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Model for Collisional Fast Ion Diffusion into Tokamak Fusion Test Reactor Loss Cone

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An analytic model is developed to estimate the classical pitch angle scattering loss of energetic fusion product ions into prompt loss orbits in a tokamak geometry. The result is applied to alpha particles produced by deuterium-tritium fusion reactions in a plasma condition relevant to Tokamak Fusion Test Reactor (TFTR). A poloidal angular distribution of collisional fast ion loss at the first wall is obtained and the numerical result from the TRANSP code is discussed. The present model includes the effect that the prompt loss boundary moves away from the slowing-down path due to reduction in banana thickness, which enables us to understand, for the first time, the dependence of the collisional loss rate on Z_{eff} .

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I. Introduction

Understanding the behavior of energetic fusion product ions and their influence on the background plasmas in a tokamak fusion reactor is essential for the success of the thermonuclear fusion research. Experimental studies of deuterium-tritium (DT) fusion produced alpha particles are being conducted in the Tokamak Fusion Test Reactor (TFTR).¹ In order to establish a proper background for the study of fusion born alpha particles in TFTR, it is important to have an adequate understanding of the classical alpha particle behavior and confinement.

One of the loss mechanisms often studied for energetic alpha particles is that due to toroidal field ripple, which will not be studied here. Numerical Monte-Carlo studies have shown that the existence of magnetic field ripples in a tokamak fusion reactor can induce large loss of energetic ions.²⁻⁵ Fortunately, magnetic field ripples in a tokamak fusion reactor are normally localized to the outside edge region of plasma column, minimizing their influence on the alpha particles in the burning core. In the present work, we assume that the field ripple strength is negligible. Hence, the analysis presented here is intended for plasma region where the single particle orbits do not suffer from significant ripple distortion.

In addition to the field ripple effect, neoclassical radial transport can also be a candidate for loss of energetic alpha particles. However, under the assumption that all the collisionless orbits are well confined, it was concluded in the literature^{6,7} that the ripple-free neoclassical radial transport by Coulomb collisional process is insignificant for the energetic fusion born ions.

In reality, the large banana excursions of the fusion born ions can make some fraction of the collisionless single particle orbits unconfined, creating a prompt loss region in velocity space (velocity space hole). This makes the prompt first-orbit loss (prompt loss) to the wall to be the most obvious and direct loss mechanism for the energetic fusion born ions. Detailed studies conducted in Refs. 8-13 showed that a significant

fraction (5 – 10%) of the new born alpha particles can be promptly lost to the first wall in a medium size tokamak like TFTR. Losses of this magnitude would not be tolerable in a fusion reactor due to potentially localized heat loading onto the first wall.

There is another loss process which is associated with the prompt loss process. It is the pitch angle scattering loss of the marginally confined particles into the prompt loss region.^{12,14,15} Normally, the 90°-pitch angle scattering rate is small compared to the slowing-down rate for energetic ions like alphas. But the marginally confined ions near the prompt loss region need only a small amount of pitch angle change to be scattered into the prompt loss region, and the small angle scattering rate is much higher than the 90°-scattering rate. Hence, a boundary layer will form in velocity space near the prompt loss region within which the distribution function of the energetic ions will decay toward the prompt loss region and vanish at the loss boundary. The slope formed by this decay in the particle distribution function will generate a net diffusive flow of fusion products into the prompt loss region by the random pitch angle scattering process (collisional non-prompt loss).

The collisional non-prompt loss has been previously studied in Refs. 12,14,15. Reference 14 used a constant electric current density throughout a circular plasma cross-section and performed a Monte-Carlo calculation including the actual banana motions of the energetic alpha particles, and concluded that the collisional non-prompt loss may be safely negligible in a DT plasma at low Z_{eff} values. However, Ref. 14 also concluded that the collisional non-prompt loss rate is rather sensitive to the effective charge number Z_{eff} (proportional to Z_{eff}), which can make the collisional non-prompt loss rate significant at higher Z_{eff} numbers. On the other hand, Ref. 12 used a thin banana approximation in a concentric circular flux surface geometry for an analytic modeling of the non-prompt loss rate and also concluded that the non-prompt loss is small. However, Ref. 12 found that the collisional non-prompt loss rate is proportional to $\sqrt{Z_{eff}}$, making it less sensitive to the impurity contamination level. The disagreement has not been understood so far in the literature. A three dimensional Fokker-Planck

study has been reported recently in Ref. 15, but the collisional non-prompt loss rate predicted for TFTR in Ref. 15 was an order of magnitude higher than those reported in Refs. 14 and 12. The non-prompt loss phenomenon has also been observed by ORBIT code,³ where the collisional loss was found to be much less than prompt loss for a typical TFTR case.¹⁶

Even though the resulting alpha loss may be small in the overall power balance, a proper understanding of the collisional non-prompt loss process can be important for two reasons. Firstly, it could provide way to test experimentally whether the energetic alpha particles are subject to classical or anomalous scattering in velocity space.¹⁷ Secondly, it could indicate if the non-prompt losses are localized in poloidally narrow wall regions, which could raise a concern about localized heating of plasma-facing components even if the non-promptly lost alphas represent only a small percentage of total alpha power.

In this work, we present an improved analytic model to calculate the collisional non-prompt loss phenomenon in a tokamak plasma condition similar to that of TFTR, including the fat banana effect which makes the prompt loss boundary move away from the confined, slowing-down particles. The moving-away of the loss boundary occurs because the banana width gets smaller as a particle slows down; hence, the turning points shift toward smaller magnetic field strength, making the trapped boundary move to smaller pitch angle values. The analytic expression obtained in this work for the collisional non-prompt loss rate makes it possible to understand the underlying physical phenomena more clearly, including the sensitivity to the Z_{eff} values. Developing a practical analytic tool to predict such a loss rate can be timely since the DT experiments in TFTR is already in progress, and it can be used to interpret the experimental data on the loss of energetic fusion born ions into a fast particle detector.^{17,18} The poloidal angular distribution of collisional fast ion loss at the first wall is obtained. A Monte-Carlo result from the TRANSP¹⁹ code analysis is also discussed.

In order to simplify the analyses, we use the following assumptions in the present work. We assume that the background electrostatic potential effect is negligible for

the energetic ions because the background electrostatic potential energy is of order background plasma temperature, which is much smaller than the energy of the fusion product ions. We assume that the plasma is quiescent, free from instabilities. We assume that the particle gyroradius is small compared to the gradient scale length of the equilibrium plasma quantities, which allows us to use the guiding center position to describe the particle orbits. We also assume that the safety factor is reasonably high ($q \gtrsim 1$), hence the poloidal magnetic field is much weaker than the toroidal magnetic field. The ripple field is assumed to be absent, as mentioned earlier, and the first wall is circular shaped. Furthermore, we assume that the plasma current is high enough so that the barely passing orbits originating from the high fusion-reaction region are confined within the first wall, and that the special types of orbits due to orbit stagnation (or pinch) at outside of magnetic axis^{8-10,12,13} do not occur within the first wall. Thus all the prompt loss orbits in the present work are of normal banana type. Detailed description of special orbits with stagnation occurring at outside of magnetic axis can be found in Refs. 8-10,12,13. Disconnected orbits are discussed in Ref. 9, which are neglected in the present work. We note here, however, that we take into consideration of the fat banana orbits and shift of flux-surface, which were absent in the previous analytic treatment.¹²

The prompt loss boundaries are described in Section II under the plasma conditions relevant to TFTR DT operation. In Section III, a simplified description of the pitch angle scattering process for high energy ions is presented, in order to help understand the basic physical phenomena for the collisional non-prompt loss. In Section IV, diffusion of the energetic particles into the prompt loss region in velocity space is formulated and a simple analytic expression for the collisional non-prompt loss rate is obtained. The result is applied, in Section V, to a standard TFTR plasma condition to calculate the non-prompt loss rates. It is predicted that the non-prompt Coulomb loss rate of energetic alpha particles is small ($\lesssim 2\%$) in TFTR supershots with moderate to high plasma current ($\gtrsim 1.65MA$). A numerical result from TRANSP is presented in Sec. VI and some discussions are contained in Sec. VII.

II. The prompt loss boundary

The boundary of the prompt loss region in phase space has been extensively studied previously in Refs. 8–13. A derivation of prompt loss boundary using a circular wall geometry is presented here for heuristic purpose, and it is intended for plasma conditions suitable to TFTR plasmas at minor radial locations where the collisionless barely passing orbits are confined. We note here that barely passing orbits of 3.5 MeV alphas traveling through the region near the plasma edge in TFTR can intersect the first wall, depending upon the toroidal field and plasma current strength. Such orbits are excluded from the present analysis under the assumption that the density of energetic alpha particles at such radial locations is negligibly small compared to the inner radial region of the plasma where the barely passing orbits are confined.

It is also assumed here that, as stated before, the 3.5 MeV alpha orbits under consideration here consist of normal types of passing and banana orbits within the first wall boundary (excluding special orbits like double stagnation or co-pinch orbits, discussed in Refs. 8–10,12). The above assumptions used here are valid for the TFTR plasma conditions analyzed in the present work, and are expected to be valid for most of the higher plasma current experiments in TFTR. Typical particle orbits in TFTR are shown in Fig. 1. The orbit marked (a) is barely passing, (b) is barely trapped, and (c) is deeply trapped intersecting the wall at the outboard midplane. Banana particles with pitch angles falling between (a) and (b) are lost to the wall.

For energetic ions, the collision frequency is small compared with the particle transit frequency around the poloidal or toroidal orbits. Thus, the lowest order particle orbit can be well approximated by the collisionless canonical angular momentum conservation equation,

$$\Delta_o \psi = I \Delta_o (v_{\parallel} / \Omega), \quad (1)$$

where Δ_o is an operator representing the change during the particle orbital motion, ψ is the usual poloidal magnetic flux (divided by 2π), Ω is the gyrofrequency of the

energetic ions, $v_{\parallel}$ is the component of the particle velocity $\vec{v}$ in the direction parallel to the equilibrium magnetic field $\vec{B}$, $I = R_c B_{Tc}$, and R_c and B_{Tc} are the major radius and the toroidal component of magnetic field at the magnetic axis, respectively. As implied in the definition of R_c and B_{Tc} , the subscript “c” will denote the value at the magnetic axis in the present work.

Because all the particle orbits can be extended to cross the outside midplane, the reference point for a flux surface is taken at the outside midplane, where we define the poloidal angle to be zero. Reference point position, particle speed, and magnetic moment together yield a complete description of the particle orbits. Variation of the major radius R , magnetic field strength B , and gyrofrequency Ω on a flux surface are given by

$$R = R_c(1 + x/R_c), \quad B = B_c/(1 + x/R_c),$$

$$\Omega = \Omega_c(B/B_c) = \Omega_c(R_c/R) = \Omega_c/(1 + x/R_c),$$

where $x = R - R_c$ is the horizontal minor radius. Then, the variation of the parallel speed $v_{\parallel}$ is derived from the conservation of the particle kinetic energy W and the magnetic moment μ :

$$v_{\parallel} = v\xi = v(1 - \lambda B/B_c)^{1/2},$$

where $\xi = (1 - \lambda B/B_c)^{1/2}$ and $\lambda = \mu B_c/W$ are used. Notice here that λ is a pitch angle variable and it is a conserved quantity along a particle orbit due to conservation of magnetic moment and kinetic energy. It is convenient to use (v, λ) to describe the guiding center velocity.

Under the present physical conditions, where all the passing orbits are confined and only the normal types of passing and banana orbits are present, the loss boundaries at a given speed v are given by $\lambda = \lambda_t$, representing the fattest banana orbit at the transition between the trapped and passing regions, and by $\lambda = \lambda_i$, representing the maximum λ value (thinnest banana) in the trapped regime which can lead to the particle orbits intersecting with the wall (see Fig. 1). The prompt loss region at a given v is then

described by $\lambda_t < \lambda < \lambda_i$ as can be seen from Fig. 2.

In order to calculate the value of λ_t , we need to know the turning location R_t of the marginally trapped particles at $\theta = \pi$. If we use $x_t \equiv R_t - R_c$ to denote the horizontal location x of the turning point where $\xi = 0$, then the relation $0 = 1 - \lambda_t B(x_t, \theta = \pi)/B_c$ at the turning point yields

$$\lambda_t = B_c/B(x = x_t, \theta = \pi) = 1 + x_t(v)/R_c. \quad (2)$$

And the value of $\xi = \xi_t$ corresponding to a marginally trapped particle, observed at the horizontal distance x_p on the outboard midplane, becomes

$$|\xi_t| = [1 - (1 + x_t/R_c)/(1 + x_p/R_c)]^{1/2} = [(x_p - x_t)/(R_c + x_p)]^{1/2},$$

where x_t is a negative value for a normal type of marginally trapped orbit. The canonical angular momentum conservation between the observation point x_p at the outside midplane and the turning point ($\xi = 0$) then becomes

$$\psi(x_t) - \psi(x_p) = \frac{Iv}{\Omega_c} |\xi_t| (1 + x_p/R_c) = \frac{Iv}{\Omega_c} \left[\left(1 + \frac{x_p}{R_c}\right) \frac{x_p - x_t}{R_c} \right]^{1/2}, \quad (3)$$

which is equivalent to the corresponding equations in Refs. 8 and 9.

Equation (3) gives a general relationship to determine the turning point x_t (for $x_t < x_p$), hence the values of ξ_t and λ_t , as a function of x_p and v . The left-hand-side can be easily evaluated from a relation between ψ and $x = R - R_c$, which is a well known relationship for a given plasma equilibrium. When x_t is known, the passing side of the prompt loss region in (v, λ) space is then given by Eq. (2).

Notice here in Eq. (3) that x_t is function of v . It can be easily seen from Eq. (3) that as v becomes smaller, $x_t < 0$ gets smaller (or $|x_t|$ gets larger). Thus, unlike the thin banana cases, λ_t (or ξ_t) is a function of particle speed. In general as a particle slows down, the banana width gets smaller and, thus, the turning points shift toward smaller magnetic field strength, making $|\xi_t|$ smaller and λ_t larger according to Eq. (2). This effect can be seen from Fig. 2: The left boundary of the prompt loss region corresponds

to $\lambda = \lambda_t$. It is sloped away from a slowing down path (which is along constant- λ path¹²). This effect, which was neglected in the previous analytic modeling,¹² can have a significant influence on the collisional non-prompt loss rate, as will be shown later. We note here that, usually, the shift of turning point during slowing-down is more pronounced in a plasma with high- β shift of magnetic flux surfaces or with low central plasma current.

Since the banana width gets smaller as the particle kinetic energy gets lower, the prompt loss region extends downward in v until the marginally trapped orbit (which yields the fattest banana orbit for a given v) can no longer intersect with the wall. We use λ_L to denote the value of λ corresponding to this orbit. The minimum loss speed, denoted by $v = v_L$, can again be calculated from the canonical angular momentum conservation between the turning point, described by $\xi = 0$ at $(x = x_t, \theta = \pi)$, for a marginally trapped particle, and the wall strike point identified by $|\xi| = |\xi_w| = [1 - \lambda_t(v_L)/(1 + x_w/R_c)]^{1/2}$ at $(x = x_w, \theta = 0)$. Using Eq. (1), we obtain

$$\psi(x_w) - \psi(x_t) = \frac{Iv_L}{\Omega_c} [1 - \lambda_t(v_L)/(1 + x_w/R_c)]^{1/2} (1 + x_w/R_c).$$

Since $\lambda_t(v_L) = 1 + x_t(v_L)/R_c$, this equation yield a relation between x_t and v_L :

$$\psi(x_w) - \psi(x_t) = \frac{Iv_L}{\Omega_c} \left[\frac{x_w - x_t(v_L)}{R_c} \right]^{1/2} \left(1 + \frac{x_w}{R_c} \right)^{1/2}. \quad (4)$$

On the other hand, Eq. (3) yields another relation between x_t and $v = v_L$:

$$\psi(x_t) - \psi(x) = \frac{Iv_L}{\Omega_c} \left[\left(1 + \frac{x_p}{R_c} \right) \frac{x_p - x_t}{R_c} \right]^{1/2}. \quad (5)$$

Equations (4) and (5) constitute two coupled equations to determine the minimum loss speed v_L and the corresponding turning point x_t , and thus, $\lambda_t(v_L)$.

The trapped-side boundary of the prompt loss region, usually located deeply inside of the trapped regime, is denoted here by $\lambda_i(v)$, and it can be obtained by identifying the thinnest banana orbit which can intersect with the wall for a given v . The thinnest unconfined banana orbits in TFTR are identified as those which intersect with the

circular first wall (limiter) at $\theta = 0$ and $x = x_w$ [see the orbit-(c) in Fig. 1]. In this case, the canonical angular momentum conservation equation (1) yields

$$\begin{aligned}\psi(x_w) - \psi(x_p) &= \frac{Iv}{\Omega_c} \xi_i(x_w)(1 + x_w/R_c) - \frac{Iv}{\Omega_c} \xi_i(x_p)(1 + x_p/R_c) \\ &= \frac{Iv}{\Omega_c} \left[\left[1 - \frac{\lambda_i(v)}{1 + x_w/R_c} \right]^{1/2} (1 + \frac{x_w}{R_c}) + \left[1 - \frac{\lambda_i(v)}{1 + x_p/R_c} \right]^{1/2} (1 + \frac{x_p}{R_c}) \right], \quad (6)\end{aligned}$$

where the two terms in the right hand side are additive because the parallel speed changes sign between $(x_p, \theta = 0)$ and $(x_w, \theta = 0)$ during the banana orbital motion. The value of $\lambda_i(v)$ is then obtained by solving Eq. (6).

Exact solutions to Eq. (6) can be found numerically, But, a simple analytic solution to Eq.(6) is possible. Using the facts that $\lambda_i = \lambda_{i0}$ at $v = v_0$ and $\lambda_i = \lambda_L$ at $v = v_L$, and assuming that the deviation of the loss boundary $\lambda_{i0} - \lambda_i(v)$ from λ_{i0} is small, Taylor expansion of the above equation in the small quantity $\lambda_{i0} - \lambda_i(v)$ yields a simple approximate solution for the deviation of $\lambda_i(v)$ from $\lambda_{i0} = \lambda_i(v = v_0)$:

$$\lambda_{i0} - \lambda_i(v) = (\lambda_{i0} - \lambda_L) \frac{1/v - 1/v_0}{1/v_L - 1/v_0}. \quad (7)$$

If we use this approximate relationship, Eq. (7), for the trapped-side of the loss boundary, then Eq. (6) needs to be solved for $\lambda_{i0} = \lambda_i(v = v_0)$ only. Once we find λ_{i0} , the value of ξ_{i0} corresponding to λ_{i0} is identified by

$$\xi_{i0} = [1 - \lambda_{i0}/(1 + x/R_c)]^{1/2}.$$

The fraction of the prompt loss region F_{prompt} relative to the whole velocity space at the reference point $(x = x_p, \theta = 0)$ is then

$$F_{prompt} = (\xi_{i0} - \xi_{i0})/2.$$

F_{prompt} can represent the prompt loss fraction of the energetic particles born at the point $(x = x_p, \theta = 0)$. The value of F_{prompt} increases toward larger minor radius, but it needs to be negligibly small over most of the hot plasma radius in a successful fusion reactor. In a typical TFTR case represented by Table I, F_{prompt} is zero for x_p (at $\theta = 0$)

less than $1/5$ of plasma minor radius, corresponding to the fattest banana lying entirely inside the first wall. But, the loss fraction increases sharply at larger x_p values. At x_p corresponding to $1/3$ of the minor radius, we obtain $F_{prompt} = 5\%$ for protons or tritons from D-D reactions and $F_{prompt} = 8.5\%$ for alphas from DT reactions, as can be obtained from Fig. 2.

It should be noted here, however, that F_{prompt} represents the prompt loss fraction for particles born at $(x = x_p, \theta = 0)$ only. It does not represent the prompt loss fraction F_{prompt} of all the particles whose orbits pass through this point. This is because the point $(x = x_p, \theta = 0)$ are shared by a set of different prompt loss orbits which travel through different flux surfaces at different poloidal angles, and the birth rate $\mathcal{S}$ of the energetic particles is function of flux surfaces. In order to obtain the global F_{prompt} , the value of F_{prompt} needs to be calculated at all points in the plasma and an $\mathcal{S}$ -weighted volume integral needs to be performed. This is a well documented procedure⁸⁻¹³ but is not attempted in the present work. The present section is presented here to define boundaries for the collisional non-prompt losses to be used in the later sections.

III. Heuristic description of pitch angle diffusion and non-prompt loss

To help understand the physical process of the collisional non-prompt loss phenomenon before a detailed analysis, we present here a simplified description using a random-walk diffusion argument. Energetic ions born at $v = v_0$ will mainly slow down by collisions with background plasma, leading to a slowing-down distribution in energy. But at the same time, they will experience small-step random walks in pitch angle direction due to weak pitch angle scatterings with the background ions. For most of the energetic ions, the small scale random walk in pitch angle is insignificant. However, for energetic ions near the prompt loss boundary, the small-scale random walk in pitch angle can lead to diffusive loss into the prompt loss region.

We first demonstrate the significance of pitch angle spread for an energetic ion born

near the prompt loss region as it slows down from the birth speed v_0 . To evaluate the expected spread in pitch angle, we recall the definition of the perpendicular scattering time $\tau_\perp$ and write,

$$\tau_\perp^{-1} \simeq \frac{1}{v^2} \frac{(\delta v_\perp)^2}{\delta t},$$

where $\delta v_\perp = |v_\perp - v_{\perp 0}|$ is the deviation of $v_\perp$ from the initial value $v_{\perp 0}$. Physically it means that after a short time δt a fast ion experiences change in the perpendicular velocity component by the amount

$$\delta v_\perp \simeq v N_\perp^{1/2},$$

where $N_\perp = \delta t / \tau_\perp$ is the number of perpendicular angle scatterings in the time interval δt . The corresponding change in $\xi \equiv v_\parallel / v$ is also proportional to $N_\perp^{1/2}$:

$$\delta \xi = \delta(1 - v_\perp^2 / v^2)^{1/2} = \frac{v_{\perp 0} \delta v_\perp}{v^2 \xi_0} = (1 - \xi_0^2)^{1/2} N_\perp^{1/2} / \xi_0, \quad (8)$$

where $\xi_0 = (1 - v_{\perp 0}^2 / v^2)^{1/2}$.

From this equation, we can obtain the spread in particle pitch angle for the barely passing particles born near the prompt loss region, while they slow down from the birth speed v_0 to the speed v_L corresponding to the bottom of the prompt loss region. In terms of the slowing down time $\tau_s \equiv 1/\nu_s$, where ν_s is the slowing down rate, the time needed for a fast particle to slow down by the amount $\delta v = v_0 - v_L$ can be written as $\delta t \simeq \tau_s \delta v / v_0$, provided $\delta v \ll v_0$. Then, $N_\perp$ becomes $N_\perp = \delta t / \tau_\perp = (\tau_s / \tau_\perp)(\delta v / v_0)$.

Let us calculate the spread $\delta \xi$ for the case shown in Fig. 2. For a confined passing particle near the loss cone, we take $\xi_0 = 0.6$. Using the alpha particle parameters relevant to TFTR (see table II) for $\tau_s = 1/\nu_s$, $\tau_\perp = 1/2\nu_{90}^0$, and $\delta v = v_0 - v_L$; we obtain $N_\perp = 0.03$ and, from Eq. (8),

$$\delta \xi = (4/3) N_\perp^{1/2} \simeq 0.07,$$

for a barely passing alpha particle. Or, in terms of the other pitch angle parameter $\lambda \equiv (v_\perp^2 / v^2)(B_c / B)$ introduced in the previous section, we obtain

$$\delta \lambda = 2 \frac{v_{\perp 0}}{v_0} \frac{\delta v_\perp}{v_0} \frac{B_c}{B} \simeq 2 \frac{v_{\perp 0}}{v_0} N_\perp^{1/2} \frac{B_c}{B} \simeq 0.1,$$

where $(v_{\perp 0}/v)B_c/B = (1 - \xi_0^2)^{1/2}B_c/B \simeq 0.86$ has been used for $x = 21$ cm and $R_c = 261$ cm.

As can be seen from a pictorial description of the pitch angle spread in Fig. 2 (shaded region), a passing energetic ion born within $\delta\lambda$ from the prompt loss boundary (to be more precise, boundary λ_L evaluated at $v = v_L$) has a reasonable probability of diffusing into the the prompt loss region in velocity space. For a birth speed v_0 and $\xi_0 = 0.6$, the amount of diffusive spreading is visually illustrated in Fig. 2b as a non-negligible fraction of energetic ions that diffuse into the left side of the prompt loss region. Diffusive spreading by pitch angle scattering on the right hand side of the loss region is not fast enough to cause the thermalizing ions to reach the loss boundary, also as shown in Fig. 2b for $\xi_0 = 0.35$.

It is possible to estimate the diffusive flow rate into the prompt loss region using a somewhat more detailed random walk argument in the variable $\xi \equiv v_{\parallel}/v$. The small angle scattering rate, $\nu_{\Delta\xi}$, is more frequent than the 90° scattering rate, ν_{90} according to the following relationship:

$$\nu_{\Delta\xi} \simeq \nu_{90}/(\Delta\xi)^2.$$

Thus, at a distance $\Delta\xi \simeq (\nu_{90}/\nu_s)^{1/2}$ from the prompt loss boundary, the rate of slowing down rate becomes the same as the rate of pitch angle scattering loss into the prompt loss region. Within $\Delta\xi$ from the prompt loss region, a boundary layer is formed in which the pitch angle scattering rate is higher than the slowing down rate. The particle distribution function in the $\Delta\xi$ -boundary layer decreases and becomes zero at the prompt loss boundary. This leads to a slope in the particle distribution function of the magnitude $\partial f/\partial\xi \simeq f_s/\Delta\xi$. The diffusion coefficient in ξ -direction is:

$$D_\xi \sim \nu_{90}v^2,$$

and the diffusive flux of particles in the pitch angle direction is

$$\Gamma_\xi \simeq -D_\xi \frac{\partial f}{v \partial \xi} \simeq \nu_{90} \frac{f_s}{v \Delta\xi},$$

where $f_s \simeq S/(4\pi\nu_s v^3)$ is the slowing-down distribution of energetic ions.

The collisional non-prompt loss fraction F_ν relative to the birth rate S is obtained by multiplying the flux Γ_ξ by the area of the prompt-loss boundary surface $2\pi v_\perp v(v_0 - v_L)$,

$$F_\nu \simeq 2\pi v_\perp v(v_0 - v_L)\Gamma_\xi/S \simeq \frac{1}{2\Delta\xi} \frac{\nu_{90}}{\nu_s} \frac{v_0 - v_L}{v_0} = \frac{1}{2} \left(\frac{\nu_{90}}{\nu_s} \right)^{1/2} \frac{v_0 - v_L}{v_0},$$

where the relation $(\Delta\xi)^2 \simeq \nu_{90}/\nu_s$ has been used. Using $v_L/v_0 \simeq 0.69^{1/2}$ from Fig. 1, and $\nu_{90}/\nu_s \simeq 45$ for a typical TFTR DT plasma (see Table I and II), we obtain

$$F_\nu \sim 1.3\%$$

for non-prompt loss fraction (compared to the birth rate) of alpha particles in DT fueled TFTR experiments. A more complete and formal expression for the collisional non-prompt loss rate will be derived in the later section. The simple description presented above is to help understand the basic physical processes behind the collisional non-prompt losses before the detailed mathematical derivation.

IV. Calculation of the collisional flux into the prompt loss region

In this section, we calculate the Coulomb pitch angle scattering loss of marginally confined particles into the prompt loss region. The small angle deflection processes, which occurs much more frequently than the nominal 90° deflection processes, can scatter marginally confined energetic ions into the prompt loss region as they slow down. As a result, a boundary layer will form in velocity space near the loss boundary, where a diffusive flow of energetic particles into the prompt loss region is established by the random Coulomb pitch angle scattering process. A qualitative description of this mechanism was presented in Section II.

Analytic modeling of this process was reported previously in Ref. 12. However, Ref. 12 neglected the fat banana effect for non-prompt losses. In the present section, another analytic model will be presented with finite banana effect included, which makes the prompt loss boundary move away from the slowing-down particles and enhances the

boundary layer width. An analytic expression for the collisional non-prompt loss rate is worth pursuing because it enables us to understand the underlying physical phenomena, as will be discussed later in this section.

We assume that the fusion process is in steady state. We also assume that the background electrostatic potential effect is negligible for the energetic ions and that the particle gyroradius is small compared with the gradient scale length of the equilibrium plasma quantities. More general prescription of the underlying assumptions are as presented in Section I. Then the guiding center Fokker-Planck equation yields

$$\vec{v}_{gc} \cdot \nabla f = C(f) + S,$$

where f is the particle distribution function of the fusion product, $\vec{v}_{gc}$ is the guiding center velocity, C represents a linearized Coulomb collision operator, and S represents the particle source at the birth energy $v = v_0$. Since the collisional effect is weak during a guiding center orbit for the energetic particles, an expansion in small collision frequency is appropriate; $f = f_0 + f_1 + \dots$. Then the zeroth order equation is

$$\vec{v}_{gc} \cdot \nabla f_0 = 0,$$

which implies that f_0 is constant along the guiding center motion. Here, the source function S is regarded as first order in collision frequency because it needs to be balanced against the Coulomb slowing-down process in steady state. The first order equation becomes

$$\vec{v}_{gc} \cdot \nabla f_1 = C(f_0) + S,$$

and f_0 is determined from orbital average version of the first order equation

$$\{C(f_0) + S\} = 0, \tag{9}$$

where $\{\dots\}$ represents orbital average operation.

Using v and $\lambda \equiv \mu B_c / W$ to represent the particle velocity, where μ is the magnetic moment, B_c is a representative value of the magnetic field strength B , and E is the

kinetic energy, the collision operator can be simplified into the following form:²⁰

$$C(f_0) = \nu_s v^2 \frac{\partial}{\partial v} (v^3 + v_c^3) f_0 + \nu_{90} 2h\xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0,$$

where $h = R/R_c$; ν_s is the alpha particle slowing-down rate (e-folding rate for speed) given as the inverse of the slowing-down time τ_s ,

$$1/\nu_s \equiv \tau_s = \frac{3m_\alpha T_e^{3/2}}{4\sqrt{2\pi m_e} Z_\alpha^2 e^4 n_e \log \Lambda_e};$$

$\nu_{90} = v_b^3/(v^3 \tau_s)$ is the pitch angle scattering rate, v_b denotes the critical pitch angle scattering speed where the the electron slowing down rate is the same as the ion pitch angle scattering rate,

$$v_b^3 = \frac{3\sqrt{\pi} T_e^{3/2} \sum_a (Z_a^2 n_a \log \Lambda_a)}{\sqrt{2m_e n_e} m_\alpha \log \Lambda_e}; \quad (10)$$

and v_c is the critical alpha particle slowing down speed where the alpha slowing down rate by plasma ions is as strong as that by electrons,

$$v_c^3 = \frac{3\sqrt{\pi} T_e^{3/2} \sum_a (Z_a^2 n_a \log \Lambda_a / m_j)}{\sqrt{2m_e n_e} \log \Lambda_e}.$$

The above collision operator is local in the sense that it is described in terms of local values of background plasma densities and temperatures. It can conveniently describe the collisional effect on a particle whose orbit lies within a flux surface, since the background plasma densities and temperatures are constant on a flux surface. However, in the present case where the particle orbits considerably deviate from a flux surface, we use a modified form of collision operator which uses average values for the background densities and temperatures:

$$C(f_0) = \bar{\nu}_s v^2 \frac{\partial}{\partial v} (v^3 + \bar{v}_c^3) f_0 + \bar{\nu}_s \Delta_\xi^2 2h\xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0,$$

where the parameters $\bar{\nu}_s$, $\Delta_\xi^2 \equiv \bar{v}_b^3/v^3$ and $\bar{v}_c^3$ are defined from the values of background densities and temperatures evaluated at the bounce averaged radial position $\bar{r} = \{r\}$. An average slowing-down time can also be defined in the same way with the relationship $\bar{\tau}_s \equiv 1/\bar{\nu}_s$.

Equation (9), then, becomes

$$\frac{\bar{\nu}_s}{v^2} \frac{\partial}{\partial v} (v^3 + \bar{v}_c^3) f_0 + \bar{\nu}_s \Delta_\xi^2 \{ 2h\xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0 \} + \{S\} = 0.$$

In the present case where a particle orbit does not follow a flux surface, the usual flux-surface average process is not very useful. Instead of the flux-surface average, we use an orbit-surface average $\langle \cdots \rangle_o$. The orbit-surface is given by an axisymmetric surface which contains the actual particle orbit. From the general mathematical relationship²¹ between the orbit average and a surface average $\{h\xi(\partial/\partial\lambda)\xi Y\} = \eta(\partial/\partial\lambda) \langle |\xi| \rangle_o Y$ for some function $Y(v, \lambda)$, this equation is changed into

$$\frac{\bar{\nu}_s}{v^2} \frac{\partial}{\partial v} (v^3 + \bar{v}_c^3) f_0 + 2\eta \bar{\nu}_s \Delta_\xi^2 \frac{\partial}{\partial \lambda} \lambda \langle |\xi| \rangle_o \frac{\partial}{\partial \lambda} f_0 + \{S\} \simeq 0, \quad (11)$$

where η is defined to be

$$\eta = (\int d\ell B_c / v B) / (\int d\ell / |v_{gc}|);$$

and the guiding center speed v_{gc} can be written as a combination of parallel (to $\vec{B}$) velocity $\vec{v}_\parallel$ and the drift velocity $\vec{v}_d$, $\vec{v}_{gc} = \vec{v}_\parallel + \vec{v}_d$. If we neglect the pitch angle scattering term in Eq. (11), we obtain the slowing-down distribution in the form

$$f_s = \frac{\{S\}}{\bar{\nu}_s (v^3 + \bar{v}_c^3)}.$$

Let us estimate the error associated with the above averaged form for the collision operator. In the bounce averaged equation (11), the source of error is in the following three approximations:

$$\{\nu_s\} \simeq \bar{\nu}_s, \quad \{v_c^3 \nu_s\} \simeq \bar{v}_c^3 \bar{\nu}_s, \quad \{\nu_s v_b^3 h\xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\} \simeq \bar{\nu}_s \bar{v}_b^3 \{h\xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\}.$$

Taylor expansion of the plasma quantities (pressure, for example) around the average radial position $\bar{r} = \{r\}$ yields

$$p = \bar{p} + p'(r - \bar{r}) + \frac{1}{2} p''(r - \bar{r})^2 + \cdots,$$

where $p' = dp/dr$ and $\bar{p} = p(\bar{r})$. Notice here that the second term is odd in $r - \bar{r}$ and of order $p\delta r/L$, where δr is the radial orbit width of the confined alphas and L is the radial scale length for the physical quantity p . The magnitude of the third term is of order $p(\delta r)^2/L^2$. Then, using $\{r - \bar{r}\} = 0$, it can be easily shown that

$$\{\nu_s\} = \bar{\nu}_s + \mathcal{O}[(\delta r)^2/L^2],$$

$$\{v_c^3 \nu_s\} = \bar{v}_c^3 \bar{\nu}_s + \mathcal{O}[(\delta r)^2/L^2].$$

Thus, the error associated with the first two approximations $\{\nu_s\} \simeq \bar{\nu}_s$, $\{v_c^3 \nu_s\} \simeq \bar{v}_c^3 \bar{\nu}_s$ is only of second order in $\delta r/L$. A larger error comes from the third approximation

$$\{\nu_s v_b^3 h \xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\} \simeq \bar{\nu}_s \bar{v}_b^3 \{h \xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\},$$

where the variation of ξ along the barely passing orbit is of zeroth order. In this case, the error is of first order in $\delta r/L$:

$$\{\nu_s v_b^3 h \xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\} = \bar{\nu}_s \bar{v}_b^3 \{h \xi \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0\} + \mathcal{O}(\delta r/L).$$

For the confined passing alphas in TFTR, the relationship $\delta r \ll L$ is true and the expected error from the averaged collision operator is small. However, for the confined trapped alphas in TFTR, the radial excursion width δr can be as large as the radial scale length L , enhancing the error margin. As was explained in Ref. 12 and will be shown later in the present work, contribution to the collisional non-prompt loss from the trapped particles is small compared to that from the passing particles; hence, our main concern is on the passing alphas only. Thus, the overall error introduced from the averaged collision operator is of order $\delta r/L$ in the present analysis. We note here that the use of an averaged collision operator can be even better justified in a tokamak fusion reactor where $\delta r \ll L$ is true even for the banana alpha particles.

The function η can be calculated numerically using a specific flux surface geometry. However, it can easily be estimated analytically from the following argument. We note here that in the case of thermal particles with $v_d/r \ll v_{||}/qR$, the value of v_{gc}

becomes vanishingly small at the banana turning points. For most of the trapped particles, this does not cause any trouble to the integral $\int d\ell/|v_{gc}|$ since the contribution from near the turning point becomes of the type $\int dx/x^{1/2}$, for $x \rightarrow 0$. Then, the value of the integral for most of the trapped and passing particles can be roughly approximated by $\int d\ell/|v_{gc}| \simeq L / \langle |v_{\parallel}| \rangle_0$, where L is the total integral path length. However, for the particles at the boundary between the trapped and passing regimes (transitional particles), the singularity becomes of the type $\int dx/x$ at $\theta \rightarrow \pi$ and the value of the integral $\int d\ell/|v_{gc}|$ becomes very large (singular behavior). For the energetic particles considered in the present work, whose banana width can be of the same order as the plasma minor radius, the drift speed becomes large ($v_d/r \sim v_{\parallel}/qR$) and the guiding center speed v_{gc} of the transitional particles does not vanish near the critical turning point ($v_{\parallel} = 0$, $\theta = \pi$), removing the singularity. Thus, the approximation $\int d\ell/|v_{gc}| \simeq L / \langle |v_{\parallel}| \rangle_0$ is valid for all the energetic particles of concern in the present work. Therefore, the function η can be estimated as

$$\eta \simeq \frac{\langle |\xi| \rangle_0}{L} \int d\ell \frac{B_c}{B} \simeq \langle |\xi| \rangle_0. \quad (12)$$

For an approximate evaluation of Eq. (11), this expression for η may be sufficient.

The operator $\mathcal{S}$ is non-zero at the birth speed only and it establishes a boundary condition to the differential equation (11). The other boundary condition to consider is that the particle distribution function vanishes at the loss boundary in the velocity space. Due to the existence of the loss boundary in velocity space, the particle distribution function will have a monotonically decreasing slope versus pitch angle toward the loss boundary, where it vanishes. Within this region (pitch angle scattering layer), the pitch angle scattering effect becomes significant. The width of pitch angle scattering layer is determined by two physical effects: one is the deviation d of the loss boundary away from the slowing-down path (see Fig. 2), which is given by $d = \lambda_{t0} - \lambda_t$ or $\lambda_{i0} - \lambda_i$; and the other is the boundary layer produced by the pitch angle scattering itself, whose width in λ -space is about Δ_{ξ} . In the present work, we define a parameter Δ to provide

a rough estimate for the width of the combined pitch angle boundary layer

$$\Delta = (\Delta_\xi^2 + d^2)^{1/2}.$$

We note here again that, since Δ is much smaller than unity for practical cases, the pitch angle scattering operator plays a significant role only within the thin boundary layer near the prompt-loss boundary. In the well confined velocity space away from the pitch-angle layer, the balance between the source operator $\mathcal{S}$ and the Coulomb slowing-down operator dominates the physical process. Thus, the particle distribution function will approach the slowing-down distribution function $f_s = \{S\}/\bar{\nu}_s(v^3 + \bar{v}_c^3)$ outside of the pitch-angle boundary layer.

Within the boundary layer, the energetic ions will experience both slowing-down and pitch angle scattering. Since the significance of the pitch angle scattering effect is inversely proportional to square of pitch angle scattering amount, the particles very near the prompt-loss boundary, ($|\lambda - \lambda_c| \ll \Delta$) experience pitch angle scattering dominantly over the energy scattering. In this case f_0 is approximated by the pitch-angle-dominant solution

$$f_0 = K g(\lambda),$$

$$g(\lambda) = \int_{\lambda_c}^{\lambda} \frac{d\lambda}{\lambda < |\xi| >_o},$$

where $K(v)$ is function of v only. The function $g(\lambda)$ is a monotonically increasing function of $\lambda - \lambda_c$.

Since we know the solution at small $|\lambda - \lambda_c|$ ($\ll \Delta$) and large $|\lambda - \lambda_c|$ ($\gg \Delta$) values, a model boundary-layer solution for the particle distribution function can then be constructed in the following form

$$f_0(v, \lambda) = f_s \frac{g(\lambda)}{[A^2 g^2(\lambda_\Delta) + g^2(\lambda)]^{1/2}}, \quad (13)$$

where $g(\lambda_\Delta)$ is the value of g at a distance Δ from λ_c in the direction away from the prompt loss region and its relation to K is given by $A g(\lambda_\Delta) = 1/K(v)$. The constant A

is introduced to compensate for the arbitrariness in defining $g(\lambda_\Delta)$. We can easily see that this model solution satisfies both of the required conditions

$$\begin{aligned} f_0 &\rightarrow f_s \text{ for } |\lambda - \lambda_c| \gg \Delta, \\ &\rightarrow g(\lambda) \text{ for } |\lambda - \lambda_c| \ll \Delta. \end{aligned}$$

We determine the constant A in such a way that the above model solution becomes an actual solution to the differential equation (11) at $\lambda = \lambda_\Delta$. In order to achieve this, we first evaluate $g(\lambda_\Delta)$ as follows:

$$g(\lambda_\Delta) = \int_0^\Delta \frac{d\zeta}{(\lambda_c + \zeta) < |\xi| >_o},$$

where a change of variable $\lambda = \lambda_c + \zeta$ has been used. In evaluating $< |\xi| >_o$ for an energetic passing ion near the trapped-passing boundary, we need to follow the actual guiding-center orbit which passes through different flux surfaces. We approximate the poloidal orbital path of a marginally passing ion by a circle whose minor radius is given by an averaged horizontal minor radius $x_* = (x_p - x_t)/2$ and the major radius corresponding to the center of the orbit $R_* = R_c - (x_p + x_t)/2$. We then have

$$\begin{aligned} < |\xi| >_o = \frac{1}{2\pi} \int d\theta (R_*/R_c + x_* \cos \theta / R_c) [1 - \lambda / (R_*/R_c + x_* \cos \theta / R_c)]^{1/2} \\ \simeq (2\epsilon_* + \frac{\zeta}{\lambda_c})^{1/2} \frac{2}{\pi} E(k^2), \end{aligned}$$

where $R_*/R_c \simeq 1$ has been used, $\epsilon_* \equiv x_*/R_c \simeq x_*/R_*$, and

$$k^2 \equiv 2\epsilon_* \frac{(1 - \zeta/\lambda_c)}{2\epsilon_* - \zeta/\lambda_c}.$$

Since $\zeta \ll \lambda_c$ within the boundary layer, we can use the approximations $k^2 \rightarrow 1$ for $\zeta \ll \lambda_c$, which yields

$$(\lambda_c + \zeta) < |\xi| >_o \rightarrow \lambda_c (2\epsilon_* + \frac{\zeta}{\lambda_c})^{1/2} \frac{2}{\pi} E(1).$$

Using $E(1) = 1$, we then obtain

$$g(\lambda_\Delta) = \frac{\pi}{2} \int_0^\Delta \frac{d\zeta}{\lambda_c (2\epsilon_* + \zeta/\lambda_c)^{1/2}} = \pi [(2\epsilon_* + \Delta/\lambda_c)^{1/2} - (2\epsilon_*)^{1/2}].$$

Notice here that we did not neglect Δ compared to another possibly small parameter ϵ_* in the denominator of the integrand. Also notice that $g(\lambda_\Delta)$ is a function of v through Δ .

Knowing $g(\lambda_\Delta)$, the constant A of Eq. (13) can now be determined. We substitute Eq. (13) into the differential equation (11) to obtain an equation for A , in a straightforward manner,

$$A^2(A^2 + 1) = \gamma(2A^2 - 1),$$

where $v_c^3/v^3 \ll 1$ has been used and $\gamma \equiv 8\lambda_c(2\epsilon_*)^{1/2} \langle \xi \rangle / 3\pi$. A physically meaningful solution ($A^2 > 0$) to this equation is trivial to obtain:

$$A^2 = (\gamma^2 + 2\gamma + 1/4)^{1/2} - \gamma - 1/2.$$

Hence our solution becomes

$$f_0(v, \lambda) = f_s \frac{g(\lambda)}{\pi A [(2\epsilon_* + \Delta/\lambda_c)^{1/2} - (2\epsilon_*)^{1/2}]^2 + g^2(\lambda)^{1/2}}.$$

The particle flux across the loss boundary by pitch angle scattering process is then obtained as follows:

$$\begin{aligned} \Gamma_\nu &= \langle \int d^3v C_\lambda(f_0) \rangle_o \\ &= \langle \frac{\pi B}{B_c} \int \frac{d\lambda dv v^2}{|\xi|} \bar{v}_s \Delta_\xi^2 \frac{2\xi B_c}{B} \frac{\partial}{\partial \lambda} \lambda \xi \frac{\partial}{\partial \lambda} f_0 \rangle_o. \end{aligned}$$

where the integration is bounded by the birth ($v = v_0$) and loss ($\lambda = \lambda_c$) boundaries in the velocity space. Using

$$\left[\lambda < |\xi| \rangle_o \frac{\partial f_0}{\partial \lambda} \right]_{\lambda=\lambda_c} = \frac{1}{\pi A} \frac{f_s}{(2\epsilon_* + \Delta/\lambda_c)^{1/2} - (2\epsilon_*)^{1/2}},$$

it is straightforward to obtain the following local ($x = x_p$) loss rate by collisional scattering into the prompt loss region:

$$\Gamma_\nu = \frac{1}{2\pi A} \sum_{\lambda_c=\lambda_t, \lambda_i} \{S\} \bar{v}_b^3 \int_{v_L}^{v_0} \frac{dv}{v} \frac{1}{v^3 + \bar{v}_c^3} \frac{1}{(2\epsilon_* + \Delta/\lambda_c)^{1/2} - (2\epsilon_*)^{1/2}}. \quad (14)$$

It should be noted here that the summation in Eq. (14) is over the two sides of the loss boundary. However, the contribution from the trapped-passing transition side ($\lambda_c = \lambda_t$) is dominant (as correctly noted in Ref. 12), and only this contribution will be kept from now on in the present work. This is because the orbit averaged values of $\{S\}$ and $\{v_b^3\}$ are much smaller for the trapped particles at $\lambda_c = \lambda_t$ than for the passing particles at $\lambda_c \gtrsim \lambda_t$, since the large banana motions take the orbit to larger minor radial locations where both the source rate S and critical speed v_b^3 is much smaller, and also the value of d , hence Δ , is larger at the $\lambda_c = \lambda_t$ side. The local collisional loss rate at $x = x_p$ compared to the orbit averaged source rate $\{S\}$, along the marginally passing trajectory, is then written as

$$F_\nu = \Gamma_\nu / \{S\}. \quad (15)$$

In the literature, we can find two contradicting reports on the Z_{eff} -dependence of the collisional non-prompt loss fraction. Reference 14, using a numerical Monte-Carlo calculation, found that the collisional non-prompt loss fraction is proportional to Z_{eff} ; while Ref. 12, using an analytic thin banana calculation, showed a less sensitive Z_{eff} dependence (non-prompt loss fraction proportional to $\sqrt{Z_{eff}}$). As a result, they had different conclusions on the importance of the collisional non-prompt loss at higher Z_{eff} values.

From the present analytic expression, Eq. (14), we can easily understand the Z_{eff} dependence of F_ν . Remember here that the boundary layer, of the width $\Delta = (\Delta_\xi^2 + d^2)^{1/2}$, is formed by two different mechanisms: One is the pitch angle scattering effect itself (Δ_ξ) and the other is the finite banana width effect (d) which makes the prompt loss boundary move away from the slowing down path. The Z_{eff} -dependence comes into F_ν through v_b^3 and Δ_ξ . From Eq. (10), keeping in mind that the Coulomb logarithm $\log \Lambda$ is almost constant, we can easily find the relationship $v_b^3 \propto Z_{eff}$. Hence, the parameter $\Delta_\xi \equiv \sqrt{v_b^3/v^3}$ is proportional to $\sqrt{Z_{eff}}$, while d is independent of Z_{eff} . Thus, if we take the limit of small boundary layer $\Delta \ll \epsilon_*$ and Taylor-expand the denominator

in the integrand of Eq. (14), we can easily find the relationship

$$F_\nu \propto v_\nu^3/\Delta = v_b^3/(\Delta_\xi^2 + d^2)^{1/2}. \quad (16)$$

From Eq. (16), we can see that in the limit of small d (which is equivalent to the thin banana approximation of Ref. 12), we get $F_\nu \propto v_b^3/\Delta_\xi \propto \sqrt{v_b^3} \propto \sqrt{Z_{eff}}$. On the other hand, we have a finite d value ($d \approx \Delta_\xi$) in reality, and the Z_{eff} dependence in the denominator of Eq. (16) can be somewhat suppressed, depending upon the relative magnitude of d . This leads to $F_\nu \propto Z_{eff}^k$, with $1/2 < k < 1$. The actual value of k is sensitive to the degree of large-banana-width effect, which is reflected in the magnitude of d . As will be shown in the following section, for TFTR plasma parameters chosen in the present work, we find $k \simeq 0.58$, which is close to the findings of Ref. 12. However, for a flatter current profile, k can be close to unity, as reported in Ref. 14.

V. Application to a model flux surface equilibrium in TFTR

As an application of the analytic formulae obtained in the previous section for the collisional non-prompt loss rate, we consider a plasma condition close to DT burning TFTR experiments. The parameters used in the present section are based upon a typical supershot plasma specified in Ref. 19. Table I shows representative operation parameters for this case. In order to describe the poloidal flux, we use a simple model q -profile which roughly resembles that of Ref. 19. We note here that, instead of a q -profile, we could use an actual expression for ψ as function of major radius, which is usually known from a numerical equilibrium analysis.

Within the plasma, we choose a simple parabolic form for the radial dependence of the cylindrical safety factor $q^{cyt}(x)$ expressed in terms of the horizontal distance $x = R - R_c$ along the outside midplane:

$$q^{cyt}(x) = q_c^{cyt} + (q_a^{cyt} - q_c^{cyt})(x/a_o)^2, \text{ for } 0 < x < a_o,$$

where q_a^{cyt} is the value of q^{cyt} at $x = a_o$ corresponding to the plasma surface at $\theta = 0$. In order to model the shift of the flux surfaces, we take a simple stretched-out form of

q^{cyl} along the inside midplane $x = R - R_c < 0$:

$$q^{cyl}(x) = q_c^{cyl} + (q_a^{cyl} - q_c^{cyl})(x/k(x)a_o)^2, \text{ for } -a_i < x < 0,$$

$$k(x) = 1 + (a_i/a_o - 1)(x/a_o),$$

where $x = -a_i$ represents the location of plasma surface at $\theta = -\pi$. For q -profile outside of the plasma, we use a simple r^2 dependence:

$$q(r) = q_a(r/a)^2, \text{ for } r > a,$$

where r is the distance from the geometric center of the plasma (not from the magnetic axis) and the plasma surface has been approximated as being circular with the representation $r = a$.

For a simple treatment of the Shafranov shift effect,²² we identify the horizontal half width of a shifted flux surface and call it x_s . Then we can easily find the relation $x_s = x[1 + k(x)]/2$. The geometric center (denoted by $R = R_s$) of the magnetic surface is shifted from the magnetic axis by the amount $x_s - x = x[k(x) - 1]/2$, hence we have $R_s = R_c - (x_s - x)$. We define a new poloidal angle θ_s with respect to R_s and use B_{ts} and B_{ps} to represent the values of the toroidal and poloidal magnetic field strengths B_t and B_p , respectively at $\theta_s = 0$. The change of ψ between two minor radial positions x_1 and x_2 along the outside midplane can, then, be written as

$$\Delta_\xi \psi(x_1, x_2) = \int_{x_1}^{x_2} dx \frac{\partial \psi}{\partial x} = \int_{x_1}^{x_2} dx (RB_p)_{\theta_s=0} = \int_{x_1}^{x_2} dx \frac{x_s(x)B_{ts}(x_s)}{q^{cyl}(x)} (1 - dR_s/dx_s),$$

where the relations $q^{cyl}(x) = x_s B_{ts}/R_s B_{ps}$ and $R(\theta=0)B_p(\theta=0)/R_s B_{ps} = 1 - dR_s/dx_s$ have been used. It is trivial to obtain $dR_s/dx_s = -1 + dx/dx_s$ with $dx_s/dx = k(x)$. It can be pointed out that the parameters x_s and R_s correspond to the minor radius and major radius of the Shafranov geometry²² with the shift parameter Λ given by $\epsilon_s(1 + \Lambda) = -dR_s/dx_s$. We note here that if we want to use $q^\psi = (\partial\phi/\partial x)/(2\pi\partial\psi/\partial x)$ instead of q^{cyl} , then $\Delta_\xi \psi$ becomes

$$\Delta_\xi \psi(x_1, x_2) = \int_{x_1}^{x_2} dx \frac{\partial \psi / \partial x}{\partial \phi / \partial x} \frac{\partial \phi}{\partial x} = \int_{x_1}^{x_2} dx \frac{x_s(x)B_{ts}(x_s)}{q^\psi(x)} \frac{dx_s}{dx},$$

Using Eq. (6) to calculate λ_{i0} at $v = v_0$, Eq. (7) for the trapped-side boundary of the prompt loss region, Eqs. (3) and (4) for v_L , and Eq. (3) for the passing side boundary, we can obtain the prompt loss boundaries in $v - \lambda$ space in a TFTR plasma. Figure 1(a) is the prompt loss boundary for 1 MeV triton loss (or, equivalently, 3 MeV proton loss) from D-D plasma, and Fig. 1(b) is for 3.5 MeV alpha loss from a DT plasma, observed at the point ($x = 21$ cm, $\theta = 0$). Since the gyroradius and banana width are proportional⁹ to $\sqrt{Em}/q$, where E is kinetic energy, q is the charge, and m is mass, the 3 MeV protons and 1 MeV tritons have identical orbits. As can be seen from Table I, the plasma minor and major radius are taken to be 80 cm and 245 cm, respectively, at the geometrical center of the whole plasma while the minor and major radius of the vacuum wall is $a_w = 99$ cm and $R_w = 261$ cm, respectively, measured at the geometrical center of the circular poloidal wall cross section. The major radius of the magnetic axis is assumed to be $R_c = 261$ cm, shifted 16 cm outward from the plasma center. Hence, we have $a_o = 64$ cm and $a_i = 96$ cm.

The pitch angle scattering loss rate into the prompt loss region is evaluated according to Eq. (14), and it is function of the background plasma density and temperatures upon which the energetic ions scatter. A proper orbital average through radial profile of plasma parameters needs to be performed in order to include the background density and temperature effect exactly. However, since a passing ion has a relatively small amount of radial excursion, we take a simplified approach and use the plasma density and temperature corresponding to the flux surface at the half radial width along the marginally passing ion orbits. The radial plasma profiles can be specified along the outside midplane as a function of the distance from the magnetic axis. A simple parabolic form is used for the electron temperature profile of a typical TFTR plasma¹⁹:

$$T_e(x) = 9.0 [1 - (x/a_o)^2]^2 \text{ keV}.$$

For the electron density profile, combination of two different parabolic functions is used

to model the highly peaked central density of a typical TFTR supershot¹⁹:

$$n_e = \{3.3 [1 - (x/a_o)^2]^6 + 3.3 [1 - (x/a_o)^2]^{0.3}\} \times 10^{13} \text{cm}^{-3},$$

and the density profiles for the main ions are assumed to follow the shape of $n_e(x)$.

For 1 MeV tritons and 3 MeV protons from D-D plasma, the local collisional loss percentage (compared to their birth rate) corresponding to the orbits passing through $x = 21$ cm are, then, calculated to be 3.4% and 1.2%, respectively, according to Eq. (15). And for 3.5 MeV α -particles from mixture of DT plasma, the local collisional loss percentage is calculated to be 2.8%. The collision properties for each case is shown in Table II for comparison.

The Z_{eff} dependence of the local loss fraction at $x = 21$ cm is listed in Table III. The second row is from Eqs. (14) and (15), and the third row shows that $F_\nu \simeq Z_{eff}^k$ with $k \simeq 0.58$. The value of k being close to $1/2$ can be explained as follows: With the plasma parameters used in the present work, d is smaller than Δ_ξ (see Fig. 3). Thus, the Z_{eff} dependence in $(\Delta_\xi^2 + d^2)^{1/2}$ roughly follows that of $\Delta_\xi \propto Z_{eff}^{1/2}$. However, if we assume a flat current profile (keeping total plasma current the same), as was done in Ref. 14, we get smaller current density in the plasma center. This leads to larger banana turning radius, hence, greater d values for the fusion product ions. This effect can suppress the Z_{eff} -dependence in $(\Delta_\xi^2 + d^2)^{1/2}$ and leads to k -value close to unity, as was concluded in Ref. 14.

The total collisional non-prompt loss fraction of the alpha particles in TFTR from a typical simulated DT plasma condition of Ref. 19 has been calculated by a radial integration of the local loss rate along the outside midplane, divided by the radially integrated alpha source rate. The source rate for the energetic alpha particles, S_α , from DT fusion can be expressed as

$$S_\alpha = n_D n_T \langle \sigma_{DT} v_i \rangle_o,$$

where n_D and n_T are the deuterium and tritium densities, respectively, σ_{DT} is the DT fusion cross-section, and $\langle \sigma_{DT} v_i \rangle_o$ is the corresponding DT reaction rate. There

are three different types of DT reactions in TFTR; beam-beam, beam-background, and background-background. The total source profile shape is similar to that from background-background reaction. Hence, for simplicity, we model the radial profile shape of $\langle \sigma_{DT} v_i \rangle_o$ from the reactions between Maxwellian D and T ions with a common temperature T_i . From an analytic fit to the curve shown in Ref. 23, we approximate the reaction rate $\langle \sigma_{DT} v_i \rangle_o$ between Maxwellian D and T ions as

$$\langle \sigma_{DT} v_i \rangle_o \simeq T_i^2 (keV) 10^{[1.1 - \{\log_{10} T_i (keV) - 1.1\}^2]} \times 10^{-19} cm^3 s^{-1}.$$

After radial integration, considering these profile effects and using radial T_i profile similar to T_e profile, it has been found that about 1.3% of the total fusion born alpha particles will be lost to the first wall by collisional diffusion into the prompt loss region. Compared to the total prompt and non-prompt loss rate of $\sim 10\%$ obtained from TRANSP code for the same TFTR parameters, the collisional non-prompt loss itself is insignificant. We note here that the exact coincidence in the total non-prompt loss rates (1.3%) between the heuristic estimate of Section III and the more complete calculation of this section is accidental.

Approximate information on the radial source distribution of collisional non-prompt losses can be obtained by using the radial dependence of S_o given in the previous paragraph, and by defining a normalized loss function $\hat{\Gamma}_\nu(x)$ as

$$\hat{\Gamma}_\nu(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta V S_o(r_{ave}) \Gamma_\nu(x)}{\Delta x \int_V dV S_o(r_{ave})} \simeq \frac{r_{ave}(x) S_o(r_{ave}) \Gamma_\nu(x)}{\int_0^a dx r_{ave}(x) S_o(r_{ave})},$$

where $\Delta V \simeq (2\pi)^2 R_s r_{ave} \Delta x$ has been used, and r_{ave} is the radius of the half width position of the marginally passing particles as explained before. Figure 4 shows the dependence of $\hat{\Gamma}_\nu$ on the outside midplane position $x = R - R_c$. It can be seen that the collisional non-prompt loss is highly localized toward $x = 19$ cm radially. This localization is due to the combined effects of the prompt loss region disappearing rapidly at smaller x values and that the source S_o decaying rapidly as the x value is increased. Due to the finite orbital thickness of the marginally passing particles, r_{ave} is considerably greater than x . This fact makes the decay of S_o in x even more rapid.

When the marginally passing ions move into the prompt loss region by small pitch-angle scatterings, they are lost to the wall along the marginally trapped orbits. The marginally trapped orbits from different outside midplane positions (x -values) intersect the wall at different poloidal locations. This means that a certain type of distribution of the collisional non-prompt loss in x appears as a corresponding type of particle flux distribution in poloidal wall strike position. The relationship between an x -value and the poloidal wall strike position can be found by requiring that the canonical angular momentum between those two points are equal. Using Eq. (1), we find

$$\begin{aligned}\psi_w - \psi_x &= (Iv/\Omega)_w - (Iv/\Omega)_x = (Iv/\Omega_c)(|\xi_w|R_w/R_c + |\xi_x|R_x/R_c) \\ &= \frac{Iv}{\Omega_c}[(1 - \lambda_t R_c/R_w)^{1/2}(R_w/R_c) + (1 - \lambda_t R_c/R_x)^{1/2}(R_x/R_c)],\end{aligned}\quad (17)$$

where R_w is the major radius at the wall strike point and $R_x = R_c + x$. The result for the nonprompt loss distribution, $\hat{\Gamma}_\nu(\theta) = (\partial x(\theta)/\partial \theta)\hat{\Gamma}_\nu[x(\theta)]$, at the TFTR wall is shown in Fig. 5. For the direction of the vertical drift chosen here, all the alpha losses will occur in the lower half plane of the torus. Thus, the loss domain is between $-180^\circ < \theta < 0^\circ$ at the wall, as shown in Fig. 5. Comparison of the theoretical prediction for collisional non-prompt loss rate with an experimentally observed rates at a certain wall strike positions can be quite useful in understanding any possible anomaly of alpha particle behavior. Such studies will be conducted in near future.

VI. Numerical Results from TRANSP code

TRANSP is an analysis code for tokamak plasmas, which is used mainly for analyzing tokamak transport phenomena based upon experimentally measured parameters.¹⁹ It, however, can also be used in a modeling mode to check the consistency of measured parameters and to calculate parameters which are difficult to measure directly. The TRANSP code contains a Monte-Carlo modeling routine to calculate the Coulomb collisional behavior of high energy ions.^{11,24} It can predict the wall strike locations and energy of lost alpha particles. However, identification of the loss mechanism for each

lost particle is not possible in the present version of TRANSP. Also, alpha loss induced by toroidal field ripple is not yet present in TRANSP.

A result from the TRANSP code for the poloidal angular distribution at the wall of nonpromptly lost alphas is shown in Fig. 6, from a plasma similar to that used in the previous section (see Table I, which is based upon the shot # 66887). The comparisons of the TRANSP poloidal distribution (Fig. 6) with those from the analytic model in this paper (Fig. 5) show reasonable agreement. The total delayed loss from TRANSP, as a fraction of the total number of alphas, is estimated to be about 0.3%; which is somewhat smaller than 1.3% obtained from the analytic model. Considering that the ordinary neoclassical loss is small,⁷ we may assume that a major contribution to this loss is from the Coulomb collisional scattering into the prompt loss region.

The accuracy of the TRANSP calculation is affected by two artificial methods in the program. One is the speed-up (“goosing”) of the collision rate for computational efficiency, and the other is the limited, linear extrapolation of the magnetic fields outside the plasma from the magnetic fields inside. The “goosing” enhances the time step differently for different particles, and the alpha particles in the passing-trapped boundary layer can be somewhat sensitive to “goosing” as they scattering around to different pitch angles and different spatial orbits. As a test to estimate the effect of “goosing” as a possible source of inaccuracy, the “goosing” was reduced by a factor of 2, and it was found that neither the poloidal distribution nor the delayed lost amount was affected much. Hence, it seems that the “goosing” does not yield a significant error, even though it is not conclusive because of the possibility of more accumulated Monte-Carlo error in the reduced goosing case. We note here that the reduction of “goosing” by half increases the computational run time by an order of magnitude.

Perhaps, a more significant problem may exist in the limited, linear extrapolation of the magnetic fields. The extrapolation of the magnetic fields is performed only up to $x = 1.3$ linearly in x , where $x = \sqrt{\phi/\phi_a}$ is a radial coordinate defined in terms of the toroidal flux ϕ normalized to its value ϕ_a at the plasma boundary (determined by the

“bumper limiter” on the inner side), while the first wall extends further on the outer side of the plasma (see Fig. 7). This provides an artificial limiter on the outer side for TRANSP calculation, and the extension of the artificial limiter to the real limiter location is possible only through another numerical extrapolation in the present version of TRANSP. In order to get the extrapolation in the limiter positions, the artificial location of the limiter on the outer side of the plasma, and thus the extent of the magnetic field extrapolation, was varied between $x=1.1$, 1.2 , and 1.3 . It was found that the collisional loss fraction was about 7% for $x=1.1$, 4% for $x=1.2$, and 3% for $x=1.3$. However, the overall shape of the poloidal distribution curve of the collisionally lost alphas at the wall remained unchanged during the x -variation. The total collisional loss fraction of about 0.3% (0.25, to be more precise) mentioned above is the result of a simple extrapolation from the available x -values (1.1, 1.2, and 1.3) to the actual bumper limiter location ($x \simeq 2$).

Comparison between the TRANSP result and the analytic result on the collisional delayed loss of alpha particles is not yet conclusive at the present stage of the TRANSP code development. However, the above observations seem to indicate that the collisional alpha loss fraction calculated from TRANSP is less than what was obtained from the analytic model, even though the poloidal distribution of the collisionally lost alphas at the wall agrees reasonably well between the TRANSP and analytic calculations.

VII. Discussion

An approximate analytic model to calculate the collisional non-prompt loss of high energy ions into the prompt loss region is presented. A simple set of algebraic equations are used to describe the prompt first-orbit loss boundaries appropriate to TFTR first wall. Description of more general loss boundaries appropriate to non-circular first wall shapes can be found in Refs. 2–4. The collisional non-prompt loss rate is obtained in the form of a one dimensional velocity integral. In addition to the plasma profiles and wall location, knowledge of q -profile or the poloidal flux ψ as function of the major radius

along the midplane is needed in order to perform the integral.

The theory is applied to calculate the collisional non-prompt loss of fusion products using a model TFTR equilibrium profiles resembling the simulated DT shot case of Ref. 19. In this case, it is found that about 1.3% of the total fusion alpha particles are lost to the wall by pitch angle scattering into the prompt loss orbits. Thus the Coulomb collisional loss process of the alpha particles directly to the wall structure should not cause a noticeable reduction in the alpha population in a DT plasma in TFTR. However, the loss is found to originate near $r \sim 0.3$ of the plasma minor radius in the case considered, resulting in the peaking of the strike points toward the outside bottom ($\sim -70^\circ$) of the wall. Since the walls of future DT tokamaks will be sensitive to the localized alpha strike power, this phenomenon needs to be examined more carefully in the future.

From Eq. (14), it can be understood that the Z_{eff} -dependence in the collisional non-prompt loss rate is sensitive to the degree of large-banana effect. It is shown that in the limit of thin banana width we have $F_\nu \propto \sqrt{Z_{eff}}$, as was found in Ref. 12; while in the limit of the thick banana width we get $F_\nu \propto Z_{eff}$, as was found in the Monte-Carlo simulation of Ref. 14. From the TFTR plasma parameters used here in the present work, we find $F_\nu \propto Z_{eff}^{0.58}$, a result close to the thin banana approximation of Ref. 12.

A possibly important, future application of the present work is in the interpretation of lost-alpha data into the detectors installed in TFTR, as described in Ref. 18. It was reported in Ref. 18 that in the case of D-D reactions, there was an unexpected type of delayed loss of fusion products into the bottom (-90°) detector. The present theory can be used to estimate how much of this loss should be due to classical Coulomb scattering processes into the prompt loss orbits, and so how much of the observed loss was due to anomalous behavior. Such study is in progress and it will be reported in near future.

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Table I. TFTR parameters used in the present work, which are approximately deduced from Ref. 19.

Symbol	Physical quantity	Value
R_0	Major radius at plasma geometric center	245 cm
R_w	Major radius at wall geometric center	261 cm
R_c	Major radius at magnetic axis	261 cm
a_p	Plasma minor radius	80 cm
a_w	Wall minor radius	99 cm
B_0	Magnetic field strength at $R = R_0$	49 kG
T_{e0}	Electron temperature at magnetic axis	9 keV
T_{i0}	Ion temperature at magnetic axis	17 keV
n_{e0}	Total electron density at magnetic axis	$6.6 \times 10^{14} \text{ cm}^{-3}$
q_0^{cyl}	Cylindrical safety factor at magnetic axis	0.9
q_a^{cyl}	Cylindrical safety factor at plasma edge	3.9
Z_{eff}	Effective ion charge number	3
I_p	Plasma current	1.65 MA

Table II. Properties of barely passing energetic ions whose orbits pass through $x=21$ cm at outside midplane in a TFTR plasma characterized by Table I. All the physical quantities in this table are evaluated using the background densities and temperatures at the orbit averaged radial position $\{r\}$. v_L is the lowest speed for the prompt loss, v_b is the critical pitch angle scattering speed, and $\nu_s = 1/\tau_s$ and ν_{90}^0 are the slowing down and 90° scattering rates at $v = v_0$. The value of $\{r\}$ given here is the minor radius of the corresponding flux surface at the outside midplane.

Species	P (3 MeV)	T (1 MeV)	α (3.5 MeV)
$v_0(cm/s)$	2.4×10^9	8.0×10^8	1.3×10^9
$v_L(cm/s)$	2.2×10^9	7.2×10^8	1.1×10^9
$v_b(cm/s)$	5.8×10^8	4.1×10^8	3.6×10^8
$\{r\}(cm)$	33.6	33.6	34.4
$\{T_e\}(keV)$	4.7	4.7	4.5
$\{n_e\}(cm^{-3})$	3.5×10^{13}	3.5×10^{13}	3.4×10^{13}
ν_{90}^0/ν_s	1.4×10^{-2}	1.3×10^{-1}	2.1×10^{-2}
$\nu_s(sec^{-1})$	3.4	1.1	3.4
$\nu_{90}^0(sec^{-1})$	4.9×10^{-2}	1.4×10^{-1}	7.3×10^{-2}

Table III. Dependence on Z_{eff} of the local collisional non-prompt loss fraction F_ν with respect to local source rate at $x = 21$ cm.

Z_{eff}	1	2	3	4	5	6
F_ν (%)	1.5	2.2	2.8	3.2	3.6	3.9
$1.5Z_{eff}^{0.58}$ (%)	1.5	2.2	2.8	3.4	3.8	4.2

Figure Caption

Fig. 1 Particle orbits for a) barely passing, b) barely trapped, and c) deeply trapped particles. The orbit c) intersects the wall at $\theta = 0$.

Fig. 2 Prompt loss boundaries at the observation point on the outside midplane 21 cm away from the magnetic axis for (a) 1 MeV triton (or 3 MeV proton) loss from D-D plasma and (b) 3.5 MeV alpha loss from 1:1 DT plasma. The plasma equilibrium parameters are as described in the text.

Fig. 3 Constituents of the boundary layer near the prompt loss region in alpha particle velocity space; $d_t = \lambda_{tc}(v) - \lambda_{tc}(v_0)$ is for the trapped-passing transition side, $d_i = \lambda_{tc}(v_0) - \lambda_{tc}(v)$ is for the deeply inside of trapped region, and $\Delta\xi = (v_b/v)^{3/2}$.

Fig. 4 Radial distribution of collisional alpha loss into the prompt loss orbits in TFTR. Normalization is chosen in such a way that the radial integration of the local loss rates will be the total collisional loss percentage.

Fig. 5 Wall distribution of collisional alpha loss into the prompt loss orbits in TFTR. Normalization is chosen in such a way that the angular integration from 0° to 180° of the local loss rates will be the total collisional loss percentage.

Fig. 6 TRANSP result for the wall distribution of collisional loss into the prompt loss orbits.

Fig. 7 Artificial numerical wall boundary ($x=1.3$) used in TRANSP for Fig. 6. Here, $x = \sqrt{\phi/\phi_a}$ is a radial coordinate defined in terms of the toroidal flux ϕ normalized to its value ϕ_a at the plasma boundary.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

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