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OF TRANSFER FUNCTION

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ADVANCES IN THE IDENTIFICATION OF TRANSFER FUNCTION MODELS USING PRONY ANALYSIS

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Abstract

This paper further advances the usefulness and understanding of Prony analysis as a tool for identification of models. The presented results allow more generality in the assumed model formulation. In addition, a comparison is made between Prony analysis and autoregressive moving-average (ARMA) modeling. Special attention is given to system conditions often encountered with power system electromechanical dynamics.

1. Introduction

A key issue in the study and control of dynamic systems is the analysis of time-domain data for model identification. For example, in studying power system electromechanical dynamics, Prony analysis has been used for data analysis and control system design [1,2]. The objectives of this paper are: 1) to further advance the potential applications of Prony analysis as a system identification tool; and 2) to compare Prony analysis with the autoregressive moving-average (ARMA) form of system identification.

2. Prony Signal Analysis

Prony analysis is a method of fitting a signal to a weighted linear-combination of exponential terms [3]. Consider a general signal $y(t)$ that is to be modeled by

$$\hat{y}(t) = \sum_{i=1}^n B_i e^{\lambda_i t} \quad (1)$$

where $B_i \in \mathbb{C}$ are the output residues and $\lambda_i \in \mathbb{C}$ are the continuous-time eigenvalues. The objective is to find the residues, eigenvalues, and n that force the output of (1) to be the least-squares fit to $y(t)$. The signal $y(t)$ is sampled at a period T , and (1) is rewritten in discrete-time form as

$$\hat{y}_k = \sum_{i=1}^n B_i z_i^k; \quad k=0,1,\dots,N-1 \quad (2)$$

where $z_i = e^{\lambda_i T}$. It is well known that the output of (2) satisfies its own characteristic equation. Let the characteristic equation of (2) be written as

$$p(z) = z^n - (a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_n) \quad (3)$$

The output of (2) satisfies

$$\hat{y}_{n+i} = a_1 \hat{y}_{n+i-1} + a_2 \hat{y}_{n+i-2} + \dots + a_n \hat{y}_i \quad (4)$$

for $i=0,1,\dots,N-n-1$. This is termed the linear regression equation.

The process of solving for the unknowns involves three main steps. These steps are as follows:

1. n is assumed large. y is substituted for $\hat{y}$ and (4) is solved in a least-squares sense for the a_i 's. Polynomial (3) is factored for the eigenvalues.
2. Again, substituting y for $\hat{y}$, equation (2) is solved in a least-squares sense for the output residues.
3. The proper subset of estimated residues and eigenvalues that adequately fit the actual signal are used as the model with the order n being the dimension of this subset.

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Many different solution techniques can be used to solve steps 1 and 2 [3]. A method for solving step 3 is contained in [4].

3. System Identification Via Prony Analysis

The Prony analysis technique described above is a signal analysis method. Under certain restrictions, it was progressively shown in [1], [2], and [5] that Prony analysis could also be used as a system identification tool. The results in this section further expand the usefulness of Prony analysis in system identification.

Consider a general system with input $u(t)$ and output $y(t)$. It is desired to obtain a linear model with the Laplace transform of

$$G(s) = K + \sum_{i=1}^n \frac{R_i}{s - \lambda_i} \quad (5)$$

where K is the feed-forward term and $R_i \in \mathbb{C}$ is the transfer-function residue corresponding to eigenvalue $\lambda_i \in \mathbb{C}$ (note: $\lambda_i \in \mathbb{R}$, $R_i \in \mathbb{R}$). The objective is to find $G(s)$ that results in $\hat{y}(t)$ (the model output with input $u(t)$) being a least-squares fit to $y(t)$. The input is assumed known and can be written as

$$U(s) = \sum_{j=0}^L c_j \frac{e^{-sD_j} - e^{-sD_{j+1}}}{s - \hat{\lambda}_1} \quad (6)$$

The input (6) is applied to (5). After considerable algebraic manipulation, the output becomes

$$\hat{y}_k = R_1 v_{0,k} z_1^k + \sum_{i=2}^n Q_i v_{i,k} z_i^k \quad (7)$$

for $k=0,1,\dots,N-1$, where $\hat{y}_k = \hat{y}(kT)$, and

$$Q_i = \frac{R_i}{\lambda_i - \hat{\lambda}_1}; \quad i=2,3,\dots,n \quad (8)$$

$$Q_1 = K - \sum_{i=2}^n Q_i \quad (9)$$

$$v_{i,k} = T \sum_{j=0}^L c_j [(k-M_j) z_1^{-M_j} \mu(k-M_j) - (k-M_{j+1}) z_1^{-M_{j+1}} \mu(k-M_{j+1})] \quad (10)$$

$$v_{i,k} = \sum_{j=0}^L c_j [z_i^{-M_j} \mu(k-M_j) - z_i^{-M_{j+1}} \mu(k-M_{j+1})] \quad (11)$$

with $TM = D_j$, and $\mu(\cdot)$ is the discrete-time step function. Note that the "ramp" effect of repeated eigenvalue λ_1 is absorbed in $v_{0,k}$. Equation (7) in combination with (8-11) are solved similar to the original Prony analysis method for the unknowns in (5).

The above identification method extends the results in [1], [2], and [5] in three distinct ways. First, a feed-forward term has been included in the model. The second improvement is the inclusion of the input eigenvalue in the system model. Use of all the data to calculate the residues is the third improvement to previous published versions described here. Previous versions used data only after the input was removed.

4. Prony Analysis versus ARMA Identification

With ARMA modeling, a system is identified as the ratio of two polynomial difference equations and then mapped to a continuous-time model. Various algorithms exist that solve the first part of this problem, depending on the objective. In [6], ARMA modeling is used to identify models for power system signals containing electromechanical swings.

Consider a continuous-time system where the input and output are sampled to form $u_k = u(kT)$ and $y_k = y(kT)$ ($k=0,1,\dots,N-1$). The process is estimated with the ARMA model

$$\hat{y}_k = -\sum_{i=1}^n a_i \hat{y}_{k-i} + \sum_{i=1}^m b_i u_{k-i} \quad (12)$$

for $k=0,1,\dots,N-1$. Least-squares estimates of the unknown coefficients are obtained by substituting y_k for $\hat{y}_k$ in (12) and solving the resulting N equations in a least-squares sense for the unknowns. The resulting coefficients are then mapped to their continuous-time counterpart. Numerical steps for solving this problem are similar to those for Prony analysis.

System identification via Prony analysis requires the input to be of the form described by (6). In many cases, such as the power system problem, this is not a major restriction because (6) allows a wide range of deterministic inputs.

The primary difference between ARMA modeling and Prony analysis is that with Prony analysis, residues are directly identified; with ARMA modeling, moving average (MA) "b" coefficients are identified. The advantage of identifying the residues directly is found in [7] and is seen by studying the sensitivity of a given b coefficient to variations in a given system zero. Consider the model (12); the sensitivity of this model is

$$\frac{\partial q_i}{\partial b_k} = \frac{-q_i^{n-k}}{b_0 \prod_{m=1}^n (q_i - q_m)} \quad (13)$$

where the q_i 's are the zeros of the b polynomial. Equation (13) is a measure of the sensitivity of the i th zero to a change (error) in the k th b coefficient of the model (12). The denominator term of (13) shows that the sensitivity is very large when zeros are close together. Such a sensitivity does not occur for the parallel form [7]. The following example demonstrates this sensitivity.

Example

Consider the linear system

$$G(s) = \left(\frac{2(s+0.5 \pm j0.9\pi)}{(s+2)(s+0.2 \pm j\pi)} \right) \left(\frac{(s+0.505 \pm j0.909\pi)}{(s+0.25 \pm j0.75\pi)} \right) \quad (14)$$

where "+" denotes a complex conjugate pair. A square-wave input with an amplitude of 1 and a duration of 0.3 s is applied to (14). Low-pass-filtered Gaussian noise is added to the system output. Prony analysis and ARMA fits are made to the noisy outputs, resulting in the models in (15) and (16).

Comparing (14-16) shows that the Prony analysis resulted in a more accurate model. Further comparison is shown in Fig. 1. The outputs of the Prony and ARMA models for the square-wave input are compared with the actual noisy system output.

$$G_{\text{Prony}}(s) = \left(\frac{2.04(s+0.44 \pm j2.67)}{(s+1.99)(s+0.21 \pm j3.16)} \right) \left(\frac{(s+0.53 \pm j3.02)}{(s+0.23 \pm j2.35)} \right) \quad (15)$$

$$G_{\text{ARMA}}(s) = \left(\frac{1.97(s+0.63 \pm j2.75)}{(s+2.38)(s+0.74 \pm j3.47)} \right) \left(\frac{(s+1.16 \pm j3.62)}{(s+0.40 \pm j2.48)} \right) \quad (16)$$

Figure 1: Model fits versus system output.

5. Conclusion

The results in this paper further advance the potential use of Prony analysis as a tool for linear system identification. Also, the advantage of Prony analysis is demonstrated through comparison with ARMA modeling.

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