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TITLE: IMPROVED APPROXIMATIONS APPLIED TO THE S_N EVEN-PARITY EQUATION

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Improved Approximations Applied to the S_N Even-Parity Equation

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Abstract

We have developed various finite element differencing schemes by applying lumping techniques in neutron streaming and removal terms of the S_N even-parity transport equation in two-dimensional x-y geometry. We have derived an analytic form of the even-parity reflective boundary condition, which along with the vacuum boundary condition can be applied directly to solve second-order even-parity boundary value problems. We have also developed a new *simplified even-parity* equation that is much more computationally efficient than the even-parity equation. The developed schemes are numerically compared with the conventional first-order diamond-differencing (DD) scheme.

Theory

The even-parity vacuum boundary condition can be easily derived[1] and after some algebraic manipulation, the even-parity *reflective boundary condition* can also be derived as

$$\hat{n} \cdot \nabla \chi(\vec{r}, \hat{\Omega}) = 0 \qquad \vec{r} \in \Gamma, \quad \text{all } \hat{\Omega}, \tag{1}$$

where $\hat{n} = \hat{\Omega} - \hat{\Omega}'$ is a unit vector normal to the reflecting boundary and χ the even-parity flux. Equation(1) is analytically correct and substituted for $\chi(\vec{r}, \hat{\Omega}) = \chi(\vec{r}, \hat{\Omega}')$ type of implicit reflective condition which has been previously used but requires an extra iterative technique. This new step significantly improves computational efficiency. It was found that the reflective boundary condition of Eq.(1), when applied to the cross-derivative terms in the even-parity equation, generate matrix asymmetries at boundary cells. Therefore, the differencing schemes for the regular even-parity equation result in an asymmetric 9-point operator. Several schemes are discussed below one of which eliminates this asymmetry.

1. Finite Element Differencing (EP959)

We can derive the discrete-ordinates, finite element differencing scheme by applying bi-linear interpolation functions to the variational form of the S_N even-parity equation. The resulting equation has a weighted 9-point discretization of the x-y derivative, a 5-point discretization of cross-derivative, and a weighted 9-point discretization of the removal term; hence we call it the 'EP 959' scheme.

2. Lumped x-y Derivative Differencing (EP559)

The 9-point discretization of the x-y derivative term can be simplified to a 5-point relationship using the common 3-point central differencings in the x- and y-directions.

3. Lumped Removal Differencing (EP951)

Alternatively, the removal term can also be lumped to a 1-point discretization.

4. Fully Lumped Differencing (EP551)

In this scheme, the lumping is applied to both the x-y derivative and removal term resulting in the 5-point and 1-point discretizations respectively.

5. Simplified Even-Parity Equation (SEP501)

We have derived a new *simplified even-parity equation* (SEP). It can be obtained by applying the even-parity equation for the direction (μ,η) and $(\mu, -\eta)$, and adding the two resultant equations using the assumption $\chi(\mu,\eta)=\chi(\mu, -\eta)$. With the cross-derivative terms thereby eliminated, it is possible to have a symmetric 5-point finite difference approximation. This equation also reduces the angular domain by another half of that required by the regular even-parity equation. This simplification is a generalization of the reduced P_3 angular approximation in the variational nodal approach developed by Lewis[2]. Logically, the justification for this simplification is that 1) SEP has the same diffusion limit[4] as the even-parity equation, 2) SEP is exact if the solution can be approximated by only the even power of μ and/or η in P_N expansion, and 3) SEP gives the correct solution for the case of a flux that is dominated by behavior in only one dimension.

Results

The developed differencing schemes for the even-parity equation are compared with the conventional first-order S_N transport code TWODANT[3] for a test problem. The problem consists of 10cm×10cm (20×20 cells) rectangular domain with reflective boundary conditions on the left and bottom, and vacuum boundary conditions on the right and top. The domain consists of the same material(σ=1.0, c=0.9) but the constant source exists in the central zone only. The symmetric S_4 quadrature set[1] is used so that the number of discrete ordinates should be equal to 12 for TWODANT, and 3 for SEP501, and 6 for the others. We performed the calculation with and without using the diffusion synthetic acceleration(DSA)[4] method. The final scalar flux distributions are all identical and the number of source iterations and the CPU consumed are summarized in Table 1. The SEP results are extremely close to the EP results for this problem.

Conclusions

We have shown that the finite element differencing and its lumped schemes can provide promising properties when compared to the conventional first-order DD scheme.[4-6] In particular, the proposed simplified even-parity equation, which is an elliptic equation, provides the economy of diffusion-level calculation in each discrete direction without causing severe errors and appears to be a candidate to replace the conventional DD scheme.

One of the drawbacks of the second-order S_N even-parity equation is the necessity to solve the matrix in each discrete direction. Therefore the right choice of the matrix solver is crucial in determining the economy of the even-parity codes. Presently, multi-grid methods[7] are known to be faster than any other methods and have been developed for non-iterative boundary value problems. At present, a 9-point asymmetric multi-grid solver is implemented on the test codes. However, to have more economic performance, the multi-grid method should be tailored to accommodate the specific types of the matrices found in these approaches and to compliment the iterative solution techniques required by the transport equation. The EP and SEP run times in Table 1 can be further reduced.

Further work will be directed toward computational efficiency and determination of the accuracy of SEP.

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Table 1 Number of Iterations and CPU time for the Test Problem

Differencing Schemes	No. of Iterations		CPU time (sec)	
	w/o DSA	w/ DSA	w/o DSA	w/ DSA
EP959	78	6	4.69	0.43
EP559	78	6	4.42	0.39
EP951	78	6	5.34	0.55
EP551	78	6	4.36	0.38
SEP501	80	6	2.00	0.19
TWODANT*	N/A	6	N/A	0.06

* pure CPU for sweeping and DSA only

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