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MICROSTRUCTURAL DESIGN OF CELLULAR MATERIALS I:HONEYCOMB BEAMS AND PLATES

J.S. Huang* and L.J. Gibson*

*Department of Civil Engineering
National Cheng Kung University
Tainan, Taiwan

*Department of Civil Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

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Abstract

Performance indices for materials describe the mechanical efficiency of a component under a given mode of loading: the higher the performance index, the lower the mass of the component for a given mechanical requirement. Material selection charts (Ashby, 1989) offer a graphical means of comparing performance indices for a wide range of materials. The performance indices are first described. Micromechanical models for the behaviour of cellular materials are then used to suggest novel microstructural designs for cellular materials with improved performance. Three novel microstructural designs, described in the two companion papers, have been fabricated and tested. The results of the tests indicate that the new microstructures have higher values of some performance indices than those of the solids from which they are made.

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MICROSTRUCTURAL DESIGN OF CELLULAR MATERIALS I:HONEYCOMB BEAMS AND PLATES1. Introduction

Wood is often cited for its high stiffness per unit weight: its value of E/ρ compares well with that for mild steel. For beams or plates, the relevant performance indices are $E^{1/2}/\rho$ and $E^{1/3}/\rho$: here, wood outperforms mild steel and is comparable to unidirectional carbon fibre reinforced plastics (CFRP). In this paper we use micromechanical models for cellular materials to understand the excellent performance of wood in beams and plates. The results suggest that materials with a honeycomb-like array of parallel cells are efficient in bending stiffness. The Young's modulus and fracture toughness of model materials we call honeycomb beams and plates (mimicking wood) are measured. The performance indices for honeycomb beams and plates are higher than those of the solid from which they are made; the indices are well described by simple micromechanics of cellular materials.

2. Analysis

Performance indices for materials describe the mechanical efficiency of a component under a given mode of loading: the higher the performance index, the lower the mass of the component for a given mechanical requirement. For an axial tie rod of a given stiffness, for instance, the material which minimizes the mass of the tie is that with the

highest value of E/ρ . But for a beam of a given flexural stiffness, the material which minimizes the mass of the beam is that with the highest value of $E^{1/2}/\rho$. And for a plate of a given flexural stiffness, the best material is that with the highest value of $E^{1/3}/\rho$. The argument can be illustrated by the example of the beam of a given bending stiffness, S_b .

$$S_b = \frac{P}{\delta} = \frac{C_1 EI}{l^3} \quad (1)$$

where P is the load applied to the beam, δ is the deflection, E is its Young's modulus, I is the second moment of area of the cross section, l is the span and C_1 is a constant related to the loading configuration. The mass, m , of the beam is given by:

$$m = \rho A l \quad (2)$$

where ρ is the density of the material the beam is made from and A is the cross sectional area. Defining a shape factor, $\Phi = 4\pi I/A^2$ and combining eqns (1) and (2) gives:

$$m = \left(\frac{4\pi S_b l^5}{C_1 \Phi E} \rho^2 \right)^{1/2} \quad (3)$$

The *material* which minimizes the mass of a beam of a given flexural stiffness is that which *maximizes* $E^{1/2}/\rho$. Table 1 lists performance indices for various mechanical requirements and loading configurations (Ashby, 1989). The *cross-section*

which minimizes the mass of the beam is that which maximizes the *shape factor*, Φ (Ashby, 1991).

Material selection charts (Ashby, 1989) offer a graphical means of comparing performance indices for a wide range of materials. Figure 1 shows a material selection chart plotting Young's modulus against density. The guidelines indicate lines of constant value of each of the performance indices described above: E/ρ , $E^{1/2}/\rho$, and $E^{1/3}/\rho$. Cellular materials are represented on the plot by woods and foams. Woods loaded along the grain have roughly equal values of E/ρ . Noting that wood cells are roughly long hexagonal prisms that deform uniaxially when loaded along the grain, it is simple to show that (Easterling et al, 1982):

$$\frac{E}{E_s} = \frac{\rho}{\rho_s} \quad (4)$$

where E_s and ρ_s , the Young's modulus along the grain and the density of the solid wood cell wall material, respectively, are constants, independent of wood species (Dinwoodie, 1981). The value of E/ρ for wood is equal to that of the cell wall material. Equation (4) can be rearranged to give:

$$\frac{E^{1/2}}{\rho} = \frac{E_s^{1/2}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{1/2} \quad (5)$$

The value of $E^{1/2}/\rho$ for wood is higher than that for the solid, by a factor of the square root of the inverse of the relative density. Low density woods have higher values of

$E^{1/2}/\rho$ than higher density woods: balsa beams have a lower mass than oak beams of the same bending stiffness. This observation suggests that for a beam of a given bending stiffness a material with parallel holes along the length of the beam is mechanically efficient: we call such beams *honeycomb beams*, as the cross section resembles a honeycomb (Fig. 2).

Equation (4) can also be rearranged to give:

$$\frac{E_s^{1/3}}{\rho} = \frac{E_s^{1/3}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{2/3} \quad (6)$$

It appears that the performance index $E^{1/3}/\rho$, for a plate of a given bending stiffness, increases with the square of the inverse of the relative density for wood: wood is even more efficient than its solid for a plate in bending. But this has neglected the fact that wood is anisotropic. Its Young's modulus is linearly proportional to the density (eqn 4) only for loading along the grain; for loading across the grain wood is much less stiff. To overcome this difficulty, engineers laminate sheets of wood together to form plywood. Here, we idealize this structure as one of laminae of a honeycomb-like material oriented at 90° to one another (Fig. 3). We next consider the performance index $E^{1/3}/\rho$ for such a *honeycomb plate*.

The Young's modulus of a single lamina of the plate, parallel to the prism axis of the honeycomb, E_1 , is simply :

$$E_1 = (\rho/\rho_s)E_s \quad (7)$$

while that normal to the prism axis, E_2 , is, for regular hexagonal cells (Gibson and Ashby, 1988):

$$E_2 = 1.5(\rho / \rho_s)^3 E_s \quad (8)$$

The Young's modulus of an orthotropic plate made up of symmetric cross-ply of this lamina can be found using laminate theory (Hull, 1981). For honeycomb laminae, with $v_{21}=0$, the equivalent plate modulus E_{eq} is simply given by the rule of mixtures:

$$E_{eq} = V_1E_1 + V_2E_2 \quad (9)$$

Assuming that the plate is made up of equal volume fractions of laminae in either orientation, arranged in symmetric cross-ply, the equivalent plate modulus becomes:

$$E_{eq} = 0.5(\rho/\rho_s)E_s + 0.75(\rho/\rho_s)^3 E_s \quad (10)$$

For low density honeycomb plates, the second term is small compared with the first and we can make the approximation that:

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$$\frac{E^{1/3}}{\rho} = 0.8 \frac{E_s^{1/3}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{2/3} \quad (11)$$

The fracture toughness K_{IC} of a honeycomb beam can also be calculated. Fracture toughness of an orthotropic material is related to the critical strain energy release rate G_c and the elastic compliance tensor a_{ij} by (Sih et al., 1965):

$$K_{IC}^2 = G_c / f(a_{ij}) \quad (12)$$

G_c is the product of the relative density of the honeycomb beam and the critical strain energy release rate of the solid G_{cs} :

$$G_c = (\rho/\rho_s) G_{cs}. \quad (13)$$

$f(a_{ij})$ is given by

$$f(a_{ij}) = \left(\frac{a_{11}a_{22}}{2} \right)^{1/2} \left[\left(\frac{a_{22}}{a_{11}} \right)^{1/2} + \frac{2a_{12} + a_{66}}{2a_{11}} \right]^{1/2}$$

where 2 is the direction normal to the crack plane and 1 is the direction of crack propagation. For a honeycomb beam with the crack plane normal to the prism axis we find $f(a_{ij})$ is $1/[0.63(\rho/\rho_s)E_s]$. Noting that the fracture toughness of the solid $K_{ICs} = (G_{cs}E_s)^{1/2}$, we find that the fracture toughness of a honeycomb beam is then:

$$K_{IC} = 0.8(\rho/\rho_s)K_{ICs} \quad (14)$$

Rearranging eqn (13) in terms of the performance index which minimizes the weight of a beam of a given fracture load, $K_{IC}^{2/3}/\rho$, we obtain:

$$\frac{K_{IC}^{2/3}}{\rho} = C \frac{K_{ICs}^{2/3}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{1/3} \quad (15)$$

Again, the honeycomb beam performs better than the solid from which it is made; for a given fracture load it will be lighter by a factor of $(\rho_s/\rho)^{1/3}$.

Here we describe the fabrication and mechanical testing of polyester resin beams with various volume fractions of parallel, longitudinal holes and of two cross-sections: square ($\Phi = 1.05$) and I section ($\Phi = 1.67$). The results of the tests confirm that the performance index $E^{1/2}/\rho$ increases with the square root of the inverse of relative density. A single honeycomb plate was fabricated and tested in bending. Its value of $E^{1/3}/\rho$ is 1.25 times that of the solid resin, in good agreement with the value calculated from eqn (10). The fracture toughness of notched honeycomb beams was also measured: the trend is less clear, but the performance index $K_{IC}^{2/3}/\rho$ does increase with decreasing relative density.

3. Experimental Methods

Honeycomb beams with parallel cylindrical holes along their length were made from polyester resin with relative densities of 0.85, 0.61, 0.51 and 0.32. The honeycomb beams

were made by pouring resin mixed with a hardener into specially designed rectangular molds through which rubber tubing (Tygon) was strung (Fig. 4). The three highest relative density beams were made using tubes of equal diameter. The lowest relative density beam was made using two different diameter tubes, with the smaller tubes in the interstices between the larger ones. The specimens were removed from the molds after curing for 4 hours under a fume hood. After curing, the rubber tubes were pulled out of the resin and the beams were either trimmed to give a square cross-section or machined to give an I section. A solid polyester resin beam was also cast to measure the solid resin properties. Photographs of three specimens are shown in Fig. 5a,b,c. The density of each specimen was calculated from measurements of the dimensions made with a micrometer accurate to 0.1 mm and of the mass, made with a balance accurate to 0.1 g.

A single honeycomb plate (Fig. 5d) was made and tested in bending to measure its value of $E^{1/3}/\rho$. The plate was made using the same technique as the beams, with the exception that the rubber tubes were arranged in 8 layers oriented at 90° to each other, as shown in Fig. 5. Each lamina was of equal thickness and there was an equal number of lamina with holes in each direction. The dimensions of the plate were 301mm x 301mm x 27mm.

The Young's moduli of the solid resin and the honeycomb beams were calculated from tests in three point bending using

an Instron testing machine (Model 4201) as shown in Fig. 6. The setup was first checked by measuring the linear elastic response of an aluminum beam of cross sectional dimensions 15.9mm x 15.9mm and span 254mm. The measured bending stiffness was 1100 N/mm, giving a Young's modulus of 70.7 GPa, in excellent agreement with handbook values for aluminum.

The fracture toughnesses of the solid resin and the honeycomb beams were calculated from the results of three point bending tests on notched beams. The beams had nominal dimensions of 25mm x 25mm x 156mm and were tested over a span of 130mm. The notch length was half the depth of each beam; the fracture surface is normal to the cylindrical holes. Six specimens of each relative density of beam were tested.

The Young's modulus of the honeycomb plate was calculated from its load deflection curve in a bending test in the same Instron testing machine (Model 4201). A central load was applied while the plate was simply supported on all 4 edges by rollers. The deflection of the plate was measured by a linear variable differential transducer (LVDT) placed under the plate at its central point.

4. Results

The results of the three point bend tests on square cross-section honeycomb beams of different relative densities are given in Table 2. Young's modulus, E , was calculated

from the measured slope of the linear elastic load-deflection curve, P/δ , using the equation:

$$E = \frac{(P/\delta)l^3}{48I} \quad (16)$$

where l is the span of the beam and I is the second moment of area. The ratio of the performance index of each honeycomb beam to that of the solid resin, $E^{1/2}/\rho/E_s^{1/2}/\rho_s$, is plotted against relative density in Fig. 6; it increases with the square root of the inverse of the relative density, as expected from eqn (5). The lowest density honeycomb beam tested ($\rho/\rho_s = 0.39$) has a mass 0.56 times that of an equivalent solid beam.

The effect of cross sectional shape on the performance of the honeycomb beams was measured by testing beams of the same relative densities as specimens 2, 3 and 4, but with an I section with a shape factor, Φ , of 1.67 rather than a square section with $\Phi = 1.05$. The results are listed in Table 3. The measured value of Φ is calculated by rearranging eqn (3):

$$\Phi = \frac{4\pi}{C_1} \frac{l^5(P/\delta)}{(E/\rho^2)m^2} \quad (17)$$

and substituting $C_1 = 48$ for three point bending and the measured values for the span, l , the stiffness, P/δ , the performance index, $E^{1/2}/\rho$, and the mass, m . Agreement between

the theoretical and measured values of the shape factor is excellent.

The density and Young's modulus of the honeycomb plate tested in bending were measured to be 0.728 Mg/m³ and 1.54 GPa, respectively, giving a value of $E^{1/3}/\rho$ of 1.58 GPa^{1/3}m³/Mg, 1.26 times the value of 1.25 GPa^{1/3}m³/Mg measured for the solid resin. The performance index $E^{1/3}/\rho$ of the honeycomb plate relative to that of the solid resin in plotted against relative density in Fig. 8. The measured value is in close agreement with that given by eqn (10).

The fracture toughness of each notched honeycomb beam specimen is calculated from the maximum load, P , in the three point bend test

$$K_{IC} = \frac{PS}{BW^{3/2}} f(a/w)$$

where S is the span, B is the width, W is the depth and $f(a/w)$, a function of the ratio of the notch length to the depth, is equal to 2.66 for $a/w = 0.5$ (Broek, 1982). The results are listed in Table 4 and are plotted in Fig. 9. The fracture toughness decreases almost linearly with relative density, as expected from eqn (14). The ratio of the performance index $K_{IC}^{2/3}/\rho$ for the honeycomb beams relative to that for the solid resin is roughly constant at a value of about 1; we expect that lower density honeycomb beams would produce improved fracture toughness results.

5. Discussion

The performance indices identifying the material minimizing mass for a beam of a given flexural stiffness, a plate of a given flexural stiffness and a beam of a given fracture load are, respectively:

$$\frac{E^{1/2}}{\rho} = \frac{E_s^{1/2}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{1/2}$$

$$\frac{E^{1/3}}{\rho} = 0.8 \frac{E_s^{1/3}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{2/3}$$

$$\frac{K_{IC}^{2/3}}{\rho} = \frac{K_{ICs}^{2/3}}{\rho_s} \left(\frac{\rho_s}{\rho} \right)^{1/3}$$

These simple equations, based on the micromechanics of cellular materials, describe the measured performance indices for beam and plate stiffness and beam fracture load well (Fig. 7-9). The mass of a honeycomb beam of a given stiffness or fracture load is lower than that of a solid beam: the lowest relative density beam made in these tests is 1/1.79 or 56% of the mass of a solid resin beam of the same stiffness, span and cross-sectional shape (Table 2). Like solid, homogeneous materials, the mass of honeycomb beams can be further reduced by the use of efficient cross-sectional shapes with high shape factors (Table 3). A honeycomb plate has a higher performance index than the solid from which it is made: the plate tested in this study is 25% lighter than the equivalent solid plate of the same

stiffness. The fracture toughness of the honeycomb beams tested decreases linearly with relative density, suggesting that at low densities, the fracture performance index should improve (Fig. 9).

The lowest relative density honeycomb beam made in this study was 0.32; only a single honeycomb plate was made, of relative density 0.59. Clearly, for both stiffness and fracture load requirements, the efficiency of honeycomb beams and plates increases as the relative density decreases. At a relative density of 0.15, the relative performance indices have the following values:

$$\begin{aligned} (E^{1/2}/\rho)/(E_s^{1/2}/\rho_s) &= 2.58 && \text{(honeycomb beam)} \\ (E^{1/3}/\rho)/(E_s^{1/3}/\rho_s) &= 2.84 && \text{(honeycomb plate)} \\ (K_{IC}^{2/3}/\rho)/(K_{ICs}^{2/3}/\rho_s) &= 1.88 && \text{(honeycomb beam)} \end{aligned}$$

The honeycomb beam is $1/2.58 = 39\%$ of the mass of a solid beam of the same flexural stiffness; the honeycomb plate is 35% of the mass of a solid plate of the same flexural stiffness; and the honeycomb beam is 53% of the mass of a solid beam of the same fracture load. Significant weight savings are possible for low density materials. In the future we plan to make lower density honeycomb beams and plates by using a gradation of void sizes.

The results of this study demonstrate that it is possible to produce honeycomb beams and plates with higher values of the performance indices $E^{1/2}/\rho$ and $E^{1/3}/\rho$ than the

solid from which they are made. Solids with already high values of the performance indices can be improved further by fabricating them into honeycomb beams or plates. Consider, for instance, a unidirectional carbon fibre reinforced epoxy with 60% volume fraction of fibres. Its density is 1.62 Mg/m³ and its Young's modulus parallel to the fibre direction is around 220 GPa, giving a value of $E^{1/2}/\rho$ of 9.2 GPa m³ Mg⁻¹ one of the highest values for any material currently available. It is an excellent material choice for beams of minimum mass. Yet the mass of a CFRP beam of a given stiffness could be lowered even further by fabricating the CFRP into a honeycomb beam. The relative performance index $E^{1/2}/\rho / E_s^{1/2}/\rho_s = (\rho_s/\rho)^{1/2}$: a CFRP honeycomb beam with a relative density of 0.2 would have a relative performance index of 2.24. The mass of such a honeycomb CFRP beam would be 1/2.24 or 45% of that of a solid CFRP beam of equal stiffness, cross-sectional shape, span and loading configuration. Further reductions in mass could be achieved at lower relative densities. Honeycomb CFRP plates could be designed with similar savings in mass. Table 5 evaluates relative performance indices for a number of potential materials.

Clearly, there is some limit to the reduction in the relative density of the beam: it is the onset of local buckling. Here we only note that local buckling can be suppressed by a number of means: the introduction of foam-like cells into the voids to act as an elastic foundation;

the introduction of membranes across the void at regular intervals; and pressurizing the voids. Microstructural design to suppress local buckling is one of our current research subjects.

6. Conclusions

Wood beams and plywood plates have high values of the performance indices $E^{1/2}/\rho$ and $E^{1/3}/\rho$: they have a lower weight, for a given stiffness, span and cross-sectional shape, than most other materials. The high mechanical efficiency of wood arises from its microstructure: a honeycomb-like array of parallel, prismatic cells. Here, we have fabricated and tested model engineering materials mimicking this microstructural feature of wood. Honeycomb beams have longitudinal voids parallel to the beam axis; honeycomb plates have laminae with parallel voids oriented at $\pm 90^\circ$ in the plane of the laminae. Both microstructures give higher performance indices $E^{1/2}/\rho$ and $E^{1/3}/\rho$ than the solid from which they are made. The performance indices increase with decreasing relative density of the honeycomb beam or plate as described by eqns (5) and (10), respectively. Low density honeycomb beams also offer improved performance for a given fracture load. Both microstructures offer potential for improved performance of engineering materials.

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Table 1

Performance Indices for Minimum Weight Design

Component Shape and Loading	Performance Index
TIE (tensile strut); stiffness, length specified, section area free	E/ρ
BEAM loaded in bending; stiffness, length specified, section area free	$E^{1/2}/\rho$
COLUMN (compression strut); failure by elastic buckling; collapse load and length specified, section area free	$E^{1/2}/\rho$
PLATE loaded in bending; stiffness, length, width specified, thickness free	$E^{1/3}/\rho$
TIE; fracture load, length, crack geometry specified, section area free	K_{IC}/ρ
BEAM; fracture load, length, crack geometry specified, section area free	$K_{IC}^{2/3}/\rho$
PLATE; fracture load, length, width, crack geometry specified, thickness free	$K^{1/2}/\rho$

ρ = density (Mg/m^3)

E = Young's modulus (GPa)

K_{IC} = fracture toughness ($\text{MPa m}^{1/2}$)

Table 2
 E/ρ^2 for Honeycomb Beams of Square Cross-section

Specimen number	ρ (Mgm ⁻³)	ρ/ρ_s (-)	E (GPa)	$E^{1/2}/\rho$ (GPa ^{1/2} Mg ⁻¹ m ³)	$(E^{1/2}/\rho) / (E_s^{1/2}/\rho_s)$ (-)
1	1.23*	1.00	3.72	1.57	1.00
2	1.05	0.85	3.22	1.71	1.09
3	0.75	0.61	2.48	2.10	1.34
4	0.63	0.51	1.90	2.19	1.39
8	0.39	0.32	1.20	2.81	1.79

* solid resin

Table 3
Measured Shape Factors for Square and
I Section Honeycomb Beams

Specimen	ρ/ρ_s	l	$(P/\delta)_{\text{meas}}$	$(E^{1/2}/\rho)_{\text{meas}}$	m_{meas}	Φ_{meas}	$\Phi_{\text{calc.}}^1$
number	(-)	(mm)	(N/mm)	(GPa ^{1/2} Mg ⁻¹ m ³)	(g)	(-)	(-)
1	1.00	254	245	1.57	162	1.05	1.05
2	0.85	254	406	1.71	191	1.06	1.05
3	0.61	254	164	2.10	99	1.05	1.05
4	0.51	254	240	2.19	114	1.07	1.05
8	0.32	554	241	2.81	636	1.03	1.05
5	0.85	254	361	1.71	147	1.58	1.67
6	0.61	254	155	2.10	79	1.56	1.67
7	0.51	254	220	2.19	91	1.54	1.67

Note: 1. Square section $\Phi = 1.05$
I section $\Phi = 1.67$

Table 4

Fracture Toughness of Honeycomb Beams

ρ	ρ/ρ_s	K_{IC}	$K_{IC}^{2/3}/\rho$	$(K_{IC}^{2/3}/\rho) / (K_{ICs}^{2/3}/\rho_s)$
(Mg/m ³)	(-)	(MPa m ^{1/2})	(MPa ^{2/3} m ^{4/3} /Mg)	-
1.20	1.00	0.832	0.74	1.00
1.01	0.84	0.546	0.66	0.89
0.73	0.61	0.444	0.80	1.08
0.61	0.51	0.336	0.80	1.08

Note: Each result is the average of 6 specimens.

Table 5 Performance indices for honeycomb materials

Material	ρ (Mg/m ³)	E (GPa)	$E/\rho^{1/2}$	$E^{1/2}/\rho^{1/4}$	$E^{1/3}/\rho^{1/6}$
CFRP (unidirectional)	1.62	220	136	9.2	2.4 ²
CFRP honeycomb ($\rho/\rho_s=0.10$)	0.16	22	136	29	14 ³
Ti alloy	4.2	110	26	2.5	1.1
Ti alloy honeycomb ($\rho/\rho_s=0.10$)	0.42	11	26	7.9	4.2
Mild steel	7.8	210	27	1.9	0.75
Mild steel honeycomb ($\rho/\rho_s=0.10$)	0.78	21	27	5.9	2.8

Notes

1. ρ - Mg/m³ ; E - GPa.
2. $E^{1/3}/\rho$ for CFRP laminate.
3. $E^{1/3}/\rho$ estimated from eq'n (11).

Figure Captions

Fig. 1 A materials selection chart plotting Young's modulus, E , against density, ρ . Guidelines indicate contours of equal values of the performance indices E/ρ , $E^{1/2}/\rho$, and $E^{1/3}/\rho$. (after Ashby, 1992)

Fig. 2 A schematic drawing of a honeycomb beam.

Fig. 3 A schematic drawing of a honeycomb plate.

Fig. 4 A schematic drawing of a honeycomb beam mold showing the end plate with drilled holes through which the rubber tubing is passed.

Fig. 5 Photographs showing the cross-sections of (a) a square sectioned honeycomb beam, $\rho/\rho_s = 0.51$ (b) a I sectioned honeycomb beam, $\rho/\rho_s = 0.51$ (c) a square sectioned honeycomb beam, $\rho/\rho_s = 0.32$ and (d) the honeycomb plate, $\rho/\rho_s = 0.59$.

Fig. 6 Three point bending jig.

Fig. 7 (a) Young's modulus plotted against density for the square cross-sectioned honeycomb beams.

Fig. 7 (b) The relative performance index $(E^{1/2}/\rho) / (E_s^{1/2}/\rho_s)$ plotted against relative density ρ/ρ_s for square sectioned honeycomb beams.

Fig. 8 The relative performance index $(E^{1/3}/\rho) / (E_s^{1/3}/\rho_s)$ plotted against relative density ρ/ρ_s for honeycomb plates.

Fig. 9(a) The fracture toughness plotted against relative density for honeycomb beams.

Fig. 9(b) The relative performance index $(K_{IC}^{2/3}/\rho) / (K_{ICs}^{2/3}/\rho_s)$ plotted against relative density.

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PHOTOS

HONEY COMB BEAM

(a)

HONEY COMB PLATE

(b)

Fig. 5 Photographs showing the cross-sections of (a) a square sectioned honeycomb beam, $\rho/\rho_s = 0.51$ (b) a I sectioned honeycomb beam, $\rho/\rho_s = 0.51$ (c) a square sectioned honeycomb beam, $\rho/\rho_s = 0.32$ and (d) the honeycomb plate, $\rho/\rho_s = 0.59$.

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Fig. 9(a) The fracture toughness plotted against density for honeycomb beams.

Fig. 9(b) The relative performance index $(K_{IC}^{2/3}/\rho) / (K_{ICs}^{2/3}/\rho_s)$ plotted against relative density.

Appendix

A single lamina with parallel cylindrical voids is shown in Fig. A1. Finite element analysis was used to calculate the Young's modulus of the lamina normal to the longitudinal axis of the voids for 4 relative densities: 0.3, 0.4, 0.6 and 0.8. The finite element mesh for a relative density of 0.4 is shown on the figure; similar meshes were used for the other relative densities. The Young's modulus and Poisson's ratio of the solid were assumed to be 2 GPa and 0.3, respectively. The results are summarized in Table A1; the moduli calculated using eqn (8) are given for comparison.

Table A1 Young's Modulus of a Single Lamina with Parallel Cylindrical Voids Normal to the Longitudinal Axis of the Voids

Relative Density	Relative Young's Modulus (FEM)	Relative Young's Modulus (eqn 8)
ρ/ρ_s	E_2/E_s	E_2/E_s
0.3	0.032	0.041
0.4	0.095	0.096
0.6	0.301	0.324
0.8	0.578	0.768

Fig. A1 Finite element grid for calculation of E_2 .

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