plane to maximize overall shielding effects.

GUARDING

A guarded instrument has an additional shield between the low and ground, effectively increasing the low to ground-leakage-impedance. The guard terminal should be connected closely to the low terminal so that the guard terminal and the low terminal are at or close to being at identical the same voltages. Therefore, no common mode(CM) current goes through any of the source resistance. The guard increases the resistance and decreases the total capacitance. It shunts the CM current paths with typical rejection of 160dB (DC),140 dB (AC).

DRIVEN GUARD

The signal line connects to the driven guard line through a buffer amplifier at the source. At the receiving end, the guarding line is left floating. Thus providing a "Faraday" shield between the guarding line and signal line. In this way, the effect of distributed capacitance of the cable is greatly reduced. Guarding is not only provided to the external cables, but each signal line on the PC board is run between two guard lines throughout, these guard lines being driven by the same buffer amplifier as the external guard. In addition, the guarding is carried through all the switches by connecting the guard lines to the internal r-f shield of each switch.

TREE SWITCHING MODEL TO REDUCE THE STRAY CAPACITANCE AND CROSS-TALK

Tree switching is often used to reduce the stray open switch capacitance in systems resulting from connecting the unused relays in parallel in the system. This method is utilized in this design.

DECOUPLING

"Decoupling" provides means to dissociate circuit functions from the power bus serving that circuit. In practice, this includes adding decoupling and power-entry capacitors, and appropriate printed-circuit board (PCB) layout. Good decoupling decreases radiated and conducted emission from equipment and minimizes susceptibility to noise. Methods include:

- Leaving as much copper as possible on the PCB.
- Using decoupling capacitors and placing them close to their respective components. These capacitors reduce noise and minimize voltage change due to load variation.

- Keeping the Vcc and ground traces reasonably close together.
- Reducing loop areas, which always corresponds to a reduction of inductances and vice-versa.
- Keeping the highest-current solid state device(DIPs) as close as possible to the power entry point in order to minimize ground noise.
- Using additional edge connector pins for ground.

FLOATING MEASUREMENTS

Measurements with differences between grounds, or common mode voltages, are called floating measurements and are said to "float" by the amount of common mode voltage. An ideal floating measurement would be insensitive to the common mode signals. It would measure only the normal mode input, no matter how much common mode noise it had. A floating meter, like HP 3458A DVM, can reduce the effects of common mode voltages and is utilized for this work.

1.1.4 Computer Interfacing

The whole system is controlled by an IBM personal computer. Matrix switching is controlled by IBM compatible personal computer (PC) through a MetraByte's PIO-96, 96-channel digital input/output (I/O) board.⁹⁰ Measurement instruments are controlled through MetraByte's GPIB interface card. MetraByte's PIO-96 is a high density parallel digital I/O card that allows up to 96 TTL/DTL/CMOS compatible digital I/O lines to be controlled by an IBM PC/XT/AT. The board consists of 4 industry-standard 8255 Programmable Peripheral Interface (PPI) chips each providing three 1-byte ports: PA, PB, and PC (8 bits). The PC port can be further subdivided into 2 nybble-wide (4 bit) ports. Each of the PA, PB, PC-upper and PC-lower ports can be programmed through the 8255 write-only control register, and each can be either an input or output port. After power up or whenever the computer's hardware RESET line is asserted, all 8255 lines are cleared to the digital input state on all ports. All of the standard features of the 8255 PPI are supported including operation in Modes 0, 1 and 2. Mode 0 means basic I/O, all ports are I/O ports; Mode 1 means strobed I/O, part of the PC port controls data transfer; Mode 3 means bidirectional I/O on PA only, part of the PC controls data transfer. Each digital output line has a sink current capability of 1.7 mA (one standard, or four LS TTL loads). Each 8255 uses four consecutive I/O addresses so that the PIO-96 requires a contiguous block of 16 I/O addresses. The base I/O address can be set on any 16 bit boundary in the PC's I/O address range of 200-3FO Hex (100-3FO Hex for PC/AT) by a switch. More than one PIO-96 may be installed in a computer; provided they are set to different base addresses, there is no conflict with memory because no memory addresses are used.

Each 8255 uses a 50 pin double row 0.1" header for its 24 I/O connections. This allows for a ground drain wire using standard flat insulation displacement cable (50 way 0.05", centers) between each I/O pin to provide maximum shielding and minimize cross talk. Power from the computer's +5v is also brought out on 1 pin of each connector. These pins are mainly intended for use with pull up resistors etc., but may also be used them for powering accessories as long as computer's available

power ratings are observed (taking into account other adapter boards) and as long as each connector pin current limit rating is 1 amp.

The DVM, function generator and oscilloscope are connected to computer through the GPIB interface. We can use GPIB to set up these instruments, control the whole measuring process and get data from DVM and scope. In order to obtain high accuracy, we calibrate instruments for every excitation, and we obtain effective data by using the oversampling method to ensure the average reading has settled to the maximum resolution of the instrument.

1.1.5 Phase-sensitive measurement

Using the 32-channel high accuracy static system, we can measure the phase difference between the exciting and responding signals.

We output two measurement system signals (i.e. excitation and response signals) to HP-54510A Digitizing Oscilloscope first, then program the oscilloscope settings and acquisition over the GPIB. The most important point is that the oscilloscope digitizes two channel signals simultaneously, the exciting signal and the responding signals. After digitizing, two wave forms, data are transferred to controller computer. Comparing those two sets data, allows the phase information to be obtained from zero voltage cross-over. While this method is slow, it is suitable to our cost-less static measurement system.

1.1.6 Accuracy

We have done several experiments to test the system accuracy. The results revealed that the accuracy of the data acquisition system is better than 0.5% with the target impedance around 200k Ω . When the target impedance increased, the resolution decreased. This phenomena is caused by low input impedance (10 M Ω) of digital voltmeter. Data are recorded and ensemble-averaged until the per-measurement variation in the average is reduced below one part in 10^4 . This error is verified by performing nodal averaging of the reconstructed images with results having the same order of magnitude error in comparison with measurements.

In addition, we compared the FFTs (Fast Fourier Transform) of original signal (Fig. 71) and the corresponding signal (Fig. 72) after passing through the excitation/measurement system. The two FFTs gave us the same result showing that the measurement system introduces no spurious noise or signals.

1.1.7 Software

The PIO-96 is programmed using simple 8 bit I/O port commands e.g. BASIC's INP and OUT command. We set three ports as output mode. When one of the pins is low, the switch controlled by this pin closes. When it is high, the switch opens. The basic control command is the following:

- 10 out &H303, &H80

Figure 71. FFT of the original signal.

Figure 72. FFT of the signal after passing through the measurement system.

- 20 out &H300, control-word1
- 30 out &H301, control-word2
- 40 out &H302, control-word3

Turning these switches on/off is effected by different control-words, thereby selecting different combinations of switching for excitation and measurement. The measurement controlling software is listed in Appendix 2.3.^{59, 60, 61}

This high accuracy system has been tested and utilized in taking all data sets reported in this report. Each PC board was installed into an individual metal module to provide inter-board shielding and reduce noise from the surroundings. All modules as well as the separate power supply module are settled in a rack. A total of eight boards were constructed to enable measurements to be taken for a total of 32 separate electrodes.

2. DESIGN OF PARALLEL, APPLIED CURRENT IMAGING SYSTEM WITH FAST INTEGRATED COMPONENTS - PACIFIC-1

2.1. Introduction

An electrical impedance tomography instrument has one or more current or voltage sources that inject AC into an object to be imaged internally. For current injection, the current sources require a control circuit. A single current source must be switched into many different electrode combinations. Multiple current sources require a control circuit that adjusts both the magnitude of each current source and its destination (multiplexing) to achieve the desired patterns of current density.

A single voltmeter consists of a differential amplifier, a phase-sensitive demodulator, a low-pass filter, and switches that can operate in parallel to speed data acquisition time. After the data are in the computer, reconstruction algorithms process them to display the image of the resistivity distribution in the object.

As we know, a four-electrodes system helps to reduce the problems with contact impedance experience by two-electrodes. Still in a four-electrodes system, contact impedance can be a factor affecting system accuracy. So a good current source has high output impedance, while a good voltage source has low output impedance. This means that while the contact impedance will be negligible for a current source, it will degrade the operation of a voltage source. Likewise, the input impedance of a voltmeter is high, while the input impedance of an ammeter should be near zero. This suggests that a system using current injection and voltage measurement would produce the best results with an unknown, varying contact impedance.

The imaging hardware has three major parts: the timing oscillator, the current source, and the voltmeter. The reactive properties are detected by the phase difference between the current signal and the resulting voltage signal. To make phase measurements, the current sources must all be coherent, meaning that all current sources have the same phase. This coherence is accomplished by having one master oscillator for the entire system. This oscillator then feeds the voltage controlled current sources.

Another consideration is data collection time. The time that is taken to collect the data for one picture is analogous to the time the shutter of a camera is open. The greater the time needed during the data collection process, the worse the temporal resolution of the pictures. For static, or slow moving events, the allowable data collection time may be longer than for quick events.

It is planned to expand the PC-Bus outside a basic computer, utilizing 32 parallel excitation, acquisition, and storage systems, one for each electrode. Each part of the system will be connected together through the PC-Bus. The host's CPU or DSP (Digital Signal Processing chip) can access any part of RAM or use DMA (Direct Memory Access) to transfer data from one block of RAM to another. Figure 73 shows the overall system block diagram.

In order to get a clear picture of distribution of vapor in the flow, it is ideal to take a "snapshot;" i.e., get the data instantaneously for a single picture. What this really means is that the data excitation and

Figure 73. Overall system block diagram

acquisition process for a single image must occur over a time span which is short relative to that characteristic of system or process changes or of fluid motions. To the extent that this is impossible, temporal blur will be found similar to that which occurs in photographing a moving target. The ideal system will have similar temporal and spatial resolutions (blur).

For imaging of two-phase flows it is, therefore, necessary to implement a high speed excitation and measurement. It is known, and we have verified, that using multiple-pole excitation provides enhanced sensitivity and better image clarity than does single-pole excitation. We also know from the work done by the Rensselaer Biomedical Imaging Group that use of spatially-sinusoidal excitation waveforms produces optimum clarity for a centrally-located target in a uniform field. For this reason, and considering the accuracy of the measurement, a spatial step function (Walsh function) generator will be utilized in our system to achieve suitable speed and accuracy instead of a sinusoid waveform. In effect, this will have both the low order sinusoidal distributions but also higher order sinusoids associated with the Fourier components of the waveform. To eliminate the DC effect on the whole region, we will implement step functions in time, and walsh function in space. The temporal step functions of current injection will provide the necessary alternating current effect to preclude polarization of the medium. Our goal is to obtain 1000 images per second.

2.2. Preliminary Specifications Overview of PACIFIC-I

The goals for PACIFIC-I are to build a much faster system with excellent measurement accuracy using oversampling methods. The one major limitation found in the research to date, the inability to obtain sufficiently large capacitive response in sizes of interest, is utilized to our advantage in this design, thereby enabling significant speed improvements in PACIFIC-I. Thus, high frequency excitation is not required to provide quadrature measurements. Furthermore, alternating current sources are not required, and ionic mobility effects (charge buildup in the system) are eliminated by

the electrode (polarity) switching patterns using fixed DC currents rather than AC currents. The following design parameters appear appropriate for PACIFIC-I at this time and have been used in our design

- The overall system should accommodate 32 electrodes operating in parallel. Individual current sources for each electrodes are used, injecting 31 current patterns in rapid succession. The total voltages are measured simultaneously on all electrodes for each current pattern.
- The instrument should be capable of gathering and storing all the data for one picture in 1 ms (this is 1000 frame per second), that means we should put 1 Meg $\times$ 16 RAM on each board.
- Overall system resolution is 14 bits.
- The current generators should have 16 bits of resolution.
- The current generator maximum output should be 1 mA.
- The current generator should have the flexibility to supply different frequency signals. The maximum frequency of the square wave should be 50 KHz.
- Current generator should have 16 Mohms output impedance, and the voltmeter should have 16 Mohms input impedance, which will be acceptable for the case where the phantom loading for the current source is near 1 K Ω .
- The voltmeter should have 14 bits of resolution (or 16–18 bits using the oversampling technique).
- Each channel contains a DSP which controls the digital synthesis of the excitation waveform, and adjusts its amplitude with 16-bit precision. The DSP also acquires and processes voltage data from an on-board ADC. Only real voltage magnitudes will be obtained. The DSP also acquires and processes voltage data from analog-to-digital converter (ADC).
- Parallel DSP's may also be added for real-time imaging.
- DMA transfer will be utilized to move data between the host computer and each data acquisition board.
- Optical isolator parts may be utilized to reduce noise.
- The entire system should operate at several speeds.
- Noise will be reduced using optical isolator parts.

2.3. Implementation of a Digital Step Function Generator

The applications of step functions are numerous. The digital approach has several advantages which include ease of control, flexibility, stability, and low cost. Using a digital signal processor (DSP) provides complete control and the speed required to generate a wide range of step function frequencies, with the flexibility to generate other types of wave form.

In this project, we have chosen a Texas Instruments TMS320C25 DSP because of its performance, popularity and low cost. A block diagram of the function generator is shown in Fig. 74. The DSP's

Figure 74. System block diagram.

program memory and data memory can be accessed by the host computer when the DSP is halted, for the purpose of loading the DSP program and transferring data. Communications from the PC to the DSP are handled via the I/O registers of the computer. Before the DSP is operated, the user must download the control program into DSP's memory. Using a look-up table (LUT) method, the DSP can create the needed waveform.

2.4. The TMS320C25 DSP

The TMS320 family's unique versatility and real time performance offer flexible design approaches in a variety of applications. Key features of TMS320C25 device which are concerned here is listed below.¹³⁹

- 100-ns instruction cycle timing
- 544-word programmable on-chip data RAM
- 128K-word total data/program memory space
- single-cycle multiply/accumulate instruction
- 32-bit ALU/accumulator
- 16- × 16-bit parallel multiplier with a 32-bit product

After the host computer transfers the control to 'C25, the 'C25 manages the whole data acquisition process including coordinating the DAC and ADC timing, and memory access.

2.5. The Analog-to-Digital Converter

The A/D converter is a key component in data acquisition portions of high accuracy measurement systems. High resolution A/D converters have typically used successive approximation techniques with complicated trimming/calibration or dual-ramp conversion techniques which required accu-

Figure 75. Internal block diagram of the DSP56ADC16

rate comparators and expensive samples-and-hold (S/H) circuits to yield over 15-bits of accuracy. In addition, the anti-aliasing filter for these A/D converters generally sets severe limitations on the attainable signal-to-noise ratio and phase linearity.

The Motorola DSP56ADC16¹⁰⁹ is a single-chip, linear, 16-bit oversampling analog-to-digital (A/D) converter, providing output samples rates up to 100 KHz. Third order noise shaping sigma-delta technology is employed utilizing 64 times oversampling which yield 96 dB dynamic range and 90dB signal-to-noise ratio (SNR) for the signal bandwidth from 0 to 45.5 KHz with an in-band ripple of less than 0.001 dB.

The DSP56ADC16 uses an advanced third order sigma-delta quantizer to implement an over-sampled noise shaping A/D converter system on a single chip. By oversampling the input signal, the overall quantization noise spectrum expands well beyond the frequency band of interest. Third order noise shaping insures that this expanded noise spectrum contains very little noise power in the pass-band. The oversampled signal is lowpass filtered, effectively removing the out-of-band quantization noise. The lowpass filtering is then followed by decimation to reduce the output sample rate commensurate with the frequency band of interest and to increase the resolution. Since the input signal is oversampled by a factor of 64, the need for a high order anti-aliasing filter can be eliminated. Another good feature is an output provided before the final FIR decimation filter, this output being available for higher speed application. This intermediate output has 12-bit resolution for AC levels. The output rate from the Comb filter is, therefore, up to 400KHz. Fig. 75 show the block diagram of DSP56ADC16. Figures 76, 77 and 78 show the response of filters inside DSP56ADC16.

The maximum input voltage range is a function of the reference input voltage, V_{refin} , which is expressed as

$$\text{Maximum Input Range} = 2V_{refin} - 0.5V \quad (159)$$

Figure 76. Magnitude response of the Comb filter (CLKIN=12 MHz).

Figure 77. Magnitude response of bypass FIR filter.

Figure 78. Magnitude response of the FIR filter with compensation.

with $V_{refin} = 2V$, input range is 3.5V full-scale differential inputs.

Using its flexible serial interface, DSP56ADC16 can easily be interfaced to the host processor TMS320C25. In addition, two DSP56ADC16's can be multiplexed on a single serial port. Thus, one DSP can control two channels. This advantage reduces the cost of system. Because the two converts must synchronously be sampled, two 8-bit shift registers may be clocked by the SCO which may be connected to the second converter SDO pin to delay 16 SCO cycles. Final configuration of the ADC and interface with DSP is shown in Fig. 79.

Figure 79. Analog-to-Digital (ADC) interface.

2 of 2

Figure 80. $\pm 10V$ bipolar voltage output with gain and offset adjustment.

2.6. Digital-to-Analog Converter

A DAC accepts an n -bit input word $b_1b_2b_3\dots b_n$ of fractional binary D_i and produces an analog signal proportional to D_i .³³ The output can be either a voltage or a current. Controlled by a μP (microprocessor), the DAC can almost produce any kind of wave form.

Analog Devices's AD669 is a complete 16-bit, monolithic D/A converter with an on-board reference and output amplifier¹. The ad669's architecture insures 15-bit monotonicity over temperature. Integral nonlinearity is maintained at $\pm 0.003\%$, while differential nonlinearity is $\pm 0.003\%$ max. The on-chip output amplifier provides a voltage output setting time of $10\ \mu s$ to within $1/2$ least significant bit (LSB) for a full step.

Data is loaded into the AD699 in a parallel 16-bit format. It is very simple to interface AD669 to the DSP chip type 'C25. The output range of the AD669 is pin programmable and must be set to provide a unipolar output range of $0V$ to $+10V$ or a bipolar output range of $-10V$ to $+10V$. Fig. 80 shows the bipolar output configuration of output range $\pm 10V$. Gain offset and bipolar zero errors can be adjusted to zero by adjusted R1 and R2.

2.7. High precision voltage reference

High resolution ADC and DAC require a high precision voltage reference to guarantee high performance. Analog Devices's AD588 is such a high precision programmable voltage reference with low temperature drift ($1.5\text{ppm}/^\circ C$) and low initial error ($\pm 1\text{ mV}$).² It offers 12-bit absolute accuracy without any user adjustments. Optional fine-trim connections are provided for higher precision requirement. A precision $\pm 5V$ tracking mode is available with no external components. Tracking error is less than 1 mV and a fine-trim is available for exact symmetry between the $+5V$ and $-5V$ outputs.

Figure 81. AD588 calibration circuit.

Output noise of AD588 is very low (typically $6\mu\text{V}$ p-p). A pin is provided for additional noise filtering using an external capacitor. Fig. 81 shows calibration circuit for AD588. Gain adjust and balance adjust can be used to trim the magnitude of the span voltage and the position of the center tap within the span. The GAIN trim should be performed first.

2.8. Signal conditioning and Voltage-to-current converter Circuit

The response voltage on the electrode as a result of the applied current cannot be applied directly to the ADC, because input impedance of DSP56ADC16 does not meet the requirement. A impedance transfer buffer is needed here. Analog Devices's precision, unit-gain differential amplifier AMP-03 seems suitable.³ The key features are:

- High CMRR 100dB Typ
- Low nonlinearity 0.001% Max
- Low Distortion 0.001% Typ
- Wide bandwidth 3Mhz Typ
- Fast slew rate 9.5V/ μs Typ
- Fast setting (0.01%) 1 μs Typ

Fast slew rate and Fast setting time are just what we need here. Another good feature the AMP-03 offers is superior performance for high speed current sources. Figure 82 and 83 show the configuration of AMP-03 for buffer circuit and differential input voltage-to-current converter³.

Another fast setting JFET instrumentation amplifier with high cost is Analog Devices's AMP-05.² The setting time for -10V to $+10\text{V}$ step within 0.0025% is 10 μs Typ, with better than 14-bit linearity at

Figure 82. Precision difference amplifier.

Figure 83. Differential input voltage-to-current converter.

all gains up to 1000. On-board dual guard drives maintain good settling time and common-mode rejection performance when shielded cable connects the input signal to AMP05. AMP05 can be used to build a high-compliance current source with high output impedance. Fig. 84 shows the configuration of high precision current source.

The input and output of the AMP05 can be transposed to make a precision bipolar current source. Reference and sense pin become differential inputs and the "old" input now monitors the voltage across a precision current-sensing resistor R_1 . Voltage gain is set to unity, so the transfer function is simply $I_{out} = V_{in}/R_1$ using a $200\ \Omega$ resistor for R_1 and limiting output current to $\pm 5\text{ mA}$. Voltage compliance is $\pm 10\text{V}$ with a typical output impedance of $50\text{ M}\Omega$. Potentiometer R_2 will trim the output current to zero with the two inputs grounded. Fine gain adjustment is accomplished by trimming R_G and R_S .

Figure 84. High compliance current source with high precision.

2.9. Negative impedance compensation

As mentioned before, the output impedance of current should be $16\text{ M}\Omega$ for $1000\text{ }\Omega$ loading. Theory predicts that the output impedance should be lower than that if compensation is not provided. Therefore the current source should have some active components to compensate for this low output impedance.

The impedance compensation takes the form of a negative capacitor and a negative resistor placed in parallel with the current source output.⁴⁴ The circuit which produces a negative capacitance is shown in Fig. 85.²⁵ Analysis of this circuit yields an impedance of

$$Z_{in} = \frac{Z(1 + A) + R_1 \parallel C_1}{sC_f(Z + (1 + A)R_1 \parallel C_1)} \quad (160)$$

where A represents the open-loop gain of the op amp and C_f implements feedback load compensation to insure the circuit stability. The real part of Z_{in} gives the resistance R_{in} , whose sign depends upon circuit component values, and the imaginary part gives the capacitance C_{in} , whose sign is negative.

Using a Burr Brown OPA602 op amp, which has an open loop gain of 100 dB , and choosing $C_f = 20\text{ pF}$, $R_1 = 10\text{ K}$, $C_c = 12\text{ pF}$, $R_2 = 10\text{ K}$ pot, this circuit gives the suitable performance.²⁵ R_2 used here is to cancel finite R_{in} .

Figure 85. A negative impedance converter.

2.10. Optoisolation

To reduce the interfere between digital circuit and analog circuit, it is better to use optocouplers to isolate those two parts. This will be accomplished by interposing opto-coupler in every digital connection between DSP and the ADC which also eliminates ground loops. Quality Technologies HCPL-2361 dual optocouplers have been chosen.¹²² It is internally shielded for high CMRR (10 kV/ μ s), and operates at high speed (10 Mbit/s). Schematic of the interface circuit is shown in Fig. 86.

Figure 86. Opto-isolation schematic diagram.

2.11. Wiring and Cable

In order to keep the highest accuracy so far achieved by careful selection of components and attention to design details, we should determine a suitable method of cabling and interconnecting for the routing of signals with minimal loss, degradation and noise pickup.

2.11.1. Coax cable

Shielded cable should be used to protect against magnetic and capacitive stray fields in all cases of potential interference (low or high frequency).¹⁴⁰ Grounded coax cable is excellent for use from 20KHz to 5GHz for most system. However, the outer conductor of a coax cable is not a true shield in that it is also the signal return. When two conductors at different voltages are placed near one another, a charge difference exists between the two conductors, and a capacitive effect also exists between them. Such capacitive effects can give rise to interference signals in both conductors.

2.11.2. Triax cable

The coupling noise from one conductor to the other is a result of the capacitance between the conductors and expressed as

$$V_n = j\omega RC V_s \quad (161)$$

where V_n is the noise voltage, V_s the signal voltage, ω the frequency, and RC the coupling time constant between the two conductors. If we could reduce the capacitance, C , then the noise voltage coupled into the conductor would be reduced.

Triax cable is coax cable with an additional outer copper braid (insulated from the signal carrying conductors) that acts as a true shield and protects the enclosed coax conductors. This braid or shield is grounded and passes both ground loop and capacitive field noise currents away from the signal carrying coax, thereby greatly improving the signal to noise ratio (SNR) over standard cable.

Because of the capacitance of the cable, driven shield is needed to decrease the overall capacitance. Triax cable is used here where the inner conductor and the first braid are driven in parallel at the transmitting end, these working against the outer braid which is insulated above ground (Fig. 87). The inner braid is left floating at the receiving end, providing a "Faraday" shield between the inner conductor and outer braid. In this way the cable's distributed capacitance is greatly reduced, thereby reducing cable losses and loading.

2.12. Shield Driver

Using triax cable to connect the current source to electrode, the center conductor carries the current signal, and the outer shield is connected to chassis ground to reduce capacitive coupling of external noise. But the triax cable introduces a high capacitive load to the source, as described above, and the value of the capacitive load depends on the length of the cable. This capacitive load must be minimized. Shield driving is one of the methods used to accomplish this reduction.

Figure 87. Triaxial driven shielded cable.

Figure 88. Shield driver with frequency compensation.

To implement this idea, the open loop bandwidth of the amp must be sufficiently high, and the closed loop gain equal to unity, to insure the voltage drop is zero. The op amp must also be stable. The capacitive load shield circuit is chosen here as shown in Fig. 88.

The op amp chosen is Burr Brown's OPA602.²⁵ It is unity-gain stable for capacitive loads up to several microfarads. The purpose of R_2 is to isolate the capacitive load from the op amp, while C_c provides a high frequency path from input to output. R_0 is the output impedance of the op amp, and R_s limits the amount of positive feedback to insure stability. C_s and C_g are the capacitances from center conductor to inner shield, and from inner shield to outer shield, respectively.

Figure 89. EICT analog circuit block diagram.

The feedback factor, b , of the circuit shown above is given by

$$b \equiv \left(\frac{V_+ - V_-}{V_{out}} \right)_{V_{in}} = - \left[\frac{sC_d(R_1R_2 + R_1Z + R_2Z) + Z}{sC_d(R_1R_2 + R_1Z + R_2Z + R_1R_0 + R_2R_0) + R_0 + Z} \right] \quad (162)$$

where

$$Z \equiv R_s + \left[\frac{1 + sR_1(C_s + C_{dc})}{s^2R_{dc}(C_{dc}C_s + C_gC_s + C_gC_{dc}) + s(C_s + C_g)} \right] \quad (163)$$

The phase margin of the circuit can be solved for numerically.²⁵

3. Summary

In summary, the whole system is showed in Fig. 89. Much attention has been paid to achieve high values for impedance, accuracy, speed and stability. High precision voltage sources supply high quality references voltage to other high precision devices. In practices, much work will required to trim and calibrate the system to achieve the desired performance. The layout of the printed circuit board will also be critical. Good ground and power layout is required and guarding for op amps must also be implemented.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

E. EXPERIMENTAL RESULTS

Both basic two-dimensional results as well as three-dimensional results are shown in this chapter. Starting with Sect. 1, results from the point-electrode, two-dimensional phantom are shown including the effect of number of excitations (Sect. 1.1). The results of using image improvement methods are shown in Sect. 1.2, while the results of using of three different excitation patterns, single-pair, multiple electrode, and Walsh function, are shown in Sect. 1.3. Comparison of results from the point-electrode model with those from plate electrode experiments is then given in Sect. 2, where the method of handling the plate electrode geometry is discussed in Sect. 2.2. Detailed information regarding numerical sensitivity of both point- and plate-electrode experiments is presented in Sect. 2.3 where the detailed description of plate electrode modeling as implemented herein is discussed in Sect. 2.4. Section 3 describes the use of the finite element model and two-dimensional scanning methods for three-dimensional imaging and presents experimental results. Quasi-three-dimensional imaging using back-projection methods is described in Sect. 4.

1. BASIC 2-DIMENSIONAL RESISTIVE IMAGING USING THE FINITE ELEMENT MODEL: POINT ELECTRODES

1.1. Comparing Reconstructed Images of Different Number of Excitations

One factor in our improved speed of computational imaging has to do with the use of incomplete excitation sets. Furthermore, a major factor in the complexity, cost, and time to design and construct the PACIFIC-I system is also the number of excitations required to obtain a satisfactory image, approximately a factor of 15 between, say 32 excitations and 496 excitations of a 32-electrode system.

In order to determine how the quality of reconstructed images varies with the number of the excitations, we reconstructed images using different four-pole excitations number (i.e. 16 excitations, 32 excitations, 64 excitations, 120 excitations). The experimental set up was as shown in Fig. 62. We used 16 electrodes in this series of experiments. The full combination of excitations is $16 \times 15 / 2$ or 120. From the results (Fig. 90), we can see qualitatively that 32 excitations gave the best image and the quality of image did not improve as we increase the number of excitations.

16 channels 16 excitations

16 channels 32 excitations

16 channels 64 excitations

16 channels 96 excitations

16 channels 120 excitations

Original locations of targets

Figure 90. Effect of number of excitations on the image reconstruction.

Figure 91. RMS error in boundary reconstruction as a function of number of excitations.

A computation of the converged relative error was obtained based on

$$\varepsilon = \frac{1}{NV} \sqrt{\sum_{n=1}^N (V_n - V_e)^2} \quad (164)$$

where V_n is the n^{th} voltage reading and V_e is the voltage on the electrode. N is the number of nodes in the model and V is the applied voltage. Thus, Eq. (164) provides an estimate of the rms error between the boundary measurements and the computations. The results are shown in Fig. 91. For the homogeneous case, increasing excitations number after 32 had negligible effect on the relative error while error decreased slightly with more than 64 excitations. Note that the data were acquired using the running error method until the average became stable to within $1:10^4$. This was more a matter of time than limitations in the hardware since averages stable to $1:10^6$ could easily have been obtained by increasing the acquisition time to many hours. However, the accuracies obtained are consistent with the best accuracy which can be expected from a high speed system having 14-bit ADC accuracy and reasonable cost.

From the results of Fig. 91, it is not clear that designing a high speed system for more than 31–32 excitations is cost effective. Nevertheless, since this result is based on a still developing set of logical and numerical imaging algorithms it seems advisable to leave open the possibility of expanded capability in the PACIFIC-I system.

Figure 92. Effect of multiple electrodes on image quality. Targets shown dashed.

1.2. Image Improvement

There is considerable room for improvement in the quality of the image based only on the processing of existing data, within the accuracy of the system. Having a high accuracy system able to obtain highly qualified data having known characteristics (i.e., known static targets in a truly two-dimensional experiment and comparison with equivalent three dimensional targets in a geometrically similar three-dimensional system). We have found that having this capability significantly enhances our ability to develop improved image restoration methods and come closer to true replication of the conductivity field. This section shall document those accomplished to date.

Using the set up shown in Fig. 62, we used multiple pole excitation to determine what improvement if any could be obtained over the four-pole method. As seen in Fig. 92, the results are somewhat improved albeit still with significant noise. In Fig. 93, we undertook two separate operations including either division of target data, or subtraction from the target data, by the homogeneous data. It is seen that the best results are obtained by dividing the target data by the homogeneous data followed by digital filtering to take advantage of the binary nature of the conductivity. While the results are still not as clean as desired, substantial improvements have been obtained and the noise is significantly reduced and almost eliminated. Only the shape of the resultant targets remains somewhat elusive.

In most of the images obtained to date, both voltages and currents have been measured. However, due to the time required to obtain a complete set of accurate data, Kirchoff's law was used to deter-

(a) Multiply homogeneous (3 or 5 electrodes per excitation) 8 excitations

(b) Multiple electrode excitation with 3 targets (3 or 5 electrodes per excitation) with 8 excitations

(c) Division by the Homogeneous Image and filtering at 0.6 of full scale. Single pair excitation.

(d) Subtraction of the Homogeneous image data followed by filtering at 0.6 of full scale. 32 excitations, 2 electrodes per excitation.

Figure 93. Use of alternate improvement methods including division or subtraction of the homogeneous image from the target data followed by filtering of the image at 60% of full scale.. Targets shown in Fig. 90.

mine some of the current-sink data. That is, current was measured on the high electrodes, and was not measured on the ground electrodes, (four-pole method). This method was also extended to the first multiply-excited measurements including those described in the preceding paragraph (Fig. 92) for the homogeneous case. This assumed that since the excitation pattern was symmetric being applied corner-to-corner, that the sink currents would be distributed symmetrically about the diagonal plane. However, the experiment is not exactly and perfectly symmetric due to, if nothing else, construction tolerances. It has been found that obtaining actual measurements of all currents improves the imaging. This can be seen in Fig. 94 where identical processing methods were used for the two cases in a homogeneous phantom. The actual measurement of all currents results in slight asymmetries which may exist in fact and produce data which yield less noisy image reconstruction.

Another idea for image processing has to do with the way in which the data are displayed. All images shown until now have been linear displays of data shaded, or patterned, or with dot densities varying linearly with conductivity. An alternate display might use logarithmic patterning as shown in Fig. 95 which has a significant effect on the display of noise.

1.3. Comparing Reconstructed Images of Three Different Types of Excitation Patterns

We have used three different excitation patterns to collect data. There include: single-pair excitation pattern; multiple excitation patterns; Walsh function excitation patterns. Here "single-pair excita-

Figure 94. Comparison of results using different measurement methods for the homogeneous case.

Figure 95. Logarithmic conductivity pattern of data shown in Fig. 93. Division by the homogeneous pattern without filtering.. Targets shown in Fig. 90.

tion pattern” means only one electrode is set to high voltage level, another one is set to low level, and all other electrodes are left floating. “Multiple excitation pattern” means 3 or 5 electrodes are chosen to set high voltage level and the opposite 3 or 5 electrodes are chose to set low voltage level. “Walsh function excitation” means all electrodes are set either high voltage level or low voltage level simultaneously, both voltage and the response current values of each electrode are measured. For Walsh function excitation patterns, there were totally 15 independence excitations when we used 16 channels. Results are compared in Fig. 96–111. Three insulators were used in each case, one 50–mm square, one 25–mm square, and one 40–mm diameter. Locations of targets are shown by the outlines. Note that filtering uses binary gating at the specified level expressed as a fraction of full scale conductivity range. Division processing indicates that the target image is divided by the homogeneous image providing relative variations in the conductivity patterns.

Obviously, single–pair excitation pattern and Walsh function excitation pattern results were better than multiple–pair excitation results. But the single–pair excitation pattern needed more excitations (32 vs. 15 in this case). Thus, Walsh function excitation pattern appears preferable at this time. Furthermore, Walsh function excitation appears to be an extension of optimal excitation methods developed by the biomedical team at Rensselaer. Walsh function excitation will also be utilized in the design of the Pacific–I system.

Figure 96. 16 electrodes with Walsh Function excitation pattern. This is homogenous case with 8 excitations.

Figure 97. 16 electrodes with multiple excitation pattern. This is homogenous case with 8 excitations.

Figure 98. 16 electrodes with multiple excitation pattern. This is 8 excitations with three targets.

Figure 99. 16 electrodes with Walsh Function excitation pattern. This is 8 excitations with three targets.

Figure 100. 16 electrodes with multiple excitation pattern. This is 8 excitations with 3 targets at different locations.

Figure 101. 16 electrodes with Walsh Function excitation pattern. This is 8 excitations with 3 targets at different locations.

Figure 102. 16 electrodes with Walsh function excitations pattern. 15 excitations with 3 targets (Compare with Fig. 100)

Figure 103. A pair-excitations pattern. This is 32 excitations with 3 targets.

Figure 104. 16 electrodes 15 excitations with Walsh function excitation pattern. Division processing.

Figure 106. A pair excitation pattern with 32 excitations. Division processing.

Figure 105. 16 electrodes 15 excitations with Walsh function excitation pattern. Division & filtering (Level=0.5)

Figure 107. A pair excitation pattern with 32 excitations. Division and filtering (Level=0.5)

Figure 110. 16 electrodes 15 excitations Walsh function excitation pattern. Division & filtering (Level=0.7)

Figure 108. A pair excitation pattern with 32 excitations. Division and filtering (Level=0.7)

Figure 109. 16 electrode with Walsh function excitation pattern. Homogenous case with 15 excitations.

Figure 111. A pair electrode excitation pattern. Homogenous case with 32 excitations.

2. BASIC 2-DIMENSIONAL RESISTIVE IMAGING USING THE FINITE ELEMENT MODEL: POINT-PLATE ELECTRODE COMPARISONS

2.1. Introduction

In iterative reconstruction techniques, the error in the boundary potential arises from, experimental, numerical, or modeling errors. As discussed previously, measurement error can come from unsteadiness in the excitation source, inaccuracy in measurement of current or voltage, contact impedances, and geometric tolerance error in construction of the phantom and placement of the electrodes. Numerical error can come from round off, truncation, and computational build-up effects and can be especially troubling in iterative methods. Precise modelling of the phantom depends on how accurately the actual geometry is modeled, especially the electrodes, in the finite element method. Using locally analytical methods and higher order approximation functions can significantly improve the accuracy.

Two types of electrode models are of interest and utilized in the FEM solution. Rod electrodes are relatively easy to model by FEM since we inject the electrode current into a point. However, the use of rod electrodes seems to provide less sensitivity than use of plate electrodes due to the high potential gradients immediately adjacent to the current injection points. Plate electrodes mitigates this effect to some extent providing more uniform current distribution and hence better sensitivity. Unfortunately, plate electrodes are difficult to model and we have had little success to date to develop analytical models to account for partial coverage of the bounding surface of a phantom with conducting surfaces. This problem, however, is still being investigated. In this section, an approximate plate electrode method is implemented and utilized in the FEM solver.

In addition to considerations of electrode type, reconstructions from experimental data have consistently shown that converged images do not give the correct contrast ratio for the case of pure insulators or pure conductors. Also, small sized targets are difficult to distinguish from the background, a fact identified also by others.

In this section of the report, we describe the results of an investigation to determine the sensitivity of our FEM model. Specifically, we will provide information relative to the following questions:

- (1) How much sensitivity do we gain if we use plate electrodes compared to rod electrodes?
- (2) Does mesh refinement improve the sensitivity?
- (3) How does the improved sensitivity affect the quality of reconstructed images?
- (4) How can the sensitivity be improved?

Using both rod and plate electrodes in our two-dimensional phantom the conductivity of a chosen finite element was varied and the relative change in the boundary voltages was determined by solving the forward problem for each case. The mesh was refined by a factor of three and the results repeated. A similar technique to determine sensitivity was used by Murai et al.¹¹⁰ They locally var-

Figure 112. Computational plate electrode model with external element layer.

ied the conductivity and calculated the relative change in mean impedance. The variation of the mean impedance is a highly nonlinear function of conductivity but quite linear for small perturbations.

2.2. Modeling the Phantom

Plate electrode phantom is modelled by adding an external FE domain to that in which the conductivity distribution will be found. This is shown in Fig. 112 for the 8 x 8 mesh where the electrode numbering and the mesh used for the computations of the forward problem are identified. This additional domain has half the thickness of internal elements but is otherwise the same. Edge admittances of adjacent elements have conductivities of 10^5 and $10^{-5} \Omega$ respectively, modeling the alternating regions of plate electrodes and insulation zones.

Figure 113 shows the application of multiple excitation currents into the electrodes (to be considered later). In Fig. 114 we show a rod electrode geometry model with a 32x32 mesh and multiple excitation current excitations. In each case the shaded element inside the model represents the perturbed element utilized for sensitivity analysis.

2.3. Numerical Sensitivity

To compare the sensitivities of two electrode models, the plate electrode mesh in Fig. 113 is also used in rod electrode model. The boundary voltage is first calculated for the homogeneous, constant conductivity, case, and then the computation is recalculated with the conductivity of the shaded element increased by a factor of ten. The linear block decomposition FEM is used in these computations. In Fig 115 the relative change in the boundary voltages are shown for both rod and plate electrode cases showing for both models that the maximum sensitivity is obtained near the excitation points. This is, of course, because, as shown in Fig. 21, the potential gradients are highest in these locations. From

Figure 113. Computational plate electrode model with multiple excitations (33 x 33 mesh).

Figure 114. Computational rod electrode model (32 x 32 mesh).

these curves we can say that the plate electrode model more than twice as sensitive as the rod electrode model in the configurations shown.

The variation of the sensitivity with the conductivity of the perturbed element is shown in Fig. 116. For small perturbations of the conductivity in the approximate range of 0.1 to 10 times the original value, the sensitivity variation is nearly linear. For larger perturbations the variation becomes highly nonlinear approaching a limiting value in each direction. This explain why the iterative method, when used for imaging pure conductors or pure insulators in a conducting medium does not converge to the very high or very low conductivity values but, rather, produces contrast ratios which are significantly lower than the true cases. It is not unusual to obtain conductivity ratios of the image of only 10:1 relative to the homogeneous medium.

The previous results were obtained with the use of a relatively coarse 11x11 mesh which introduces discretization error into the solution of the potential function. This error is due to the fact that the large elements in the vicinity of the rod electrodes can not adequately describe the high potential gradients in these zones.

Figure 115. Comparison of sensitivity of rod electrode phantom to plate electrode phantom with corner-to-corner pair excitation. Center perturbed element had factor of ten conductivity increase. (a) Rod electrode model; (b) plate electrode model.

Figure 116. Comparison of rode-electrode sensitivity to changes in the central conductivity element with plate electrodes. (a) Rod electrode model; (b) Plate electrode model.

The variations in sensitivity due to sequential mesh refinement was obtained starting with an 8x8 mesh rod electrode model. The results are shown in Fig. 117. It is seen that we get a very optimistic sensitivity analysis with high discretization error if the number of mesh elements is small, predominantly due to the high potential gradients near the excited electrodes. Coarse mesh results show relatively more uniform gradient inside the phantom than the exact solution causing more sensitivity (see Fig. 118). For the plate electrode model this situation is not as severe as in the rod electrode model. Refining the mesh after 22x22, as shown in Fig. 119 does not change the sensitivity much.

Figure 117. Variation of electrode sensitivity for the rod-electrode model due to mesh refinement.

Figure 118. Comparison of solutions of potential distribution using triangular and quadrilateral elements for the rod electrode model.

Figure 119. Effect of mesh refinement on electrode sensitivity for the plate electrode model.

Using multiple excitations has shown a significant increase by a factor of three in the sensitivity of the rod electrode model and an about 40% increase for plate electrode model as shown in Fig. 120. It is also seen that the sensitivity is quite uniform around the periphery of the phantom in each case.

Figure 120. Sensitivity variation for multiple excitations.
(a) Point electrode model, 32 x 32 mesh; (b) Plate electrode model, 33 x 33 mesh.

Figure 121. Approximate modeling of plate electrodes. (a) Schematic of current distribution on a plate electrode with adjacent finite element; (b) Equivalent circuits.

2.4. Modeling of Plate Electrodes

Up to this time we have been unsuccessful in obtaining a workable analytical basis for numerical modeling of variable-width plate electrodes. Consider the schematic of a plate shown in Fig. 121(a). The potential field will be relatively flat across the broad expanse of the electrode. However, near each corner, the potential field will curve and the gradient will steepen. This causes the electron current density to increase near the corners. Thus, rather than being uniform across an electrode, the current distribution is U-shaped being lower in the center and higher at the edges. It is a combination

Figure 122. Simple linear array to check behavior of plate electrode model.

Figure 123. Full FEM network model of the linear array with electrode admittance taken as $1000 \Omega^{-1}$

of the variable length of the electrodes relative to the boundary circumference and the nonuniform current distribution across any electrode which produces the most difficulty in modeling.

While we have not succeeded in obtaining an acceptable analytical model, an approximate model would ignore the variable current distribution and simply consider the very low impedance of the electrode relative to the field being imaged. Shown in Fig. 121(b), such a concept can take on at least two possibilities. Rather than using a triangular-shaped element and allowing the entire current to go into the field through this point, we chose the one with four admittances so that we could model the electrode impedance adjacent to an insulated portion of the boundary with alternatively low and high admittances and let the current split evenly between the two end-points of the electrode admittance. We then added this additional ring of elements around the field elements in our FEM.

To determine how this modeling would actually behave in a simple system we chose a simple linear array of elements as shown in Fig. 122 which produced the element admittance array shown in Fig. 123. The results of computations using this FEM in comparison with exact calculations are shown in Fig. 124 shows the comparison between the exact voltage distribution and the approximate distribution computed by the FEM. There is 9% difference between the two, reasonable agreement.

When the approximate plate electrode model is used to compute an image from experimental data, the results are striking. Figure 125 shows two examples of an image computed with an 8×8 block-decomposed procedure, but without bilinear exponential shaping of the conductivity. The raw, unfiltered results, for the first time in any of our image computations, have virtually no noise in the homo-

Figure 124. Comparison between plate-model FEM and exact calculation for the simple linear array shown in Figs. 122 and 123. The overall resistance was exactly 4.00Ω whereas the approximate value determined by the FEM was 3.64Ω , an error of 9%.

geneous image. Likewise, the image produced is exceptionally clear in comparison with the other results shown previously. No image enhancement methods have been used in this result. The filtered results are sharper at the boundaries than the unfiltered result, otherwise quite similar to the former.

In both cases shown in Fig. 125, the boundaries do not conform well to the actual boundaries of the square element. This is because of the linear nature of the shape function used to describe the conductivity in the decomposed subspace.

A comparison showing the improvement resulting from the bilinear exponential conductivity shape function may be seen with reference to Fig. 126. In this figure, two experimentally determined shape images are shown, one for a 50-mm square insulator and one for a 75-mm diameter cylindrical insulator, both centered in the two-dimensional phantom. The former has boundaries coinciding with boundaries of the FEM. The latter, of course, does not. It is readily seen that in both cases the noise level is virtually nonexistent and the actual shapes in both cases are well reproduced. The reason for the jagged edges in the cylindrical image is because the shape function, while providing very high contrast, is still quasi-linear. That is, the $\exp(\alpha x + \beta y)$ will still produce a line having a sharp edge rather than a curve which can be matched at element boundaries with adjacent elements. We are currently working on methods to improve this situation.

Figure 125. Experimental result from the two-dimensional phantom using the approximate plate electrode model. A 50 x 50 mm square insulator was centered at (75, 75) and the 8 x 8 linear block decomposition method was used for analysis. (a) unfiltered raw computation; (b) filtered results using the conductivity scale as shown.

Figure 126. Experimental results of imaging using plate electrodes and approximate plate electrode model in conjunction with the 8 x 8 mesh, bilinear exponential, block decomposition model. (a) Image of a 50 x 50 mm square insulator centered in the phantom (target boundaries match finite element boundaries). (b) Image of a 75-mm diameter cylindrical insulator centered in the phantom.

3. BASIC 3-DIMENSIONAL IMAGING--FEM

3.1. 3-D Imaging by 2-D Scanning Technique

In previous reports^{81,83} we presented converged results from a 2-D square phantom. This section of the report will concentrate on 3-D imaging problems. Here, the "3-D problem" means that the conductivity and electrode geometry vary along the axis of the phantom.

It is not feasible at this time to undertake a true 3-D FEM reconstruction from a set of 3-D boundary electrodes, because of the intensive computational resources required. In this part of the report, rather, the method of 2-D scanning of a 3-D target has been used. The idea of a 2-D scanning technique is based on moving the scanning plane (electrode plane) upwards or downwards along the axis of a 3-D stationary target, or keeping the scanning plane at a fixed location and acquiring data in short period of time intervals while the moving target passes through the electrode plane. It is, however, convenient to keep the scanning plane at a fixed location for multiphase flow applications because the volume of fluid we are interested in passes through the cross section of the pipe.

Applying the method above we get a set of data for every time interval. Reconstruction of images from these data gives us a set of 2-D pictures of a target or volume of fluid passing through the scanning plane. In other words, we divide the imaging volume into n -thin slices and consider the 3-D imaging volume as n slices. The faster the data acquisition speed and the smaller the electrode size along the phantom axis, the more detailed picture of a 3-D target is obtained to the extent that true 2-D slices can actually be obtained.

With the data acquisition system available we are now able to simulate the scanning of moving targets by keeping the target stationary during data acquisition, and then moving it to its new location for a subsequent image. In the future, the data acquisition speed will be increased with a target of up to 1000 frames/sec. Therefore, this method will be applicable to dynamic imaging problems.

3.1.1. The Experimental Results and Conclusions

Different sizes of 3-D targets have been tested for 3-D imaging. The results are obtained for both conductive and resistive targets.

The first experiment was completed for a moving conductive cone target along the axis of the phantom. The object was imaged when it was in three different positions as shown in Fig. 127. The conic part of the target is 6 inches and 1 inch in bottom and top diameters respectively and 4 inches high. The top cylindrical part is 2 inches high.

The reconstructed successive images for the positions I, II and III are shown in Fig. 128. The results in the left column of Fig. 128 were obtained by using the first experimental method explained before. The results in the right column were obtained by subtracting the background image in Fig. 128(a) from the results in the left column. We can easily see that subtracting the background image eliminates background noise. However, it cannot be completely eliminated for targets with large cross-sectional area.

Figure 127. Positions of cone target moving upward between the electrodes. The bottom diameter is 6" and upper diameter 1".

The results of the other two methods explained earlier are shown in Fig. 129. The results of the second method which corrects v/c data are shown in the left column of Fig. 129. The second method results in high contrast ratio which is about one order of magnitude but noisy background. In contrast, the results of the third method which is also called the "numerical correction method" gives less contrast ratio between the background and the target but gives quite a uniform background.

To see how these three data correction methods improve the 3-D image, we directly reconstructed the 3-D data without any correction. Fig. 130(a) shows the result with no correction in the data of a conductive square target located close to the left bottom corner of the phantom. The target looks about 4 times larger than it really is and it appears rounded rather of a square. Fig. 130(b) shows the result after the 3-D reconstructed background image is subtracted from Fig. 130(a). It improves the quality of the image but there is still a very noisy background.

The results of the three methods for the above mentioned target compared with a resistive circular cylindrical shell target are shown in Fig. 130(c),(d),(e) and (f) and Fig. 131 (see figure captions for explanations).

The numerical correction method can also be applied to 2-D problems. The computer program determines the average conductivity from the homogeneous data collected from a 3-D phantom and using this conductivity in the homogeneous phantom it solves the electrode voltages for the same

Figure 128. Reconstructed images of 3-D conductive cone target.

Figure 129. Reconstructed images of 3-D conductive cone target. In this case, the voltage / current ratio is corrected. (a), (c) and (e) are the computed images for Position I, II, and III respectively. The voltage/current ratio is corrected using the experimental method. (b), (d), and (f) correspond to the same target but the voltage/current ratio is corrected numerically.

Figure 130. Reconstructed 3-D images. (a) 2'' x 2'' x 4'' conductive target located at (3'',3'') with no 3-D error eliminated in data. (b) The image after background is subtracted from (a). (c) Reconstructed image with 3-D error eliminated from data of the target in (a). (d) Image after subtracting the background from (c). (e) The Reconstructed image of a cylindrical shell ,with a diameter of 4 1/2'', height of 3'' and shell thickness of 1/4'', located at the center of the phantom. 3-D error is eliminated in data. (f) The image after subtracting the background from (e).

Figure 131. Reconstructed 3-D images. (a) the center of 2"x2"x4" conductor target located at (3",3") with voltage/current data corrected experimentally. (b) voltage/current ratio is corrected numerically. (c) The Reconstructed image of a cylindrical shell, with a diameter of 4 1/2", height of 3" and shell thickness of 1/4", located at the center of the phantom. 3-D error is experimentally eliminated in voltage/current ratio. (d) The error eliminated numerically.

excitation combinations. Then, computed electrode voltages are subtracted from the experimentally measured voltages of the homogeneous phantom to calculate an error vector. Finally, the non-homogeneous 2-D data are then corrected by this error vector in the same way explained above. The results of a 2-D problem are shown in Fig. 132. For the target location see the caption of the figure and Fig. 132(c). As seen from the images, the background noise has been significantly improved by this method; however, the actual shapes and sizes of the reconstructed images still leave a lot to be desired and smaller images merge with the larger ones.

Comparing the results of these three methods, the best results have been achieved from the third method but without the hoped-for accuracy for the small size targets. In all experiments we used 8 excitations; however, better success is obtained with other methods as explained in Sect. 1 of Ch. E.

Figure 132. Reconstructed 2-D image of three resistive targets. The 50-mm square target is centered at (75,75) and two 25-mm square targets are centered at (63.5,140) and (165,114). (a) Reconstruction of 32-excitation data. (b) Reconstruction of the same data with numerical correction of background noise. (c) The original location of the resistive targets.

4. QUASI 3-DIMENSIONAL IMAGING--BACK PROJECTION

Data were obtained with the 3-D back-projection phantom shown in Fig. 133. Figure 134 shows the imaged result for this case where a 50 x 50 x 37.5-mm target was utilized and data were obtained using levels A and B as the source-sink levels for excitation. The square outline shows the limits of the image output whereas the circular outline shows the limits of the inside of the phantom, the region represented by the data. As seen, the results are quite good and are indicative of a reasonable chance for successful imaging when full three-dimensional data are tested. It is hoped that further development of this method, while probably being insufficient to provide detailed phase distribution information, would provide an excellent starting image for the finite element method, thereby reducing the work required to obtain final images.

Level D
Level C
Level B
Level A

Figure 133. 3-D Cylindrical Phantom, 150-mm diameter, 600-mm high, Plexiglass.

Z = 4 — 6 — 8 — 10 — 12 — 14

Figure 134. 3-D back-projection image of a 2-D square target 50 x 50 mm in the circular, 150-mm diameter phantom.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

F. SUMMARY AND RECOMMENDATIONS

1. SUMMARY

This report summarizes three years of development effort for obtaining images of binary fluid mixtures, such as liquid and gas, using electrical excitation and measurement on the boundary. The report is broken into six major parts: [A] Theoretical Development; [B] Numerical Results; [C] Experimental Design--Mechanical; [D] Experimental Design--Electronic; [E] Experimental Results.

The first chapter on theoretical development, Chapt. A, summarizes the analytical methods utilized and developed to obtain impedance images from boundary-excited fields of variable conductivity. Section 1 summarizes the basic theory of the finite element method. First, the Newton-Raphson finite element method (FEM) developed by Yorkey is described in Sect. 1.1 followed by a significant new development which eliminates the need to compute the Jacobian in order to obtain the Hessian matrix (Sect. 1.2). Section 2 then outlines several innovations which, together, have both increased the accuracy of the FEM used (exponential shape functions) and reduced the time required to undertake computations by more than three orders (block decomposition). Section 3 describes errors involved in impedance imaging and methods developed to improve the basic images obtained from the FEM exponential block decomposition method. In Sect. 3.1, various sources of error are identified and discussed. Methods of error correction are developed and discussed including that for 2-D imaging (Sect. 3.2), image enhancement for 3-D imaging correcting for both voltage and current distortions (Sect. 3.3). Backprojection theory is outlined in Sects. 4 and 5 including both two-dimensional methods and quasi-three-dimensional applications. The use of multiplexing as a possible method to speed the data excitation and acquisition process is considered and basic theoretical methods are developed in Sect. 6. Finally, Sect. 7 describes the use of neural nets as an alternative approach to obtaining images from impedance data.

In the second part of the report, Chapt. B, we summarize the results obtained through the development of numerical methods. This includes the implementation of the basic finite element method (Sect. 1), considerations of the effects of noise (Sect. 2), the use of block reconstruction methods (Sect. 3) including both linear and bilinear exponential shape functions for the conductivity in any given element. While the use of block decomposition methods has resulted in major improvements in computational resources required for imaging, the use of incomplete excitation sets (Sect. 4) has also had a significant effect in time savings during image computations.

Chapter C of the report summarizes the various mechanical equipment designed and constructed as part of the program. First described is the two-dimensional phantom (Sect. 1) and several three-dimensional phantoms including one of rectangular geometry (Sect. 2.1), and several of cylindrical geometry for both resistive (Sects. 2.2.1 and 2.2.2) and capacitive measurements (Sects. 2.2.3 and 2.2.4). In addition to phantoms designed for use with finite element modeling, a phantom was specially designed for three-dimensional back projection analysis (Sect. 2.3). Finally, a natural circulation geometry was designed and constructed to test the ability of the methods developed to image isotherms as they affect the local conductivity (Sect. 3).

In Chapter D, we summarize the electronic design results undertaken to complete the imaging measurement. This includes the high accuracy system (Sect. 1) including detailed circuit board design and layout, noise reduction methods, computer interfacing, phase measurement, accuracy considerations, and software. Initial design considerations for the PACIFIC-1 high speed system is also discussed in Sect. D.2.

Finally, Chapter E summarizes both two-dimensional imaging as well as three-dimensional results. Starting with Sect. 1, results from the point-electrode, two-dimensional phantom are shown including the effect of number of excitations (Sect. 1.1). The results of using image improvement methods are shown in Sect. 1.2, while the results of using of three different excitation patterns, single-pair, multiple electrode, and Walsh function, are shown in Sect. 1.3. Comparison of results from the point-electrode model with those from plate electrode experiments is then given in Sect. 2, where the method of handling the plate electrode geometry is discussed in Sect. 2.2. Detailed information regarding numerical sensitivity of both point- and plate-electrode experiments is presented in Sect. 2.3 where the detailed description of plate electrode modeling as implemented herein is discussed in Sect. 2.4. Section 3 describes the use of the finite element model and two-dimensional scanning methods for three-dimensional imaging and presents experimental results. Quasi-three-dimensional imaging using back-projection methods is described in Sect. 4.

The first major conclusion we can draw from this work is that accurate excitation and measurement of boundary voltages and currents appears sufficient to obtain images of the real conductivity distribution within a body, that the quality of these images is heavily dependent on the theoretical and numerical implementation of imaging algorithms, and that this method *has the potential* of being both fast and cost effective in comparison with alternative methods. **The second major conclusion** drawn is that at this point considerable effort must be expended before the the potential becomes the reality. We have found that we can get excellent images of large objects within a homogeneous region but that our ability to correctly determine the boundary of the object depends on its shape, its relationship to the finite element geometry, the algorithm(s) chosen to represent the conductivity behavior within the elements, and perhaps the image enhancement algorithms utilized to treat the image following reconstructive iteration.

Specific conclusions may be drawn from the research as itemized below:

1. The mechanical design of the excitation and measurement geometry is not trivial. Plate electrodes appear to be optimum for producing the most sensitive and uniform electrical field

distribution in the body. Close attention must be paid to minimizing the interactive coupling between electrodes which requires they be extremely thin if they are to be in close proximity to one another, and the electrical leads to each electrode must be shielded and preferably guarded up to the immediate vicinity of the connection to the electrode.

2. Using thirty two square electrodes in geometries of the order of 50-mm characteristic dimension produces insufficient capacitance and capacitance change to provide meaningful results in the reactive impedance domain. Capacitance between opposing 5-mm square electrodes, 50-mm apart in air is of the order of 1 fF (10^{-15} F). We were unable to design circuitry to provide sufficient sensitivity to determine changes on this order.
3. The use of neural net methods to image two-phase flows does not seem reasonable in view of the extraordinary number of training patterns needed to provide a learned recognition capability. For an 8 x 8 array, 25,000 training patterns would be necessary simply to recognize a random placement of three individually black elements in a white background (i.e., a binary field having three different elements).
4. Accuracy of reconstruction is quite noise sensitive. Care must be given in both electronic and mechanical design to minimize susceptibility to noise. In our case, we provided complete driven guarding for all lines from the signal generator through all circuitry and out to the phantom and to the measurement devices. This included using r-f-shielded switching with the driven guard connected through the shielding, and surrounding each signal lead on the circuit board on both sides with driven guard lines. Resultant wide-band noise in our static electronic system was 80-100 dB below the signal level.
5. Using block reconstruction methods allows us to rapidly compute a converged image, approximately 20 seconds now being required on our IBM 3090-200G main frame computer. Using incomplete excitation sets is possible with our least squares minimization algorithms with potential reduction in computational effort with reduction in number of excitation patterns. More than 32 excitations does not appear to significantly enhance our per-electrode error following iterative convergence which was found to be of the same order as our measurement error.
6. Reconstructions from data obtained with round (point) electrodes is quite noisy, the images showing considerable variation even in the homogeneous case. By comparison, reconstructions using plate electrodes was quite noise free showing a relatively uniform homogenous images.
7. Different methods were utilized to improve images. Factors which have an improving effect on images include: increasing the number of excitations up to a point; using multiple simultaneous excitations; Using spatial Walsh function (plus/minus) excitation; dividing a target image by the homogeneous image results (ratioing the image to the homogeneous case); subtracting the homogeneous image from the target image (providing a difference image); logarithmic imaging. No method produces exact images with point-electrode data. The best results were obtained with multiple excitations ratioed to the homogeneous case. Trigger

filtering of the final result to take advantage of the binary nature of gas-liquid systems offered additional improvement.

8. Using point electrodes, it was impossible to produce definitive separation between shape and position of differently sized targets. The use of multiple-electrode excitation with division correction and level filtering produced results which were, however, quite clear and almost of correct shape and position.
9. The largest improvement in our experimental methods utilized to gain acceptable images was the development of an acceptable method of modeling plate electrodes. A simple impedance model utilized in conjunction with plate electrode data produced excellent results. A comparison between boundary sensitivity to central-element perturbations showed pair-excited plate electrode geometry to be more than twice as sensitive as point electrodes.
10. It is judged impossible to obtain perfect delineation of target conductivity if it varies by more than approximately a factor of ten from the homogeneous values. This is because the change in boundary voltage diminishes as the conductivity change exceeds this magnitude, saturating in both directions with infinitely increasing or decreasing conductivity.
11. It was found that the computed sensitivity to perturbations diminishes as the size of the element used in the model decreases. This is because smaller elements do a better job of correctly modeling the potential field gradient adjacent to the electrodes. Thirty two-electrode results were almost identical to the twenty four-electrode case for point electrodes. This effect was much less noticeable for plate electrodes than for rod (point) electrodes because the field was flatter with the former than with the latter. Virtually no difference was found between the use of twenty two vs. thirty three electrodes in the plate electrode case. Use of multiple-electrode excitation in both cases of plate and point electrodes produced a much more uniform boundary sensitivity, both results being quite similar.
12. Use of the linear-conductivity, block decomposition, plate-electrode model produced excellent homogeneous field results with virtually no noise. Results of imaging an off-element resistive target were also very noise free in comparison with other imaging methods with the image of the target being both definitive and in the correct location. However, the shape was not reproduced adequately. Filtering the result had some improvement on the shape reconstruction but still left something to be desired.
13. Use of the bilinear-exponential conductivity shape function with the plate-electrode model produced excellent results for the case of a square target. Results with a round target were also quite acceptable but slightly ragged on the edges where the conductivity calculated for adjacent elements produced discontinuities. This is obviously a limitation in the modeling of the conductivity. The form chosen, $\sigma = \sigma_0 \exp(\alpha x + \beta y)$, while having the ability to yield very rapid conductivity changes within a single element thus simulating discontinuous behavior, produces a constant value along the straight line $y = -(\alpha/\beta)x + \text{const}$, where the value of the constant determines the value of the exponential. In retrospect, a better shape function or a better method of handling discontinuities will be required in the future.

14. Based on the previous conclusions, it is concluded that the finite element model, of all those known and examined to date, provides the best possibility to yield the best quality impedance images.
15. Numerical methods have been developed to provide image enhancement under various circumstances. This includes corrections for noise as well as correcting for three-dimensional effects in two-dimensional reconstructions using finite-length boundary-electrode elements. Through the use of these methods, the first, crude images of a moving target were obtained showing that sequential slicing of a three-dimensional target may be possible to build three-dimensional images. This method should be further enhanced by application of additional methods as described above.
16. Finally, the use of quasi-three-dimensional back projection has been demonstrated showing that the image of a cube-shaped element in a cylindrical phantom could be imaged. While this method is not expected to be an end in itself since it provides neither generally clear nor unique results, it may provide a starting point for the finite element method which is expected to be more accurate.

2. RECOMMENDATIONS

There are four major developmental items which are the minimum required to provide quality images:

1. Development of a general purpose boundary-element model for electrodes and spacers in the overall context of the finite element model;
2. Develop a better internal shape function to replace the bilinear-exponential shape function representation of conductivity within a computational element;
3. Include the decomposed elements within the iteration loop rather than after the iteration sequence of the model;
4. Develop an adaptive quadrilateral element formation method which can be used to place element boundaries along the edges of "large" discontinuities in the field.

The latter, if accomplished, should remove some of the emphasis from the shape function which can then be used to image smaller discontinuities. There seems to be no reason, in principal, why at least one discontinuity per block element could not be imaged correctly. That is, with $8 \times 8 = 64$ block elements, we should be able to image 60 or more individual discrete elements given the proper combination of conductivity shape function and adaptive meshing. This seems entirely reasonable as a target upper bound since more elements might indeed produce sufficient interfacial area that interfacial processes occur in equilibrium and the area density need not be known.

In addition, since it now appears possible to accurately image binary fluid mixtures as proven by static tests

5. Construct an electronic system to acquire transient data needed to sequentially image a three-dimensional body passing a plane. The outline of such a system has been described herein.

In view of the usefulness of the two-dimensional phantom in methods development, the use of this should be continued. Specifically, the following methods should be pursued:

6. Determine the best methods of improving overall current density and sensitivity within the phantom using plate geometry;
7. Determine the optimum fluid conductivity which would provide maximum sensitivity and minimum error due to electrode contact impedance;

Further the development of three-dimensional imaging methods having as the goal the computation of true tomographs--slices which can then be pieced together to display the whole field. To do this,

8. Develop a truly three-dimensional back projection method which can utilize a 3-D array of boundary electrodes to image discrete elements in the measurement volume
9. Further develop the finite element method with emphasis on 3-D/2-D correction methods to obtain clear tomographs.
10. Search for a method or methods of training the imaging system to better correct for 3-D effects in an image of a planar distribution within a three-dimensional field.

Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

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Development of an Electrical Impedance Computed Tomographic Two-Phase Flows Analyzer

Annual Technical Report for Program Renewal

H. APPENDIX 2: PUBLICATIONS

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