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A S_2-LIKE ACCELERATION METHOD FOR THE NONLINEAR CHARACTERISTIC TRANSPORT SCHEME

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I. INTRODUCTION

Recently a nonlinear spatial differencing scheme for solving the discrete-ordinate equations was introduced[1,2]. This scheme, referred to as the Nonlinear Characteristic (NC) scheme, is accurate for both optically thin and optically thick spatial meshes and is strictly positive. The NC discrete-ordinate equations can be solved using the source iteration (SI) method. It is well known that the SI method converges infinitely slowly for optically thick problems with scattering ratios near unity. In this summary, we describe a S_2-like acceleration scheme for accelerating the convergence of the SI method as applied to the NC scheme and provide results to show how effective it is.

II. THE NC SCHEME

The NC scheme is based upon the analytic solution to the discrete-ordinate equations. It preserves the zeroth and first spatial moments of the angular flux. The scalar flux within each cell is approximated by an exponential representation, derived from information theory[3]. Using the SI method, the discrete-ordinate equations for the NC scheme with isotropic scattering and a spatially constant, fixed source take the following form:

$$\frac{\mu_m}{\Delta x_i}\left(\psi_{m,i+1/2}^{l+1/2} - \psi_{m,i-1/2}^{l+1/2}\right) + \sigma_{t,i}\psi_{m,i}^{l+1/2} = \frac{\sigma_{s,i}\phi_i^l}{2} + \frac{Q_i}{2} \quad , \tag{1}$$

$$\frac{3\mu_m}{\Delta x_i}\left(\psi_{m,i+1/2}^{l+1/2} + \psi_{m,i-1/2}^{l+1/2} - 2\psi_{m,i}^{l+1/2}\right) + \sigma_{t,i}\hat{\psi}_{m,i}^{l+1/2} = \frac{\sigma_{s,i}\hat{\phi}_i^l}{2} \quad , \tag{2}$$

$$\psi_{m,i\pm1/2}^{l+1/2} = \psi_{m,i\mp1/2}^{l+1/2} e^{-|\varepsilon_{m,i}|} + \frac{\sigma_{s,i}\Delta x_i}{2}\phi_i^l \xi_{m,i}(\pm\lambda_i^l) + \frac{\Delta x_i Q_i (1 - e^{-|\varepsilon_{m,i}|})}{2|\mu_m|} \quad , \quad \mu_m \gtrless 0 \ , \tag{3}$$

$$\phi_i^{l+1} = \sum_{m=1}^{N} \psi_{m,i}^{l+1/2} w_m \ , \ \hat{\phi}_i^{l+1} = \sum_{m=1}^{N} \hat{\psi}_{m,i}^{l+1/2} w_m \ , \tag{4,5}$$

and

$$\frac{\hat{\phi}_i^{l+1}}{\phi_i^{l+1}} = 3\left(\coth(\lambda_i^{l+1}) - 1/\lambda_i^{l+1}\right) \equiv \beta(\lambda_i^{l+1}) \ . \tag{6}$$

Here, for the i-th cell, $\psi_{m,i\pm 1/2}^{l+1/2}$, $\psi_{m,i}^{l+1/2}$, and $\hat{\psi}_{m,i}^{l+1/2}$ are the edge angular fluxes, the average angular flux, and the first angular flux moment respectively; ϕ_i^l and $\hat{\phi}_i^l$ are the average scalar flux and slope and Q_i is the fixed source. We have defined $\varepsilon_{m,j} = (\sigma_{t,i}\Delta x_i)/\mu_m$. The nonlinear function $\xi_{m,i}(\lambda_i^l)$ is given by the following equation:

$$\xi_{m,i}(\lambda_i^l) = \frac{1}{|\mu_m|} \frac{\lambda_i^l}{\sinh(\lambda_i^l)} e^{\lambda_i^l} \left[\frac{1 - e^{-(\lambda_i^l + |\varepsilon_{m,i}|)}}{\lambda_i^l + |\varepsilon_{m,i}|} \right] \ . \tag{7}$$

IV. THE S2-LIKE ACCELERATION SCHEME

The S2 acceleration scheme replaces Eqs.(4)-(5) with a more complicated yet more efficient set of equations, which are basically NC S2 equations with different discrete-ordinates. Similar methods have been applied to linear spatial differencing schemes[4,5], but to our knowledge, ours is the first for nonlinear spatial differencing schemes. Our acceleration equations are given by:

$$\frac{1}{\Delta x_i}\left(\mu_{i+1/2}^{\pm,l+1/2}\psi_{i+1/2}^{\pm,l+1} - \mu_{i-1/2}^{\pm,l+1/2}\psi_{i-1/2}^{\pm,l+1}\right) + \sigma_{t,i}\psi_i^{\pm,l+1} = \frac{\sigma_{s,i}\phi_i^{l+1}}{2} + \frac{Q_i}{2} \ , \ \mu_m \gtrless 0 \ , \tag{8}$$

$$\frac{3}{\Delta x_i}\left(\mu_{i+1/2}^{\pm,l+1/2}\psi_{i+1/2}^{\pm,l+1} + \mu_{i-1/2}^{\pm,l+1/2}\psi_{i-1/2}^{\pm,l+1} - 2\mu_i^{\pm,l+1/2}\psi_i^{\pm,l+1}\right) + \sigma_{t,i}\hat{\psi}_i^{\pm,l+1} = \frac{\sigma_{s,i}\hat{\phi}_i^{l+1}}{2} \ , \ \mu_m \gtrless 0 \ , \tag{9}$$

$$\begin{aligned} \psi_{i\pm 1/2}^{\pm,l+1} = \psi_{i\mp 1/2}^{\pm,l+1} \frac{\sum_{\mu_m \gtrless 0} \mu_m e^{-|\varepsilon_{m,i}|}\psi_{m,i+1/2}^{l+1/2} w_m}{\sum_{\mu_m \gtrless 0} \psi_{m,i+1/2}^{l+1/2} w_m} + \frac{\sigma_{s,i}\Delta x_i \phi_i^{l+1}}{2} \sum_{\mu_m \gtrless 0} \xi_{m,i}^{\pm}(\lambda_i^{l+1}) w_m \\ + \frac{\Delta x_i Q_i}{2} \sum_{\mu_m > 0} \frac{1 - e^{-|\varepsilon_{m,i}|}}{|\mu_m|} w_m \ , \ \mu_m \gtrless 0 \ , \end{aligned} \tag{10}$$

$$\phi_i^{l+1} = \frac{\left(\psi_i^{+,l+1} + \psi_i^{-,l+1}\right)}{2} \quad , \quad \hat{\phi}_i^{l+1} = \frac{\left(\hat{\psi}_i^{+,l+1} + \hat{\psi}_i^{-,l+1}\right)}{2} \quad , \tag{11,12}$$

and Eq.(6). Here we have defined:

$$\mu_{i+k/2}^{\pm,l+1/2} = \frac{\sum\limits_{\mu_m \gtrless 0} \mu_m \psi_{m,i+1/2}^{l+1/2} w_m}{\sum\limits_{\mu_m \gtrless 0} \psi_{m,i+1/2}^{l+1/2} w_m} \quad , \quad k = 0,1 \quad , \tag{13}$$

Upon convergence, the solution to Eqs.(8)-(10) is equivalent to the unaccelerated S_N solution. To solve these S_2-like equations we could sweep them as we do the S_N equations, however; this is not efficient for problems with scattering ratios at or near unity. These S_2-like equations are nonlinear, thus; we cannot directly invert them. To solve this problem, we expand Eq.(6) and (7) in a Taylor series about $\lambda_i^{l+1/2}$ to give:

$$\frac{\hat{\phi}_i^{l+1}}{\phi_i^{l+1}} = \beta(\lambda_i^{l+1/2}) + \Delta\lambda_i^{l+1} \frac{d\beta(\lambda_i^{l+1/2})}{d\lambda_i^{l+1/2}} \quad . \tag{14}$$

$$\xi_{m,i}(\lambda_i^{l+1}) = \xi_{m,i}(\lambda_i^{l+1/2}) + \Delta\lambda_i^{l+1} \frac{d\xi_{m,i}(\lambda_i^{l+1/2})}{d\lambda_i^{l+1/2}} \quad , \tag{15}$$

$$\lambda_i^{l+1} = \lambda_i^{l+1/2} + \Delta\lambda_i^{l+1} \quad , \tag{16}$$

This is equivalent to one Newton-Raphson iteration on all nonlinear functions of λ_i^{l+1} . Using Eq.(14), we can eliminate $\Delta\lambda_i^{l+1}$ in Eq.(15). Now the S_2-like equations make up a set of linearly-independent equations which can be directly inverted.

V. RESULTS AND CONCLUSIONS

In this section, we give results to show the effectiveness of our S_2-like acceleration method. The test problem is a homogeneous slab, 60 cm wide with a total cross section of 1 cm^{-1} and a uniform source of 1 cm^{-3} s^{-1}. Table 1 gives the number of iterations to converge the problem to a relative error of 10^{-4} at various values for the scattering ratio and cell widths. We provide results for both the S_4 and S_{16} Guass-Legrendre quadrature sets.

The results show that the S_2-like acceleration scheme is stable and effective at reducing the number of transport iterations, especially for c=1.0. We do see a slight degradation in the method for problems with high scattering ratios and very optically thin cells with high quadrature sets. This can be corrected if one performs two S_N sweeps per one S_2 calculation. We see from Table 1 that this extra transport sweep is very effective for such problems. We have tested our method on a variety of heterogeneous problems and we find that it is just effective as it is for homogeneous problems.

ACKNOWLEDGMENTS

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REFERENCES

1. W. F. Walters and T. A. Wareing, "A Nonlinear Positive Method for Solving the Transport Equation on Coarse Meshes", Proc. Eighth International Conference on Radiation Shielding", April 24-28, 1994, Arlington, Texas, Vol 1, pp. 173-177 (1994).

2. W. F. Walters and T. A. Wareing, "An Accurate, Strictly-Positive, Nonlinear Characteristic Scheme for the Discrete-Ordinate Equations", *Transport Theory and Statistical Physics*, submitted (1994).

3. E.T. Jaynes, "Information Theory and Statistical Mechanics", *Phys. Rev.*, 106, 620, (1957).

4. D.Yu. Anistratov and V. Ya. Gol'din, "Nonlinear Methods for Solving Particle Transport Problems", *Transport Theory and Statistical Physics*, Vol. 22, Numbers 2 & 3, pp 125-163, (1993).

5. M.L. Adams, "New Nonlinear Methods for Linear Transport Problems", Proc. Joint International Conference on Mathematical Methods and Supercomputing in Nuclear Applications, Karlsruhe, Germany, April 19-23, 1993, Vol. 1, pp 683-694 (1993)

TABLE 1
Number of Iterations to Convergence for Test Problem.

	Number of Iterations			
	S_4		S_{16}	
Δx (*mfp*)	c=1.0	c=0.9	c=1.0	c=0.9
0.1	6	6	16 [4][a]	14 [4]
1.0	4	3 (36)[b]	4 (4329)	3
2.0	4	3	5	4
5.0	5	3	6	3
10.0	5	3	6	3
15.0	5 (6294)	3	6	3 (60)
20.0	5	3	6	3
30.0	5	3	6	3

a- values in [] are when one iteration consists of two S_N sweeps per one S_2 calculation.
b- values in () are for the number of unaccelerated (SI) iterations.

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