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A NOVEL APPROACH TO MODELING UNSTABLE EOR DISPLACEMENTS

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OBJECTIVES

This research is aimed at developing a methodology for predicting the performance of unstable displacements in heterogeneous reservoirs. A performance prediction approach that combines numerical modeling with laboratory imaging experiments is being developed.

Flow visualization experiments are being performed on laboratory corefloods using X-ray Computed Tomography (CT) and other imaging technologies to map the insitu fluid saturations in time and space. A systematic procedure is being developed to replicate the experimental image data with high-resolution numerical models of the displacements. The well-tuned models will then be used to scale the results of the laboratory coreflood experiments to heterogeneous reservoirs in order to predict the performance of unstable displacements in such reservoirs.

SUMMARY OF TECHNICAL PROGRESS

In this report, we continue our examination of modeling miscible displacements. In a previous report, we presented a quantitative description of dispersion in porous media (Peters, 1993). We presented an improved method for determining the longitudinal dispersion coefficient of a porous medium from CT imaging of a tracer test in the medium. The method was demonstrated by measuring the dispersion coefficients for a sandpack and a Berea sandstone. The solvent concentration in the two media was visualized enabling the effects of dispersion and gross heterogeneity to be distinguished.

In this report, we examine the problem of hydrodynamic instability in miscible displacements. We present the mathematical model for unstable first-contact miscible displacements. The mathematical model is nondimensionalized to derive the dimensionless similarity scaling groups for unstable displacements. Although the mathematical model is not integrated in this report, the dimensionless form of the model reveals the similarity groups that control the displacement performance. CT images of two unstable, first-contact miscible displacements are presented to demonstrate the effects of some of the similarity groups on the displacement performance.

Theory

The mathematical model for a first-contact miscible displacement in a heterogeneous porous medium consists of the following mass transport, continuity, flow and state equations:

$$\frac{\partial(\phi C)}{\partial t} + \nabla \cdot (\vec{u}C) - \nabla \cdot (\phi \bar{D} \nabla C) = 0 \quad (1)$$

$$\frac{\partial(\phi \rho)}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (2)$$

$$\vec{u} = -\frac{\bar{k}}{\mu} \nabla (P \pm \rho gh) \quad (3)$$

$$\rho = \rho(C) \quad (4)$$

$$\mu = \mu(C) \quad (5)$$

To simplify the model, we consider a two-dimensional displacement in a medium of length L and width H oriented in space as shown in Figure 1. Let us assume (a) a homogeneous and anisotropic medium, (b) incompressible fluids, (c) ideal mixing of the solvent and oil, (d) constant-rate injection of the solvent at the inlet ($x = 0$), and (e) coordinate axes that coincide with the principal axes of the permeability and dispersion coefficient tensors. Under these conditions, Equations 1 to 5 can be written in scalar forms with the appropriate initial and boundary conditions as:

$$\phi \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} - \phi D_L \frac{\partial^2 C}{\partial x^2} - \phi D_T \frac{\partial^2 C}{\partial y^2} = 0 \quad (6)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$u = -\frac{k_x}{\mu} \left(\frac{\partial P}{\partial x} + \rho g \sin \alpha \right) \quad (8)$$

$$v = -\frac{k_y}{\mu} \left(\frac{\partial P}{\partial y} + \rho g \cos \alpha \right) \quad (9)$$

$$\rho = \rho_o + C \Delta \rho \quad (10)$$

$$\mu = \mu_o + C \Delta \mu \quad (11)$$

where $\Delta \rho = (\rho_s - \rho_o)$ and $\Delta \mu = (\mu_s - \mu_o)$.

Initial Condition:

$$C(x, y, t) = 0 \quad \text{at} \quad t = 0 \quad (12)$$

Boundary Conditions:

$$C(x, y, t) = 1 \quad \text{at} \quad x = 0 \quad (13)$$

$$u(x, y, t) = U \quad \text{at} \quad x = 0 \quad (14)$$

$$\phi \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = 0 \quad \text{at} \quad x = L \quad (15)$$

$$P(x, y, t) = P_1 - \int_0^y \rho g \cos \alpha d\xi \quad \text{at} \quad x = L \quad (16)$$

$$\frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (17)$$

$$v(x, y, t) = 0 \quad \text{at} \quad y = 0 \quad (18)$$

$$\frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = H \quad (19)$$

$$v(x, y, t) = 0 \quad \text{at} \quad y = H \quad (20)$$

Equations 6 through 20 can put in dimensionless forms by defining the following dimensionless variables based on the initial and boundary conditions:

$$x_D = \frac{x}{L}, y_D = \frac{y}{H}, u_D = \frac{u}{U}, v_D = \frac{vL}{UH}, P_D = \frac{k_x P}{\mu_o UL}, \rho_D = \frac{\rho}{\rho_o}, \mu_D = \frac{\mu}{\mu_o} \quad (21)$$

Substituting Equation 21 into Equations 6 through 20 gives the following dimensionless equations:

$$\frac{\partial C}{\partial t_D} + u_D \frac{\partial C}{\partial x_D} + v_D \frac{\partial C}{\partial y_D} - \left[\frac{\phi D_L}{UL} \right] \frac{\partial^2 C}{\partial x_D^2} - \left[\frac{\phi D_T L}{UH^2} \right] \frac{\partial^2 C}{\partial y_D^2} = 0 \quad (22)$$

$$\frac{\partial u_D}{\partial x_D} + \frac{\partial v_D}{\partial y_D} = 0 \quad (23)$$

$$u_D = -\frac{1}{\mu_D} \left(\frac{\partial P_D}{\partial x_D} + \left[\frac{k_x \rho_o g}{\mu_o U} \right] \rho_D \sin[\alpha] \right) \quad (24)$$

$$v_D = -\frac{1}{\mu_D} \left[\frac{k_y L^2}{k_x H^2} \right] \left(\frac{\partial P_D}{\partial y_D} + \left[\frac{k_x \rho_o g H}{\mu_o UL} \right] \rho_D \cos[\alpha] \right) \quad (25)$$

$$\rho_D = 1 + C \left[\frac{\Delta \rho}{\rho_o} \right] \quad (26)$$

$$\mu_D = 1 + C \left(\left[\frac{\mu_s}{\mu_o} \right] - 1 \right) \quad (27)$$

$$C(x_D, y_D, t_D) = 0 \quad \text{at} \quad t_D = 0 \quad (28)$$

$$C(x_D, y_D, t_D) = 1 \quad \text{at} \quad x_D = 0 \quad (29)$$

$$u_D(x_D, y_D, t_D) = 1 \quad \text{at} \quad x_D = 0 \quad (30)$$

$$\frac{\partial C}{\partial t_D} + u_D \frac{\partial C}{\partial x_D} + v_D \frac{\partial C}{\partial y_D} = 0 \quad \text{at} \quad x_D = 1 \quad (31)$$

$$P_D(x_D, y_D, t_D) = P_{1D} - \left[\frac{k_x \rho_o g H}{\mu_o UL} \right] \int_0^{y_D} \rho_D \cos[\alpha] d\xi_D \quad \text{at} \quad x_D = 1 \quad (32)$$

$$\frac{\partial C}{\partial y_D} = 0 \quad \text{at} \quad y_D = 0 \quad (33)$$

$$v_D = 0 \text{ at } y_D = 0 \quad (34)$$

$$\frac{\partial C}{\partial y_D} = 0 \text{ at } y_D = 1 \quad (35)$$

$$v_D = 0 \text{ at } y_D = 1 \quad (36)$$

The terms in square brackets in Equations 22 to 32 are dimensionless numbers that will be the same at corresponding points in similar systems. These dimensionless similarity scaling groups control the performance of miscible displacements. Based on the above inspectional analysis, we identify the following dimensionless similarity groups for miscible displacements:

$$G_1 = \frac{\phi D_L}{UL} \quad (37)$$

$$G_2 = \frac{\phi D_T L}{UH^2} \quad (38)$$

$$G_3 = \frac{k_x \rho_o g}{\mu_o U} \quad (39)$$

$$G_4 = \alpha \quad (40)$$

$$G_5 = \frac{k_y L^2}{k_x H^2} \quad (41)$$

$$G_6 = \frac{k_x \rho_o g H}{\mu_o UL} \quad (42)$$

$$G_7 = \frac{\Delta \rho}{\rho_o} \quad (43)$$

$$G_8 = \frac{\mu_s}{\mu_o} \quad (44)$$

There are several things to note about the dimensionless scaling groups G_1 to G_8 . First, they are dimensionless power products which could have been obtained from dimensional analysis of the displacement problem. Second, they may not be linearly independent and can therefore be further manipulated to obtain the minimum set of independent dimensionless groups needed to describe the displacement. Third, they are not unique and can therefore be combined to form other legitimate dimensionless groups. It can be shown from linear algebra theory that the above groups do form a complete set of independent dimensionless numbers (Shook et al., 1992; Gharbi, 1993).

The dimensionless groups can be further manipulated to obtain the following more familiar dimensionless numbers (Gharbi, 1993):

$$N_{PeL} = \frac{1}{G_1} = \frac{UL}{\phi D_L} = \text{Longitudinal Peclet Number} \quad (45)$$

$$N_d = \sqrt{\frac{G_2}{G_1}} = \frac{L}{H} \sqrt{\frac{D_T}{D_L}} = \text{Dispersion Number} \quad (46)$$

$$A_r = \frac{G_6}{G_3} = \frac{H}{L} = \text{Aspect Ratio} \quad (47)$$

$$N_\alpha = G_4 = \alpha = \text{Dip Angle} \quad (48)$$

$$R_L = \sqrt{G_5} = \frac{L}{H} \sqrt{\frac{k_y}{k_x}} = \text{Effective Aspect Ratio} \quad (49)$$

$$N_g = G_6 \times G_7 = \frac{k_x \Delta \rho g H}{\mu_o U L} = \text{Gravity (Buoyancy) Number} \quad (50)$$

$$N_\rho = G_7 = \frac{\Delta \rho}{\rho_o} = \text{Density Number} \quad (51)$$

$$M = \frac{1}{G_8} = \frac{\mu_o}{\mu_s} = \text{Mobility Ratio (Viscosity Ratio)} \quad (52)$$

The performance of an unstable miscible displacement is determined by the above dimensionless groups. The effects of the groups on the displacement performance may be obtained experimentally or by solving Equations 22 to 36 for various values of the dimensionless numbers. It can be shown by a simple stability analysis that the conditions for the perturbation of Figure 1 to diminish or grow with time are given by the following inequalities (Bear, 1972; Hill, 1952; and Marle, 1981).

$$(M - 1)(U - U_c) < 0 \quad \text{Stable Displacement} \quad (53)$$

$$(M - 1)(U - U_c) > 0 \quad \text{Unstable Displacement} \quad (54)$$

where U_c is a critical velocity given by

$$U_c = \frac{k_x(\rho_s - \rho_o)g \sin \alpha}{\mu_o - \mu_s} \approx \frac{N_g \sin \alpha}{A_r} U \quad (55)$$

If the solvent is less dense and less viscous than the oil, as is normally the case in practice, then $M > 1$, $U_c < 0$ and $(M - 1)(U - U_c) > 0$, indicating that the displacement will always be unstable. The instability will manifest itself as viscous fingering and/or gravity override.

Experiments

Two unstable first-contact miscible displacements in unconsolidated sandpacks were imaged by CT to demonstrate the effect of the gravity number on the displacements. Table 1 shows the pertinent experimental data and the values of the various dimensionless groups. Experiment 1 had a gravity number of - 0.0001438 whereas Experiment 2 had a gravity number of - 0.0000272. Because the magnitude of the gravity number for Experiment 1 was

about five times that for Experiment 2, it may be expected that the tendency for gravity override of the oil by the solvent would be greater in Experiment 1 than Experiment 2. Although the mobility ratios for the two experiments were different, they were sufficiently high to promote viscous fingering in addition to gravity override. For flow in porous media, inertial terms are negligible. Therefore, the density number will not affect the displacement. Density affects the displacement only through the gravity or buoyancy number.

Figures 1 and 2 show the solvent concentration images for Experiments 1 and 2 at 0.5, 1.0 and 2.0 pore volumes injected. The images show a vertical slice through the center of the coreflood experiments. It is apparent from the images that the two experiments were affected by hydrodynamic instabilities in the forms of viscous fingering and gravity override. However, the gravity override problem is more severe in Experiment 1 than Experiment 2 as expected from their higher gravity numbers. Because of its higher gravity number (and higher mobility ratio), Experiment 1 is less efficient than Experiment 2 at the same pore volumes injected.

Concluding Remarks

In this report, we have examined the problem of hydrodynamic instabilities in first-contact miscible displacements. Inspectional analysis was used to derive the dimensionless similarity or scaling groups for miscible displacements. Two unstable first-contact miscible displacements were presented to demonstrate the effects of gravity or buoyancy number and mobility ratio on the unstable displacements. The results indicate that for displacements in horizontal porous media, the higher the gravity number, the more severe will be the gravity override problem. In the next report, we will solve the mathematical model for the unstable first-contact miscible displacement numerically in order to examine the impact of heterogeneity on the displacement.

NOMENCLATURE

C	=	Solvent concentration
D_L	=	Longitudinal dispersion coefficient
D_T	=	Transverse dispersion coefficient
H	=	Height of porous medium
k	=	Permeability tensor
k_x	=	Permeability in the x direction
k_y	=	Permeability in the y direction
L	=	Length of porous medium
P	=	Pressure
t	=	Time
u	=	Darcy velocity vector
u	=	Darcy velocity in the x direction
U_c	=	Critical velocity
U	=	Constant solvent injection velocity
v	=	Darcy velocity in the y direction
x, y	=	Cartesian coordinates

Greek Symbols

α	=	Dip angle
ρ	=	Density
ϕ	=	Porosity
μ	=	Viscosity

Subscripts

D = dimensionless variable
 o = oil
 s = solvent

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TABLE 1
EXPERIMENTAL CONDITIONS FOR UNSTABLE FIRST-CONTACT
MISCIBLE DISPLACEMENTS

	Symbol	Experiment 1	Experiment 2
Oil		85% Glycerol +15% Water	84% Glycerol +15% Water
Solvent		Water +10% BaCl ₂	Water + 14.87% NaCl + 7.74% BaCl ₂
Length (cm)	L	54.3	54.2
Height (cm)	H	4.8	4.8
Porosity (%)	ϕ	31.15	30.39
Permeability (Darcies)	k_x, k_y	10.28	9.50
Oil Viscosity (cp)	μ_o	92.78	88.15
Solvent Viscosity (cp)	μ_s	1.11	1.63
Oil Density (g/cm ³)	ρ_o	1.218	1.216
Solvent Density (g/cm ³)	ρ_s	1.089	1.191
Injection Velocity (cm/s)	U	0.008504	0.008504
Longitudinal Peclet Number	N_{PeL}	1779	1820
Dispersion Number	N_d	3.58	3.57
Aspect Ratio	A_r	0.088	0.089
Dip Angle	N_α	0	0
Effective Aspect Ratio	R_L	11.3	11.3
Gravity (Buoyancy) Number	N_g	-0.0001438	-0.0000272
Density Number	N_ρ	-0.16	-0.02
Mobility (Viscosity) Ratio	M	84	54

Figure 1. First-Contact Miscible Displacement in a Two-Dimensional Porous Medium

Figure 2. Solvent Concentration Images for Experiment 1
 A. 0.50 Pore Volume Injected
 B. 1.0 Pore Volume Injected
 C. 2.0 Pore Volumes Injected

Figure 3. Solvent Concentration Images for Experiment 2

A. 0.50 Pore Volume Injected

B. 1.0 Pore Volume Injected

C. 2.0 Pore Volumes Injected

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