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AN ELECTROMAGNETIC AND THERMODYNAMIC LUMPED PARAMETER
MODEL OF AN EXPLOSIVELY DRIVEN REGENERATIVE
MAGNETOHYDRODYNAMIC GENERATOR

A dissertation

Presented in partial fulfillment of the requirements for the
Degree of Doctor of Philosophy
with a
Major in Electrical Engineering

in the

College of Graduate Studies
at the
University of Idaho

by
John L. Morrison
December 1992.

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MASTER

AUTHORIZATION TO SUBMIT DISSERTATION

This dissertation of John L. Morrison, submitted for the degree of Doctor of Philosophy, with a major in Electrical Engineering and titled "An Electromagnetic and Thermodynamic Lumped Parameter Model of an Explosively Driven Regenerative Magnetohydrodynamic Generator", has been reviewed in final form, as indicated by the signatures and dates given below. Permission is now granted to submit final copies to the College of Graduate Studies for approval.

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ABSTRACT

The objective of this research is to develop a simple, yet accurate, lumped parameter mathematical model for an explosively driven magnetohydrodynamic generator that can predict the pulse power variables of voltage and current from startup through regenerative operation. The inputs to the model will be the plasma properties entering the generator as predicted by the explosive shock model of Reference [1]. The strategy used was to simplify electromagnetic and thermodynamic three dimensional effects into a zero dimensional model. The model will provide a convenient tool for researchers to optimize designs to be used in pulse power applications. The model is validated using experimental data of Reference [1].

An overview of the operation of the explosively driven generator is first presented. Then a simplified electrical circuit model that describes basic performance of the device is developed. Then a lumped parameter model that incorporates the coupled electromagnetic and thermodynamic effects that govern generator performance is described and developed. The model is based on fundamental physical principles and parameters that were either obtained directly from design data or estimated from experimental data. The model was used to obtain parameter sensitivities and predict beyond the limits observed in the experiments to the levels desired by the potential Department of Defense sponsors. The model identifies process limitations that provide direction for future research.

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DEDICATION

This work is dedicated to the memories of my mother Frances B. Morrison and my good friend, the late Rollie Stewart.

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1.0 INTRODUCTION

The Department of Defense (DoD) has funded research for many years in technologies related to explosively-driven, electrical, pulse-power sources [1,16 to 27]. Their interest is in a power source for new technology weapons such as: rail guns or lasers. One of the explosively driven pulse power sources uses the concept of a Magnetohydrodynamic (MHD) generator. Recently the U.S. Marine Corps Research and Development Command funded a research effort with the Idaho National Engineering Laboratory (INEL) to demonstrate the feasibility of a conventional explosively driven MHD generator with magnetic field regeneration (boot-up) producing magnetic and electric fields of sufficient strength to be of interest to the DoD. This program conducted 14 tests over a six-month period. While the desired high strength fields were not achieved, generator dynamics that are precursors to such fields were observed.

The purpose of the research described in this dissertation is to develop a simple, yet accurate, lumped parameter model of the coupled electromagnetic and thermodynamic effects of an explosively driven MHD generator. The model will predict the pulse-power variables of generator voltage and current from process startup through regeneration, but not process shutdown. The development strategy was to simplify three dimensional or one dimensional effects to obtain a lumped parameter model of the system. The model was validated with data from INEL experiments. The validated model will become a useful tool for researchers to develop advanced designs that can achieve the high pulse power outputs that potential sponsors desire.

In Chapter 2 a description of the device and its operation is presented. Chapter 3 is a review of the literature that is related to the technology of this device and research that is directly associated to this type of device. Chapter 4 develops a simple electrical circuit model of the device that neglects many electromagnetic and thermodynamic constraints, yet simply and clearly illustrates the basic functions. Chapter 5 develops the complete model that incorporates the full electromagnetic and thermodynamic effects homogenized into lumped parameters to simulate the device operation. Chapter 6 validates the model by imputing parameters from the INEL experiments that are either measured or estimated and simulating the INEL experimental results. In Chapter 7 the model is used to simulate high power performance. Selected parameter values are altered to achieve the desired levels of performance. A discussion is provided to assess the feasibility of realizing designs that will offer the required

parameter values. Chapter 8 provides a discussion that summarizes the development effort and the results of the various simulations. Observations and conclusions from those results are provided.

The significant contributions of this research are:

1. A full effects lumped parameter model of an explosively driven MHD generator was developed that considers electromagnetic and thermodynamic effects.
2. The full effects models defining equations were described as electrical equivalent circuits and were analyzed with a PC version of the electrical simulation code, SPICE [2].
3. A ladder electrical network was developed to implement a one dimensional magnetic flux diffusion equation description of the magnetic flux/plasma interactions. The magnetic flux that propagates to the end of the ladder is at a plasma penetration distance to the center of the MHD channel and is the value of flux that was used to excite the full effects model.
4. A unique approach was developed to model the systems thermodynamic equations in that thermodynamic equations were described as system diagrams. The blocks in the system diagrams were implemented as electrical circuits of SPICE elements.
5. The Taylor Series was used to implement the Gamma Gas Law to obtain an equation of state for the plasma that is compatible with the lumped parameter analysis strategy and mitigates the errors associated with an ideal gas description of a highly ionized plasma. The Taylor Series projected the known input plasma properties across the MHD channel to the outlet.

2.0 OVERVIEW OF GENERATOR OPERATION

The fundamental principles of MHD can be understood by considering the basic relation given by Equation 2.1 [Ref. 8]. It describes the forces acting on a positively charged particle q , traveling with a velocity U , in an electric field E , and a magnetic field B .

$$\vec{F} = q(\vec{E} + \vec{U} \times \vec{B}) \quad 2.1$$

If the electric field is not present, then the configuration is illustrated in Figure 2.1

Figure 2.1: MHD fundamental principles.

Equation 2.2 can be obtained from equation 2.1 by setting the electric field E , to zero.

$$\vec{F} = q\vec{U} \times \vec{B} \quad 2.2$$

If B and E are constant over region of interest then Equation 2.3 can be obtained as follows:

$$\frac{1}{q} \int_0^x \vec{F} \cdot d\vec{\ell} = \int_0^x \vec{U} \times \vec{B} \cdot d\vec{\ell}$$

$$V = UBx \quad 2.3$$

Equation 2.3 gives the potential difference between two points in Figure 2.1 that are separated by a distance X , that is in the same direction as the force F .

A highly ionized gas (plasma) flowing through a magnetic field with an average particle velocity U will develop a potential difference across its flow according to Equation 2.3. If conductive plates are positioned in the plasma flow separated by the distance X , that is in the same direction as the force F , then the potential difference appears on the plates and they become terminals of a generator. Electrical power can then be drawn from the plates and applied to an external load. Figure 2.2 illustrates this exact situation with the elements of a MHD generator consisting of a plasma flowing at very high velocity between two electrodes separated by a distance ℓ , and cutting through a magnetic field perpendicular to the direction of flow. An electrical potential will develop on the electrodes according to Equation 2.3. Equation 2.4 gives this potential relative to the geometry of Figure 2.2.

$$V_{mhd} = UB\ell$$

2.4

Where: V_{mhd} is the voltage between the electrodes (V).
 U is the Plasma velocity (ms^{-1}).
 B is the magnetic field (T).
 ℓ is the distance between the electrodes (m).

Figure 2.2: MHD generator elements.

When power is drawn from the generator the same current must flow in the plasma as flows in the external load. The current density in the plasma causes an electric field given by [Ref. 8]:

$$\vec{J} = \sigma \vec{E} \quad 2.5$$

Where: E is the electric field (Vm^{-1}).
 J is the plasma current density (Am^{-2}).
 σ is the plasma conductivity (Sm^{-1}).

Equation 2.6 is derived from Equation 2.5 to obtain the resulting potential drop between the electrodes due to current flow:

$$E = \frac{J}{\sigma} = \frac{I_{mhd}}{\sigma A'}, \quad \int \vec{E} \cdot d\vec{\ell} = V_p = \frac{I_{mhd} \ell}{\sigma A'}$$

$$V_p = I_{mhd} \frac{\ell}{\sigma A'} = I_{mhd} R_p \quad 2.6$$

Where: E is the electric field (Vm^{-1}).
 J is the plasma current density (Am^{-2}).
 σ is the plasma conductivity (Sm^{-1}).
 V_p is the plasma ohmic voltage drop (V).
 I_{mhd} is the MHD current provided by the generator (A).
 A' is the electrode surface area (m^2).
 R_p is the plasma resistance between the electrodes (Ω).
 ℓ is the distance between the electrodes (m).

This potential difference is of opposite polarity of the MHD potential of Equation 2.4 and describes the source impedance of the generator. Thus Equation 2.3 gives the open circuit voltage of the generator, while Equation 2.7 gives the terminal voltage when the generator is loaded:

$$V_{mhd} = UB\ell - I_{mhd} R_p \quad 2.7$$

An additional effect occurs as power is drawn from the generator. As with a standard electrical generator a force will develop that will oppose the rotation of the armature which must be overcome if the generator is to output power. That force at a point in the plasma of the MHD generator is given by [Ref. 5]:

$$d\vec{F} = \vec{J} \times \vec{B} dv \quad 2.8$$

Where: J is the plasma current density ($A m^{-2}$).
 B is the magnetic field in the plasma (T).
 dv is a differential volume element (m^3).
 dF is the differential force at a point (N).

If J and B are assumed constant within the plasma volume between the electrodes then, Equation 2.8 can be integrated over the complete volume between the electrodes:

$$F = I B \ell \quad 2.9$$

Where: I is the plasma current (A).
 B is the magnetic field (T).
 ℓ is the distance between the electrodes (m).

Equation 2.9 gives the force that will oppose the plasma flow within the MHD generator caused by the generator providing a current I , to power an external load.

The principle of a regenerative magnetic field (boot-up) MHD generator is illustrated in Figure 2.3. Within the plasma generator an explosive shaped charge is detonated in a reservoir of Argon gas shocking it to a plasma state and forcing it from the generator into the MHD channel. The plasma leaves the generator by bursting through a rupture disk and flowing into the MHD channel through a nozzle that helps accelerate the flow to tens of kilometers/second. As the plasma passes through the channel it cuts an externally imposed magnetic field (provided by a permanent magnet) generating an electrical potential. This potential is used to excite the coil of an electromagnet that is arranged so that its field reinforces the initial magnetic field set up by the permanent magnet. Since the MHD generated potential is proportional to the magnetic field strength, a regenerative positive feedback condition is established. If the regeneration gains exceed the losses then the MHD potential, the electromagnet current and magnetic field will all grow to very large values and "boot-up" will be considered to have

occurred. This condition continues for the duration of the plasma flow, causing a large amount of energy to be transferred from the plasma to the magnetic field. If the plasma flow were to rapidly become nonconductive the interruption of current in the coil of the electromagnet would result in a rapidly collapsing magnetic field. If the magnetic field collapses rapidly enough it will initiate a large propagating electromagnetic pulse (EMP). The purpose of the INEL program [Ref. 1.] was to investigate this phenomenon. The focus of this research is a model of the boot-up characteristics of the regenerative MHD generator.

Figure 2.3: Schematic of a boot-up MHD generator.

Figure 2.4 illustrates the INEL boot-up MHD generator. The view shown is a cross section down the length of the device. The following observations of Figure 2.4 will help the reader relate features of this device to the schematic of Figure 2.3:

- o In the plasma generator, the tuning fork structure is the reservoir of argon gas. That container is really shaped like a bottom-less thermos bottle where the vacuum part of the thermos bottle is pressurized with argon gas.
- o The plasma generator functions by an explosive plane wave detonation front passing left to right through the device and crushing the argon reservoir and causing the argon gas to be shocked into a plasma. Figure 2.6 illustrates this.
- o The high pressure plasma blows through a rupture disk and exhausts at high velocity into the MHD channel.
- o The high velocity plasma within the MHD channel encounters the permanent magnets field and the MHD electrodes (The MHD electrodes are the large pair to the left).
- o Ceramic permanent magnets (outline shown) are on each side of channel and cause a magnetic field perpendicular to the plasma flow and perpendicular to a center line through the electrodes.
- o The MHD potential developed on the MHD electrodes is applied to the electromagnet coils.
- o The electromagnet is not shown in this drawing. The electromagnet is shown in Figure 2.5, a cross sectional end view slicing through the MHD channel exactly through the center line of the large MHD electrodes.
- o The right set of electrodes are for a plasma impedance experiment (not part of the MHD generator).
- o Spent plasma is exhausted out the right end of the device.

A perspective on size can be obtained from the following:

Plasma generator length is 17.5".

MHD electrode diameter is .56".

Distance between the electrodes is 1".

Electromagnet diameter is 3".

Figure 2.4: Linear MHD Assembly

Figure 2.5: End View of MHD Generator Electromagnet

Figure 2.6: INEL Plasma Generator Explosive Dynamics

In summary, this chapter has introduced the basic physical principals that describe the MHD power conversion process. It presented the concept of a regenerative explosively driven system. It described the INEL design configuration and explained the operation of the explosively powered plasma generator.

The next chapter will review the literature for relevant research.

3.0 REVIEW OF THE LITERATURE

Related research has occurred outside the arena of explosively driven MHD. IEEE Transactions on Plasma Science has some reports of research on plasma aspects of MHD generators [28, 29]. However, the focus of the research is in support of low temperature, steady state, coal fired large power plants. The plasma is low conductivity seeded combustion products (typical temperature $< .3\text{ev}$ and typical conductivity 100Si/m). For that application the analysis of the effects of the magnetic field and plasma interactions are greatly simplified. A measure of the validity of this simplification is the Magnetic Reynolds Number R_m [9]. The simplifications can be made if R_m is less than 1. The focus of this research is associated with high temperature, high conductivity, and high plasma velocity (temperature 3ev , conductivity 20000 Si/m , and velocity 60km/sec). This results in an R_m much greater than 1. In this research the magnetic diffusion and eddy current effects can not be ignored. Some of the non-MHD research in the IEEE Transactions on Plasma Science is associated with a high R_m . References [30, 31] report on MHD analysis performed on railgun plasma armatures. The analysis in both of these reports is of interest because reference [30] used the same relationship for conductivity used herein and reference [31] also uses the Gamma Gas Law [3] as an equation of state for the plasma that was used herein. There are many other reports of related research. Reference [11] is such a paper and reports on electrical conductivity of highly ionized plasma. That work, while applicable herein, was not used. Instead the simpler relation [6] was chosen.

All the applied research performed specifically in the area of explosively driven MHD in the U.S. has been sponsored directly or indirectly by the DoD. A few contractors have performed the majority of the research. Artec Associates was one of the principle researchers (S. P. Gill, D. W. Baum, W. L. Shimmin, D. Mukherjee). They were active from about 1976 until 1987 and performed much of their explosive testing on Lawrence Livermore National Laboratory (LLNL) test ranges with LLNL technical support. It is also noted that during this same time period that LLNL developed an argon plasma equation of state [16] and implemented it in a computer code. The development was performed on argon in the region of operation of these generators. References [17, 20, 21, 22, 23, 26, 27] report on the Artec work. Most of this research deals with non-regenerative explosively driven MHD with some discussion and analysis of the regenerative concept. Reports of this work are often cryptic with critical details lacking. Some of this is likely due to the information being classified. In 1990 INEL

performed some research sponsored by the USMC to specifically investigate regeneration. Reference [1] provides complete details of their results. Additional systems analysis and feasibility studies have been performed by various other contractors. Coal Technology (B. Zauderer [18]) performed a system investigation of regeneration. Mountain States Energy (D. Cott [16, 19]) did system studies of applications of the concept. STD Corp (D. A. Oliver [25]) did an analytical study of a very high Magnetic Reynolds Number MHD system. These works describe the technology as a viable pulse power source and have probably encouraged the DoD to continue funding research in this technology.

The next chapter will perform a simple analysis that illustrates the regenerative characteristic behavior.

4.0 DEVELOPMENT OF A SIMPLE CIRCUIT MODEL

A great deal of insight into the operation of the boot-up MHD generator can be obtained from a simplified equivalent circuit analysis. Figure 2.3 is a schematic of the system. For a simplified analysis the complex effects of magnetic field diffusion into the plasma, eddy current losses, and plasma thermodynamic considerations will be ignored. These areas will be investigated in depth and integrated in a full-effects model in Chapter 5. The significance of these simplifying assumptions is that the plasma flow into the MHD channel is considered as a constant source of velocity and conductivity, and is independent of any aspect of the MHD process. Furthermore, the magnetic field diffuses into the plasma filled MHD channel with zero time delay. This results in Equation 2.7 describing the MHD generator as a voltage source with a source impedance.

$$V_{mhd} = UB\ell - I_{mhd} R_p \quad 2.7$$

An equivalent circuit of the system illustrated in Figure 2.3 is shown in Figure 4.1.

Where:

- V_0 is the MHD voltage from the permanent magnet (V).
- V_{bu} is the regenerative MHD voltage from the electromagnet (V).
- i_L is the current in the Electromagnet coil (A).
- L_B is the Electromagnet inductance (H).
- R_{cu} is the Copper winding resistance of the electromagnet coil (Ω).
- R_p is the resistance of the plasma between electrodes (Ω).

Figure 4.1: Simplified equivalent circuit of the MHD generator.

The MHD potentials V_o and V_{bu} are obtained from Equation 2.4.

$$V_{mhd} = UB\ell \quad 2.4$$

$$V_o = UB_o\ell \quad 4.1$$

$$V_{bu} = UB_u\ell \quad 4.2$$

Where: U is the plasma velocity, considered constant (ms^{-1}).
 B_o is the magnetic field from the permanent magnet (T).
 B_u is the magnetic field from the electromagnet (T).
 ℓ is the distance between the electrodes (m).

Bu, the in channel magnetic field from the electromagnet is produced by the current in the inductor L_B and is given by Equation 4.3.

$$B_u = Ki_L \quad 4.3$$

Where K is a geometry constant (see Appendix C and Equation C4) and i_L is the current in L_B . From the equivalent circuit of Figure 4.1 the following equation can be obtained:

$$V_o + V_{bu} = i_L R_p + i_L R_{cu} + L_B \frac{di_L}{dt} \quad 4.4$$

Combining Equations 4.1, 4.2 and 4.3 with 4.4:

$$UB_o\ell = i_L R_p + i_L R_{cu} - i_L KU\ell + L_B \frac{di_L}{dt} \quad 4.5$$

Eqn. 4.5 is a first order ordinary D.E.Q. with an input given by Eqn. 4.1 and a time constant of:

$$\tau = \frac{L_B}{R_p + R_{cu} - KU\ell} \quad 4.6$$

Observe that $-KU\ell$ has the form of a negative resistance. Equation 4.5 has the initial condition: $i_L(t=0) = 0$. If $\tau < 0$ then the solution of Equation 4.5 for a unit step input (at $t = 0$ and of magnitude $UB_0\ell$) is given by Equation 4.7 :

$$i_L(t) = \frac{B_0 U \ell}{R_p + R_{cu} - KU\ell} \left(1 - e^{\frac{-t}{\tau}} \right) \quad t \geq 0 \quad 4.7$$

This system is non-regenerative and a sketch of its inductor current waveform is given in Figure 4.2.

Figure 4.2: Non-regenerative response.

If $\tau < 0$ then the solution of Equation 4.5 is given by:

$$i_L(t) = \frac{B_0 U \ell}{KU\ell - (R_p + R_{cu})} \left(e^{\frac{-t}{\tau}} - 1 \right) \quad t \geq 0 \quad 4.8$$

Equation 4.8 is the same solution as Equation 4.7. The sign change in τ causes Equation 4.8 to become unstable, while in Equation 4.7, it is stable. Additionally if the negative resistance term just equals the losses then the step input sees only a pure inductor and the current will be the integral of that step, a ramp. The unstable solution of Equation 4.8 is regenerative and a sketch of its inductor current waveform is given in Figure 4.3.

Figure 4.3: Regenerative response.

The characteristic of regeneration or boot-up is well illustrated by Equation 4.8 and Figure 4.3 with exponential growth. If the negative resistance term exceeds the losses, then the enabling conditions for boot-up are established. Equation 4.9 states this condition.

$$KU\ell > R_{cu} + R_p \quad 4.9$$

The simplified model of the system is enlightening but also misleading. For example Equation 4.8 indicates that current growth is unlimited and power can be extracted from the plasma at an ever increasing rate. By neglecting the thermodynamic effects where plasma power is converted into electrical power through the MHD process, the simplified model has assumed the characteristic of a perpetual motion machine. The full-effects model of Chap. 5 will address this limitation. However, in spite of this limitation this model does illustrate the characteristic and enabling condition for boot-up.

In summary, this chapter has examined the regenerative MHD system with all of the thermodynamic interactions and magnetic flux diffusion effects neglected. This simplified approach illustrated the basic regenerative characteristics. It also identified the key issue of minimizing system losses to enable regeneration (In this simple analysis losses were due to copper and plasma resistance.).

In the next chapter the analysis was expanded to include the electromagnetic and thermodynamic interactions with the development of the full effects model.

5.0 DEVELOPMENT OF THE FULL-EFFECTS MODEL

5.1 Description of the Full-effects Model

A full-effects model was developed that considers magnetic field diffusion effects, plasma eddy current effects, and MHD channel plasma thermodynamic effects. The strategy used to develop the model was to simplify three dimensional effects into a zero dimensional model so as to obtain the desired lumped parameter responses (MHD voltage, current, inductor voltage, current etc.). A model that considers full-effects in three dimensions is extremely complicated. Although, such solutions could be averaged over the MHD volume to obtain the desired responses. Thus the following simplifying assumptions were made:

1. MHD channel inlet plasma was assumed to be an independent forcing function for the model. Its properties coming into the channel were assumed uniform over the inlet channel cross sectional area and were assumed independent of any MHD channel effects.
2. Plasma properties within the channel were assumed to respond to channel effects but, were assumed homogeneous throughout the MHD channel volume.
3. Plasma viscosity, boundary layer effects with the channel walls, and gravitational effects were all ignored.
4. Magnetic field strength was assumed uniform throughout the channel at the magnitude that would exist at the center of the channel.
5. The mass flow into the MHD channel was assumed equal to the mass flow out of the MHD channel and the mass within the MHD channel was assumed constant.
6. The power into the MHD channel was assumed equal to the power out of the MHD channel and power loss to the channel walls was ignored.

Figure 5.1 illustrates a schematic of the way that the system will be analyzed and Figure 5.2 illustrates a block diagram of the full-effects model.

Figure 5.1: MHD channel schematic

Figure 5.2: Full-effects model block diagram.

Referring to the block diagram, Figure 5.2, the model has three basic Blocks:

1. The Generator and Electromagnet Model
2. The Magnetic Flux Model
3. The Thermodynamic Model

Block 1 is very similar to the equivalent circuit used in the simplified analysis. Significant differences are that it accounts for eddy current loss and its parameters are dynamically modified by the plasma properties. Blocks 2 and 3 specifically address areas that the simplified analysis neglected. Figure 5.1 shows all the inputs and outputs for each Block. The inputs marked with a *, are exogenous inputs. All the other variables are computed in one of the various Blocks and fed back as inputs to other Blocks. Table 5.1 lists and defines all the model variables. The table also gives where they were computed and to which blocks they are provided as inputs. It is emphasized that this model makes no attempt to describe the initial plasma generation by way of attempting to model the explosive shock physics within the plasma generator. A model that simulates the plasma generator explosive shock behavior was developed by INEL and is described in Reference [1]. The full-effects model will use the plasma properties as predicted by the INEL explosive shock model as inputs.

Table 5.1. Full-effects model variables (all units are MKS).

Variable	Description
A_0^*	A_0^* is the MHD channel input plasma properties are the independent variables that are the forcing functions to the full-effects model. They are an input to Block 3.
G_s^*	G_s^* is the MHD generator geometry constants represent the collection of physical measurements such as cross sectional areas and distance between electrodes. They are independent inputs that characterize the full-effects model. They are input to all 3 Blocks.
B_0^*	B_0^* Is the magnetic field (T) in the center of the MHD channel that would exist if the plasma were not present. This is an input to Block 2 and is used to compute $B_1(t)$.
$B_1(t)$	$B_1(t)$ is the magnetic field (T) within the plasma filled MHD channel that is the result of the permanent magnets field interacting with the conductive plasma flow. This variable was computed by Block 2 and was an input to Blocks 1 and 3.
$B_2(t)$	$B_2(t)$ is the magnetic field (T) within the plasma filled MHD channel that is the result of the electromagnets field interacting with the conductive plasma flow. This variable was computed by Block 2 and was an input to Blocks 1 and 3.
$i_L(t)$	$i_L(t)$ is the current (A) in the electromagnet coil. It was computed in Block 1 and was an input to Block 2.
$i_{mhd}(t)$	$i_{mhd}(t)$ is the current (A) produced by the MHD potential. It was computed in Block 1 and was an input to Block 3.
$\dot{W}_e(t)$	$\dot{W}_e(t)$ is the eddy current power (W) dissipation within the plasma flowing through the MHD channel that is caused by the time varying magnetic field passing through the conductive plasma. It was computed in Block 1 and input to Block 3.
$\dot{W}_{mhd}(t)$	$\dot{W}_{mhd}(t)$ is the MHD power (W) developed by the MHD generator. It was computed in Block 1 and was an input to Block 3.

Table 5.1 Cont.

$\dot{W}_{plasma}(t)$	$\dot{W}_{plasma}(t)$ is the power (W) dissipated within the plasma source resistance. It is caused by ohmic losses of the MHD generator current, $i_{mhd}(t)$, flowing through the conductive plasma between the electrodes. It was computed in Block 1 and was an input to Block 3.
$\sigma(t)$	$\sigma(t)$ is the average plasma conductivity (Sm^{-1}) within the MHD channel corrected for thermodynamic effects. It was computed in Block 3 and used in Blocks 1 and 2.
$U_{eff}(t)$	$U_{eff}(t)$ is the average plasma velocity (ms^{-1}) within the MHD channel corrected for thermodynamic effects was computed by Block 3 and used in Block 1.
V_0	V_0 is the MHD potential (V) due to the magnetic field from the permanent magnets. It was computed in Block 1.
V_{bu}	V_{bu} is the MHD potential (V) due to the regenerative magnetic field from the electromagnet. It was computed in Block 1.

The basic Blocks of the full-effects model are illustrated in Figure 5.2. As previously stated the full-effects model will be based on a lumped parameter analysis. Some of the geometry constraints, however, do use a three dimensional analysis to obtain a lumped parameter. Each Block of the model, as shown in Figure 5.2, is very dependent on the other Blocks. The development of the Blocks is presented separately with these interdependencies pointed out. The model is highly nonlinear with extensive multiplication and division of variables. Other operations, such as powers and roots also occur. The computer code SPICE [Ref. 2] with its feature of nonlinear transient analysis was selected to implement the model. SPICE is an electrical circuit analysis code. The development of the first two Blocks makes extensive use of electrical equivalent circuits in order to obtain the system equations. Thus for these two Blocks SPICE is the logical approach for a computer model. The third Block is the thermodynamic effects and equations from these are not normally thought of in terms of electrical equivalent circuits. Thus its describing equations were configured as a system diagram. The elements represent operations, such as multiply or square root.

The equivalent circuit implementation of the elements in the SPICE format is provided in Appendix A.

5.2 Development of the Generator and Electromagnet Model

Referring to Figure 5.2, Block 1 is the electrical part of the model. Its function is to model the generator electromagnetics. It computes the MHD generator voltages and currents. It is the center of the full-effects model. Blocks 2 and 3 support Block 1 by correcting the velocity, magnetic field and conductivity for plasma/MHD channel interactions.

The starting place for Block 1 is the electrical equivalent circuit of the simplified analysis as shown in Figure 4.1. This circuit is correct except that it does not account for the eddy current losses in the plasma. This effect is similar to core losses in a transformer (transformer core losses also include hysteresis, which does not occur in this plasma) and is accounted for in a transformer model with a resistor in parallel with the primary winding. The core, in this case, is the conductive plasma within the MHD channel and the primary winding is the electromagnet inductor L_B . Thus Figure 4.1 was corrected to include this effect as shown in Figure 5.3.

Figure 5.3: MHD generator equivalent circuit.

Table 5.2 defines the variables used in the following relations.

Table 5.2: Generator and Electromagnet Model Variables

$B_1(t)$	$B_1(t)$ is the magnetic field (T) within the plasma filled MHD channel that was the result of the permanent magnets field interacting with the conductive plasma flow.
$B_2(t)$	$B_2(t)$ is the magnetic field (T) within the plasma filled MHD channel that was the result of the electromagnets field interacting with the conductive plasma flow.
$U_{eff}(t)$	$U_{eff}(t)$ is the average plasma velocity (ms^{-1}) within the MHD channel corrected for thermodynamic effects.
V_0	V_0 is the MHD potential (V) due to the magnetic field from the permanent magnets.
V_{bu}	V_{bu} is the MHD potential (V) due to the magnetic field from the electromagnet.
$\sigma(t)$	$\sigma(t)$ is the average plasma conductivity (Sm^{-1}) within the MHD channel corrected for thermodynamic effects.
L_B	L_B is the Inductance (H) of the electromagnet.
R_p	R_p is the resistance (Ω) of the plasma between the electrodes
R_{cu}	R_{cu} is the Copper winding and lead resistance (Ω) of the electromagnet coil
R_e	R_e is the eddy current loss resistance (Ω) caused by the time varying flux from the electromagnet
ℓ	ℓ is the distance (m) between the MHD electrodes.

Equations 5.1 and 5.2 can be obtained from Figure 5.3.

$$V_0 + V_{bu} = i_{mhd} (R_p + R_{cu} + R_s) - i_L R_s \quad 5.1$$

$$0 = -i_{mhd} R_s + i_L R_s + L_B \frac{di_L}{dt} \quad 5.2$$

The parameters in Figure 5.3 and the equations are the same as Figure 4.1, except for R_s which is the eddy current loss resistance. The parameters in Figure 5.2, however, are more complex than was assumed for the simplified analysis. These parameters are described as follows:

V_0 , the non-regenerative MHD potential is given by Equation 5.3.

$$V_0 = B_1(t) U_{eff} \ell \quad 5.3$$

V_{bu} , the regenerative MHD potential, is given by Equation 5.4.

$$V_{bu} = B_2(t) U_{eff} \ell \quad 5.4$$

R_{cu} and L_B are unchanged from the simple model of Figure 4.1.

R_p and R_s are respectively the plasma resistance (Ω) between the electrodes and the eddy current loss in the plasma contained in the MHD channel. They are given by Equations 5.5 and 5.6.

$$R_p = (\sigma(t) G_p)^{-1} \quad 5.5$$

$$R_s = (\sigma(t) G_s)^{-1} \quad 5.6$$

Where G_p and G_s are geometric constants. The expression for G_p is given in Equation 5.7.

$$G_p = \frac{A'}{\ell} \quad 5.7$$

Where A' was the electrode surface area (m^2).

The expression for G_e is given as Eqn. B21 in Appendix B, Derivation of Eddy Current Resistance. The derivation and the expression are very involved and were left in Appendix B. The conductivity was computed in the Thermodynamic Model, Block 3. The SPICE format of the Generator and Electromagnet Model (Block 1) equivalent circuit are in Appendix A. It also includes auxiliary circuits that multiply different circuit voltages and currents to obtain MHD developed power and ohmic dissipation in R_e and R_p . These additional computations are used by the Thermodynamic Model, Block 3. The remaining Blocks, the Magnetic Flux Model, Block 2, and the Thermodynamic Model, Block 3, are quite complicated, but only support Block 1, the Generator and Electromagnet Model and are derived in the following sections.

5.3 Development of the Magnetic Flux Model

The function of the Magnetic Flux Model, (Block 2 Fig. 5.2), was to describe the magnetic field penetration into the plasma and provide corrected magnetic field variables, $B_1(t)$ & $B_2(t)$, that were used in the Generator and Electromagnet Model (Block 1 Fig. 5.2), and the Thermodynamic Model (Block 3 Fig. 5.2). The situation analyzed was that of a time varying magnetic field in a conductive medium. Table 5.3 defines the variables that were used in the following relations.

Table 5.3 Magnetic Flux Model Variables

B	The magnetic field penetrating the plasma (T).
H	The magnetic field intensity within the plasma (Am^{-1}).
E	The electric field within the plasma caused by the time varying magnetic field (Vm^{-1}).
D	The electric flux density within the plasma (Cm^{-2})
J	The current density within the plasma caused by the electric field (Am^{-2})
σ	The conductivity of the plasma (Sm^{-1})
μ_0	The magnetic permeability of the plasma (assumed same as free space $4\pi \times 10^{-7} Hm^{-1}$ [7])

The analysis begins with one of Maxwell's equations to describe how the changing magnetic field induces an electric field within the plasma:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad 5.8$$

Equation 5.9 gives Ohm's Law at a point in a conductive medium.

$$\vec{E} = \frac{\vec{J}}{\sigma} \quad 5.9$$

Combining Eqns. 5.8 and 5.9:

$$\nabla \times \frac{\vec{J}}{\sigma} = -\frac{\partial \vec{B}}{\partial t} \quad 5.10$$

Also from Maxwell's Equations [8], Eqn. 5.11 describes the relationship between the magnetic field intensity and the current within the plasma.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad 5.11$$

Observe that the displacement current term $(\frac{\partial \vec{D}}{\partial t})$ was set to zero. This was due to the very large value of plasma conductance causing a very small effective value of resistance to be in parallel with a capacitance between any two points in the plasma. Thus the current through that capacitance, accounted for by the displacement current term $(\frac{\partial \vec{D}}{\partial t})$, was considered negligible compared with the current flow due to plasma conductivity.

Combining Eqns. 5.10, 5.11:

$$\frac{\nabla \times \nabla \times \vec{H}}{\sigma} = -\frac{\partial \vec{B}}{\partial t} \quad 5.12$$

Assuming constant μ_0 within the plasma [7]:

$$\vec{B} = \mu_0 \vec{H} \quad 5.13$$

Combining Eqns. 5.12 and 5.13

$$\nabla \times \nabla \times \vec{B} = -\mu_0 \sigma \frac{\partial \vec{B}}{\partial t} \quad 5.14$$

Using the identity [8]:

$$\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} \quad 5.15$$

Also from Maxwell [8]:

$$\nabla \cdot \vec{B} = 0 \quad 5.16$$

Combining Eqns. 5.15, 5.16:

$$\nabla \times \nabla \times \vec{B} = -\nabla^2 \vec{B} \quad 5.17$$

And finally combining Eqns. 5.14, 5.17:

$$\nabla^2 \vec{B} = \mu_0 \sigma \frac{\partial \vec{B}}{\partial t} \quad 5.18$$

Equation 5.18 indicates that the magnetic flux penetration into the plasma obeys a relationship that is of the form of a three dimensional diffusion equation. Equation 5.18 was simplified to the diffusion equation in one dimension from an examination of the geometry of the electromagnet of Figure 5.4.

Figure 5.4: Cross section of the MHD channel

Referring to Figure 5.4, a cross section through the MHD channel, the magnetic flux between the magnets was assumed to be only in the direction of the centerline across the channel, thus a one dimensional analysis along the center line of the magnets through the channel as described in Equation 5.19 was considered valid.

$$\frac{\partial^2 B}{\partial t^2} = \mu_0 \sigma \frac{\partial B}{\partial t} \quad 5.19$$

An equation of this same form was obtained from a distributed RC network [9,30]. Figure 5.5 illustrates the distributed RC network as a ladder network.

Figure 5.5: A ladder network of R's and C's

In a given cell the following node equation was written:

$$\frac{E(x) - E(x + \Delta x)}{R\Delta x} = C\Delta x \frac{d}{dt} E(x + \Delta x) + \frac{E(x + \Delta x) - E(x + 2\Delta x)}{R\Delta x}$$

$$\frac{E(x) + E(x + 2\Delta x) - 2E(x + \Delta x)}{R\Delta x} = C\Delta x \frac{d}{dt} E(x + \Delta x)$$

$$\frac{E(x) + E(x + 2\Delta x) - 2E(x + \Delta x)}{\Delta x^2} = CR \frac{d}{dt} E(x + \Delta x) \quad 5.20$$

In Eqn. 5.20 taking the limit as $\Delta x \rightarrow 0$ [28] and letting $\frac{d}{dt} \rightarrow \frac{\partial}{\partial t}$:

$$\frac{\partial^2 E}{\partial x^2} = RC \frac{\partial E}{\partial t} \quad 5.21$$

In comparing Eqns. 5.21 and 5.19, E is the analog for B , R is the analog for σ , and C is the analog for μ_0 . Thus the electrical circuit of Figure 5.5 was used to model magnetic flux diffusion into a conductive medium in one dimension. This electrical circuit model was used in the analysis with ladder network of N cells (Fig. 5.6) to model Eqn. 5.19.

$$R_i = \sigma \Delta x$$

$$C_i = \mu \Delta x$$

$$\Delta x = x/N$$

Figure 5.6: Circuit for magnetic field penetration in plasma.

If x is the distance to the center of the channel, through which the magnetic flux penetration is desired, then the incremental distance step is:

$$\Delta x = \frac{x}{N} \quad 5.22$$

The resistance and capacitance of the ladder network are:

$$R_{cell} = \sigma \Delta x \quad 5.23$$

$$C_{cell} = \mu_0 \Delta x \quad 5.24$$

Where: $\sigma(t)$ is the plasma conductivity ($S m^{-1}$) within the MHD channel

μ_0 is the magnetic permeability [7] ($4\pi \times 10^{-7} H m^{-1}$).

The excitation for the circuit was chosen to be a voltage source that is the analog to the regenerative magnetic field penetrating the plasma, Eqn. 4.3. That source gives the magnitude of the regenerative flux that would appear at the center of the channel (Assumption 4 Sec. 5.1). In addition that value of flux would be reasonably constant along the center line of the electrodes since the distance between the electrodes was inside the diameter of the electromagnet (Fig. 5.4). The response of the model is the voltage at the capacitance of each of the cells which is an analog of the magnetic field at that number of distance steps, Δx into the plasma. The voltage at the last capacitor in the ladder network is the magnetic field over the penetration distance of $\ell/2$ at mid-

channel into the plasma. For the regenerative magnetic field this is $B_2(t)$ and it determines the regenerative MHD voltage source, V_{bu} (Eqn. 5.4). Appendix A gives the development of the SPICE circuit with the details of how an adequate number of cells, N , that were used in the ladder were determined. Appendix A also gives the computations for the model parameters, K and ℓ (The constant K is derived in Appendix C, Eqn. C4), from the design of the INEL MHD device tested.

Thus far the development has presented how the plasma and regenerative magnetic flux interactions were incorporated into the Magnetic Flux Model. The complete model must also include the interaction between the permanent magnet's field and the plasma. Most likely the onset of highly conductive plasma flow will significantly alter the initial magnetic field in a complex 3 dimensional disturbance until a new steady state magnetic field is established. This effect was accounted for with the one dimensional analysis approach. An examination of the recorded electrode MHD voltage response of an INEL regenerative test (Fig. 5.7) indicated a rise time of about 5usec.

Figure 5.7: INEL regeneration test showing the voltage rise time.

This voltage rise time can only be caused by:

- o Instrumentation response.
- o Plasma velocity response.
- o Initial magnetic flux disturbance.

The INEL instrumentation had a rise time of 1usec (it was measured). The plasma velocity was due to a shock front, which has a very fast rise time ($< < 5\text{usec}$). A fast shock front rise time is due to the characteristic of most gasses where the velocity increases with pressure. Thus a shock front that starts out with distinct rise time will quickly develop a rise time that has the characteristic of a discontinuity [3]. The argon plasma was assumed to behave in this fashion and a 60 kms^{-1} shock front passing over the electrodes (.56" Dia.) would take only 0.25 uSec. Therefore the rise time was assumed due to a disturbance of the permanent magnets field. Consider Figure 5.8, the response of the SPICE Magnetic Flux Diffusion Model to a unit step input of amplitude 1.0T and a typical plasma conductivity of $20 \times 10^3\text{ Sm}^{-1}$ (Chap. 6).

Figure 5.8: SPICE Flux diffusion model step response.

The step response of the SPICE Flux Diffusion Model can account for the rise time observed in the INEL data. This approach was used to model the initial flux penetration with the one dimension SPICE model for the diffusion equation. The assumption was that if the step response of the Magnetic Flux Model could account for

the rise time observations in the MHD voltage then it was acceptable. It may have been, that with the initial transient, where by the highly conductive plasma swept the magnetic field from the permanent magnets out of the channel, then the magnetic field was reestablished with the rise time that was observed on the MHD voltage. The one dimensional model will not describe any such three dimensional effects, but it does account for the rise time that was actually measured in the MHD electrode voltage response. Thus a second RC ladder network was implemented to model the one dimension flux diffusion equation (5.19) with a step input of magnetic field with a magnitude equal to what INEL measured pretest in the MHD channel from the permanent magnets. The two ladder networks completed the Magnetic Flux Model. They were implemented to accommodate dynamic feedback of plasma conductivity corrected due to thermodynamic effects computed in the Thermodynamic Model, Block 3. The complete SPICE implementation details are given in Appendix A.

5.4 Development of the Thermodynamic Model

The function of Block 3 (Fig. 5.2) is to model the thermodynamic effects of the plasma interactions with the MHD channel. Specifically Block 3 computed corrected plasma velocity and conductivity. These variables are inputs to the Generator and Electromagnet Model. The corrected conductivity also feeds into the Magnetic Flux Model. The equations that govern this effect were derived from the following constraints:

- o Conservation of mass
- o Conservation of momentum
- o Conservation of energy

In addition, an equation of state for argon plasma was employed. Figure 5.1, repeated here, and Figure 5.9 illustrate the simplified geometry that was used in the analysis.

Figure 5.1: MHD channel schematic

Figure 5.9: Schematic of the MHD Channel and Generator Circuit.

The variables used in the analysis are defined in Table 5.4.

Table 5.4: Thermodynamic Variables and Parameters (units MKS).

E :	Plasma internal energy (Jkg^{-1}) E_0 input, E_1 output
h :	Plasma enthalpy (Jkg^{-1}) h_0 input, h_1 output
U :	Plasma velocity (ms^{-1}) U_0 input, U_1 output
P :	Plasma pressure (Nm^{-2}) P_0 input, P_1 output
ρ :	Plasma density (kgm^{-3}) ρ_0 input, ρ_1 output
σ :	Plasma conductivity (Sm^{-1}) σ_0 input, σ_1 output
R_p :	Plasma resistance between the electrodes (Ω)
R_e :	Plasma eddy current resistance from the electromagnet (Ω)
L_B :	Electromagnet inductance (H)
i_{mhd} :	MHD generator current (A)
i_e :	Eddy current (A)
i_L :	Electromagnet current (A)
$\dot{W}_{mhd}$:	MHD generator power (W)
$\dot{W}_e$:	Eddy current ohmic dissipation (W)
$\dot{W}_p$:	Plasma power dissipation (W)
A :	MHD channel cross sectional area (m^2)
A' :	Electrode cross sectional area (m^2)
B_{tot} :	Total of initial and regenerative magnetic field (T)
F :	Plasma mass flow in or out of the MHD channel ($kg s^{-1}$)
M' :	Total plasma mass in the MHD channel (kg)

As shown in Figure 5.1, it was assumed (Assumption 5) that the mass flow into the MHD channel equaled the mass flow out of the channel. To facilitate the analysis the channel geometry is shown as rectangular. Thus, to remain consistent, the area A' was set equal to the actual device electrode area and, the area A was set equal to the actual device cross sectional area. Figure 5.9 also is a representation of the channel. However, in this Figure the generator elements, R_p , R_e , and L_B are shown schematically connected to the MHD channel. The resistors R_p and R_e are actually distributed within the MHD channel. All the variables used in the thermodynamic relations are as defined in Table 5.4. The first thermodynamic constraint to be considered is conservation of mass. It was assumed that the mass flow into and out of

the MHD channel is constant and the mass of plasma in the channel was also constant (Assumption 5). This assumption simplified the model and is credible during the regeneration period. However, the consequences are that the start up transient of the explosive shock front passing through the MHD channel were not be modeled correctly. This event occurs so rapidly ($< 1\text{usec}$) compared to the boot-up transient that it was neglected. In addition this assumption also caused the model not to be valid during shutdown. It is likely that the shutdown mechanism with the INEL design was extremely complex. The mass that started to flow during shutdown was explosive products and shreds of the generator and is beyond the capability of a simple 1 dimension model. Thus the model will be valid for the regeneration period from about $t = 0.5\text{usec}$ until the start of plasma shutdown. Equation 5.25 states this assumption.

$$\rho_0 U_0 A = \rho_1 U_1 A = F \quad 5.25$$

The second thermodynamic constraint is the conservation of momentum. This principle states that the rate of change of momentum of a body is equal to the sum of the forces acting on the body. A convenience for stating the momentum equation is the following definition for relative velocity between MHD channel output and input:

$$U_1 = U_0 + \delta U \quad 5.26$$

Equating the momentum relation and rearranging terms:

$$\underbrace{\sum \frac{d}{dt}(MU)}_{\text{momentum change}} = \underbrace{A(P_0 - P_1)}_{\text{plasma pressure}} - \underbrace{J \times B_{tot} \Delta v}_{\text{electrical generator force}}, \quad \Delta v = \text{Channel volume}$$

$$J = \frac{i_{mhd}}{A'}, \quad \Delta v = A' \ell, \quad A' = \text{Electrode surface area}, \ell = \text{Distance between electrodes}$$

$$J \times B_{tot} \Delta v = \frac{i_{mhd} A'}{A'} \ell B_{tot}$$

Eqn. 5.25 and expansion of the momentum term:

$$\sum \frac{d}{dt}(MU) = \sum (\dot{M}U + M\dot{U}) = \overbrace{FU_0 - FU_1}^{\text{flow term}} + \overbrace{M'\dot{U}_0 - M'\dot{U}_1}^{\text{f=ma term}}$$

Plugging in Eqn. 5.26:

$$\sum \frac{d}{dt}(MU) = FU_0 - FU_0 - F\delta U + M'\dot{U}_0 - M'\dot{U}_0 - M'\delta\dot{U}$$

$$F\delta U + M'\delta\dot{U} = A(P_1 - P_0) + i_{mhd} B_{tot} \ell \quad 5.27$$

Equation 5.27 is the resultant momentum equation. An analysis similar to this was performed in Reference [4] Chapter 7. A constant velocity assumption was made (expansion of the plasma in the channel was assumed to provide an acceleration that prevents the velocity from decreasing), which causes the left side of Equation 34 to vanish (This occurs because in Equation 5.26 the plasma relative velocity δU is zero). That assumption was not used but, the model predictions were examined to see if it could have been used (Chap. 7, Section 7.3).

The next constraint is the conservation of energy. A similar assumption was made with the energy flow as was made with the mass flow. The power was in set equal to power out and the MHD channel power loss to the walls was neglected (Assumption 6). This assumption is not valid during start-up and shutdown for the exact same reasons as in the case of mass flow. However, during the regeneration period of sustained plasma flow it was considered valid. Although, the consequences of neglecting boundary layer effects probably caused the plasma conductivity to be slightly overestimated. The tendency for the predicted conductivity to possibly be higher than the actual value was compensated for by using the MHD channel outlet conductivity, rather than averaging the inlet and outlet as was done with velocity. Implementing the assumption sets power into the MHD channel equal to power out of the channel. From [Ref. 5] the power carried by a flowing plasma is given by:

$$\text{Plasma power} = F \left(\frac{U^2}{2} + h \right) \quad 5.28$$

F = Mass flow, U = Plasma velocity, h = Plasma enthalpy

Using the relationship of Eqn. 5.28, the energy balance for the plasma flow in the channel can be established as follows:

Energy into the channel over time Δt :

$$\underbrace{\frac{1}{2} \rho_0 U_0 A U_0^2 \Delta t}_{\text{kinetic}} + \underbrace{\rho_0 U_0 A h_0 \Delta t}_{\text{enthalpy}} + \underbrace{(i_{mhd}^2 R_p + i_e^2 R_e) \Delta t}_{\text{ohmic dissipation}}$$

Energy out of the channel over time Δt :

$$\underbrace{\frac{1}{2} \rho_1 U_1 A U_1^2 \Delta t}_{\text{kinetic}} + \underbrace{\rho_1 U_1 A h_1 \Delta t}_{\text{enthalpy}} + \underbrace{V_{mhd} i_{mhd} \Delta t}_{\text{MHD energy generated}}$$

Using equation 5.25 and simplifying:

$$\frac{1}{2} F U_0^2 \Delta t + F h_0 \Delta t + (i_{mhd}^2 R_p + i_e^2 R_e) \Delta t = \frac{1}{2} F U_1^2 \Delta t + F h_1 \Delta t + V_{mhd} i_{mhd} \Delta t$$

Thus the energy balance equation is given by:

$$\frac{1}{2} F (U_0^2 - U_1^2) + F (h_0 - h_1) + (i_{mhd}^2 R_p + i_e^2 R_e) = V_{mhd} i_{mhd} \quad 5.29$$

In order to solve for the thermodynamic effects additional relationships are needed. Enthalpy is defined as:

$$h = E + \frac{P}{\rho} \quad 5.30$$

The final relationship needed was an equation of state. The Gamma Gas Law (GGL) [Ref. 3] was chosen. This approach greatly simplifies the model. It allowed a complete closed set of system equations which were compatible with the strategy used in modeling this system. An alternative would have been to use the LLNL equation of state [Ref. 24]. It would have ended up in the form of a computerized look up table and a closed set of system equations would not be possible. The use of the GGL to obtain a closed set of system equations with gas conditions similar to this research has been demonstrated [4], [31]. However, the GGL for these applications can introduce

significant errors [24]. To minimize the potential for errors a Taylor series expansion of the GGL was used to project from known MHD channel inlet plasma conditions to the relatively close channel outlet plasma conditions [28]. The GGL is:

$$E = \frac{1}{\Gamma - 1} \frac{P}{\rho}, \quad P = \rho E(\Gamma - 1), \quad \Gamma = 1.6667 \text{ for a monatomic gas.} \quad 5.31$$

The Taylor series expansion is truncated beyond the first order terms and applied to the GGL to solve for outlet pressure:

$$P_1 = \left. \frac{\partial P}{\partial E} \right|_0 (E_1 - E_0) + \left. \frac{\partial P}{\partial \rho} \right|_0 (\rho_1 - \rho_0) + P_0$$

$$\text{Eqn. 5.31} \Rightarrow \left. \frac{\partial P}{\partial E} \right|_0 = \rho_0(\Gamma - 1), \quad \left. \frac{\partial P}{\partial \rho} \right|_0 = E_0(\Gamma - 1) \quad 5.32$$

$$P_1 = P_0 + \rho_0(\Gamma - 1)(E_1 - E_0) + E_0(\Gamma - 1)(\rho_0 - \rho_1) \quad 5.33$$

$$\text{Eqns. 5.30 \& 5.33} \Rightarrow P_1 = P_0 + \rho_0(\Gamma - 1)\left(-\frac{P_1}{\rho_1} + h_1 - E_0\right) + E_0(\Gamma - 1)(\rho_0 - \rho_1)$$

$$P_1 = \frac{\rho_1}{\rho_0} \left[\frac{P_0 + \rho_0 h_1(\Gamma - 1) - 2\rho_0 E_0(\Gamma - 1) + \rho_1 E_0(\Gamma - 1)}{\frac{\rho_1}{\rho_0} + (\Gamma - 1)} \right] \quad 5.34$$

Equation 5.34 is the GGL projection from MHD channel inlet to outlet conditions applied to a solution for MHD channel outlet pressure. The complete set of thermodynamic equations are summarized in Table 5.5.

Table 5.5: Thermodynamic equation summary

Equation	Comments
5.25. $\rho_0 U_0 A = \rho_1 U_1 A = F$	conservation of mass
5.26 $U_1 = U_0 + \delta U$	definition
5.27 $F \delta U + M' \delta \dot{U} = A(P_1 - P_0) + i_{mhd} B_{tot} \ell$	conservation of momentum
5.29 $\frac{1}{2} F(U_0^2 - U_1^2) + F(h_0 - h_1) + (i_{mhd}^2 R_p + i_e^2 R_p) = V_{mhd} i_{mhd}$	conservation of energy
5.30 $h = E + \frac{P}{\rho}$	definition
5.34 $P_1 = \frac{\rho_1}{\rho_0} \left[\frac{P_0 + \rho_0 h_1 (\Gamma - 1) - 2 \rho_0 E_0 (\Gamma - 1) + \rho_1 E_0 (\Gamma - 1)}{\frac{\rho_1}{\rho_0} + (\Gamma - 1)} \right]$	solution for pressure using the gamma gas law projection

The objective of the thermodynamic analysis is to obtain corrections for plasma velocity and conductivity caused by thermodynamic plasma interactions in the MHD channel. Plasma velocity and conductivity feed back into the Generator and Electromagnet Model. Plasma conductivity also feeds back into the Magnetic Flux Model.

The unknowns in the equations of Table 5.5 are:

$$\rho_1, U_1, \delta U, E_1, h_1, P_1$$

The following variables are fed into this Block from other Blocks:

$$\begin{aligned} V_{mhd} \quad i_{mhd} \quad i_e &: \text{from the Generator and Electromagnet} \\ B_{tot} &: \text{the sum of } B_1(t) \text{ and } B_2(t) : \text{from the Magnetic Flux} \end{aligned}$$

The Equations in Table 5.5 were solved in the form that can be configured as a system diagram so as to be compatible with the implementation in the computer code SPICE. The system diagrams and the SPICE implementation are given in Appendix A. The velocity δU , was obtained from Eqn. 5.27 as follows:

$$\delta \dot{U} = \frac{1}{M'} [A(P_1 - P_0) + i_{mhd} B_{tot} \ell - F \delta U]$$

Resulting in:

$$\delta U = \int \frac{1}{M'} [A(P_1 - P_0) + i_{mhd} B_{tot} \ell - F \delta U] dt \quad 5.35$$

Where: $B_{tot} = B_1(t) + B_2(t)$, $F = \rho_0 U_0 A$, and the mass in the MHD channel, M' , was assumed: $M' = \rho_0 \Delta v$, Δv is the volume of the channel: $\Delta v = A' \ell$

The relative MHD channel plasma velocity given by Equation 5.35 was input into Equation 5.26 to obtain the MHD channel output velocity, U_1 . The plasma velocity that the MHD channel voltage (Eqn. 5.3, V_0 ; Eqn. 5.4, V_{bu}) responds to was assumed to be the average of U_0 and U_1 :

$$U_{eff} = U_0 + \frac{\delta U}{2}, \quad U_1 = U_0 + \delta U \quad 5.36$$

The MHD channel plasma output density, ρ_1 was obtained from Equation 5.25:

$$\rho_1 = \rho_0 \frac{U_0}{U_1} \quad 5.37$$

The MHD channel output enthalpy, h_1 , was obtained by combining Equations 5.26 and 5.29. Solving 5.29 for h_1 :

$$h_1 = \frac{\frac{1}{2} F (U_1^2 - U_0^2) + i_{mhd}^2 R_p + i_s^2 R_s - V_{mhd} i_{mhd} + F h_0}{F}$$

Then plugging in 5.26:

$$h_1 = h_0 - (U_0 \delta U + \frac{1}{2} \delta U^2) - \frac{V_{mhd} i_{mhd} - i_{mhd}^2 R_p - i_s^2 R_s}{F} \quad 5.38$$

The channel output plasma internal energy, E_1 , was obtained from Equation 5.30:

$$E_1 = h_1 - \frac{P_1}{\rho_1} \quad 5.39$$

To complete the Thermodynamic Model the MHD channel output plasma conductivity, σ_1 was required. Equation 5.40 [Ref 6] gives an expression for conductivity of a highly ionized plasma.

$$\sigma = \underbrace{\frac{3m_e}{16\sqrt{\pi}Z_eLn\Lambda}}_{\Phi} \left(\frac{3k}{m_e}\right)^{\frac{3}{2}} (T)^{\frac{3}{2}} \quad , \quad T = {}^\circ K \quad 5.40$$

In order to obtain a correction for conductivity the following assumptions were made:

Assume that Φ is constant from channel inlet to outlet.

Assume that in projecting from σ_0 to σ_1 that E is proportional to T in ${}^\circ K$.

Thus from 5.40 Equation 5.41 can be obtained.

$$\sigma_1 = \sigma_0 \left(\frac{E_1}{E_0}\right)^{\frac{3}{2}} \quad 5.41$$

These Equations were compatible with system diagram solutions and are summarized in Table 5.6. Appendix A will configure the system diagrams and implement them in the computer code SPICE.

Table 5.6: System Diagram Format of Thermodynamic Equations.

Eqn. No.	Equation
5.34	$P_1 = \frac{\rho_1}{\rho_0} \left[\frac{P_0 + \rho_0 h_1 (\Gamma - 1) - 2 \rho_0 E_0 (\Gamma - 1) + \rho_1 E_0 (\Gamma - 1)}{\frac{\rho_1}{\rho_0} + (\Gamma - 1)} \right]$
5.35	$\delta U = \int \frac{1}{M'} \left[A(P_1 - P_0) + i_{mhd} B_{tot} \ell - F \delta U \right] dt$
5.36	$U_{eff} = U_0 + \frac{\delta U}{2}, \quad U_1 = U_0 + \delta U$
5.37	$\rho_1 = \rho_0 \frac{U_0}{U_1}$
5.38	$h_1 = h_0 - (U_0 \delta U + \frac{1}{2} \delta U^2) - \frac{V_{mhd} i_{mhd} - i_{mhd}^2 R_p - i_s^2 R_s}{F}$
5.39	$E_1 = h_1 - \frac{P_1}{\rho_1}$
5.41	$\sigma_1 = \sigma_0 \left(\frac{E_1}{E_0} \right)^{\frac{3}{2}}$

In summary this chapter has described the full effects model as three basic blocks:

1. The Generator Electromagnet Model
2. The Magnetic Flux Model
3. The Thermodynamic Model

The system equations for each Block were described in lumped parameter form. The Generator Electromagnet Model and the Magnetic Flux Model were described as electrical equivalent circuits. The Thermodynamic Model was described as a system of lumped parameter equations. The complete details of realizing the various Blocks as SPICE format equivalent circuits was given in Appendix A. Additionally, Appendix A describes the system diagrams of the Thermodynamic Model equations and the details of the SPICE format equivalent circuits for the system diagrams.

In the next chapter the model is applied to match the INEL experimental data.

6.0 FULL-EFFECTS MODEL FIT TO EXPERIMENTAL DATA

6.1 Test Article Descriptions

The full-effects model was used to simulate the two INEL regenerative MHD tests by predicting the MHD electrode voltage response. Figure 6.1 illustrates the MHD generator system configuration for both tests. Both tests used the same plasma generator and MHD channel design. Figures 6.2 and 6.3 illustrate these parts. Note that the large diameter pipe threads are the MHD electrodes and the small diameter pipe threads are electrodes for a plasma impedance experiment. The principal difference between the two systems was in the regenerative magnet design. Figures 6.4 and 6.5 illustrate the two electromagnet designs for, respectively, the first and second regenerative tests. The drawings are to scale, but not dimensioned. Note that the channel diameter was 1.25", thus dimensional data is present. Also note that the wiring shown is approximate and the exact wiring path may have been a little longer or shorter than shown.

Figure 6.1: MHD generator assembly used in INEL tests.
(for clarity electromagnets not shown)

Figure 6.2: MHD plasma generator used in INEL tests.

Figure 6.3: MHD channel used in INEL tests.

Figure 6.4: Electromagnet used in INEL regeneration test 1.

Figure 6.5: Electromagnet used in INEL regeneration test 2.

6.2 Experimental Data

Figure 6.6 illustrates wave-form characteristics of the observed INEL boot-up wave-forms. Table 6.1 lists the wave-form characteristic data from each of the INEL regenerative tests.

Figure 6.6: Boot-up MHD wave-form characteristics.

Table 6.1. INEL boot-up MHD wave-form data.

Test	T_R	T_D	V_{mhd0}	V_{mhdu}
1	4.7usec	14.7usec	1472V	1952V
2	5usec	15usec	1104V	1520V

Figures 6.7 and 6.8 illustrate the MHD voltage response observed at the electrodes in INEL regenerative MHD tests 1 and 2.

Figure 6.7: INEL regenerative test 1 MHD electrode voltage.

Figure 6.8: INEL regenerative test 2 MHD electrode voltage.

6.3 Estimations of Model Parameters

Table 6.2 illustrates pertinent data from the INEL tests that was needed for the full-effects SPICE model.

Table 6.2: Data from INEL tests

Parameter	Test 1	Test 2
Distance between electrodes ℓ	1"	1"
Electrode diameter	0.56"	0.56"
Electromagnet diameter	3"	3"
MHD channel diameter	1.25"	1.25"
Electromagnet inductance, L_B	0.56uH	0.445uH
Initial magnetic field, B_0	0.97T	0.9T
Initial plasma velocity, U_0	$59.74kms^{-1}$	$48.29kms^{-1}$
Current to regeneration flux, H_1 (Appendix C)	$1.66uAT^{-1}$	$3.5uAT^{-1}$
Total copper resistance, R_{cu} (Appendix C)	$>0.49m\Omega$	$>0.49m\Omega$

Table 6.3 lists estimates of the thermodynamic plasma properties that were used in the full-effects model.

Table 6.3: Estimates of input plasma thermodynamic properties

Parameter	Value
Conductivity, σ_0	$20 \times 10^3 Sm^{-1}$
Internal energy, E_0	$25 \times 10^6 jkg^{-1}$
Enthalpy, h_0	$32.5 \times 10^6 jkg^{-1}$
Pressure, P_0	$150 \times 10^6 Nm^{-2}$

The INEL final report [1] didn't provide any of the thermodynamic data listed in Table 6.3. The report did give plasma temperature. However, the temperature reported ($T > 3ev$) was from a simulation. Furthermore, that simulation stopped while still in the "thermos bottle" of the plasma generator and did not continue on into the MHD

channel inlet. The INEL report did attempt to measure plasma resistance and reported a value of $.028\Omega$ for the first test and $.017\Omega$ for the second test. Using the full-effects model geometry factor of $.00625$ (Appendix C) for the plasma resistance between the electrodes, the estimated conductivity for the plasma in the MHD channel for both of these tests was backed out as $5714.3Sm^{-1}$ for Test 1 and $9411.8Sm^{-1}$ for Test 2. If the assumption that the initial MHD voltage rise time was caused from magnetic field diffusion into the plasma, then the observed voltage rise time was not compatible with the backed out estimates of conductivity. The observed rise time for both tests was about 5usec. Using a conductivity of $20 \times 10^3 Sm^{-1}$ in the magnetic diffusion model, with a step input of magnitude equal to the initial magnetic field, does provide a result that agrees with the observed rise time. Since the observed rise time for both tests was about the same, and the reported value for plasma resistance differed by almost a factor of 2 between both tests, and the backed out conductivity from those tests was not compatible with the rise time, the compatible conductivity of approximately $20 \times 10^3 Sm^{-1}$ was used as σ_0 in the full-effects model.

The remaining thermodynamic properties were obtained from [Ref. 20 and 17]. The internal energy and pressure were obtained from Figure 1 of [Ref. 20] which illustrates plots of argon plasma properties generated using the LLNL equation of state for argon plasma [Ref. 24]. The assumed temperature of about 2ev and the conductivity of about $20 \times 10^3 Sm^{-1}$ were used to estimate from the curves internal energy E_0 , and the pressure, P_0 . [Ref. 17] listed typical in channel plasma properties obtained using that same equation of state and the density of $19kgm^{-3}$ was listed with temperature, pressure, and internal energy all very close to what was selected from Figure 1 of [Ref. 20]. Thus a value of $20 kgm^{-3}$ was selected for inlet plasma density.

6.4 Model Fit to Experimental Data

The data from Table 6.1 was used as a reference to determine the model fit to the test results. Figures 6.9, 6.10, 6.11, and 6.12 illustrate the full-effects model simulations of these tests. The match to the test observations was obtained by adjustments to the model parameters. The initial values to the geometry factors listed in Table 6.4 were computed in Appendix C. The initial magnetic field, B_0 , and the initial plasma velocity, U_0 , were from INEL measurements [Ref. 1]. The attempt at a fit was made by keeping the various parameters in a neighborhood that was plausible and then comparing the simulation MHD voltage with the test observations of Table 6.1. The duration of the

plasma properties parameters was set to match the observed MHD voltage duration. Table 6.5 gives the final parameters and the match MHD voltage. Table 6.6 gives sensitivities of the model parameters that obtained the final fit to the first INEL regeneration test. Appendix C gives the complete listing of the SPICE input data files for both simulations.

Table 6.4. Initial value of model parameters being adjusted.

Boot-up test 1						
B_0	U_0	σ_0	H_1	GG_p	GG_{ed}	R_{cu}
.97	59K	20K	1.66u	6.25m	104n	>.5m
Boot-up test 2						
.9	48K	20K	3.5u	6.25m	451n	>.5m

Where:

H_1 is a current controlled voltage source that is slaved to the electromagnet current. H_1 drives the regenerative Magnetic Flux Model. H_1 gain is shown.

GG_p is a nonlinear voltage controlled current source that models the plasma resistance. The gain of GG_p is actually the geometry constant G_p which is shown.

GG_{ed} is a nonlinear voltage controlled current source that models the eddy current resistance, R_e . The gain of GG_{ed} is the geometry constant G_{ed} which is shown.

Table 6.5: Fit data (adjusted parameters and V_{mhd}).

Boot-up test 1								
B_0	U_0	σ_0	H1	GG_p	R_{cu}	V_{mhd0}	V_{mhd1}	ΔV
.99	68K	25K	1.7u	75m	1m	1458	1945	487
Boot-up test 2								
.95	50K	same	3.15u	75m	1m	1118	1531	413

Table 6.6: Parameter Sensitivities

Parameter	%Vmhd/%Pi	%Imhd/%Pi
P_0	0	1.8×10^{-3}
ρ_0	0	0
U_0	1.16	1.07
σ_0	-0.046	-0.184
E_0	0	0
H_1	0.139	0.066
B_0	1.01	1.01
R_{cu}	-5.14×10^{-3}	-1.48×10^{-2}
GG_e	0	0
GG_p	0.01	7.38×10^{-3}

Table 6.7 compares the initial and the final adjusted parameter simulations for the two INEL tests with the actual test values.

Table 6.7: Comparison of simulation and test MHD voltages

Initial/Final	Sim V_{mhd0}	V_{mhd0}	Sim V_{mhd1}	V_{mhd1}
Test 1.				
Initial	1282	1472	1298	1952
Final	1458	1472	1945	1952
Test 2.				
Initial	970	1104	1072	1520
Final	1118	1104	1531	1520

Figures 6.9 through 6.12 illustrate the simulated MHD voltage and current for both of the INEL tests using the non-adjusted parameters. Figures 6.13 through 6.16 illustrate the simulated MHD voltage and current for both of the INEL tests using the final adjusted parameters.

Figure 6.9: MHD voltage initial simulation of test 1.
(● indicates measured data point)

Figure 6.10: MHD voltage initial simulation of test 2.
(● indicates measured data point)

Figure 6.11: MHD current initial simulation of test 1.

Figure 6.12: MHD current initial simulation of test 2.

Figure 6.13: MHD voltage final simulation of test 1.
(● indicates measured data point)

Figure 6.14: MHD voltage final simulation of test 2.
(● indicates measured data point)

Figure 6.15: MHD current final simulation of test 1

Figure 6.16: MHD current final simulation of test 2

Another approach to validate the model was performed by which the correction factors that were required to modify the initial parameters for a good fit to the INEL Test 1 data were applied to modify the initial parameters for the Test 2 data. A simulation was performed with these "open loop" corrected parameters and the results compared with the simulation with unmodified parameters. Table 6.8 applies the correction factors to the initial parameters for the Test 2 data. It is noted that for conciseness Table 6.8 omits the units and the power of ten multiplier for the parameters.

Table 6.8: Test 2 parameters adjusted with Test 1 correction factors.

$$\begin{aligned}
 U_0 &: \frac{68}{59} \times 48 = 55.32 & \sigma_0 &: \frac{25}{20} \times 20 = 25 \\
 H_1 &: \frac{1.7}{1.66} \times 3.15 = 3.23 & GG_p &: \frac{75}{6.25} \times 6.25 = 75 & R_{cu} &: \frac{1}{.5} \times .5 = 1
 \end{aligned}$$

Table 6.9 gives the "open loop parameters" used and the simulation results. Table 6.10 compares the simulation results with the Test 2 data.

Table 6.9: Boot-up Test 2 "open loop" parameters and simulation results.

B_0	U_0	σ_0	H1	GG_p	R_{cu}	V_{mhd0}	V_{mhd1}	ΔV
.92	55.3K	25K	3.23u	75m	1m	1112	1670	558

Table 6.10: Boot-up Test 2 and "open loop" simulation compared.

Initial/Final	Sim V_{mhd0}	V_{mhd0}	%Er	Sim V_{mhd1}	V_{mhd1}	%Er
Initial	970	1104	12	1072	1520	29
Final	1112	1104	-0.72	1670	1520	-10

Figure 6.17 and 6.18 illustrates the MHD voltage and current "open loop" simulations.

Figure 6.17: MHD voltage "open loop" simulation of Test 2.
(● indicates measured data point)

Figure 6.18: MHD current "open loop" simulation of Test 2

6.5 Evaluation of Model Performance

Comparing the tabulated results of Table 6.5 with the test data in Table 6.1 a close fit was achieved. It is likely that with further adjustment of the parameters a closer fit could be achieved. The major issue is whether the values the parameters were adjusted to were plausible. For the first test fit, the parameters that appear implausible are GG_p , the geometry factor of the plasma resistance, and U_0 the input plasma velocity. The expression for GG_p (as developed in Appendix C) is given by:

$$R_p = \frac{\ell}{\sigma A'}, \quad GG_p = \frac{A'}{\ell} \quad 6.1$$

The distance between the electrodes, ℓ can not plausibly decrease. Thus in order to have GG_p increase by a factor of 10 the effective area, A' , must increase by a factor of 10. Consider the geometry of Figure 27 with a rectangular area of 3/4 inch height by 3.33 inch depth (1.67 inch each way on the electrode surface). That would provide a factor of 10 increase in area. The model of a cylindrical shaft of plasma connecting the electrodes would represent a conservative worst case maximum value for R_p and a smaller value is likely.

It is possible that the plasma velocity obtained from the INEL tests was in error by at least 15%. They assumed that the initial jump was all due to the non-regenerative MHD voltage thus allowing them to back out velocity. Their recorded response does not have a distinct transition and the signal was noisy. Thus an uncertainty of 15% is quite plausible. It is possible that the conductivity, σ_0 , could have been 25K instead of the estimated 20K since the 20K was only an estimate.

Additionally it was observed that when the parameter correction factors from the Test 1 fit were applied to the Test 2 initial parameters that a significant reduction in the deviation between the test data and simulation occurred: 12% to -0.7% at the 5usec time and 29% to -10% at the 20usec time.

Thus the adjusted parameters are all plausible and the model fit appears to be reasonable.

In summary, this chapter obtained model parameters from INEL design information, INEL experimental data, or other sources [16, 20]. The model was exercised with an actual set of parameters for each of the INEL regenerative tests. The simulation results indicated that the model was more pessimistic of regeneration than the test data indicated. The model parameters were adjusted within a range that was credible and the resultant match to the data was much improved. At the parameter settings of the improved fit parameter sensitivities were computed and tabulated.

In the next chapter some liberties were taken with model parameters and plasma properties in order to simulate high power response.

7.0 FULL-EFFECTS MODEL HIGH POWER SIMULATION

The full-effects model was used to model enhanced performance regeneration. Enhanced performance is considered to be electromagnet current at the mega-amp level.

7.1 Estimation of Model Parameters

Table 7.1 lists the values of the initial input parameters that were used to attempt significant boot-up. These are the same parameters that were used to fit the INEL test data. In addition to these parameters the plasma flow duration was expanded out to 75usec. All of the remaining model parameters and plasma input properties not listed are unchanged.

Table 7.1. Significant boot-up initial model parameters

B_0	U_0	σ_0	H_1	GG_p	GG_{ed}	R_{cu}	L_B
1	60K	25K	3.3u	75m	1.6u	1m	.5u

Where: H_1 is a constant, given in Appendix A, that is directly proportional to the electromagnet current to regeneration flux constant K of Equation. 4.3.

GG_p is a nonlinear voltage controlled current source that models the plasma resistance. The gain of GG_p is actually the geometry constant G_p which is shown.

GG_{ed} is a nonlinear voltage controlled current source that models the eddy current resistance, R_e . The gain of GG_{ed} is the geometry constant G_{ed} which is shown.

7.2 Model Fit to High Power Performance

Table 7.2 lists the values of the adjusted parameters that cause the model to achieve significant boot-up. The SPICE input data is shown in Appendix C. Figures 7.1 and 7.2 illustrate the simulated MHD voltage and current.

Table 7.2: Fit data (adjusted parameters and Vmhd).

B_0	U_0	σ_0	H_1	GG_p	R_{cu}	V_{mhd0} (5usec)	V_{mhd1} (60usec)	I_m
1	60K	25K	6.6u	75m	1m	1298	10150	.483Meg

Where:

H_1 is a constant, given in Appendix A, that is directly proportional to the electromagnet current to regeneration flux constant K of Equation. 4.3.

GG_p is a nonlinear voltage controlled current source that models the plasma resistance. The gain of GG_p is actually the geometry constant G_p which is shown.

GG_{ed} is a nonlinear voltage controlled current source that models the eddy current resistance, R_e . The gain of GG_{ed} is the geometry constant G_{ed} which is shown.

I_m is the electromagnet current.

Additional plots of simulation variables at the MHD channel outlet were obtained and are shown in Figures 7.3 (conductivity), 7.4 (density), 7.5 (pressure), 7.6 (enthalpy), and 7.7 (velocity).

Figure 7.1: MHD voltage simulation for significant regeneration.

Figure 7.2: MHD current simulation for significant regeneration.

Figure 7.3: MHD channel outlet plasma conductivity.
(As a fraction of the input conductivity)

Figure 7.4: MHD channel outlet plasma density.

Figure 7.5: MHD channel outlet plasma pressure.

Figure 7.6: MHD channel outlet plasma enthalpy.

Figure 7.7: MHD channel outlet plasma velocity.

7.3 Evaluation of Model High Power Simulation

The results gave some very enlightening insight of what is happening in the energy conversion process. The first observation was that the power came from enthalpy in the plasma. Figure 7.6 shows a significant decrease in this variable. The prediction of a large pressure drop across the channel was compatible with this conversion mechanism. As shown in Figure 7.5 the channel outlet pressure was significantly less than the inlet pressure. The other surprise was that the plasma velocity across the channel was very close to constant (Figure 7.7). Reference [4], in the development of their system of MHD equations for a continuous system had assumed a constant velocity. This result substantiates the author's assumption that the pressure drop would cause an expansion of the plasma that would maintain constant velocity. That expansion causes an acceleration that compensates the deceleration from MHD power extraction. In this model the thermodynamic momentum equation has a term that represents the acceleration of the plasma mass in the channel. This causes an integration operation with respect to time to occur to obtain velocity. That integration is what holds the velocity almost constant. Also notice, in Figure 7.5, that at about 65 usec the outlet pressure was predicted as going negative. Since this is impossible, it implies that, for this geometry, the design limit for maximum power extraction occurs at 65 usec. At this

point the pressure drop is greater than what the channel can sustain and the regeneration process will start to shutdown. The pressure drop phenomenon is analogous to the electrical load on a generator that stalls its mechanical power source.

The parameters in Table 7.2 can be realized such that the model predictions will be credible. The only parameter in Table 7.2 that is a significant upgrade is H_1 . INEL increased H_1 by a factor of 2 between tests 1 and 2. Thus another factor of 2 does not appear impossible.

8.0 SUMMARY AND CONCLUSIONS

A one dimensional full-effects model of an explosively driven MHD generator was developed. The model includes electrical, electromagnetic, and thermodynamic effects. The MHD generator that was modeled was a regenerative magnetic field configuration that was the subject of some development and testing at INEL. The INEL generator design parameters were input into the model along with some estimates of the properties of the argon plasma driving the generator and the model simulated the INEL measured performance. The model was used to simulate extended duration plasma flow to determine if significant energy conversion from the plasma is feasible. The model successfully fit the measured data with a set of parameter values that were credible. The model predicts that significant energy conversion can occur with extended duration plasma flow and with model parameters that are feasible. The model does not address feasibility of extended plasma flow (INEL was only able to demonstrate 20usec flow) or whether or not a device will maintain structure integrity for extended durations.

The conclusions that were drawn from the development of the model and it's predictions are:

1. The electrical circuit approach using SPICE works and provides a unique insight to the process, especially in regard to electrical and magnetic effects.
2. The model indicates that power conversion from the plasma comes from the thermodynamic energy, enthalpy, of the plasma and not the kinetic energy, velocity.
3. The model predictions and the sensitivity analysis of model parameters indicates that the role of plasma velocity is that of a catalyst, in that, if it is present, power will be converted from thermodynamic enthalpy to electrical power.
4. The model indicated that feasible parameters can yield significant regeneration.
5. The model predictions of negative outlet pressure established a theoretical upper limit for the power conversion.

6. This model can be used to tune advanced designs of MHD generators for specific performance objectives.

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APPENDIX A

SPICE IMPLEMENTATION

A1 Introduction

The objective of this Appendix is to provide the details of the implementation of the full-effects model in the electrical circuit simulation code SPICE. The discussion is quite detailed and knowledge of SPICE was assumed. The version of SPICE used was 2G.6 [Ref. 2]. The analysis feature of SPICE that was employed is non-linear transient analysis. This feature in SPICE has the capability of obtaining the time response to systems of non-linear differential equations. The interface to SPICE is an electrical circuit. Therefore the system equations must be expressed as an equivalent electrical circuit with the electrical variables of voltage, current, or power being analogs for the actual variables. In the main body of the report the Generator and the Electromagnet Model, and the Magnetic Flux Model were described as electrical equivalent circuits. For those blocks SPICE is the natural tool to effect a solution. However, in the Thermodynamics Model, the Equations were described in the form of a system diagram with elements that represent operations such as multiply, sum, divide, integrate, raise to a power, and raise to a non-integer power. For this application SPICE was applied like an analog computer. Equivalent circuits were designed that realized each of the required operations. The system was interconnected and the gain coefficients for each stage were adjusted for numerical stability. The overall effect of gain adjustments was unity. Additional coefficients for the various blocks that represent geometry constants (cross sectional area etc.) were also included and not adjusted out. General features of SPICE, such as the resistor, capacitor, inductor, controlled sources, etc, that are employed were assumed valid. Non-standard elements that were realized as custom circuits, such as divide, raise to non-integer powers, and integrate, were derived and evaluated for performance.

A2 Generator and Electromagnet Model

The MHD generator equivalent circuit is illustrated in Figure A1 and is a repeat of the Figure 5.3 from the main body text in the development of the Generator and Electromagnet Model of the full-effects Model. For clarity an explanation of the elements of Figure A1 are also repeated from the main body text in Table A1.

Figure A1: MHD generator equivalent circuit.

Table A1 MHD equivalent circuit parameters.

Element	Explanation
V_{bu}	Regenerative MHD voltage source.
V_0	Permanent magnet MHD voltage source.
R_p	MHD channel electrode to electrode plasma resistance.
R_{cu}	Total MHD generator copper resistance.
R_e	MHD channel eddy current resistance.
L_B	Electromagnet inductance.

Figure A2 is the SPICE realization of Figure A1. It is electrically equivalent to Figure A1 and the voltage labeled V_{out} in both figures is the same. Table A2 explains the elements of Figure A2 and relates them to Figure A1.

Table A2: Parameters of Figure A2.

Element	Explanation
V_{mhd}, V_{ss}, V_e	Zero valued independent voltage sources used by SPICE to reference currents (ie $I(V_{mhd})$ is the current through the voltage source V_{mhd}).
EE_2	Identical to the permanent magnet MHD voltage source V_0 .
EE_1	Identical to the regenerative MHD voltage source V_{bu} . EE_1 was relocated in the series circuit because SPICE has numerical instabilities with the two voltage sources EE_1 and EE_2 on top of each other.
GG_p	Identical to R_{cu} (see the following derivation).
R_{cu}	Identical to R_{cu} .
GG_d	Identical to R_s (see the following derivation).
L_B	Identical to L_B .

The implementation of the voltage source V_0 of Figure A1 in SPICE format EE_2 of Figure A2 is as follows.

Equation A1 gives the expression for the source V_0 :

$$V_0 = EE_2 = U_{eff} B_1(t) \ell \quad A1$$

Where: U_{eff} is the corrected plasma velocity.
 $B_1(t)$ is the corrected in channel permanent magnet field.
 ℓ is the distance between the electrodes.

Equation A2 gives the implementation of V_0 as EE_2 in the SPICE variables:

$$EE_2 = V(22)V(16)\ell \quad A2$$

Where: $V(22)$ is the voltage from node 22 in Block 3 that is the corrected plasma velocity. Block 3 is the thermodynamic Block that computes corrected plasma velocity and corrected conductivity.

$V(16)$ is the voltage from node 16 in Block 2 that is the corrected in channel permanent magnet field.

Block 2 computes the corrected magnetic field due to plasma flow in the MHD channel.

The line of data that describes the source EE_2 in the SPICE format is:

$$\underbrace{EE2}_{\text{name}} \quad \underbrace{1 \ 2}_{\text{connect nodes}} \quad \underbrace{POLY(2)}_{\text{dimension of power series}} \quad \underbrace{16 \ 0}_{B_1(t)} \quad \underbrace{22 \ 0}_{U_{eff}} \quad \underbrace{0 \ 0 \ 0 \ 0}_{\text{terms} = 0} \quad \underbrace{25.4E-03}_{\ell}$$

Equation A3 gives the expression for R_p :

$$R_p = (\sigma(t)G_p)^{-1} \quad A3$$

Where: $\sigma(t)$ is the in channel plasma conductivity.
 G_p is the geometry portion of R_p

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2 of 2

GG_p in the SPICE equivalent circuit is a non-linear voltage controlled current source with it's current given by A4:

$$GG_p = V(9)V(68)V(1, 2)P_{14} \quad A4$$

Where: $V(9)$ is the voltage from node 9 that represents the MHD channel input plasma conductivity.

$V(68)$ is the voltage from node 68 from Block 3 that represents the conductivity correction factor. Block 3 is the thermodynamic Block and computes the corrected plasma velocity and the corrected plasma conductivity.

$V(1, 2)$ is the voltage drop across the current source GG_p .

P_{14} is a geometry constant.

A4 can be rewritten as A5:

$$R_p = \frac{V(1, 2)}{GG_p} = (V(9)V(68)P_{14})^{-1} \quad A5$$

Since $V(9)V(68)$ is the conductivity in the channel.

Thus: $P_{14} = G_p$

To compute the geometry constant it was assumed that current flow occurs in a cylinder of plasma connecting the two electrodes across the channel. Thus the geometry constant P_{14} is given by:

$$R_p = \frac{\ell}{\sigma(t)A'}, \quad G_p = \frac{A'}{\ell} \quad A6$$

Where: A' is the electrode surface area.
 ℓ is the distance across the channel.

The line of code that describes the current source GG_p in SPICE format is given by:

$$\underbrace{GGP}_{\text{name}} \quad \underbrace{1 \ 2}_{\text{connect nodes}} \quad \underbrace{POLY(3)}_{\substack{\text{dimension} \\ \text{of power} \\ \text{series}}} \quad \underbrace{1 \ 2}_{V(1 \ 2)} \quad \underbrace{9 \ 0}_{V(9)} \quad \underbrace{68 \ 0}_{V(68)} \quad \underbrace{0 \dots 0}_{\substack{P_0 \text{ to } P_{13} \\ \text{terms} = 0}} \quad \underbrace{75E-03}_{P_{14}}$$

Equation A7 gives the expression for R_e

$$R_e = (\sigma(t)G_e)^{-1} \quad \text{A7}$$

Where: $\sigma(t)$ is the in channel plasma conductivity.
 G_e is the geometry portion of R_e

GG_{ed} in the SPICE circuit is a non-linear voltage controlled current source given by A8:

$$GG_{ed} = V(9)V(68)V(6)P_{14} \quad \text{A8}$$

Where: $V(9)$ is the voltage from node 9 that represents the MHD channel input plasma conductivity.

$V(68)$ is the voltage from node 68 from Block 3 that represents the conductivity correction factor. Block 3 is the thermodynamic Block and computes the corrected plasma velocity and the corrected plasma conductivity.

$V(6)$ is the voltage drop across the current source GG_{ed} .
 P_{14} is a geometry constant.

A8 can be rewritten as A9:

$$R_e = \frac{V(6)}{GG_e} = \left(\underbrace{V(9)V(68)P_{14}}_{\sigma(t)} \right)^{-1} \quad \text{A9}$$

Since $V(9)V(68)$ is the conductivity in the channel.

Thus: $P_{14} = G_e$

The geometry constant for the eddy current resistance is far more complex than the plasma resistance model of a simple cylinder of plasma connecting the two electrodes across the channel. It was derived in Appendix B and its numerical value is represented by P_{14} in the SPICE format of data for the power series expression of the non-linear voltage controlled current source GG_{ed} . The SPICE data for GG_{ed} is:

<u>GGED</u>	<u>6 0</u>	<u>POLY(3)</u>	<u>6 0</u>	<u>9 0</u>	<u>68 0</u>	<u>0 0</u>	<u>75E-03</u>
name	connect nodes	dimension of power series	V(6)	V(9)	V(68)	P_0 to P_{13} terms = 0	P_{14}

The resistor R_{cu} represents the copper lead and winding resistance. INEL measured this as 40mohm. This value is not credible because the INEL device used 1/8" solid copper leads with a length of approximately 18". Additionally 2 of these were in parallel. Appendix C computes the resistance of such a solid length of copper wire as < 1mohm (neglecting skin effect). Thus 2 of these leads in parallel will be closer to 1mohm than 40mohm. The remaining element in the MHD generator equivalent circuit is the electromagnet inductance L_B . This parameter was measured in the INEL experiments and a representative value is: .56uH and it was assumed valid.

Separate from the generator equivalent circuit but included in Block 1 are the auxiliary circuits that were used to compute the power dissipated in the plasma caused by R_p & R_e , and the power developed by the MHD generator. The group of these circuits are illustrated in Figure A3. The method used to compute power was the product of voltage and current. In SPICE the non-linear controlled sources are multidimensional power series with input variables either all voltage or all current but not mixed. Thus for each power computation circuit there is an additional circuit that converts current to voltage. The power computation circuit then uses a non-linear source to multiply the two voltages to obtain power. For the computation of total MHD power an auxiliary circuit was used to sum the two MHD sources EE_1 and EE_2 . A non-linear controlled source was used for the sum. The resultant sum was multiplied with the voltage that represents the generator current to obtain total MHD generated power.

Figure A3: Power computation auxiliary circuits.

A3 Magnetic Flux Model

The derivation of the RC ladder network model for the one dimension system of magnetic flux diffusion into the plasma was accomplished in the main body of the report. The objective here is to implement that model in SPICE. The two issues that must be resolved in order to realize the SPICE model are:

- o Determine the number, N , of RC cells that are adequate.
- o Determine how conductivity correction can be accommodated.

In order to determine the number of cells, N , that are adequate, an analysis was performed with a ramp input of $2.0T/usec$ and a depth of penetration of $0.5"$, which is the distance to the center of the MHD channel of the INEL devices. $2.0T/usec$ is about rate of change of magnetic flux that is expected during an MHD test that exhibits

significant magnetic field regeneration. Multiple simulations were performed with $N = 10, 20$, and 40 cells in order to evaluate resolution.

$$\text{Let } \mu_0 = C = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\text{Let } R = 2 \times 10^4 \text{ Sm}^{-1}$$

$$\text{Use 10 cells; } x = 0.5", \Delta x = (0.5/39.37)/10$$

$$R_k = \sigma \Delta x = 2 \times 10^4 (0.5/39.37)/10 = 25.4 \Omega$$

$$C_k = \mu_0 \Delta x = 4\pi \times 10^{-7} (0.5/39.37)/10 = 1.6 \text{ nF}$$

Do a second run with 20 cells

$$R_k = 12.7 \Omega$$

$$C_k = .798 \text{ nF}$$

Do a third run with 40 cells

$$R_k = 6.35 \Omega$$

$$C_k = .399 \text{ nF}$$

A plot of the result for ten cells is shown in Figures A4. Table A3 compares the performance of the various number of cell models.

Ramp Excitation

Figure A4: SPICE ten cell flux diffusion response

Table A3: Comparison of the time to 80T for different number of cell models.

Ramp input 80T @ 40usec

1. 10 Cell Model 79.5T @ 42usec, 80.5T @ 42.5usec
2. 20 Cell Model 79.7T @ 42usec, 80.7T @ 42.5usec
3. 40 Cell Model 79.9T @ 42usec, 80.8T @ 42.5usec

An examination of Table A3 shows that the SPICE model for flux diffusion based on 10 cells was adequate and therefore the 10 cell model was integrated into the SPICE MHD model. The 10 cell equivalent circuit for regenerative flux diffusion was excited by the controlled voltage source that is a geometry constant times the current in the electromagnet. This source represents the regenerative flux at the center of the channel that would occur if the highly conductive plasma were not present. Figure A5 illustrates the equivalent circuit.

Figure A5. 10 Cell flux diffusion model

The voltage at the capacitor of cell 10 is the regenerative flux ($B_2(t)$) that along with the effective plasma velocity (U_{eff}) enables the regenerative MHD potential. As developed in the main body of the report, this same 10 cell model was also be used to predict the initial magnetic field, from the permanent magnets, interaction with the start of highly conductive plasma flow. For that case the 10 cell model will be excited by an independent source that is a step function with a magnitude equal to the value of magnetic flux at the center of the channel from the permanent magnets as measured pretest at system assembly.

The issue that is to be resolved next is to incorporate conductivity correction into each of the cell resistors. Equation A10 gives the relation for the cell resistor as a function of plasma conductivity, $\sigma(t)$, the number of cells, N , and the distance to the center of the channel, x .

$$R_{cell} = \sigma(t) \frac{x}{N} \quad A10$$

The conductivity in A10 is not a constant. It is a variable that was computed in Thermodynamic Model and fed into this model and the Generator and Electromagnet Model. Thus a resistor can not be used to model Equation A10. That cell resistor was modeled by a SPICE non-linear current controlled voltage source. Equation A11 gives the expression that the source implemented.

$$HH_{pi} = I(V_{pi})I(V_{con}), \quad I(V_{con}) = \sigma(t) \frac{x}{N} \quad A11$$

Where: $I(V_{pi})$ is the current through the source.
 x is the distance to the center of the channel.
 N is the number of cells.

The SPICE data for the "ith" source, HH_{pi} , is:

$$\underbrace{HH_{pi}}_{\text{name}} \quad \underbrace{m \quad n}_{\substack{\text{connect} \\ \text{nodes}}} \quad \underbrace{POLY(2)}_{\substack{\text{dimension} \\ \text{of power} \\ \text{series}}} \quad I(V_{pi}) \quad I(V_{CON}) \quad \underbrace{0 \ 0 \ 0 \ 0}_{\text{terms} = 0} \quad \underbrace{1}_{P_4}$$

The complete circuit for the flux diffusion model is given by Figure A6. The function of the auxiliary circuit aside from the main ladder network is to input the voltage representing corrected conductivity times Δx and convert it into a current form as needed by HH_{pi} . This model is implemented as a subcircuit for the permanent magnet flux and the electromagnet flux. This feature in SPICE is the same as a software subroutine.

Figure A6: The complete SPICE flux diffusion subcircuit model.

A4 The Thermodynamic Model

The equations describing the thermodynamic effects for velocity and conductivity are not normally modeled as electrical equivalent circuits. Thus the development in the main body of the text took the approach of a system diagram as used to program an analog computer. In this case SPICE becomes the analog computer and electrical equivalent circuits are designed to realize each element of the analog block diagram. This discussion will implement Thermodynamic Equations as a system diagram and describe how the different elements were realized in SPICE. Table A4 illustrates the symbols for each of elements, defines what their function is, and describes how it was realized in SPICE.

Table A4: System diagram elements.

Element	Symbol	Implementation
Constant gain		Linear controlled source
Multiplier		Non-linear controlled source
Two input summer		Non-linear controlled source
Two input subtraction		Non-linear controlled source
Multi-input summer		Resistor array

Table A4 Cont.

Integrator		Subcircuit
Non-integer power		Subcircuit
Divider		Subcircuit

Table A5 is a repeat of Table 5.5. It lists all of the Thermodynamic Equations in system diagram format.

Table A5: System Diagram Format of Thermodynamic Equations.

Eqn. No.	Equation
5.34	$P_1 = \frac{\rho_1}{\rho_0} \left[\frac{P_0 + \rho_0 h_1 (\Gamma - 1) - 2 \rho_0 E_0 (\Gamma - 1) + \rho_1 E_0 (\Gamma - 1)}{\frac{\rho_1}{\rho_0} + (\Gamma - 1)} \right]$
5.35	$\delta U = \int \frac{1}{M'} [A(P_1 - P_0) + i_{mhd} B_{tot} \ell - F \delta U] dt$
5.36	$U_{eff} = U_0 + \frac{\delta U}{2}, \quad U_1 = U_0 + \delta U$
5.37	$\rho_1 = \rho_0 \frac{U_0}{U_1}$
5.38	$h_1 = h_0 - (U_0 \delta U + \frac{1}{2} \delta U^2) - \frac{V_{mhd} i_{mhd} - i_{mhd}^2 R_p - i_e^2 R_e}{F}$
5.39	$E_1 = h_1 - \frac{P_1}{\rho_1}$
5.41	$\sigma_1 = \sigma_0 \left(\frac{E_1}{E_0} \right)^{\frac{3}{2}}$

Figures A7 through A13 provide the associated system diagrams.

Figure A7: System diagram of Equation 5.34, P_1 .

Figure A8: System diagram for the solution to δU , Equation 5.35.

Figure A9: System diagram for Equation 5.36, U_{eff} and U_1 .

Figure A10: System diagram of Equation 5.37, ρ_1 .

Figure A11: System diagram for Equation 5.38, h_1 .

Figure A12: System diagram for Equation 5.39, E_1 .

Figure A13: System diagram for Equation 5.41, σ_1 .

The first 3 elements in Table A4 are standard elements in SPICE [Ref. 2]. The remaining elements, the resistor array and the 3 subcircuits are derived. The SPICE standard elements and the resistor arrays are assumed valid. The subcircuits, integrate, divide, and non-integer power, were validated with an error analysis. For an explanation of the resistor array, consider the example of Figure A14.

Figure A14: Resistor array as a 3 input summer.

Three individual variables, that are modeled as voltages are fed through equal resistors into a summing node. The voltage at the summing node will be 1/3 the sum of the individual voltages. Variations of this were implemented with different value resistors to obtain a weighted sum.

The integrator element is illustrated in Figure A15.

Figure A15: Integration circuit.

This circuit is an inverting amplifier configured as an integrator. The gain is 1×10^6 . If it were infinite, the voltage at summing node would be zero and the output is:

$$V_{out} = -\frac{1}{RC} \int V_{in} dt \quad A12$$

The next element is the non-integer power function. Positive integer exponent operations are easily implemented in SPICE with the features of the multidimensional power series non-linear controlled sources. Negative integer power or non-integer power operations are not straight forward. A negative integer power can be realized by dividing the result of a positive integer power operation into one. The method chosen to realize the non-integer positive power is the method of Log's.

Equation A13 illustrates the non-integer power, r operator.

$$(V_{in})^r = \text{Log}^{-1}(r \text{Log}(V_{in})) \quad \text{A13}$$

SPICE has internal to it the diode equation model for the semiconductor diode. A simplified form of that equation is given by A14:

$$I_D = I_s e^{\frac{V_D}{V_T r}}, \quad \text{A14a}$$

$$V_D = V_T \text{Ln}\left(\frac{I_D}{I_s}\right) \quad \text{A14b}$$

Equation A14a was the basis for an antilog function and equation A14b realized the log function. Figure 16 illustrates the diode and its parameters:

Figure A16: The diode and parameters.

It should be noted that for numerical stability in SPICE, the default diode model that was used for the log and anti-log diodes had the model parameter $\text{TT} = .1\text{nsec}$. The non-integer power function that is needed is $r = 3/2$. Figure A17 illustrates the equivalent circuit that was used with log and antilog diode circuits to obtain the desired $3/2$ power function.

Figure A17: $3/2$ power function subcircuit.

The input is a termination resistor R_{f1} . The log circuit has a current source G_1 that converts the input voltage signal into a current to feed diode D_1 . Equation A15 describes the log circuit. As seen in Figure A15 the output of the log circuit is the diode voltage $V(3)$. The next stage is the $3/2$ power and the anti-log circuit.

$$V(3) = V_T \ln\left(\frac{V_{in}}{I_s}\right) \quad A15$$

The controlled voltage source E_1 has a gain of $3/2$ and is slaved to $V(3)$. The anti-log signal is the current in diode D_2 as given by A16.

$$I(V_s) = I_s \exp\left(\frac{3}{2} \frac{V_T}{V_T} \ln\left(\frac{V_{in}}{I_s}\right)\right)$$

$$I(V_s) = I_s \left(\frac{V_{in}}{I_s}\right)^{\frac{3}{2}} = I_s^{-\frac{1}{2}} (V_{in})^{\frac{3}{2}} \quad A16$$

The output stage is the current controlled voltage source H_1 . The gain of H_1 was set to normalize the (I_s) term and any other constant that came in with the input signal and needs to be normalized out. Equation A17 gives the final result.

$$V_{out} = (I_s)^{\frac{1}{2}} I(V_s) = (I_s)^{-\frac{1}{2}}$$

$$V_{out} = (I_s)^{\frac{1}{2}} I(V_s) = (I_s)^{\frac{1}{2}} (I_s)^{-\frac{1}{2}} (V_{in})^{\frac{3}{2}} = (V_{in})^{\frac{3}{2}} \quad A17$$

The last element to be considered is the division function. Figure A18 illustrates the operation.

Figure A18: The division element.

In addition to performing the division function the implementing circuit must mitigate the problem of division by zero. Figure A19 illustrates a circuit configuration that can perform this function. The input for the numerator terminates the signal, V_x , in the resistor, R_{F1} , and converts it to a current that is sensed in the SPICE ammeter, V_s . The input for the denominator terminates the signal, V_y , in resistor, R_{F2} .

Figure A19: The division circuit.

The current controlled current source is slaved to the current through V_s as described in Equation A18.

$$FIX = I(V_s) = V_x \quad A18$$

The current in the non-linear voltage controlled current source is slaved to the denominator voltage V_y and the voltage at node 4 as given by Equation A19.

$$GG_y = V_y V(4) \quad A19$$

The node equation at node 4 is given by Equation A20.

$$FIX = \frac{V(4)}{R_{F3}} + GG_y = V(4) \frac{1}{R_{F3}} + V(4)V_y \quad A20$$

Combining A18, A19, and A20:

$$V(4) = \frac{V_x}{\frac{1}{R_{F3}} + V_y} \quad \text{A21}$$

As seen in Equation A21 resistor R_{F3} mitigates the problem of division by zero. On the other hand, the function is valid only if the constraint of A22 is met.

$$V_y \gg \frac{1}{R_{F3}} \Rightarrow V(4) = \frac{V_x}{V_y} \quad \text{A22}$$

The output stage E_1 is a voltage controlled voltage source that provides zero output impedance and the option of a normalizing gain constant. It is noted that in Equation A22, the magnitude of V_y must greatly exceed the inverse of resistor R_{F3} . SPICE convergence problems occur if R_{F3} is greater than 1meg. Thus, to realize the constraint of Equation A22, V_y was given a gain of a factor of 4 and the gain normalized out by a gain factor of 4 in E_1 . The times 4 gain factor increases the performance of the divider but doesn't compromise SPICE convergence. Figure A20a, A20b, and A20c illustrates the complete SPICE model. Appendix C gives the SPICE input data listings for the various applications of the model.

MHD generator circuit

A19a: FULL EFFECTS MODEL SPICE CIRCUIT

A19b: FULL EFFECTS MODEL SPICE CIRCUIT

A19c: FULL EFFECTS MODEL SPICE CIRCUIT

The remaining issue to be considered in the SPICE model is the validity of some of the subcircuit functions. The integration block, the division block, and the 3/2 power block were checked.

The integration block is checked by examining it separate from the model. It is excited with a 1 volt step input and its response compared with the analytical results over a time period of 50usec. Figure A20 illustrates the integration block test circuit and Table A6 compares the SPICE results with the analytical results. As seen in table A6 the error of the integrator is much less than the resolution of 4 significant figures of the SPICE output.

Figure A20: Integrator test circuit.

Table A6: SPICE integrator compared with analytical results

Time (usec)	SPICE	Analytical
0	0	0
10	4.000E-05	4.000E-05
20	8.000E-05	8.000E-05
30	12.000E-05	12.000E-05
40	16.000E-05	16.000E-05
50	20.000E-05	20.000E-05

The next block to be evaluated is the division block. This was accomplished by examining the inputs and outputs for each of the divider blocks used with the complete model simulation of advanced performance. The calculated quotient (HP 25C pocket calculator) was compared with the SPICE quotient. This particular simulation was chosen because the SPICE model is more thoroughly exercised in this simulation than with any of the others. The full details of the simulation are covered in Chapter 7.0. Table A7a tabulates the results at 20usec and A7b is at 50usec. The Xi designator in the two Tables refers to each of the particular divider subcircuits in the complete SPICE schematic of the full-effects model as illustrated in Figures A19a,b,c.

Table A7a: Comparison of SPICE and calculated division @ 20usec.

Divider	Numerator	Denominator	SPICE	HP25	Error
X3	1.853E-05	3.229E-04	5.734E-02	5.739E-02	-.08%
X4	1.824E+02	6.080E-01	3.001E+02	3.000E+02	-.033%
X5	6.000E+04	6.000E+04	1.000E-00	1.000E-00	0
X6	4.398E+01	3.000E-01	1.466E+02	1.466E+02	0
X7	4.949E-03	5.000E-03	9.898E-01	9.898E-01	0

Table A7b: Comparison of SPICE and analytical division @ 50usec.

Divider	Numerator	Denominator	SPICE	HP25	Error
X3	1.857E-04	3.229E-04	5.746E-01	5.751E-01	-.087
X4	2.692E+03	6.080E-01	4.428E+03	4.428E+03	0
X5	6.000E+04	6.003E+04	9.995E-01	9.995E-01	0
X6	3.010E+01	3.001E-01	1.003E+02	1.003E+02	0
X7	4.255E-03	4.998E-03	8.513E-01	8.513E-01	0

During the evaluation of this division error an excessive error in the divider X3 was uncovered. The cause was from the denominator of X3 having a magnitude that was inconsistent with the constraint of equation A22. This was corrected by a generic modification of the divider subcircuit by the addition of the times 4 gain factor previously discussed. Additionally another times 4 gain was applied to the denominator input signal and backed out with a gain change of times 4 in the integrator by setting $C = .25F$. This resulted in the reduction of the error in X3 by a factor of 10 to an acceptable level. SPICE convergence was not compromised.

The same SPICE simulation used to evaluate the division block was used to evaluate the $3/2$ power block. Table A8 provides the results at various event times. The analytical computation was also obtained from an HP 25C pocket calculator with the number of significant digits set at the same as the SPICE results.

Table A8: SPICE $3/2$ power evaluation.

Time(usec)	Input	SPICE	HP25	Error
10	9.974E-01	9.962E-01	.9961	+.01%
20	9.898E-01	9.847E-01	.9847	0
30	9.722E-01	9.586E-01	.9586	0
50	8.513E-01	7.855E-01	.7855	0

As seen in the Tables the error of the SPICE divider and the SPICE $3/2$ power is not a significant impact on the model results. The SPICE input data files for these test circuits are listed in APPENDIX D.

APPENDIX B
DERIVATION OF EDDY CURRENT RESISTANCE

B1 Introduction

The objective of this Appendix is to analytically obtain a numerical constant that represents the geometry portion of the plasma eddy current resistance. The plasma eddy current is exactly the same as the core loss in a transformer. It is caused from a time varying magnetic flux dissipating power from loop currents that are induced in a conductive medium.

B2 Analysis

The analysis approach taken was based on the transformer core loss analysis of Reference [9]. The situation is illustrated in Figure B1.

Figure B1: The situation of core loss.

If voltage V_{in} is applied to the system shown in Figure B1, then a current $i(t)$ and a core flux density will occur (neglecting winding resistance) as given by equation B1a and B1b.

$$i(t) = \frac{1}{L} \int V_{in} dt \quad \text{B1a}$$

$$B = \frac{Li(t)}{A} \quad \text{B1b}$$

A more realistic view of the situation is given by Figure B2, which considers winding resistance and core loss.

R_w = winding resistance

R_c = core loss

Figure B2: A model of the concept of core loss.

R_c is the effective resistance that represents core loss. For simple geometries and known conductivities it is possible to compute R_c from the relationship:

$$P_c = \frac{V_L^2}{R_c}, \quad \text{But } V_L = \dot{\lambda}, \quad R_c = \frac{\dot{\lambda}^2}{P_c}, \quad \lambda = \text{Flux turns} \quad \text{B2}$$

The required analysis is in three dimensions. However, the magnetic flux can be kept in one dimension. Figure B3 illustrates the analysis geometry.

Figure B3: MHD generator sketch illustrating eddy current loss

The analysis is based on a set of assumptions that will tend to over estimate the eddy current losses. It will be shown that this conservatism can be backed out and realism brought in when the model is matched to the data. The assumptions are as follows:

1. One side of a one turn magnet is considered and the results doubled to account for the other side.
2. All of the flux is assumed to remain within the radius of the electromagnet.
3. The flux density within this radius is considered uniform.

The analysis approach was to slice the affected conductive plasma volume into a number, N , of thin slices and then analyze a typical slice. The total ohmic dissipation in the slice was computed and then all of the different slice dissipation was totaled to obtain the half total power. That value was doubled, as per assumption 1, to get the total power. Figures B4a and B4b illustrate the simplified geometry from which relationships were obtained.

Figure B4a: Analysis geometry

Figure B4b: Analysis geometry of the nth slice

$$K = \frac{r_2}{r_1}, \quad r_2 = Kr_1, \quad r_1 = \frac{r_2}{K}$$

B3

Where: r_1 is the inside MHD channel radius.
 r_2 is the electromagnet radius.

$$\text{Let: } \Delta r_1 = \frac{r_1}{N}$$

B4

Where: N is the number of analysis sliced considered.

$$\text{From Figure B4a: } Y_n = r_1 \sqrt{1 - \frac{n^2}{N^2}}$$

B5

Where: n is the specific slice number.

$$\text{From Figure B4b: } X_n = \sqrt{r_2^2 - Y_n^2} = \sqrt{r_2^2 - r_1^2 \left(1 - \frac{n^2}{N^2}\right)}$$

B6

$$\text{Plugging in } r_1 = \frac{r_2}{K}$$

$$Y_n = \frac{r_2}{K} \sqrt{1 - \frac{n^2}{N^2}} \quad \text{B7}$$

$$X_n = r_2 \sqrt{1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)} \quad \text{B8}$$

$$dX_n = dr_2 \sqrt{1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)} \quad \text{B9}$$

$$\alpha = \text{TAN}^{-1} \frac{Y_n}{X_n} = \text{TAN}^{-1} \left\{ \frac{\frac{1}{K} \sqrt{1 - \frac{n^2}{N^2}}}{\sqrt{1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)}} \right\} \quad \text{B10}$$

The path length of the loop is: $4X_n + \frac{4\alpha}{2\pi} \times 2\pi r_2$ B11

The path resistance is: $\frac{C}{\sigma D \Delta W}$ B12

Where: C is the path length.
 ΔW is the path width.
 D is the path thickness.
 σ is the path conductivity.

Combining B11 & B12 to obtain the total path resistance:

$$R_T = \frac{4X_n}{\sigma D dX_n} + \frac{\frac{4\alpha}{2\pi} (2\pi r_2)}{\sigma D dX_n} \quad \text{B13}$$

Plugging B9 into B13:

$$R_T = \frac{4r_2}{\sigma D dr_2} (1 + \alpha) \quad \text{B14}$$

The voltage that is causing the loop current is caused by, $\frac{d\Phi}{dt}$ the time changing magnetic flux within the loop:

$$\frac{d\Phi}{dt} = B A = i_{loop} R_T, \quad i_{loop} = \frac{BA}{R_T} \quad \text{B15}$$

The ohmic power in the loop is:

$$dP = i_{loop}^2 R_T$$

B16

To complete the analysis one additional assumption was needed:

Assume that $\frac{dB}{dt}$ was independent of the depth into the conductive plasma. If this assumption is invalid then $\frac{dB}{dt}$ can not be brought outside of the integration to obtain total slice power. To determine if this assumption was valid, the response of the flux diffusion SPICE model was examined. The current through any of the capacitors is a measure of $\frac{dB}{dt}$ at that cell location:

$$i_c = C \frac{dV_{cell}}{dt}, \quad i_c = C \dot{B}$$

B17

Where: C is the cell capacitance.
 V_{cell} is the cell voltage, the analog for B .
 i_c is the cell capacitor current.

The ramp response of the 10 cell SPICE model was repeated and the cell capacitor currents were computed. Figure B5 compares the first cell capacitor current with the last cell capacitor current.

Figure B5: Constant $\frac{dB}{dt}$ for a ramp in a conductive medium.

During the period that would corresponds to regeneration the wave-forms overlap. At start-up they diverge. Because they match during the period of regeneration the assumption that $\frac{dB}{dt}$ is independent of depth of penetration will be considered valid and the analysis continues:

The area inside the loop, A of Eqn. B15 is given by:

$$A = \frac{4\alpha}{2\pi} \pi r_2^2 + 4\left(\frac{1}{2} X_n Y_n\right) \quad \text{B18}$$

Combining Eqns. B7 & B8 with B18:

$$A = 2\alpha r_2^2 + \frac{2r_2^2}{K} \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)\right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2}\right)^{\frac{1}{2}} \quad \text{B19}$$

Combining Eqns. B15 & B19:

$$i_{loop} = \frac{B}{R_T} = \frac{B 2r_2^2 \left(\alpha + \frac{1}{K} \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)\right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2}\right)^{\frac{1}{2}} \right)}{\frac{4}{\sigma D dr_2} r_2 (1 + \alpha)} \quad \text{B20}$$

The ohmic power in the loop, dP_n is given by:

$$dP_n = i_{loop}^2 R_T \quad \text{B21}$$

The total ohmic power in the analysis slab is given by:

$$P_n = \left(\alpha + \frac{1}{K} \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)\right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2}\right)^{\frac{1}{2}} \right)^2 \frac{B^2 \sigma D}{(1 + \alpha)} \int_0^{r_2} r_2^3 dr_2 \quad \text{B22}$$

The total eddy current power, P_{TOT} in the plasma is two times the sum of the slab power because of the analysis symmetry:

$P_{TOT} = 2 \sum_{n=1}^N P_n$, The $\times 2$ because of 2 sides of channel

$$P_{TOT} = \sum_{n=1}^N \frac{B^2 \sigma D r_2^4}{2(1 + \alpha)} \left(\alpha + \frac{1}{K} \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N^2}\right)\right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2}\right)^{\frac{1}{2}} \right)^2 \quad \text{B23}$$

Plugging: $D = \frac{r_1}{N}$ and $B = \frac{V_L}{\pi r_2^2}$ into P_{TOT} and solving for R_e from: $P_{TOT} = \frac{V_L^2}{R_e}$

$$R_e = \left\{ \frac{\sigma r_1}{2N\pi^2} \sum_{n=1}^N \left\{ \frac{1}{(1+\alpha)} \left[\alpha + \left(\frac{1}{K} \right) \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N} \right) \right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2} \right)^{\frac{1}{2}} \right]^2 \right\} \right\}^{-1} \quad \text{B24}$$

This derivation assumes:

- 1 B is constant through the plasma
- 2 B is constant over a given analysis slab.
- 3 Fringing is neglected so that: $V_L = B \pi r_2^2$

A "C" program was written to implement Equation B24. When typical values are plugged in (magnet Dia=3", channel ID=1", $\sigma = 10^4$) the program with a large value, $N = 100$, for the number of slices, then R_e is about .7 ohms . Figure B6 is a listing of the source code.

```
/* This program computes the eddy current resistance for the MHD generator SPICE
model.*/
/* The method used is from /*Zahn's text: "Electromagnetic Field Theory" Section
6-1-3 */ /*pages 401,402. published by Krieger 1987*/
#include <stdio.h>
#include <math.h>
#define pi 3.1415926
main()
{
int i,n;
float a,k,r1,g,gc,ge,re,temp,temp2,nn,ii,x,y,xx,yy;
printf("Enter the total # of Zahn slices n =");
scanf("%d", &n);
nn = n;
printf("Enter the mag. dia. in inches r2 =");
scanf("%f", &temp);
printf("Enter the MHD chan. I.D. in inches r1 =");
scanf("%f", &temp2);
k=temp/temp2;
r1=.5*temp2/39.37;
printf("Enter the plasma conductivity in MKS units g =");
scanf("%cg", &g);
gc = (g*r1)/(2.*pi*pi*nn);
ge = 0.;
```

```

for(i=1;i<=n;i++)
{
ii=i;
x = 1.-(ii*ii)/(nn*nn);
y = k*k-x;
temp = sqrt(x/y);
a = atan(temp);
xx = 1.-x/(k*k);
yy = a+(sqrt(xx)/k)*sqrt(x);
temp = (yy*yy)/(1+a);
ge = ge+temp;
}
re = 1./(gc*ge);
printf("The eddy current resistance re is: %f \n",re);
}

```

Figure B6: "C" source code for MHD eddy current loss resistance.

Consider the effect on R_e if not all the regenerative flux passes through the channel.

$$\Phi_{CHANNEL} = \zeta \Phi_{TOT}$$

Where: ζ is the fraction of the total flux that gets into the MHD channel.

Φ_{TOT} is the total electromagnet flux.

$\Phi_{CHANNEL}$ is the flux that gets to the MHD channel.

Thus B becomes:

$$B = \zeta \frac{\Phi_{TOT}}{A} = \zeta \frac{V_L}{\pi r_2^2} \quad \text{B25}$$

When this new value for B is input into Eqn. B23 a modified R_e^* is obtained from Eqn.

B24:

$$R_e^* = \frac{R_e}{\zeta^2} \quad \text{B26}$$

Since $\zeta < 1$, the effect is a reduction of the eddy current loss.

The value of R_e is estimated from Equations B24 and B26. It was incorporated into the SPICE model exactly like the SPICE non-linear plasma resistance current source GG_p . When the numerical value of conductivity used to evaluate Equation B24 (the "C" program) and Equation B26 is backed out, the result becomes the geometry portion of R_e and it was input to the nonlinear eddy current resistance current source GG_{ed} as the constant P14 used in the SPICE model (see Appendix A).

$$P_{14} \text{ of } GG_{ed} = \frac{\zeta^2 r_1}{2\pi^2 N} \sum_{n=1}^N \left\{ \frac{1}{(1+\alpha)} \left[\alpha + \left(\frac{1}{K} \right) \left(1 - \frac{1}{K^2} \left(1 - \frac{n^2}{N} \right) \right)^{\frac{1}{2}} \left(1 - \frac{n^2}{N^2} \right)^{\frac{1}{2}} \right]^2 \right\} \quad \text{B27}$$

APPENDIX C
DERIVATION OF GEOMETRY CONSTANTS

Calculation of Magnet Current to Regeneration Flux Constant K

From Reference [9] the expression for the magnetic field along the center line of two identical facing loops carrying the same current is given by Equation C1. Figure C2 illustrates the geometry.

Figure C1: Electromagnet geometry.

$$B_z = I' \mu_0 \frac{a^2}{2} \left(\frac{1}{(Z^2 + a^2)^{\frac{3}{2}}} + \frac{1}{((Z - d)^2 + a^2)^{\frac{3}{2}}} \right) \quad \text{C1}$$

$$B_{\frac{d}{2}} = KI', \quad \text{Where: } I = 2I' \quad \text{C2}$$

$$\text{Thus: } B_{\frac{d}{2}} = \frac{I\mu_0 a^2}{2 \left(\left(\frac{d}{2} \right)^2 + a^2 \right)^{\frac{3}{2}}} \quad \text{C3}$$

The regenerative flux constant K is given by Eqn. C4:

$$K = \frac{\mu_0 a^2}{2 \left(\left(\frac{d}{2} \right)^2 + a^2 \right)^{\frac{3}{2}}} \quad \text{C4}$$

Table C1 lists the results of the analysis.

Table C1: Electromagnet gain constants for INEL regeneration tests.

Test#	a	d/2	K
1	1.5"	3"	1.475×10^{-6} T/amp
2	1.5"	1.78"	4.413×10^{-6} T/amp

The results in Table C1 are based on magnets with one turn, while the actual test articles had 1.5 turns. K has the characteristics of inductance and inductance varies with the number of turns squared. Thus a better estimate of the gain K :

$$(1.5)^2 K = \begin{matrix} 3.32 \times 10^{-6} \text{ test \#1} \\ 6.90 \times 10^{-6} \text{ test \#2} \end{matrix} \quad \text{C5}$$

This results in the gain of H_1 :

$$H_1 \text{ test1} = 1.66 \times 10^{-6}$$

$$H_1 \text{ test2} = 3.50 \times 10^{-6}$$

Where H_1 is the gain of the current to voltage controlled source that drives the regenerative magnetic flux subcircuit in the Magnetic Flux SPICE model. The factor of 2 between K and H_1 was required for SPICE numerical stability. The gain factor of 2 was normalized out in the gain of the non-linear voltage source EE_1 .

Calculation of Copper Loss Resistance R_{cu}

An estimate can be made of the total winding and lead resistance R_{cu} .

Figure C2 illustrates the geometry for establishing the total lead and winding length for one of the magnet bobbins.

Figure C2: Estimate of bobbin winding length.

$$\text{Total wire length: } \ell = \frac{1}{39.37} (1.5 \times \pi \times 3'' + 4'') = 0.46 \text{ m}$$

$$\text{Wire cross sectional area: } A = \pi r^2 = \left(\frac{1}{16}'' \right)^2 \pi \left(\frac{1}{39.37} \right)^2 = 7.917 \times 10^{-6} \text{ m}^2$$

$$R_{cu} = \frac{\ell}{A \sigma_{\text{copper}}} = .983 \times 10^{-3} \Omega \quad \text{C6}$$

However, the two windings are in parallel. Thus:

$$R_{cu} = .4917 \times 10^{-3} \Omega \quad \text{C7}$$

Correction to R_c Due to Current to Flux Regeneration Constant K .

R_c was derived in Appendix B for the geometry of the INEL regeneration tests as

$R_c = 0.711 \Omega$. This is based on the assumption that 100% of the magnetic flux in the magnet gets into the conductive plasma and a value of 10^3 for the plasma σ . Also in

Appendix B it was shown that if only part of the flux makes its way into the plasma the eddy current loss will decrease. Equation C8 corrects R_e for this effect. Equation C9 separates the conductivity out from R_e . Equation C10 applies the correction to the gain constant for GG_{ed} , the nonlinear voltage controlled current source in the full-effects SPICE model that is R_e .

$$K_{MAX} = \frac{L_B}{A} = \frac{.556 \times 10^{-6}}{\pi \left(\frac{1.5}{39.37} \right)^2} = 1.219 \times 10^{-4} \text{ T/Amp} \quad \text{C8}$$

$$\zeta = \frac{K_{ACTUAL}}{K_{MAX}} \quad \text{C9}$$

$$R_e = \frac{1}{\sigma G_e}, \quad G_e = \frac{1}{\sigma R_e} \quad \text{C10}$$

$$G_{e,CORRECTED} = \frac{\zeta^2}{\sigma R_e} = \frac{\zeta^2}{2 \times 10^4 \times .711} \quad \text{C11}$$

Where: G_e is the geometry factor of R_e .

σ is the plasma conductivity.

APPENDIX D
SPICE INPUT DATA LISTINGS

Introduction

The objective of this appendix is to document the various SPICE input data files of the full-effects model used in this report. Figure C1 illustrates the complete SPICE equivalent circuit for the following SPICE input data.

SPICE Data to Simulate INEL Regeneration Test 1:

```

REGENTST1   CREATED   08/30/92
*
***          .SUBCKT CONTROL          ***
.SUBCKT Sqrt1 1 6
*
*****          SUBCIRCUIT FILE Sqrt1          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS      4 5
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1      4 0 3 0 1.5
*
***          CURRENT CONTROLLED VOLTAGE SOURCES          ***
H1      6 0 VS 3.162277K
*
***          RESISTORS          ***
RF1     1 0 1
*
***          DIODES          ***
D1      3 0 DIODE   IC=0
D2      5 0 DIODE   IC=0
*
***          VOLTAGE-CONTROLLED CURRENT SOURCES          ***
G1      0 3 1 0 .1U
*
***          DIODE MODELS          ***
.MODEL  DIODE D   TT=.1NS
.ENDS   Sqrt1
.SUBCKT FLUX 2 1 23
*
*****          SUBCIRCUIT FILE FLUX          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VCON    24 0
VP1     3 4
VP10    21 23
VP2     5 6
VP3     7 8
VP4     9 10
VP5     11 12

```

```

VP6    13 14
VP7    15 16
VP8    17 18
VP9    19 20
*
***          NONLINEAR DEPENDENT CCV SOURCES          ***
HHP1    2 3 POLY(2) VP1 VCON 0 0 0 0 1 IC = 0, 0
HHP10   20 21 POLY(2) VP10 VCON 0 0 0 0 1 IC = 0, 0
HHP2    4 5 POLY(2) VP2 VCON 0 0 0 0 1 IC = 0, 0
HHP3    6 7 POLY(2) VP3 VCON 0 0 0 0 1 IC = 0, 0
HHP4    8 9 POLY(2) VP4 VCON 0 0 0 0 1 IC = 0, 0
HHP5   10 11 POLY(2) VP5 VCON 0 0 0 0 1 IC = 0, 0
HHP6   12 13 POLY(2) VP6 VCON 0 0 0 0 1 IC = 0, 0
HHP7   14 15 POLY(2) VP7 VCON 0 0 0 0 1 IC = 0, 0
HHP8   16 17 POLY(2) VP8 VCON 0 0 0 0 1 IC = 0, 0
HHP9   18 19 POLY(2) VP9 VCON 0 0 0 0 1 IC = 0, 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
*
***          CAPACITORS          ***
C1     4 0 1.6N IC=0
C10    23 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4     10 0 1.6N IC=0
C5     12 0 1.6N IC=0
C6     14 0 1.6N IC=0
C7     16 0 1.6N IC=0
C8     18 0 1.6N IC=0
C9     20 0 1.6N IC=0
.ENDS    FLUX
.SUBCKT  DIVIS 1 3 5
*
*****          SUBCIRCUIT FILE  DIVIS          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS     2 0
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1     5 0 4 0 4
*
***          RESISTORS          ***
RF1    1 2 1
RF2    3 0 1
RF3    4 0 1MEG
*
***          CURRENT-CONTROLLED CURRENT SOURCES          ***
FIX    0 4 VS 1
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGIY   4 0 POLY(2) 4 0 3 0 0 0 0 0 4 IC = 0, 0
.ENDS    DIVIS

```

```

*
***          SUBCIRCUIT CALLS          ***
X1   14 15 16  FLUX
X2   17 15 18  FLUX
X3   23 58 25  DIVIS
X4   32 61 33  DIVIS
X5   8 36 37  DIVIS
X6   43 44 63  DIVIS
X7   46 47 48  DIVIS
X8   48 50  SQRT1
*
***          INDEPENDENT VOLTAGE SOURCES      ***
VBEFF  52 21
VBO    14 0      PULSE( 0.0 .99 0.0 0.0 0.0  )
VCD    9 0      EXP( 0.0 25K 0.0 100N 20U 30U )
VDNS   11 0      EXP( 0.0 20 0.0 100N 20U 20U )
VE      5 6
VEE    10 0      EXP( 0.0 25 0.0 100N 20U 20U )
VH     13 0      EXP( 0.0 32.5 0.0 100N 20U 20U )
VMHD   2 3
VOS    65 0      PULSE( 0.0 1 0.0 10N 0.0  )
VP     12 0      EXP( 0.0 150 0.0 100N 20U 20U )
VSS    5 7
VU     8 0      EXP( 0.0 68K 0.0 100N 20U 2U )
VCDC   63 45
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES      ***
E2    42 0 12 0 6
E3    23 0 55 0 1U
E4    61 0 24 0 1M
E5    64 0 25 0 1MEG
E6    58 0 11 0 16.144U
E7    36 0 59 0 1
E8    0 57 56 0 1MEG
*
***          CURRENT CONTROLLED VOLTAGE SOURCES      ***
H1    17 0 VSS 1.7U
H2    26 0 VMHD 1
H3    28 0 VE 1
*
***          NONLINEAR DEPENDENT VCV SOURCES      ***
EE1    4 3 POLY(2) 18 0 22 0 0 0 0 0 50.8M IC = 0, 0
EE10   41 0 POLY(2) 11 0 10 0 0 0 0 0 -8 IC = 0, 0
EE11   43 0 POLY(2) 8 0 53 0 0 0 0 0 2U IC = 0, 0
EE12   44 0 POLY(2) 8 0 59 0 0 3U 2U IC = 0, 0
EE13   46 0 POLY(3) 38 0 35 0 45 0 0 0 0 -10U 0 10U
+IC = 0, 0, 0
EE14   47 0 POLY(2) 38 0 10 0 0 0 0 0 10U IC = 0, 0
EE15   15 0 POLY(2) 9 0 68 0 0 0 0 0 1.27M IC = 0, 0
EE16   66 0 POLY(3) 0 65 50 0 65 0 0 1 0 0 0 0 0 1
+IC = 0, 0, 0
EE17   59 0 POLY(2) 8 0 0 57 0 1 1 IC = 0, 0
EE18   0 60 POLY(2) 24 0 0 57 0 0 0 0 4 IC = 0, 0
EE19   68 0 POLY(2) 67 0 65 0 1 1 IC = 0

```



```

EE2    1 0 POLY(2) 16 0 22 0 0 0 0 0 25.4M IC = 0, 0
EE4    24 0 POLY(2) 8 0 11 0 0 0 0 0 .5067M IC = 0, 0
EE5    32 0 POLY(3) 31 0 29 0 27 0 0 1U -1U -1U IC = 0, 0, 0
EE6    34 0 POLY(2) 57 0 8 0 0 0 0 .5U -1U IC = 0, 0
EE7    39 0 POLY(2) 38 0 10 0 0 0 0 0 4 IC = 0, 0
EE77   35 0 POLY(3) 13 0 33 0 34 0 0 1 -1M 1 IC = 0, 0, 0
EE8    40 0 POLY(2) 11 0 35 0 0 0 0 0 4 IC = 0, 0
EE9    38 0 POLY(2) 37 0 11 0 0 0 0 0 1 IC = 0, 0
EEBEF  52 0 POLY(2) 16 0 18 0 0 1 2 IC = 0, 0
EED    29 0 POLY(2) 28 0 7 0 0 0 0 0 1 IC = 0, 0
EEMHD  31 0 POLY(2) 26 0 30 0 0 0 0 0 1 IC = 0, 0
EEP    0 20 POLY(2) 12 0 45 0 0 2.027K -2.027K IC = 0, 0
EEPL   27 0 POLY(2) 26 0 2 1 0 0 0 0 1 IC = 0, 0
EETOT  30 0 POLY(2) 1 0 4 3 0 1 1 IC = 0, 0
EEUEF  22 0 POLY(2) 8 0 0 57 0 1 .5 IC = 0, 0
HHVEL  19 0 POLY(2) VMHD VBEFF 0 0 0 0 101.6M IC = 0, 0

```

*

RESISTORS

```

RCU    4 5 1M
RF01   8 0 1
RF02   9 0 1
RF03   10 0 1
RF04   11 0 1
RF05   12 0 1
RF06   13 0 1
RF07   19 55 1
RF08   20 55 1
RF09   21 0 1
RF10   22 0 1
RF11   25 0 1
RF12   26 0 1
RF13   27 0 1
RF14   28 0 1
RF15   29 0 1
RF16   30 0 1
RF17   31 0 1
RF18   33 0 1
RF19   34 0 1
RF20   35 0 1
RF21   37 0 1
RF22   38 0 1
RF23   39 53 1
RF24   40 53 1
RF25   41 53 1
RF26   42 53 1
RF27   48 0 1
RF28   50 0 1
RF29   65 0 1
RF30   45 0 1
RF31   64 56 1
RF32   66 67 1
RF33   24 0 1
RF34   55 0 1
RF35   60 55 1

```

```

RF36  59 0 1
RF37  68 0 1
***          CAPACITORS          ***
C1    16 0 .8N IC=0
C2    18 0 .8N IC=0
C3    56 57 .25 IC=0
C4    67 0 .1U IC=0
*
***          INDUCTORS           ***
LB    7 0 .56U IC=0
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGED 6 0 POLY(3) 6 0 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 104N
+IC = 0, 0, 0
GGP  1 2 POLY(3) 1 2 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 75M
+IC = 0, 0, 0
*
***          ANALYSIS CONTROL INFORMATION          ***
*
***          TRANSIENT ANALYSIS          ***
.TRAN .500000E-6 20U 0 UIC
*
***          PLOT CONTROL          ***
.PLOT  TRAN  V(0,4)
.PLOT  TRAN  I(VMHD)
.PLOT  TRAN  V(16)
.PLOT  TRAN  V(18)
.PLOT  TRAN  V(21)
.PLOT  TRAN  V(57)
.PLOT  TRAN  V(59)
.PLOT  TRAN  V(22)
.PLOT  TRAN  V(35)
.PLOT  TRAN  V(38)
.PLOT  TRAN  V(45)
.PLOT  TRAN  V(64)
.PLOT  TRAN  V(48)
.PLOT  TRAN  V(68)
*
***          OPTIONS CONTROL          ***
.OPTIONS          ITL5=0
+
**
***** END OF DATA DECK *****
.END
*

```

SPICE Input Data to Simulate INEL Regeneration Test 2:

```

REGENTST2   CREATED   08/30/92
*
***               .SUBCKT CONTROL               ***
.SUBCKT SQRT1 1 6
*
*****          SUBCIRCUIT FILE SQRT1          *****
*
***               INDEPENDENT VOLTAGE SOURCES               ***
VS      4 5
*
***               VOLTAGE CONTROLLED VOLTAGE SOURCES               ***
E1      4 0 3 0 1.5
*
***               CURRENT CONTROLLED VOLTAGE SOURCES               ***
H1      6 0 VS 3.162277K
*
***               RESISTORS               ***
RF1     1 0 1
*
***               DIODES               ***
D1      3 0 DIODE IC=0
D2      5 0 DIODE IC=0
*
***               VOLTAGE-CONTROLLED CURRENT SOURCES               ***
G1      0 3 1 0 .1U
*
***               DIODE MODELS               ***
.MODEL DIODE D TT=.1NS
.ENDS SQRT1
.SUBCKT FLUX 2 1 23
*
*****          SUBCIRCUIT FILE FLUX          *****
*
***               INDEPENDENT VOLTAGE SOURCES               ***
VCON    24 0
VP1     3 4
VP10    21 23
VP2     5 6
VP3     7 8
VP4     9 10
VP5     11 12
VP6     13 14
VP7     15 16
VP8     17 18
VP9     19 20
*
***               NONLINEAR DEPENDENT CCV SOURCES               ***
HHP1    2 3 POLY(2) VP1 VCON 0 0 0 0 1 IC = 0, 0
HHP10   20 21 POLY(2) VP10 VCON 0 0 0 0 1 IC = 0, 0
HHP2    4 5 POLY(2) VP2 VCON 0 0 0 0 1 IC = 0, 0
HHP3    6 7 POLY(2) VP3 VCON 0 0 0 0 1 IC = 0, 0

```

```

HHP4  8 9 POLY(2) VP4 VCON 0 0 0 0 1 IC = 0, 0
HHP5  10 11 POLY(2) VP5 VCON 0 0 0 0 1 IC = 0, 0
HHP6  12 13 POLY(2) VP6 VCON 0 0 0 0 1 IC = 0, 0
HHP7  14 15 POLY(2) VP7 VCON 0 0 0 0 1 IC = 0, 0
HHP8  16 17 POLY(2) VP8 VCON 0 0 0 0 1 IC = 0, 0
HHP9  18 19 POLY(2) VP9 VCON 0 0 0 0 1 IC = 0, 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
*
***          CAPACITORS          ***
C1     4 0 1.6N IC=0
C10    23 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4     10 0 1.6N IC=0
C5     12 0 1.6N IC=0
C6     14 0 1.6N IC=0
C7     16 0 1.6N IC=0
C8     18 0 1.6N IC=0
C9     20 0 1.6N IC=0
.ENDS  FLUX
.SUBCKT  DIVIS 1 3 5
*
*****          SUBCIRCUIT FILE  DIVIS          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS     2 0
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1     5 0 4 0 1
*
***          RESISTORS          ***
RF1    1 2 1
RF2    3 0 1
RF3    4 0 1MEG
*
***          CURRENT-CONTROLLED CURRENT SOURCES          ***
FIX    0 4 VS 1
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGIY   4 0 POLY(2) 4 0 3 0 0 0 0 0 1 IC = 0, 0
.ENDS  DIVIS
*
***          SUBCIRCUIT CALLS          ***
X1     14 15 16  FLUX
X2     17 15 18  FLUX
X3     23 58 25  DIVIS
X4     32 61 33  DIVIS
X5     8 36 37  DIVIS
X6     43 44 63  DIVIS
X7     46 47 48  DIVIS
X8     48 50  SQRT1

```

```

***          INDEPENDENT VOLTAGE SOURCES          ***
VBEFF  52 21
VBO    14 0      PULSE( 0.0 .95 0.0 0.0 0.0 )
VCD    9 0      EXP( 0.0 20K 0.0 100N 25.85U 30U )
VDNS   11 0      EXP( 0.0 20 0.0 100N 25.85U 20U )
VE     5 6
VEE    10 0      EXP( 0.0 25 0.0 100N 25.85U 20U )
VH     13 0      EXP( 0.0 32.5 0.0 100N 25.85U 20U )
VMHD   2 3
VOS    65 0      PULSE( 0.0 1 0.0 0.0 0.0 )
VP     12 0      EXP( 0.0 150 0.0 100N 25.85U 20U )
VSS    5 7
VU     8 0      EXP( 0.0 50K 0.0 100N 25.85U 2U )
VCDC   63 45
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES      ***
E2     42 0 12 0 6
E3     23 0 55 0 1U
E4     61 0 24 0 1M
E5     64 0 25 0 1MEG
E6     58 0 11 0 .159M
E7     36 0 59 0 1
E8     0 57 56 0 1MEG
*
***          CURRENT CONTROLLED VOLTAGE SOURCES      ***
H1     17 0 VSS 3.15U
H2     26 0 VMHD 1
H3     28 0 VE 1
*
***          NONLINEAR DEPENDENT VCV SOURCES        ***
EE1    4 3 POLY(2) 18 0 22 0 0 0 0 0 50.8M IC = 0, 0
EE10   41 0 POLY(2) 11 0 10 0 0 0 0 0 -8 IC = 0, 0
EE11   43 0 POLY(2) 8 0 53 0 0 0 0 0 2U IC = 0, 0
EE12   44 0 POLY(2) 8 0 59 0 0 2U 3U IC = 0, 0
EE13   46 0 POLY(3) 38 0 35 0 45 0 0 0 0 -10U 0 10U
+IC = 0, 0, 0
EE14   47 0 POLY(2) 38 0 10 0 0 0 0 0 10U IC = 0, 0
EE15   15 0 POLY(2) 9 0 68 0 0 0 0 0 1.27M IC = 0, 0
EE16   66 0 POLY(3) 0 65 50 0 65 0 0 1 0 0 0 0 0 1
+IC = 0, 0, 0
EE17   59 0 POLY(2) 8 0 0 57 0 1 1 IC = 0, 0
EE18   0 60 POLY(2) 24 0 0 57 0 0 0 0 4 IC = 0, 0
EE19   68 0 POLY(2) 67 0 65 0 1 1 IC = 0
EE2    1 0 POLY(2) 16 0 22 0 0 0 0 0 25.4M IC = 0, 0
EE4    24 0 POLY(2) 8 0 11 0 0 0 0 0 .5067M IC = 0, 0
EE5    32 0 POLY(3) 31 0 29 0 27 0 0 1U -1U -1U IC = 0, 0, 0
EE6    34 0 POLY(2) 57 0 8 0 0 0 0 .5U -1U IC = 0, 0
EE7    39 0 POLY(2) 38 0 10 0 0 0 0 0 4 IC = 0, 0
EE77   35 0 POLY(3) 13 0 33 0 34 0 0 1 -1M 1 IC = 0, 0, 0
EE8    40 0 POLY(2) 11 0 35 0 0 0 0 0 4 IC = 0, 0
EE9    38 0 POLY(2) 37 0 11 0 0 0 0 0 1 IC = 0, 0
EEBEF  52 0 POLY(2) 16 0 18 0 0 1 2 IC = 0, 0
EED    29 0 POLY(2) 28 0 7 0 0 0 0 0 1 IC = 0, 0
EEMHD  31 0 POLY(2) 26 0 30 0 0 0 0 0 1 IC = 0, 0

```

```

EEP      0 20 POLY(2) 12 0 45 0 0 2.027K -2.027K IC = 0, 0
EEPL     27 0 POLY(2) 26 0 2 1 0 0 0 0 1 IC = 0, 0
EETOT    30 0 POLY(2) 1 0 4 3 0 1 1 IC = 0, 0
EEUEF    22 0 POLY(2) 8 0 0 57 0 1 .5 IC = 0, 0
HHVEL    19 0 POLY(2) VMHD VBEFF 0 0 0 0 101.6M IC = 0, 0

```

*

RESISTORS

```

RCU      4 5 1M
RF01     8 0 1
RF02     9 0 1
RF03     10 0 1
RF04     11 0 1
RF05     12 0 1
RF06     13 0 1
RF07     19 55 1
RF08     20 55 1
RF09     21 0 1
RF10     22 0 1
RF11     25 0 1
RF12     26 0 1
RF13     27 0 1
RF14     28 0 1
RF15     29 0 1
RF16     30 0 1
RF17     31 0 1
RF18     33 0 1
RF19     34 0 1
RF20     35 0 1
RF21     37 0 1
RF22     38 0 1
RF23     39 53 1
RF24     40 53 1
RF25     41 53 1
RF26     42 53 1
RF27     48 0 1
RF28     50 0 1
RF29     65 0 1
RF30     45 0 1
RF31     64 56 1
RF32     66 67 1
RF33     24 0 1
RF34     55 0 1
RF35     60 55 1
RF36     59 0 1
RF37     68 0 1

```

*

CAPACITORS

```

C1       16 0 .8N IC=0
C2       18 0 .8N IC=0
C3       56 57 1 IC=0
C4       67 0 .1U IC=0

```

*

INDUCTORS

```

LB       7 0 ..445U IC=0

```

```

*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGED 6 0 POLY(3) 6 0 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 .9143U +IC = 0, 0, 0
GGP  1 2 POLY(3) 1 2 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 6.25M +IC = 0, 0, 0
*
***          ANALYSIS CONTROL INFORMATION          ***
*
***          TRANSIENT ANALYSIS          ***
.TRAN .500000E-6 20U 0 UIC
*
***          PLOT CONTROL
*
***
.PLOT  TRAN  V(0,4)
.PLOT  TRAN  I(VMHD)
.PLOT  TRAN  V(16)
.PLOT  TRAN  V(18)
.PLOT  TRAN  V(21)
.PLOT  TRAN  V(57)
.PLOT  TRAN  V(59)
.PLOT  TRAN  V(22)
.PLOT  TRAN  V(35)
.PLOT  TRAN  V(38)
.PLOT  TRAN  V(45)
.PLOT  TRAN  V(64)
.PLOT  TRAN  V(48)
.PLOT  TRAN  V(68)
*
*
***          OPTIONS CONTROL          ***
.OPTIONS          ITL5=0
+
*
**
***** END OF DATA DECK *****
.END
*
*

```

SPICE Input Data File to Simulate Advanced Performance

```

ADVANPERF  CREATED  08/30/92
*
***          .SUBCKT CONTROL          ***
.SUBCKT SQRT1 1 6
*
*****          SUBCIRCUIT FILE  SQRT1          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS      4 5
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1      4 0 3 0 1.5
*
***          CURRENT CONTROLLED VOLTAGE SOURCES          ***
H1      6 0 VS 3.162277K
*
***          RESISTORS          ***
RF1     1 0 1
*
***          DIODES          ***
D1      3 0 DIODE  IC=0
D2      5 0 DIODE  IC=0
*
***          VOLTAGE-CONTROLLED CURRENT SOURCES          ***
G1      0 3 1 0 .1U
*
***          DIODE MODELS          ***
.MODEL  DIODE D  TT=.1NS
.ENDS    SQRT1
.SUBCKT FLUX 2 1 23
*
*****          SUBCIRCUIT FILE  FLUX          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VCON    24 0
VP1     3 4
VP10    21 23
VP2     5 6
VP3     7 8
VP4     9 10
VP5     11 12
VP6     13 14
VP7     15 16
VP8     17 18
VP9     19 20
*
***          NONLINEAR DEPENDENT CCV SOURCES          ***
HHP1    2 3 POLY(2) VP1 VCON 0 0 0 0 1 IC = 0, 0
HHP10   20 21 POLY(2) VP10 VCON 0 0 0 0 1 IC = 0, 0
HHP2    4 5 POLY(2) VP2 VCON 0 0 0 0 1 IC = 0, 0
HHP3    6 7 POLY(2) VP3 VCON 0 0 0 0 1 IC = 0, 0

```



```

HHP4  8 9 POLY(2) VP4 VCON 0 0 0 0 1 IC = 0, 0
HHP5  10 11 POLY(2) VP5 VCON 0 0 0 0 1 IC = 0, 0
HHP6  12 13 POLY(2) VP6 VCON 0 0 0 0 1 IC = 0, 0
HHP7  14 15 POLY(2) VP7 VCON 0 0 0 0 1 IC = 0, 0
HHP8  16 17 POLY(2) VP8 VCON 0 0 0 0 1 IC = 0, 0
HHP9  18 19 POLY(2) VP9 VCON 0 0 0 0 1 IC = 0, 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
*
***          CAPACITORS          ***
C1     4 0 1.6N IC=0
C10    23 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4     10 0 1.6N IC=0
C5     12 0 1.6N IC=0
C6     14 0 1.6N IC=0
C7     16 0 1.6N IC=0
C8     18 0 1.6N IC=0
C9     20 0 1.6N IC=0
.ENDS  FLUX
.SUBCKT  DIVIS 1 3 5
*
*****          SUBCIRCUIT FILE  DIVIS          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS     2 0
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1     5 0 4 0 1
*
***          RESISTORS          ***
RF1    1 2 1
RF2    3 0 1
RF3    4 0 1MEG
*
***          CURRENT-CONTROLLED CURRENT SOURCES          ***
FIX    0 4 VS 1
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGIY   4 0 POLY(2) 4 0 3 0 0 0 0 0 1 IC = 0, 0
.ENDS  DIVIS
*
***          SUBCIRCUIT CALLS          ***
X1     14 15 16  FLUX
X2     17 15 18  FLUX
X3     23 58 25  DIVIS
X4     32 61 33  DIVIS
X5     8 36 37  DIVIS
X6     43 44 63  DIVIS
X7     46 47 48  DIVIS
X8     48 50  SQRT1

```

*** INDEPENDENT VOLTAGE SOURCES ***

VBEFF 52 21
 VBO 14 0 PULSE(0.0 1 0.0 0.0 0.0)
 VCD 9 0 EXP(0.0 25K 0.0 100N 50U 30U)
 VDNS 11 0 EXP(0.0 20 0.0 100N 50U 20U)
 VE 5 6
 VEE 10 0 EXP(0.0 25 0.0 100N 50U 20U)
 VH 13 0 EXP(0.0 32.5 0.0 100N 50U 20U)
 VMHD 2 3
 VOS 65 0 PULSE(0.0 1 0.0 0.0 0.0)
 VP 12 0 EXP(0.0 150 0.0 100N 50U 20U)
 VSS 5 7
 VU 8 0 EXP(0.0 60K 0.0 100N 50U 2U)
 VCDC 63 45

*** VOLTAGE CONTROLLED VOLTAGE SOURCES ***

E2 42 0 12 0 6
 E3 23 0 55 0 1U
 E4 61 0 24 0 1M
 E5 64 0 25 0 1MEG
 E6 58 0 11 0 .159M
 E7 36 0 59 0 1
 E8 0 57 56 0 1MEG

*** CURRENT CONTROLLED VOLTAGE SOURCES ***

H1 17 0 VSS 6.6U
 H2 26 0 VMHD 1
 H3 28 0 VE 1

*** NONLINEAR DEPENDENT VCV SOURCES ***

EE1 4 3 POLY(2) 18 0 22 0 0 0 0 0 50.8M IC = 0, 0
 EE10 41 0 POLY(2) 11 0 10 0 0 0 0 0 -8 IC = 0, 0
 EE11 43 0 POLY(2) 8 0 53 0 0 0 0 0 2U IC = 0, 0
 EE12 44 0 POLY(2) 8 0 59 0 0 2U 3U IC = 0, 0
 EE13 46 0 POLY(3) 38 0 35 0 45 0 0 0 0 -10U 0 10U IC = 0, 0, 0
 EE14 47 0 POLY(2) 38 0 10 0 0 0 0 0 10U IC = 0, 0
 EE15 15 0 POLY(2) 9 0 68 0 0 0 0 0 1.27M IC = 0, 0
 EE16 66 0 POLY(3) 0 65 50 0 65 0 0 1 0 0 0 0 0 1 IC = 0, 0, 0
 EE17 59 0 POLY(2) 8 0 0 57 0 1 1 IC = 0, 0
 EE18 0 60 POLY(2) 24 0 0 57 0 0 0 0 4 IC = 0, 0
 EE19 68 0 POLY(2) 67 0 65 0 1 1 IC = 0
 EE2 1 0 POLY(2) 16 0 22 0 0 0 0 0 25.4M IC = 0, 0
 EE4 24 0 POLY(2) 8 0 11 0 0 0 0 0 .5067M IC = 0, 0
 EE5 32 0 POLY(3) 31 0 29 0 27 0 0 1U -1U -1U IC = 0, 0, 0
 EE6 34 0 POLY(2) 57 0 8 0 0 0 0 .5U -1U IC = 0, 0
 EE7 39 0 POLY(2) 38 0 10 0 0 0 0 0 4 IC = 0, 0
 EE77 35 0 POLY(3) 13 0 33 0 34 0 0 1 -1M 1 IC = 0, 0, 0
 EE8 40 0 POLY(2) 11 0 35 0 0 0 0 0 4 IC = 0, 0
 EE9 38 0 POLY(2) 37 0 11 0 0 0 0 0 1 IC = 0, 0
 EEDEF 52 0 POLY(2) 16 0 18 0 0 1 2 IC = 0, 0
 EED 29 0 POLY(2) 28 0 7 0 0 0 0 0 1 IC = 0, 0
 EEMHD 31 0 POLY(2) 26 0 30 0 0 0 0 0 1 IC = 0, 0
 EEP 0 20 POLY(2) 12 0 45 0 0 2.027K -2.027K IC = 0, 0
 EEP 27 0 POLY(2) 26 0 2 1 0 0 0 0 1 IC = 0, 0

EETOT 30 0 POLY(2) 1 0 4 3 0 1 1 IC = 0, 0
 EEUEF 22 0 POLY(2) 8 0 0 57 0 1 .5 IC = 0, 0
 HHVEL 19 0 POLY(2) VMHD VBEFF 0 0 0 0 101.6M IC = 0, 0

*

RESISTORS

RCU 4 5 1M
 RF01 8 0 1
 RF02 9 0 1
 RF03 10 0 1
 RF04 11 0 1
 RF05 12 0 1
 RF06 13 0 1
 RF07 19 55 1
 RF08 20 55 1
 RF09 21 0 1
 RF10 22 0 1
 RF11 25 0 1
 RF12 26 0 1
 RF13 27 0 1
 RF14 28 0 1
 RF15 29 0 1
 RF16 30 0 1
 RF17 31 0 1
 RF18 33 0 1
 RF19 34 0 1
 RF20 35 0 1
 RF21 37 0 1
 RF22 38 0 1
 RF23 39 53 1
 RF24 40 53 1
 RF25 41 53 1
 RF26 42 53 1
 RF27 48 0 1
 RF28 50 0 1
 RF29 65 0 1
 RF30 45 0 1
 RF31 64 56 1
 RF32 66 67 1
 RF33 24 0 1
 RF34 55 0 1
 RF35 60 55 1
 RF36 59 0 1
 RF37 68 0 1

*

CAPACITORS

C1 16 0 .8N IC=0
 C2 18 0 .8N IC=0
 C3 56 57 1 IC=0
 C4 67 0 .1U IC=0

*

INDUCTORS

LB 7 0 .56U IC=0

*

```

***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGED 6 0 POLY(3) 6 0 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.6U
+IC = 0, 0, 0
GGP  1 2 POLY(3) 1 2 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 75M
+IC = 0, 0, 0
*
***          ANALYSIS CONTROL INFORMATION          ***
*
***          TRANSIENT ANALYSIS          ***
.TRAN 1.000000E-6 75U 0 UIC
*
***          PLOT CONTROL
.PLOT  TRAN  V(0,4)
.PLOT  TRAN  I(VMHD)
.PLOT  TRAN  V(16)
.PLOT  TRAN  V(18)
.PLOT  TRAN  V(21)
.PLOT  TRAN  V(57)
.PLOT  TRAN  V(59)
.PLOT  TRAN  V(22)
.PLOT  TRAN  V(35)
.PLOT  TRAN  V(38)
.PLOT  TRAN  V(45)
.PLOT  TRAN  V(64)
.PLOT  TRAN  V(48)
.PLOT  TRAN  V(68)
*
***          OPTIONS CONTROL          ***
.OPTIONS                                ITL5=0
+
***** END OF DATA DECK *****
.END
*
*

```

SPICE Input Data File for Integrater Test Circuit.

```

INTERGRATE
*
***      INDEPENDENT VOLTAGE SOURCES      ***
VIN      1 0      PULSE( 0.0 1.0 0.0 1NS 0.0  )
*
***      VOLTAGE CONTROLLED VOLTAGE SOURCES      ***
E1      0 3 2 0 1MEG
E2      0 4 3 0 1
*
***      RESISTORS      ***
RF1      1 2 1
RF2      4 0 1
*
***      CAPACITORS      ***
C1      2 3 1
*
***      ANALYSIS CONTROL INFORMATION      ***
*
***      TRANSIENT ANALYSIS      ***
.TRAN 5.000000E-6 50U 0
*
***      PLOT CONTROL      ***
.PLOT  TRAN  V(3)
.PLOT  TRAN  V(4)
.PLOT  TRAN  V(1)
**
***** END OF DATA DECK *****
.END
*
*

```

SPICE Input Data File for Division and 3/2 Power Test Circuit.

```

ADVANPERF   CREATED   08/30/92
*
***               .SUBCKT CONTROL               ***
.SUBCKT SQRT1 1 6
*
*****               SUBCIRCUIT FILE SQRT1               *****
*
***               INDEPENDENT VOLTAGE SOURCES               ***
VS      4 5
*
***               VOLTAGE CONTROLLED VOLTAGE SOURCES               ***
E1      4 0 3 0 1.5
*
***               CURRENT CONTROLLED VOLTAGE SOURCES               ***
H1      6 0 VS 3.162277K
*
***               RESISTORS               ***
RF1     1 0 1
*
***               DIODES               ***
D1      3 0 DIODE   IC=0
D2      5 0 DIODE   IC=0
*
***               VOLTAGE-CONTROLLED CURRENT SOURCES               ***
G1      0 3 1 0 .1U
*
***               DIODE MODELS               ***
.MODEL  DIODE D  TT=.1NS
.ENDS    SQRT1
.SUBCKT FLUX 2 1 23
*
*****               SUBCIRCUIT FILE FLUX               *****
*
***               INDEPENDENT VOLTAGE SOURCES               ***
VCON    24 0
VP1     3 4
VP10    21 23
VP2     5 6
VP3     7 8
VP4     9 10
VP5     11 12
VP6     13 14
VP7     15 16
VP8     17 18
VP9     19 20
*
***               NONLINEAR DEPENDENT CCV SOURCES               ***
HHP1    2 3 POLY(2) VP1 VCON 0 0 0 0 1 IC = 0, 0
HHP10   20 21 POLY(2) VP10 VCON 0 0 0 0 1 IC = 0, 0
HHP2    4 5 POLY(2) VP2 VCON 0 0 0 0 1 IC = 0, 0
HHP3    6 7 POLY(2) VP3 VCON 0 0 0 0 1 IC = 0, 0

```

```

HHP4  8 9 POLY(2) VP4 VCON 0 0 0 0 1 IC = 0, 0
HHP5  10 11 POLY(2) VP5 VCON 0 0 0 0 1 IC = 0, 0
HHP6  12 13 POLY(2) VP6 VCON 0 0 0 0 1 IC = 0, 0
HHP7  14 15 POLY(2) VP7 VCON 0 0 0 0 1 IC = 0, 0
HHP8  16 17 POLY(2) VP8 VCON 0 0 0 0 1 IC = 0, 0
HHP9  18 19 POLY(2) VP9 VCON 0 0 0 0 1 IC = 0, 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
*
***          CAPACITORS          ***
C1     4 0 1.6N IC=0
C10    23 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4     10 0 1.6N IC=0
C5     12 0 1.6N IC=0
C6     14 0 1.6N IC=0
C7     16 0 1.6N IC=0
C8     18 0 1.6N IC=0
C9     20 0 1.6N IC=0
.ENDS  FLUX
.SUBCKT  DIVIS 1 3 5
*
*****          SUBCIRCUIT FILE  DIVIS          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS     2 0
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1     5 0 4 0 4
*
***          RESISTORS          ***
RF1    1 2 1
RF2    3 0 1
RF3    4 0 1MEG
*
***          CURRENT-CONTROLLED CURRENT SOURCES          ***
FIX    0 4 VS 1
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGIY   4 0 POLY(2) 4 0 3 0 0 0 0 4 IC = 0, 0
.ENDS  DIVIS
*
***          SUBCIRCUIT CALLS          ***
X1     14 15 16  FLUX
X2     17 15 18  FLUX
X3     23 58 25  DIVIS
X4     32 61 33  DIVIS
X5     8 36 37  DIVIS
X6     43 44 63  DIVIS
X7     46 47 48  DIVIS
X8     48 50  SQRT1

```

```

*
***          INDEPENDENT VOLTAGE SOURCES          ***
VBEFF  52 21
VBO    14 0      PULSE( 0.0 1 0.0 0.0 0.0 )
VCD    9 0      EXP( 0.0 25K 0.0 100N 75U 30U )
VDNS   11 0      EXP( 0.0 20 0.0 100N 75U 20U )
VE     5 6
VEE    10 0      EXP( 0.0 25 0.0 100N 75U 20U )
VH     13 0      EXP( 0.0 32.5 0.0 100N 75U 20U )
VMHD   2 3
VOS    65 0      PULSE( 0.0 1 0.0 0.0 0.0 )
VP     12 0      EXP( 0.0 150 0.0 100N 75U 20U )
VSS    5 7
VU     8 0      EXP( 0.0 60K 0.0 100N 75U 2U )
VCDC   63 45
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES    ***
E2     42 0 12 0 6
E3     23 0 55 0 1U
E4     61 0 24 0 1M
E5     64 0 25 0 1MEG
E6     58 0 11 0 16.144U
E7     36 0 59 0 1
E8     0 57 56 0 1MEG
*
***          CURRENT CONTROLLED VOLTAGE SOURCES     ***
H1     17 0 VSS 6.6U
H2     26 0 VMHD 1
H3     28 0 VE 1
*
***          NONLINEAR DEPENDENT VCV SOURCES       ***
EE1    4 3 POLY(2) 18 0 22 0 0 0 0 0 50.8M IC = 0, 0
EE10   41 0 POLY(2) 11 0 10 0 0 0 0 0 -8 IC = 0, 0
EE11   43 0 POLY(2) 8 0 53 0 0 0 0 0 2U IC = 0, 0
EE12   44 0 POLY(2) 8 0 59 0 0 3U 2U IC = 0, 0
EE13   46 0 POLY(3) 38 0 35 0 45 0 0 0 0 -10U 0 10U IC = 0, 0, 0
EE14   47 0 POLY(2) 38 0 10 0 0 0 0 0 10U IC = 0, 0
EE15   15 0 POLY(2) 9 0 68 0 0 0 0 0 1.27M IC = 0, 0
EE16   66 0 POLY(3) 0 65 50 0 65 0 0 1 0 0 0 0 0 1 IC = 0, 0, 0
EE17   59 0 POLY(2) 8 0 0 57 0 1 1 IC = 0, 0
EE18   0 60 POLY(2) 24 0 0 57 0 0 0 0 4 IC = 0, 0
EE19   68 0 POLY(2) 67 0 65 0 1 1 IC = 0
EE2    1 0 POLY(2) 16 0 22 0 0 0 0 0 25.4M IC = 0, 0
EE4    24 0 POLY(2) 8 0 11 0 0 0 0 0 .5067M IC = 0, 0
EE5    32 0 POLY(3) 31 0 29 0 27 0 0 1U -1U -1U IC = 0, 0, 0
EE6    34 0 POLY(2) 57 0 8 0 0 0 0 .5U -1U IC = 0, 0
EE7    39 0 POLY(2) 38 0 10 0 0 0 0 0 4 IC = 0, 0
EE77   35 0 POLY(3) 13 0 33 0 34 0 0 1 -1M 1 IC = 0, 0, 0
EE8    40 0 POLY(2) 11 0 35 0 0 0 0 0 4 IC = 0, 0
EE9    38 0 POLY(2) 37 0 11 0 0 0 0 0 1 IC = 0, 0
EEBEF  52 0 POLY(2) 16 0 18 0 0 1 2 IC = 0, 0
EED    29 0 POLY(2) 28 0 7 0 0 0 0 0 1 IC = 0, 0
EEMHD  31 0 POLY(2) 26 0 30 0 0 0 0 0 1 IC = 0, 0
EEP    0 20 POLY(2) 12 0 45 0 0 2.027K -2.027K IC = 0, 0

```


EEPL 27 0 POLY(2) 26 0 2 1 0 0 0 0 1 IC = 0, 0
 EETOT 30 0 POLY(2) 1 0 4 3 0 1 1 IC = 0, 0
 EEUEF 22 0 POLY(2) 8 0 0 57 0 1.5 IC = 0, 0
 HHVEL 19 0 POLY(2) VMHD VBEFF 0 0 0 0 101.6M IC = 0, 0

*

RESISTORS

RCU 4 5 1M
 RF01 8 0 1
 RF02 9 0 1
 RF03 10 0 1
 RF04 11 0 1
 RF05 12 0 1
 RF06 13 0 1
 RF07 19 55 1
 RF08 20 55 1
 RF09 21 0 1
 RF10 22 0 1
 RF11 25 0 1
 RF12 26 0 1
 RF13 27 0 1
 RF14 28 0 1
 RF15 29 0 1
 RF16 30 0 1
 RF17 31 0 1
 RF18 33 0 1
 RF19 34 0 1
 RF20 35 0 1
 RF21 37 0 1
 RF22 38 0 1
 RF23 39 53 1
 RF24 40 53 1
 RF25 41 53 1
 RF26 42 53 1
 RF27 48 0 1
 RF28 50 0 1
 RF29 65 0 1
 RF30 45 0 1
 RF31 64 56 1
 RF32 66 67 1
 RF33 24 0 1
 RF34 55 0 1
 RF35 60 55 1
 RF36 59 0 1
 RF37 68 0 1

*

CAPACITORS

C1 16 0 .8N IC=0
 C2 18 0 .8N IC=0
 C3 56 57 .25 IC=0
 C4 67 0 .1U IC=0

*

INDUCTORS

LB 7 0 .5U IC=0

*

```

***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGED 6 0 POLY(3) 6 0 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.6U
+IC = 0, 0, 0
GGP 1 2 POLY(3) 1 2 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 75M
+IC = 0, 0, 0
*
***          ANALYSIS CONTROL INFORMATION              ***
*
***          TRANSIENT ANALYSIS                        ***
.TRAN 1.000000E-6 70U 0 UIC
*
***          PLOT CONTROL
.PLOT  TRAN  V(0,4)
.PLOT  TRAN  I(VMHD)
.PLOT  TRAN  V(23)
.PLOT  TRAN  V(58)
.PLOT  TRAN  V(25)
.PLOT  TRAN  V(32)
.PLOT  TRAN  V(61)
.PLOT  TRAN  V(33)
.PLOT  TRAN  V(8)
.PLOT  TRAN  V(36)
.PLOT  TRAN  V(37)
.PLOT  TRAN  V(43)
.PLOT  TRAN  V(44)
.PLOT  TRAN  V(45)
.PLOT  TRAN  V(46)
.PLOT  TRAN  V(47)
.PLOT  TRAN  V(48)
.PLOT  TRAN  V(50)
*
***          OPTIONS CONTROL                            ***
.OPTIONS                                ITL5=0
***** END OF DATA DECK *****
.END

```

SPICE Input Data to Simulate Flux Diffusion Model Step Response.

CHKFLUXSTEP

*** INDEPENDENT VOLTAGE SOURCES ***

V6IN 1 0 PULSE(0.0 25.4 0.0 1N 0.0)

VBIN 2 0 PULSE(0.0 1 0.0 0.0 0.0)

VCON 24 0

VDBIN 4 25

VDBOT 23 26

VP1 3 4

VP10 21 23

VP2 5 6

VP3 7 8

VP4 9 10

VP5 11 12

VP6 13 14

VP7 15 16

VP8 17 18

VP9 19 20

*

*** CURRENT CONTROLLED VOLTAGE SOURCES ***

HIN 28 0 VDBIN 625MEG

HOUT 27 0 VDBOT 1.25G

*

*** NONLINEAR DEPENDENT CCV SOURCES ***

HHP1 2 3 POLY(2) VP1 VCON 0 0 0 0 1 0 0

HHP10 20 21 POLY(2) VP10 VCON 0 0 0 0 1 0 0

HHP2 4 5 POLY(2) VP2 VCON 0 0 0 0 1 0 0

HHP3 6 7 POLY(2) VP3 VCON 0 0 0 0 1 0 0

HHP4 8 9 POLY(2) VP4 VCON 0 0 0 0 1 0 0

HHP5 10 11 POLY(2) VP5 VCON 0 0 0 0 1 0 0

HHP6 12 13 POLY(2) VP6 VCON 0 0 0 0 1 0 0

HHP7 14 15 POLY(2) VP7 VCON 0 0 0 0 1 0 0

HHP8 16 17 POLY(2) VP8 VCON 0 0 0 0 1 0 0

HHP9 18 19 POLY(2) VP9 VCON 0 0 0 0 1 0 0

*

*** RESISTORS ***

RF1 2 0 1

RF2 1 24 1

RF3 27 0 1

RF4 28 0 1

*

*** CAPACITORS ***

C1 25 0 1.6N IC=0

C10 26 0 .8N IC=0

C2 6 0 1.6N IC=0

C3 8 0 1.6N IC=0

C4 10 0 1.6N IC=0

C5 12 0 1.6N IC=0

C6 14 0 1.6N IC=0

C7 16 0 1.6N IC=0

C8 18 0 1.6N IC=0

C9 20 0 1.6N IC=0

```
***          ANALYSIS CONTROL INFORMATION          ***
*
***          TRANSIENT ANALYSIS          ***
.TRAN  100N 20U 0
*
***          PLOT CONTROL          ***
.PLOT  TRAN V(27)
.PLOT  TRAN V(28)
***
***
***** END OF DATA DECK *****
.END
```

SPICE Input Data to Simulate Flux Diffusion Model Ramp Response.

CHKFLUXRAMP

```

*
***          INDEPENDENT VOLTAGE SOURCES          ***
V6IN  1 0      PULSE( 0.0 25.4 0.0 1N 0.0  )
VBIN  2 0      PULSE( 0.0 50  0.0 50U 0.0  )
VCON   24 0
VDBIN  4 25
VDBOT  23 26
VP1    3 4
VP10   21 23
VP2    5 6
VP3    7 8
VP4    9 10
VP5   11 12
VP6   13 14
VP7   15 16
VP8   17 18
VP9   19 20
*
***          CURRENT CONTROLLED VOLTAGE SOURCES      ***
HIN    28 0 VDBIN 625MEG
HOUT   27 0 VDBOT 1.25G
*
***          NONLINEAR DEPENDENT CCV SOURCES          ***
HHP1   2 3 POLY(2) VP1 VCON 0 0 0 0 1 0 0
HHP10  20 21 POLY(2) VP10 VCON 0 0 0 0 1 0 0
HHP2   4 5 POLY(2) VP2 VCON 0 0 0 0 1 0 0
HHP3   6 7 POLY(2) VP3 VCON 0 0 0 0 1 0 0
HHP4   8 9 POLY(2) VP4 VCON 0 0 0 0 1 0 0
HHP5  10 11 POLY(2) VP5 VCON 0 0 0 0 1 0 0
HHP6  12 13 POLY(2) VP6 VCON 0 0 0 0 1 0 0
HHP7  14 15 POLY(2) VP7 VCON 0 0 0 0 1 0 0
HHP8  16 17 POLY(2) VP8 VCON 0 0 0 0 1 0 0
HHP9  18 19 POLY(2) VP9 VCON 0 0 0 0 1 0 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
RF3    27 0 1
RF4    28 0 1
*
***
***
***          CAPACITORS          ***
C1     25 0 1.6N IC=0
C10    26 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4    10 0 1.6N IC=0

```

```
C5    12  0 1.6N IC=0
C6    14  0 1.6N IC=0
C7    16  0 1.6N IC=0
C8    18  0 1.6N IC=0
C9    20  0 1.6N IC=0
```

```
***          ANALYSIS CONTROL INFORMATION          ***
*
***          TRANSIENT ANALYSIS                      ***
.TRAN  100N 20U 0
*
***          PLOT CONTROL                          ***
.PLOT  TRAN V(27)
.PLOT  TRAN V(28)
*
**
***** END OF DATA DECK *****
.END
```

SPICE input for TEST 2 "open loop" simulation.

```

TST2TR  CREATED  11/3/92 07:23
*
***          .SUBCKT CONTROL          ***
.SUBCKT SQRT1 1 6
*
*****          SUBCIRCUIT FILE SQRT1          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS      4 5
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1      4 0 3 0 1.5
*
***          CURRENT CONTROLLED VOLTAGE SOURCES          ***
H1      6 0 VS 3.162277K
*
***          RESISTORS          ***
RF1      1 0 1
*
***          DIODES          ***
D1      3 0 DIODE IC=0
D2      5 0 DIODE IC=0
*
***          VOLTAGE-CONTROLLED CURRENT SOURCES          ***
G1      0 3 1 0 .1U
*
***          DIODE MODELS          ***
MODEL DIODE D TT=.1US
.ENDS SQRT1
.SUBCKT FLUX 2 1 23
*
*****          SUBCIRCUIT FILE FLUX          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VCON      24 0
VP1      3 4
VP10     21 23
VP2      5 6
VP3      7 8
VP4      9 10
VP5      11 12
VP6      13 14
VP7      15 16
VP8      17 18
VP9      19 20
*
***          NONLINEAR DEPENDENT CCV SOURCES          ***
HHP1     2 3 POLY(2) VP1 VCON 0 0 0 0 1 IC = 0, 0
HHP10    20 21 POLY(2) VP10 VCON 0 0 0 0 1 IC = 0, 0
HHP2     4 5 POLY(2) VP2 VCON 0 0 0 0 1 IC = 0, 0
HHP3     6 7 POLY(2) VP3 VCON 0 0 0 0 1 IC = 0, 0

```

```

HHP4  8 9 POLY(2) VP4 VCON 0 0 0 0 1 IC = 0, 0
HHP5  10 11 POLY(2) VP5 VCON 0 0 0 0 1 IC = 0, 0
HHP6  12 13 POLY(2) VP6 VCON 0 0 0 0 1 IC = 0, 0
HHP7  14 15 POLY(2) VP7 VCON 0 0 0 0 1 IC = 0, 0
HHP8  16 17 POLY(2) VP8 VCON 0 0 0 0 1 IC = 0, 0
HHP9  18 19 POLY(2) VP9 VCON 0 0 0 0 1 IC = 0, 0
*
***          RESISTORS          ***
RF1    2 0 1
RF2    1 24 1
*
***          CAPACITORS         ***
C1     4 0 1.6N IC=0
C10    23 0 .8N IC=0
C2     6 0 1.6N IC=0
C3     8 0 1.6N IC=0
C4     10 0 1.6N IC=0
C5     12 0 1.6N IC=0
C6     14 0 1.6N IC=0
C7     16 0 1.6N IC=0
C8     18 0 1.6N IC=0
C9     20 0 1.6N IC=0
.ENDS FLUX
.SUBCKT DIVIS 1 3 5
*
*****          SUBCIRCUIT FILE DIVIS          *****
*
***          INDEPENDENT VOLTAGE SOURCES          ***
VS     2 0
*
***          VOLTAGE CONTROLLED VOLTAGE SOURCES          ***
E1     5 0 4 0 4
*
***          RESISTORS          ***
RF1    1 2 1
RF2    3 0 1
RF3    4 0 1MEG
*
***          CURRENT-CONTROLLED CURRENT SOURCES          ***
FIX    0 4 VS 1
*
***          NONLINEAR DEPENDENT VCC SOURCES          ***
GGIY   4 0 POLY(2) 4 0 3 0 0 0 0 4 IC = 0, 0
.ENDS DIVIS
*
***          SUBCIRCUIT CALLS          ***
X1     14 15 16 FLUX
X2     17 15 18 FLUX
X3     23 58 25 DIVIS
X4     32 61 33 DIVIS
X5     8 36 37 DIVIS
X6     43 44 63 DIVIS
X7     46 47 48 DIVIS
X8     48 50 SQRT1

```



```

*
***      INDEPENDENT VOLTAGE SOURCES      ***
VBEFF  52 21
VBO    14 0      PULSE( 0.0 .92 0.0 0.0 0.0 )
VCD    9 0      EXP( 0.0 25K 0.0 100N 25.85U 30U )
VDNS   11 0      EXP( 0.0 20 0.0 100N 25.85U 20U )
VE     5 6
VEE    10 0      EXP( 0.0 25 0.0 100N 25.85U 20U )
VH     13 0      EXP( 0.0 32.5 0.0 100N 25.85U 20U )
VMHD   2 3
VOS    65 0 PULSE( 0.0 1 0.0 10N 0.0 )
VP     12 0      EXP( 0.0 150 0.0 100N 25.85U 20U )
VSS    5 7
VU     8 0      EXP( 0.0 55.32K 0.0 100N 25.85U 2U )
VCDC   63 45
*
***      VOLTAGE CONTROLLED VOLTAGE SOURCES      ***
E2     42 0 12 0 6
E3     23 0 55 0 1U
E4     61 0 24 0 1M
E5     64 0 25 0 1MEG
E6     58 0 11 0 16.144U
E7     36 0 59 0 1
E8     0 57 56 0 1MEG
*
***      CURRENT CONTROLLED VOLTAGE SOURCES      ***
H1     17 0 VSS 3.23U
H2     26 0 VMHD 1
H3     28 0 VE 1
*
***      NONLINEAR DEPENDENT VCV SOURCES      ***
EE1    4 3 POLY(2) 18 0 22 0 0 0 0 0 50.8M IC = 0, 0
EE10   41 0 POLY(2) 11 0 10 0 0 0 0 0 -8 IC = 0, 0
EE11   43 0 POLY(2) 8 0 53 0 0 0 0 0 2U IC = 0, 0
EE12   44 0 POLY(2) 8 0 59 0 0 3U 2U IC = 0, 0
EE13   46 0 POLY(3) 38 0 35 0 45 0 0 0 0 -10U 0 10U IC = 0, 0, 0
EE14   47 0 POLY(2) 38 0 10 0 0 0 0 0 10U IC = 0, 0
EE15   15 0 POLY(2) 9 0 68 0 0 0 0 0 1.27M IC = 0, 0
EE16   66 0 POLY(3) 0 65 50 0 65 0 0 1 0 0 0 0 0 1 IC = 0, 0, 0
EE17   59 0 POLY(2) 8 0 0 57 0 1 1 IC = 0, 0
EE18   0 60 POLY(2) 24 0 0 57 0 0 0 0 4 IC = 0, 0
EE19   68 0 POLY(2) 67 0 65 0 0 1 1 IC = 0, 0
EE2    1 0 POLY(2) 16 0 22 0 0 0 0 0 25.4M IC = 0, 0
EE4    24 0 POLY(2) 8 0 11 0 0 0 0 0 .5067M IC = 0, 0
EE5    32 0 POLY(3) 31 0 29 0 27 0 0 1U -1U -1U IC = 0, 0, 0
EE6    34 0 POLY(2) 57 0 8 0 0 0 0 .5U -1U IC = 0, 0
EE7    39 0 POLY(2) 38 0 10 0 0 0 0 0 4 IC = 0, 0
EE77   35 0 POLY(3) 13 0 33 0 34 0 0 1 -1M -1 IC = 0, 0, 0
EE8    40 0 POLY(2) 11 0 35 0 0 0 0 0 4 IC = 0, 0
EE9    38 0 POLY(2) 37 0 11 0 0 0 0 0 1 IC = 0, 0
EEBEF  52 0 POLY(2) 16 0 18 0 0 1 2 IC = 0, 0
EED    29 0 POLY(2) 28 0 7 0 0 0 0 0 1 IC = 0, 0
EEMHD  31 0 POLY(2) 26 0 30 0 0 0 0 0 1 IC = 0, 0
EEP    0 20 POLY(2) 12 0 45 0 0 2.027K -2.027K IC = 0, 0

```

EEPL 27 0 POLY(2) 26 0 2 1 0 0 0 0 1 IC = 0, 0
 EETOT 30 0 POLY(2) 1 0 4 3 0 1 1 IC = 0, 0
 EEUEF 22 0 POLY(2) 8 0 0 57 0 1.5 IC = 0, 0
 HHVEL 19 0 POLY(2) VMHD VBEFF 0 0 0 0 101.6M IC = 0, 0

*

RESISTORS

RCU 4 5 1M
 RF01 8 0 1
 RF02 9 0 1
 RF03 10 0 1
 RF04 11 0 1
 RF05 12 0 1
 RF06 13 0 1
 RF07 19 55 1
 RF08 20 55 1
 RF09 21 0 1
 RF10 22 0 1
 RF11 25 0 1
 RF12 26 0 1
 RF13 27 0 1
 RF14 28 0 1
 RF15 29 0 1
 RF16 30 0 1
 RF17 31 0 1
 RF18 33 0 1
 RF19 34 0 1
 RF20 35 0 1
 RF21 37 0 1
 RF22 38 0 1
 RF23 39 53 1
 RF24 40 53 1
 RF25 41 53 1
 RF26 42 53 1
 RF27 48 0 1
 RF28 50 0 1
 RF29 65 0 1
 RF30 45 0 1
 RF31 64 56 1
 RF32 66 67 1
 RF33 24 0 1
 RF34 55 0 1
 RF35 60 55 1
 RF36 59 0 1
 RF37 68 0 1

*

CAPACITORS

C1 16 0.8N IC=0
 C2 18 0.8N IC=0
 C3 56 57 .25 IC=0
 C4 67 0.1U IC=0

*

INDUCTORS

LB 7 0.445U IC=0

*

```

***      NONLINEAR DEPENDENT VCC SOURCES      ***
GGED 6 0 POLY(3) 6 0 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 451N IC = 0, 0, 0
GGP 1 2 POLY(3) 1 2 9 0 68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 75M IC = 0, 0, 0
*
***      ANALYSIS CONTROL INFORMATION      ***
*
***      TRANSIENT ANALYSIS      ***
.TRAN .500000E-6 20U 0 UIC
*
***      PLOT CONTROL      ***
.PLOT TRAN V(0,4)
.PLOT TRAN I(VMHD)
.PLOT TRAN V(16)
.PLOT TRAN V(18)
.PLOT TRAN V(21)
.PLOT TRAN V(57)
.PLOT TRAN V(59)
.PLOT TRAN V(22)
.PLOT TRAN V(35)
.PLOT TRAN V(38)
.PLOT TRAN V(64)
.PLOT TRAN V(48)
.PLOT TRAN V(50)
.PLOT TRAN V(66)
.PLOT TRAN V(67)
.PLOT TRAN V(68)
*
***      OPTIONS CONTROL      ***
.OPTIONS      ITL5=0
+
*
***** END OF DATA DECK *****
.END
*
*
```

**DATE
FILMED**

9 / 16 / 93

END