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NUCLEAR PAIRING: GLOBAL TRENDS AND LOCAL FLUCTUATIONS

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Abstract: We calculate nuclear pairing gaps for ground-state configurations for 8979 nuclei from ^{16}O to $A = 339$ in the Lipkin-Nogami pairing model. The energy levels required for the calculation are obtained from the folded-Yukawa single-particle model for ground-state shapes obtained in the macroscopic-microscopic approach by minimizing the total potential energy with respect to ϵ_2 , ϵ_4 , ϵ_3 and ϵ_6 shape degrees of freedom. We study the behavior of the calculated pairing gap Δ and the number-fluctuation constant λ_2 on neutron number N and proton number Z . We discuss alternative methods for comparing calculated pairing gaps to odd-even experimental mass differences and study the trends of the deviations between calculated and measured quantities versus N and Z . In particular we discuss the many non-smooth contributions to the so-called experimental pairing gap that is extracted from odd-even measured mass differences and the resulting difficulties that are present when these are compared to calculated results. In recent years several studies have investigated whether nuclear pairing depends on neutron excess. A resolution of this problem requires that the question is posed very precisely. Furthermore, any dependence on neutron excess is small compared to both the random and the correlated errors in the pairing gaps that are extracted from experimental odd-even mass differences. Therefore appropriate care is required before conclusions can be drawn from comparisons between calculated and experimental quantities. We use our global calculations of nuclear pairing quantities as a basis for a discussion of what conclusions can be drawn about the accuracy of current pairing models and the behavior of nuclear pairing with neutron excess. We also suggest what new nuclear mass measurements would serve as useful tests of pairing models. To provide the most informative presentation of the trends of the quantities studied over the entire periodic chart we display several of our results in the form of color contour diagrams versus N and Z .

1 Introduction

The deformed single-particle nuclear-structure model has been extremely useful since its introduction almost 40 years ago¹⁾. In its initial form it described well nuclear ground-state quadrupole deformations and the single-particle level order of deformed nuclei^{1,2)}.

Soon after the introduction of the deformed single-particle model several enhancements were added to the original formulation. Strutinsky's shell-correction method^{3,4)} made possible the calculation of the nuclear potential energy of deformation to an accuracy of about one MeV. Some deviations of real nuclei from the simple single-particle picture could often be accounted for by adding additional, *residual* interactions to the simple single-particle potential. The most important residual interaction is the pairing interaction⁵⁻¹¹⁾ which is essential in, for example, nuclear mass calculations and studies of nuclear rotation. Other examples of residual interactions are interactions specific to particular nuclear decays, for example the residual Gamow-Teller interaction¹²⁻¹⁴⁾ which is necessary in studies of β -decay.

Here we discuss the nuclear pairing interaction, its global trends and local fluctuations. Some background about pairing models is given in the next section, but for more complete details about pairing models we refer to our earlier study¹⁵⁾.

2 Background

In an earlier study¹⁵⁾ we investigated both a macroscopic pairing model and a microscopic pairing model, which was solved in both the the BCS⁵⁻⁸⁾ and Lipkin-Nogami (LN) approximations⁹⁻¹¹⁾. For each model we determined a preferred form of the effective pairing interaction. Optimum values of the parameters of the effective pairing interaction were obtained from least-squares minimization of the difference between calculated pairing gaps and experimental odd-even mass differences.

2.1 Pairing-gap models and parameters

An important result of our previous study is that it is crucial to differentiate between several pairing-gap concepts. The most simple concept is the *average* pairing gap $\bar{\Delta}$. It is an algebraic function, for example $c/\sqrt{A}$ where c is a parameter and A the number of nucleons in the system studied. The average pairing gap may be considered to be a macroscopic model for the nuclear pairing gap and it can therefore be directly compared with experimental odd-even mass differences.

When a microscopic approach is used the situation is considerably more complicated. In this case the quantities that are compared to experimental odd-even mass differences are obtained as solutions to microscopic pairing equations, for example the BCS or LN equations. In the LN approximation it is the sum of the pairing gap Δ and the number-fluctuation constant λ_2 , where Δ and λ_2 are obtained as solutions to the LN equations, that should be compared to odd-even mass differences. We denote this sum by Δ_{LN} . Thus $\Delta_{\text{LN}} = \Delta + \lambda_2$. In the BCS method it is Δ that should be directly compared to the odd-even mass differences. For consistency we therefore introduce in the BCS case the redundant notation $\Delta_{\text{BCS}} = \Delta$. To solve the usual pairing equations¹⁵⁾ one needs in addition to single-particle energies also the value of the pairing-strength parameter G . This parameter depends in a complicated way on the number of levels included in the calculation and on the particular nuclear region considered. However, it may be determined from an effective-interaction pairing gap Δ_G by use of a Strutinsky-like

Table 1: Final model constants. For lines 2, 4 and 6 we have determined r for t fixed to zero.

Pairing model	rms (MeV)	r (MeV)	t
Macro	0.167	5.55	6.07
Macro	0.198	4.77	0
BCS	0.195	4.96	1.23
BCS	0.196	4.80	0
LN	0.167	3.50	2.36
LN	0.169	3.30	0

procedure¹⁵). This may seem as an unnecessary complication, but the advantage is that Δ_G does not depend on the particular truncation of the single-particle level spectrum that is chosen in the calculation. Furthermore, it depends in a very simple way on Z and N . Therefore a significant simplification is achieved if one considers Δ_G to be the primary quantity that is needed for pairing calculations and the parameters that enter in the function that defines Δ_G to be the pairing-model effective-interaction parameters.

In our earlier studies¹⁵) we obtained as the preferred functional form of the effective-interaction pairing gap Δ_G :

$$\begin{aligned}\Delta_{Gn} &= \frac{rB_s}{N^{1/3}} e^{-tI^2} \\ \Delta_{Gp} &= \frac{rB_s}{Z^{1/3}} e^{-tI^2}\end{aligned}\quad (1)$$

where Z and N are the number of protons and neutrons, respectively, and $I = (N - Z)/(N + Z)$ is the relative neutron excess. From root-mean-square minimizations results consistent with $t = 0$ for the BCS and LN models were obtained. For these cases eq. 1 simplifies to

$$\begin{aligned}\Delta_{Gn} &= \frac{rB_s}{N^{1/3}} \\ \Delta_{Gp} &= \frac{rB_s}{Z^{1/3}}\end{aligned}\quad (2)$$

and the effective-interaction pairing gap Δ_G is determined by one parameter for the entire nuclear chart, for both neutrons and protons. A summary of the parameter values are given in table 1

In a macroscopic model it is $\bar{\Delta}$ that should be compared to odd-even mass differences. If we assume that in the macroscopic case $\bar{\Delta}$ is defined by the same function as Δ_G in eq. (1) one obtains the result in the top two lines of table 1. In this case the rms deviation is decreased by 15% if one allows $t \neq 0$. Thus, it is clear that one cannot find the optimum parameters of Δ_G , which in turn determines G , by fitting Δ_G to odd-even experimental mass differences. This procedure would yield an rms deviation between calculated microscopic pairing gaps and odd-even mass differences that is about 20% higher than the true minimum for the BCS model and an even higher rms deviation in the LN model. To determine optimum parameters of Δ_G it is instead necessary to perform much more extensive calculations in a brute-force method. It is necessary to solve the microscopic pairing equations for a sufficiently large grid of the unknown parameters that enter Δ_G and determine the optimum parameters from this calculation. This approach was used in our previous work to obtain the results on the last four lines of table 1.

The results presented in fig. 2 in our earlier work¹⁵⁾ show that when the old expression $c/\sqrt{A}$ is used for the average pairing gap $\bar{\Delta}$ and for the effective-interaction pairing gap Δ_G one obtains about the same value of c in a fit of $\bar{\Delta}$ to odd-even mass differences as is obtained for c when Δ_G is used to provide the pairing-strength parameter G used in microscopic BCS calculations. This accidental results is the reason why earlier no distinction was made between the average and effective-interaction pairing gap concepts.

We emphasize the most important results of our previous study¹⁵⁾

1. The preferred form of the pairing gap given by eq. 1 lowers the rms deviation by about 20% relative to the rms deviation obtained with the standard choice $c/\sqrt{A}$ for the average ($\bar{\Delta}$) or effective-interaction (Δ_G) pairing gaps.
2. The Lipkin-Nogami pairing model yields an rms deviation that is 14% lower than the rms deviation in the BCS approximation.
3. One cannot deduce the optimum parameters for a microscopic pairing model by simplified macroscopic calculations.
4. It is necessary to distinguish between several pairing-gap concepts, notably the average pairing gap $\bar{\Delta}$, the effective-interaction pairing gap Δ_G used as input in microscopic calculations, and the microscopic pairing gap Δ obtained as a solution to the BCS or Lipkin-Nogami pairing equations.
5. The effective-interaction pairing-gap Δ_G does not explicitly depend on the relative neutron excess I .

2.2 Pairing-gap neutron dependence

In our earlier work we also addressed the question: does nuclear pairing depend on neutron excess? An important outcome of our previous study was the realization that the question above can only be answered if the question is posed more exactly. However, for pedagogical reasons we will as a starting point of our discussion use the above imprecise formulation, and provide the formulation that is required to give a complete answer in the discussion of microscopic pairing models.

We first discuss the simplest case of a macroscopic, average pairing model. Several investigators¹⁶⁻¹⁸⁾ tried to answer the above question by comparing the average pairing gap to experimental odd-even mass differences. Root-mean-square minimizations performed in such investigations resulted in an explicit neutron excess dependence. For example, when eq. (1) is fitted to data then one obtains the result in the top line of table 1, that is t is significantly different from zero, which results in an explicit I^2 dependence of the average pairing gap $\bar{\Delta}$. Similar results are obtained for other functional forms of $\bar{\Delta}$, which allow for an explicit neutron-excess dependence.

However, a more detailed comparison¹⁵⁾ of the behaviour of $\bar{\Delta}$ and the odd-even mass differences with neutron excess revealed a surprising feature. Although the parameters corresponding to the minimum rms deviation had been found, this does not guarantee a correct trend of $\bar{\Delta}$ with neutron excess. In fact, line 3 of table 4 in our previous work shows that for neutrons $\bar{\Delta}$ has an excess decrease of 0.01 MeV per added neutron relative to the behaviour of the odd-even mass differences. This does not sound like much, but let us consider the effect over a relatively long chain of isotopes, say from ^{103}Sn to ^{133}Sn . Over 30 neutrons a 0.01 MeV excess slope per neutron would result in a buildup of a 0.3 MeV error, a significant number when we consider that the pairing gap is about 1.2 MeV in this region. However, we seem to have a paradox: how can a 0.3 MeV error build up, when we apply a model that has been optimized through a least-squares minimization? The resolution is that in the region of known nuclei the calculated pairing gap is higher than the experimental result at the beginning of the isotope chain, whereas it is lower at the end of the known elements in the isotope chain. In an actual case such as in our Sn example the calculated gap would typically be too high by 0.15 MeV in the beginning of the chain and too low by 0.15 in the end of the chain. This results in minimum rms deviation, although the trend with neutron excess is quite incorrect, leading to a buildup of large errors beyond the region of known nuclei, in the vicinity of the r-process line and neutron drip line.

In real nuclei microscopic structure strongly influences pairing properties. It is therefore meaningful to use the average, macroscopic pairing model $\bar{\Delta}$ only for very rough survey studies. It is highly plausible that it is much too inaccurate to provide insight into whether neutron pairing depends on neutron excess. The macroscopic approach also does not provide occupation probabilities and other quantities that enter into most microscopic calculations. We therefore will here spend very little time on discussing the macroscopic pairing model and instead focus on our microscopic pairing model of choice, the Lipkin-Nogami pairing model.

We now consider how a microscopic approach may allow us to obtain a more complete answer to our original question: "Does nuclear pairing depend on neutron excess?" Obviously we want to consider some particular pairing property so we reformulate the questions slightly: "Does the nuclear pairing gap depend on neutron excess?" However, even phrased this way the question cannot be answered. Recall that in the microscopic approach there occur at least two gaps, namely Δ_G , which is an *input* quantity, and Δ which is obtained as a *solution* to the LN equations. In the LN model one must also remember that it is $\Delta_{\text{LN}} = \Delta + \lambda_2$ that should be compared to odd-even mass differences.

We now realise that the question posed above must be further specified; does pairing gap refer to Δ_G or to Δ_{LN} We do not here consider the third possibility, Δ , since it

is an intermediate quantity. The relevant point to consider is: what question is more fundamental, is it whether the *input* effective force parameters depend on neutron excess, or is it whether the *calculated* quantity Δ_{LN} depends on neutron excess?

A guide to the correct answer is obtained by recalling that basic principle physics is that complex behaviour should be understood in terms of solutions to equations of very simple structure and with few parameters. Therefore the fundamental problem is to formulate the equations and find the input parameters of the problem. Of course the study and solution of these equations may also yield new insight, but the first task is the more fundamental one. Therefore we now realize that our original question should be reformulated: "Does the effective-interaction pairing gap explicitly depend on neutron excess?" By explicitly we mean that there occurs in the function that defines Δ_G an explicit dependence on the relative neutron excess I . Some type of implicit neutron dependence has always been present. Consider for example the standard model for the average pairing $\bar{\Delta} = c/\sqrt{Z+N}$.

Our original investigation of the effect of neutron excess on nuclear pairing yielded the following answers. A rms fit of the parameters of $\bar{\Delta}$ seemed to yield a strong dependence on neutron excess of this macroscopic model, in complete agreement with earlier results of ref. ¹⁸⁾ who also derived the specific functional form of Δ_G and $\bar{\Delta}$ which we have adopted. However, as discussed above this result leaves us with two problems. First, although the rms fit yielded a fairly strong I^2 dependence one finds ¹⁵⁾ that the trend with neutron excess in each individual isotope chain is quite erroneous. So there is an internal model contradiction in this case. Second, the effect investigated is so small that it is clearly of the order of microscopic effects that contribute to the pairing gap. Therefore one must assume that when this type of feature is studied that it is insufficient to use a macroscopic approach, and that one must instead adopt a microscopic approach.

In our previous investigation of the microscopic Lipkin-Nogami we found the following answer to our question. In an rms fit along the lines discussed above we obtained results consistent with $t = 0$ in eq. (1), that is *no* explicit neutron-excess dependence. Furthermore we showed in an investigation of the trends of the calculated pairing gaps along individual isotope sequences that it agreed with the trend of the experimental pairing gaps. Thus the inconsistency present in the macroscopic results were entirely absent in the microscopic results.

Whereas we in our earlier studies summarized results of global calculations in terms of two statistical measures, the mean and standard deviation of quantities studied we here present results in the form of color contour diagrams versus N and Z . This provides the opportunity to simultaneously survey both global trends versus N and Z and look at results for individual nuclei or small groups in nuclei in the set of several thousand nuclei included in the calculation. It is the first time such global nuclear pairing results have been presented in a form so suitable for detailed and overall analysis. The calculation presented here is based on our latest nuclear structure model and corresponding mass calculation.

We use the presented material to compare in detail both the global and local agreement between calculated and experimental pairing gaps. We also address in greater detail than earlier the problems associated with extracting experimental pairing gaps from odd-even mass differences. We present an alternative method for obtaining the theoretical pairing

gaps that are compared to experimental pairing gaps.

3 Calculations

Microscopic nuclear pairing calculations require as input energy levels of the nuclear ground-state configuration. Thus, both the ground-state shapes and the energy levels corresponding to this shape must be known. In our approach we obtain the shape from a nuclear mass calculation. The single-particle energy levels are then determined from the same folded-Yukawa single-particle potential that was used in the nuclear mass calculations, for the appropriate ground-state shapes.

In our 1990 pairing studies we obtained the required shapes and energy levels from an interim 1990 mass model. After this interim mass table was produced, several improvements^{19,20)} to the calculations have resulted in a new mass model that will be submitted for publication in a forthcoming issue of Atomic Data and Nuclear Data Tables²¹⁾. The most notable improvement is that the potential energy has been minimized with respect to additional shape degrees of freedom, namely ϵ_3 and ϵ_6 . Although this refinement leads to somewhat different ground-state levels relative to those obtained in the interim 1990 mass calculation the difference is usually small and it had only a small effect on the pairing gap calculated in the Lipkin-Nogami model. We see in table 2 that the rms deviation for $r = 3.3$, the value used in the 1990 calculation is now 0.1675 MeV compared to 0.169 MeV in the 1990 calculation.

In table 2 we also list Δ_{thmass} and S_n . The first quantity is a theoretical pairing gap determined from odd-even mass differences based on theoretical masses and the second quantity is the neutron separation energy. Although the discrepancy between the experimental pairing gap and the pairing gap obtained from theoretical odd-even mass differences is larger than the discrepancy between Δ_{LN} and the experimental pairing gaps one would anticipate that the former comparison has certain advantages relative to the latter. This will be further discussed later.

The FRDM, which includes Coulomb redistribution effects, is now our preferred nuclear mass model. Relative to the work described in ref.¹⁹⁾ further improvements have been incorporated into the model. First, it was found that the γ zero-point energy could not be sufficiently accurately calculated in our current model. It is therefore no longer included, whereas the ϵ_2 zero-point energy is retained. Second, we have also returned to the original prescription of including basis functions corresponding to 12 oscillator shells for all A values, instead of using somewhat fewer basis functions for lighter nuclei¹⁹⁾. Third, we now use an eighth-order Strutinsky shell-correction with a range $\gamma = 1.0\hbar\omega$ instead of our earlier choice of a sixth-order Strutinsky shell correction with the same range. The change in zero-point energy reduced the error in the calculated neutron separation energies from 0.551 MeV to 0.444 MeV and the mass error from 0.778 MeV¹⁹⁾ to 0.773 MeV. The second and third improvement further reduced the separation-energy error to 0.411 MeV and the mass model error to 0.669 MeV. The rms error for Δ_{thmass} has decreased in a similar manner as the error in S_n . Although the effect of the mass model improvements on Δ_{LN} are small, the effect on Δ_{thmass} is dramatic. Relative to the 1990 calculation the improvement is more than 20%. It is no accident that both S_n and Δ_{thmass} showed similar

Table 2: Pairing parameter determination.

r (MeV)	σ Mass model (MeV)	rms Δ_{LN} (MeV)	rms Δ_{thmass} (MeV)	rms S_n (MeV)
3.1	0.6746	0.1740	0.2035	0.409
3.2	0.6694	0.1691	0.2044	0.411
3.3	0.6695	0.1675	0.2091	0.416
3.5	0.6733	0.1745	0.2287	0.432

improvements. Both are determined from mass differences between nearby masses, and such differences dramatically improved when the inaccurate γ zero-point energies were excluded from the calculations.

In our 1990 pairing model studies we determined r in eq. (2) by minimizing the rms deviations between pairing gaps calculated in the LN model and experimental pairing gaps. An alternative possibility is to find r by minimizing the *mass model* error. Because our change in the Strutinsky calculation does slightly influence the pairing calculations through the determination of G from the effective-interaction pairing gap Δ_G , a small change in r may be required to obtain an optimum pairing calculation. We have therefore studied the dependence of several nuclear-structure quantities on r . Specifically we calculated the ground-state shell-plus-pairing corrections for several r values. For each of these calculations we determined a set of macroscopic model parameters and generated a full-fledged mass table. In this process we also obtained microscopic pairing quantities and neutron separation energies. The results are summarized in table 2.

Ideally all quantities should be minimum for the same r value. This is almost true. All quantities studied in table 2 have minima that are close to the mass table minimum $r = 3.2$. We therefore chose this r value in our pairing calculations. One may argue that it is more correct to determine the r parameter value from minimizing the mass model error than the rms deviation between Δ_{LN} and “experimental” pairing gaps determined from odd-even mass differences. The latter may have large errors arising from non-smooth contributions other than pairing effects, for example shape transitions and gaps in the deformed single-particle level spectra. Since such contributions are equally present in Δ_{exp} and Δ_{thmass} they cancel out in the difference between these two quantities. Consequently they do not affect an rms minimization of Δ_{thmass} . It is therefore of interest to note that our chosen value of r is about intermediate between what would have been obtained from considering Δ_{thmass} and Δ_{LN} deviations.

Since we are interested in global trends and local fluctuations of nuclear pairing features it is somewhat incomplete to characterize a model by a single number as above. For additional insight into pairing properties more detailed information is required. Since we have available several calculated properties for each of 8979 nuclei it is important to present the results so that one does not become overwhelmed by detail. We therefore

choose to present some of our results in the form of color contour diagrams versus N and Z .

3.1 Microscopic pairing calculation

Our first 6 figures represent results of theoretical calculations of pairing quantities in the Lipkin-Nogami approximation. In this model a pairing gap Δ and number-fluctuation constant λ_2 are obtained as solutions of the pairing equations. It is the sum $\Delta + \lambda_2$ that should be compared to odd-even mass differences. This sum, which is designated Δ_{LN} is shown in fig. 1 for neutrons. The calculated pairing gap varies rather smoothly over the periodic system. The pairing gap decreases with increasing A and with increasing N . There is no collapse at the magic numbers as is the case with BCS solutions. The areas enclosed in solid black lines are areas where experimental pairing gaps can be extracted from experimental masses by use of fourth-order finite-difference expressions. Because magic numbers and the $N = Z$ Wigner cusp give non-smooth contributions to the mass surface and corresponding contributions to the finite-difference expressions, it is not possible to extract experimental pairing gaps in certain regions near magic numbers and $N = Z$ ^{15,18}).

The pairing gap Δ_{LN} shown in fig. 1 is the sum of the two terms Δ and λ_2 shown in figs. 2 and 3, respectively. The Lipkin-Nogami pairing gap Δ decreases considerably near magic neutron numbers. This decrease is compensated for by a strong increase in λ_2 so that their sum behaves relatively smoothly at magic neutron numbers as is seen in fig. 1.

In figs. 4–6 we show $\Delta_{LN} = \Delta + \lambda_2$, Δ , and λ_2 , for protons. The region where pairing gaps can be extracted for protons in fig. 4 is slightly different from that for neutrons in fig. 1. The proton pairing gap Δ_{LN} decreases with increasing A and Z . There is a decrease of Δ at magic proton numbers. This decrease is compensated for by a strong increase of λ_2 which results in a smooth appearance of Δ_{LN} at magic numbers.

As already discussed above one may also determine pairing gaps directly from odd-even mass differences based on *theoretical* masses. This type of theoretical pairing gap we denote by Δ_{thmass} . There are several strong reasons to expect that it is more correct to compare Δ_{thmass} to Δ_{exp} than to compare Δ_{LN} to Δ_{exp} . The odd-even mass differences pick up any nonsmooth contribution to the nuclear mass surface, not just the pairing gap. Although we have excluded regions near magic numbers from consideration there are other gaps in the level spectra that are expected to give non-smooth contributions in the odd-even mass differences, for example the $N = 56$ spherical subshell near $Z = 40$ and the $N = 152$ deformed gap in the actinide region. Shape transitions also give rise to nonsmooth components in the odd-even mass differences. Therefore one expects such additional contributions to Δ_{exp} near $N = 88$ in the transition region to the deformed rare earth region, for example. Nonsmooth contributions of this kind are present in both Δ_{exp} and Δ_{thmass} , at least if the theoretical calculation accurately describes the nuclear mass surface. Although neither Δ_{exp} nor Δ_{thmass} represent the true pairing gap because of these nonsmooth contributions, they are absent in the discrepancy $\Delta_{exp} - \Delta_{thmass}$, whose behaviour therefore is expected to well indicate the accuracy of the pairing model.

Calculated Δ_{thmass} are displayed in figs. 7 and 8. For heavy nuclei, above about $N = 50$, Δ_{thmass} is fairly similar to Δ_{LN} in figs. 1 and 4, but for lighter regions there are

large differences.

3.2 Experimental pairing gaps

In figs. 9 and 10 we show pairing gaps determined from experimental masses for the regions bordered by black lines in figs. 1 and 4. It is important to consider how accurately the pairing gap can be determined from experimental odd-even mass differences. One type of error has its origins in the errors associated with the measured masses. The errors from this source in the extracted experimental pairing gaps are presented in figs. 11 and 12. We note that the errors are considerable in the light region and close to the borders of the known regions. A few of the errors even exceed 0.5 MeV. Other sources of errors, for example shape transitions and gaps other than magic gaps, further increase the error in the extracted "experimental" pairing gaps.

A comparison of fig. 9 with either of figs. 1 or 7 show rough general agreement. The same holds in a comparison of the proton pairing gap in figs. 10 with either of figs. 4 or 8. For a more detailed comparison we discuss in the next section the differences between the experimental and theoretical gaps plotted on a smaller scale.

4 Analysis

Experimental pairing gaps and pairing gaps calculated in the Lipkin-Nogami approximation are compared in figs. 13 and 14. A comparison based on calculated gaps determined from *theoretical* odd-even mass differences are shown in figs. 15 and 16.

Before we discuss the details of the discrepancy plots we give some overall characteristics of the pairing model error in table 3. Here we give errors for protons and neutrons separately, as well as the total error for both neutrons and protons. Theories are often characterized by their rms error with respect to data points. This is often unsuitable since the rms deviation contains contributions from the errors in the experimental data points, and is therefore always larger than the intrinsic error of the theory²²). However, it is possible to determine the intrinsic error of the theory in a way that contains no contributions from the experimental error²²). It is most natural to characterize the deviation between measured data points and theoretical calculations by the mean deviation μ_{th} of the theory from the *true* data points, without the experimental error contribution, and a standard deviation σ_{th} ^{20,23}) of the theoretical results around this mean. Alternatively or complimentary one may characterize the error of the theory by a single number, the second central moment of the error term of the theory. This measure is similar to the rms deviation, but without the deficiency of experimental error contributions. We list these four error measures in table 3 for both Δ_{LN} and Δ_{thmass} .

In contrast to our expectations we find better agreement between Δ_{exp} and Δ_{LN} than between Δ_{exp} and Δ_{thmass} . In the latter comparison the second central moment is about 20% larger than in the former. The reason that we obtain such a large error for Δ_{thmass} is that the deviation of the calculated mass surface from the true mass surface is sufficiently large to cancel the advantages of using Δ_{thmass} as a theoretical pairing gap instead of Δ_{LN} . It was in particular in the transition regions between spherical and deformed nuclei near

Table 3: Errors of pairing gap calculations. The data base of measured masses consist of one large data base of 1654 masses compiled by Audi²⁴) and a smaller data base of recent experiments in the region $N = 20$ to 28.

Nucleons	N_{nuc}	LN				Mass model			
		rms (MeV)	μ_{th} (MeV)	σ_{th} (MeV)	2nd cm (MeV)	rms (MeV)	μ_{th} (MeV)	σ_{th} (MeV)	2nd cm (MeV)
neutron	780	0.1759	0.0202	0.1619	0.1634	0.2132	-0.0452	0.1934	0.1979
protons	666	0.1691	0.0438	0.1528	0.1593	0.2036	-0.0123	0.1725	0.1728
total	1446	0.1728	0.0310	0.1581	0.1634	0.2088	-0.0300	0.1848	0.1979

magic numbers that we had expected such advantages. However, it is here that the error in the calculated mass surface is particularly large as can be seen in the comparison of measured and calculated masses in fig. 17.

Both transitions between spherical and deformed shapes and between prolate and oblate shapes may influence the the pairing gap. To allow studies of the correlations between shape transitions and deviations between calculated and experimental pairing gaps we present in fig. 18 calculated ground-state quadrupole deformations. We have indicated oblate regions by horizontal black lines.

Obviously, when a large deviation between a calculated and experimental pairing gap occurs it does not mean that the calculated pairing gap is in error by the amount of the discrepancy because apart from the pairing gap itself there are many other possible nonsmooth contributions the experimental odd-even mass differences. We have earlier mentioned three sources of nonsmooth contributions: uncertainties in measured masses, shape transitions and gaps in level spectra, other than magic gaps. We now discuss each effect separately.

4.1 Effect of experimental mass uncertainties

The errors in the experimental pairing gaps due to mass uncertainties are, as seen in figs. 11 and 12, quite large in a few special regions and more generally along the borders of the known regions. However, the regions of large errors do not seem strikingly correlated with large errors in the plots of the discrepancy between calculated and experimental pairing gaps in figs. 13–16. A region of particularly large errors is the neutron-rich region between $N = 20$ and $N = 28$. We have in our pairing studies here included in our experimental database recent nuclear mass measurements in this region. The uncertainties of these measurements are larger than in the data base of 1654 nuclei²⁴) that we use in our nuclear mass studies^{19–21}) and which we also use here. To understand the effect of uncertainties in the measured masses on our analysis here we exclude the new masses in the region between $N = 20$ and 28 and perform an error analysis corresponding to the one shown in table 3 now based only on a database of 1654 masses²⁴). The results are shown in table 4.

Table 4: Errors of pairing gap calculations. The data base of measured masses consist only of one large data base of 1654 masses compiled by Audi²⁴).

Nucleons	N_{nuc}	LN				Mass model			
		rms (MeV)	μ_{th} (MeV)	σ_{th} (MeV)	2nd cm (MeV)	rms (MeV)	μ_{th} (MeV)	σ_{th} (MeV)	2nd cm (MeV)
neutron	756	0.1712	0.0220	0.1610	0.1627	0.2088	-0.0427	0.1929	0.1972
protons	648	0.1670	0.0455	0.1529	0.1596	0.1994	-0.0096	0.1728	0.1730
total	1446	0.1692	0.0328	0.1577	0.1627	0.2045	-0.0274	0.1846	0.1972

The errors are now all smaller than the errors in table 3 corresponding to the larger data set. However, the effect on the rms deviation is much larger than on the second central moments. For example the second central moment for Δ_{LN} decreased from 0.1634 MeV in table 3 to 0.1627 MeV in table 4, a decrease by only 0.0007 MeV. The rms deviation on the other hand decreased from 0.1728 MeV to 0.1692 MeV, a decrease by 0.0036 MeV, a decrease that is more than 5 times larger than the decrease in the second central moment. The second central moment is by construction of our theory insensitive to the errors in the experimental data set. The 0.0007 MeV decrease we observed here is just a normal random statistical fluctuation that results from using a different data set. One could in fact investigate the results for several other restricted choices of data and in this manner obtain an estimate of the uncertainty in our estimate of σ_{th} , that is we could estimate the standard deviation of σ_{th} . The rms deviation is, in contrast to σ_{th} , sensitive to the errors associated with the data. This is the reason for the substantial decrease in the rms deviation when a few data points with large errors were removed.

Thus, we have established that the second central moments that we use as a measure of the overall agreement between theory and experimental pairing gaps are insensitive to the experimental error. However, if the experimental errors are large one would need more data points to determine the second central moment accurately than would be required if the data points were known to higher accuracy.

Although the second central moment is insensitive to experimental errors, there are other contexts where the experimental errors as they are now distributed make some studies difficult. For example, if one decides to plot experimental and theoretical pairing gaps for individual isotope chains and compare their trend with neutron number some difficulties arise because the experimental errors are building up in the beginning and end of each isotope chain. It is exactly here one would expect to see if some systematic difference with neutron excess started to appear between calculated and experimental gaps.

4.2 Effect of shape transitions

There are several regions in the discrepancy plots where the deviations that appear, probably show the effect of shape transitions on the experimental pairing gap rather than

an error in the calculated pairing gap. A particularly illustrative example is the beginning and end of the rare earth region. Here at about $N = 88$ and 114 the discrepancy of the neutron Δ_{LN} pairing gap in fig. 13 is particularly large. Figure 18 indicates that the region of well-deformed rare-earth nuclei starts and ends at these neutron numbers. Thus, at these neutron numbers shape transitions occur. Therefore, the experimental pairing gap in these regions is likely to contain nonsmooth contributions resulting from this shape transition, and, in consequence, not represent the true pairing gap. We mentioned earlier that shape transitions make the same contributions to pairing gaps extracted from odd-even mass differences based on calculated masses as to the experimental odd-even mass differences. Therefore errors associated with shape transitions would not be expected to in plots of $\Delta_{\text{exp}} - \Delta_{\text{thmass}}$. We see that in fig. 15 the error in the rare-earth transition regions are much smaller than in fig. 13, so we are partially borne out. Obviously the remaining errors in the calculated mass surface prevents perfect agreement in these transition regions.

Fairly large discrepancies occur for both neutrons and protons near $Z = 40$ and $N = 40$ in figs. 13 and 14. We see in fig. 18 that there are rapid changes in deformation here, both in the magnitude and in sign of the deformations; that is there are sudden changes between prolate and oblate shapes in this region. Therefore one cannot expect the extracted experimental pairing gap to be very accurate in this region. However, a comparison between Δ_{exp} and Δ_{thmass} may be less affected by such shape changes, as discussed above. This is indeed the case, the discrepancy in figs. 15 and 16 is smaller than the corresponding discrepancy in figs 13 and 14. Thus the theoretical calculations pick up these shape changes fairly well. However in figs. 15 and 16 there are large discrepancies near $Z = 40$ and $N = 42$. They occur because of the large errors in the calculated masses near these nucleon numbers.

4.3 Effect of gaps in level spectrum

Magic numbers give discontinuities in the mass surface that give contributions in odd-even mass-difference expressions. However, precisely for this reason regions around magic numbers are not included in our comparison between calculated and experimental pairing gaps. But just as magic gaps in the single-particle level spectra give nonsmooth contributions to the odd-even mass surface, any gap for spherical or deformed shapes will give some nonsmooth contribution to the odd-even mass differences. The magnitude of the contribution depends on the magnitude of the gap.

In the plot of the discrepancy between Δ_{exp} and Δ_{LN} for neutrons there is a large discrepancy near $Z = 40$ and $N = 56$. It is well-known that 56 is a submagic neutron gap, reinforced by proton number 40. Thus one can expect that the odd-even mass-difference expression does not exactly give the true pairing gap here. A comparison between Δ_{exp} and Δ_{thmass} exhibits equally large errors in fig. 15. But the mass model has some of its largest errors in this region as is seen in fig. 17. One must therefore expect that Δ_{thmass} is a very inaccurate model for the theoretical pairing gap in this localized region.

The proton pairing-gap discrepancy in this region in figs. 14 and 16 is much smaller. It is conceivable that the submagic gap $Z = 40$ is much less pronounced than $N = 56$, leading to a more accurate Δ_{exp} .

Other regions with relatively large neutron pairing-gap discrepancies occur near $N =$

108 and $N = 152$. However, these neutron numbers are clearly associated with gaps in the level spectra of deformed rare-earth and actinide nuclei and also manifest themselves as irregularities in the two-neutron separation energies²⁵).

Although we have shown that several of the large discrepancies between Δ_{exp} and Δ_{LN} occur at well-known subshell closures, which are expected to give rise to errors in Δ_{exp} we have not been able to prove that all the discrepancy comes from this effect. It is indeed possible that some of the discrepancy is due to errors in the calculated Δ_{LN} , which may be somewhat too high at shell closures.

4.4 Favorable regions for theory–experiment comparisons

We have emphasized that comparisons between Δ_{exp} and Δ_{LN} are often complicated by nonsmooth contributions to Δ_{exp} that are not related to the pairing gap. These other nonsmooth contributions arise mainly from shape transitions and from sub-shell closures. Ideally one would like to compare Δ_{exp} and Δ_{LN} in regions where such non-pairing contributions are small or negligible.

Since Δ_{thmass} picks up contributions from sub-shell closures and shape transitions in a similar way as does Δ_{exp} one can identify regions where these effects are absent in the following way. One expects that the two theoretical pairing gaps Δ_{thmass} and Δ_{LN} are roughly identical, except where shape transitions and sub-shell closures give extra contributions to Δ_{thmass} . In plots versus N and Z of $\Delta_{\text{thmass}} - \Delta_{\text{LN}}$ one can therefore identify regions that exhibit large differences with shape transitions or sub-shell closures. In regions where such effects are absent Δ_{thmass} and Δ_{LN} are expected to be nearly equal.

We plot the difference $\Delta_{\text{thmass}} - \Delta_{\text{LN}}$ for neutrons and protons in figs. 19 and 20, respectively. Regions where the plotted function is close to zero indicate regions where Δ_{exp} is expected to represent most accurately the true pairing gap. However, this conclusion is only correct if the calculated mass surface accurately represents shape transitions and sub-shell closures. A well-known failure of the model in this respect occurs in the region near $Z = 40$ and $N = 56$. Here fig. 19 suggests that the region is well suited for a comparison between Δ_{exp} and Δ_{LN} . It is not, because although the model does not exhibit a strong $N = 56$ subshell closure, real nuclei do.

4.5 Alternative approaches to comparing theoretical and experimental pairing effects

Almost all our comparisons above between our pairing model and pairing effects in real nuclei have focussed on comparing calculated pairing gaps with odd-even mass differences based on experimental masses. Because large uncertainties are present in the experimental pairing gaps it is desirable to compare pairing models to effects in real nuclei also by other means.

As one such alternative we compared directly in table 2 calculated masses with experimental masses. We found that such a comparison allowed the accurate determination of the effective-interaction pairing-gap parameter r . Indeed, the value we obtained was only 1.5% or so different from what is obtained in an rms minimization of $\Delta_{\text{exp}} - \Delta_{\text{LN}}$ or $\Delta_{\text{exp}} - \Delta_{\text{thmass}}$.

Quantities related to nuclear rotation show strong dependences on pairing properties. A complimentary approach to studying odd-even mass differences is therefore to ... (more later)

5 Summary and conclusions

We have presented results of global nuclear pairing calculations. Nuclear ground-state shapes and energy levels used in our pairing studies were obtained from a recent nuclear-structure model that reproduces known masses from ^{16}O to the heaviest elements with an error of only 0.669 MeV. Calculated pairing quantities have been presented in the form of color contour diagrams versus N and Z . Comparisons between calculated pairing gaps and experimental odd-even mass differences have been presented and analyzed in detail. Our findings are:

- The pairing gaps extracted from odd-even experimental mass differences are subject to large uncertainties, because the difference expressions pick up any nonsmooth component of the mass surface, for example effects related to subshell closures and shape transitions.

References

- 1) S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **29**:No. 16 (1955).
- 2) B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Skr. . **1**:No. 8 (1959).
- 3) V. M. Strutinsky, Nucl. Phys. **A95** (1967) 420.
- 4) V. M. Strutinsky, Nucl. Phys. **A122** (1968) 1.
- 5) Å. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110** (1958) 936.
- 6) S. T. Belyaev, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **31**:No. 11 (1959).
- 7) S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **32**:No. 16 (1961).
- 8) W. Ogle, S. Wahlborn, R. Piepenbring, and S. Fredriksson, Rev. Mod. Phys. **43** (1971) 424.
- 9) H. J. Lipkin, Ann. Phys. (N. Y.) **9** (1960) 272.
- 10) Y. Nogami, Phys. Rev. **134** (1964) B313.
- 11) H. C. Pradhan, Y. Nogami, and J. Law, Nucl. Phys. **A201** (1973) 357.
- 12) I. Hamamoto, Nucl. Phys. **62** (1965) 49.
- 13) J. A. Halbleib, Sr. and R. A. Sorensen, Nucl. Phys. **A98** (1967) 542.

- 14) J. Krumlinde and P. Möller, Nucl. Phys. **A417** (1984) 419.
- 15) P. Möller and J. R. Nix, Nucl. Phys. **A536** (1992) 20.
- 16) P. Vogel, B. Jonson, and P. G. Hansen, Phys. Lett. **139B** (1984) 227.
- 17) A. S. Jensen, P. G. Hansen, and B. Jonson, Nucl. Phys. **A431** (1984) 393.
- 18) D. G. Madland and J. R. Nix, Nucl. Phys. **A476** (1988) 1.
- 19) P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Nucl. Phys. **A536** (1992) 61.
- 20) P. Möller, J. R. Nix, K.-L. Kratz, A. Wöhr, and F.-K. Thielemann, Proc. 1st Symp. on nuclear physics in the universe, Oak Ridge, 1992 (IOP Publishing, Bristol, 1993) to be published.
- 21) P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Atomic Data Nucl. Data Tables (1993) to be published.
- 22) P. Möller and J. R. Nix, Atomic Data Nucl. Data Tables **39** (1988) 213.
- 23) P. Möller and J. R. Nix, Atomic Data Nucl. Data Tables (1993) to be published.
- 24) G. Audi, Midstream atomic mass evaluation, private communication (1989), with four revisions.
- 25) R. Bengtsson, P. Möller, J. R. Nix, and Jing-ye Zhang, Phys. Scr. **29** (1984) 402.

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Figure Captions

- Fig. 1 Global microscopic pairing calculation for neutrons. The energy levels entering the pairing calculation correspond to ground-state shapes that have been determined by minimizing the total potential energy with respect to ϵ_2 , ϵ_4 , ϵ_3 and ϵ_6 shape degrees of freedom. In the LN model it is the sum $\Delta_n + \lambda_{2n}$ that should be compared to odd-even mass differences. The solid black lines in the middle of the colored region indicate the regions where experimental neutron pairing gaps may be extracted from odd-even mass differences. In contrast to the behaviour of BCS solution the sum $\Delta_n + \lambda_{2n}$ plotted here shows no sign of collapse at magic neutron numbers.
- Fig. 2 Global behaviour of the neutron pairing gap Δ_n , a term in the sum $\Delta_n + \lambda_{2n}$ plotted in fig. 1. Whereas the sum $\Delta_n + \lambda_{2n}$ showed no sign of collapse at magic numbers, the LN pairing gap Δ_n exhibits a significant decrease near most magic neutron numbers.
- Fig. 3 Global behavior for the neutron number-fluctuation constant λ_{2n} a term in the sum $\Delta_n + \lambda_{2n}$ plotted in fig. 1. In contrast to Δ_n the number-fluctuation constant λ_{2n} becomes very large at magic neutron numbers.
- Fig. 4 Global microscopic pairing calculation for protons. The energy levels entering the pairing calculation correspond to ground-state shapes that have been determined by minimizing the total potential energy with respect to ϵ_2 , ϵ_4 , ϵ_3 and ϵ_6 shape degrees of freedom. In the LN model it is the sum $\Delta_p + \lambda_{2p}$ that should be compared to odd-even mass differences. The solid black lines in the middle of the colored region indicate the regions where experimental proton pairing gaps may be extracted from odd-even mass differences. In contrast to the behaviour of BCS solution the sum $\Delta_p + \lambda_{2p}$ plotted here shows no sign of collapse at magic proton numbers.
- Fig. 5 Global behaviour of the proton pairing gap Δ_p , a term in the sum $\Delta_p + \lambda_{2p}$ plotted in fig. 4. Whereas the sum $\Delta_p + \lambda_{2p}$ showed no sign of collapse at magic numbers, the LN pairing gap Δ_p exhibits a significant decrease near most magic proton numbers.
- Fig. 6 Global behavior for the proton number-fluctuation constant λ_{2p} a term in the sum $\Delta_p + \lambda_{2p}$ plotted in fig. 4. In contrast to Δ_p the number-fluctuation constant λ_{2p} becomes very large at magic proton numbers.
- Fig. 7 Neutron pairing gaps extracted from odd-even *theoretical* mass differences. If the theoretical mass model correctly describes shape changes and deformed gaps in the single-particle level spectra, and since these effects would give a similar contribution to gaps extracted from odd-even experimental mass differences, one anticipates that some discrepancies that are present in fig. 13 would not appear in the difference between the function in fig. 9 and the function in this figure. This is further discussed in the text.

- Fig. 8 Proton pairing gaps extracted from odd-even *theoretical* mass differences. If the theoretical mass model correctly describes shape changes and deformed gaps in the single-particle level spectra, and since these effects would give a similar contribution to gaps extracted from odd-even experimental mass differences, one anticipates that some discrepancies that are present in fig. 14 would not appear in the difference between the function in fig. 10 and the function in this figure. This is further discussed in the text.
- Fig. 9 Neutron pairing gaps extracted from odd-even mass differences. Globally the pairing gap decreases with increasing A . However, there are strong local deviations from this general trend. For example in the two regions between the magic neutron numbers $N = 28$ and 50 the pairing gap exhibits an increase with A or N in the lower region and a decrease in the upper region. The same behaviour occurs in the two regions between the magic neutron numbers $N = 50$ and 82 . This behaviour illustrates that it is probably very hard to deduce general pairing trends from a study limited to a single region between magic numbers. Instead a global investigation would normally be required to allow reasonably definite conclusions to be drawn.
- Fig. 10 Proton pairing gaps extracted from odd-even mass differences. Globally the pairing gap decreases with increasing A . However, there are strong local deviations from this general trend. To establish the general trend of the pairing gap it is necessary to study several of the regions between magic proton and neutron numbers.
- Fig. 11 Errors in experimental neutron pairing gaps extracted from measured masses by use of fourth-order finite difference expressions. The errors plotted here are those that arise from uncertainties in the measured masses. Other sources of errors further enhance the error in the experimental pairing gaps.
- Fig. 12 Errors in experimental proton pairing gaps extracted from measured masses by use of fourth-order finite difference expressions. The errors plotted here are those that arise from uncertainties in the measured masses. Other sources of errors further enhance the error in the experimental pairing gaps.
- Fig. 13 Discrepancy between experimental neutron pairing gaps determined from odd-even mass differences and microscopic pairing gaps $\Delta_n + \lambda_{2n}$ obtained in the LN model. The figure is the difference between the functions plotted in figs. 9 and 1. Fairly large discrepancies occur in several places. However, these discrepancies do not necessarily mean that the calculated gaps are incorrect. Instead, it may be that the experimental pairing gap is not determined properly from odd-even mass differences as discussed in the text.
- Fig. 14 Discrepancy between experimental proton pairing gaps determined from odd-even mass differences and microscopic pairing gaps $\Delta_p + \lambda_{2p}$ obtained in the LN model. The figure is the difference between the functions plotted

in figs. 10 and 4. Fairly large discrepancies occur in several places. However, these discrepancies do not necessarily mean that the calculated gaps are incorrect. Instead, it may be that the experimental pairing gap is not determined properly from odd-even mass differences. In particular when large errors in this figure occur in the same region where large errors occur in fig. 13 one may suspect that the reasons are sudden shape transitions.

- Fig. 15 Discrepancy between experimental neutron pairing gaps from fig. 9 and calculated pairing gaps from fig. 7, which have been determined from odd-even *theoretical* mass differences. In the transitional regions between spherical and deformed regions at the beginning and at the end of the rare-earth region at $N \approx 90$ and $N \approx 116$, respectively, the discrepancy here is smaller than in fig. 13.
- Fig. 16 Discrepancy between experimental proton pairing gaps from fig. 10 and calculated pairing gaps from fig. 8, which have been determined from odd-even *theoretical* mass differences.
- Fig. 17 Discrepancy between experimental masses and our current FRDM. For the whole region the model error is 0.669 MeV, but for nuclei with $N > 65$ the error is only 0.448 MeV. Although the overall agreement between calculated and measured masses is quite good there are correlated local fluctuations of the error that are larger in magnitude, in particular near $N = 82$ and in the vicinity of $Z = 40$ and $N = 60$ and in the very light region. Such short-range local fluctuations in the error will give rise to errors in the pairing gaps extracted from odd-even *theoretical* mass differences in these regions.
- Fig. 18 Calculated ground-state quadrupole deformations for nuclei throughout the periodic system. Oblate nuclei are indicated by horizontal black lines. Nuclear pairing gaps are not properly determined from odd-even mass differences when there are non-smooth contributions to the nuclear ground state mass. Such contributions occur where there are sudden shape changes, for example from prolate to oblate shape or from deformed to spherical shape. Such transition regions may be identified in this figure.
- Fig. 19 Difference between neutron pairing gaps calculated in two different ways: (a) from odd-even *theoretical* mass differences (cf. fig. 7) and (b) from the LN microscopic pairing equations (cf. fig. 1). If the mass surface were smoothly varying, except for odd-even effects, both methods would yield similar results because the LN pairing gap is one of contributions to the odd-even and odd-odd calculated nuclear mass in the FRDM. When the difference plotted here is significantly different from zero it therefore signals the presence in the calculated mass surface of other non-smooth contributions.
- Fig. 20 Difference between proton pairing gaps calculated in two different ways: (a) from odd-even *theoretical* mass differences (cf. fig. 8) and (b) from the LN microscopic pairing equations (cf. fig. 4). As in fig. 19 regions significantly

different from zero show the presence in the mass surface of non-smooth contributions other than those present in our pairing model.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

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