AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

2
DE/PC/90180--T7

QUARTERLY REPORT
(01/01/93 TO 03/31/93)

RECEIVED
JAN 15 1993
0011

STABILITY ANALYSIS FOR THE FLOW OF GRANULAR MATERIALS DOWN AN INCLINED PLANE USING KINETIC MODEL

Project Director : K. R. Rajagopal
Department of Mechanical Engineering
University of Pittsburgh
Pittsburgh, PA 15261

Contract No : DE-AC22-91PC90180

Contract Period : 12/13/1990 - 12/13/1993

Student(s) and the degree for which they are registered : Mr. R. Gudhe, (Ph.D)

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED.

SUMMARY

In the previous report the linearized stability equations for the flow of granular materials down an inclined plane, modeled by the kinetic constitutive theory [cf. Richman & Marciniec (1990)] were derived. Here, we use the approximate solution of Richman & Marciniec (1990) as the base solution for the linearized stability analysis. The governing equations obtained are solved numerically to obtain the marginal stability curves which are presented in this report.

GOVERNING EQUATIONS

The stress tensor governing the flow of granular materials down an inclined plane is given by [cf. Richman and Marciniak (1990)].

$$\mathbf{P} = \{ 4 \rho G F T \} \mathbf{1} - 2 \mu E \hat{\mathbf{D}} \quad (1)$$

$$\text{Where,} \quad F(v) = 1 + \frac{1}{4G} \quad (2)$$

$$E(v) = 1 + \pi \frac{(1+5/8G)^2}{12} \quad (3)$$

$$G(v) = \frac{v(2-v)}{2(1-v)^3} \quad (4)$$

$$\mu = \frac{8 \sigma \rho G T^{1/2}}{5 \pi^{1/2}} \quad (5)$$

In the above equation $\mathbf{P}$ denotes the Cauchy Stress, v the volume fraction of the solid, $\hat{\mathbf{D}}$ denotes the deviatoric part of stretching tensor $\mathbf{D}$ associated with the solid motion and T the granular temperature.

Consider the flow of granular material modeled by the above kinetic model down an inclined plane (cf. Figure 1) due to the action of gravity [Savage (1979), Johnson and Jackson (1987), Johnson, et al. (1990), Hui, et al. (1984), Richman and Marciniak (1990), Hutter, et al. (1986a, b)]. In this problem we consider steady one dimensional flow of incompressible granular materials (*i.e.* $\rho_p = \text{constant}$) down an inclined plane, where the angle of inclination is α . The governing equations of motion are the conservation of mass, momentum, and energy. The final non-dimensional equations corresponding to the order of ε are

$$\begin{aligned} P_1 \frac{d^2 U_{px}}{d\bar{y}^2} + i G_7 \frac{d^2 U_{py}}{d\bar{y}^2} + G_8 \frac{d U_{px}}{d\bar{y}} + (G_{10} + i G_9) \frac{d U_{py}}{d\bar{y}} + P_3 \frac{d W_p}{d\bar{y}} \\ + (G_{11} + i G_{12}) U_{px} + (G_{13} + i G_{14}) U_{py} + (P_4 + i G_{15}) W_p = 0 \end{aligned} \quad (6)$$

$$\begin{aligned}
& (G_{16} + i G_{17}) \frac{d^2 U_{py}}{d\bar{y}^2} + (G_{18} + i G_{19}) \frac{dU_{py}}{d\bar{y}} + (G_{20} + i G_{21}) \frac{dU_{px}}{d\bar{y}} \\
& - S_2 \frac{dW_p}{d\bar{y}} + (G_{22} + i G_{23}) U_{py} + G_{24} U_{px} + (G_{25} + i G_{26}) W_p = 0
\end{aligned} \tag{7}$$

$$\begin{aligned}
& G_{27} \frac{d^2 W_p}{d\bar{y}^2} + i G_{30} \frac{d^2 U_{py}}{d\bar{y}^2} + (G_{31} + i G_{32}) \frac{dU_{py}}{d\bar{y}} + G_{33} \frac{dU_{px}}{d\bar{y}} + (G_{28} + i G_{29}) \frac{dW_p}{d\bar{y}} \\
& + (G_{34} + i G_{35}) W_p + (G_{36} + i G_{37}) U_{py} + (G_{38} + i G_{39}) U_{px} = 0
\end{aligned} \tag{8}$$

and the boundary conditions become

$$U_{px} = \sqrt{\frac{\pi}{2}} \tan \alpha \bar{W}_0 \left\{ f_p v_p + f \frac{1}{2 \bar{W}_0^2} W_p \right\}$$

$$U_{py} = 0 \tag{9}$$

$$\int_0^\beta \left\{ i v_0 \frac{dU_{py}}{d\bar{y}} + i \frac{dv_0}{d\bar{y}} U_{py} - \sigma v_0 U_{px} \right\} d\bar{y} = 0 \tag{10}$$

$$\begin{aligned}
& S_1 \frac{dU_{px}}{d\bar{y}} + i \frac{A_1 v_0}{\sigma \bar{u}_0} \frac{d\bar{u}_0}{d\bar{y}} \frac{dU_{py}}{d\bar{y}} + i \left\{ \sigma S_1 + \frac{A_1}{\sigma \bar{u}_0} \frac{d\bar{u}_0}{d\bar{y}} \frac{dv_0}{d\bar{y}} \right\} U_{py} \\
& - \frac{A_1 v_0}{\bar{u}_0} \frac{d\bar{u}_0}{d\bar{y}} U_{px} + \frac{S_1}{2 \bar{W}_0^2} \frac{d\bar{u}_0}{d\bar{y}} W_p = 0
\end{aligned} \tag{11}$$

$$2 \bar{W}_0 \frac{dW_p}{d\bar{y}} + \frac{d\bar{W}_0}{d\bar{y}} W_p = 0 \tag{12}$$

$$W_p = 0 \tag{13}$$

$$\text{With,} \quad \bar{y} = \frac{L-y}{\sigma}; \quad \bar{u}_0 = \frac{u}{\sqrt{\sigma g}}; \quad \bar{W}_0 = \left\{ \frac{T_0}{\sigma g} \right\}^{1/2}; \quad \bar{x} = \frac{L-x}{\sigma},$$

$$\bar{u}_1 = \frac{u_1}{\sqrt{\sigma g}}; \quad \bar{u}_2 = \frac{u_2}{\sqrt{\sigma g}}; \quad \bar{W}_1 = \left\{ \frac{T_1}{\sigma g} \right\}^{1/2}; \quad \bar{t} = t \sqrt{\frac{g}{\sigma}} \tag{14}$$

where L is the characteristic length. Equations (9)_{1,2} are the boundary conditions at $\bar{y} = \beta$ on the inclined plane and equations (11), (12) and (13) are the boundary conditions at $\bar{y} = 0$ at the free surface.

RESULTS AND DISCUSSION

The system of equations (6), (7) and (8) with the boundary conditions (9) through (13) are solved numerically to obtain marginal stability curves. For the base solution we use Richman and Marciniec (1990) approximate solution in the stability analysis.

The manner in which the base solution for volume fraction, velocity and granular temperature change with the angle of inclination (α) and Q are shown in Figures 2, 3 and 4 when $e = 0.95$ and $e_w = 0.8$ ($r = .5$ and $\bar{r} = .414$). Notice, that the volume fraction decreases from the surface of the plane to the free surface. Similarly, in the second case profiles of volume fraction, velocity and granular temperature are shown in Figures 5, 6 and 7 when $e = 0.8$ and $e_w = .95$ ($r = .5$ and $\bar{r} = .414$). In Figure 8, Q Vs σ (marginal stability curve) is plotted for different values of angle of inclination (α)

LIST OF FIGURES

<u>Figure No.</u>		<u>Page</u>
1	Flow Down An Inclined Plane	5
2	Volume Fraction Profiles When $E = .95$ and $E_w = .8$	6
3	Velocity Profiles When $E = .95$ and $E_w = .8$	6
4	Granular Temperature Profiles When $E = .95$ and $E_w = .8$	7
5	Volume Fraction Profiles When $E = .8$ and $E_w = .95$	7
6	Velocity Profiles When $E = .8$ and $E_w = .95$	8
7	Granular Temperature Profiles When $E = .8$ and $E_w = .95$	8
8	Marginal Stability Curve [Q vs σ]	9

Figure 1. Flow Down An Inclined Plane

Figure 2. Volume Fraction Profiles When $E = .95$ and $E_w = .8$.

Figure 3. Velocity Profiles When $E = .95$ and $E_w = .8$.

Figure 4. Granular Temperature Profiles When $E = .95$ and $E_w = .8$.

Figure 5. Volume Fraction Profiles When $E = .8$ and $E_w = .95$.

Figure 6. Velocity Profiles When $E = .8$ and $E_w = .95$.

Figure 7. Granular Temperature Profiles When $E = .8$ and $E_w = .95$.

Figure 8. Marginal Stability Curve [Q vs σ]

REFERENCES

1. Hui, K., P.K. Haff, J.E. Unger and R. Jackson, 1984 "Boundary Condition's for High-Shear Grain Flows", *J. of Fluid Mechanics*, 145: 223-233.
2. Hutter, K., F. Szidarovszky, and S. Yakowitz, 1986a "Plane Steady Shear Flow of a Cohesionless Granular Material Down an Inclined Plane: A Model for Flow Avalanches Part-I: Theory," *Acta Mechanica*, 63: 87-112.
3. Hutter, K., F. Szidarovszky, and S. Yakowitz, 1986b "Plane Steady Shear Flow of a Cohesionless Granular Material Down an Inclined Plane: A Model for Flow Avalanches Part-II: Numerical Results," *Acta Mechanica*, 65: 239-261.
4. Johnson, P.C. and R. Jackson, 1987 "Frictional-Collisional Constitutive Relations for Granular Materials, with Application to Plane Shear", *J. of Fluid Mechanics*, 176: 67-93.
5. Johnson, P.C., P. Nott and R. Jackson, 1990 "Frictional-Collisional Equations of Motion for Particular Flows and their Application to Chutes", *J. of Fluid Mechanics*, 210: 501-535.
6. Richmond, M.W., and R.P. Marciniec, 1990 "Gravity-Driven Granular Flows of Smooth, Inelastic Spheres Down Bumpy Inclines", *J. of Applied Mechanics*, 57: 1036-1043.
7. Savage, S.B., 1979 "Gravity Flow of Cohesionless Granular Materials in Chutes and Channels", *J. Fluid Mech.*, 92: 53-96.

**DATE
FILMED**

8 / 18 / 93

END