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VISCOSITY IN THE EDGE OF TOKAMAK PLASMAS

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ABSTRACT: A fluid representation of viscosity has been incorporated into a set of fluid equations that are maximally ordered in the "short-radial-gradient-scale-length" (srgsl) ordering that is appropriate for the edge of tokamak plasmas. The srgsl ordering raises viscous drifts and other viscous terms to leading order and fundamentally alters the character of the fluid equations. A leading order viscous drift is identified. Viscous-driven radial particle and energy fluxes in the scrape-off layer and divertor channel are estimated to have an order unity effect in reducing radial peaking of energy fluxes transported along the field lines to divertor collector plates.

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I. INTRODUCTION

The calculation of multi-component plasma flows within confining magnetic fields in the presence of large neutral populations is a problem that is receiving increased emphasis in the modeling of processes in the edge region of high-temperature fusion plasma confinement devices. Modeling the three-dimensional plasma flows that are created in the edge regions of tokamaks has become a large-scale undertaking in recent years, as it has become apparent that an understanding of how to control these flows may lead to an ability to control the L-to-H transition in energy confinement and to reduce the (predicted) unacceptably high heat load on the divertor collector plates in future devices. Most calculations (e.g. refs. 1-6) use a fluid model of the plasma and an approximate representation of viscosity. Results from the computational models reveal complex flow patterns which would produce a variety of viscous forces not usually allowed for in the computational models.

Approximate treatments of viscosity in plasma fluid calculations have in the past been justified by the relatively small order of viscous terms in the large-radial-gradient-scale-length ordering that is appropriate for the central core of a confined plasma. However, in the edge plasma the radial gradient scale lengths are of order of the gyroradius. In a short-radial-gradient-scale-length (srgsl) ordering, the viscous terms become of leading order.

The first purpose of this paper is to present explicit forms for the viscous force and energy flux terms that enter the fluid equations describing a plasma, in generalized cylindrical or toroidal curvilinear coordinates. These equations generally apply to a wide range of plasma systems. The second purpose of the paper is to use the maximal short-radial-gradient-scale length ordering and the toroidal symmetry of the tokamak edge region to reduce these equations to a form appropriate for tokamak edge region modeling. The third purpose is to analyze these equations to estimate the effect of

viscous terms on particle and energy flows in the plasma edge, scrape-off layer and divertor channel regions.

The development and analysis are carried out within the framework of collisional fluid plasma transport theory. Questions have been raised about the inability of fluid theory to properly treat the flow of electrons along field lines, and the validity of the conventional derivation of fluid theory from kinetic theory for ions flowing along field lines in the scrape-off layer of tokamaks has been challenged⁷. We further note the need to examine at the kinetic theory level the effect on the resulting plasma transport coefficients of large, interacting neutral and impurity particle concentrations. Resolution of the electron question has led to a hybrid fluid-ion kinetic electron treatment in radio frequency discharges⁸, and this is a likely solution for tokamak edge plasmas as well. However, the viscous effects with which we are concerned in this paper are associated with the ions, which are relatively collisional in the plasma edge. It is likely that resolution of the proper fluid theory for ions will effect the value of the plasma transport coefficients of the present theory, rather than the form of the viscosity representation. Thus, the analysis of this paper is conducted within existing collisional fluid theory, with the expectation that the results will be readily extendible to accommodate any new fluid transport coefficients that may be developed.

The paper is organized as follows. The viscosity representation and fluid equations are developed in a generalized cylindrical or toroidal curvilinear coordinate system with axes along and perpendicular to the magnetic field, in section II. These results are specialized to the scrape-off layer and divertor channel of an axisymmetric tokamak in section III by invoking the orderings and symmetry appropriate to those regions to develop a reduced viscosity representation and a reduced set of fluid equations under the maximal ordering that could exist in those regions. The importance of the various viscous terms in the reduced fluid equations is discussed, with regard both to the determination of particle and heat flows and to the reduction

of the peak energy flux incident on the divertor collector plate, also in section III. In section IV, a viscosity representation and reduced fluid equations are developed in the toroidal-poloidal flux surface coordinate system normally employed in analyzing the confined plasma inside of the separatrix, but in the ordering appropriate for the plasma edge. Finally, summarizing remarks are presented in section V.

II. GENERALIZED CYLINDRICAL OR TOROIDAL SYSTEMS

The expressions for viscosity and the moments equations will first be developed for a generalized cylinder-like or generalized toroid-like geometry which is distinguished by an axial or toroidal direction and by a generalized radial direction which is orthogonal to the magnetic field.

A. $(\psi, \perp, \parallel)$ Coordinate Systems

We adopt a right-hand, orthogonal $(\psi, \perp, \parallel)$ coordinate system where: ψ is a (radial) coordinate in the direction orthogonal to both the magnetic field line and to the axial or toroidal direction; $\parallel$ is the direction along the magnetic field line; and $\perp$ is the direction orthogonal to both the ψ - and $\parallel$ - and directions. The volume element is $dV = dl_\psi dl_\perp dl_\parallel \equiv h_\psi d\xi_\psi h_\perp d\xi_\perp h_\parallel d\xi_\parallel$, where dl_x is the length element in the ξ_x direction and h_x is the metric coefficient. The unit vectors are $\hat{n}_\psi, \hat{n}_\perp$ and $\hat{n}_\parallel$.

B. Rate-of-Strain-Tensor

The rate-of-strain tensor of the flow field ($\mathbf{v}$) is defined⁹

$$W_{\alpha\beta} \equiv \hat{n}_\alpha \cdot \nabla \bar{\mathbf{v}} \cdot \hat{n}_\beta + \hat{n}_\beta \cdot \nabla \bar{\mathbf{v}} \cdot \hat{n}_\alpha - \frac{2}{3} \delta_{\alpha\beta} \nabla \cdot \bar{\mathbf{v}} \quad (1)$$

In this $(\psi, \perp, \parallel)$ coordinate system, the tensor elements are

$$W_{\psi\psi} = 2 \left[\frac{\partial v_\psi}{\partial l_\psi} + \Gamma_{\psi\perp}^\psi v_\perp + \Gamma_{\psi\parallel}^\psi v_\parallel \right] - \frac{2}{3} \nabla \cdot \bar{\mathbf{v}}$$

$$W_{\perp\perp} = 2 \left[\frac{\partial v_\perp}{\partial l_\perp} + \Gamma_{\perp\psi}^\perp v_\psi + \Gamma_{\perp\parallel}^\perp v_\parallel \right] - \frac{2}{3} \nabla \cdot \bar{\mathbf{v}}$$

$$\begin{aligned}
W_{\parallel\parallel} &= 2 \left[\frac{\partial v_{\parallel}}{\partial l_{\parallel}} + \Gamma_{\parallel\psi}^{\parallel} v_{\psi} + \Gamma_{\parallel\perp}^{\parallel} v_{\perp} \right] - \frac{2}{3} \nabla \cdot \vec{v} \\
W_{\psi\perp} &= h_{\perp} \frac{\partial(v_{\perp}/h_{\perp})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi}/h_{\psi})}{\partial l_{\perp}} = W_{\perp\psi} \\
W_{\psi\parallel} &= h_{\parallel} \frac{\partial(v_{\parallel}/h_{\parallel})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi}/h_{\psi})}{\partial l_{\parallel}} = W_{\parallel\psi} \\
W_{\perp\parallel} &= h_{\parallel} \frac{\partial(v_{\parallel}/h_{\parallel})}{\partial l_{\perp}} + h_{\perp} \frac{\partial(v_{\perp}/h_{\perp})}{\partial l_{\parallel}} = W_{\parallel\perp}
\end{aligned} \tag{2}$$

where the Christoffel symbols which we need are given by

$$\Gamma_{xy}^x \equiv \frac{1}{h_x} \frac{\partial h_x}{\partial l_y}. \tag{3}$$

C. Viscosity Tensor

The viscosity tensor is defined⁹ in terms of a decomposition of the rate-of-strain tensor into parallel (η_o), perpendicular (η_1, η_2) and gyroviscous (η_3, η_4) components

$$\begin{aligned}
\pi_{\alpha\beta} &\equiv -\eta_o W_{\alpha\beta}^o - [\eta_1 W_{\alpha\beta}^1 + \eta_2 W_{\alpha\beta}^2] + [\eta_3 W_{\alpha\beta}^3 + \eta_4 W_{\alpha\beta}^4] \\
&\equiv \pi_{\alpha\beta}^o + \pi_{\alpha\beta}^{\perp} + \pi_{\alpha\beta}^{\Omega}
\end{aligned} \tag{4}$$

where

$$W_{\alpha\beta}^o \equiv \frac{3}{2} \left(f_{\alpha} f_{\beta} - \frac{1}{3} \delta_{\alpha\beta} \right) \left(f_{\mu} f_{\nu} - \frac{1}{3} \delta_{\mu\nu} \right) W_{\mu\nu},$$

$$W_{\alpha\beta}^1 \equiv \left(\delta_{\alpha\mu}^{\perp} \delta_{\beta\nu}^{\perp} + \frac{1}{2} \delta_{\alpha\beta}^{\perp} f_{\mu} f_{\nu} \right) W_{\mu\nu},$$

$$W_{\alpha\beta}^2 \equiv \left(\delta_{\alpha\mu}^{\perp} f_{\beta} f_{\nu} + \delta_{\beta\nu}^{\perp} f_{\alpha} f_{\mu} \right) W_{\mu\nu},$$

$$W_{\alpha\beta}^3 \equiv \frac{1}{2} (\delta_{\alpha\mu}^\perp \epsilon_{\beta\gamma\nu} + \delta_{\beta\nu}^\perp \epsilon_{\alpha\gamma\mu}) f_\gamma W_{\mu\nu},$$

$$W_{\alpha\beta}^4 \equiv (f_\alpha f_\mu \epsilon_{\beta\gamma\nu} + f_\beta f_\nu \epsilon_{\alpha\gamma\mu}) f_\gamma W_{\mu\nu},$$

and

$$\delta_{\alpha\beta}^\perp \equiv \delta_{\alpha\beta} - f_\alpha f_\beta,$$

$\epsilon_{\alpha\beta\gamma}$ is the antisymmetric unit tensor, $f_\alpha \equiv B_\alpha / B$, and the Einstein summation convention is employed. Here $W_{\mu\nu}$ is given in Eq. (2).

Anticipating that the values of the viscosity coefficients will change from the Braginskii values when rederived from kinetic theory, we take some small liberty with Braginskii's results to write

$$\pi_{\alpha\beta} \simeq -\eta_o W_{\alpha\beta}^o - \eta_\perp W_{\alpha\beta}^\perp + \eta_\Omega W_{\alpha\beta}^\Omega \quad (5)$$

where

$$\eta_o \equiv p\tau, \quad \eta_\Omega = p/\Omega, \quad \eta_\perp = p/\Omega^2\tau, \quad (6)$$

where p is the pressure, τ is the self-collision frequency, $\Omega = \frac{eB}{m}$ is the gyrofrequency,

and

$$\begin{aligned} W_{\alpha\beta}^\perp &\equiv \left[\frac{1}{4} W_{\alpha\beta}^1 + W_{\alpha\beta}^2 \right] \\ W_{\alpha\beta}^\Omega &\equiv \left[\frac{1}{2} W_{\alpha\beta}^3 + W_{\alpha\beta}^4 \right] \end{aligned} \quad (7)$$

We note that the collisional result for η_o given by Eq. (6) has been extended¹⁰ to other collisionality regimes, and that this extension can be incorporated into the formalism of this paper. In general $\eta_o \gg \eta_\Omega \gg \eta_\perp$; however, as we will see, $W_{\alpha\beta}^o = 0$ for $\alpha \neq \beta$ and we need to retain the (η_Ω) gyroviscous term. The

perpendicular viscous ($\eta_{\perp}$) terms are always smaller than other viscous terms and only need be retained when the gyroviscous terms are small.

The "parallel" viscous tensor elements are

$$\begin{aligned}\pi_{\psi\psi}^o &= -\frac{1}{2}\eta_o W^o \\ \pi_{\perp\perp}^o &= -\frac{1}{2}\eta_o W^o \\ \pi_{\parallel\parallel}^o &= \eta_o W^o \\ \pi_{\alpha\beta}^o &= 0, \quad \alpha \neq \beta\end{aligned}\tag{8}$$

where

$$\begin{aligned}W_o = \frac{2}{3} \bigg[& \left\{ 2 \frac{\partial v_{\parallel}}{\partial l_{\parallel}} - v_{\parallel} \frac{\partial \ln(h_{\psi} h_{\perp})}{\partial l_{\parallel}} \right\} - \left\{ \frac{h_{\parallel}^2}{h_{\psi}} \frac{\partial(h_{\psi} v_{\perp} / h_{\parallel}^2)}{\partial l_{\perp}} \right\} \\ & - \left\{ \frac{h_{\parallel}^2}{h_{\perp}} \frac{\partial(h_{\perp} v_{\psi} / h_{\parallel}^2)}{\partial l_{\psi}} \right\} \bigg].\end{aligned}\tag{9}$$

The gyroviscous tensor elements are

$$\begin{aligned}W_{\psi\psi}^{\Omega} &= -\frac{1}{2} \left[h_{\perp} \frac{\partial(v_{\perp} / h_{\perp})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi} / h_{\psi})}{\partial l_{\perp}} \right] \equiv -W_{\perp\perp}^{\Omega} \\ W_{\psi\perp}^{\Omega} &= \frac{1}{2} \left[h_{\perp} \frac{\partial(v_{\psi} / h_{\perp})}{\partial l_{\psi}} - h_{\psi} \frac{\partial(v_{\perp} / h_{\psi})}{\partial l_{\perp}} + \frac{\partial \ln(h_{\psi} / h_{\perp})}{\partial l_{\parallel}} v_{\parallel} \right] = W_{\perp\psi}^{\Omega} \\ W_{\psi\parallel}^{\Omega} &= - \left[h_{\parallel} \frac{\partial(v_{\parallel} / h_{\parallel})}{\partial l_{\perp}} + h_{\perp} \frac{\partial(v_{\perp} / h_{\perp})}{\partial l_{\parallel}} \right] = W_{\parallel\psi}^{\Omega} \\ W_{\perp\parallel}^{\Omega} &= \left[h_{\parallel} \frac{\partial(v_{\parallel} / h_{\parallel})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi} / h_{\psi})}{\partial l_{\parallel}} \right] = W_{\parallel\perp}^{\Omega} \\ W_{\parallel\parallel}^{\Omega} &= 0\end{aligned}\tag{10}$$

The perpendicular viscosity tensor elements are

$$\begin{aligned}
W_{\psi\psi}^{\perp} &= \frac{1}{4} \left[h_{\perp} \frac{\partial(v_{\psi}/h_{\perp})}{\partial l_{\psi}} - h_{\psi} \frac{\partial(v_{\perp}/h_{\psi})}{\partial l_{\perp}} + (\Gamma_{\psi\parallel}^{\psi} - \Gamma_{\perp\parallel}^{\perp}) v_{\parallel} \right] = -W_{\perp\perp}^{\perp} \\
W_{\psi\perp}^{\perp} &= \frac{1}{4} \left[h_{\perp} \frac{\partial(v_{\perp}/h_{\perp})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi}/h_{\psi})}{\partial l_{\perp}} \right] = W_{\perp\psi}^{\perp} \\
W_{\psi\parallel}^{\perp} &= \frac{5}{4} \left[h_{\parallel} \frac{\partial(v_{\parallel\perp}/h_{\parallel})}{\partial l_{\psi}} + h_{\psi} \frac{\partial(v_{\psi}/h_{\psi})}{\partial l_{\parallel}} \right] = W_{\parallel\psi}^{\perp} \\
W_{\perp\parallel}^{\perp} &= \frac{5}{4} \left[h_{\parallel} \frac{\partial(v_{\parallel}/h_{\parallel})}{\partial l_{\perp}} + h_{\perp} \frac{\partial(v_{\perp}/h_{\perp})}{\partial l_{\parallel}} \right] = W_{\parallel\perp}^{\perp} \\
W_{\parallel\parallel}^{\perp} &= 0
\end{aligned} \tag{11}$$

D. Fluid Equations

The standard, textbook (e.g. ref. 11) derivation of the fluid moments equations in generalized curvilinear geometry may be specialized to the $(\psi, \perp, \parallel)$ coordinate system

to obtain a particle continuity equation

$$\frac{\partial n}{\partial t} + \frac{1}{h_{\perp} h_{\parallel}} \frac{\partial(h_{\perp} h_{\parallel} n v_{\psi})}{\partial l_{\psi}} + \frac{1}{h_{\psi} h_{\parallel}} \frac{\partial(h_{\psi} h_{\parallel} n v_{\perp})}{\partial l_{\perp}} + \frac{1}{h_{\psi} h_{\perp}} \frac{\partial(h_{\psi} h_{\perp} n v_{\parallel})}{\partial l_{\parallel}} = S^0, \tag{12}$$

the three components $(\psi, \perp, \parallel)$ of the momentum balance equation

$$\begin{aligned}
m \frac{\partial(n v_{\psi})}{\partial t} + \left[\left\{ \frac{1}{h_{\perp} h_{\parallel}} \frac{\partial(h_{\perp} h_{\parallel} M_{\psi\psi})}{\partial l_{\psi}} - \Gamma_{\perp\psi}^{\perp} M_{\perp\perp} - \Gamma_{\parallel\psi}^{\parallel} M_{\parallel\parallel} \right\} + \frac{1}{h_{\psi}^2 h_{\parallel}} \frac{\partial(h_{\psi}^2 h_{\parallel} M_{\perp\psi})}{\partial l_{\perp}} \right. \\
\left. + \frac{1}{h_{\psi}^2 h_{\perp}} \frac{\partial(h_{\psi}^2 h_{\perp} M_{\parallel\psi})}{\partial l_{\parallel}} \right] - n e (E_{\psi} + v_{\perp} B) = R_{\psi}^1 + S_{\psi}^1,
\end{aligned} \tag{13}$$

$$m \frac{\partial(n v_{\perp})}{\partial t} + \left[\frac{1}{h_{\perp}^2 h_{\parallel}} \frac{\partial(h_{\perp}^2 h_{\parallel} M_{\psi\perp})}{\partial l_{\psi}} + \left\{ \frac{1}{h_{\psi} h_{\parallel}} \frac{\partial(h_{\psi} h_{\parallel} M_{\perp\perp})}{\partial l_{\perp}} - \Gamma_{\parallel\perp}^{\parallel} M_{\parallel\parallel} - \Gamma_{\psi\perp}^{\psi} M_{\psi\psi} \right\} + \frac{1}{h_{\psi} h_{\perp}^2} \frac{\partial(h_{\psi} h_{\perp}^2 M_{\parallel\psi})}{\partial l_{\parallel}} \right] - ne(E_{\perp} - v_{\psi} B) = R_{\perp}^1 + S_{\perp}^1, \quad (14)$$

$$m \frac{\partial(n v_{\parallel})}{\partial t} + \left[\frac{1}{h_{\perp} h_{\parallel}^2} \frac{\partial(h_{\perp} h_{\parallel}^2 M_{\psi\parallel})}{\partial l_{\psi}} + \frac{1}{h_{\psi} h_{\parallel}^2} \frac{\partial(h_{\psi} h_{\parallel}^2 M_{\perp\parallel})}{\partial l_{\perp}} + \left\{ \frac{1}{h_{\psi} h_{\perp}} \frac{\partial(h_{\psi} h_{\perp} M_{\parallel\parallel})}{\partial l_{\parallel}} - \Gamma_{\psi\parallel}^{\psi} M_{\psi\psi} - \Gamma_{\perp\parallel}^{\perp} M_{\perp\perp} \right\} \right] - ne E_{\parallel} = R_{\parallel}^1 + S_{\parallel}^1, \quad (15)$$

and an energy balance equation

$$\frac{\partial\left(\frac{3}{2}p + \frac{1}{2}nmv^2\right)}{\partial t} + \frac{1}{h_{\perp} h_{\parallel}} \frac{\partial(h_{\perp} h_{\parallel} Q_{\psi})}{\partial l_{\psi}} + \frac{1}{h_{\psi} h_{\parallel}} \frac{\partial(h_{\psi} h_{\parallel} Q_{\perp})}{\partial l_{\perp}} + \frac{1}{h_{\psi} h_{\perp}} \frac{\partial(h_{\psi} h_{\perp} Q_{\parallel})}{\partial l_{\parallel}} = ne(v_{\psi} E_{\psi} + v_{\perp} E_{\perp} + v_{\parallel} E_{\parallel}) + R^2 + S^2. \quad (16)$$

In these equations E_x is the x -th component of the electric field; B is the magnetic field; $n, m, \bar{v}$ and e are the number density, mass, average fluid velocity, and charge of the particular ion species; S^0 is the ion source density rate from external sources and from atomic processes such as ionization and charge exchange, and $\bar{S}^1$ and S^2 are the net momentum and energy input or loss associated with these sources; and $\bar{R}^1$ is the collisional momentum exchange of the particular ion species with other ion species and with electrons. The momentum tensor ($\tilde{M}$) and the energy flux ($\tilde{Q}$) are

$$\tilde{M} \equiv nm\bar{v}\bar{v} + p\tilde{I} + \tilde{\pi} \quad (17)$$

and

$$\tilde{Q} \equiv \frac{1}{2}nmv^2\bar{v} + \frac{5}{2}p\bar{v} + \bar{v} \cdot \tilde{\pi} + \tilde{q}, \quad (18)$$

with elements

$$M_{\alpha\beta} \equiv \hat{n}_\alpha \cdot \vec{M} \cdot \hat{n}_\beta, \quad Q_\alpha \equiv \hat{n}_\alpha \cdot \vec{Q}, \quad (19)$$

where $\vec{q}$ is the heat conduction.

With the representation of viscosity discussed previously, the viscous components may be written explicitly

$$\begin{aligned} \Pi_{\psi\psi} &= -\frac{1}{2}\eta_o W^o - \eta_\perp W_{\psi\psi}^\perp + \eta_\Omega W_{\psi\psi}^\Omega \\ \Pi_{\psi\perp} &= -\eta_\perp W_{\psi\perp}^\perp + \eta_\Omega W_{\psi\perp}^\Omega = \Pi_{\perp\psi} \\ \Pi_{\psi\parallel} &= -\eta_\perp W_{\psi\parallel}^\perp + \eta_\Omega W_{\psi\parallel}^\Omega = \Pi_{\parallel\psi} \\ \Pi_{\perp\perp} &= -\frac{1}{2}\eta_o W^o - \eta_\perp W_{\perp\perp}^\perp + \eta_\Omega W_{\perp\perp}^\Omega \\ \Pi_{\perp\parallel} &= -\eta_\perp W_{\perp\parallel}^\perp + \eta_\Omega W_{\perp\parallel}^\Omega = \Pi_{\parallel\perp} \\ \Pi_{\parallel\parallel} &= \eta_o W^o \end{aligned} \quad (20)$$

and

$$\begin{aligned} Q_\psi^\pi &\equiv \hat{n}_\psi \cdot (\vec{v} \cdot \vec{\pi}) = v_\psi \pi_{\psi\psi} + v_\perp \pi_{\perp\psi} + v_\parallel \pi_{\parallel\psi} \\ Q_\perp^\pi &\equiv \hat{n}_\perp \cdot (\vec{v} \cdot \vec{\pi}) = v_\psi \pi_{\psi\perp} + v_\perp \pi_{\perp\perp} + v_\parallel \pi_{\parallel\perp} \\ Q_\parallel^\pi &\equiv \hat{n}_\parallel \cdot (\vec{v} \cdot \vec{\pi}) = v_\psi \pi_{\psi\parallel} + v_\perp \pi_{\perp\parallel} + v_\parallel \pi_{\parallel\parallel} \end{aligned} \quad (21)$$

Using Eqs. (20) and (21), the momentum and energy balance equations may be written in a form that makes more explicit the different phenomena that are involved.

$$\begin{aligned} m \frac{\partial(n v_\psi)}{\partial t} + \left[\left\{ \frac{\partial \left(n m v_\psi^2 + p - \frac{1}{2} \eta_o W^o - \eta_\perp W_{\psi\psi}^\perp + \eta_\Omega W_{\psi\psi}^\Omega \right)}{\partial l_\psi} + n m \left(v_\psi^2 (\Gamma_{\perp\psi}^\perp + \Gamma_{\parallel\psi}^\parallel) - \right. \right. \right. \\ \left. \left. v_\perp^2 \Gamma_{\perp\psi}^\perp - v_\parallel^2 \Gamma_{\parallel\psi}^\parallel \right) - \frac{3}{2} \eta_o W^o \Gamma_{\parallel\psi}^\parallel + (\eta_\Omega W_{\psi\psi}^\Omega - \eta_\perp W_{\psi\psi}^\perp) (\Gamma_{\parallel\psi}^\parallel + 2 \Gamma_{\perp\psi}^\perp) \right\} + \end{aligned}$$

$$\left\{ \frac{1}{h_\psi^2 h_\parallel} \frac{\partial \left(h_\psi^2 h_\parallel (nm v_\psi v_\perp + (\eta_\Omega W_{\psi\psi}^\Omega - \eta_\perp W_{\psi\psi}^\perp)) \right)}{\partial l_\perp} \right\} + \left\{ \frac{1}{h_\psi^2 h_\perp} \frac{\partial \left(h_\psi^2 h_\perp (nm v_\psi v_\parallel + (\eta_\Omega W_{\psi\parallel}^\Omega - \eta_\perp W_{\psi\parallel}^\perp)) \right)}{\partial l_\parallel} \right\} \Bigg]$$

$$-ne(E_\psi + v_\perp B) = R_\psi^1 + S_\psi^1 \quad (22)$$

$$\begin{aligned} & m \frac{\partial(nv_\perp)}{\partial t} + \left[\left\{ \frac{1}{h_\perp^2 h_\parallel} \frac{\partial \left(h_\perp^2 h_\parallel (nm v_\psi v_\perp + (\eta_\Omega W_{\psi\perp}^\Omega - \eta_\perp W_{\psi\perp}^\perp)) \right)}{\partial l_\psi} \right\} + \right. \\ & \left\{ \frac{\partial \left(nm v_\perp^2 + p - \frac{1}{2} \eta_o W^\circ + (\eta_\Omega W_{\perp\perp}^\Omega - \eta_\perp W_{\perp\perp}^\perp) \right)}{\partial l_\perp} \right\} \\ & + nm (v_\perp^2 (\Gamma_{\psi\perp}^\psi + \Gamma_{\parallel\perp}^\parallel) - v_\parallel^2 \Gamma_{\parallel\perp}^\parallel - v_\psi^2 \Gamma_{\psi\perp}^\psi) - \frac{3}{2} \eta_o W^\circ \Gamma_{\parallel\perp}^\parallel + \\ & \left. (\eta_\Omega W_{\perp\perp}^\Omega - \eta_\perp W_{\perp\perp}^\perp) (\Gamma_{\parallel\perp}^\parallel + 2\Gamma_{\psi\perp}^\psi) \right\} \\ & + \left\{ \frac{1}{h_\psi h_\perp^2} \frac{\partial \left(h_\psi h_\perp^2 (nm v_\parallel v_\perp + (\eta_\Omega W_{\perp\perp}^\Omega - \eta_\perp W_{\perp\perp}^\perp)) \right)}{\partial l_\parallel} \right\} \Bigg] - ne(E_\perp - v_\psi B) = R_\perp^1 + S_\perp^1 \quad (23) \end{aligned}$$

$$\begin{aligned} & m \frac{\partial(nv_\parallel)}{\partial t} + \left[\left\{ \frac{1}{h_\perp^2 h_\parallel} \frac{\partial \left(h_\perp^2 h_\parallel (nm v_\psi v_\parallel + (\eta_\Omega W_{\psi\parallel}^\Omega - \eta_\perp W_{\psi\parallel}^\perp)) \right)}{\partial l_\psi} \right\} + \right. \\ & \left\{ \frac{1}{h_\psi h_\parallel^2} \frac{\partial \left(h_\psi h_\parallel^2 (nm v_\perp v_\parallel + (\eta_\Omega W_{\perp\parallel}^\Omega - \eta_\perp W_{\perp\parallel}^\perp)) \right)}{\partial l_\perp} \right\} + \end{aligned}$$

$$\left\{ \frac{\partial(nmv_{\parallel}^2 + p + \eta_o W^o)}{\partial l_{\parallel}} + nm(v_{\parallel}^2(\Gamma_{\psi\parallel}^{\psi} + \Gamma_{\perp\parallel}^{\perp}) - v_{\perp\parallel}^2\Gamma_{\perp\perp}^{\perp} - v_{\psi\parallel}^2\Gamma_{\psi\parallel}^{\psi}) + \right. \\ \left. \eta_o W^o(\Gamma_{\psi\parallel}^{\psi} + \Gamma_{\perp\parallel}^{\perp}) - (\eta_{\perp} W_{\psi\psi}^{\perp} - \eta_{\parallel} W_{\perp\perp}^{\perp})(\Gamma_{\psi\parallel}^{\psi} - \Gamma_{\perp\parallel}^{\perp}) \right\} - neE_{\parallel} = R_{\parallel}^1 + S_{\parallel}^1 \quad (24)$$

and

$$\frac{\partial\left(\frac{3}{2}p + \frac{1}{2}nmv^2\right)}{\partial t} + \frac{1}{h_{\perp}h_{\parallel}} \frac{\partial\left(h_{\perp}h_{\parallel}\left(\frac{1}{2}nmv^2v_{\psi} + \frac{5}{2}pv_{\psi} + Q_{\psi}^{\pi} + q_{\psi}\right)\right)}{\partial l_{\psi}} + \\ \frac{1}{h_{\psi}h_{\parallel}} \frac{\partial\left(h_{\psi}h_{\parallel}\left(\frac{1}{2}nmv^2v_{\perp} + \frac{5}{2}pv_{\perp} + Q_{\perp}^{\pi} + q_{\perp}\right)\right)}{\partial l_{\perp}} + \\ \frac{1}{h_{\psi}h_{\perp}} \frac{\partial\left(h_{\psi}h_{\perp}\left(\frac{1}{2}nmv^2v_{\parallel} + \frac{5}{2}pv_{\parallel} + Q_{\parallel}^{\pi} + q_{\parallel}\right)\right)}{\partial l_{\parallel}} \\ = ne\bar{v} \cdot \bar{E} + R^2 + S^2 \quad (25)$$

III. TOKAMAK SCRAPE-OFF LAYER & DIVERTOR CHANNEL

The scrape-off layer and divertor channel in a tokamak form a toroidally symmetric annular region which wraps around the confined plasma (the scrape-off layer) but which, rather than connecting to form a toroidal annulus, then continues as a toroidally symmetric layer (the divertor channel) extending from the confined plasma to the divertor collector plate. The helical magnetic field lines within the scrape-off layer and divertor channel spiral about the torus in the toroidal direction many times while traversing poloidally from one divertor collection plate to another. The helical magnetic field lines within the confined plasma also spiral about the torus in the toroidal direction, but remain within the confinement chamber rather than intersecting a material surface. The field lines that form the boundary between the confined plasma

and the scrape-off layer lie in a surface known as the separatrix. An idealization of this configuration is illustrated in the $\phi = \text{const.}$ cross section view shown in Fig. 1.

A. Ordering and Symmetry

Density and temperature radial gradients in the scrape-off layer of a tokamak are predicted,¹² from first principles, to be quite steep, and this result is generally confirmed by experiment.¹³⁻¹⁶ There is also recent experimental evidence^{13,14} of strongly (radially) sheared poloidal flows in the plasma edge, both inside and outside of the separatrix.

Defining the general scale length parameter

$$(L_\xi^y)^{-1} \equiv \frac{1}{y} \frac{\partial y}{\partial l_\xi}, \quad (26)$$

the experimental data indicate that

$$L_\psi^n \sim L_\psi^T \sim L_\psi^{v_0} \sim r_L \equiv m v_{th} / eB \quad (27)$$

in the tokamak scrape-off layer, where v_{th} is the thermal speed and r_L is the gyroradius. This ordering has a profound effect upon the fluid theory in the scrape-off layer and divertor relative to the fluid theory in the plasma core, where $L_\psi^n \sim L_\psi^T \sim L_\psi^{v_0} \gg r_L$.

The ion flow along the field lines increases from some fraction of the sound speed in the scrape-off layer to approximately the sound speed just in front of the divertor collector plate.

To a good approximation, tokamaks have toroidal symmetry, and this assumption is invoked to further simplify the fluid equations. When the toroidal derivative

vanishes, $\partial/\partial\phi = 0$, both $\partial/\partial l_\perp$ and $\partial/\partial l_\parallel$ can be related to the poloidal derivative, $\partial/\partial\theta$, hence to each other

$$\frac{\partial}{\partial l_\perp} = \frac{f_\phi}{f_\theta} \frac{\partial}{\partial l_\parallel} \quad (28)$$

where $f_x \equiv B_x / B$ is the normalized field component in the x-direction.

B. Maximally Ordered Momentum and Heat Flux Elements

A "maximal ordering" procedure is adopted to decide which terms to retain in the fluid equations. Each term will be ordered according to the maximum value that it can take on in the scrape-off layer or divertor. The order of quantities, such as the flows, which are determined by a multi-term equation will be taken as the order of the largest term.

With the ordering of Eq. (27) and the observation $v_\parallel \sim v_{ih}$, Eqs. (13) and (14) can be used to determine the maximal ordering

$$\begin{aligned} v_\psi &\sim O\left(\frac{f_\phi}{f_\theta} \frac{r_L}{L_\parallel^p} v_{ih}\right) \ll v_{ih} \\ v_\perp &\sim O\left(\frac{r_L}{L_\psi^p} v_{ih}\right) \sim O(v_{ih}) \\ v_\parallel &\sim O(v_{ih}) \end{aligned} \quad (29)$$

in the scrape-off layer and divertor channel.

Using the orderings of Eqs. (27) and (29) enables the three contributions to the elements of the viscosity tensor given by Eq. (20) to be maximally ordered

$$\begin{aligned}
\pi_{\psi\psi}^o : \pi_{\psi\psi}^\Omega : \pi_{\psi\psi}^\perp &= 1 : (\Omega\tau)^{-1} \frac{L_\parallel^v}{L_\psi^v} : (\Omega\tau)^{-2} \frac{L_\parallel^v}{L_\parallel^h} \\
\pi_{\psi\perp}^o : \pi_{\psi\perp}^\Omega : \pi_{\psi\perp}^\perp &= 0 : 1 : (\Omega\tau)^{-1} \frac{L_\parallel^v}{L_\psi^v} \frac{f_\phi}{f_\theta} \\
\pi_{\psi\parallel}^o : \pi_{\psi\parallel}^\Omega : \pi_{\psi\parallel}^\perp &= 0 : 1 : (\Omega\tau)^{-1} \frac{L_\parallel^v}{L_\psi^v} \\
\pi_{\perp\perp}^o : \pi_{\perp\perp}^\Omega : \pi_{\perp\perp}^\perp &= 1 : (\Omega\tau)^{-1} \frac{L_\parallel^v}{L_\psi^v} : (\Omega\tau)^{-2} \frac{L_\parallel^v}{L_\parallel^h} \\
\pi_{\perp\parallel}^o : \pi_{\perp\parallel}^\Omega : \pi_{\perp\parallel}^\perp &= 0 : 1 : (\Omega\tau)^{-1} \frac{L_\psi^v}{L_\parallel^v} \\
\pi_{\parallel\parallel}^o : \pi_{\parallel\parallel}^\Omega : \pi_{\parallel\parallel}^\perp &= 1 : 0 : 0
\end{aligned} \tag{30}$$

Noting that $L_\psi^v / L_\parallel^v \sim r_L / L_\parallel^v$ and $L_\parallel^v / L_\parallel^h \sim O(1)$, and using Braginskii's expressions⁹ to estimate

$$\Omega\tau \simeq \frac{2.6 B_T T_{ev}^{3/2}}{\sqrt{A} Z^2 n_{14}} \tag{31}$$

to vary from $0(10^2)$ – for $(n=2 \times 10^{14} / \text{cm}^3, B=4T, T=10\text{ev})$ – to $O(10^4)$ –

for $(n=5 \times 10^{13} / \text{cm}^3, B=4T, T=100\text{ev})$ – in the scrape-off layer and divertor, it is clear that the leading terms in the elements of the viscosity tensor are

$$\begin{aligned}
\pi_{\psi\psi} &\simeq \pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega \\
\pi_{\psi\perp} &\simeq \pi_{\psi\perp}^\Omega + \pi_{\psi\perp}^\perp \\
\pi_{\psi\parallel} &\simeq \pi_{\psi\parallel}^\Omega + \pi_{\psi\parallel}^\perp \\
\pi_{\perp\perp} &\simeq \pi_{\perp\perp}^o + \pi_{\perp\perp}^\Omega \\
\pi_{\perp\parallel} &\simeq \pi_{\perp\parallel}^\Omega \\
\pi_{\parallel\parallel} &= \pi_{\parallel\parallel}^o
\end{aligned} \tag{32}$$

It is seen from Eqs. (30) or (32) that the gyroviscous tensor elements are comparable to the parallel viscous elements in the maximal ordering, when the former

exist, and that in two instances the perpendicular viscous element is comparable to the gyroviscous element, despite large differences in viscosity coefficients. This situation arises because the "radial" velocity gradient is much larger than the parallel velocity gradient. The actual magnitudes of the tensor elements depend, of course, on the actual magnitudes of these velocity gradients.

Similar reasoning reveals that the leading order terms in a maximal ordering of the viscous heat flux terms are

$$\begin{aligned}
Q_{\psi}^{\pi} &\simeq v_{\psi} (\pi_{\psi\psi}^{\circ} + \pi_{\psi\psi}^{\Omega}) + v_{\perp} (\pi_{\psi\perp}^{\Omega} + \pi_{\psi\perp}^{\perp}) + v_{\parallel} (\pi_{\psi\parallel}^{\Omega} + \pi_{\psi\parallel}^{\perp}) \\
Q_{\perp}^{\pi} &\simeq v_{\perp} (\pi_{\perp\perp}^{\circ} + \pi_{\perp\perp}^{\Omega}) + v_{\parallel} \pi_{\perp\parallel}^{\Omega} \\
Q_{\parallel}^{\pi} &\simeq v_{\perp} \pi_{\perp\parallel}^{\Omega} + v_{\parallel} \pi_{\parallel\parallel}^{\circ}
\end{aligned} \tag{33}$$

C. Maximally Ordered Fluid Equations

In the maximal ordering of the previous sections, Eqs. (22) - (24) reduce to

$$m \frac{\partial(n v_{\psi})}{\partial t} + \frac{\partial \left(p - \frac{1}{2} \eta_o W^{\circ} + \eta_{\Omega} W_{\psi\psi}^{\Omega} \right)}{\partial l_{\psi}} - n e \left(- \frac{\partial \Phi}{\partial l_{\psi}} + v_{\perp} B \right) = 0, \tag{34}$$

$$\begin{aligned}
&m \frac{\partial(n v_{\perp})}{\partial t} + \left[\left\{ \frac{1}{h_{\perp}^2 h_{\parallel}} \frac{\partial (h_{\perp}^2 h_{\parallel} n m v_{\psi} v_{\perp} + \eta_{\Omega} W_{\psi\perp}^{\Omega} + \eta_{\perp} W_{\psi\perp}^{\perp})}{\partial l_{\psi}} \right\} + \right. \\
&\left. \left\{ \frac{f_{\phi}}{f_{\theta}} \frac{\partial \left(n m v_{\perp}^2 + p - \frac{1}{2} \eta_o W^{\circ} + \eta_{\Omega} W_{\perp\perp}^{\Omega} \right)}{\partial l_{\parallel}} + \frac{1}{h_{\psi} h_{\perp}^2} \frac{\partial (h_{\psi} h_{\perp}^2 (n m v_{\parallel} v_{\perp} + \eta_{\Omega} W_{\parallel\perp}^{\Omega}))}{\partial l_{\parallel}} \right\} \right] \\
&+ n e \left(\frac{f_{\phi}}{f_{\theta}} \frac{\partial \Phi}{\partial l_{\parallel}} + v_{\psi} B \right) = R_{\perp}^1 + S_{\perp}^1, \tag{35}
\end{aligned}$$

$$\begin{aligned}
& m \frac{\partial(n v_{\parallel})}{\partial t} + \left[\left\{ \frac{1}{h_{\perp} h_{\parallel}^2} \frac{\partial(h_{\perp} h_{\parallel}^2 (n m v_{\psi} v_{\parallel} + \eta_{\Omega} W_{\psi\parallel}^{\Omega} + \eta_{\perp} W_{\psi\parallel}^{\perp}))}{\partial l_{\psi}} \right\} + \right. \\
& \left. \left\{ \frac{\partial(n m v_{\parallel}^2 + p + \eta_o W^o)}{\partial l_{\parallel}} + \frac{f_{\phi}}{f_{\theta}} \frac{1}{h_{\psi} h_{\parallel}^2} \frac{\partial(h_{\psi} h_{\parallel}^2 (n m v_{\perp} v_{\parallel} + \eta_{\Omega} W_{\perp\parallel}^{\Omega}))}{\partial l_{\parallel}} + \right. \right. \\
& \left. \left. n m (v_{\parallel}^2 (\Gamma_{\psi\parallel}^{\psi} + \Gamma_{\perp\parallel}^{\perp}) - v_{\perp}^2 \Gamma_{\perp\parallel}^{\perp}) + \eta_o W^o (\Gamma_{\psi\parallel}^{\psi} + \Gamma_{\perp\parallel}^{\perp}) - \eta_{\Omega} W_{\psi\psi}^{\Omega} (\Gamma_{\psi\parallel}^{\psi} - \Gamma_{\perp\parallel}^{\perp}) \right\} \right] \\
& + n e \frac{\partial \Phi}{\partial l_{\parallel}} = R_{\parallel}^1 + S_{\parallel}^1, \tag{36}
\end{aligned}$$

Both the radial and parallel derivative terms in Eq. (12) must be retained in the maximal ordering. In the maximal ordering, the representation of the viscous energy flux given by Eqs. (33) is used in the energy balance Eq. (16). In writing these equations, it has been further assumed that the electric field is electrostatic to leading order, Eq. (20) has been used, and it has been assumed on physical grounds that any radial momentum input to the plasma ions is negligible.

Equation (12) is unchanged. Equation (25) is modified only by the use of Eqs. (28) and (33).

D. Discussion of the Maximally Ordered Fluid Equations

It is instructive to compare certain terms in Eqs. (34) - (36). The second term in Eq. (34) is a generalization of the familiar pressure gradient term to include viscosity terms which are usually ordered out in plasma core applications. These terms are in the ratio

$$\begin{aligned}
p : \eta_o W^o : \eta_{\Omega} W_{\psi\psi}^{\Omega} &= 1 : \tau W^o : \Omega^{-1} W_{\psi\psi}^{\Omega} \\
&= 1 : \left(\frac{\tau v_{th}}{L_{\parallel}^v} \right) \left(\frac{v_{\parallel}}{v_{th}} \right) : \left(\frac{r_L}{L_{\psi}^v} \right) \left(\frac{v_{\perp}}{v_{th}} \right)
\end{aligned}$$

The parameter $\tau v_{th} \sim O(10^{-2})$ in the scrape-off layer where $\frac{v_{\parallel}}{v_{th}} \sim O(10^{-1})$ and $L_{\parallel}^v \sim O(10)$. Deep in the divertor, $\tau v_{th} \sim O(10^{-2} - 1)$, while $\frac{v_{\parallel}}{v_{th}} \sim O(1)$ and $L_{\parallel}^v \sim O(10^{-1} - 1)$. Thus, the parallel viscosity should be comparable to the pressure gradient throughout the scrape-off layer and divertor.

The parameter $r_L / L_{\psi}^v \sim O(1)$. The ratio $(v_{\perp} / v_{th}) \sim O(10^{-1})$ in the scrape-off layer and $O(1)$ deep in the divertor. Thus, the gyroviscous term should be most important in the presheath region immediately in front of the divertor plate, but also significant in the scrape-off layer.

The standard procedure (e.g. refs. 6, 17, 18) is to solve the steady-state form of Eq. (34) for the "drift" velocity, $v_{\perp}$. Formally,

$$v_{\perp} = \frac{1}{neB} \left[\frac{\partial p}{\partial l_{\psi}} + ne \frac{\partial \Phi}{\partial l_{\psi}} - \frac{1}{2} \frac{\partial(\eta_o W^o)}{\partial l_{\psi}} + \frac{\partial(\eta_{\Omega} W_{\psi\psi}^{\Omega})}{\partial l_{\psi}} \right]. \quad (37)$$

The first two terms in Eq. (37) correspond to the conventional expression for the drift velocity, which is usually included in fluid codes that model the plasma edge (e.g. refs. 6, 17, 18). It is clear from the above discussion, however, that "viscous drift velocities" - the last two terms in Eq. (37) - are comparable to diamagnetic and ExB drift velocities - the first two terms in Eq. (37). Thus, the neglect of the last two terms in Eq. (37) in codes that model the plasma edge could be expected to lead to order unity errors in the calculation of the flow fields.

It is normal practice (e.g. ref. 6) to neglect the second $\partial / \partial l_{\parallel}$ term in Eq. (36) relative to the first $\partial / \partial l_{\parallel}$ term. The ratio of inertial contributions to these two terms is

$$\frac{inert_2}{inert_1} = \frac{f_{\phi}}{f_{\theta}} \frac{v_{\perp}}{v_{\parallel}} \sim O\left(\frac{f_{\phi}}{f_{\theta}} \frac{r_L}{L_{\psi}^p}\right) \geq O(1),$$

and the ratio of viscous terms is

$$\frac{(\text{viscous})_2}{(\text{viscous})_1} = \frac{\frac{\partial(\eta_\Omega W_{\parallel\Omega})}{\partial l_\parallel}}{\frac{\partial(\eta_o W^o)}{\partial l_\parallel}} \sim (\Omega\tau)^{-1} \frac{W_{\parallel\Omega}}{W^o}$$

$$\sim (\Omega\tau)^{-1} \frac{(v_\parallel / L_\psi^\nu)}{(v_\parallel / L_\parallel^\nu)} \sim (\Omega\tau)^{-1} \frac{L_\parallel^\nu}{L_\psi^\nu}$$

As noted above, $(\Omega\tau)^{-1}$ varies from about 10^{-2} to 10^{-4} through the scrape-off layer and divertor, while $L_\psi^\nu / L_\parallel^\nu \sim 10^{-3}$ to 10^{-4} , except in regions where the parallel flow stagnates. Thus, neglect of the second $\partial/\partial l_\parallel$ term and, hence, of the gyroviscous term, could be expected to lead to order unity errors in the calculation of the flow field.

The larger implication of these comparisons of terms is that since the viscosity terms can have an order unity effect on the calculation of flow fields in the scrape-off layer and divertor, their partial neglect or approximate representation (as is done in the fluid codes presently being used for analysis of this region) can result in substantial error in the calculated flow fields. Given that the nature of the flow field is a major determiner of the effectiveness of the divertor, omission or misrepresentation of the viscous terms could cause misleading predictions to be made.

When $v_\perp$ can be represented by the first two terms in Eq. (37), or neglected, then the first two terms in Eq. (37) can be used to eliminate $v_\perp$ from Eqs. (34) and (36), and Eq. (35) becomes superfluous. If, in addition, terms leading to mixed derivatives (e.g. $\frac{\partial}{\partial l_\psi} \left(\frac{\partial v}{\partial l_\parallel} \right)$) are omitted, a reduced 2D formulation is obtained. Such a formulation is the basis for many 2D codes used in the modeling of tokamak scrape-off layers and divertors. It has been argued above why it is inappropriate to omit the viscous terms in Eq. (37). As an example of why it is inappropriate to omit the cross derivative terms, compare two such terms in Eq. (36)

$$\frac{\partial(\eta_\Omega W_{\psi\parallel}^\Omega)/\partial l_\psi}{\partial(\eta_o W^o)/\partial l_\parallel} \sim \frac{\frac{\partial}{\partial l_\psi} \left(\eta_\Omega \frac{\partial v_\parallel}{\partial l_\parallel} \frac{f_\phi}{f_\theta} \right) / \partial l_\psi}{\frac{\partial}{\partial l_\parallel} \left(\eta_o \frac{\partial v_\parallel}{\partial l_\parallel} \right)} \sim \frac{f_\phi}{f_\theta} \frac{L_\parallel^p}{L_\psi^p} (\Omega\tau)^{-1}.$$

It has been estimated above that $(\Omega\tau)^{-1}$ varies over 10^{-2} to 10^{-4} in the scrape-off layer and divertor, while $L_\parallel^p / L_\psi^p \sim 10^3 - 10^4$ and $\frac{f_\phi}{f_\theta} \simeq 10$. Thus, the form of the 2D fluid

equations is different when important cross derivative viscous terms are retained than when they are omitted, which has important implications for numerical solution procedures.

E. Reduction of Peak Energy Flux on Divertor Plates

The magnitude and sharply peaked "radial" distribution of the energy flux incident on the divertor collector plate is predicted to be a serious, perhaps limiting, problem for future tokamaks. Accordingly, it is of interest to use the above equations to assess if the inclusion of viscosity in the predictions could alter their outcome; i.e. if viscous effects could ameliorate the severity of the peaked energy loading on the divertor collector plates.

An enhancement of the radial flow of particles or energy would tend to radially level the heat flux distribution on the divertor plate and to transfer some of the energy that would otherwise flow along the field lines to the divertor plates radially out of the divertor channel and subsequently to the divertor walls. Both of these effects would ameliorate the sharply peaked and intense heat flux that is predicted by models that omit viscosity or treat it approximately. Thus, an estimate of the relative magnitudes of viscous-driven radial particle flows and parallel particle flows is of interest. Similarly, an estimate of the magnitude of viscous-driven radial energy flows relative

to convective and conductive energy flows along the field lines is relevant. Noting that the radial dimension of the scrape-off layer is measured in centimeters, while the flow path length along field lines is measured in tens of meters, averaged radial-to-parallel flow ratios greater than about 10^{-3} will be significant.

The steady-state version of Eq. (35) can be solved formally to obtain

$$\mathbf{v}_\psi = \frac{1}{neB} \left[R_\perp^1 + S_\perp^1 - \frac{f_\phi}{f_\theta} \left\{ \frac{\partial p}{\partial l_\parallel} + ne \frac{\partial \Phi}{\partial l_\parallel} + \frac{\partial}{\partial l_\parallel} \left(nm v_\perp^2 - \frac{1}{2} \eta_o W^o + \eta_\Omega W_{\perp\perp}^\Omega \right) \right\} \right. \\ \left. - \frac{1}{h_\psi h_\perp^2} \frac{\partial (h_\psi h_\perp^2 (nm v_\parallel v_\perp + \eta_\Omega W_{\perp\perp}^\Omega))}{\partial l_\parallel} - \frac{1}{h_\perp^2 h_\parallel} \frac{\partial (h_\perp^2 h_\parallel (nm v_\psi v_\perp + \eta_\Omega W_{\psi\perp}^\Omega + \eta_\perp W_{\psi\perp}^\perp))}{\partial l_\psi} \right] \quad (38)$$

The components on the RHS containing viscous tensor elements can be considered as viscosity-driven components of $\mathbf{v}_\psi$. The order of magnitude of these terms can be related to the gradient scale lengths and other parameters (without invoking the maximal ordering)

$$\frac{1}{neB} \left(\frac{f_\phi}{f_\theta} \right) \frac{\partial (\eta_o W^o)}{\partial l_\parallel} \sim O \left[\left(\frac{f_\phi}{f_\theta} \right) \left(\frac{r_L}{L_\psi^\nu} \right) \left(\frac{\tau v_{th}}{L_\parallel^\tau} \right) \left(\frac{5}{2} + \frac{L_\parallel^\tau}{L_\psi^\nu} \right) v_\parallel \right] \quad (39a)$$

$$\frac{1}{neB} \left(\frac{f_\phi}{f_\theta} \right) \frac{\partial (\eta_\Omega W_{\perp\perp}^\Omega)}{\partial l_\parallel} \sim O \left[\left(\frac{r_L}{L_\psi^\nu} \right) \left(\frac{r_L}{L_\psi^p} \right) \left(\frac{r_L}{L_\parallel^\nu} \right) \left(\frac{f_\phi}{f_\theta} \right) \left(\frac{v_{th}}{v_\parallel} \right) \left(1 + \frac{L_\parallel^\nu}{L_\psi^p} \right) v_\parallel \right] \quad (39b)$$

$$\frac{1}{neB} \frac{\partial (\eta_\Omega W_{\perp\perp}^\Omega)}{\partial l_\parallel} \sim O \left[\left(\frac{r_L}{L_\psi^\nu} \right) \left(\frac{r_L}{L_\parallel^p} \right) \left(1 + \frac{L_\parallel^p}{L_\psi^\nu} \right) v_\parallel \right] \quad (39c)$$

$$\frac{1}{neB} \frac{\partial}{\partial l_\psi} (\eta_\Omega W_{\psi\perp}^\Omega) \sim O \left[\left(\frac{f_\phi}{f_\theta} \right) \left(\frac{r_L}{L_\psi^p} \right) \left(\frac{r_L}{L_\parallel^\nu} \right) \left(\frac{r_L}{L_\psi^\nu} \right) \left(1 + \frac{L_\psi^p}{L_\psi^\nu} \right) v_\parallel \right] \quad (39d)$$

$$\frac{1}{neB} \frac{\partial}{\partial l_\psi} (\eta_\perp W_{\psi\perp}^\perp) \sim O \left[\left(\frac{r_L}{L_\psi^\nu} \right)^2 \left(\frac{r_L}{L_\psi^p} \right) (\tau \Omega)^{-1} \left(\frac{v_{th}}{v_\parallel} \right) \left(1 + \frac{L_\psi^\nu}{L_\psi^p} \right) v_\parallel \right] \quad (39e)$$

Since $\left(\frac{r_L}{L_\psi}\right) \sim O(1)$ and $\left(\frac{f_\phi}{f_\theta}\right) \sim O(10)$, the ratios of these viscous-driven radial flows to $v_\parallel$ depend on the local values of $\left(\frac{r_L}{L_\psi^*}\right)$, (τv_{th}) and $\left(\frac{v_{th}}{v_\parallel}\right)$. The quantity (τv_{th}) varies from $\sim O(10^2)$ at $(n = 10^{13} / \text{cm}^3 \text{ and } T = 200 \text{ eV})$ to $\sim O(10^{-2})$ at $(n = 3 \times 10^{14} / \text{cm}^3 \text{ and } T = 10 \text{ eV})$; correspondingly, the first term varies from $\sim O\left(10^3 \left(\frac{r_L}{L_\psi^*}\right) \left(\frac{1}{L_\psi^*}\right)\right) x v_\parallel$ in the scrape-off layer to $\sim O\left(10^{-1} \left(\frac{r_L}{L_\psi^*}\right) \left(\frac{1}{L_\psi^*}\right)\right) x v_\parallel$ just in front of the divertor plate. In the scrape-off layer $L_\psi \sim O(1-10m)$, unless the flow stagnates, but $L_\psi \sim O(10^{-1}m)$ just in front of the divertor plate. The ion gyroradius is $r_L \sim O(10^{-3})$ in the scrape-off layer and $r_L \sim O(10^{-4})$ in front of the divertor plate. Combining, the radial flow component given by Eq. (39a) is estimated as $v_\psi \simeq 10^{-2} v_\parallel$ in the scrape-off layer and as $v_\psi \simeq 10^{-3} v_\parallel$ in front of the collector plate. This could lead to a significant effect, because of the different length flow paths in the parallel and radial directions. Similar arguments lead to similar conclusions for the other three components, except that Eq. (39b) indicates $v_\psi \simeq 10^{-2} v_\parallel$ in front of the collector plate. Thus, it would appear that viscous-driven radial ion flows could be as large as about 10^{-2} times the parallel ion flows. Given the 1-3 orders of magnitude difference in radial and parallel flow paths, this could lead to an order unity or more reduction in the peak radial energy flux convected to the divertor collector plate by ions.

Comparing the different terms in the radial component, Q_ψ^π , of the viscous energy flux, given by the first of Eqs. (33), with the convective, $\frac{5}{2} p v_\parallel$, and conductive,

$$q_\parallel \simeq \frac{4nT\tau}{m} \frac{\partial T}{\partial l_\parallel} \quad (40)$$

parallel energy fluxes, it is found that

$$\frac{v_\parallel \pi_\psi^\Omega}{\frac{5}{2} p v_\parallel} \sim O\left[\left(\frac{f_\phi}{f_\theta}\right) \left(\frac{r_L}{L_\psi^*}\right) \frac{v_\parallel}{v_{th}}\right] \quad (41a)$$

$$\frac{v_{\parallel} \pi_{\psi \parallel}^{\perp}}{\frac{5}{2} p v_{\parallel}} \sim O \left[\left(\frac{r_L}{L_{\psi}^v} \right) \left(\frac{v_{\parallel}}{v_{th}} \right) (\Omega \tau)^{-1} \right] \quad (41b)$$

$$\frac{v_{\perp} \pi_{\psi \perp}^{\Omega}}{\frac{5}{2} p v_{\parallel}} \sim O \left[\left(\frac{r_L}{L_{\psi}^p} \right)^2 \left(\frac{r_L}{L_{\psi}^v} \right) \left(\frac{f_{\phi}}{f_{\theta}} \right) \left(\frac{v_{th}}{v_{\parallel}} \right) \right] \quad (41c)$$

and

$$\frac{v_{\parallel} \pi_{\psi \parallel}^{\Omega}}{q_{\parallel}} \sim O \left[\left(\frac{v_{\parallel}}{v_{th}} \right)^2 \left(\frac{f_{\phi}}{f_{\theta}} \right) (\Omega \tau)^{-1} \left(\frac{L_{\parallel}^T}{L_{\parallel}^v} \right) \right] \quad (42a)$$

$$\frac{v_{\parallel} \pi_{\psi \parallel}^{\perp}}{q_{\parallel}} \sim O \left[\left(\frac{v_{\parallel}}{v_{th}} \right)^2 (\Omega \tau)^{-2} \left(\frac{L_{\parallel}^T}{L_{\psi}^v} \right) \right] \quad (42b)$$

$$\frac{v_{\perp} \pi_{\psi \perp}^{\Omega}}{q_{\parallel}} \sim O \left[\left(\frac{r_L}{L_{\psi}^p} \right)^2 \left(\frac{f_{\phi}}{f_{\theta}} \right) (\Omega \tau)^{-1} \left(\frac{L_{\parallel}^T}{L_{\parallel}^v} \right) \right]. \quad (42c)$$

The quantity $(\Omega \tau)^{-1} \sim O(10^{-4})$ in the scrape-off layer ($n = 5 \times 10^{13} / \text{cm}^3, T = 200 \text{ eV}$), and $(\Omega \tau)^{-1} \sim O(10^{-2})$ in front of the divertor plate ($n = 3 \times 10^{14} / \text{cm}^3, T = 10 \text{ eV}$). Arguments similar to those following Eqs. (39) lead to the estimate that the radial viscous energy flux would be 1-2 orders of magnitude smaller than the parallel convective and conductive energy fluxes in front of the divertor plate, and 2-3 orders of magnitude smaller in the scrape-off layer. Again, given the 1-3 orders of magnitude differences in parallel and radial path lengths, the viscous energy flux is estimated to lead to an order unity reduction in the peak energy flux conducted to the divertor collector plate by ions.

IV. PLASMA EDGE INSIDE SEPARATRIX

The same orderings and symmetries discussed in section III.A are appropriate for the plasma edge inside the separatrix. However, in the confined plasma region it is more natural to use a (ψ, θ, ϕ) flux surface coordinate system (e.g. ref. 11). Such a coordinate system, with a generalization of ψ as described previously, could also be used in the scrape-off layer and divertor channel, and this may in fact be advantageous from the viewpoint of coupling the fluid calculations across the separatrix, which must be an ultimate objective. We will develop the formalism in a general (ψ, θ, ϕ) coordinate system, where ψ may be the flux surface coordinate or the generalized coordinate described in previous sections. The resulting equations are then valid both in the plasma edge inside the separatrix and in the scrape-off layer and divertor channel.

A. Viscosity Representation

The viscosity representation has been given¹⁹ in a flux surface coordinate system. This may be generalized to a general (ψ, θ, ϕ) coordinate system by replacing $(RB_\theta)^{-1} \rightarrow h_\theta, R \rightarrow h_\phi, f_\theta \rightarrow f_\theta / f_\phi$ in Ref. 19. The viscous flux is given by a tensor with elements

$$\begin{aligned}\pi_{\alpha\beta} &= \pi_{\alpha\beta}^o + \pi_{\alpha\beta}^\Omega + \pi_{\alpha\beta}^\perp \\ &= -\eta_o W_{\alpha\beta}^o + \eta_\Omega W_{\alpha\beta}^\Omega - \eta_\perp W_{\alpha\beta}^\perp\end{aligned}\tag{43}$$

where $W_{\alpha\beta}^o, W_{\alpha\beta}^\Omega$ and $W_{\alpha\beta}^\perp$ are decompositions of the rate-of-strain tensor of the flow field constructed according to the prescriptions given previously, for a generalized, curvilinear (ψ, θ, ϕ) coordinate system.

In a general orthogonal curvilinear (ψ, θ, ϕ) coordinate system, with ϕ -symmetry, the first three moments equations are the continuity equation

$$\frac{\partial n}{\partial t} + \frac{1}{h_\phi h_\psi} \frac{\partial(h_\phi h_\psi n v_\theta)}{\partial l_\theta} + \frac{1}{h_\phi h_\theta} \frac{\partial(h_\phi h_\theta n v_\psi)}{\partial l_\psi} = S^\circ, \quad (44)$$

the three $(\hat{n}_\psi, \hat{n}_\theta, \hat{n}_\phi)$ components of the momentum balance

$$\begin{aligned} & \frac{\partial(n v_\psi)}{\partial t} + \frac{1}{h_\phi h_\theta} \frac{\partial(h_\phi h_\theta M_{\psi\psi})}{\partial l_\psi} + \frac{1}{h_\phi h_\psi^2} \frac{\partial(h_\phi h_\psi^2 M_{\psi\theta})}{\partial l_\theta} - \Gamma_{\theta\psi}^\theta M_{\theta\theta} \\ & - \Gamma_{\phi\psi}^\phi M_{\phi\phi} = ne(E_\psi + v_\theta B_\phi - v_\phi B_\theta) + R_\psi^1 + S_\psi^1 \end{aligned} \quad (45)$$

$$\begin{aligned} & \frac{\partial(n v_\theta)}{\partial t} + \frac{1}{h_\phi h_\theta^2} \frac{\partial(h_\phi h_\theta^2 M_{\psi\theta})}{\partial l_\psi} + \frac{1}{h_\psi h_\phi} \frac{\partial(h_\psi h_\phi M_{\theta\theta})}{\partial l_\theta} - \Gamma_{\psi\theta}^\psi M_{\psi\psi} \\ & - \Gamma_{\phi\theta}^\phi (M_{\phi\phi} + M_{\psi\phi}) = ne(E_\theta - v_\psi B_\phi) + R_\theta^1 + S_\theta^1 \end{aligned} \quad (46)$$

$$\begin{aligned} & \frac{\partial(n v_\phi)}{\partial t} + \frac{1}{h_\theta h_\phi^2} \frac{\partial(h_\theta h_\phi^2 M_{\psi\phi})}{\partial l_\psi} + \frac{1}{h_\psi h_\phi^2} \frac{\partial(h_\psi h_\phi^2 M_{\theta\phi})}{\partial l_\theta} \\ & = ne(E_\phi + v_\psi B_\theta) + R_\phi^1 + S_\phi^1 \end{aligned} \quad (47)$$

and the energy balance equation

$$\begin{aligned} & \frac{\partial(\frac{3}{2} p + \frac{1}{2} n m v^2)}{\partial t} + \frac{1}{h_\phi h_\psi} \frac{\partial(h_\phi h_\psi Q_\theta)}{\partial l_\theta} + \frac{1}{h_\phi h_\theta} \frac{\partial(h_\phi h_\theta Q_\psi)}{\partial l_\psi} \\ & = ne \vec{v} \cdot \vec{E} + R^2 + S^2 \end{aligned} \quad (48)$$

where $dl_\theta = h_\theta d\theta$, $dl_\phi = h_\phi d\phi$, $dl_\psi = h_\psi d\psi$, the elements $M_{\alpha\beta}$ and Q_α are defined by Eqs.

(17) and (18), respectively, and the other quantities have been previously defined.

C. Maximal Ordering

The orderings of Eqs. (27) pertain just inside the separatrix, as well as in the scrape-off layer and divertor. These orderings and the momentum balance equations define a maximal ordering of the flow speeds

$$\begin{aligned}
 v_\psi &\sim O\left(\frac{r_L}{L_\theta^P} v_{th}\right) \\
 v_\theta &\sim O\left(\frac{r_L}{L_\psi^P} v_{th}\right) \\
 v_\phi &\sim O\left(\frac{f_\phi}{f_\theta} \frac{r_L}{L_\psi^P} v_{th}\right) \sim O\left(\frac{r_{L\theta}}{L_\psi^P} v_{th}\right)
 \end{aligned} \tag{49}$$

The ratio of the parallel: gyro: perpendicular viscous terms in this maximal ordering is

$$\begin{aligned}
 \pi_{\psi\psi}^\parallel : \pi_{\psi\psi}^\Omega : \pi_{\psi\psi}^\perp &\sim 1 : (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) : (\Omega\tau)^{-2} \\
 \pi_{\psi\theta}^\parallel : \pi_{\psi\theta}^\Omega : \pi_{\psi\theta}^\perp &\sim 0 : 1 : (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) \\
 \pi_{\psi\phi}^\parallel : \pi_{\psi\phi}^\Omega : \pi_{\psi\phi}^\perp &\sim 0 : 1 : (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) \\
 \pi_{\theta\theta}^\parallel : \pi_{\theta\theta}^\Omega : \pi_{\theta\theta}^\perp &\sim 1 : (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) : (\Omega\tau)^{-2} \\
 \pi_{\theta\phi}^\parallel : \pi_{\theta\phi}^\Omega : \pi_{\theta\phi}^\perp &\sim 1 : (f_\phi / f_\theta)^2 (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) : (f_\phi / f_\theta)^2 (\Omega\tau)^{-2} \\
 \pi_{\phi\phi}^\parallel : \pi_{\phi\phi}^\Omega : \pi_{\phi\phi}^\perp &\sim 1 : (\Omega\tau)^{-1} (L_\theta^\nu / L_\psi^\nu) : (\Omega\tau)^{-2}
 \end{aligned} \tag{50}$$

For the plasma edge region just inside the separatrix $n \simeq 1-5 \times 10^{13} \text{ cm}^{-3}$, $T \simeq 100-200 \text{ kev}$. For $B_\phi \simeq 4T$, $(\Omega\tau)^{-1} \simeq 10^{-4}$. For these same parameters, $(\tau v_{th}) \simeq 10^0-10^2$. If $O(\epsilon)$ poloidal asymmetries exist, then $L_\theta^z \simeq \epsilon^{-1} \pi a \simeq \pi R \simeq 10$. $L_\psi^z \simeq 10^{-2}$. Thus, to leading order

$$\begin{aligned}
\pi_{\psi\psi} &\simeq \pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega \\
\pi_{\psi\theta} &= \pi_{\psi\theta}^\Omega + \pi_{\psi\theta}^\perp \\
\pi_{\psi\phi} &= \pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp \\
\pi_{\theta\theta} &\simeq \pi_{\theta\theta}^o + \pi_{\theta\theta}^\Omega \\
\pi_{\theta\phi} &\simeq \pi_{\theta\phi}^o + \pi_{\theta\phi}^\Omega \\
\pi_{\phi\phi} &\simeq \pi_{\phi\phi}^o + \pi_{\phi\phi}^\Omega
\end{aligned} \tag{51}$$

A similar analysis leads to

$$\begin{aligned}
M_{\psi\psi} &\simeq p + \pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega \\
M_{\psi\theta} &\simeq \pi_{\psi\theta}^\Omega + \pi_{\psi\theta}^\perp = M_{\theta\psi} \\
M_{\psi\phi} &= \pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp + nm v_\psi v_\phi = M_{\phi\psi} \\
M_{\theta\theta} &\simeq p + \pi_{\theta\theta}^o + \pi_{\theta\theta}^\Omega + nm v_\theta^2 \\
M_{\theta\phi} &\simeq \pi_{\theta\phi}^o + \pi_{\theta\phi}^\Omega + nm v_\theta v_\phi = M_{\phi\theta} \\
M_{\phi\phi} &\simeq p + \pi_{\phi\phi}^o + \pi_{\phi\phi}^\Omega + nm v_\phi^2
\end{aligned} \tag{52}$$

and to

$$\begin{aligned}
Q_\psi &\equiv v_\psi \pi_{\psi\psi} + v_\theta \pi_{\theta\psi} + v_\phi \pi_{\phi\psi} \simeq v_\psi (\pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega) + v_\theta (\pi_{\psi\theta}^\Omega + \pi_{\psi\theta}^\perp) + v_\phi (\pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp) \\
Q_\theta &\equiv v_\psi \pi_{\psi\theta} + v_\theta \pi_{\theta\theta} + v_\phi \pi_{\phi\theta} \simeq v_\theta (\pi_{\theta\theta}^o + \pi_{\theta\theta}^\Omega) + v_\phi (\pi_{\theta\phi}^o + \pi_{\theta\phi}^\Omega) \\
Q_\phi &\equiv v_\psi \pi_{\psi\phi} + v_\theta \pi_{\theta\phi} + v_\phi \pi_{\phi\phi} \simeq v_\theta (\pi_{\theta\phi}^o + \pi_{\theta\phi}^\Omega) + v_\phi (\pi_{\phi\phi}^o + \pi_{\phi\phi}^\Omega)
\end{aligned} \tag{53}$$

D. Maximally Ordered Fluid Equations

The maximally ordered fluid equations are Eq. (44), Eq. (48) using Eqs. (53), and

$$\frac{\partial(nv_\psi)}{\partial t} + \frac{\partial(p + \pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega)}{\partial l_\psi} = ne \left(-\frac{\partial\Phi}{\partial l_\psi} + v_\theta B_\phi - v_\phi B_\theta \right) \quad (54)$$

$$\begin{aligned} \frac{\partial(nv_\theta)}{\partial t} + \frac{1}{h_\phi h_\theta^2} \frac{\partial(h_\phi h_\theta^2 (nmv_\psi v_\theta + \pi_{\psi\theta}^\Omega + \pi_{\psi\theta}^\perp))}{\partial l_\psi} + \frac{\partial(nmv_\theta^2 + p + \pi_{\theta\theta}^o + \pi_{\theta\theta}^\Omega)}{\partial l_\theta} \\ + \Gamma_{\psi\theta}^\psi (nmv_\theta^2 + \pi_{\theta\theta}^\Omega - \pi_{\psi\psi}^\Omega) + \Gamma_{\phi\theta}^\phi (nmv_\theta^2 - nmv_\phi^2 + \pi_{\theta\theta}^o - \pi_{\phi\phi}^o + \pi_{\theta\theta}^\Omega - \pi_{\phi\phi}^\Omega) \\ = -ne \left(\frac{\partial\Phi}{\partial l_\theta} + v_\psi B_\phi \right) + R_\theta^1 + S_\theta^1 \end{aligned} \quad (55)$$

$$\begin{aligned} \frac{\partial(nv_\phi)}{\partial t} + \frac{1}{h_\theta h_\phi^2} \frac{\partial(h_\theta h_\phi^2 (nmv_\psi v_\phi + \pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp))}{\partial l_\psi} + \frac{1}{h_\psi h_\phi^2} \frac{\partial(h_\psi h_\phi^2 (nmv_\theta v_\phi + \pi_{\theta\phi}^o + \pi_{\theta\phi}^\Omega))}{\partial l_\theta} \\ = nev_\psi B_\theta + R_\phi^1 + S_\phi^1 \end{aligned} \quad (56)$$

In deriving these equations, the maximal ordering results of the previous section have been used. In addition, it has been assumed that the electric field is electrostatic (hence $\partial\Phi \setminus \partial\phi = 0$) and that the "radial" momentum input (S_ψ^1) and collisional momentum exchange (R_ψ^1) were ordered smaller than the leading terms.

E. Discussion of Fluid Equations

The steady-state version of Eq. (54) can be solved formally for

$$v_\phi = \frac{f_\phi}{f_\theta} v_\theta - \frac{1}{B_\theta} \frac{\partial\Phi}{\partial l_\psi} - \frac{1}{neB_\theta} \frac{\partial p}{\partial l_\psi} - \frac{1}{neB_\theta} \frac{\partial(\pi_{\psi\psi}^o + \pi_{\psi\psi}^\Omega)}{\partial l_\psi} \quad (57)$$

Equation (57) is entirely consistent with Eq. (37). The first term corresponds to motion along field lines, the next two terms correspond to the familiar (e.g. refs. 6, 17) ExB and grad-p drifts, and the last term is a viscous drift of the same order of magnitude. This viscous drift, which arises in the ordering appropriate for the plasma edge, has been noted previously²⁰, but it has not heretofore been included in calculations. In the ordering $(r_L / L_\psi) \ll 1$ that is appropriate in the plasma core, the viscous drift orders out, and Eq. (57) reduces to the familiar form (e.g. Ref. 10)

In the usual solution procedure that has been developed (e.g. Ref. 10) for the core of a confined tokamak plasma, the "radial" gradient term is ordered out of Eq. (44) and that equation is then integrated to obtain v_θ to within an arbitrary constant of integration. Then this v_θ is used in Eq. (57) minus the viscous term to obtain v_ϕ to within an arbitrary constant. The arbitrary constant is determined by using these forms in the parallel momentum balance, which is a weighted sum of Eqs. (46) and (47) minus the viscous terms. This procedure is not appropriate in the plasma edge: first because the "radial" gradient term in the continuity equation is of the same order as the poloidal gradient term in the maximal ordering of the plasma edge; and second because the viscous terms cannot be ordered out of the momentum balance equations in the maximal ordering of the plasma edge. Instead, the continuity, momentum and energy balance equations must be solved together²⁰ (and numerically).

The steady-state version of Eq. (56) can be solved formally for

$$v_\psi = \frac{1}{neB_\theta} \left[-R_\phi^1 - S_\phi^1 - \frac{1}{h_\theta h_\phi^2} \frac{\partial (h_\theta h_\phi^2 (nmv_\psi v_\phi + \pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp))}{\partial l_\psi} \right. \\ \left. + \frac{1}{h_\psi h_\phi^2} \frac{\partial (h_\psi h_\phi^2 (nmv_\theta v_\phi + \pi_{\theta\phi}^\Omega + \pi_{\theta\phi}^\perp))}{\partial l_\theta} \right] \quad (58)$$

In this form, it appears that "radial" particle fluxes are driven by toroidal momentum input or sinks (S_ϕ^1), toroidal collisional momentum exchange among species (R_ϕ^1), radial and poloidal gradients in inertial terms (inertial forces), and radial and poloidal gradients in viscous terms (which themselves arise from radial and poloidal gradients in the flow field).

The steady-state version of Eq. (56) can be multiplied through by $h_\psi h_\phi^2$ and integrated over a closed period in θ to obtain an expression for the net "radial" particle flow across the surface denoted by ψ .

$$\oint_\psi h_\psi h_\phi^2 n e v_\psi B_\theta dl_\theta = -\oint_\psi h_\psi h_\phi^2 [R_\phi^1 + S_\phi^1] dl_\theta + \oint_\psi \frac{h_\psi}{h_\theta} \frac{\partial (h_\theta h_\phi^2 (n m v_\psi v_\phi + \pi_{\psi\phi}^\Omega + \pi_{\psi\phi}^\perp))}{\partial l_\psi} dl_\theta$$

This quantity is related by a normalization factor to the familiar flux surface average of the "radial" particle flux. The last term, due to inertial and viscous forces, arises in the short-radial-gradient-scale-length ordering of the plasma edge, but is of higher order in the plasma core.

V. SUMMARY AND CONCLUSIONS

The fluid theory representation of viscosity has been developed in the "short-radial-gradient-scale-length" (srgsl) ordering that is appropriate to the edge plasma in a tokamak. In a maximal ordering, the viscosity is the same order as the pressure and other terms that are normally of leading order in the core of a confined plasma where the radial gradient scale length is long. Fluid equations are developed in this maximal srgsl ordering, both in a coordinate system with axes along and perpendicular to the magnetic field lines and in a coordinate system with axes in the toroidal and poloidal directions. Analysis of these equations reveals the existence of a viscous drift term of the same order as the ExB and diamagnetic drift terms and a complex interrelation among the components of the flow field that is characterized by cross-derivative terms

which fundamentally alter (relative to the situation in the core of a confined plasma) the character of the fluid equations that must be solved to obtain the flow field in the plasma edge.

The radial energy fluxes due to viscosity are estimated to be about 10^{-2} to 10^{-3} times the parallel energy fluxes due to ion conduction and convection in the scrape-off layer and divertor channel. Since the parallel flow path lengths are 10^{+2} to 10^{+3} times the radial dimensions of the scrape-off layer and divertor channel, this implies that viscosity has an order unity effect in smoothing out peaked radial distributions of energy that are transported along field lines from the plasma to the divertor collector plate. It also implies that viscosity has a significant effect in removing energy radially to the wall, which energy would otherwise be transported along the field lines to the divertor collector plates. These important viscous effects will profoundly alter modeling studies of the divertor in tokamaks, since such studies are presently performed with approximate representations of the viscous tensor.

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LIST OF FIGURES

1. Idealized Tokamak Geometry ($\phi = \text{const.}$ cross-section view)

Fig. 1: Idealized Tokamak Geometry
($\phi = \text{const.}$ cross-section view)

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