**REVIEW OF ELECTRON BEAM MICROINSTABILITIES AND
THEIR RELEVANCE TO EBIS DEVICES***

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Abstract

Plasma kinetic theory and some examples of microinstabilities are briefly reviewed. Although the velocity space configuration of any EBIS is inherently susceptible to a number of classical beam microinstabilities, the small radial dimension of an EBIS plasma may prevent modes from occurring in EBIS traps due to physical limitation. In EBIS devices with high electron beam compression, where the potential for beam microinstabilities is great, the radial dimension is smaller than the Debye length, which renders plasma kinetic theory invalid. Potential for parametric instabilities exists due to coupling of structure excited modes into plasma modes by various mode-mode coupling "decay" processes.

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1. INTRODUCTION

Stability has been one of the most controversial [1] topics in EBIS research. Poor performance in some EBIS devices, especially when coupled with observation of rf signals, was occasionally blamed on plasma instabilities. When trying to attribute anomalous behavior in an EBIS to various plasma instabilities, questions were raised as to whether there is a simpler explanation like misalignment. Or, is the generated rf noise due to excitation and/or amplification of electromagnetic waves in the electron beam and trap structure rather than plasma type instabilities.

Part of the reason for skepticism, to quickly attribute anomalous behavior to microinstabilities, stems from the experience of the U.S. fusion community with regard to stability of stellarators. During the 1950s, an anomalously high rate of particle loss, referred to as particle pumpouts, was observed in the stellarators at Princeton. This phenomenon became a catalyst for major developments in plasma kinetic theory, since plasma microinstabilities were believed to be the reason for the pumpouts. In the 1970s, it was discovered that the reason for the pumpouts was the poor quality of the magnetic field of the early stellarators, which caused the particles to drift out. With the greatly improved computer codes (for computing magnetic fields and particle orbits) of the 1970s, a new generation of stellarators was built that matched the performance of comparable tokamaks.

Nevertheless, there were also great achievements like those in the subfield of plasma physics that deals with the stability of magnetic mirror devices. Both MHD and kinetic theories successfully explained major plasma macro and

microinstabilities, for which remedies were eventually found.

Unlike atomic physics where the Schroedinger equation can be used to define and become the basis for solving a problem, plasma physics does not have a master equation. Instead, a set of equations must be chosen based on best guess as to which equations are most relevant. There are two basic levels of theoretical description: (1) microscopic description, also referred to as kinetic theory, which is based on phase space distribution of plasma particles, utilizes mainly the Vlasov and some of Maxwell's equations. One of the main virtues of this theory is the natural manner in which finite-temperature effects are included. (2) Macroscopic fluid description based on moments of the Liouville or the Boltzmann equations, some of Maxwell's equations, and Ohm's law. The choice of equations in either theory is based on the problem to be solved.

Although microscopic quantities are more difficult to measure, they frequently play a dominant role in determining macroscopic properties of a plasma. However, it is very difficult to incorporate effects due to small beam dimensions, and it is next to impossible to account for plasma wall interactions and other phenomena that is often referred to as "sourcery".

In this paper, a short review of plasma kinetic theory will be presented and some electron beam microinstabilities will be evaluated for EBIS devices.

2. ELEMENTS OF PLASMA KINETIC THEORY

Before proceeding, it is important to define a plasma. The most common definition is that a plasma is a collection of charged particles whose statistical properties are determined by multiple Coulomb interactions. Mathematically, it

can be expressed as $n\lambda_D^3 > 1$, where λ_D is the Debye length. In essence, there

should be many particles in a Debye sphere to have plasma collective effects.

Modern plasma kinetic theory has its foundation in the Vlasov model [2] of plasma waves. In this model, particle discreteness is lost, i.e., the number of particles $N \rightarrow \infty$, while their charge and mass q and $m \rightarrow 0$. But qN and mN remain constant. It is as if particles are being chopped into smaller and smaller fragments to reach the so-called must limit. Since a plasma does not have discrete particles in this limit, the plasma is collisionless. However, the plasma is not correlationless. The various components (particles) of the plasma interact with each other via waves, i.e., their electric fields.

Like any many-body system, a collection of charged particles can be described by a chain of statistical equations, which could or could not be closed.

However, in a plasma (where $n\lambda_D^3 > 1$) taken to the must limit, statistical chains

of Liouville equations can be simplified by neglecting terms of order $1/n\lambda_D^3$.

To this order, the kinetic equation for the plasma distribution for the j^{th} species is

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q_i}{m_j} \mathbf{E} \cdot \nabla_{\mathbf{v}} \right) f_j(\mathbf{r}, \mathbf{v}, t) = 0. \quad (1)$$

Where $f(\underline{r}, \underline{v}, t)$ is the distribution function for the j^{th} species. Equation (1) is called the Vlasov equation. The interactions among the particles appear in the equation through $\underline{E}$, the average electric field, which can be computed self-consistently through Maxwell's equations. With

$$\nabla \cdot \underline{E} = \sum_j \frac{q_j \bar{n}_j}{\epsilon_0} \int f_j(\underline{r}, \underline{v}, t) d\underline{v}. \quad (2)$$

Equations (1) and (2) form a closed set of equations for $f_j(\underline{r}, \underline{v}, t)$.

This procedure can be generalized to include magnetic fields as well as externally produced fields. In this more general case, Equations (3), (4), (5), and (6) form a closed set.

$$\left[\frac{\partial}{\partial t} + \underline{v} \cdot \nabla + \frac{q_j}{m_j} (\underline{E} + \underline{v} \times \underline{B}) \cdot \nabla_{\underline{v}} \right] f_j(\underline{r}, \underline{v}, t) = 0 \quad (3)$$

$$\nabla \cdot \underline{E} = \sum_j \frac{q_j \bar{n}_j}{\epsilon_0} \int f_j(\underline{r}, \underline{v}, t) d\underline{v} + \frac{\rho_{\text{ext}}}{\epsilon_0} \quad (4)$$

$$\nabla \times \underline{B} = \frac{\partial \underline{E}}{\partial t} + \sum_j \frac{q_j \bar{n}_j}{\epsilon_0} \int \underline{v} \cdot f_j(\underline{r}, \underline{v}, t) d\underline{v} + \frac{\underline{J}_{\text{ext}}}{E_0} \quad (5)$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad (6)$$

An additional advantage of this theory is the ease of treating non-neutral, or even single species plasmas. A summation over species in these equations is all that is required.

A. Solution of the Linearized Vlasov Equation, the Landau Problem.

Next, a solution to the simplest case is outlined. More complex cases involve more complex mathematics, and in many practical cases, numerical solutions are needed. Landau [3] was the first to solve the Vlasov equation. He considered an infinite homogeneous plasma, with no external electric or magnetic fields. Only electrostatic modes are considered. The effect of a small perturbation, whether it is damped (stable mode) or will it grow (unstable mode), is examined. Mathematically, Equations (1) and (2) are solved as a linearized initial-value problem. Both equations are linearized with

$$f(\underline{r}, \underline{v}, t) = f_o(\underline{v}) + \tilde{f}(\underline{r}, \underline{v}, t), \text{ and } \tilde{\underline{E}} = -\nabla\phi ,$$

to yield

$$\frac{\partial}{\partial t} \tilde{f}_j(\underline{r}, \underline{v}, t) + \underline{v} \cdot \nabla \tilde{f}_j(\underline{r}, \underline{v}, t) - \frac{q}{m} \nabla \phi \cdot \nabla_{\underline{v}} f_j(\underline{v}) = 0 \quad (7)$$

$$\nabla^2 \phi = \sum_j \frac{q\bar{n}}{\epsilon_o} \int d\underline{v} f_j(\underline{r}, \underline{v}, t) \quad (8)$$

To solve for $\tilde{f}(\underline{r}, \underline{v}, t)$ and ϕ , equations (7) and (8) are to be reduced to algebraic equations by Fourier-Laplace transforms. After solving the algebraic equations for ϕ , the transform inversion must be carried out (in principle) to find the system response. There is a problem with the Laplace inversion since its integration contour and some of the singularities of $\phi(\underline{k}, \omega)$ are excluded. Landau developed the technique for analytic continuation of the contour. Now the time asymptotic response can be found by considering the highest pole, which is due to the zero of the denominator of $\phi(\underline{k}, \omega)$. This limit defines the dispersion relation,

$$1 - \sum_j \frac{\omega_{pj}}{k^2} \int d\underline{v} \frac{\underline{k} \cdot \nabla_{\underline{v}} f_{j0}(\underline{v})}{\underline{k} \cdot \underline{v} - \omega} = 0, \quad (9)$$

where $\underline{k}$ is the wave number, the frequency ω can be complex, and ω_p is the plasma frequency.

To examine stability properties of a system, Equation (9) needs to be solved for the system's distribution function. Complex values of ω indicate that growth or damping will occur. Landau showed [3] that in a plasma with a Maxwellian distribution function modes are damped.

For

$$f_o = \frac{1}{\sqrt{\pi} v_{th}} e^{-\left(\frac{v^2}{v_{th}^2}\right)}$$

when substituted in Equation (9), yields a damping rate of

$$\gamma \approx -\omega_p \frac{\sqrt{\pi}}{(k\lambda_D)^3} e^{-\left(\frac{1}{k^2\lambda_D^2}\right)}.$$

Another classical example is the bump-on-tail distribution with

$$f = \frac{n_1}{n} \left(\frac{1}{\pi v_{th1}} \right)^{3/2} e^{-\left(\frac{v}{v_{th1}}\right)^2} + \frac{n_2}{n} \left(\frac{1}{\pi v_{th2}} \right)^{1/2} e^{-\frac{(v-v_o)^2}{v_{th2}^2}}$$

$n = n_1 + n_2$ where n_1 is the bulk density and n_2 is the bump density; when substituted in Equation (9), yields a growth rate of

$$\gamma \approx \sqrt{\frac{\pi}{8}} \omega_p e^{0.5 n_2 v_o^2 / n_1 v_{th2}^2}.$$

B. Nonlinear Effects.

Wave-particle interactions are either resonant, i.e., localized in velocity space, or non-resonant usually localized in configuration space. It is clear from the dispersion relation that the resonance condition is $\omega - \mathbf{k} \cdot \mathbf{v} = 0$. A wave

with a phase velocity ω/k interacts with particles moving with velocity v . The wave has an equal probability of either accelerating slightly slower particles or decelerating slightly faster particles. If the velocity distribution function has a negative slope (e.g., a Maxwellian), more slower particles are accelerated than faster particles decelerated, therefore, wave damping occurs. The bump-on-tail distribution function has positive slope. A wave interacting with particles on that slope gains on the average energy, since it decelerates more particles than it accelerates. Hence, the wave grows, i.e., the mode is unstable.

For non-resonant instabilities, the mechanism is "bunching"; a local increase in charge density causes a stream of particles to slow down and further enhance the density of that clump. This mechanism dominates beam instabilities.

Linear theory deals with small amplitude oscillations. Hence, within the limits of linear theory, a wave affects only a localized region in phase space, and the bulk plasma parameters remain unchanged. As the wave grows, it interacts with an increasing number of particles (in a larger region of phase space), as well as modifying plasma parameters. Quasilinear and nonlinear theories deal with these effects. Nonlinear phenomena is vast (at least theoretical) due to the large number of possible channels of evolution from linear phenomena. A select number of nonlinear effects are described next.

Evolution of Plasma Parameters. As waves grow due to instabilities, particles are scattered in both real and velocity space, thus modifying the plasma distribution. A numerical computation by O'Neil and Malmberg [4] showed how a non-resonant beam-plasma instability evolves, as the beam temperature increases into a far less violent bump-on-tail instability.

However, the classical "textbook" theoretical example [5,6] involves saturation of this bump-on-tail instability. Unstable waves resonate with particles on the positive slope of the bump. On the average, particles on the bump are being scattered to lower energy until the bump flattens. The instability then saturates due to this "plateau" formation.

Reproducible experimental examples of "classical" beam instabilities are next to impossible to find (since they are unstable at wavelengths larger than most devices). One of the few examples is that of the half-cyclotron frequency instability, which occurs in counterstreaming electron beams in a magnetic field as a consequence of interaction between the forward electrostatic wave on one beam and the backward cyclotron wave on the other. It was first reported by Etievant and Perulli.[7] Maxum and Trivelpiece [8] found that the instability "disappears" after a short time. Murakami and Lidsky [9] measured the time evolution of the beam temperatures and tried to explain the disappearance by some velocity space saturation. Hershcovitch and Politzer [10] showed both theoretically and experimentally that as the beam temperature increases, the largest growth rate shifts to lower frequency modes. This is known as mode competition.

Resonance Broadening and Strong Turbulence Effects. [11] As an unstable wave grows, the resonance condition changes from $\omega - k \cdot v = 0$ to $1/2 m (\omega/k - v)^2 \leq e\phi$. Consequently, a larger number of particles resonantly interact with the wave. Referring again to the bump-on-tail instability, the wave saturates when it interacts with an equal number of particles on both the negative and positive slope of the velocity distribution function. Furthermore, as the

turbulence grows, secular diffusion of the particle orbits occur to a point where coherent wave particle fluctuations is disrupted and saturation is reached. Mathematically, the two effects have the same manifestation in the dispersion relation as a stabilization contribution. An experiment with external induced turbulence provided some backing to the theory.[12]

Mode Mode Coupling. In fusion it is also referred to as parametric wave phenomena. If more than one wave is present in a plasma, particles can resonate with the beat frequency of these waves, if there are two waves with frequencies ω_1 and ω_2 and wave numbers k_1 , and k_2 , the resonance condition becomes

$$\omega_1 \pm \omega_2 = (k_1 \pm k_2) \cdot v .$$

These waves can now drive a third mode with ω_3 and k_3 . This interaction can be very strong if the wave satisfies the resonance conditions.

$$\omega_3 = \omega_1 \pm \omega_2$$

and

$$k_3 = k_1 \pm k_2 .$$

Since the dispersion relation needs now to be summed for the perturbed fields of all the modes, it is obvious that the mathematics becomes rather complex. Additionally, the number of modes and possible interactions is vast. In the next section, some interactions that are relevant for EBIS are mentioned.

3. APPLICATIONS TO EBIS

Equation (9) is a dispersion relation obtained from Equation (1) for an infinite homogeneous plasma with no external electric or magnetic fields. When analyzing an experimental configuration, analysis based on this dispersion relation is usually inadequate even though it may seem to be a good approximation. Instead, the proper geometry and distributions need to be incorporated in Equations (3) - (6) to yield an adequate dispersion relation. A good example is the Buneman instability.[13] This classical instability is very relevant to EBIS devices since it is due to an electron beam streaming through ions. In principle, a one-dimensional analysis should be adequate since it is an electrostatic instability due to an electron beam drifting parallel to the magnetic field. Substituting the electron and ion distribution functions

$$f_e = \frac{1}{\sqrt{\pi} v_{th_e}} e^{-\frac{(v-v_d)^2}{v_{th_e}^2}} ; f_i = \frac{1}{\sqrt{\pi} v_{th_i}} e^{-\frac{v^2}{v_{th_i}^2}}$$

into Equation (9), yields instability conditions that are met for many EBIS devices. However, inclusion of the magnetic field stabilizes the EBIS.[14]

To adequately describe the plasma configuration in an EBIS trap, Equations (3) - (6) must be used in order to arrive at a dispersion relation that is roughly [15]

$$0 = 1 - \sum_{j=i,e} \frac{\omega_{pj}^2}{k^2} \int d\mathbf{v} \left(iR \left[k_{\parallel} \frac{\partial}{\partial v_{\parallel}} + (\omega - k_{\parallel} v_{\parallel}) \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \right. \right. \\ \left. \left. - \frac{1}{\Omega_j} \left(k_x \frac{\partial}{\partial \tilde{y}} - k_y \frac{\partial}{\partial \tilde{x}} \right) \right] + \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \right) f_{oj}(\tilde{x}, \tilde{y}, v_{\perp}, v_{\parallel}) \quad , \quad (10)$$

where

$$R = \sum_{n=-\infty}^{\infty} \frac{iJ_n^2(k_{\perp} v_{\perp} / \Omega_j)}{\omega - k_{\parallel} v_{\parallel} - n \frac{\Omega_j}{2} \{1 \pm [1 - \frac{2\tilde{\omega}_{pj}^2}{r_j} (1-f)]^{1/2}\}} \quad .$$

The term in the last { } brackets was added by inspection as an orbit correction due to ExB rotation as a consequence of the radial plasma potential. This term was added by inspection as a particle orbit correction to replace a correction due to turbulent diffusion in Reference 15. The superscript circumflex ($\sim$) indicates guiding center coordinates. The electron and ion distribution that must be substituted in Equation (10) are

$$f_{oi} = \frac{1}{\pi^{3/2} v_{thi}^3} e^{-\left(\frac{v_{\parallel}^2 + v_{\perp}^2}{v_{thi}^2}\right)} \left[1 - \frac{\tilde{r}}{R_i}\right] \quad (11)$$

$$f_{oe} = \frac{1}{\pi^{3/2} v_{the} v_{the}^2} e^{-\left(\frac{v_{\perp}^2}{v_{the}^2}\right)} e^{-\frac{(v_{\parallel} - v_d)^2}{v_{the}^2}} \left[1 - \frac{\tilde{r}}{R_e}\right] \quad ,$$

where

$$\tilde{r} = \sqrt{\tilde{x}^2 + \tilde{y}^2} .$$

Here there is unistropy in the parallel and perpendicular electron temperature, as one would expect due to electron beam compression and conservation of magnetic moment. As an example $1 - \frac{\tilde{r}}{R_e}$ is used for the electron spatial profile, R_i and

R_e are the radial extent of ions and electrons, respectively. However, any other spatial profile can be used in Equations (11). The reasons for indicating guiding center coordinates for the spatial distribution function is to distinguish them from the actual particle orbits due to finite Larmor radius (FLR), which can have an important effect on stability. Due to FLR, fields "seen" by particles are different from those seen by the guiding centers, i.e., particle-wave interaction is not as efficient, thus yielding a stabilizing effect.

Equations (11) can now be substituted in Equation (10) to yield a fairly comprehensive dispersion relation, which will include a number of Z and Bessel functions that are tabulated. As is the case with other plasma configurations, a number of instabilities are possible. An EBIS device has a source of free energy due to the drift velocity of the electron beam. Due to beam compression, anisotropic temperature is most likely occurring in an EBIS trap. In addition, compression causes steep density gradients. Whenever there is a density gradient perpendicular to the magnetic field, drift waves (also known as universal modes)

occur. A number of sources of free energy can drive these modes unstable. The most obvious source of free energy in an EBIS is the electron beam current. However, the "jury is still out" on drift waves, and their frequency is low, i.e., oscillation time is much longer than a beam electron transit time through an EBIS trap. But very steep gradients, due to beam non-uniformities, [16] can substantially increase that frequency.

A major problem exists in the applicability of kinetic theory (or most plasma theories) to an EBIS configuration due to the very small radial dimension of the beam. For example, in an EBIS with a 10 kV electron beam, 100 mA electron current, with a current density with 1 kA/cm² or higher (due to a compression of about 1000), the potential for instabilities to exist is great due to a high electron density (10¹² cm⁻³) and most likely a large temperature unistropy. However, the Debye length in this case exceeds the beam radius (by about a factor of 2). It does not mean that instabilities do not occur, it does, however, mean that collective effects are not well accounted by conventional theories.

In a large current, immersed or partially immersed EBIS, (with relatively low current densities), the potential for beam microinstabilities is not large. However, kinetic theory is applicable since the beam $R_b \gg \lambda_D$. There are some general criteria for linear stability like the Penrose criterion [17] which require evaluation of the particle distributions only. However, their applicability is limited to electrostatic modes, and it does not give any information on the instabilities themselves. Nonlinear plasma theory is on even shakier ground, since even classical models like plateau formation fail [18] when straight extrapolation from one dimension into two dimensions is done.

4. DISCUSSION

Equations (10) and (11) can form a basis for a formalism to examine beam microinstabilities in an EBIS. The analysis is rather complex. It requires some knowledge of a suspected instability since approximations are needed for solving such a complex dispersion relation. Otherwise, many terms in the expansion of Bessel and Z functions would not converge. In addition, the theory is not applicable in cases of most interest. One big potential problem can occur due to stripped electrons left in the trap. Without even performing the necessary calculations, it is obvious that these electrons can provide a number of new channels for instabilities to occur, most important of which are beam plasma instabilities and the very serious diocotron instability. The latter requires that the density gradient slope change sign. Since these electrons are likely to be pushed to the outer perimeter of the electron beam due to its negative potential, they are likely to accumulate at the edge and change the slope of the density gradient.

Although macroscopic analysis was not discussed in this paper, the validity of applying dispersion relations must be examined based on assumptions made in its derivation.

Finally, a caveat, parametric instabilities can occur due to "decay" of excited structure modes into plasma modes. For example, a structure excited electromagnetic wave can decay into an electron plasma oscillation and an ion acoustic wave (this is a classical example of a mode-mode coupling decay process). If such an instability does occur, much confusion can be caused when trying to identify the instability since both electron plasma oscillations and beam instabilities have frequencies close to the electron plasma frequency.

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