12/31/93 9501

UCRL-ID-114259

A Program for Passively Tracking a Target Using an Array of Sensors

Michael R. Buhl

January 1993

Lawrence
Livermore
National
Laboratory

This is an internal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the Laboratory.

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7408-Eng-48.

THE DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the Laboratory.

This report has been reproduced
directly from the best available copy.

Available to DOE and DOE contractor from the
Office of Scientific and Technical Information
P.O. Box 62, Oak Ridge, TN 37831
Prices available from (615) 576-8401, FTS 626-8401

Available to the public from the
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Rd.,
Springfield, VA 22161

ABSTRACT

A PROGRAM FOR PASSIVELY TRACKING A TARGET USING AN ARRAY OF SENSORS

The objective of this document is to present a program which passively tracks a target using an array of sensors. This program is available in MATLAB, version 3.5. The algorithm which is implemented consists of three main parts: time delay estimation, passive localization, and data post processing. Each of these parts are discussed, and the mathematical foundation for their solution given. Following this, the organization of the program is presented, and an example of its usage is given.

MASTER

ACKNOWLEDGEMENTS

I would like to thank James V. Candy, James A. Markevitch, and Larry C. Ng for the knowledge and guidance provided. This work was funded by O Program.

TABLE OF CONTENTS

ABSTRACT	i
ACKNOWLEDGEMENTS	ii
TABLE OF CONTENTS	iii
LIST OF FIGURES	vi
LIST OF TABLES	vii
 Section 1 Introduction	 1
 Section 2 Time Delay Estimation	 3
2.1. Introduction	3
2.2. Mathematical Theory	3
2.2.1. Unity Gain Processor	5
2.2.2. Roth Processor.....	5
2.2.3. Smoothed Coherence Transform (SCOT) Processor	6
2.2.4. Phase Transform (PHAT) Processor.....	7
2.2.5. Maximum Likelihood (ML) Processor	7
2.3. Summary.....	9
 Section 3 Passive Localization	 10
3.1. Introduction	10
3.2. Mathematical Theory	10
 Section 4 Post Processing	 14
4.1. Introduction	14
4.2. Mathematical Theory	14

Section 5 Program Description	17
5.1. Introduction	17
5.2. Program Organization	17
5.2.1. Physical Parameters	19
5.2.2. Sensor Signals	20
5.2.2.1. Source Signal Characteristic Parameters	21
5.2.2.2. Source Signal Trajectory Parameters	22
5.2.3. Tracking Parameters	23
5.2.4. Tracking.....	24
5.2.5. Display	24
5.3. Example Problem	25
5.3.1. Problem Specifications	25
5.3.2. Programming Session	26
5.4. Additional Considerations	34
5.5. Availability	40
References	41

LIST OF FIGURES

Section 1 Introduction

1.1. Passive tracking block diagram	1
---	---

Section 2 Time Delay Estimation

2.1. Filtered cross-correlation	4
---------------------------------------	---

Section 5 Program Description

5.1. Flow diagram of program sequences	18
5.2. Sensor signals for the example problem	35-36
5.3. Time delay between sensor signals and the reference sensor for the example problem	36-37
5.4. Estimated and actual source trajectories for the example problem	38

LIST OF TABLES

Section 2 Time Delay Estimation

2.1. Summary of time delay processors	9
---	---

Section 5 Program Description

5.1. Physical parameters.....	19
5.2. Source signal characteristic parameters for signals which are comprised of colored noise events.....	21
5.3. Source signal characteristic parameters for signals which are comprised of second order events	22
5.4. Source signal trajectory parameters.....	23
5.5. Tracking parameters.....	23
5.6. Variables which are saved into result file	39

1. Introduction

1.1. Introduction

The problem of passively tracking a moving signal source has importance in a variety of applications such as radar, sonar, seismology, and radio astronomy. In many applications, only limited information is available about the signal source. For the algorithm discussed in this document, it will be assumed that only the signals which are detected by the sensors and the velocity of the source signal are known.

The problem of passively tracking a moving target using an array of sensors is shown below.

Fig. 1.1. Passive tracking block diagram.

In this figure, the signal source is moving in three dimensions. This motion can be specified by

$$x(t) = f_x(t) \quad (1.1)$$

$$y(t) = f_y(t) \quad (1.2)$$

$$z(t) = f_z(t). \quad (1.3)$$

Each sensor is fixed in position and represented by a coordinate location, also in three dimensions. Since the sensor signals are continuous, they must be sampled before they are

stored in memory. These signals can then be digitally processed to compute the source trajectory. This computation is accomplished in three steps. First, the time delays are computed relative to a reference sensor. Second, a passive localization algorithm uses the time delays, the sensor array geometry, and the signal velocity to compute the source location. These two steps are repeated over and over to generate a source trajectory. Lastly, the source locations from the previous two steps are processed to smooth the trajectories and interpolate intermediate points.

Sections 2 through 4 of this document discuss this algorithm in more detail. The second section describes the time delay estimation. In this section, the computation of the time delays utilizing the cross-correlation is discussed. In an effort to improve upon the accuracy of these estimates, a method of filtering the sensor signals prior to computing the cross-correlation is presented. The third section describes the passive localization algorithm which estimates the source location. This algorithm computes the least squares solution to a set of linear equations which are derived from the estimated time delays between the sensor signals, the signal velocity, and the sensor geometry. The fourth section describes post processing of the data. The purpose of this step is to decrease the variance of source locations; thereby improving the trajectories. This step also interpolates source locations between those computed by the passive localization algorithm. Lastly, the fifth section describes a computer program which can be used to study the effectiveness of this approach to particular problems. This program can also be used to study the effects various parameters have on the tracking ability of this algorithm. An example problem is included and results are given.

2. Time Delay Estimation

2.1. Introduction

In this section, the computation of the time delays is discussed. Five techniques are presented which are based on finding the time at which the cross-correlations between two sensor signals peak. The mathematical theory for these techniques is given in section 2.2, and a summary of them is given in section 2.3.

2.2. Mathematical Theory

Consider the problem of estimating the delay between two sensor signals. These sampled signals can be modeled as

$$x_1(n) = s_1(n) + v_1(n) \quad (2.1)$$

$$x_2(n) = s_2(n) + v_2(n) = \alpha \cdot s_1(n - D) + v_2(n), \quad (2.2)$$

where $s_i(n)$ ($i = 1$ and 2) are the sampled and attenuated signals arriving at sensor i due solely to the source, and $v_1(n)$ and $v_2(n)$ are noise. Furthermore, it is assumed that $s_1(n)$, $v_1(n)$, and $v_2(n)$ are uncorrelated, stationary, and have zero mean. Given these assumptions, the cross-correlation of $x_1(n)$ and $x_2(n)$ is given by

$$R_{x_1x_2}(k) = R_{s_1s_2}(k) = \alpha \cdot E\{s_1(n)s_1(n - D + k)\} = \alpha \cdot R_{s_1s_1}(k - D). \quad (2.3)$$

From the properties of auto-correlation,

$$R_{s_1s_1}(l) = E\{s_1(n)s_1(n + l)\} \leq R_{s_1s_1}(0). \quad (2.4)$$

Indicating that $R_{x_1x_2}(k)$ must peak at k equal to the time delay, D . This is the simplest technique used to estimate the time delay.

An alternative representation of Eqn. (2.3) is given by

$$R_{x_1x_2}(k) = \alpha \cdot R_{s_1s_1}(k - D) = \alpha \cdot \{R_{s_1s_1}(k) * \delta(k - D)\}, \quad (2.5)$$

where $*$ is the convolution operator, and δ is the delta function. From Eqn. (2.5), it is evident that $R_{x_1x_2}(k)$ can be interpreted as a "smeared" delta function. From the property of autocorrelations given by Eqn. (2.4), it appears from Eqns. (2.3) and (2.5) that fairly accurate estimates of the time delays are easily achieved. However, since only finite record lengths are available, the correlations given above can only be estimated, which in turn may lead to errors in computing the time delay. Additionally, $s_1(n)$, $v_1(n)$, and $v_2(n)$ may not be uncorrelated.

One way to alleviate these effects, thereby increasing time delay estimate accuracy, is to filter $x_1(n)$ and $x_2(n)$ prior to computing the cross-correlations. In doing so, sharper peaks may result at the time delay, which in turn may lead to more accurate time delay estimates. This process is shown below.

Figure 2.1. Filtered cross-correlation

The filtered cross-correlation from Fig. 2.1 is given by

$$R_{y_1y_2}(k) = E\{y_1(n)y_2(n+k)\} = \text{iDFT}\{S_{y_1y_2}(\omega)\}, \quad (2.6)$$

where $S_{y_1y_2}(\omega)$ is the cross-power spectrum of $y_1(n)$ and $y_2(n)$, and iDFT is the inverse discrete Fourier transform [1]. In order to determine the time delays between $x_1(n)$ and

$x_2(n)$, the relationship between these signals and $R_{y_1y_2}(k)$ is necessary. From the properties of random linear systems [1], the cross-power spectral relationship is given by

$$S_{y_1y_2}(\omega) = H_1(\omega)H_2^*(\omega)S_{x_1x_2}(\omega). \quad (2.7)$$

Substituting Eqn. (2.7) into Eqn. (2.6) yields

$$R_{y_1y_2}(k) = \text{iDFT}\{\Psi(\omega)S_{x_1x_2}(\omega)\}, \quad (2.8)$$

where

$$\Psi(\omega) = H_1(\omega)H_2^*(\omega). \quad (2.9)$$

Therefore, $\Psi(\omega)$ should be chosen in such a way so as to "sharpen" the peaks of the cross-correlations at the time delay, leading to more accurate time delay estimates. Much effort has been spent to study the problem of selecting processors, $\Psi(\omega)$, which will optimize performance criteria. A brief discussion of five processors follows in sections 2.2.1 to 2.2.5. For more detailed discussions refer to [1-9].

2.2.1. Unity Gain Processor

The unity gain processor [1-4] is given by

$$\Psi(\omega) = 1. \quad (2.10)$$

This processor does not provide filtering of $x_1(n)$ and $x_2(n)$ prior to the cross-correlation computation. It is therefore the simplest time delay technique.

2.2.2. Roth Processor

The Roth processor [4,6] is given by

$$\Psi(\omega) = \frac{1}{S_{x_1x_1}(\omega)}. \quad (2.11)$$

In order to better interpret the effect of this processor, the power spectrum of $x_1(n)$ can be specified in terms of the power spectrum of $s_1(n)$ and $v_1(n)$ with the aid of Eqn. (2.1). Holding to the earlier assumption that $s_1(n)$ and $v_1(n)$ are uncorrelated, this yields

$$\Psi(\omega) = \frac{1}{S_{s_1 s_1}(\omega) + S_{v_1 v_1}(\omega)} \quad (2.12)$$

Substituting Eqn. (2.12) into Eqn. (2.8) results in

$$R_{y_1 y_2}(k) = \text{iDFT} \left\{ \frac{S_{x_1 x_2}(\omega)}{S_{s_1 s_1}(\omega) + S_{v_1 v_1}(\omega)} \right\} \quad (2.13)$$

From this equation, the effect of the Roth processor becomes apparent. At frequencies where the power of $v_1(n)$ is large relative to the power of $s_1(n)$ (low SNR), $S_{y_1 y_2}(\omega)$ is suppressed. Therefore at these frequencies, $R_{y_1 y_2}(k)$ contains smaller frequency components than $R_{x_1 x_2}(k)$.

2.2.3. Smoothed Coherence Transform (SCOT) Processor

The smoothed coherence transform processor (SCOT) [4,5,7,8] is given by

$$\Psi(\omega) = \frac{1}{\sqrt{S_{x_1 x_1}(\omega) S_{x_2 x_2}(\omega)}} \quad (2.14)$$

The Roth processor suppresses frequencies where $S_{v_1 v_1}(\omega)$ is large, but it doesn't account for frequencies where $S_{v_2 v_2}(\omega)$ is large. To remedy this problem, the SCOT processor takes the power spectral density of both $v_1(n)$ and $v_2(n)$ into account. With the aid of Eqn. (2.9) and Fig. 2.1, it is evident that the time delay computation can be viewed as two filters followed by a cross correlator, where the filters are given by

$$H_i(\omega) = \frac{1}{\sqrt{S_{x_i x_i}(\omega)}} \quad (i = 1, 2) \quad (2.15)$$

The effect of these filters is to whiten the data prior to computing the cross-correlation, thereby sharpening the function peak.

2.2.4. Phase Transform (PHAT) Processor

The PHAT processor [4,8] is given by

$$\Psi(\omega) = \frac{1}{|S_{x_1x_2}(\omega)|} \quad (2.16)$$

Substituting Eqn. (2.16) into Eqn. (2.8) results in

$$R_{y_1y_2}(k) = \text{iDFT} \left\{ \frac{S_{x_1x_2}(\omega)}{|S_{x_1x_2}(\omega)|} \right\} \quad (2.17)$$

With the aid of Eqn. (2.5), $S_{x_1x_2}(\omega)$ can be given as

$$S_{x_1x_2}(\omega) = \text{DFT}\{R_{x_1x_2}(k)\} = \alpha \cdot S_{s_1s_1}(\omega) \cdot e^{-j\omega D} = |S_{x_1x_2}(\omega)| \cdot e^{-j\omega D} \quad (2.18)$$

$S_{s_1s_1}(\omega)$ is real-valued since $R_{s_1s_1}(k)$ is an even function [1]. Substituting Eqn. (2.18) into Eqn. (2.17) yields

$$R_{y_1y_2}(k) = \text{iDFT} \{ e^{-j\omega D} \} = \delta(k - D). \quad (2.19)$$

From Eqn. (2.19) it appears as if the PHAT processor eliminates the "smearing" in the cross correlations. However, since only finite record lengths are available, the spectral densities can only be estimated. Additionally, the sensor signal noise may not be uncorrelated. For these reasons, the cross-correlation will not be a perfect delta function, but will exhibit "smearing".

2.2.5. Maximum Likelihood (ML) Processor

The maximum likelihood processor is given by

$$\Psi(\omega) = \frac{|\gamma|^2}{(1 - |\gamma|^2) |S_{x_1 x_2}(\omega)|}, \quad (2.20)$$

where $|\gamma|^2$ is the magnitude square coherence specified by

$$|\gamma|^2 = \frac{|S_{x_1 x_2}(\omega)|^2}{S_{x_1 x_1}(\omega) S_{x_2 x_2}(\omega)}. \quad (2.21)$$

This processor attempts to maximize the probability of obtaining discrete Fourier coefficients for $x_1(n)$ and $x_2(n)$ given the power and cross power spectral densities. This is summarized below. For more detailed discussions refer to [4,5,9].

The discrete Fourier coefficients of $x_1(n)$ and $x_2(n)$ are given by

$$a(m) = \frac{1}{N} \sum_n x_1(n) e^{-j\Omega_m n} \quad (2.22)$$

$$b(m) = \frac{1}{N} \sum_n x_2(n) e^{-j\Omega_m n}. \quad (2.23)$$

where there are N points in the signals, n is the summation over N , and $\Omega_m = \frac{2\pi m}{N}$. If it is assumed that $x_1(n)$ and $x_2(n)$ are gaussian, then the discrete coefficients are also gaussian since they are linear transformations of $x_1(n)$ and $x_2(n)$. Given this assumption and the earlier assumption of $s_1(n)$, $v_1(n)$, and $v_2(n)$ being uncorrelated, the probability of discrete Fourier coefficients can be determined as a function of spectral densities, time delay, and signal attenuation. This can be represented mathematically by

$$p(a(m), b(m)) = f(\alpha, S_{s_1 s_1}(\omega), S_{v_1 v_1}(\omega), S_{v_2 v_2}(\omega), D). \quad (2.24)$$

The estimated time delay between $x_1(n)$ and $x_2(n)$ is the value of D which maximizes Eqn. (2.24). This is equivalent to selecting the delay where $R_{y_1 y_2}(k)$ peaks given the processor of Eqns. (2.20) and (2.21). Substituting these equations into Eqn. (2.8) yields

$$R_{y_1 y_2}(k) = \text{iDFT} \left\{ \frac{|\gamma|^2}{(1 - |\gamma|^2)} \frac{S_{x_1 x_2}(\omega)}{|S_{x_1 x_2}(\omega)|} \right\} = \text{iDFT} \left\{ \frac{|\gamma|^2}{(1 - |\gamma|^2)} e^{-j\omega D} \right\} \quad (2.25)$$

This processor is similar to the PHAT processor. However, it weights the phase given in Eqn. (2.19) by a function of the squared coherence. At frequencies where $x_1(n)$ and $x_2(n)$ are highly correlated, the weighting is large, and at frequencies where these signals are less correlated, the weighting is smaller.

2.3. Summary

A summary of five different time delay processors is given in Table 2.1.

Processor Name	$\Psi(\omega)$
Unity Gain	1
Roth	$\frac{1}{S_{x_1 x_1}(\omega)}$
SCOT	$\frac{1}{\sqrt{S_{x_1 x_1}(\omega) S_{x_2 x_2}(\omega)}}$
PHAT	$\frac{1}{ S_{x_1 x_2}(\omega) }$
ML	$\frac{ \gamma(\omega) ^2}{ S_{x_1 x_2}(\omega) [1 - \gamma(\omega) ^2]}$

Table. 2.1. Summary of time delay processors

This completes the discussion of time delay estimators. The next section will discuss the passive localization algorithm.

3. Passive Localization

3.1. Introduction

In this section, the passive localization method is discussed. This algorithm computes the least squares solution to a set of linear equations. If there are "N" sensor signals, there will be "N-1" linear equations. These equations are derived from the estimated time delays between a reference sensor signal and the remaining "N-1" sensor signals, the signal velocity, and the sensor geometry. For this method, there must be at least "N" sensors, where "N" is greater than the number of spatial dimensions plus one. The discussion below assumes there are three spatial dimensions. Therefore, there must be at least five sensors. The mathematical theory for this method is given in section 3.2. For a more detailed discussion refer to [10].

3.2. Mathematical Theory

Let the location of the sensors and the source be specified in three dimensions as $q_i = [x_i, y_i, z_i]^T$ and $q_s = [x_s, y_s, z_s]^T$, respectively. Furthermore, assume that the reference for these coordinate locations is the origin of the three dimensional space [0,0,0]. With this notation the range difference between two sensors "i" and "j" is given by

$$r_{ij} = \|q_i - q_s\| - \|q_j - q_s\|, \quad (3.1)$$

where r_{ij} can be computed simply by multiplying the time delay between sensors "i" and "j" by the signal velocity. Using this definition of range difference, $\|q_i - q_s\|^2$ is given by

$$\|q_i - q_s\|^2 = (r_{ij} + \|q_j - q_s\|)^2 = r_{ij}^2 + 2\|q_j - q_s\|r_{ij} + \|q_j - q_s\|^2. \quad (3.2)$$

Additionally, $\|q_i - q_s\|^2$ can also be given by

$$\|q_i - q_s\|^2 = \|q_i\|^2 + \|q_s\|^2 - 2q_i^T q_s. \quad (3.3)$$

Equating the right hand side of Eqns. (3.2) and (3.3) yields

$$2q_i^T q_s = \|q_i\|^2 + \|q_s\|^2 - r_{ij}^2 - 2\|q_j - q_s\|r_{ij} - \|q_j - q_s\|^2. \quad (3.4)$$

Setting $i = j$, and noting that $r_{ij} = 0$ for $i = j$, results in

$$2q_j^T q_s = \|q_j\|^2 + \|q_s\|^2 - \|q_j - q_s\|^2. \quad (3.5)$$

Subtracting Eqn. (3.5) from Eqn. (3.4) yields

$$2(q_i - q_j)^T q_s = (\|q_i\|^2 - \|q_j\|^2) - r_{ij}^2 - 2\|q_j - q_s\|r_{ij}. \quad (3.6)$$

This equation can be written in matrix form as given by

$$S_j x_s = \mu_j - \|q_j - q_s\| \rho_j, \quad (3.7)$$

where

$$S_j = \begin{bmatrix} (x_1 - x_j) & (y_1 - y_j) & (z_1 - z_j) \\ \vdots & \vdots & \vdots \\ (x_{j-1} - x_j) & (y_{j-1} - y_j) & (z_{j-1} - z_j) \\ (x_{j+1} - x_j) & (y_{j+1} - y_j) & (z_{j+1} - z_j) \\ \vdots & \vdots & \vdots \\ (x_N - x_j) & (y_N - y_j) & (z_N - z_j) \end{bmatrix} \quad (3.8)$$

$$\mu_j = \frac{1}{2} \begin{bmatrix} \|q_1\|^2 & -\|q_j\|^2 & -r_{ij}^2 \\ \vdots & \vdots & \vdots \\ \|q_{j-1}\|^2 & -\|q_j\|^2 & -r_{j-1j}^2 \\ \|q_{j+1}\|^2 & -\|q_j\|^2 & -r_{j+1j}^2 \\ \vdots & \vdots & \vdots \\ \|q_N\|^2 & -\|q_j\|^2 & -r_{Nj}^2 \end{bmatrix}, \text{ and} \quad (3.9)$$

$$\rho_j = \begin{bmatrix} r_{ij} \\ \vdots \\ r_{j-1j} \\ r_{j+1j} \\ \vdots \\ r_{Nj} \end{bmatrix} \quad (3.10)$$

Sensor signal "j" is the reference in these equations. All quantities in Eqn. (3.7) are known except for x_s and $\|q_j - q_s\|$. To simplify the solution of this equation, it is multiplied by a matrix M_j , where M_j is in the null-space of ρ_j . One such matrix is given by

$$M_j = V_k^T D_j, \quad (3.11)$$

where D_j is a diagonal matrix given by

$$D_j = \begin{bmatrix} r_{ij} & & & & 0 \\ & \ddots & & & \\ & & r_{j-1j} & & \\ & & & r_{j+1j} & \\ & & & & \ddots \\ 0 & & & & & r_{Nj} \end{bmatrix}^{-1} \quad (3.12)$$

V_k^T is computed using the singular value decomposition as

$$(I - Z^k) = [U_k, u_k] \begin{bmatrix} n_1^k & & & 0 \\ & \ddots & & \\ & & n_{N-2}^k & \\ 0 & & & 0 \end{bmatrix} [V_k, v_k]^T \quad (3.13)$$

where I is the "N-1" x "N-1" identity matrix and Z^k is the "N-1" x "N-1" circular shift matrix given by

$$Z^k = \begin{bmatrix} 0 & 1 & & 0 & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & \ddots & \ddots & 0 \\ & \ddots & & \ddots & \\ 0 & & & \ddots & 1 \\ 1 & 0 & & 0 & 0 \end{bmatrix} \quad (3.14)$$

Multiplying Eqn. (3.7) by M_j yields

$$M_j S_j x_s = M_j \mu_j. \quad (3.15)$$

A least squares technique can be used to solve for x_s . A closed-form solution is given by

$$x_s = (S_j^T M_j^T M_j S_j)^{-1} S_j^T M_j^T M_j \mu_j \quad (3.16)$$

In order for $M_j S_j$ to be nonsingular, it is necessary (but not sufficient) that the number of rows of S_j be greater than the number of columns. Therefore to obtain a unique solution, the number of sensors, "N", must be greater than the number of dimensions plus one.

This completes the discussion of the passive localization. The next section will discuss the post processing of the estimated source locations.

4. Post Processing

4.1. Introduction

In this section, the post processing of the computed trajectory points is discussed. The post processing can be broken into two steps. First, the source locations computed by the passive localization algorithm are smoothed. This is accomplished by fitting a second order polynomial to the data. Second, the source locations are interpolated at times intermediate to those computed by the passive localization. The mathematical theory is given in section 4.2.

4.2. Mathematical Theory

This algorithm is given the computed source location and the corresponding time from the passive localization algorithm. These points will be represented by $x_i(t_i)$, $y_i(t_i)$, and $z_i(t_i)$. The polynomial curve fitting algorithm fits second degree polynomials through these sets of data points. These polynomials are given by

$$x(t) = c_{x2}t^2 + c_{x1}t + c_{x0}, \quad (4.1)$$

$$y(t) = c_{y2}t^2 + c_{y1}t + c_{y0}, \text{ and} \quad (4.2)$$

$$z(t) = c_{z2}t^2 + c_{z1}t + c_{z0}. \quad (4.3)$$

Substituting $x_i(t_i)$, $y_i(t_i)$, and $z_i(t_i)$ into Eqns. (4.1) to (4.3) yields the linear equations

$$x_i(t_i) = c_{x2}t_i^2 + c_{x1}t_i + c_{x0}, \quad (4.4)$$

$$y_i(t_i) = c_{y2}t_i^2 + c_{y1}t_i + c_{y0}, \text{ and} \quad (4.5)$$

$$z_i(t_i) = c_{z2}t_i^2 + c_{z1}t_i + c_{z0}. \quad (4.6)$$

These can be represented in matrix form as

$$\begin{bmatrix} x_1(t_1) \\ \vdots \\ x_M(t_M) \end{bmatrix} = \begin{bmatrix} t_1^2 & t_1 & 1 \\ \vdots & \vdots & \vdots \\ t_M^2 & t_M & 1 \end{bmatrix} \begin{bmatrix} c_{x2} \\ c_{x1} \\ c_{x0} \end{bmatrix}, \quad (4.7)$$

$$\begin{bmatrix} y_1(t_1) \\ \vdots \\ y_M(t_M) \end{bmatrix} = \begin{bmatrix} t_1^2 & t_1 & 1 \\ \vdots & \vdots & \vdots \\ t_M^2 & t_M & 1 \end{bmatrix} \begin{bmatrix} c_{y2} \\ c_{y1} \\ c_{y0} \end{bmatrix}, \text{ and} \quad (4.8)$$

$$\begin{bmatrix} z_1(t_1) \\ \vdots \\ z_M(t_M) \end{bmatrix} = \begin{bmatrix} t_1^2 & t_1 & 1 \\ \vdots & \vdots & \vdots \\ t_M^2 & t_M & 1 \end{bmatrix} \begin{bmatrix} c_{z2} \\ c_{z1} \\ c_{z0} \end{bmatrix}. \quad (4.9)$$

A least squares technique can be used to solve for the coefficients in Eqns. (4.7) to (4.9).

By defining Eqns. (4.7) to (4.9) as

$$\bar{x} = T_x \bar{c}_x \quad (4.10)$$

$$\bar{y} = T_y \bar{c}_y, \text{ and} \quad (4.11)$$

$$\bar{z} = T_z \bar{c}_z, \quad (4.12)$$

the least squares solution is given by

$$\bar{c}_x = (T_x' T_x)^{-1} T_x' \bar{x} \quad (4.13)$$

$$\bar{c}_y = (T_y' T_y)^{-1} T_y' \bar{y} \quad (4.14)$$

$$\bar{c}_z = (T_z' T_z)^{-1} T_z' \bar{z}. \quad (4.15)$$

Once the coefficients are computed, the points intermediate to t_i are interpolated directly from the polynomials, as given by Eqns. (4.1) to (4.3).

This completes the discussion of the post-processing. The next section will present a computer program which can be used to study the performance of the passive tracking algorithm.

5. Program Description

5.1. Introduction

This section presents a computer program which can be used to study the performance of the passive tracking algorithm to particular problems. It also may be used to study the effects various parameters have on the tracking ability of this algorithm. This menu-driven program is available in MATLAB, version 3.5. Section 5.2 presents the program organization. Following this section, an example is given in section 5.3. Section 5.4 presents additional considerations relating to data files. Finally, section 5.5 provides information regarding the availability of this program.

5.2. Program Organization

The following steps must be taken prior to applying the tracking algorithm.

1. Physical parameters must be initialized.
2. Sensor signals must be initialized.
3. Tracking parameters must be initialized.

The physical parameters consist of the sensor array geometry, the signal velocity, the signal attenuation constant, the sampling period, and the variance of the sensor signal noise. These parameters may be specified by the user or loaded in from a file. After the physical parameters are initialized, the user must initialize the sensor signal. These signals can also be specified by the user or loaded in from a file. If the user wishes to specify these signals, additional parameters must be initialized. These include the source signal characteristic parameters and the source trajectory parameters. Once these are specified, the sensor signals can be generated. Signals which are generated in this manner will be referred to as simulated sensor signals. In contrast, sensor signals which are obtained in a real environment will be referred to as real sensor signals. Both types of sensor signals may be

loaded from a file. After the sensor signals are initialized, the user can specify the tracking algorithm parameters. These parameters consist of the reference sensor, the period of trajectory point computations, the window length of the signal used to estimate the time delays, and the time delay processor. Following this, all necessary variable have been initialized, and the tracking can be performed. After the tracking has been performed, the user may go back and change any parameters before performing the tracking again. However, because the steps listed above are not decoupled, there are restrictions as to the sequence of steps. In addition to the steps given above, the user may display the sensor signals, the relative time delays, or the estimated and actual source trajectories. All possible sequences of steps are shown in Fig. 5.1. If the user should violate this sequence, an error message will be displayed. The blocks shown in Fig. 5.1 are presented in more detail in sections 5.2.1 to 5.2.5.

Fig. 5.1. Flow diagram of program sequences

5.2.1. Physical Parameters

The physical parameters are specifications which are independent of the sensor signal characteristics. As stated above, these parameters may be specified by the user or loaded from a file. These parameters are defined below, in Table 5.1.

Physical Parameter Name	Description
Number of sensors	The number of sensors in the array (at least five).
Sensor coordinates	The three dimensional (x,y,z) sensor coordinates (meters).
Signal velocity	The velocity of the source signal (meters/second).
Source signal attenuation constant	The constant which determines the signal magnitude at the sensors (meters).
Sampling period	The sampling period of the sensor signals (seconds).
Variance of sensor signal noise	The variance of normally distributed, zero mean noise which is added to the sensor signals.

Table 5.1. Physical parameters

All of these parameters self explanatory with the exception of the source signal attenuation constant. This constant determines the attenuation of the source signal as it arrives at each sensor. The attenuation is given by

$$\text{Signal attenuation} = \frac{\text{Source signal attenuation constant}}{\text{distance between source and sensor}} . \quad (5.1)$$

The way in which this quantity is used in the generation of the sensor signals is explained in section 5.2.2.

It should also be noted that the variance of the sensor signal noise is only utilized if the sensor signals are to be specified (generated). If the sensor signals are loaded, this parameter is not used.

5.2.2. Sensor Signals

Similar to the physical parameters, the sensor signals may be specified by the user or loaded from a file. If the user chooses to specify these signals, the source signal characteristic parameters and the source signal trajectory parameters must be initialized. These are discussed in section 5.2.2.1 and 5.2.2.2, respectively. Once these parameters are initialized, the sensor signals can be generated. The generation of these signals is a two step process. First, the source signal is generated from the source signal characteristic parameters. Next, the sensor signals can be generated from the source signal, the physical parameters, and the source trajectory. These signals are generated by summing up the delayed and attenuated impulses which make up the source signal. The delay is determined by the distance between the source and the sensor at the time of the impulse and the signal velocity, and the attenuation is determined by Eqn. 5.1. Due to movement of the source, the sensor signals will be distorted with respect to one another. If the user wishes to load simulated sensor signals from a file, care must be taken to ensure that the sensor signals were generated using the physical parameters as specified for the current problem. This is due to the fact that the generation of the sensor signals is dependent upon the physical parameters. Likewise, if the user wishes to load real sensor signals from a file, care must be taken to ensure that the sensor signals were gathered using the physical parameters as specified for the current problem. The signal processing is identical for simulated and real data. Their only difference is in the display of the trajectories. If simulated data is processed, the actual trajectories are displayed along with the estimated trajectories. If real data is processed, the actual trajectories are unknown, and therefore only estimated trajectories are displayed.

5.2.2.1. Source Signal Characteristic Parameters

The source signal consists of events which are attenuated and delayed version of a characteristic signal. All the events in a signal have identical characteristics, with the exception of their power. They represent echoes in the signal. Additionally, these events may be either colored noise or second order in nature.

The colored noise events are normally distributed, zero mean noise which is filtered. The characteristics which describe such a source signal are given in Table 5.2.

"Colored Noise" Source Signal Characteristic Parameters	Description
Low cutoff frequency	The low cutoff frequency of the bandpass filter which filters the events (Hertz).
High cutoff frequency	The high cutoff frequency of the bandpass filter which filters the events (Hertz).
Bandpass filter order	The order of the bandpass filter.
Length of signal	The length of the source signal (seconds).
Number of events	The number of colored noise events in the source signal.
Length of events	The length of the events (seconds).
Time of events	The time at which each event occurs (seconds).
Power of events	The average power of each events (dBs).
Variance of source signal noise	The variance of normally distributed, zero mean noise which is added to the source signal.

Table. 5.2. Source signal characteristic parameters for signals which are comprised of colored noise events

A signal which consists of second order events is generated by passing delayed and attenuated impulses through a second order system. The characteristics which describe this type of source signal are given in Table 5.3.

"Second Order" Source Signal Characteristic Parameters	Description
Damping ratio	The damping ratio of the second order system.
Natural frequency	The natural frequency of the second order system (Hertz).
Length of signal	The length of the source signal (seconds).
Number of events	The number of second order events in the source signal.
Time of events	The time at which each event occurs (seconds).
Power of events	The total power of each events which is computed over the length of the signal (dBs).
Variance of source signal noise	The variance of normally distributed, zero mean noise which is added to the source signal.

Table 5.3. Source signal characteristic parameters for signals which are comprised of second order events

It should be noted that the power of events for the two types of signals is computed differently.

5.2.2.2. Source Signal Trajectory Parameters

Before the sensor signal can be generated, the source trajectory must be initialized. This trajectory is specified by the parameters given in Table 5.4.

Source Signal Trajectory Parameters	Description
X-plane trajectory coefficients	Coefficients a_2 , a_1 , and a_0 , where $x(t) = a_2 t^2 + a_1 t + a_0$
Y-plane trajectory coefficients	Coefficients b_2 , b_1 , and b_0 , where $y(t) = b_2 t^2 + b_1 t + b_0$
Z-plane trajectory coefficients	Coefficients c_2 , c_1 , and c_0 , where $z(t) = c_2 t^2 + c_1 t + c_0$

Table. 5.4. Source signal trajectory parameters

5.2.3. Tracking Parameters

After the sensor signals have been initialized, the user may initialize the tracking parameters. These variables, which effect the passive tracking algorithm, are given in Table 5.5.

Tracking Parameters	Description
Reference sensor	The sensor signal to which all other time delays are computed relative to.
Period of trajectory point computations	The period between the estimated trajectory points (seconds).
Window length (fixed windows)	The length of the sensor signal sections which are used to estimate the time delays (seconds).
Initial window length (adaptive windows)	The length of the sensor signal sections which is used to estimate the first set of time delays (seconds).
Minimum window length (adaptive windows)	The minimum length of the sensor signal sections which are used to estimate the time delays (seconds).
Time delay processor	The processor used to compute the time delay (unity gain, Roth, SCOT, PHAT, or ML).

Table. 5.5. Tracking parameters

For the tracking parameters, two different window length methods can be chosen. The first uses a fixed window length. This method uses the same length of data to compute the time delays for every trajectory point computation. The second uses an adaptive window length. For this method a variable length of data is used to compute the time delays for every trajectory point computation. The length, in points, is determined as the smallest number which is a power of two and is larger than eight times the largest relative delay computed in the previous computation. To ensure that this number isn't too small, a minimum window length is specified. If the computed window length is smaller than this value, the minimum window length of data is used to compute the time delays. In order to increase the speed of the tracking algorithm, the user may wish to choose window lengths which result in a power of two (points) when divided by the sampling period.

5.2.4. Tracking

When all the variables described above are initialized, the passive tracking algorithm can be applied. This algorithm computes the estimated time delays and the estimated source trajectories. These results may be saved into a file which can be used as input into additional signal processing algorithms, such as Kalman Filters. The variables which are saved are described in section 5.4.

5.2.5. Display

The user may display the sensor signals, the estimated time delays, or the estimated and actual source trajectory. The steps in which these results can be viewed are given in Fig. 5.1.

5.3. Example Problem

In this section, an example will be presented. Section 5.3.1 will specify the parameters for this particular problem. Following this, a session is given to demonstrate the use of this program in section 5.3.2.

5.3.1. Problem Specifications

The following example, which processes simulated sensor signals, is a hypothetical one, presented only to demonstrate the use of this program. The parameters for this example are given below.

Physical Parameters

Number of sensors = 5
Coordinates of sensor 1 (meters) = [0, 0, 0]
Coordinates of sensor 2 (meters) = [300, 300, 100]
Coordinates of sensor 3 (meters) = [-250, 150, -75]
Coordinates of sensor 4 (meters) = [-487, -365, 10]
Coordinates of sensor 5 (meters) = [278, -576, 25]
Source signal velocity = 343 meters/second
Source signal attenuation constant = 1.0
Sampling period = 0.001 seconds
Variance of sensor noise = 1.00

Sensor Signal - Source Signal Characteristic Parameters

Event type = colored noise
Low cutoff frequency = 25 Hertz
High cutoff frequency = 250 Hertz
Bandpass filter order = 4
Length of signal = 16.0 seconds
Number of events = 1
Length of events = 16.0 seconds
Time of events = 0.0 seconds
Power of events = 160 dBs
Variance of signal noise = 10.0

Sensor Signal - Trajectory Parameters

X-plane trajectory coefficients (m/s², m/s, m) = [0, 40, 0]

Y-plane trajectory coefficients (m/s², m/s, m) = [0, 40, 0]

Z-plane trajectory coefficients (m/s², m/s, m) = [5, 0, 15000]

Tracking Parameters

Reference sensor = 1

Period of trajectory point computations = 1.0 seconds

Window length method = adaptive

Initial window length = 1.024 seconds

Minimum window length = 0.512 seconds

Time delay processor = unity gain

5.3.2. Programming Session

Following is the programming session. The program is started by typing "track". To aid in the use of this program, a flow diagram of the menu structure is given in Appendix

A. The input which is typed by the user is italicized.

»*track*

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: *1*

----- Physical Parameters Menu -----

- 1) Load physical parameters
- 2) Specify physical parameters
- 3) Goto main menu

Select a menu number: *2*

Enter the number of sensors (at least five): 5
Enter the coordinates of sensor 1 (meters) : [0,0,0]
Enter the coordinates of sensor 2 (meters) : [300,300,100]
Enter the coordinates of sensor 3 (meters) : [-250,150,-75]
Enter the coordinates of sensor 4 (meters) : [-487,-365,10]
Enter the coordinates of sensor 5 (meters) : [278,-576,25]
Enter the signal velocity (meters/second): 343
Enter the source signal attenuation constant (meters): 1.0
Enter the sampling period (seconds): 0.001
Enter the variance of the sensor signal noise: 1.0
Do you want to save these parameters to a file [y/n] ? y
Enter the parameter file name: *para_example*

----- Physical Parameters Menu -----

- 1) Load physical parameters
- 2) Specify physical parameters
- 3) Goto main menu

Select a menu number: 3

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 2

----- Sensor Signal Menu -----

- 1) Load sensor signals
- 2) Specify sensor signals
- 3) Goto main menu

Select a menu number: 2

----- Simulate Sensor Signal Menu -----

- 1) Specify source signal characteristics
- 2) Specify source trajectory
- 3) Generate sensor signals
- 4) Goto sensor signal menu

Select a menu number: 1

----- Event Type -----

- 1) Colored noise events
- 2) Second order events

Select a menu number: 1

Enter the low cutoff frequency (Hertz): 25

Enter the high cutoff frequency (Hertz): 250

Enter the bandpass filter order: 4

Enter the length of the signal (seconds): 16.0

Enter the number of events: 1

Enter the length of the events (seconds): 16.0

Enter the time of events (seconds): 0.0

Enter the signal power of the events (dBs): 160

Enter the variance of the source signal noise: 10.0

----- Simulate Sensor Signal Menu -----

- 1) Specify source signal characteristics
- 2) Specify source trajectory
- 3) Generate sensor signals
- 4) Goto sensor signal menu

Select a menu number: 2

The trajectories are computed as follows, where distance is in meters and time in seconds:

$$x(t) = a2*t*t + a1*t + a0$$

$$y(t) = b2*t*t + b1*t + b0$$

$$z(t) = c2*t*t + c1*t + c0$$

Enter the X-plane trajectory coefficients [a2, a1, a0]: [0,40,0]
Enter the Y-plane trajectory coefficients [b2, b1, b0]: [0,40,0]
Enter the Z-plane trajectory coefficients [c2, c1, c0]: [5,0,15000]

----- Simulate Sensor Signal Menu -----

- 1) Specify source signal characteristics
- 2) Specify source trajectory
- 3) Generate sensor signals
- 4) Goto sensor signal menu

Select a menu number: 3

The sensor signals are being generated ...

The sensor signals have been generated.

Do you want to save these sensor signals to a file [y/n] ? y

Enter the sensor signal file name: *sig_example*

----- Simulate Sensor Signal Menu -----

- 1) Specify source signal characteristics
- 2) Specify source trajectory
- 3) Generate sensor signals
- 4) Goto sensor signal menu

Select a menu number: 4

----- Sensor Signal Menu -----

- 1) Load sensor signals
- 2) Specify sensor signals
- 3) Goto main menu

Select a menu number: 3

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 5

----- Display Menu -----

- 1) Sensor signals
- 2) Time delay estimates
- 3) Source trajectories
- 4) Goto main menu

Select a menu number: 1

Enter the sensor number: 1

(Sensor signal 1, shown in Fig. 5.2, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number: 2

(Sensor signal 2, shown in Fig. 5.2, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number: 3

(Sensor signal 3, shown in Fig. 5.2, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number: 4

(Sensor signal 4, shown in Fig. 5.2, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number: 5

(Sensor signal 5, shown in Fig. 5.2, appears in the graph window)

Another plot (y/n) ? n

----- Display Menu -----

- 1) Sensor signals
- 2) Time delay estimation
- 3) Source trajectory
- 4) Goto main menu

Select a menu number: 4

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 3

Enter the reference sensor: 1

Enter the period of trajectory point computations (seconds): 1.0

----- Select a window length method -----

- 1) Fixed window length
- 2) Adaptive window length

Select a menu number: 2

Enter the initial window length (seconds): 1.024

Enter the minimum window length (seconds): 0.512

----- Select a time delay processor -----

- 1) Unity gain
- 2) Roth impulse response
- 3) Smoothed coherence transform (SCOT)
- 4) Phase transform (PHAT)
- 5) Maximum likelihood (ML)

Select a menu number: 1

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 4

The estimated relative delay vectors in seconds:

At time =	0.51 seconds:	0.000	-0.276	0.227	0.010	-0.032
At time =	1.51 seconds:	0.000	-0.278	0.228	0.019	-0.030
At time =	2.51 seconds:	0.000	-0.287	0.228	0.025	-0.028
At time =	3.51 seconds:	0.000	-0.287	0.229	0.041	-0.026
At time =	4.51 seconds:	0.000	-0.291	0.230	0.037	-0.024
At time =	5.51 seconds:	0.000	-0.296	0.230	0.042	-0.022
At time =	6.51 seconds:	0.000	-0.302	0.231	0.040	-0.020
At time =	7.51 seconds:	0.000	-0.306	0.232	0.052	-0.019
At time =	8.51 seconds:	0.000	-0.309	0.232	0.052	-0.017
At time =	9.51 seconds:	0.000	-0.313	0.233	0.062	-0.015
At time =	10.51 seconds:	0.000	-0.316	0.233	0.066	-0.014
At time =	11.51 seconds:	0.000	-0.319	0.234	0.066	-0.012

At time = 12.51 seconds:	0.000	-0.323	0.234	0.074	-0.011
At time = 13.51 seconds:	0.000	-0.326	0.235	0.079	-0.010
At time = 14.51 seconds:	0.000	-0.329	0.235	0.080	-0.008

The estimated source locations in meters:

At time = 0.51 seconds:	19.60	19.74	15036.62
At time = 1.51 seconds:	73.29	42.13	14262.89
At time = 2.51 seconds:	124.83	119.10	15550.00
At time = 3.51 seconds:	216.12	129.07	13818.58
At time = 4.51 seconds:	181.28	148.85	14379.72
At time = 5.51 seconds:	218.34	197.43	14824.26
At time = 6.51 seconds:	203.88	240.92	15770.02
At time = 7.51 seconds:	290.41	275.61	15317.40
At time = 8.51 seconds:	286.96	306.70	15724.06
At time = 9.51 seconds:	348.47	338.53	15268.90
At time = 10.51 seconds:	383.80	373.20	15518.39
At time = 11.51 seconds:	370.63	389.57	15604.95
At time = 12.51 seconds:	440.87	438.84	15791.49
At time = 13.51 seconds:	471.52	457.41	15640.96
At time = 14.51 seconds:	478.32	492.83	15980.95

Do you want to save these results to a file [y/n] ? y

Enter the result file name: *res_example*

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 5

----- Display Menu -----

- 1) Sensor signals
- 2) Time delay estimates
- 3) Source trajectories
- 4) Goto main menu

Select a menu number: 2

Enter the sensor number (excluding reference) : 2

(The relative delay for sensor 2, shown in Fig. 5.3, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number (excluding reference) : 3

(The relative delay for sensor 3, shown in Fig. 5.3, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number (excluding reference) : 4

(The relative delay for sensor 4, shown in Fig. 5.3, appears in the graph window)

Another plot (y/n) ? y

Enter the sensor number (excluding reference) : 5

(The relative delay for sensor 5, shown in Fig. 5.3, appears in the graph window)

Another plot (y/n) ? n

----- Display Menu -----

- 1) Sensor signals
- 2) Time delay estimates
- 3) Source trajectories
- 4) Goto main menu

Select a menu number: 3

(The X-plane trajectories, shown in Fig. 5.4, appear in the graph window)

Hit any key to continue ...

(The Y-plane trajectories, shown in Fig. 5.4, appear in the graph window)

Hit any key to continue ...

(The Z-plane trajectories, shown in Fig. 5.4, appear in the graph window)

----- Display Menu -----

- 1) Sensor signals
- 2) Time delay estimates
- 3) Source trajectories
- 4) Goto main menu

Select a menu number: 4

----- Main Menu -----

- 1) Initialize physical parameters
- 2) Initialize sensor signals
- 3) Initialize tracking parameters
- 4) Perform tracking
- 5) Display results
- 6) Exit

Select a menu number: 6

This concludes the example.

For this particular problem, the passive tracking algorithm performed fairly well. However, this may not always be the case. As previously stated, the sensor signals are distorted relative to one another due to Doppler effects. This signal distortion causes time delay estimate errors which in turn cause source localization errors. Therefore, this algorithm may perform well only for cases where this distortion is small.

5.4. Additional Considerations

A couple points relating to the internal workings of this program are worth noting.

First, the sensor signal files may contain simulated or real data. The signal processing is identical for both data types. Their only difference is in the display of the trajectories. If simulated data is processed, the actual trajectories are displayed along with the estimated trajectories. If real data is processed, the actual trajectories are unknown, and therefore only estimated trajectories are displayed. Due to this difference, the sensor signal files must contain a variable, referred to as "Data_Type", which indicates the type of data. If this data represents real sensor signals, this variable should be 1, and if this data represents simulated sensor signals, this variable should be 2. Additionally, if this data represents simulated sensor signals, the actual trajectory coefficients, which were used to generate these signals, are stored in variables "X_Coef", "Y_Coef", and "Z_Coef".

Fig. 5.2. Sensor signals for the example problem (con't)

Fig. 5.2. Sensor signals for the example problem

Fig. 5.3. Time delay between sensor signals and the reference sensor (sensor 1) for the example problem (con't)

Fig. 5.3. Time delay between sensor signals and the reference sensor (sensor 1) for the example problem

Fig. 5.4. Estimated and actual source trajectories for the example problem

Second, the results may be saved into a file which can be used as input into additional signal processing algorithms such as Kalman Filters. The variables which are saved are given in Table 5.6.

Variable Name	Description
Time_Comp	The time (offset) at which the source locations are estimated by the passive localization (seconds).
X_Comp	The X-plane locations of the source estimated by the passive localization (meters).
Y_Comp	The Y-plane locations of the source estimated by the passive localization (meters).
Z_Comp	The Z-plane locations of the source estimated by the passive localization (meters).
Rel_Delay	The relative time delay between the sensor signals and the reference sensor signals (seconds).
Time	The time (offset) at which the source locations are estimated after post processing (seconds).
Traj_X_Comp	The X-plane locations of the source estimated after post processing (meters).
Traj_Y_Comp	The Y-plane locations of the source estimated after post processing (meters).
Traj_Z_Comp	The Z-plane locations of the source estimated after post processing (meters).

Table. 5.6. Variable which are saved into result file

The variables "X_Comp", "Y_Comp", and "Z_Comp" are one-dimensional arrays which contain the estimated source locations which are computed by the passive localization algorithm. These may be referred to as the pre-processed source locations. By comparison, the post-processed source locations are stored in the one-dimensional arrays "Traj_X_Comp", "Traj_Y_Comp", and "Traj_Z_Comp". The times which correspond to the pre-processed source locations are given in "Time_Comp", while the

times which correspond to the post-processed source locations are given in "Time". Finally, "Rel_Delay" is a two-dimensional array where the entry (i,j) represents the i^{th} relative time delay between sensor signal "j" and the reference signal. This i^{th} relative time delay occurs at the time given by the i^{th} entry of "Time_Comp".

5.5. Availability

For more information regarding this program and its availability contact

Michael Buhl
Electronics Engineering
Engineering Research Division
Signal and Image Processing Research Group.

REFERENCES

- [1] J.V. Candy, *Signal Processing - The Modern Approach*. McGraw-Hill, 1988.
- [2] G.M. Jenkins and D.G. Watts, *Spectral Analysis and Its Applications*. San Francisco, CA: Holden-Day, 1968.
- [3] L.H. Koopmans, *The Spectral Analysis of Time Series*. New York: Academic, 1974.
- [4] G.C. Carter and C.H. Knapp, "The generalized correlation method for estimation of time delays", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-24, pp. 320-327, August 1976.
- [5] K. Scarbrough, N. Ahmed, G.C. Carter, "On the simulation of a class of time delay estimation algorithms", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-29, pp. 534-539, June 1981.
- [6] P.R. Roth, "Effective measurements using digital signal analysis", *IEEE Spectrum*, vol. 8, pp. 62-70, April 1971.
- [7] G.C. Carter, A.H. Nuttall, and P.G. Cable, "The smoothed coherence transform", *Proc. IEEE (Lett.)*, vol. 61, pp. 1497-1498, October 1973.
- [8] G.C. Carter, A.H. Nuttall, and P.G. Cable, "The smoothed coherence transform", Naval Underwater Systems Center, New London, CT, Tech. Memo TC-15972, August 8, 1972.
- [9] E.J. Hannan and P.J. Thomson, "Estimating group delay", *Biometrika*, vol. 60, pp. 241-253, 1973.
- [10] Benjamin Friedlander, "A Passive Localization Algorithm and Its Accuracy Analysis", *IEEE Journal of Oceanic Engineering*, vol. OE-12, pp. 234-244, January 1987.
- [11] Lawrence C. Ng and Yaakov Bar-Shalom, "Multisensor Multitarget Time Delay Vector Estimation", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 669-678, August 1986.

A. Menu Structure

Fig. A.1. Main menu structure

Fig. A.2. Physical parameter menu structure

Fig. A.3. Sensor signals menu structure (con't)

Fig. A.3. Sensor signals menu structure

Fig. A.4. Display menu structure

B. Source Code

B.1. track.m

```
%%%%%%%%%%
% Title:      track.m
% Author:     Mike Buhl
% Date:       9/92
%
% Purpose:    The purpose of this program is to study an algorithm which tracks a moving
%              source given the signals present at sensors, which monitor the source.
%              The user specifies physical parameters, sensor signals, and tracking para-
%              meters. Once all these variables are initialized, the tracking algorithm can
%              be applied, and results displayed. A complete discription of this algorithm
%              and program can be found in sections 1 to 5 of this document.
%
%%%%%%%%%%

%%%%%%%%%
%
% Define menu selection strings.
%
%%%%%%%%%%

menu0_1 = 'Initialize physical parameters';
menu0_2 = 'Initialize sensor signals';
menu0_3 = 'Initialize tracking parameters';
menu0_4 = 'Perform tracking';
menu0_5 = 'Display results';
menu0_6 = 'Exit';

menu1_1 = 'Load physical parameters';
menu1_2 = 'Specify physical parameters';
menu1_3 = 'Goto main menu';

menu2_1 = 'Load sensor signals';
menu2_2 = 'Specify sensor signals';
menu2_3 = 'Goto main menu';

menu3_1 = 'Specify source signal characteristics';
menu3_2 = 'Specify source trajectory';
menu3_3 = 'Generate sensor signals';
menu3_4 = 'Goto sensor signal menu';

menu4_1 = 'Colored noise events';
menu4_2 = 'Second order events';

menu5_1 = 'Fixed window length';
menu5_2 = 'Adaptive window length';

menu6_1 = 'Unity gain';
menu6_2 = 'Roth impulse response';
menu6_3 = 'Smoothed coherence transform (SCOT)';
menu6_4 = 'Phase transform (PHAT)';
menu6_5 = 'Maximum likelihood (ML)';

menu7_1 = 'Sensor signals';
```

```

menu7_2 = 'Time delay estimates';
menu7_3 = 'Source trajectories';
menu7_4 = 'Goto main menu';

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Initialize flag variables.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

Para_Spec = 0;
Sig_Spec = 0;
Traj_Spec = 0;
Track_Spec = 0;

```

```

Event_Type = 0;
Sen_Sig_Valid = 0;
Track_Valid = 0;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Display the Main Menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

Select_0 = 0;
while (Select_0 ~= 6)
    clc;
    Select_0 = menu('Main Menu',menu0_1,menu0_2,menu0_3,menu0_4,menu0_5, ...
                    menu0_6);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Main Menu => Specify physical parameters
% Display the Physical Parameters Menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

    if (Select_0 == 1)
        Select_1 = 0;
        while (Select_1 ~= 3)
            clc;
            Select_1 = menu('Physical Parameters Menu',menu1_1,menu1_2,menu1_3);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Physical Parameters Menu => Load physical parameters
% Load the number of sensors, sensor coordinates, signal velocity, signal
% attenuation constant, sampling period, and variance of sensor signal noise
% from a parameter file.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

        if (Select_1 == 1)
            clc;
            File_Name = input('Enter the parameter file name: ','s');
            Ins_Arg = [File_Name '.mat'];

```

```

Load_Ins = ['load ' Ins_Arg];
eval(Load_Ins);
fprintf('The parameters have been loaded. ');
fprintf('\n Number of sensors: %3.0f ,Number_Sen);
for Sen_Num = 1: Number_Sen
    fprintf('\n . Coordinates of sensor %3.0f (meters): ', Sen_Num);
    fprintf('%7.2f %7.2f %7.2f', Sen_Coordinates(Sen_Num,1), ...
        Sen_Coordinates(Sen_Num,2), Sen_Coordinates(Sen_Num,3));
end
fprintf('\n Signal velocity (meter/second): %6.2f, Signal_Vel);
fprintf('\n Signal attenuation constant (meters): %6.2f, Signal_Att);
fprintf('\n Sampling period (seconds): %6.4f, Sampling_Per');
fprintf('\n Variance of sensor noise: %6.2f\n\n', ...
    Var_Sen_Noise);
Para_Spec = 1;
Sen_Sig_Valid = 0;
Track_Spec = 0;
Track_Valid = 0;
dummy = input('Hit any key to continue ...');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Physical Parameters Menu => Specify physical parameters
% Enter the number of sensors, sensor coordinates, signal velocity, signal
% attenuation constant, sampling period, and variance of sensor signal noise.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_1 == 2)
    clc;
    Number_Sen = 0;
    while (Number_Sen < 5)
        Number_Sen = input('Enter the number of sensors (at least five): ');
        if (Number_Sen < 5)
            fprintf('There must be at least five sensors!');
        end
    end
    for Sen_Num = 1: Number_Sen
        Sen_Coordinates(Sen_Num,1:3) = input(['Enter the coordinates of sensor ',
int2str(Sen_Num), ' (meters): ']);
    end
    Sen_Coordinates = Sen_Coordinates(1: Number_Sen, 1:3);
    Signal_Vel = input('Enter the signal velocity (meters/second): ');
    Signal_Att = input('Enter the source signal attenuation constant (meters): ');
    Sampling_Per = input('Enter the sampling period (seconds): ');
    Var_Sen_Noise = input('Enter the variance of the sensor signal noise: ');
    Para_Spec = 1;
    Sen_Sig_Valid = 0;
    Track_Spec = 0;
    Track_Valid = 0;

    answer = 'a';
    while ((answer ~= 'y') & (answer ~= 'Y') & ...
        (answer ~= 'n') & (answer ~= 'N'))
        answer = input('Do you want to save these parameters to a file [y/n] ? ','s');
        if ((answer == 'y') | (answer == 'Y'))
            File_Name = input('Enter the parameter file name: ','s');

```

```

        Ins_Arg = [File_Name '.mat Number_Sen Sen_Coordinates Signal_Vel
Signal_Att Sampling_Per Var_Sen_Noise'];
        Save_Ins = ['save ' Ins_Arg];
        eval(Save_Ins);
    end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Main Menu => Specify sensor signals
%       Display the Sensor Signal Menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_0 == 2)
    if (Para_Spec == 1)
        Select_2 = 0;
        while (Select_2 ~= 3)
            clc;
            Select_2 = menu('Sensor Signal Menu',menu2_1,menu2_2,menu2_3);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Sensor Signal Menu => Load sensor signals
%       Load sensor signals
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            if (Select_2 == 1)
                clc;
                File_Name = input('Enter the sensor signal file name: ','s');
                Ins_Arg = [File_Name '.mat'];
                Load_Ins = ['load ' Ins_Arg];
                eval(Load_Ins);
                [Num_Sen_Data,Num_Points] = size(Sensor_Sig);
                if (Num_Sen_Data == Number_Sen)
                    fprintf('The sensor signals have been loaded. ');
                    Signal_Len = Num_Points*Sampling_Per;
                    Sen_Sig_Valid = 1;
                    Track_Valid = 0;
                    Track_Spec = 0;
                else
                    clc;
                    fprintf('\nThe number of sensor corresponding to the data is different than that
specified as a physical parameter');
                    dummy= input('Hit any key to continue ...');
                end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Sensor Signal Menu => Specify sensor signals
%       Display Simulate Sensor Signals Menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

                elseif (Select_2 == 2)

```



```

        Select_3 = 0;
        while (Select_3 ~= 4)
            clc;
            Select_3 = menu('Simulate Sensor Signal Menu',menu3_1, ...
                           menu3_2,menu3_3,menu3_4);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Simulate Sensor Signal Menu => Specify source signal characteristics
%       Display Event Type menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            if (Select_3 == 1)
                clc;
                Select_4 = menu('Event Type',menu4_1,menu4_2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Event Type Menu => Colored noise events
%       Initialize the source signal characteristic parameters for colored noise events.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

                    if (Select_4 == 1)
                        clc;
                        Low_Cutoff = input('Enter the low cutoff frequency (Hertz): ');
                        High_Cutoff = input('Enter the high cutoff frequency (Hertz): ');
                        Bandpass_Order = input('Enter the bandpass filter order: ');
                        Sig_Len_Tmp = input('Enter the length of the signal (seconds): ');
                        Numb_of_Events = 0;
                        while (Numb_of_Events < 1)
                            Numb_of_Events = input('Enter the number of events: ');
                            if (Numb_of_Events < 1)
                                fprintf('There must be at least one event. ');
                            end
                        end
                        Leng_of_Events = input('Enter the length of the events (seconds): ');
                        Time_of_Events = input('Enter the time of the events (seconds): ');
                        Power_of_Events = input('Enter the signal power of the events (dBs): ');
                        Var_Sig_Noise = input('Enter the variance of the source signal noise: ');

                        Sig_Spec = 1;
                        Event_Type = 1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Event Type Menu => Second order events
%       Initialize the source signal characteristic parameters for second order events.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

                                else
                                    clc;
                                    Damping_Ratio = input('Enter the damping ratio: ');
                                    Natural_Freq = input('Enter the natural frequency (Hertz): ');
                                    Sig_Len_Tmp = input('Enter the length of the signal (seconds): ');
                                    Numb_of_Events = 0;

```

```

while (Numb_of_Events < 1)
    Numb_of_Events = input('Enter the number of events: ');
    if (Numb_of_Events < 1)
        fprintf('There must be at least one event. ');
    end
end
Time_of_Events = input('Enter the time of the events (seconds): ');
Power_of_Events = input('Enter the signal power of the events (dBs): ');
Var_Sig_Noise = input('Enter the variance of the source signal noise: ');

Sig_Spec = 1;
Event_Type = 2;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Simulate Sensor Signal Menu => Specify source trajectory
% Initialize the source trajectory coefficients.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_3 == 2)
    clc;
    fprintf('The trajectories are computed as follows, where distance is in');
    fprintf('\nmeters and time in seconds. ');
    fprintf('\n\n x(t) = a2*t*t + a1*t + a0');
    fprintf('\n y(t) = b2*t*t + b1*t + b0');
    fprintf('\n z(t) = c2*t*t + c1*t + c0\n\n');
    X_Coef_Temp = input('Enter the X-plane trajectory coefficients [a2, a1, a0]: ');
    Y_Coef_Temp = input('Enter the Y-plane trajectory coefficients [b2, b1, b0]: ');
    Z_Coef_Temp = input('Enter the Z-plane trajectory coefficients [c2, c1, c0]: ');
    Traj_Spec = 1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Signal Specifications => Generate sensor signals.
% Generate the sensor signals.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_3 == 3)
    if ((Sig_Spec == 1) & (Traj_Spec == 1))
        clc;
        fprintf('The sensor signals are being generated ...');
        X_Coef = X_Coef_Temp;
        Y_Coef = Y_Coef_Temp;
        Z_Coef = Z_Coef_Temp;
        Signal_Len = Sig_Len_Tmp;
        if (Event_Type == 1)
            Src_Sig = src_sig_color(Signal_Len, Sampling_Per, ...
                Leng_of_Events, Low_Cutoff, High_Cutoff, ...
                Bandpass_Order, Time_of_Events, ...
                Power_of_Events, Var_Sig_Noise);
        else
            Src_Sig = src_sig_2order(Signal_Len, Sampling_Per, ...

```

```

Damping_Ratio,Natural_Freq, ...
Time_of_Events, Power_of_Events, ...
Var_Sig_Noise);
end
Sensor_Sig = sen_sig(Sen_Coordinates, ...
    Signal_Vel,Sampling_Per,X_Coef, Y_Coef, ...
    Z_Coef,Var_Sen_Noise,Signal_Att,Src_Sig);
Num_Points = length(Sensor_Sig(1,:));
Data_Type = 2;
Sen_Sig_Valid = 1;
Track_Valid = 0;
Track_Spec = 0;
fprintf('\nThe sensor signals have been generated.\n');

answer = 'a';
while ((answer ~= 'y') & (answer ~= 'Y') & ...
    (answer ~= 'n') & (answer ~= 'N'))
    answer = input('Do you want to save these sensor signals to a file
[y/n] ? ','s');

    if ((answer == 'y') | (answer == 'Y'))
        File_Name = input('Enter the sensor signal file name: ','s');
        Ins_Arg = [File_Name '.mat Data_Type X_Coef Y_Coef
Z_Coef Sensor_Sig'];

        Save_Ins = ['save ' Ins_Arg];
        eval(Save_Ins);
    end
end

else
    clc;
    if (Sig_Spec == 0)
        fprintf('Source signal characteristics must be initialized first!\n');
    end
    if (Traj_Spec == 0)
        fprintf('\nSource trajectory must be initialized first!');
    end
    dummy = input('Hit any key to continue ...');
end
end
end
end
end
else
    clc;
    fprintf('\nPhysical parameters must be initialized first!');
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Main Menu => Specify tracking parameters
% Initialize the tracking parameters.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_0 == 3)
    if (Sen_Sig_Valid == 1)
        clc;

```

```

%% Initialize the reference sensor.

```

```

Ref_Sen_Temp = 0;
while ((Ref_Sen_Temp < 1) | (Ref_Sen_Temp > Number_Sen))
    Ref_Sen_Temp = input('Enter the reference sensor: ');
    if ((Ref_Sen_Temp < 1) | (Ref_Sen_Temp > Number_Sen))
        fprintf('Sensor number is invalid. ');
    end
end

```

```

%% Initialize the period of the computed trajectory points.

```

```

Comp_Per = input('Enter the period of trajectory point computations (seconds): ');

```

```

%% Display the "Window Length" menu.

```

```

clc;
Select_5 = menu('Select a window length method', menu5_1, menu5_2);

```

```

%% Window Length menu => Fixed window length
% Initialize the window length.

```

```

clc;
if (Select_5 == 1)
    Win_Len_Min = Signal_Len;
    while (Win_Len_Min > (Signal_Len/2.0))
        Win_Len_Min = input('Enter the window length (seconds): ');
        if (Win_Len_Min > (Signal_Len/2.0))
            fprintf('The window length must be less than %6.2f seconds!', (Signal_Len/2.0));
        end
    end
    Win_Len_Init = Win_Len_Min;
    Win_Meth = 1;

```

```

%% Window Length menu => Adaptive window length
% Initialize the initial window length and the minimum window length.

```

```

else
    Win_Len_Init = Signal_Len;
    while (Win_Len_Init > (Signal_Len/2.0))
        Win_Len_Init = input('Enter the initial window length (seconds): ');
        if (Win_Len_Init > (Signal_Len/2.0))
            fprintf('The window length must be less than %6.2f seconds!',(Signal_Len/2.0));
        end
    end

    Win_Len_Min = input('Enter the minimum window length (seconds): ');
    Win_Meth = 2;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Select a time delay processor.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc;
Select_6 = menu('Select a time delay processor',menu6_1,menu6_2, ...
               menu6_3,menu6_4,menu6_5);
Time_Est_Meth = Select_6;
Track_Spec = 1;
Track_Valid = 0;

else
    clc;
    fprintf('Sensor signals must be initialized first!');
    dummy = input('Hit any key to continue ...');
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Main Menu => Perform tracking
%   Apply the tracking algorithm.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_0 == 4)
    clc;
    if (Track_Spec == 1)

        Win_Len_Init_P = round(Win_Len_Init/Sampling_Per);
        Win_Len_Min_P = round(Win_Len_Min/Sampling_Per);

        Ref_Sen = Ref_Sen_Temp;
        Part = 0;
        Start = 1;
        Stop = Win_Len_Init_P;

        fprintf('The estimated relative delay vectors in seconds:\n');
        while (Stop < Num_Points)
            Part = Part + 1;
            x1 = Sensor_Sig(Ref_Sen,Start:Stop);
            for Sen_Num = 1:Number_Sen
                if (Sen_Num ~= Ref_Sen)
                    x2 = Sensor_Sig(Sen_Num,Start:Stop);

```

```

    if (Time_Est_Meth == 1)
        Sx1x2 = spec_est_corr(x2,x1);
        [Est_Delay,Sqr_Ry1y2]=time_est_cross(Sx1x2,Sampling_Per);
    elseif (Time_Est_Meth == 2)
        Sx1x1 = spec_est_corr(x1,x1);
        Sx1x2 = spec_est_corr(x2,x1);
        [Est_Delay,Sqr_Ry1y2]= time_est_roth(Sx1x1,Sx1x2, ...
            Sampling_Per);
    elseif (Time_Est_Meth == 3)
        Sx1x1 = spec_est_corr(x1,x1);
        Sx2x2 = spec_est_corr(x2,x2);
        Sx1x2 = spec_est_corr(x2,x1);
        [Est_Delay,Sqr_Ry1y2] = time_est_scot(Sx1x1,Sx2x2, ...
            Sx1x2,Sampling_Per);
    elseif (Time_Est_Meth == 4)
        Sx1x2 = spec_est_corr(x2,x1);
        [Est_Delay,Sqr_Ry1y2] = time_est_phat(Sx1x2,Sampling_Per);
    elseif (Time_Est_Meth == 5)
        Sx1x1 = spec_est_corr(x1,x1);
        Sx2x2 = spec_est_corr(x2,x2);
        Sx1x2 = spec_est_corr(x2,x1);
        [Est_Delay,Sqr_Ry1y2] = time_est_ml(Sx1x1,Sx2x2, ...
            Sx1x2,Sampling_Per);
    end
    Rel_Delay(Sen_Num,Part) = Est_Delay;
end
Rel_Delay(Ref_Sen,Part) = 0.0;

if (Part == 1)
    Time_Comp(1) = Win_Len_Init/2.0;
else
    Time_Comp(Part) = Time_Comp(Part-1) + Comp_Per;
end

if (Win_Meth == 1)
    Win_Len_Pnts = Win_Len_Min_P;
else
    Max_Delay = 0.0;
    for Sen_Num = 1:Number_Sen
        if (Rel_Delay(Sen_Num,Part) > Max_Delay)
            Max_Delay = Rel_Delay(Sen_Num,Part);
        end
    end
    Win_Delay_P = round((8.0*Max_Delay)/Sampling_Per);
    Win_Len_Pnts = power_of_two(Win_Delay_P);
    if (Win_Len_Pnts < Win_Len_Min_P)
        Win_Len_Pnts = Win_Len_Min_P;
    end
end

Next_Time_P = round((Time_Comp(Part) + Comp_Per)/Sampling_Per);
Start = Next_Time_P - round(Win_Len_Pnts/2);
if (Start < 1)
    Start = 1;
    Stop = 2*Next_Time_P;
else

```

```

        Stop = Next_Time_P + round(Win_Len_Pnts/2) - 1;
    end

    fprintf('\nAt time = %6.2f seconds:',Time_Comp(Part));
    for Sen_Num = 1:Num_Sen
        fprintf('%9.3f',Rel_Delay(Sen_Num,Part));
    end

end

Num_Parts = Part;
Rel_Delay = Rel_Delay(:,1:Num_Parts);
Time_Comp = Time_Comp(1:Num_Parts);

clc;
fprintf('\n\n\nThe estimated source locations in meters:\n');
for Part = 1:Num_Parts
    Time_Delay = Rel_Delay(:,Part);
    Est_Src_Cor = locate_source(Sen_Coordinates,Ref_Sen,Time_Delay, ...
                                Signal_Vel);

    X_Comp(Part) = Est_Src_Cor(1);
    Y_Comp(Part) = Est_Src_Cor(2);
    Z_Comp(Part) = Est_Src_Cor(3);
    fprintf('\nAt time = %6.2f seconds:',Time_Comp(Part));
    fprintf('%8.2f %8.2f %8.2f',X_Comp(Part), Y_Comp(Part), ...
            Z_Comp(Part));
end

X_Comp = X_Comp(1:Num_Parts);
Y_Comp = Y_Comp(1:Num_Parts);
Z_Comp = Z_Comp(1:Num_Parts);

X_Coef_Comp = polyfit(Time_Comp,X_Comp,2);
Y_Coef_Comp = polyfit(Time_Comp,Y_Comp,2);
Z_Coef_Comp = polyfit(Time_Comp,Z_Comp,2);

Num_Pnts_Trk = round(Time_Comp(Num_Parts)/Sampling_Per);
n = 0:Num_Pnts_Trk;
Time = n * Sampling_Per;
Traj_X_Comp = polyval(X_Coef_Comp,Time);
Traj_Y_Comp = polyval(Y_Coef_Comp,Time);
Traj_Z_Comp = polyval(Z_Coef_Comp,Time);

if (Data_Type == 2)
    Traj_X = polyval(X_Coef,Time);
    Traj_Y = polyval(Y_Coef,Time);
    Traj_Z = polyval(Z_Coef,Time);
end

Track_Valid = 1;

clc;
fprintf('\n\nThe tracking has been performed.\n');
answer = 'a';
while ((answer ~= 'y') & (answer ~= 'Y') & ...
        (answer ~= 'n') & (answer ~= 'N'))
    answer = input('Do you want to save these results to a file [y/n] ? ','s');
    if ((answer == 'y') | (answer == 'Y'))

```

```

        File_Name = input('Enter the result file name: ','s');
        Ins_Arg = [File_Name '.mat Time_Comp X_Comp Y_Comp Z_Comp Rel_Delay
Time Traj_X_Comp Traj_Y_Comp Traj_Z_Comp'];
        Save_Ins = ['save ' Ins_Arg];
        eval(Save_Ins);
    end
end

else
    clc;
    fprintf('Tracking parameters must be initialized first!');
    dummy = input('Hit any key to continue ...');
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Main Menu => Display Results
% Display the Display Menu.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif (Select_0 == 5)
    Select_7 = 0;
    while (Select_7 ~= 4)
        clc;
        Select_7 = menu('Display Menu',menu7_1,menu7_2,menu7_3,menu7_4);
        if (Select_7 == 1)
            if (Sen_Sig_Valid == 1)
                n = 0:(Num_Points-1);
                answer = 'y';
                while ((answer == 'y') | (answer == 'Y'))
                    clc;
                    Sen_Num = 0;
                    while ((Sen_Num < 1) | (Sen_Num > Number_Sen))
                        Sen_Num = input('Enter the sensor number: ');
                        if ((Sen_Num < 1) | (Sen_Num > Number_Sen))
                            fprintf('Invalid sensor number!');
                        end
                    end
                    plot(n*Sampling_Per,Sensor_Sig(Sen_Num,:));
                    title(['Sensor Signal ',int2str(Sen_Num)]);
                    xlabel('offset time (seconds)');
                    grid;
                    clc;
                    answer = input('Another plot (y/n) ? ','s');
                end
            else
                clc;
                fprintf('Sensor signals must be initialized first!');
                dummy = input('Hit any key to continue ...');
            end
        end

        elseif (Select_7 == 2)
            if (Track_Valid == 1)
                answer = 'y';
                while ((answer == 'y') | (answer == 'Y'))
                    clc;
                    Sen_Num = 0;

```



```

while ((Sen_Num < 1) | (Sen_Num > Number_Sen) | (Sen_Num == Ref_Sen))
    Sen_Num = input('Enter the sensor number (excluding referenece) : ');
    if ((Sen_Num < 1) | (Sen_Num > Number_Sen) | (Sen_Num == Ref_Sen))
        fprintf('Invalid sensor number!');
    end
end
plot(Time_Comp,Rel_Delay(Sen_Num,:));
title(['Relative Delay Between Sensor ',int2str(Sen_Num), ' and the Reference']);
xlabel('offset time (seconds)');
ylabel('relative time delay (seconds)');
grid;
clc;
answer = input('Another plot (y/n) ? ','s');
end
else
    clc;
    fprintf('The tracking algorithm has not been applied!');
    dummy = input('Hit any key to continue ...');
end
elseif (Select_7 == 3)
    if (Track_Valid == 1)
        n = 0:Num_Pnts_Trk;
        if (Data_Type == 1)
            plot(n*Sampling_Per,Traj_X_Comp,'g');
            title('Estimated Source Trajectory in the X-Plane');
            xlabel('offset time (seconds)');
            ylabel('meters');
            grid;
            clc;
            dummy = input('Hit any key to continue ...');
            plot(n*Sampling_Per,Traj_Y_Comp,'g');
            title('Estimated Source Trajectory in the Y-Plane');
            xlabel('offset time (seconds)');
            ylabel('meters');
            grid;
            clc;
            dummy = input('Hit any key to continue ...');
            plot(n*Sampling_Per,Traj_Z_Comp,'g');
            title('Estimated Source Trajectory in the Z-Plane');
            xlabel('offset time (seconds)');
            ylabel('meters');
            grid;
        else
            plot(n*Sampling_Per,Traj_X,'r',n*Sampling_Per, ...
                Traj_X_Comp,'g');
            title('Actual and Estimated Source Trajectory in the X-Plane');
            xlabel('offset time (seconds)');
            ylabel('meters');
            grid;
            clc;
            dummy = input('Hit any key to continue ...');
            plot(n*Sampling_Per,Traj_Y,'r',n*Sampling_Per, ...
                Traj_Y_Comp,'g');
            title('Actual and Estimated Source Trajectory in the Y-Plane');
            xlabel('offset time (seconds)');
            ylabel('meters');
            grid;
        end
    end
end

```

```

        clc;
        dummy = input('Hit any key to continue ...');
        plot(n*Sampling_Per,Traj_Z,'r',n*Sampling_Per, ...
            Traj_Z_Comp,'g');
        title('Actual and Estimated Source Trajectory in the Z-Plane');
        xlabel('offset time (seconds)');
        ylabel('meters');
        grid;
    end

else
    clc;
    fprintf('The tracking algorithm has not been applied!');
    dummy = input('Hit any key to continue ...');
end
end
end
end
end
end

```

B.2. src_sig_color.m

```

function Src_Sig = src_sig_color(Signal_Len,Sampling_Per, ...
    Leng_of_Events,Low_Cutoff,High_Cutoff, ...
    Bandpass_Order,Time_of_Events, ...
    Power_of_Events,Var_Sig_Noise);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Title:      src_sig_color
% Author:     Mike Buhl
% Date:       9/92
%
% Purpose:    This function generates the source signal. This signal consists of white
%              noise events. The time at which these events occur and the signal power of
%              these events are passed as arguments. Once the signal is constructed, unit
%              variance white noise is added to it.
%
% Inputs:     Signal_Len      = length of source signal (seconds)
%              Sampling_Per   = sample period (seconds)
%              Leng_of_Events = length of each event (seconds)
%              Low_Cutoff     = low cutoff frequency for bandpass filter (Hz)
%              High_Cutoff    = high cutoff frequency for bandpass filter (Hz)
%              Bandpass_Order = bandpass filter order
%              Time_of_Events = time of events in x1
%              Power_of_Events= power of events in dBs
%              Var_Sig_Noise  = variance of noise added to signal
%
% Outputs:    Src_Sig        = source signal
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Event_Pnts = round(Leng_of_Events/Sampling_Per);
Num_Pnts = round(Signal_Len/Sampling_Per);
Loc1_Pnts = round(Time_of_Events/Sampling_Per);
Loc1_Pnts = Loc1_Pnts + 1;

```

```

rand('normal');
Filter_In = rand(1:Event_Pnts);
Filter_Out = filter(b,a,Filter_In);
test = Filter_Out.^2;
Sig_Pow = sum(test)/length(test);
Amp_1 = sqrt((10.^(Power_of_Events/10))/Sig_Pow);

q1 = zeros(1,(Num_Pnts+Event_Pnts));
for i = 1:length(Loc1_Pnts)
    start = Loc1_Pnts(i);
    q2 = q1(start:(Event_Pnts+start-1));
    q2 = q2 + Amp_1(i)*Filter_Out;
    q1(start:(Event_Pnts+start-1)) = q2;
end
Src_Sig = q1(1:Num_Pnts) + sqrt(Var_Sig_Noise)*rand(1:Num_Pnts);

```

```
function Src_Sig = src_sig_2order(Signal_Len,Sampling_Per,Damping_Ratio, ...
    Natural_Freq,Time_of_Events,...
    Power_of_Events,Var_Sig_Noise);
```

```

Num_Pnts = round(Signal_Len/Sampling_Per);
Loc1_Pnts = round(Time_of_Events/Sampling_Per);
Loc1_Pnts = Loc1_Pnts + 1;

```

```

a(1) = 1;
if Damping_Ratio > 1
    temp = WnT*sqrt((Damping_Ratio^2) - 1);
    a(2) = -exp(-Damping_Ratio*WnT)*(exp(temp) + exp(-temp));

```

```

else
    temp = WnT*sqrt(1 - Damping_Ratio^2);
    a(2) = -2*exp(-Damping_Ratio*WnT)*cos(temp);
end
a(3) = exp(-2*Damping_Ratio*WnT);

b(1) = 1;

Filter_In = [1 zeros(1:(Num_Pnts-1))];
test = filter(b,a,Filter_In);
test = test.^2;
Sig_Pow = sum(test);
Amp_1 = sqrt((10.^(Power_of_Events/10))/Sig_Pow);

Filter_In = zeros(1,Num_Pnts);
for i = 1:length(Loc1_Pnts)
    Filter_In(Loc1_Pnts(i)) = Amp_1(i);
end
Src_Sig = filter(b,a,Filter_In);
rand('normal');
Src_Sig = Src_Sig + sqrt(Var_Sig_Noise)*rand(1:Num_Pnts);

```

B.4. sen_sig.m

```

function Sensor_Sig = sen_sig(Sen_Coordinates,Signal_Vel,Sampling_Per, ...
                              X_Coef,Y_Coef,Z_Coef,Var_Sen_Noise, ...
                              Signal_Att,Src_Sig)

%% Title: sen_sig
%% Author: Mike Buhl
%% Date: 9/92
%%
%% Purpose: This function generates the signal which is present at each sensor. These
%%           signals are generated by summing up the delayed and attenuated impulses
%%           which make up the source signal. The delay is determined by the distance
%%           between the source and the sensor at the time of the impulse and the signal
%%           velocity, and the attenuation is determined by
%%
%%           Impulse attenuation = "Signal_Att" / Distance between source and sensor.
%%
%%           Due to the movement of the source, the sensor signals will be distorted with
%%           respect to one another.
%%
%% Inputs:  Sen_Coordinates = sensor coordinates (meters)
%%           Signal_Vel     = source signal velocity (meters/second)
%%           Sampling_Per    = sampling period (seconds)
%%           X_Coef         = x-trajectory coefficients of source
%%           Y_Coef         = y-trajectory coefficients of source
%%           Z_Coef         = z-trajectory coefficients of source
%%           Var_Sen_Noise  = variance of the sensor noise
%%           Signal_Att     = signal attenuation constant
%%           Src_Sig        = source signal
%%
%% Outputs: Sensor_Sig     = array in which each row represents a sensor signal
%%
%%

```

```

%% Compute the number of points/source signal and the number of sensors. Then initialize
%% the Sensor_Sig array and the number of responses array to zeros and the "rand"
%% function to normal.
%%

[Num_Sensors, Num_Cor] = size(Sen_Coordinates);
Num_Points = length(Src_Sig);
Sensor_Sig = zeros(Num_Sensors, Num_Points);
Num_Resp = zeros(Num_Sensors, Num_Points);

%% Compute the minimum initial delay between the source and the sensor so that the offset
%% time delay can be computed.
%%

x_source = X_Coef(3);
y_source = Y_Coef(3);
z_source = Z_Coef(3);
Src_Coordinates = [x_source y_source z_source];
for i = 1:Num_Sensors
    Dist = Src_Coordinates - Sen_Coordinates(i,:);
    Dist_Src_Sen(i) = sqrt(Dist*Dist');
    if (i == 1)
        Min_Dist = Dist_Src_Sen(i);
    else
        if (Dist_Src_Sen(i) < Min_Dist)
            Min_Dist = Dist_Src_Sen(i);
        end
    end
end
Min_Delay = Min_Dist/Signal_Vel;

%% Compute the source coordinates.
%%

for n=1:Num_Points
    time = (n - 1)*Sampling_Per;
    time_vector = [time^2 time 1];
    x_source = X_Coef*time_vector';
    y_source = Y_Coef*time_vector';
    z_source = Z_Coef*time_vector';
    Src_Coordinates = [x_source y_source z_source];

    %% Compute the time delays from the source to the sensor.
    %%

```

```

for i = 1:Num_Sensors
    Dist = Src_Coordinates - Sen_Coordinates(i,:);
    Dist_Src_Sen = sqrt(Dist*Dist');
    Delay_Src_Sen = Dist_Src_Sen/Signal_Vel;

%% Compute the time delays in points.

    Rel_Delay_P = round((Delay_Src_Sen-Min_Delay)/Sampling_Per) + 1;
    Offset_P = Rel_Delay_P + n;

    Determine the attenuation at the sensors. Then place impulse at correct position in the
    sensor signal.

    if (Dist_Src_Sen < 1)
        Att = 1;
    else
        Att = Signal_Att/Dist_Src_Sen;
    end
    if ((Offset_P > 0) & (Offset_P <= Num_Points))
        Sensor_Sig(i,Offset_P) = Att*Src_Sig(n) + Sensor_Sig(i,Offset_P);
        Num_Resp(i,Offset_P) = Num_Resp(i,Offset_P)+1;
    end
end

    Average samples which occur at the specified sampling instant.

    zero_ind = find(Num_Resp == 0);
    Num_Resp(zero_ind) = ones(zero_ind);
    Sensor_Sig = Sensor_Sig ./ Num_Resp;

    Filter the sensor signals by interpolating to find values which are erroneously zero.

for i = 1:Num_Sensors
    j = 1;
    Point_Zero = 1;
    while ((Point_Zero == 1) & (j < Num_Points))
        if (Sensor_Sig(i,j) ~= 0)
            Point_Zero = 0;
        else
            j = j + 1;
        end
    end
end

```

```

end

zero_ind = find(Sensor_Sig(i,j:Num_Points) == 0);
for k= 1:length(zero_ind)
    index = zero_ind(k) + j - 1;
    right_ind = index;
    while (Sensor_Sig(i,right_ind) == 0 & right_ind < Num_Points)
        right_ind = right_ind + 1;
    end
    slope = (Sensor_Sig(i,right_ind) - Sensor_Sig(i,(index-1)))/ ...
        (right_ind - index + 1);
    Sensor_Sig(i,index) = slope + Sensor_Sig(i,(index-1));
end
end

rand('normal');
Sensor_Sig = Sensor_Sig + sqrt(Var_Sen_Noise)*rand(Num_Sensors,Num_Points);

```

B.5. spec_est_corr.m

```
function Sx1x2 = spec_est_corr(x1,x2)
```

```

%%
%% Title:      spec_est_corr
%% Author:     Mike Buhl
%% Date:       9/92
%%
%% Purpose:    This function generates the power spectrum and cross spectrum of x1 and
%%             x2. This is done using the Blackman-Tukey method. The correlation is
%%             windowed with a hamming window.
%%
%% Inputs:     x1      =   array containing points of x1
%%             x2      =   array containing points of x2
%%
%% Outputs:    Sx1x2   =   cross spectrum between x1 and x2
%%
%%
%%
N = length(x1);
X1 = fft(x1,2*N);
X2 = fft(x2,2*N);

Px1x2 = X1 .* conj(X2);
Rx1x2 = ifft(Px1x2);
Rx1x2 = fftshift(Rx1x2);
Rx1x2 = Rx1x2 .* (hamming(2*N));
Sx1x2 = fft(Rx1x2);

```

B.6. time_est_cross.m

```
function [Est_Delay, Sqr_Ry1y2] = time_est_cross(Sx1x2,Sampling_Fcr);
```

```

%%

```

```

% Title:      time_est_cross
% Author:     Mike Buhl
% Date:       9/92
%
% Purpose:    This function determines the time delay between two signals, x1 and x2
%              using a unity gain processor. This procedure is given below.
%
%              1.  $R_{x1x2} = R_{y1y2} = \text{iDFT}\{S_{x1x2}\}$ 
%              2. Determine the peak of  $\text{sqr}(R_{y1y2})$ . This peak should occur
%                  at the delay.
%
% Inputs:       $S_{x1x2}$           = cross-spectrum of x1 and x2
%              Sampling_Per      = sampling period
%
% Outputs:     Est_Delay         = computed time delay between x1 (reference) and x2 (
%              Sqr_Ryly2        = squared and filtered cross-correlation
%
% %%%

```

```

Num_Points_Sx1x2 = length(Sx1x2);
Num_Points = Num_Points_Sx1x2/2;

```

```

Ryly2 = ifft(Sx1x2);
Ryly2 = real(Ryly2);
Sqr_Ryly2 = Ryly2.^2;

```

```

Largest_Mag = -1;
for n = 1:Num_Points_Sx1x2
    if (Sqr_Ryly2(n) > Largest_Mag)
        Largest_Mag = Sqr_Ryly2(n);
        Largest_Ind = n;
    end
end

```

```

Est_Delay = ((Largest_Ind-1)-Num_Points)*Sampling_Per;

```

B.7. time_est_roth.m

```

function [Est_Delay, Sqr_Ryly2] = time_est_roth(Sx1x1, Sx1x2, Sampling_Per);

```

```

% %%%
% Title:      time_est_roth
% Author:     Mike Buhl
% Date:       9/92
%
% Purpose:    This function determines the time delay between two signals, x1 and x2
%              using a Roth processor. This procedure is given below.
%
%              1.  $S_{y1y2} \approx S_{x1x2}/S_{x1x1}$ 
%              2.  $R_{y1y2} = \text{iDFT}\{S_{y1y2}\}$ 
%              3. Determine the peak of  $\text{sqr}(R_{y1y2})$ . This peak should occur
%                  at the delay.
%
% Inputs:       $S_{x1x1}$           = power-spectrum of x1
%               $S_{x1x2}$           = cross-spectrum of x1 and x2
%              Sampling_Per      = sampling period
%
% %%%

```



```

Largest_Mag = -1;
for n = 1:Num_Points_Sx1x2
    if (Sqr_Ryly2(n) > Largest_Mag)
        Largest_Mag = Sqr_Ryly2(n);
        Largest_Ind = n;
    end
end

Est_Delay = ((Largest_Ind-1)-Num_Points)*Sampling_Per;

```

B.9. time_est_phat.m

```
function [Est_Delay, Sqr_Ryly2]= time_est_phat(Sx1x2,Sampling_Per);
```

```

%% Title:      time_est_phat
%% Author:     Mike Buhl
%% Date:       9/92
%%
%% Purpose:    This function determines the time delay between two signals, x1 and x2
               using a phat processor. This procedure is given below.
%%
               1. Syly2 = Sx1x2/ abs(Sx1x2)
               2. Ryly2 = iDFT(Syly2)
               3. Determine the peak of sqr(Ryly2). This peak should occur at the
                  delay.
%%
%% Inputs:     Sx1x2      = cross-spectrum of x1 and x2
               Sampling_Per = sampling period
%%
%% Outputs:    Est_Delay  = computed time delay between x1 (reference) and x2 (
               Sqr_Ryly2  = squared and filtered cross-correlation
%%
%%
%%

```

```

Num_Points_Sx1x2 = length(Sx1x2);
Num_Points = Num_Points_Sx1x2/2;

```

```

Syly2 = Sx1x2 ./ abs(Sx1x2);
Ryly2 = ifft(Syly2);
Ryly2 = real(Ryly2);
Sqr_Ryly2 = Ryly2.^2;

```

```

Largest_Mag = -1;
for n = 1:Num_Points_Sx1x2
    if (Sqr_Ryly2(n) > Largest_Mag)
        Largest_Mag = Sqr_Ryly2(n);
        Largest_Ind = n;
    end
end

```

```
Est_Delay = ((Largest_Ind-1)-Num_Points)*Sampling_Per;
```

```
function [Est_Delay, Sqr_Ryly2]= time_est ml(Sx1x1,Sx2x2,Sx1x2,Sampling Per);
```

```
Num_Points_Sx1x2 = length(Sx1x2);  
Num_Points = Num_Points_Sx1x2/2;
```

```

Largest_Mag = -1;
for n = 1:Num_Points_Sx1x2
    if (Sqr_Ry1y2(n) > Largest_Mag)
        Largest_Mag = Sqr_Ry1y2(n);
        Largest_Ind = n;
    end
end

```

B.11. locate_source.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Title:      locate_source
%
%   Author:     Mike Buhl
%
```

```

% Date:      9/92
%
% Purpose:    This function performs a non-weighted passive localization algorithm on
%             sensor array data. This algorithm is described in "A Passive Localization
%             Algorithm and Its Accuracy Analysis" by Benjamin Friedlander. This paper
%             appeared in the IEEE Journal of Oceanic Engineering on January 1987.
%
% Inputs:     Sen_Cor    = array of sensor coordinates
%             Ref_Sen    = reference sensor
%             Est_Delay  = array of estimated delays between the sensors and
%             the reference
%             Sig_Vel    = signal velocity
%
% Outputs:    x,y, and z = estimated source location
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Compute the number of sensors, the range difference between sensors, and the
% range between the sensors and the origin.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Num_Sensors = length(Est_Delay);
Range_Diff = Sig_Vel*Est_Delay;
for i=1:Num_Sensors
    Range_Org(i) = Sen_Cor(i,:)*Sen_Cor(i,:);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Compute the matrices S, Mu, and Row which are equations (7b), (7c), and (7d),
% respectively.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

j = 0;
for i= 1:Num_Sensors
    if (i ~= Ref_Sen)
        j = j + 1;
        S(j,:) = Sen_Cor(i,:) - Sen_Cor(Ref_Sen,:);
        Mu(j) = .5*(Range_Org(i) - Range_Org(Ref_Sen) - (Range_Diff(i)^2));
        Row(j) = Range_Diff(i);
    end
end

Mu = Mu';
Row = Row';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Compute the inverse diagonal matrix D and the circular shift matrix Z,
% which are given by (8a) and (8b). These matrices are necessary to compute
% the matrix M, which has Row in its null-space.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

D = inv(diag(Row));
Z = [zeros(Num_Sensors-2,1) eye(Num_Sensors-2); 1 zeros(1,Num_Sensors-2)];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Compute the matrix Vk and the matrix Mbar, which are given by (18) and (19).
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[U,N,V] = svd(eye(Num_Sensors-1) - Z);

Vk = V(:,1:(Num_Sensors-2));
Mbar = Vk'*D;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Compute the source coordinates. These are computed using equation (12).
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Est_Src_Cor = (inv(S'*Mbar'*Mbar*S))*S'*Mbar'*Mbar*Mu;

```

B.12. power_of_two.m

```

function Win_Len_Pnts = power_of_two(Win_Delay_P)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   Title:      power_of_two
%   Author:     Mike Buhl
%   Date:       9/92
%
%   Purpose:    This function computes the smallest number which is a power of two and
%               greater than "Win_Delay_P".
%
%   Inputs:     Win_Delay_P    =   number which must be less than return value
%
%   Outputs:    Win_Len_Pnts   =   smallest number which is a power of two and greater
%                                   than "Win_Delay_P".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Current_Num = 2;
Not_Found = 1;
while (Not_Found == 1)
    if (Current_Num > Win_Delay_P)
        Not_Found = 0;
    else
        Current_Num = 2*Current_Num;
    end
end

Win_Len_Pnts = Current_Num;

```

END

DATE
FILMED
10 / 6 / 93