1 of 1

**PROBABILITY COMPUTATIONS USING THE
SIGMA-PI METHOD ON A PERSONAL COMPUTER**

by
F. Eric Haskin, Maximo S. Lazo, and A. Sharif Heger
Department of Chemical and Nuclear Engineering
University of New Mexico

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1 INTRODUCTION 1

2 INSTALLATION AND TESTING OF VAX VERSION OF SIGPI 3

3 DEVELOPMENT AND TESTING OF PC VERSION OF SIGPI 6

3.1 DEVELOPMENT OF VERSION UNM-B 6

        FORTRAN Compilers 6

        Boolean Operations 7

        Transfers into IF Blocks 7

        Opening/Closing Files (Character Increment) 8

        Machine Specific Constants 8

3.2 TESTING OF VERSION UNM-B 9

3.3 FUTURE TASKS 10

        Multiple Probability Sets 10

        Problems with Dependencies 10

        Common Blocks Mismatched 10

        Optimal Linkage 11

4 SIGPI LIMITATIONS AND POTENTIAL ENHANCEMENTS 12

4.1 RESTRICTION TO GAUSSIAN DISTRIBUTIONS FOR DEPENDENT EVENTS 12

4.2 EASE OF INPUT PREPARATION 13

4.3 ALTERNATIVE LOWER BOUND 13

4.4 SENSITIVITY/UNCERTAINTY ANALYSIS CAPABILITY 16

5 CONCLUSIONS AND RECOMMENDATIONS 17

REFERENCES 19

APPENDICES 20

A. CONVERT PROGRAM 20

B. DESCRIPTION OF TEST RUNS 5, 6, AND 7 21

C. HAND CALCULATION FOR TEST RUN 6 23

D. PROOF OF ALTERNATIVE LOWER BOUND 29

E. SIGPI CALL TREE 32

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2

ABSTRACT

The SIGMA-PI ($\Sigma\Pi$) method as implemented in the SIGPI computer code, is designed to accurately and efficiently evaluate the probability of Boolean expressions in disjunctive normal form given the base event probabilities. The method is not limited to problems in which base event probabilities are small, nor to Boolean expressions that exclude the compliments of base events, nor to problems in which base events are independent. The feasibility of implementing the $\Sigma\Pi$ method on a personal computer has been evaluated, and a version of the SIGPI code capable of quantifying simple Boolean expressions with independent base events on the personal computer has been developed. Tasks required for a fully functional personal computer version of SIGPI have been identified together with enhancements that could be implemented to improve the utility and efficiency of the code.

1 INTRODUCTION

In July, 1990, Los Alamos National Laboratory Division N-6 contracted with the University of New Mexico (UNM) Department of Chemical and Nuclear Engineering to develop a personal computer (PC) version of the SIGPI computer code. The results of the UNM effort to date are presented in this report.

The SIGMA-PI ($\Sigma\Pi$) method as implemented in the SIGPI computer code, is a method for accurately and efficiently evaluating the probability of a Boolean expressions in disjunctive normal form (top events) when the base event probabilities are specified via user input. Unlike other top event quantification codes, SIGPI is not limited to problems in which base event probabilities are small, nor to Boolean expressions that exclude the compliments of base events, nor to problems in which base events are independent (although SIGPI requires dependent events be normally distributed).

It is beyond the scope of this report to discuss the $\Sigma\Pi$ method or the SIGPI computer code in detail. The reader is presumed to be familiar with the descriptions provided in References 1 and 2.

A decision was made to conduct the UNM project by a) assuming a PC version would be developed and b) initiating the development process. The process followed is depicted in Figure 1 and discussed in detail in this report. To date the process has yielded PC Version UNM-B of SIGPI, which successfully runs the simpler SIGPI test problems. Conclusions and recommendations regarding further developmental efforts are discussed in Section 5.

FIGURE 1 - UNM Process for Developing PC Version of SIGPI

2 INSTALLATION AND TESTING OF VAX VERSION OF SIGPI

The VAX version of SIGPI and copies of References 1 and 2 were obtained from Cherie Jo Patenaude of Lawrence Livermore National Laboratory (LLNL). Ms. Patenaude is the original programmer of SIGPI.[2] SIGPI was installed on the UNM VAX system and compiled without error. However, in order to get the sample problems to run, the LLNL CFTAP program¹ had to be modified to convert ASCII cut set input files to the binary (unformatted in FORTRAN) form required by SIGPI. A listing of the modified program, named CONVERT is included as Appendix A. In addition, a timing routine was added to calculate run times, and a subroutine call tree was generated (see Appendix E).

All of the test runs from the SIGPI User's Manual [1] were run successfully on the UNM VAX. These test runs are based on four top event files: TEST1, TEST2, TEST3, and TEST4 listed in the SIGPI User's Manual. Three additional test runs were devised based on the following three additional top event expressions:

- TEST5 - Corresponds to the example shown in Figure 3.8 of Reference 1
(probability file PTEST5),
- TEST6 - Corresponds to the example shown in Figure 9 of the SIGPI User's
Manual [2] (probability file PTEST6),
- TEST7 - Corresponds to example problem in TEMAC User's Manual [3]
(probability file PTEST7).

Listings of the SIGPI input for these three additional test runs are included as Appendix B.

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¹The CFTAP program, also obtained from Ms. Patenaude converts a file in FTAP format to SIGPI cut set and probability files.

Figure 2 summarizes many features (number of cut sets, number of base and compliment events appearing in the top event expression, number of independent events, and number of dependent events) for the seven top event expressions. It also presents the test results obtained from the original VAX version (version UNM-A) of SIGPI for all of the test runs. Note that the upper bounds predicted for Cases 3B2 and 3B3, were closer to the lower bound than listed in the User's Manual. This is apparently attributable to changes made at LLNL subsequent to the User's Manual. Note also that the run times increase with the size of the problem, and with the introduction of dependencies. The run times actually decrease slightly as compliment events are introduced, apparently because this speeds the SIGPI decomposition process. For Test 6, which corresponds to Figure 9 of the User's Manual, no reference quantification was available; therefore, a hand calculation was performed and is included as Appendix C. The SIGPI version UNM-A result listed in Figure 2 (approximately 0.597) was found to be slightly below the result calculated in Appendix C (0.613). The reason for the discrepancy has not yet been determined. Test 7 corresponds to the sample problem in the TEMAC User's Manual [3]. The difference between the TEMAC result (0.234×10^{-7}) and that obtained using SIGPI (0.2194×10^{-7}) may be attributable to the fact that TEMAC assumes the validity of the small event approximation for the union of cut sets.

Test Run	Ref.	No. Cut Sets	EVENTS APPEARING AS		Test Input File	Indep. Prob. File	Dep. Prob. File	Upper Bound	Agree with Ref. ?	Lower Bound	Agree with Ref. ?	Run Time (s)
			Base Events/Compliments	Independent/Dependent								
1A	2	5	17 / 0	17 / 0	TEST1	PTEST	---	0.1523	Yes	0.1523	Yes	0.20
1B	2	5	17 / 0	14 / 3	TEST1	PTEST	DTEST	0.1546	Yes	0.1546	Yes	28.43
2A	2	5	14 / 3	17 / 0	TEST2	PTEST	---	0.5786	Yes	0.5776	Yes	0.13
2B	2	5	14 / 3	14 / 3	TEST2	PTEST	DTEST	0.5781	Yes	0.5781	Yes	0.72
3A1	2	20	38 / 0	38 / 0	TEST3	PTEST	---	0.1667	Yes	0.1654	Yes	3.98
3A2					TEST3	PTEST	---	0.1311	(1)	0.1311	Yes	
3A3					TEST3	PTEST	---	0.1667	(1)	0.1654	Yes	
3B1	2	20	38 / 0	34 / 4	TEST3	PTEST	DTEST	0.1715	Yes	0.1706	Yes	390.98
3B2					TEST3	PTEST		0.1311	Yes	0.1345	Yes	
3B3					TEST3	PTEST		0.1787	Yes	0.1773	Yes	
4A1	2	20	34 / 4	38 / 0	TEST4	PTEST		0.5117	Yes	0.5068	Yes	1.86
4A2					TEST4	PTEST		0.8476	Yes	0.8476	Yes	
4A3					TEST4	PTEST		0.5117	Yes	0.5068	Yes	
4B1	2	20	34 / 4	34 / 4	TEST4	PTEST	DTEST	0.5254	Yes	0.5224	Yes	262.36
4B2					TEST4	PTEST	DTEST	0.1085		0.1076		
5	1	5	4 / 1	5 / 0	TEST5	PTEST5		0.2760	Yes	0.2760	Yes	0.09
6	2	7	9 / 0	9 / 0	TEST6	PTEST6		0.5980	No	0.5964	No	0.15
7	3	18	16 / 0	16 / 0	TEST7	PTEST7		0.2194	No	0.2194	---	0.18

(1) Closer to lower bound than indicated in Ref. 2.

FIGURE 2 – SIGPI Test Runs with Results from Original VAX Version

3 DEVELOPMENT AND TESTING OF PC VERSION OF SIGPI

3.1 DEVELOPMENT OF VERSION UNM-B

FORTRAN Compilers

Two FORTRAN compilers that are widely used for PC applications were available for this project. Lahey FORTRAN is capable of utilizing extended memory for 386 PC's operating under DOS (Version 3.0 or above). Microsoft FORTRAN is not capable of utilizing extended or expanded memory under DOS; however, Microsoft has a version of their FORTRAN that will run with IBM's Operating System 2 (OS/2), which does not have the 640K RAM limitation associated with DOS.

The PC provided by LANL for UNM to use on this project is an IBM Personal System 2, Model 70 with 6 MB of memory and a DOS 3.3 operating system. Although OS/2 could be installed on this machine, a decision was made to retain DOS and use Lahey FORTRAN, at least for the initial UNM efforts described in this report. It was determined that the changes required to get SIGPI to compile in Lahey FORTRAN would also be compatible with Microsoft FORTRAN. Fewer changes would have been required for Microsoft FORTRAN because it accepts the Boolean operations on integer variables contained in the original VAX version of SIGPI; however, the resulting version would not be compatible with Lahey FORTRAN. The DOS/Lahey approach is consistent with the intent to develop as portable a version of SIGPI as possible. Specifically the objective is that (except for machine specific constants) the PC version of SIGPI should compile without fatal errors, link and run using VMS/VAX FORTRAN, DOS/Lahey FORTRAN, or Microsoft FORTRAN. The coding changes initially identified as being required to meet this objective fell into four areas as discussed in the following subsections. SIGPI with these changes now exists as version UNM-B.

Boolean Operations

SIGPI declares all variables INTEGER by default through an IMPLICIT INTEGER (A-Z) statement. Non-integer variables are declared through the use of REAL, LOGICAL, or CHARACTER statements.

The vast majority of SIGPI FUNCTIONS and SUBROUTINES have Boolean operations of the form

I1 = IA.AND.IB

I2 = IA.OR.IB

I3 = .NOT.IA

I4 = IA.XOR.IB

where IA, IB, I1, I2, I3, and I4 are integer variables. Although VAX and Microsoft FORTRAN compilers permit these usages, Lahey FORTRAN does not. As a result it was necessary to change the preceding forms to

I1 = IAND(IA,IB)

I2 = IOR(IA,IB)

I3 = NOT(IA)

I4 = IEOR(IA,IB).

These forms are supported by all three compilers: Lahey, Microsoft, and VAX. An estimated 350 lines of FORTRAN had to be changed in the original VAX version of SIGPI to accomplish this task.

Transfers into IF Blocks

In four locations, the original version of SIGPI had GO TO statements outside an IF, THEN, ENDIF block that transferred the program to a numbered statement between the THEN and ENDIF. Such constructions are not acceptable in Lahey FORTRAN. A typical example of a modification to this type of structure is presented below.

Original Version

```
GO TO 95
...
IF (IERR) THEN
95  ...
ENDIF
JCT=LVSUM(JSTAR,NCT)
```

Modified Version

```
GO TO 95
...
IF (.NOT.IERR) GOTO 96
95  ...
96  CONTINUE
JCT=LVSUM(JSTAR,NCT)
```

Opening/Closing Files (Character Increment)

The SIGPI subroutine ADDONE increments an ASCII file name by one ASCII character. For example, file name TEMPAAAA becomes TEMPAAAB. The scheme used in the original version of SIGPI was not portable and would not compile in Lahey FORTRAN. The subroutine was rewritten to make it portable.

Machine Specific Constants

The SIGPI subroutine DMACH1 is used to return double precision machine dependent constants. The following table summarizes the machine dependent constants used in the VAX and PC (see Ref. 6):

<u>Constant</u>	<u>Interpretation</u>

DIMACH(1)	= B**(EMIN-1), the smallest positive magnitude
DIMACH(2)	= B*EMAX*(1-B**(-T)), the largest magnitude
DIMACH(3)	= B**(-T), the smallest relative spacing
DIMACH(4)	= B**(1-T), the largest relative spacing
DIMACH(5)	= LOG10(B)

	<u>VAX Value</u>	<u>PC Value</u>
DIMACH(1)	2.9387358770557188E-39	2.23D-308
DIMACH(2)	1.7014118346046923E+38	1.79D+308
DIMACH(3)	1.3877787807814457E-17	1.11D-16
DIMACH(4)	2.7755575615628914E-17	2.22D-16
DIMACH(5)	0.3010299956639812	0.301029995663981195D0

3.2 TESTING OF VERSION UNM-B

As indicated above, the FORTRAN changes made in developing SIGPI version UNM-B are compatible with Lahey, Microsoft, and VAX FORTRAN compilers. The coding changes were verified in two ways. First, a listing of all differences between the original VAX version (UNM-A) and version UNM-B was obtained and checked. This revealed a few errors, which were easily fixed. Next, version UNM-B was compiled, linked, and run on the VAX. The upper and lower bounds obtained for the eleven test runs were the same as presented in Table 1; however, for runs involving more than one set of probabilities (test runs 3A, 3B, 4A, and 4B), version UNM-B ran only the first case. The cause of this difficulty is currently under investigation. Other than slight timing changes, which are small compared to the accuracy of the timing routine, no other differences were found between the results obtained using version UNM-A and those obtained using the version UNM-B on the VAX.

After performing a line by line check of all UNM-originated changes and verifying that the test problems ran using version UNM-B on the VAX, version UNM-B was compiled and linked and the test problems were run on the PS/2 Model 70. The PC results for test runs 1, 2, 5, 6, and 7 are identical to the VAX results (see Figure 2). These test runs are relatively simple, in particular they involve no dependent events. The PC results (upper and lower bounds) for test runs 3A1 and 4B1 (larger problems but without dependent events) are slightly less than the corresponding VAX results indicated in Figure 2. The reason for these differences is currently being investigated. Finally, the PC results for test runs 1B, 2B, 3B, and 4B, which involve dependent events, failed to execute properly on the PC. The reason for such failures has not yet been determined. The quantitative PC results obtained with version UNM-B are tabulated below.

Test Run	Upper Bound	Lower Bound	Run Time (s)
1A	0.1523	0.1523	0.05
2A	0.5786	0.5786	0.06
3A1	0.1613	0.1601	10.88
4A1	0.5010	0.4993	5.11
5	0.2760	0.2760	0.05
6	0.5980	0.5964	0.06
7	0.2194E-7	0.2194E-7	0.11

3.3 FUTURE TASKS

Multiple Probability Sets

When the test runs were made using version UNM-B (see Section 3.2) those test runs involving multiple sets of input probabilities ran only for the first set. At the time of this report, the cause for this failure to process multiple input sets had not been determined.

Problems with Dependencies

When the test runs were made on the PC using version UNM-B (see Section 3.2) those test runs involving dependent events failed to execute properly. At the time of this report, the cause for this failure has not been determined.

Common Blocks Mismatched

Compilation of SIGPI version UNM-B using Lahey FORTRAN on the PC revealed that the original SIGPI COMMON blocks are not matched in size from subroutine to subroutine. For example COMMON \BLK1\ A, B, C in one subroutine might be COMMON \BLK1\ A, B in another subroutine. However, the mismatched common blocks apparently caused no problems in running on the VAX, and they seem unlikely to be the cause of problems encountered with the PC test runs.

Optimal Linkage

The size of the executable file for the PC version of SIGPI, as generated by Lahey FORTRAN without optimization, is approximately 1 MB and the memory requirement is approximately 2.3 MB. Lahey FORTRAN has a compilation option that is designed to reduce the size of the executable file. This option could be investigated in the future (see Section 5).

4 SIGPI LIMITATIONS AND POTENTIAL ENHANCEMENTS

In the process of becoming familiar with the $\Sigma\Pi$ method and the SIGPI computer code, four areas were identified in which SIGPI has limitations that could be remedied either by modifications to the SIGPI code or by developing interfaces between SIGPI and other well established codes. These SIGPI limitations and potential enhancements are discussed in this section.

4.1 RESTRICTION TO GAUSSIAN DISTRIBUTIONS FOR DEPENDENT EVENTS

The probabilities of dependent events are currently calculated in SIGPI by integrating over user-specified Gaussian (normal) distributions. Dependencies are introduced via a user-supplied covariance matrix. Probabilities of the intersections of two or more dependent events are also calculated by integrating over the multivariate Gaussian distribution as indicated in the equations on page 10 of the User's Manual [2]. A reasonable option to this approach, at least for the limited number of dependencies currently allowed in SIGPI, would be to specify the probabilities of all potential intersections of dependent events directly via user input. This would allow alternative methods for modeling multivariate probability structures to be used external to SIGPI. For example the Latin Hypercube Sampling approach [5] can accommodate a wide variety of marginal distribution types and will permit rank correlations. To illustrate, for test runs 3B and 4B, there are four dependent events; call them A, B, C, and D. The most information SIGPI could possibly require regarding dependent probabilities would be $\phi(A \cap B)$, $\phi(A \cap C)$, $\phi(A \cap D)$, $\phi(B \cap C)$, $\phi(B \cap D)$, $\phi(C \cap D)$, $\phi(A \cap B \cap C)$, $\phi(A \cap B \cap D)$, $\phi(A \cap C \cap D)$, $\phi(B \cap C \cap D)$, and $\phi(A \cap B \cap C \cap D)$. Alternatively, one could to specify the dependent data via conditional probabilities: $\phi(A|B)$, $\phi(A|C)$, $\phi(A|D)$, etc. (see also Section 4.3).

4.2 EASE OF INPUT PREPARATION

The minimal cut sets that are defined via SIGPI input files are usually generated by codes such as SETS [4] that reduce Boolean expressions (e.g. fault trees logic) to disjunctive normal form. Disjunctive normal output files from codes such as SETS cannot currently be used as input files for SIGPI. Consequently, output from SETS or similar codes must be manually converted to the format read by SIGPI. To circumvent this limitation, a stand-alone preprocessor (e.g., SETS-to-SIGPI) could be written, and, if warranted by high usage, incorporated into SIGPI.²

4.3 ALTERNATIVE LOWER BOUND

The upper bounds on the probabilities calculated by SIGPI are based on the fact that

$$\phi \left(\bigcup_{i=1}^m S_i \right) \leq \sum_{i=1}^m \phi(S_i)$$

where $\phi(S_i)$ denotes the probability of the i -th cut set in the current disjunctive normal form. The lower bounds on the probabilities calculated by SIGPI are based on the fact that

$$\phi \left(\bigcup_{i=1}^m S_i \right) \geq \max_i \left(\phi(S_i) \right)$$

Based on the additive law for non-mutually exclusive events, an alternative lower bound exists (see Appendix D for proof):

---

²The LLNL program CFTAP is apparently such a program.

$$\rho \left(\bigcup_{i=1}^m S_i \right) \geq \sum_{i=1}^m \rho(S_i) - \binom{m}{2} \max_i \left(\rho(S_i) \right) \max_{i,j} \left(\rho(S_i | S_j) \right)$$

If the cut sets are independent, which (as explained later) is usually not the case, $\rho(S_i | S_j) = \rho(S_i)$ and this expression reduces to:

$$\rho \left(\bigcup_{i=1}^m S_i \right) \geq \sum_{i=1}^m \rho(S_i) - \binom{m}{2} \left(\max_i \rho(S_i) \right)^2.$$

When the cut set probabilities are small (that is, when the small probability approximation is applicable), the alternative lower bound is much closer to the true value than the lower bound currently used in SIGPI. This can be demonstrated by considering a simple problem in which the top event is the union of four independent cut sets each having the same probability; that is,

$$TE = S_1 \cup S_2 \cup S_3 \cup S_4$$

$$\rho(S_1) = \rho(S_2) = \rho(S_3) = \rho(S_4)$$

The following table compares the two lower bounds for this problem:

<u>Current Lower Bound</u>	<u>Alternative Lower Bound</u>	<u>Exact Probability</u>
1	-2	1
0.9	-1.26	0.9999
0.8	-0.64	0.9984
0.7	-0.14	0.9919
0.66667	0	0.9877
0.6	0.24	0.9744
0.5	0.50	0.9375
0.4	0.64	0.8704
0.3	0.66	0.7599
0.2	0.56	0.5904
0.1	0.34	0.3439
0.01	0.0394	0.039403

As indicated in the table, the alternative lower bound is much closer to the true value when the cut set probabilities are small. However, when large probabilities are

involved, the current SIGPI lower bound should clearly be used. One approach would be to combine the two as follows:

$$\text{Lower Bound} = \max \left(\begin{array}{cc} \text{Current} & \text{Alternative} \\ \text{Lower} & \text{Lower} \\ \text{Bound} & \text{Bound} \end{array} \right)$$

Note that the alternative lower bound requires the conditional probabilities $\phi(S_i|S_j)$ be known for all pairs of cut sets at the current decomposition level. Any two cut sets S_i and S_j that share one or more base events are neither independent, $\phi(S_i|S_j) \neq \phi(S_i)$, nor mutually exclusive, $\phi(S_i|S_j) \neq 0$. For example, if A, B, C, D, and E denote base events and

$$S_1 = A \cap B \cap C$$

$$S_2 = C \cap D \cap E$$

then

$$\phi(S_1|S_2) = \phi(A \cap B),$$

$$\phi(S_2|S_1) = \phi(D \cap E).$$

If the base events are independent the computation is based on the multiplicative law for independent events:

$$\phi(A \cap B \cap \dots \cap Z) = \phi(A) \phi(B) \dots \phi(Z)$$

If the two or more of the base events are dependent, integration over the joint Gaussian probability distribution is involved. Therefore, to implement the alternative lower bound in SIGPI as suggested above would require the conditional probabilities $\phi(S_i|S_j)$ be computed for each SIGPI subproblem at each decomposition level. Of course, at the end of the SIGPI decomposition process $\phi(S_i|S_j) = 0$, that is, all cut sets are disjoint and the lower and upper bound expressions presented above reduce to the additive law for mutually exclusive events.

4.4 SENSITIVITY/UNCERTAINTY ANALYSIS CAPABILITY

SIGPI has the capability of quantifying the same top event expression over and over when multiple sets of probabilities are specified via user input. However, there is no capability inherent in SIGPI for analyzing the results of such multiple runs. Specifically sensitivity and uncertainty measures such as partial derivatives, risk increase, risk decrease, and the uncertainty importance measure (see Refs. 3 and 7) are not calculated in SIGPI.

For problems involving independent events it would be possible to write a code to interface SIGPI to TEMAC, which could then be used to calculate any of the aforementioned sensitivity and uncertainty measures. This is possible because 1) at each decomposition level SIGPI stores a disjunctive normal form internally, and 2) the equation used to quantify the upper bound probability of the disjunctive normal form at each level is, in essence, the small probability approximation assumed by TEMAC.

For problems involving dependent events, the key question is: "For which input parameters does one want sensitivity and uncertainty measures defined?" Certainly one would want sensitivities with respect to the marginal probabilities; but, for the current version of SIGPI would one want sensitivities with respect to covariances? This question is tied to the options posed in Section 4.2 for expressing dependent probabilities via user input.

5 CONCLUSIONS AND RECOMMENDATIONS

Based on the success achieved to date, the primary conclusion of this report is that a PC version of the SIGPI code is feasible. In fact the existing version UNM-B appears to handle simple problems involving independent base events correctly. The main difficulty encountered to date appears to lie in the routines for evaluating dependent probabilities via integration over the multivariate (joint) Gaussian probability distribution. At the very worst, a PC version of SIGPI in which dependent probabilities are user-specified could be created, and such a version might prove as useful as the original SIGPI, which is restricted by the assumption of a joint Gaussian distribution for dependent events (see Section 4.2).

Assuming that a decision is made to proceed further with the development of a PC version of SIGPI, it is recommended that the following tasks be undertaken in the order listed:

1. Debug the UNM-B version to enable the SIGPI capability for processing multiple probability sets (see Section 3.?).
2. Standardize the COMMON blocks (see Section 3.?).
3. Debug the UNM-B version to enable the SIGPI capability for handling Gaussian distributed dependent probabilities (see Section 3.). Note there exists some possibility that this task may prove to be infeasible (see item 6 below).

These tasks would result in a PC version with the same capabilities as the original SIGPI. In addition, the following two tasks are recommended:

4. Resolve the discrepancy between the SIGPI results obtained for Test Run 6 and the results obtained by hand calculation in Appendix C.
5. Assure that the PC version of SIGPI compiles and runs in Microsoft FORTRAN.

Four potential enhancements to SIGPI are described in Section 4 of this report in order of their importance as currently perceived by UNM; that is, UNM recommends the following prioritization of SIGPI enhancements from a developmental perspective:

6. As an input option and/or an alternative should task 3 prove infeasible, add a capability to specify dependent probabilities directly via user input (see Section 4.2)
7. Construct a SETS-to-SIGPI preprocessor to reduce the effort involved in preparing SIGPI input.
8. Investigate the alternative lower bound defined in Section 4.3 further to determine if SIGPI convergence could be accelerated for problems involving small probabilities.
9. Develop a SIGPI-to-TEMAC postprocessor to facilitate sensitivity uncertainty analyses.

It is recognized that others may have different priorities or require different enhancements. UNM cost estimates for performing the tasks listed above (or other SIGPI-related tasks) can be developed upon request.

In principal it would be possible to merge the various quantification codes that are currently being used into a single, comprehensive, yet efficient and user friendly code. Certainly the algorithms exist to accomplish this objective. However, the main reason for undertaking such a project would be to save users' time, consequently, user friendliness and computational efficiency should be carefully considered in the design of any such composite quantification code. The interim approach of improving the effectiveness of existing codes via enhancements and code-to-code interfaces is consistent with this ultimate objective in that it reveals the positive and negative aspects of all codes while maintaining a state-of-the-art capability (if not efficiency).

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APPENDICES

APPENDIX A. CONVERT PROGRAM

The CONVERT program reads a SIGPI user's cut set input file in ASCII format and converts it to binary (unformatted in FORTRAN) form as required by SIGPI.

```
PROGRAM CONVERT
C      CONVERT DATA FILE TO SEQUENTIAL FILE FOR INPUT TO SIGPI
      IMPLICIT INTEGER (A-Z)
      DIMENSION SET(80)
      CHARACTER*16 IFILE,OFIL,PFIL,OPFIL
      CHARACTER*8 DUMMY
      CHARACTER*8 NAMES(1000),TNAME
      REAL*8 PROB
C
      WRITE(6, '(A, \') ' INPUT CUT SET FILE= '
      READ(6, '(A16)') IFILE
      WRITE(6, '(A, \') ' OUTPUT CUT SET FILE= '
      READ(6, '(A16)') OFIL
      OPEN(1, FILE=IFIL, FORM='FORMATTED', STATUS='OLD')
      OPEN(2, FILE=OFIL, FORM='UNFORMATTED', ACCESS='SEQUENTIAL',
X      STATUS='UNKNOWN')
C
C      READ NUMBER OF SETS
      WRITE(6, *) 'NSETS = '
      READ(5, 10) NSETS
10      FORMAT(I10)
C      READ SETS FOR TOP EVENT
50      DO 1000 I=1, NSETS
      READ(1, 101) NS, (SET(N), N=1, NS)
101     FORMAT(16I5)
      WRITE(2) NS, (SET(N), N=1, NS)
1000    CONTINUE
C
      ENDFIL(2)
      CLOSE(2)
      STOP
      END
```


APPENDIX B. DESCRIPTION OF TEST RUNS 5, 6, AND 7

This appendix contain the input data for SIGPI test runs 5, 6, and 7. The input data for other SIGPI test runs are listed in the SIGPI User's Manual [2].

B.1 Cut Set Data

File TEST5 - Corresponds to the noncoherent problem illustrated in Figures 3.6 and 3.8 of Ref. 1.

Size   Base Events in Set

2	1	2	
2	1	5	
3	2	3	4
2	4	5	
3	-1	2	4

File Test6 - Corresponds to the problem depicted in Figure 9 of the User's Manual [2].

Size   Base Events in Set

3	1	3	4
3	1	2	4
3	1	2	3
3	1	5	7
3	1	6	7
3	5	8	9
3	6	8	9

File TEST7 - Corresponds to INPUT FILE 2 for the TEMAC Sample Problem, see Table 9 of Ref. 3.

Size   Base Events in Set

8	1	2	6	7	10	13	14	16	
8	1	2	6	7	10	13	15	16	
8	1	2	6	8	10	13	14	16	
8	1	2	3	7	10	13	14	16	
8	1	2	4	7	10	13	14	16	
8	1	2	5	7	10	13	14	16	
8	1	2	6	7	9	13	14	16	
8	1	2	6	7	9	13	15	16	
8	1	2	6	8	9	13	14	16	
8	1	2	3	7	9	13	14	16	
8	1	2	4	7	9	13	14	16	
8	1	2	5	7	9	13	14	16	
9	1	2	6	7	11	12	13	14	16
9	1	2	6	7	11	12	13	15	16
9	1	2	6	8	11	12	13	14	16
9	1	2	3	7	11	12	13	14	16
9	1	2	4	7	11	12	13	14	16
9	1	2	5	7	11	12	13	14	16

B.2 Independent Probability Data

File PTEST5 - Corresponds to the data used to quantify the noncoherent problem illustrated in Figures 3.6 and 3.8 of Ref. 1.

<u>Base Event</u>	<u>Probability</u>
1	.1
2	.2
3	.3
4	.4
5	.5

File PTEST6 - Corresponds to the data used to quantify the problem depicted in Figure 9 of the User's Manual (Ref. 2). A hand calculation based on this data is included as Appendix C to this report.

<u>Base Event</u>	<u>Probability</u>
1	.1
2	.2
3	.3
4	.4
5	.5
6	.6
7	.7
8	.8
9	.9

File PTEST7 - Corresponds to INPUT FILE 3 for the TEMAC Sample Problem, see Table 10 of Ref. 3. Note, initiating event probabilities have been decreased by one order of magnitude to avoid probabilities greater than one in the SIGPI input. SIGPI output bounds should correspondingly be increased by one order of magnitude in making comparisons to TEMAC.

<u>Base Event</u>	<u>Probability</u>
1	.05
2	.0833
3	.00323
4	.00266
5	.00266
6	.1
7	.0484
8	.016
9	.07
10	.08
11	.24
12	.01
13	.9
14	.0484
15	.016
16	.01

APPENDIX C - HAND CALCULATION FOR TEST RUN 6

Test run 6 corresponds to the problem depicted in Figure 9, page 37 of the SIGPI User's Manual [2]. A copy of this figure is reproduced on the next page. The seven original cut sets involve no compliment events. The problem is run assuming independent base events, and the probabilities specified in the PTEST6 file are (in the nomenclature of User's Manual Figure 9):

PC1 = 0.1,
PC2 = 0.2,
PC3 = 0.3,
PC4 = 0.4,
PC5 = 0.5,
PC6 = 0.6,
PC7 = 0.7,
PC8 = 0.8,
PC9 = 0.9.

The exact solution was first obtained using the expressions for the five disjoint sets resolved in Figure 9 of the User's Manual [2]:

$$\begin{aligned}
 \text{DP-1A1} &= \text{PC1} * \text{PC2} * [1 - (1 - \text{PC3})(1 - \text{PC4})] &= 0.0116 \\
 \text{DP-1A2} &= \text{PC1} * (1 - \text{PC2}) * \text{PC3} * \text{PC4} &= \underline{0.0096} \\
 &\text{SUBTOTAL} &= 0.0212 \\
 \\
 \text{DP-1B1} &= \text{PC1} * \text{PC7} * [1 - (1 - \text{PC5})(1 - \text{PC6})] &= 0.056 \\
 \text{DP-1B2} &= \text{PC1} * [1 - (1 - \text{PC5})(1 - \text{PC6})] * (1 - \text{PC7}) * \text{PC8} * \text{PC9} &= \underline{0.01728} \\
 &\text{SUBTOTAL} &= 0.07328 \\
 \\
 \text{DP-2} &= (1 - \text{PC1}) * [1 - (1 - \text{PC5})(1 - \text{PC6})] * \text{PC8} * \text{PC9} &= \underline{0.5184} \\
 \\
 &\text{TOTAL} &= 0.61288
 \end{aligned}$$

Next the problem was solved using a spreadsheet at each of the five decomposition levels depicted in Figure 9 of the User's Manual. The spreadsheet results are included as sheets 25, 26, and 27 of this report.

Because the SIGPI results for this test did not agree with the hand calculation results, subproblems 1A, 1B, and 2 as indicated below were run separately, and resolved by SIGPI with ACC=2 (two place accuracy requested). The SIGPI results presented below agree with those calculated by hand above:

TEST6-1A.DAT:

3 1 3 4
3 1 2 4
3 1 2 3

TEST6-1B.DAT:

3 1 5 7
3 1 6 7
4 1 5 8 9
4 1 6 8 9

TEST6-2.DAT:

4 -1 5 8 9
4 -1 6 8 9

SIGPI Results:

DP = 0.0212

DP = 0.07328

DP = 0.05184

The reason for SIGPI calculating UB = 0.5980 and LB = 0.5964 for the overall TEST6 has not yet been determined.

Level 1

Set	C1	C2	C3	C4	C5	C6	C7	C8	C9	
S1	0.1	1	0.3	0.4	1	1	1	1	1	0.012
S2	0.1	0.2	1	0.4	1	1	1	1	1	0.008
S3	0.1	0.2	0.3	1	1	1	1	1	1	0.006
S4	0.1	1	1	1	0.5	1	0.7	1	1	0.035
S5	0.1	1	1	1	1	0.6	0.7	1	1	0.042
S6	1	1	1	1	0.5	1	1	0.8	0.9	0.360
S7	1	1	1	1	1	0.6	1	0.8	0.9	0.432

@sum(P(Sj))										0.895
@max(P(Sj))										0.432

Level 2

Subprob	Set	C1	C2	C3	C4	C5	C6	C7	C8	C9	
1	S1	0.1	1	0.3	0.4	1	1	1	1	1	0.0120
	S2	0.1	0.2	1	0.4	1	1	1	1	1	0.0080
	S3	0.1	0.2	0.3	1	1	1	1	1	1	0.0060
	S4	0.1	1	1	1	0.5	1	0.7	1	1	0.0350
	S5	0.1	1	1	1	1	0.6	0.7	1	1	0.0420
	S6	0.1	1	1	1	0.5	1	1	0.8	0.9	0.0360
	S7	0.1	1	1	1	1	0.6	1	0.8	0.9	0.0432

UB = @sum(P(Sj))											0.1822
LB = @max(P(Sj))											0.0432
Subprob	Set	C1	C2	C3	C4	C5	C6	C7	C8	C9	
2	S6	0.9	1	1	1	0.5	1	1	0.8	0.9	0.3240
	S7	0.9	1	1	1	1	0.6	1	0.8	0.9	0.3888

UB = @sum(P(Sj))											0.7128
LB = @max(P(Sj))											0.3888
UB =											0.8950
LB =											0.4320

Level 3

Subprob	Set	C1	C2	C3	C4	C5	C6	C7	C8	C9	
1A	S1	0.1	1	0.3	0.4	1	1	1	1	1	0.01200
	S2	0.1	0.2	1	0.4	1	1	1	1	1	0.00800
	S3	0.1	0.2	0.3	1	1	1	1	1	1	0.00600

UB = @sum(P(Sj))											0.02600
LB = @max(P(Sj))											0.00800
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B1	S4	0.1	1	1	1	0.5	1	0.7	1	1	0.03500
	S5	0.1	1	1	1	1	0.6	0.7	1	1	0.04200
	S6	0.1	1	1	1	0.5	1	0.7	0.8	0.9	0.02520
	S7	0.1	1	1	1	1	0.6	0.7	0.8	0.9	0.03024

UB = @sum(P(Sj))											0.13244
LB = @max(P(Sj))											0.04200
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B2	S6	0.1	1	1	1	0.5	1	0.3	0.8	0.9	0.01080
	S7	0.1	1	1	1	1	0.6	0.3	0.8	0.9	0.01296

UB = @sum(P(Sj))											0.02376
LB = @max(P(Sj))											0.01296
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
2	S6	0.9	1	1	1	0.5	1	1	0.8	0.9	0.32400
	S7	0.9	1	1	1	1	0.6	1	0.8	0.9	0.38880
	S6or7	0.9	1	1	1	-0.5	0.6	1	0.8	0.9	-0.19440

DP =											0.51840
UB =											0.70060
LB =											0.58136

Level 4

Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1A1	S2	0.1	0.2	0.3	0.4	1	1	1	1	1	0.00240
	S3	0.1	0.2	1	0.4	1	1	1	1	1	0.00800
	S2or3	0.1	0.2	0.3	1	1	1	1	1	1	0.00600

											UB = @sum(P(Sj)) 0.01640
											LB = @max(P(Sj)) 0.00800
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1A2	S1	0.1	0.8	0.3	0.4	1	1	1	1	1	0.00960

											DP = 0.00960
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B1	S4	0.1	1	1	1	0.5	1	0.7	1	1	0.03500
	S5	0.1	1	1	1	1	0.6	0.7	1	1	0.04200
	S4or5	0.1	1	1	1	-0.5	0.6	0.7	1	1	-0.02100

											DP = 0.05600
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B2	S6	0.1	1	1	1	0.5	1	0.3	0.8	0.9	0.01080
	S7	0.1	1	1	1	1	0.6	0.3	0.8	0.9	0.01296
	S6or7	0.1	1	1	1	-0.5	0.6	0.3	0.8	0.9	-0.00648

											DP = 0.01728
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
2	S6	0.9	1	1	1	0.5	1	1	0.8	0.9	0.32400
	S7	0.9	1	1	1	1	0.6	1	0.8	0.9	0.38880
	S6or7	0.9	1	1	1	-0.5	0.6	1	0.8	0.9	-0.19440

											DP = 0.51840
											UB = 0.61768
											LB = 0.60928

Level 5

Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1A1	S2	0.1	0.2	0.3	1	1	1	1	1	1	0.00600
	S3	0.1	0.2	1	0.4	1	1	1	1	1	0.00800
	S2or3	0.1	0.2	-0.3	0.4	1	1	1	1	1	-0.00240

											DP = 0.01160
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1A2	S1	0.1	0.8	0.3	0.4	1	1	1	1	1	0.00960

											DP = 0.00960
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B1	S4	0.1	1	1	1	0.5	1	0.7	1	1	0.03500
	S5	0.1	1	1	1	1	0.6	0.7	1	1	0.04200
	S4or5	0.1	1	1	1	-0.5	0.6	0.7	1	1	-0.02100

											DP = 0.05600
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
1B2	S6	0.1	1	1	1	0.5	1	0.3	0.8	0.9	0.01080
	S7	0.1	1	1	1	1	0.6	0.3	0.8	0.9	0.01296
	S6or7	0.1	1	1	1	-0.5	0.6	0.3	0.8	0.9	-0.00648

											DP = 0.01728
Subprob		C1	C2	C3	C4	C5	C6	C7	C8	C9	
2	S6	0.9	1	1	1	0.5	1	1	0.8	0.9	0.32400
	S7	0.9	1	1	1	1	0.6	1	0.8	0.9	0.38880
	S6or7	0.9	1	1	1	-0.5	0.6	1	0.8	0.9	-0.19440

											DP = 0.51840
TOTAL											DP = 0.61288

APPENDIX D. PROOF OF ALTERNATIVE LOWER BOUND

This appendix provides a proof of the assertion that

$$\rho \left(\bigcup_{i=1}^m S_i \right) \geq \sum_{i=1}^m \rho(S_i) - \binom{m}{2} \max_i \left(\rho(S_i) \right) \max_{i \neq j} \left(\rho(S_i | S_j) \right) \quad (1)$$

where the S_i are Boolean events (simple sets of subproblem k in SIGPI) and m is any integer greater than or equal to 2; that is, $i = 1, 2, \dots, m$.

For $m = 2$, Eq. (1) will hold if

$$\rho(S_1 \cup S_2) \leq \rho(S_1) + \rho(S_2) - \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (2)$$

But the left hand side can be expanded to yield

$$\begin{aligned} \rho(S_1) + \rho(S_2) &\geq \rho(S_1) + \rho(S_2) \\ - \rho(S_1 \cap S_2) &- \max \rho(S_i) \max \rho(S_i | S_j) \end{aligned} \quad (3)$$

or

$$\rho(S_1) \rho(S_2 | S_1) \leq \max \rho(S_i) \max \rho(S_i | S_j) \quad (4)$$

which is clearly true. Thus Eq. (1) holds for $m = 2$.

Similarly, for $m = 3$, Eq. (1) will hold if

$$\begin{aligned} \rho(S_1) + \rho(S_2) + \rho(S_3) &\geq \rho(S_1) + \rho(S_2) + \rho(S_3) \\ - \rho(S_1 \cap S_2) - \rho(S_1 \cap S_3) - \rho(S_2 \cap S_3) &- 3 \max \rho(S_i) \max \rho(S_i | S_j) \\ + \rho(S_1 \cap S_2 \cap S_3) & \end{aligned} \quad (5)$$

or

$$\begin{aligned} \rho(S_1) \rho(S_2 | S_1) &\leq 3 \max \rho(S_i) \max \rho(S_i | S_j) \\ + \rho(S_1) \rho(S_3 | S_1) & \\ + \rho(S_2) \rho(S_3 | S_2) & \\ - \rho(S_1 \cap S_2 \cap S_3) & \end{aligned} \quad (6)$$

which is clearly true. Thus Eq. (1) holds for $m = 3$.

The balance of the proof is by induction. Assume Eq.(1) holds for $m-1$ Boolean events (simple sets); that is, assume

$$\rho(U_{m-1}) = \sum_{i=1}^{m-1} \rho(S_i) - \frac{(m-1)(m-2)}{2} \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (7)$$

where U_{m-1} is used to denote the union of the first $m-1$ Boolean events:

$$U_{m-1} = S_1 \cup S_2 \cup \dots \cup S_{m-1} \quad (8)$$

Given this assumption, Eq. (1) will be proved if it can be shown that

$$\rho(U_m) \geq \sum_{i=1}^m \rho(S_i) - \frac{m(m-1)}{2} \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (9)$$

where

$$U_m = U_{m-1} \cup S_m = S_1 \cup S_2 \cup \dots \cup S_m \quad (10)$$

By the expression for the union of two events

$$\rho(U_m) = \rho(U_{m-1}) + \rho(S_m) - \rho(U_{m-1} \cap S_m) \quad (11)$$

Substituting Eq. (11) into Eq. (9), Eq. (1) will be proved if it can be shown that

$$\rho(U_{m-1}) + \rho(S_m) - \rho(U_{m-1} \cap S_m) \geq \sum_{i=1}^m \rho(S_i) - \frac{m(m-1)}{2} \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (12)$$

Subtracting Eq. (7) from Eq. (12), Eq. (1) will be proved if it can be shown that

$$-\rho(U_{m-1} \cap S_m) \geq -(m-1) \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (13)$$

or

$$\rho(U_{m-1} \cap S_m) \leq (m-1) \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (14)$$

or

$$\rho(S_m) \rho(U_{m-1} | S_m) \leq (m-1) \max_i \rho(S_i) \max_{i \neq j} \rho(S_i | S_j) \quad (15)$$

Clearly

$$\rho(S_m) \leq \max_i \rho(S_i) \quad (16)$$

So Eq. (1) will be proved if

$$\rho(U_{m-1} | S_m) \leq (m-1) \max_{i \neq j} \rho(S_i | S_j) \quad (17)$$

and this is true because

$$\rho(U_{m-1} | S_m) \leq \sum_{i=1}^{m-1} \rho(S_i | S_m) \leq (m-1) \max_{i \neq j} \rho(S_i | S_j) \quad (18)$$

Q.E.D.

APPENDIX E. SIGPI CALL TREE

		CPCHECK
		DMIN1
		PROD
		CREMOVE1
		CLNA
		EQVEC
		LVSUM
		MOD
		PROD
		DUMP
		LARGER
		ADDONE
		CLOSE
		ESTIMATE
		DMIN1
		PROD
		LOAD
		ADDONE
		LOADCUR
		LVSUM
		MOD
		MOD
		PROD
		SPOOL
		CLNCHECK
		MAXA
		PROD
		CPCHECK
		DMIN1
		PROD
		DUMP
		LARGER
		ADDONE
		CLOSE
		ESTIMATE
		DMIN1
		PROD
		GETNEXT
		DUMP
		LARGER
		ADDONE
		CLOSE
		LOAD
		ADDONE
		LOADCUR
		LNCHECK
		MAXA
		PROD
		LOAD
		ADDONE
		PCHECK
		DMIN1
		LVSUM
		MOD
		PROD

		PROD			
		VSEEK			
		STORCUR			
		ROOM			
		DUMP			
		LARGER			
		ADDONE			
		CLOSE			
		STORNEW			
		ROOM			
		DUMP			
		LARGER			
		ADDONE			
		CLOSE			
		VSEEK			
	DMLOOP				
	DCPROC				
	CDOM				
	CLNA				
	DCANAL				
	DCOPT				
		DCREM2			
		EQVEC			
		LVSUM			
		MOD			
		MOD			
		EQVEC			
		FILL			
		FIRSTJ			
		MOD			
		LVSUM			
		MOD			
		MAXA			
		MAXV			
		FIRSTJ			
		MOD			
		MOD			
		MOD			
		VSEEK			
		FILL			
		JMIX			
		DPROB			
		ADAPT			
		ABS			
		DSQRT			
		FUNCTN			
		EXP			
		MATMUL			
		DSQRT			
		FILL			
		INVERT			
		LVLIST			
		MOD			
		LVSUM			
		MOD			

[illegible]

							MOD
						LVLIST	
						MOD	
						PROD	
						SETPROB	
						SMIX	
						DPROB	
						ADAPT	
						ABS	
						DSQRT	
						FUNCTN	
						EXP	
						MATMUL	
						DSQRT	
						FILL	
						INVERT	
						LVLIST	
						MOD	
						LVSUM	
						MOD	
						LVLIST	
						MOD	
						PROD	
						DCREMOVI	
						EQVEC	
						LVSUM	
						MOD	
						DMAX1	
						DSTORCUR	
						ROOM	
						DUMP	
						LARGER	
						ADDONE	
						CLOSE	
						DSTORNEW	
						ROOM	
						DUMP	
						LARGER	
						ADDONE	
						CLOSE	
						LVSUM	
						MOD	
						MOD	
						PLO	
						FILL	
						LVSUM	
						MOD	
						SETPROB	
						SMIX	
						DPROB	
						ADAPT	
						ABS	
						DSQRT	
						FUNCTN	
						EXP	

[illegible]

						LVSUM
						MOD
						LVSUM
						MOD
						PROD
						LVLIST
						MOD
						LVSUM
						MOD
						MOD
						DBLKTEST
						CLNA
						DREMOVE1
						EQVEC
						LVSUM
						MOD
						DSTORNEW
						ROOM
						DUMP
						LARGER
						ADDONE
						CLOSE
						LVLIST
						MOD
						LVSUM
						MOD
						PLO
						FILL
						LVSUM
						MOD
						SETPROB
						SMIX
						DPROB
						ADAPT
						ABS
						DSQRT
						FUNCTN
						EXP
						MATMUL
						DSQRT
						FILL
						INVERT
						LVLIST
						MOD
						LVSUM
						MOD
						LVLIST
						MOD
						PROD
						VSEEK
						DLNCHECK
						DPROB
						ADAPT
						ABS
						DSQRT

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							INVERT
							LVLIST
							MOD
							LVSUM
							MOD
							LVLIST
							MOD
							PROD
							SETPROB
							SMIX
							DPROB
							ADAPT
							ABS
							DSQRT
							FUNCTN
							EXP
							MATMUL
							DSQRT
							FILL
							INVERT
							LVLIST
							MOD
							LVSUM
							MOD
							LVLIST
							MOD
							PROD
							DREMOVE1
							EQVEC
							LVSUM
							MOD
							DSTORCUR
							ROOM
							DUMP
							LARGER
							ADDONE
							CLOSE
							DSTORNEW
							ROOM
							DUMP
							LARGER
							ADDONE
							CLOSE
							LVSUM
							MOD
							MOD
							PLO
							FILL
							LVSUM
							MOD
							SETPROB
							SMIX
							DPROB
							ADAPT
							ABS

						DSQRT
						FUNCTN
						EXP
						MATMUL
						DSQRT
						FILL
						INVERT
						LVLIST
						MOD
						LVSUM
						MOD
						LVLIST
						MOD
						PROD
						SETPROB
						VSEEK
						DSPool
						DEPUP
						LVSUM
						MOD
						MOD
						PFF
						DERFC
						DIMACH
						DABS
						DCSEVL
						DEXP
						DLOG
						DSQRT
						INITDS
						ABS
						SNGL (intrinsic function)
						SNGL (intrinsic function)
						SQRT (intrinsic function)
						VSEEK
						DEST
						DMIN1 (intrinsic function)
						SETPROB
						DUMP
						LARGER
						ADDONE
						CLOSE (intrinsic function)
						GETNEXTD
						DLOADCUR
						DUMP
						LARGER
						ADDONE
						CLOSE (intrinsic function)
						LOAD
						ADDONE
						JBLKUP
						PROD
						REDOJS
						LVSUM
						MOD (intrinsic function)

					SMIX	
					DPROB	
					ADAPT	
					ABS (intrinsic function)	
					DSQRT (intrinsic function)	
					FUNCTN	
					EXP (intrinsic function)	
					MATMUL	
					DSQRT (intrinsic function)	
					FILL	
					INVERT	
					LVLIST	
					MOD (intrinsic function)	
					LVSUM	
					MOD (intrinsic function)	
					LVLIST	
					MOD (intrinsic function)	
					PROD	
				LOAD		
				ADDONE		
				REMIX		
				CLNA		
				DCLNCHCK		
				DPROB		
				ADAPT		
				ABS (intrinsic function)		
				DSQRT (intrinsic function)		
				FUNCTN		
				EXP (intrinsic function)		
				MATMUL		
				DSQRT (intrinsic function)		
				FILL		
				INVERT		
				LVLIST		
				MOD (intrinsic function)		
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				MOD (intrinsic function)		
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				MOD (intrinsic function)		
				LVSUM		
				MOD (intrinsic function)		
				MAXA		
				PROD		
				SETPROB		
				DLNCHECK		
				DPROB		
				ADAPT		
				ABS (intrinsic function)		
				DSQRT (intrinsic function)		
				FUNCTN		
				EXP (intrinsic function)		
				MATMUL		
				DSQRT (intrinsic function)		
				FILL		
				INVERT		

							LVLIST
							MOD (intrinsic function)
							LVSUM
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							LVLIST
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							LVSUM
							MOD (intrinsic function)
							MAXA
							PROD
							SETPROB
							DSTORCUR
							ROOM
							DUMP
							LARGER
							ADDONE
							CLOSE (intrinsic function)
							LVSUM
							MOD (intrinsic function)
							PLO
							FILL
							LVSUM
							MOD (intrinsic function)
							SETPROB
							SMIX
							DPROB
							ADAPT
							ABS (intrinsic function)
							DSQRT (intrinsic function)
							FUNCTN
							EXP (intrinsic function)
							MATMUL
							DSQRT (intrinsic function)
							FILL
							INVERT
							LVLIST
							MOD (intrinsic function)
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							MOD (intrinsic function)
							LVLIST
							MOD (intrinsic function)
							PROD
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							REDOJS
							LVSUM
							MOD (intrinsic function)
							SMIX
							DPROB
							ADAPT
							ABS (intrinsic function)
							DSQRT (intrinsic function)
							FUNCTN
							EXP (intrinsic function)
							MATMUL
							DSQRT (intrinsic function)

						FILL
						INVERT
						LVLIST
						MOD (intrinsic function)
						LVSUM
						MOD (intrinsic function)
						LVLIST
						MOD (intrinsic function)
						PROD
						SETPROB
						VSEEK
						DUMP
						LARGER
						ADDONE
						CLOSE (intrinsic function)
						LOAD
						ADDONE
						MAINLOOP
						BLKTEST
						CLNA
						LVSUM
						MOD (intrinsic function)
						MOD (intrinsic function)
						PCHECK
						DMIN1 (intrinsic function)
						LVSUM
						MOD (intrinsic function)
						PROD
						REMOVE1
						CLNA
						EQVEC
						LVSUM
						MOD (intrinsic function)
						PROD
						STORNEW
						ROOM
						DUMP
						LARGER
						ADDONE
						CLOSE (intrinsic function)
						VSEEK
						CLNA
						DOM
						DUMP
						LARGER
						ADDONE
						CLOSE (intrinsic function)
						ESTIMATE
						DMIN1 (intrinsic function)
						PROD
						LNCHECK
						MAXA
						PROD
						LOAD
						ADDONE

LOADCUR	
LVSUM	
MOD (intrinsic function)	
MOD (intrinsic function)	
OPT	
EQVEC	
FILL	
FIRSTJ	
MOD (intrinsic function)	
LVSUM	
MOD (intrinsic function)	
MAXA	
MAXV	
FIRSTJ	
MOD (intrinsic function)	
MOD (intrinsic function)	
MINV	
FIRSTJ	
MOD (intrinsic function)	
MOD (intrinsic function)	
MOD (intrinsic function)	
REMOVE2	
EQVEC	
LVSUM	
MOD (intrinsic function)	
MOD (intrinsic function)	
VSEEK	
PCHECK	
DMIN1 (intrinsic function)	
LVSUM	
MOD (intrinsic function)	
PROD	
PROD	
REMOVE1	
CLNA	
EQVEC	
LVSUM	
MOD (intrinsic function)	
PROD	
SPOOL	
CLNCHECK	
MAXA	
PROD	
CPCHECK	
DMIN1 (intrinsic function)	
PROD	
DUMP	
LARGER	
ADDONE	
CLOSE (intrinsic function)	
ESTIMATE	
DMIN1 (intrinsic function)	
PROD	
GETNEXT	
DUMP	

					LARGER
					ADDONE
					CLOSE (intrinsic function)
					LOAD
					ADDONE
					LOADCUR
					LNCHECK
					MAXA
					PROD
					LOAD
					ADDONE
					PCHECK
					DMIN1 (intrinsic function)
					LVSUM
					MOD (intrinsic function)
					PROD
					PROD
					VSEEK
					STORCUR
					ROOM
					DUMP
					LARGER
					ADDONE
					CLOSE (intrinsic function)
					STORNEW
					ROOM
					DUMP
					LARGER
					ADDONE
					CLOSE (intrinsic function)
					VSEEK
					PREPROS
					DEPREAD
					EXIT (intrinsic function)
					LVSUM
					MOD (intrinsic function)
					MOD (intrinsic function)
					PFF
					DERFC
					DIMACH
					DABS (intrinsic function)
					DCSEVL
					DEXP (intrinsic function)
					DLOG (intrinsic function)
					DSQRT (intrinsic function)
					INITDS
					ABS (intrinsic function)
					SNGL (intrinsic function)
					SNGL (intrinsic function)
					SQRT (intrinsic function)
					VSEEK
					FILL
					GETSYM
					LEN (intrinsic function)
					MOD (intrinsic function)

___	___	PSET
___	___	CLNA
___	___	LVSUM
___	___	___MOD (intrinsic function)
___	___	___MOD (intrinsic function)
___	___	VSEEK
___	___	VSEEK
___	___	SECNDS (intrinsic function)

**DATE
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10 / 12 / 93

END