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Error Propagation Equations and Tables for Estimating the Uncertainty in High-Speed Wind Tunnel Test Results

Edward L. Clark
Aerodynamics Department
Sandia National Laboratories
Albuquerque, NM 87185

Abstract

Error propagation equations, based on the Taylor series model, are derived for the nondimensional ratios and coefficients most often encountered in high-speed wind tunnel testing. These include pressure ratio and coefficient, static force and moment coefficients, dynamic stability coefficients, calibration Mach number and Reynolds number. The error equations contain partial derivatives, denoted as sensitivity coefficients, which define the influence of free-stream Mach number, M_∞ , on various aerodynamic ratios. To facilitate use of the error equations, sensitivity coefficients are derived and evaluated for nine fundamental aerodynamic ratios, most of which relate free-stream test conditions (pressure, temperature, density or velocity) to a reference condition. Tables of the ratios, R , absolute sensitivity coefficients, $\partial R / \partial M_\infty$, and relative sensitivity coefficients, $(M_\infty / R) (\partial R / \partial M_\infty)$, are provided as functions of M_∞ .

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Nomenclature¹

Primary symbols

a	speed of sound
A	cross-sectional area of stream tube or channel
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C	constant in linear viscosity relation, Eq. (136)
C_F	force coefficient, Eq. (106)
C_l	rolling moment coefficient, Eq. (120)
C_{l_p}	rolling moment dynamic stability coefficient, Eq. (120)
C_m	pitching moment coefficient, Eq. (109)
C_{m_q}	pitching moment dynamic stability coefficient, Eq. (111)
C_p	pressure coefficient, $(p-p_\infty)/q_\infty$
d	reference diameter
E	error, either bias or precision
E'	error from single source
f	function
F	aerodynamic force
l	reference length
L	aerodynamic rolling moment
L_p	derivative of rolling moment with respect to roll rate, $\partial L / \partial p$
M	Mach number, V/a
M	aerodynamic pitching moment
M_q	derivative of pitching moment with respect to pitch rate, $\partial M / \partial q$
p	pressure
p	vehicle roll rate
P	pressure ratio, p/p_r
q	dynamic pressure, $\rho V^2/2$
q	vehicle pitch rate
R	aerodynamic ratio
R	calculated test result
R_A	stream-tube area ratio, A_∞/A^*
Re	Reynolds number, $\rho V l / \mu$
R_g	gas constant, $c_p - c_v$
S	reference area
T	absolute temperature
T_s	reference temperature for Sutherland's viscosity relation, Eq. (137)
V	flow velocity
x_i	i th parameter in test result
β	constant in Sutherland's viscosity relation, Eq. (137)
γ	ratio of specific heats, c_p/c_v ($=1.40$ for all calculations)
Δx_i	interval for numerical evaluation of sensitivity coefficient

¹Where feasible, the nomenclature of NACA Report 1135, "Equations, Tables, and Charts for Compressible Flow," is used.

θ	absolute sensitivity coefficient, Eq. (5)
θ'	relative sensitivity coefficient, Eq. (8)
μ	dynamic (absolute) viscosity
ρ	mass density
ρ_{x,x_j}	coefficient of correlation

Subscripts

a	atmospheric
g	gage pressure measurement
r	reference condition
t	total (stagnation) condition
1	conditions just upstream of a normal shock wave
2	conditions just downstream of a normal shock wave
∞	free-stream condition

Superscripts

*	critical conditions (local speed equal to local speed of sound, i.e., $M=1.0$)
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Error Propagation Equations and Tables for Estimating the Uncertainty in High-Speed Wind Tunnel Test Results

1. Introduction

Evaluation of the uncertainty in an experimental result has always been an important, but frequently neglected, part of the test process. With recent emphasis on quality and traceability of results, it has become essential. Some journals, such as the *Journal of Fluids Engineering* published by the American Society of Mechanical Engineers (ASME), will no longer accept experimental results unless they include uncertainty estimates. The experimentalist who has learned to use uncertainty estimates finds them invaluable both before and after testing. Pre-test estimates are very useful in selecting the instrumentation and test techniques which will best accomplish the test goals. Post-test estimates provide the user with error bounds which can be indispensable if the data are to be used in design studies. Several organizations including the ASME, Instrument Society of America (ISA), and the International Organization for Standardization (ISO) have established standards which define the uncertainty estimation process. Aerospace organizations will soon be joining this group as the Advisory Group for Aerospace Research & Development (AGARD) and the American Institute of Aeronautics and Astronautics (AIAA) develop their standards for uncertainty analysis.

Procedures for estimating the uncertainty in experimental measurements are well documented¹⁻⁶. Although there are some differences in the methodologies recommended by various authors, it is generally agreed that the procedure can be divided into three steps: (1) estimate measurement errors, (2) propagate measurement errors to final results and (3) calculate uncertainty. In the first step, the experimentalist defines all independent measurements involved in the experiment. The elemental errors associated with each measurement are then defined, according to categories such as (a) calibration, (b) data acquisition and (c) data reduction. At this point the errors must be classified as bias (fixed or systematic) or precision (random) errors. Finally, the elemental errors are combined by root-sum-squaring, to provide an estimate of the total bias and precision error for each measurement. In most cases, the desired test result is not the actual measurement, but is a calculated result incorporating one or more measurements, for example, a nondimensional ratio or coefficient. Thus, the second step in the uncertainty analysis is to estimate the error in the test result using an error propagation method based on a Taylor series model. The bias and precision errors are propagated separately. Finally, in the third step, the bias and precision errors obtained in the previous step are combined to provide an uncertainty estimate for the test result. The two factors to be combined are the bias error and the precision error multiplied by t , the 95th percentile point for the two-tailed Student t distribution. The combined uncertainty is estimated by direct addition of the two factors, or by a root-sum-squared addition.

The purpose of the present work is to facilitate, for high-speed aerodynamic testing, the calculations required in step two, that is, propagation of measurement errors to the final result. The Taylor series method requires that the test result functional relationship be differentiated, with respect to each of its parameters, x_i , to define a set of sensitivity coefficients, θ_{x_i} . For most experimental aerodynamic results, the calculus and algebra required to evaluate the sensitivity coefficients is difficult. This is especially true for the sensitivity coefficient, θ_{M_∞} , of the free-stream Mach number, M_∞ . In this report, error propagation equations are derived for many of the nondimensional ratios and coefficients used in high-speed wind tunnel testing. Most of the results are also applicable to low speed wind tunnel tests, but emphasis is on high-speed testing where Mach number is an important parameter. Equations and tabulated values of the sensitivity coefficients, θ_{M_∞} , are provided as functions of M_∞ .

2. Error Propagation Equations

2.1 Taylor series model

The previously cited references ¹⁻⁶ all recommend that the Taylor series model be used for error propagation into a result. Detailed derivations of the model are provided in Refs. 1, 2, 4 and 5 and will not be repeated here. Consider a test result, R , which is a function of n parameters, x_i ,

$$R = R(x_1, x_2, \dots, x_n). \quad (1)$$

Then, the bias or precision error, E_R , in R is approximated by

$$E_R = \left[\left(\frac{\partial R}{\partial x_1} E_{x_1} \right)^2 + \left(\frac{\partial R}{\partial x_2} E_{x_2} \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} E_{x_n} \right)^2 + 2 \frac{\partial R}{\partial x_1} \frac{\partial R}{\partial x_2} \rho_{x_1 x_2} E_{x_1} E_{x_2} + \dots \right]^{1/2} \quad (2)$$

where E_{x_i} is the bias or precision error in x_i . As was mentioned in the previous section, bias and precision errors are propagated separately and are then combined to estimate the uncertainty in the result. The cross product terms provide the error contribution due to correlated errors and $\rho_{x_1 x_2}$ is the coefficient of correlation between x_1 and x_2 . Evaluation of ρ is difficult, and Coleman and Steele ⁷ have suggested an alternate equation,

$$E_R = \left[\left(\frac{\partial R}{\partial x_1} E_{x_1} \right)^2 + \left(\frac{\partial R}{\partial x_2} E_{x_2} \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} E_{x_n} \right)^2 + 2 \frac{\partial R}{\partial x_1} \frac{\partial R}{\partial x_2} E'_{x_1} E'_{x_2} + \dots \right]^{1/2} \quad (3)$$

where E'_{x_1} and E'_{x_2} are the portions of the error in x_1 and x_2 that arise from the same source and are presumed to be perfectly correlated ($\rho=1.0$). This equation is easier to apply than Eq. (2). If the correlation between parameters is a result of the measurements, precision errors are assumed to be independent ($\rho=0$). However, if the correlation occurs as the result of multiple occurrences of a parameter within the equation for the test result, even precision errors will be correlated and Eq. (2) or (3) must be used. An example of this is given in Section 2.3.2. In any case, if the bias errors are not independent, Eq. (2) or (3) must be used. In evaluating the partial derivatives, it is easy to ignore the sign of the derivatives since

most of the terms are squared. However, it is essential that the signs be retained for the correlation terms which are not squared.

If the errors are independent, Eq. (2) can be simplified to,

$$E_R = \left[\left(\frac{\partial R}{\partial x_1} E_{x_1} \right)^2 + \left(\frac{\partial R}{\partial x_2} E_{x_2} \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} E_{x_n} \right)^2 \right]^{1/2} \quad (4)$$

The partial derivatives are denoted as absolute sensitivity coefficients, θ_i , that is,

$$\theta_i = \frac{\partial R}{\partial x_i} \quad (5)$$

Then,

$$E_R = \left[(\theta_1 E_{x_1})^2 + (\theta_2 E_{x_2})^2 + \dots + (\theta_n E_{x_n})^2 \right]^{1/2} \quad (6)$$

Dividing each side of Eq. (4) by R gives the “relative error” form of the propagation equation,

$$\frac{E_R}{R} = \left[\left(\frac{x_1}{R} \frac{\partial R}{\partial x_1} \frac{E_{x_1}}{x_1} \right)^2 + \left(\frac{x_2}{R} \frac{\partial R}{\partial x_2} \frac{E_{x_2}}{x_2} \right)^2 + \dots + \left(\frac{x_n}{R} \frac{\partial R}{\partial x_n} \frac{E_{x_n}}{x_n} \right)^2 \right]^{1/2} \quad (7)$$

This form of the equation is especially useful if the measurement errors are defined as relative errors, E_{x_i}/x_i , and if the test result parameters are related only by multiplication and division, e.g., $R = x_1 x_2 / x_3$. When parameters are added or subtracted, e.g., $R = (x_1 + x_2) / x_3$, the relative error form is less useful. This will be seen in the example described in Section 2.3.2. The relative sensitivity coefficient, θ'_i , is defined as

$$\theta'_i = \frac{x_i}{R} \frac{\partial R}{\partial x_i} \quad (8)$$

Then,

$$\frac{E_R}{R} = \left[\left(\theta'_1 \frac{E_{x_1}}{x_1} \right)^2 + \left(\theta'_2 \frac{E_{x_2}}{x_2} \right)^2 + \dots + \left(\theta'_n \frac{E_{x_n}}{x_n} \right)^2 \right]^{1/2} \quad (9)$$

In many cases, analytical evaluation of the sensitivity coefficients is difficult or impossible. Examples include results which involve: (1) complex and/or implicit functions; (2) interpolation of tabulated functions and (3) iterative calculations. The procedure in this case is to use a numerical evaluation of the partial derivatives. Using a central differencing scheme, the derivatives can be approximated by,

$$\frac{\partial R}{\partial x_i} = \lim_{\Delta x_i \rightarrow 0} \left[(R_{x_i + \Delta x_i/2} - R_{x_i - \Delta x_i/2}) / \Delta x_i \right]_{x_j = \text{const} (j \neq i)} \quad (10)$$

With this method, nominal values are selected for all of the parameters, x_j . Then, R is sequentially evaluated by perturbing each x_i ($i=1, n$) by $\pm \Delta x_i / 2$ while the remaining $n-1$ x_j

values remain constant. The two perturbed values of R are differenced and divided by Δx_i . The approximate derivatives are used with Eq. (4) to estimate the error in the result. This approach is discussed in detail in Refs. 8 and 9. The advantages of this method are that it avoids complex differentiation and algebra, automatically calculates the effect of correlated variables, and may be the only solution possible. The disadvantage is that it provides only a “point” solution and the functional variation of test result error with a given variable cannot be determined. A simple code, DFDX, which demonstrates this technique is listed in the Appendix. The method is very powerful and was used in the present report to verify the relative sensitivity coefficients for each of the applications listed in Section 4. For these checks, the increment was $\Delta x_i = 0.005 x_i$ and the numerical results were in excellent agreement (4 to 5 significant digits) with the analytical values.

2.2 Application of the model

Application of the Taylor series model is straightforward. However, there are two cautions. First, the correct model must be used. If the measurements are independent, Eq. (4) can be used, but if they are not independent, i.e., if two or more of the measurements are correlated, then either Eq. (2) or (3) must be used. This is very important and will be emphasized again in later sections. Second, it is essential that the data reduction equation be solved for the experimental result before performing an uncertainty analysis¹⁰. This mistake is most likely to occur during a pre-test analysis to estimate the allowable error in a measurement for a given error in the result. For example, let

$$R = xy. \quad (11)$$

Estimate the allowable error, E_x , in the measurement, x, for a given error, E_R , in the result, R. From Eq. (4),

$$\begin{aligned} E_R^2 &= \left(\frac{\partial R}{\partial x} E_x \right)^2 + \left(\frac{\partial R}{\partial y} E_y \right)^2 \\ &= (y E_x)^2 + (x E_y)^2 \\ &= R^2 \left[\left(\frac{E_x}{x} \right)^2 + \left(\frac{E_y}{y} \right)^2 \right] \end{aligned}$$

Then,

$$E_x = x \left[\left(\frac{E_R}{R} \right)^2 - \left(\frac{E_y}{y} \right)^2 \right]^{1/2}. \quad (12)$$

This is the correct estimate of E_x . However, if the original equation for R is solved for x and E_x is estimated from the resulting equation, the result will be in error. Solving Eq. (11) for x gives,

$$x = \frac{R}{y}. \quad (13)$$

The error in x is given by,

$$\begin{aligned}
E_x^2 &= \left(\frac{\partial x}{\partial R} E_R \right)^2 + \left(\frac{\partial x}{\partial y} E_y \right)^2 \\
&= \left(\frac{E_R}{y} \right)^2 + \left(\frac{R}{y^2} E_y \right)^2
\end{aligned}$$

Then,

$$E_x = x \left[\left(\frac{E_R}{R} \right)^2 + \left(\frac{E_y}{y} \right)^2 \right]^{1/2} \quad (14)$$

Comparison of Eqs. (12) and (14) shows that the second result is incorrect.

2.3 Example - pressure ratio

2.3.1 Independent pressure measurements

As an example of the use of error propagation equations, consider the simple pressure ratio, P ,

$$P = \frac{p}{p_r} \quad (15)$$

where p and p_r are two independent absolute pressure measurements. Typically, in a high-speed wind tunnel test, the reference pressure, p_r , could be p_∞ (measured on the sidewall or with a probe), p_t or p_{t_2} . The measurements are assumed to be independent - this may not be the case if the same standard was used to calibrate both transducers and/or if they share common signal conditioning. From Eq. (4),

$$E_P = \left[\left(\frac{\partial P}{\partial p} E_p \right)^2 + \left(\frac{\partial P}{\partial p_r} E_{p_r} \right)^2 \right]^{1/2} \quad (16)$$

Differentiating Eq. (15) and substituting the results in Eq. (16) gives

$$E_P = \left[\left(\frac{1}{p_r} E_p \right)^2 + \left(-\frac{p}{p_r^2} E_{p_r} \right)^2 \right]^{1/2},$$

and simplifying,

$$E_P = \frac{1}{p_r} \left[E_p^2 + (P E_{p_r})^2 \right]^{1/2} \quad (17)$$

In the relative error form,

$$\frac{E_P}{P} = \left[\left(\frac{E_p}{p} \right)^2 + \left(\frac{E_{p_r}}{p_r} \right)^2 \right]^{1/2}. \quad (18)$$

This is the simplest form of error propagation and shows that the relative error in the result is the root-sum-square of the relative errors in the two measurements.

2.3.2 Correlated pressures

Now, assume that the two pressure measurements are gage, not absolute. Then, the two absolute pressures required for the ratio are given by

$$\begin{aligned} p &= p_g + p_a \\ p_r &= p_{r,g} + p_a \end{aligned} \quad (19)$$

where p_g and $p_{r,g}$ are the two independent gage pressure measurements and p_a is the measured atmospheric pressure. In this case, the calculated pressures p and p_r are correlated by p_a . The pressure ratio, P , is given by,

$$P = \frac{p_g + p_a}{p_{r,g} + p_a}. \quad (20)$$

Calculation of the error in P is much more complicated than in the previous case, where p and p_r were independent. The error will be evaluated analytically, using two techniques, and numerically with the code DFDX, which was described earlier.

2.3.2.1 Analytical evaluation—-independent variables

If the required calculus and algebra associated with determining the sensitivity coefficients permits, it is *always* best to express the result in terms of the individual measurements, that is, in its most elementary form. This avoids the problems which can occur with correlated variables as is demonstrated in the next section. Since p_g , $p_{r,g}$, and p_a are independent measurements, Eq. (20) is the desired form and the error estimate is given by Eq. (4),

$$E_P = \left[\left(\frac{\partial P}{\partial p_g} E_{p_g} \right)^2 + \left(\frac{\partial P}{\partial p_{r,g}} E_{p_{r,g}} \right)^2 + \left(\frac{\partial P}{\partial p_a} E_{p_a} \right)^2 \right]^{1/2}. \quad (21)$$

Performing the indicated partial differentiation and simplifying gives,

$$E_P = P \left[\left(\frac{E_{p_g}}{p} \right)^2 + \left(\frac{E_{p_{r,g}}}{p_r} \right)^2 + \left(\frac{1}{p} - \frac{1}{p_r} \right)^2 E_{p_a}^2 \right]^{1/2} \quad (22)$$

or,

$$E_P = F \left[\left(\frac{E_{p_g}}{p} \right)^2 + \left(\frac{E_{p_{r,g}}}{p_r} \right)^2 + \left(\frac{1}{p^2} + \frac{1}{p_r^2} - \frac{2}{pp_r} \right) E_{p_a}^2 \right]^{1/2} \quad (23)$$

Notice that in this equation, the pressures in the denominators are the absolute pressures p and p_r , not the gage pressures p_g and $p_{r,g}$. Therefore, this is not the usual relative error form.

2.3.2.2 Analytical evaluation—dependent variables

Frequently, it is easier to differentiate the unexpanded equation, Eq. (15) in this case. If we were to take this easier course and treat p and p_r as independent variables, we would obtain the error in P as,

$$E_P = \left[\left(\frac{\partial P}{\partial p} E_p \right)^2 + \left(\frac{\partial P}{\partial p_r} E_{p_r} \right)^2 \right]^{1/2} \quad (24)$$

Then, differentiating Eq. (15) and simplifying,

$$E_P = P \left[\left(\frac{E_p}{p} \right)^2 + \left(\frac{E_{p_r}}{p_r} \right)^2 \right]^{1/2} \quad (25)$$

The effect of errors in p_a is concealed in this equation. Expanding p and p_r , Eq. (19), and estimating their errors gives,

$$E_p^2 = E_{p_g}^2 + E_{p_a}^2 \quad (26)$$

and

$$E_{p_r}^2 = E_{p_{r,g}}^2 + E_{p_a}^2 \quad (27)$$

Substituting these expressions into Eq. (25) gives

$$E_P = P \left[\left(\frac{E_{p_g}}{p} \right)^2 + \left(\frac{E_{p_{r,g}}}{p_r} \right)^2 + \left(\frac{1}{p^2} + \frac{1}{p_r^2} \right) E_{p_a}^2 \right]^{1/2} \quad (28)$$

Comparing Eq. (28) to Eq. (23) shows that there is a missing term, $-2E_{p_a}^2/pp_r$. Therefore, Eq. (28) will overpredict the error in E_P . The reason for this is that p and p_r were treated as independent variables, whereas they are actually correlated by p_a .

For a correct error estimate with correlated variables, use Eq. (2) or (3). Using Eq. (3),

$$E_P = \left[\left(\frac{\partial P}{\partial p} E_p \right)^2 + \left(\frac{\partial P}{\partial p_r} E_{p_r} \right)^2 + 2 \frac{\partial P}{\partial p} \frac{\partial P}{\partial p_r} E'_p E'_{p_r} (p_a) \right]^{1/2} \quad (29)$$

where E'_p and E'_{p_r} are the portions of the error in p and p_r that originate from the correlating factor, p_a . From Eqs. (26) and (27),

$$E'_p = E_{p_a}$$

$$E'_{p_r} = E_{p_a}.$$

The sensitivity coefficients are,

$$\frac{\partial P}{\partial p} = \frac{1}{p_r} \quad \text{and} \quad \frac{\partial P}{\partial p_r} = -\frac{p}{p_r^2}.$$

Substituting these relations into Eq. (29) and simplifying gives,

$$E_P = P \left[\left(\frac{E_{p_g}}{p} \right)^2 + \left(\frac{E_{p_{r,g}}}{p_r} \right)^2 + \left(\frac{1}{p^2} + \frac{1}{p_r^2} - \frac{2}{pp_r} \right) E_{p_a}^2 \right]^{1/2} \quad (30)$$

which agrees with Eq. (23).

2.3.2.3 Numerical evaluation

Program DFDX described earlier was used with Eq. (20) to estimate the three sensitivity coefficients - θ_{p_g} , $\theta_{p_{r,g}}$ and θ_{p_a} . The numerical results agreed with Eq. (23).

2.3.3 Pre-test analysis benefits

In addition to demonstrating various error propagation techniques, this example shows the value of pre-test uncertainty analysis. The error in P due to p_a is given by,

$$E_P = \frac{\partial P}{\partial p_a} E_{p_a}.$$

Then, the relative error is

$$\frac{E_P}{P} = \frac{p_a}{P} \frac{\partial P}{\partial p_a} \frac{E_{p_a}}{p_a},$$

or, with the definition of the relative sensitivity coefficient,

$$\frac{E_P}{P} = \theta'_{p_a} \frac{E_{p_a}}{p_a}. \quad (31)$$

Therefore, a relative error of 0.01 (1%) in p_a will appear as an $0.01 \theta'_{p_a}$ relative error in the ratio, P . Differentiating Eq. (20) with respect to p_a gives,

$$\frac{\partial P}{\partial p_a} = \frac{1}{p_r} - \frac{p}{p_r^2} = P \left(\frac{1}{p} - \frac{1}{p_r} \right) \quad (32)$$

which is the expression used earlier in Eq. (22). Then,

$$\theta'_{p_a} = \frac{p_a}{P} \frac{\partial P}{\partial p_a} = \left(\frac{1}{p} - \frac{1}{p_r} \right) p_a. \quad (33)$$

This relation has been plotted below for $p_a = 15$ psia and values of p and p_r ranging from 1 to 50 psia. This figure is useful for pre-test instrumentation selection. If p and p_r are based on gage pressure measurements, then, for small relative error in P , one or more of three conditions must exist: (1) E_p/p_a is very small; (2) $p \approx p_r$, or (3) p and p_r must be large (>10 psia for $\theta'_{p_a} < 1$). If these conditions cannot be met, absolute pressure transducers must be used.

Variation of relative sensitivity coefficient $(p_a/P)(\partial P/\partial p_a)$ with p and p_r

3. Sensitivity Coefficients - Aerodynamic Ratios

The aerodynamic ratios considered in this section are basic to high-speed aerodynamics and, in general, relate calculated free-stream conditions to measured total (stagnation) conditions. There are two reasons for interest in these ratios. First, they provide a means of estimating the uncertainty in the calculated free-stream conditions. Second, they are required for evaluating the uncertainty in the more complex applications which are discussed in Section 4. In all cases, these aerodynamic ratios are functions of only the free-stream Mach number, M_∞ , and the ratio of specific heats, γ . In Section 2.3, the uncertainty in the pressure ratio, $R = p/p_r$, was examined. In that case, both pressures were measured values. In the present section, only the total condition is measured—the free-stream value is to be

calculated from the aerodynamic ratio, R , and the total condition. For example, consider the ratio of free-stream pressure, p_∞ , to total pressure, p_t ,

$$R = \frac{p_\infty}{p_t} = f(M_\infty, \gamma). \quad (34)$$

Then, the free-stream static pressure is calculated with

$$p_\infty = R(M_\infty, \gamma) p_t. \quad (35)$$

Assuming that the variables are independent, the error in p_∞ is given by

$$E_{p_\infty} = \left[\left(\frac{\partial p_\infty}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(\frac{\partial p_\infty}{\partial \gamma} E_\gamma \right)^2 + \left(\frac{\partial p_\infty}{\partial p_t} E_{p_t} \right)^2 \right]^{1/2}. \quad (36)$$

Letting $E_\gamma = 0$ and expanding the partial derivative in the first term (chain rule),

$$E_{p_\infty} = \left[\left(\frac{\partial p_\infty}{\partial R} \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(\frac{\partial p_\infty}{\partial p_t} E_{p_t} \right)^2 \right]^{1/2}. \quad (37)$$

Differentiating Eq. (35) and substituting in Eq. (37) gives the absolute error,

$$E_{p_\infty} = \left[\left(p_t \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + (R E_{p_t})^2 \right]^{1/2}. \quad (38)$$

The sensitivity coefficient for Mach number is,

$$\theta_{M_\infty} \equiv \frac{\partial p_\infty}{\partial M_\infty} = p_t \frac{\partial R}{\partial M_\infty}. \quad (39)$$

Dividing both sides of Eq. (38) by p_∞ gives the relative error,

$$\frac{E_{p_\infty}}{p_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2}. \quad (40)$$

The relative sensitivity coefficient for M_∞ is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}. \quad (41)$$

In evaluating Eq. (38) or (40), if p_t is calculated from a gage pressure measurement, $p_{t,g}$, then

$$p_t = p_{t,g} + p_a \quad (42)$$

where p_a is measured atmospheric pressure. The error in p_t is

$$E_{p_t} = \left[\left(\frac{\partial p_t}{\partial p_{t,g}} E_{p_{t,g}} \right)^2 + \left(\frac{\partial p_t}{\partial p_a} E_{p_a} \right)^2 \right]^{1/2}$$

$$E_{p_t} = \left[E_{p_{t,g}}^2 + E_{p_s}^2 \right]^{1/2}. \quad (43)$$

This value of E_{p_t} would then be substituted in Eq. (38) or (40).

The analysis in this section makes several assumptions. First, it will be assumed that p_t is an absolute pressure measurement (or equivalently, that E_{p_s} is negligible). If this is not the case, Eq. (43) can be used with Eq. (38) or (40). Second, it will be assumed that there is negligible error in γ , i.e., $E_\gamma = 0$. Finally, it is assumed that M_∞ and p_t are independent. This may not be true in many tunnels where the free-stream Mach number is determined from a calibration relation which is a function of total pressure. In this case there are three options. First, expand the equation of interest to its elemental parameters, which are independent, and differentiate it with respect to each parameter to evaluate the sensitivity coefficients. Equation (4) can then be used to define the error equation. Second, modify the error equation to include the effect of correlated parameters, e.g., use Eq. (3). Then use the sensitivity coefficients for Mach number, which are given in the following sections, with the appropriate error estimates defined by the M_∞ calibration relation. Third, use a numerical evaluation which defines sensitivity coefficients that include the effects of correlated parameters. These coefficients can be used with an error equation based on Eq. (4).

In the following subsections, the derivative $\partial R / \partial M_\infty$ and the relative sensitivity coefficient, $\theta'_{M_\infty} = (M_\infty / R) (\partial R / \partial M_\infty)$, will be evaluated as functions of M_∞ for nine fundamental aerodynamic ratios. Values of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} are tabulated for $\gamma = 1.40$. The calculated values of R were checked by comparing them to the values given in Ref. 11. Analytical values of $\partial R / \partial M_\infty$ were checked by numerical differentiation of the tabulated values of R . The reciprocals of the derivative and relative sensitivity coefficient are also tabulated and are useful in estimating the sensitivity of M_∞ to the various ratios.

3.1 Free-stream static pressure ratio, $R = p_\infty / p_t$

The equations for free-stream static pressure, p_∞ , and its absolute and relative errors are given above by Eqs. (35), (38) and (40), respectively:

$$p_\infty = R (M_\infty, \gamma) p_t \quad (35)$$

$$E_{p_\infty} = \left[\left(p_t \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + (R E_{p_t})^2 \right]^{1/2} \quad (38)$$

$$\frac{E_{p_\infty}}{p_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2} \quad (40)$$

For an isentropic flow of a perfect gas¹, Eq.(44) of Ref. 11 gives,

$$R = \frac{p_\infty}{p_t} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{-\frac{\gamma}{\gamma-1}}. \quad (44)$$

Differentiating Eq. (44) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = -\frac{\gamma M_\infty R}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (45)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = -\frac{\gamma M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (46)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 1 and tabulated in Table 1. The free-stream static pressure ratio can be used, with measured values of p_∞ and p_t , to calculate Mach number. This application is discussed in Section 4.5.

3.2 Free-stream dynamic pressure ratio, $R = q_\infty / p_t$

The free-stream dynamic pressure, q_∞ , is calculated from

$$q_\infty = R (M_\infty, \gamma) p_t \quad (47)$$

and the error equations are

$$E_{q_\infty} = \left[\left(p_t \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + (R E_{p_t})^2 \right]^{1/2} \quad (48)$$

and

$$\frac{E_{q_\infty}}{q_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2}. \quad (49)$$

For an isentropic flow of a perfect gas, Eq. (48) of Ref. 11 gives,

$$R = \frac{q_\infty}{p_t} = \frac{\gamma}{2} M_\infty^2 \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{-\frac{\gamma}{\gamma-1}}. \quad (50)$$

Differentiating Eq. (50) with respect to M_∞ gives

¹Following the notation of Ref. 11, a "thermally perfect" gas is one which obeys the thermal equation of state, $p = \rho R_g T$. A "calorically perfect" gas is one which has constant specific heats, c_p and c_v . A "perfect" gas is both thermally and calorically perfect.

$$\frac{\partial R}{\partial M_\infty} = \frac{R}{M_\infty} \left(\frac{2 - M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2} \right). \quad (51)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{2 - M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (52)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 2 and tabulated in Table 2.

3.3 Free-stream dynamic pressure ratio, $R = q_\infty / p_{t_2}$

In some instances, it may be desirable to calculate free-stream dynamic pressure from the measured pitot pressure. This is especially true at hypersonic Mach numbers where R is insensitive to M_∞ . The free-stream dynamic pressure, q_∞ , is calculated from

$$q_\infty = R(M_\infty, \gamma) p_{t_2}. \quad (53)$$

The error equations are

$$E_{q_\infty} = \left[\left(p_{t_2} \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(R E_{p_{t_2}} \right)^2 \right]^{1/2} \quad (54)$$

and

$$\frac{E_{q_\infty}}{q_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_{t_2}}}{p_{t_2}} \right)^2 \right]^{1/2}. \quad (55)$$

For $M = M_\infty$, $p_{t_1} = p_t$ and the ratio can be expressed as

$$R = \frac{q_\infty}{p_{t_2}} = \frac{q_\infty p_{t_1}}{p_t p_{t_2}} \quad (56)$$

At free-stream Mach numbers less than 1.0 there is no shock wave ahead of the pitot tube so $p_{t_2} = p_t$ and $R = q_\infty / p_t$ (see Section 3.2). For $M_\infty \geq 1.0$, with adiabatic flow of a perfect gas, Eqs. (48) and (99) of Ref. 11 are substituted into Eq. (56) giving

$$R = \frac{\gamma}{2} M_\infty^2 \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{-\frac{\gamma}{\gamma-1}} \left[\frac{(\gamma+1) M_\infty^2}{(\gamma-1) M_\infty^2 + 2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_\infty^2 - (\gamma-1)} \right]^{-\frac{1}{\gamma-1}}.$$

Simplifying this equation,

$$R = \frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{2\gamma M_\infty^2 - (\gamma-1)}{2M_\infty^2} \right)^{\frac{1}{\gamma-1}}. \quad (57)$$

Differentiating Eq. (57) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = \frac{2R}{M_\infty} \left[\frac{1}{2\gamma M_\infty^2 - (\gamma-1)} \right]. \quad (58)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{2}{2\gamma M_\infty^2 - (\gamma-1)}. \quad (59)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 3 and tabulated in Table 3. With the tabulated values of Table 3, the earlier statement concerning the insensitivity of q_∞ / p_{t_2} to M_∞ can be quantified. For example, with $M_\infty = 5.0$, $\theta'_{M_\infty} = 0.02874$ and a 5% uncertainty in Mach number ($E_{M_\infty} = 0.25$) will give a relative error in q_∞ of only $0.02874 \times 0.25 / 5.0 = 0.0014$. The error decreases with increasing Mach number. Thus, a pitot tube measurement can be used to accurately calculate the local dynamic pressure in hypersonic flow fields even when the local Mach number is not known with high accuracy.

3.4 Pitot pressure ratio, $R = p_{t_2} / p_{t_1}$

The pitot pressure, p_{t_2} , is calculated from

$$p_{t_2} = R(M_\infty, \gamma) p_{t_1}. \quad (60)$$

The error equations are

$$E_{p_{t_2}} = \left[\left(p_{t_1} \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(R E_{p_{t_1}} \right)^2 \right]^{1/2} \quad (61)$$

and

$$\frac{E_{p_{t_2}}}{p_{t_2}} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(\frac{E_{p_{t_1}}}{p_{t_1}} \right)^2 \right]^{1/2}. \quad (62)$$

For free-stream Mach numbers less than 1.0, $p_{t_2} = p_{t_1}$ and $R = 1.0$. For $M_\infty \geq 1.0$ with adiabatic flow of a perfect gas, Eq (99) of Ref. 11 gives,

$$R = \frac{p_{t_2}}{p_{t_1}} = \left[\frac{(\gamma + 1) M_\infty^2}{(\gamma - 1) M_\infty^2 + 2} \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{\gamma + 1}{2\gamma M_\infty^2 - (\gamma - 1)} \right]^{\frac{1}{\gamma - 1}}. \quad (63)$$

Differentiating Eq. (63) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = \frac{4\gamma R}{\gamma - 1} \left[\frac{1}{M_\infty [(\gamma - 1) M_\infty^2 + 2]} - \frac{M_\infty}{2\gamma M_\infty^2 - (\gamma - 1)} \right]. \quad (64)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{4\gamma M_\infty}{\gamma - 1} \left[\frac{1}{M_\infty [(\gamma - 1) M_\infty^2 + 2]} - \frac{M_\infty}{2\gamma M_\infty^2 - (\gamma - 1)} \right]. \quad (65)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 4 and tabulated in Table 4. The pitot pressure ratio can be used, with measured values of p_{t_1} and p_{t_2} , to calculate Mach number. This application is discussed in Section 4.5.

3.5 Rayleigh pitot ratio, $R = p_\infty / p_{t_2}$

The Rayleigh pitot ratio is normally used, not to calculate p_∞ or p_{t_2} , but to calculate M_∞ from the two measured pressures. This application will be discussed in more detail in Section 4.5. At free-stream Mach numbers less than 1.0, $p_{t_2} = p_{t_1}$ and $R = p_\infty / p_{t_1}$ (see Section 3.1). For $M_\infty \geq 1.0$ with adiabatic flow of a perfect gas, Eq. (100) of Ref. 11 gives,

$$R = \frac{p_\infty}{p_{t_2}} = \frac{p_1}{p_{t_2}} = \left[\frac{2}{(\gamma + 1) M_\infty^2} \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{2\gamma M_\infty^2 - (\gamma - 1)}{\gamma + 1} \right]^{\frac{1}{\gamma - 1}}. \quad (66)$$

Differentiating Eq. (66) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = \frac{R}{M_\infty} \left[\frac{2\gamma(1 - 2M_\infty^2)}{2\gamma M_\infty^2 - (\gamma - 1)} \right]. \quad (67)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{2\gamma(1 - 2M_\infty^2)}{2\gamma M_\infty^2 - (\gamma - 1)}. \quad (68)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 5 and tabulated in Table 5.

3.6 Stream-tube area ratio, $R = A_\infty / A^*$

The stream-tube area ratio is used to evaluate the mass flow sensitivity (Section 3.7). It will also be used in the applications (Section 4.4) to estimate the error in q_∞ / V_∞ . For an isentropic flow of a perfect gas, Eq. (80) of Ref. 11 gives

$$R = \frac{A_\infty}{A^*} = \frac{1}{M_\infty} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}. \quad (69)$$

Differentiating Eq. (69) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = R \left[\frac{M_\infty^2 - 1}{M_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)} \right]. \quad (70)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{M_\infty^2 - 1}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (71)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 6 and tabulated in Table 6.

The stream-tube area ratio can also be used to estimate the sensitivity of M_∞ to changes in the sonic throat cross-sectional area and to changes in the test section cross-sectional area (e.g., blockage at subsonic speeds). Since

$$R = f(M_\infty, \gamma),$$

$$E_R = \frac{\partial R}{\partial M_\infty} E_{M_\infty}.$$

Except for $M_\infty = 1.0$, where $\partial R / \partial M_\infty = 0$,

$$E_{M_\infty} = \left(\frac{\partial R}{\partial M_\infty} \right)^{-1} E_R \quad (72)$$

Then, with the reciprocal of $\partial R / \partial M_\infty$ from Eq. (70) or Table 6, the sensitivity of M_∞ to stream-tube area ratio, $R = A_\infty / A^*$, can be estimated with Eq. (72).

3.7 Mass flow ratio, $R = \rho_\infty V_\infty / \rho^* V^*$

The stream-tube area relation is given in Eq. (79) of Ref. 11. By continuity,

$$\rho_\infty V_\infty A_\infty = \rho^* V^* A^*.$$

Then,

$$R = \frac{\rho_{\infty} V_{\infty}}{\rho^* V^*} = \frac{A^*}{A_{\infty}} = \frac{1}{R_A} \quad (73)$$

where $R_A = A_{\infty} / A^*$ (see Section 3.6). The derivative of R with respect to M_{∞} is

$$\frac{\partial R}{\partial M_{\infty}} = -\frac{1}{R_A^2} \frac{\partial R_A}{\partial M_{\infty}}. \quad (74)$$

The relative sensitivity coefficient is

$$\theta'_{M_{\infty}} = \frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}} = -\frac{M_{\infty}}{R_A} \frac{\partial R_A}{\partial M_{\infty}}. \quad (75)$$

Equations (73), (74) and (75) can be evaluated with Eqs. (69), (70) and (71) from Section 3.6. For $\gamma = 1.40$, values of R , $\partial R / \partial M_{\infty}$ and $(M_{\infty} / R)(\partial R / \partial M_{\infty})$ are plotted in Figure 7 and tabulated in Table 7.

To evaluate the reference condition, $\rho^* V^*$, the following equations are used. For a thermally perfect gas,

$$\rho^* = \frac{p^*}{R_g T^*} \quad (76)$$

and

$$V^* = \sqrt{\gamma R_g T^*} \quad (77)$$

where R_g is the gas constant. Multiplying Eqs. (76) and (77) and substituting the isentropic relations for p^* / p_t and T^* / T_t (Eqs. (44) and (82) with $M_{\infty} = 1.0$),

$$\rho^* V^* = p^* \sqrt{\frac{\gamma}{R_g T^*}} = \sqrt{\frac{\gamma}{R_g}} \left(\frac{\gamma + 1}{2} \right)^{-\frac{(\gamma + 1)}{2(\gamma - 1)}} \frac{p_t}{\sqrt{T_t}}. \quad (78)$$

3.8 Free-stream static temperature ratio, $R = T_{\infty} / T_t$

The free-stream static temperature, T_{∞} , is calculated from

$$T_{\infty} = R(M_{\infty}, \gamma) T_t \quad (79)$$

and the error equations are

$$E_{T_{\infty}} = \left[\left(T_t \frac{\partial R}{\partial M_{\infty}} E_{M_{\infty}} \right)^2 + (R E_{T_t})^2 \right]^{1/2} \quad (80)$$

and

$$\frac{E_{T_\infty}}{T_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{T_1}}{T_1} \right)^2 \right]^{1/2}. \quad (81)$$

For an adiabatic flow of a perfect gas, Eq. (43) of Ref. 11 gives,

$$R = \frac{T_\infty}{T_1} = \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{-1}. \quad (82)$$

Differentiating Eq. (82) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = \frac{-(\gamma-1) R M_\infty}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (83)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{-(\gamma-1) M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (84)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 8 and tabulated in Table 8.

3.9 Free-stream velocity ratio, $R = V_\infty / a_t$

The free-stream static velocity, V_∞ , is calculated from

$$V_\infty = R (M_\infty, \gamma) a_t \quad (85)$$

and the error equations are

$$E_{V_\infty} = \left[\left(a_t \frac{\partial R}{\partial M_\infty} E_{M_\infty} \right)^2 + (R E_{a_t})^2 \right]^{1/2} \quad (86)$$

and

$$\frac{E_{V_\infty}}{V_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{a_t}}{a_t} \right)^2 \right]^{1/2}. \quad (87)$$

For a thermally perfect gas, the speed of sound, a , is given by Eq. (29b) of Ref. 11 as

$$a = \sqrt{\gamma R_g T}. \quad (88)$$

where R_g is the gas constant. For stagnation condition,

$$a_t = \sqrt{\gamma R_g T_1}.$$

The error in a_t can then be expressed in terms of the error in T_t as,

$$\frac{E_{a_t}}{a_t} = \frac{E_{T_t}}{2T_t}.$$

Substituting this relation into Eq. (87) gives an alternative error equation,

$$\frac{E_{V_\infty}}{V_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{T_t}}{2T_t} \right)^2 \right]^{1/2}. \quad (87a)$$

From the definition of Mach number, $M_\infty = V_\infty / a_\infty$,

$$R = \frac{V_\infty}{a_t} = M_\infty \frac{a_\infty}{a_t}.$$

Then, with Eq. (88),

$$R = M_\infty \sqrt{\frac{T_\infty}{T_t}}. \quad (89)$$

Substituting Eq. (82) into Eq. (89) gives

$$R = M_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{-1/2}. \quad (90)$$

Differentiating Eq. (90) with respect to M_∞ gives

$$\frac{\partial R}{\partial M_\infty} = \frac{R}{M_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)}. \quad (91)$$

The relative sensitivity coefficient is

$$\theta'_{M_\infty} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{1}{1 + \frac{\gamma-1}{2} M_\infty^2}. \quad (92)$$

For $\gamma = 1.40$, values of R , $\partial R / \partial M_\infty$ and $(M_\infty / R) (\partial R / \partial M_\infty)$ are plotted in Figure 9 and tabulated in Table 9.

Frequently, the velocity ratio is given as

$$R^* = \frac{V_\infty}{a^*}. \quad (93)$$

From Eq. (33) of Ref. 11,

$$\frac{a_t}{a^*} = \sqrt{\frac{\gamma+1}{2}}. \quad (94)$$

Then,

$$R^* = \sqrt{\frac{\gamma+1}{2}} R \quad (95)$$

where, $R = V_\infty / a_t$. Differentiating Eq. (95),

$$\frac{\partial R^*}{\partial M_\infty} = \sqrt{\frac{\gamma+1}{2}} \frac{\partial R}{\partial M_\infty} \quad (96)$$

and

$$\theta'_{M_\infty} = \frac{M_\infty}{R^*} \frac{\partial R^*}{\partial M} = \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}. \quad (97)$$

4. Applications

In this section, error estimates will be given for several nondimensional ratios and coefficients which are frequently used in presenting the results of high-speed wind tunnel tests. These estimates will rely heavily on the results of Section 3. It will be assumed that the variables are independent. For those results which involve both an arbitrary measured pressure and a measured tunnel condition pressure, such as total pressure, p_t , this implies that the two pressures are absolute measurements or are gage measurements with a negligible error in the measured atmospheric pressure. If this is not the case, a more complicated analysis, similar to that in Section 2.3.2 will be required. Also it is assumed that M_∞ and p_t are not correlated through the tunnel calibration. If this is not true, the equations may be much more complex and a numerical estimate (Section 2.1) may be required. Finally, it is assumed that there is no error in the ratio of specific heats, γ .

4.1 Pressure ratio, P

For the first application we will again examine that ubiquitous parameter, the pressure ratio, P . In Section 2.3 the error in P was examined, where $P = p / p_t$ and both p and p_t were measured pressures. In Section 3.1 the error in p_∞ was evaluated, where $p_\infty = P(M_\infty, \gamma) p_t$, with p_t a measured pressure

In the present application, the error in the ratio, P , will be determined for

$$P = \frac{p}{p_\infty}. \quad (98)$$

In this equation, p is an arbitrary measured pressure and p_∞ is calculated from M_∞ , γ , and the measured total pressure, p_t , using Eq. (44). Assuming that the variables in Eq. (98) are independent, the error in P is given by

$$E_p = P \left[\left(\frac{E_p}{P} \right)^2 + \left(\frac{E_{p_\infty}}{P_\infty} \right)^2 \right]^{1/2} \quad (99)$$

where E_p is the error in the measured pressure and E_{p_∞} was given by Eq. (40) as,

$$\frac{E_{p_\infty}}{P_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{P_t} \right)^2 \right]^{1/2} \quad (40)$$

where $R = p_\infty/p_t$, E_{M_∞} is the error in the free-stream Mach number, and E_{p_t} is the error in the measured total pressure. The relative sensitivity coefficient, $(M_\infty/R)(\partial R/\partial M_\infty)$, is given by Eq. (46) and is tabulated in Table 1 for $\gamma=1.40$.

4.2 Pressure coefficient, C_p

The pressure coefficient, C_p , is defined as

$$C_p = \frac{p - p_\infty}{q_\infty} \quad (100)$$

For high-speed wind tunnels, the dynamic pressure is usually calculated from the measured total pressure and the free-stream Mach number with Eq. (50). The static pressure, p_∞ is calculated with Eq. (44). For a thermally perfect gas, q_∞ and p_∞ are related by Eq. (47) of Ref. 11,

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2 \quad (101)$$

Then, substituting Eq. (101) in Eq. (100),

$$\begin{aligned} C_p &= \frac{p - p_\infty}{\frac{\gamma}{2} p_\infty M_\infty^2} \\ &= \frac{2}{\gamma M_\infty^2} \left(\frac{p}{p_\infty} - 1 \right) \end{aligned}$$

and substituting $p_\infty = R(M_\infty, \gamma) p_t$ gives

$$C_p = \frac{2}{\gamma M_\infty^2} \left(\frac{p}{p_t R} - 1 \right) \quad (102)$$

Assuming p , p_t and M_∞ are independent, the error in C_p is given by

$$E_{C_p} = \left[\left(\frac{\partial C_p}{\partial p} E_p \right)^2 + \left(\frac{\partial C_p}{\partial p_t} E_{p_t} \right)^2 + \left(\frac{\partial C_p}{\partial M_\infty} E_{M_\infty} \right)^2 \right]^{1/2} \quad (103)$$

$$E_{C_p} = C_p \left(\frac{P}{P-1} \right) \left\{ \left(\frac{E_p}{p} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 + \left[2 \left(\frac{P-1}{P} \right) + \frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \right]^2 \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 \right\}^{1/2} \quad (104)$$

where $P = p / p_\infty$ and $R = p_\infty / p_t$. If desired, P can be replaced with C_p by using the relation,

$$P = 1 + \frac{\gamma M_\infty^2}{2} C_p. \quad (105)$$

The relative sensitivity coefficient, $(M_\infty/R)(\partial R/\partial M_\infty)$, is given by Eq. (46) and is tabulated in Table 1 for $\gamma=1.40$.

4.3 Force and moment coefficients, C_F and C_m

Nondimensional force coefficients are defined by

$$C_F = \frac{F}{q_\infty S} \quad (106)$$

where F is an aerodynamic force, q_∞ the free-stream dynamic pressure and S a reference area. The error in the force coefficient is given by

$$E_{C_F} = C_F \left[\left(\frac{E_F}{F} \right)^2 + \left(\frac{E_{q_\infty}}{q_\infty} \right)^2 + \left(\frac{E_S}{S} \right)^2 \right]^{1/2} \quad (107)$$

where E_F is the error in the measured force, E_S the error in the reference area and E_{q_∞} is given by Eq. (49) as

$$\frac{E_{q_\infty}}{q_\infty} = \left[\left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2} \quad (49)$$

where $R = q_\infty / p_t$, E_{M_∞} is the error in the free-stream Mach number and E_{p_t} is the error in the measured total pressure. The relative sensitivity coefficient, $(M_\infty/R)(\partial R/\partial M_\infty)$, is given by Eq. (52) and is tabulated in Table 2 for $\gamma=1.40$. Frequently, $S = \pi d^2/4$, where d is a reference diameter. Then,

$$\frac{E_S}{S} = 2 \frac{E_d}{d}. \quad (108)$$

Nondimensional moment coefficients are defined by

$$C_m = \frac{M}{q_\infty S l}, \quad (109)$$

where M is an aerodynamic moment and l is a reference length. The error in C_m is given by

$$E_{C_m} = C_m \left[\left(\frac{E_M}{M} \right)^2 + \left(\frac{E_{q_\infty}}{q_\infty} \right)^2 + \left(\frac{E_S}{S} \right)^2 + \left(\frac{E_l}{l} \right)^2 \right]^{1/2} \quad (110)$$

This equation is similar to Eq. (107) with E_M the error in the measured moment, M , and E_l the error in the reference length. Frequently, $l = d$ and $S = \pi d^2/4$. Then, l and S are correlated and the last two terms of Eq. (110) must be replaced by

$$\left(\frac{3E_d}{d} \right)^2$$

to avoid underestimating the error contribution due to d .

4.4 Dynamic stability coefficients, C_{m_q} and C_{l_p}

Nondimensional dynamic stability coefficients are defined as derivatives of the moment coefficient with respect to a normalized angular rate. For example, the pitching moment coefficient derivative, C_{m_q} , is defined by

$$C_{m_q} = \frac{\partial C_m}{\partial (ql/V_\infty)} = \frac{\partial (M/q_\infty S l)}{\partial (ql/V_\infty)} = \frac{\partial M / \partial q}{q_\infty S l^2 / V_\infty} = \frac{M_q}{q_\infty S l^2 / V_\infty} \quad (111)$$

The derivative, M_q , is determined from the experiment. Note that in these equations, M is the aerodynamic pitching moment, whereas M_∞ is the free-stream Mach number, and q is the angular pitch rate, whereas q_∞ is the free-stream dynamic pressure. The estimated error in C_{m_q} is

$$E_{C_{m_q}} = C_{m_q} \left[\left(\frac{E_{M_q}}{M_q} \right)^2 + \left(\frac{E_{q_\infty/V_\infty}}{q_\infty/V_\infty} \right)^2 + \left(\frac{E_S}{S} \right)^2 + \left(\frac{2E_l}{l} \right)^2 \right]^{1/2} \quad (112)$$

As was noted in the previous section, if $l = d$ and $S = \pi d^2/4$, l and S are correlated and the last two terms in Eq. (112) should be replaced with $(4E_d/d)^2$. By definition,

$$\frac{q_\infty}{V_\infty} = \frac{\rho_\infty V_\infty^2}{2V_\infty} = \frac{\rho_\infty V_\infty}{2} \quad (113)$$

For a thermally perfect gas,

$$\rho_\infty = \frac{p_\infty}{R_g T_\infty} \quad (114)$$

and

$$V_\infty = M_\infty \sqrt{\gamma R_g T_\infty} \quad (115)$$

Substituting these equations into Eq. (113) and using Eqs. (44) and (82) to relate p_∞ to p_t and T_∞ to T_t , respectively, gives

$$\frac{q_\infty}{V_\infty} = \frac{1}{2} \sqrt{\frac{\gamma}{R_g}} \frac{p_t}{\sqrt{T_t}} M_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \quad (116)$$

The relative error in q_∞/V_∞ is given by

$$\frac{E_{q_\infty/V_\infty}}{q_\infty/V_\infty} = \left[\left(\frac{E_{p_t}}{p_t} \right)^2 + \left(\frac{E_{T_t}}{2T_t} \right)^2 + \left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \frac{E_{M_\infty}}{M_\infty} \right)^2 \right]^{1/2} \quad (117)$$

where $R = q_\infty/V_\infty$. Differentiating Eq. (116) with respect to M_∞ gives

$$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{1 - M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2} \quad (118)$$

Comparing Eq. (118) with Eq. (71) shows that

$$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = - \frac{M_\infty}{R_A} \frac{\partial R_A}{\partial M_\infty} \quad (119)$$

where $R_A = A_\infty/A^*$. Then, Eq. (71) and the results tabulated in Table 6 can be used to evaluate $(M_\infty/R)(\partial R/\partial M_\infty)$ for $R = q_\infty/V_\infty$. Equations (117) and (112) are used to estimate the error in C_{m_q} .

The rolling moment coefficient derivative, C_{l_p} , is defined by,

$$C_{l_p} = \frac{\partial C_l}{\partial (pl/V_\infty)} = \frac{\partial (L/q_\infty Sl)}{\partial (pl/V_\infty)} = \frac{\partial L/\partial p}{q_\infty Sl^2/V_\infty} = \frac{L_p}{q_\infty Sl^2/V_\infty} \quad (120)$$

The error in C_{l_p} is given by

$$E_{C_{l_p}} = C_{l_p} \left[\left(\frac{E_{L_p}}{L_p} \right)^2 + \left(\frac{E_{q_\infty/V_\infty}}{q_\infty/V_\infty} \right)^2 + \left(\frac{E_S}{S} \right)^2 + \left(\frac{2E_l}{l} \right)^2 \right]^{1/2} \quad (121)$$

which is evaluated with Eqs. (71) and (117) as indicated above for Eq. (112).

4.5 Mach number, M_∞

In high-speed wind tunnels, the free-stream Mach number is usually determined during tunnel calibration by one of three methods, each involving a measured pressure ratio, R . First, at subsonic and transonic Mach numbers, a static pipe may be used to measure free-stream pressure, p_∞ . The measured tunnel total pressure, p_t , is used in the ratio, $R = p_\infty/p_t$ with the isentropic flow equation to determine Mach number. From Eq. (44),

$$R = \frac{P_\infty}{P_t} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{-\frac{\gamma}{\gamma-1}}. \quad (122)$$

Solving for M_∞ gives

$$M_\infty^2 = \frac{2}{\gamma-1} \left[\left(\frac{P_\infty}{P_t} \right)^{-\frac{\gamma-1}{\gamma}} - 1 \right]. \quad (123)$$

Second, at supersonic and hypersonic Mach numbers, the static pressure can be very low and the Mach number is usually determined with a pitot tube which measures the total pressure, P_{t_2} , behind the tube's normal shock. This pressure is divided by the tunnel total pressure, P_t , to give a ratio, R , which is related to Mach number by the normal shock relation, Eq. (63),

$$R = \frac{P_{t_2}}{P_t} = \left[\frac{(\gamma+1) M_\infty^2}{(\gamma-1) M_\infty^2 + 2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_\infty^2 - (\gamma-1)} \right]^{\frac{1}{\gamma-1}}. \quad (124)$$

This equation is solved numerically for M_∞ , by using an iterative root solver. Finally, a pitot-static probe may be used to determine Mach number. In this case, the ratio and Mach number dependency is given by the Rayleigh formula, Eq. (66),

$$R = \frac{P_\infty}{P_{t_2}} = \left[\frac{2}{(\gamma+1) M_\infty^2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma M_\infty^2 - (\gamma-1)}{\gamma+1} \right]^{\frac{1}{\gamma-1}}. \quad (125)$$

Again, this equation is solved numerically for M_∞ .

The error equation for the Mach number calculated from any of these three methods is the same. Let

$$R(M_\infty, \gamma) = \frac{P_x}{P_y}. \quad (126)$$

The uncertainty in the measured ratio is given by,

$$E_R = R \left[\left(\frac{E_{P_x}}{P_x} \right)^2 + \left(\frac{E_{P_y}}{P_y} \right)^2 \right]^{1/2}. \quad (127)$$

Although an explicit relation for M_∞ is not possible for two of the ratios, the uncertainty in M_∞ can be determined implicitly, when $\partial R / \partial M_\infty \neq 0$, by solving the error equation,

$$E_R = \frac{\partial R}{\partial M_\infty} E_{M_\infty} \quad (128)$$

to give

$$E_{M_\infty} = \frac{\partial M_\infty}{\partial R} E_R. \quad (129)$$

Substituting Eq. (127) in Eq. (129) gives

$$\frac{E_{M_\infty}}{M_\infty} = \frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R} \left[\left(\frac{E_{p_x}}{p_x} \right)^2 + \left(\frac{E_{p_y}}{p_y} \right)^2 \right]^{1/2} \quad (130)$$

Substituting the measured pressures for p_x and p_y gives the following estimates of error in calibration Mach number for the three methods:

(1) For $R = p_\infty/p_t$, the relative error in Mach number is

$$\frac{E_{M_\infty}}{M_\infty} = \frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R} \left[\left(\frac{E_{p_\infty}}{p_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2}. \quad (131)$$

For nonzero values of M_∞ and $\partial R / \partial M_\infty$,

$$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R} = \left(\frac{M_\infty \partial R}{R \partial M_\infty} \right)^{-1}. \quad (132)$$

Then, $(R/M_\infty)(\partial M_\infty/\partial R)$ is given by the reciprocal of Eq. (46) and is tabulated in Table 1.

(2) For $R = p_{t_2}/p_t$, the relative error in Mach number is

$$\frac{E_{M_\infty}}{M_\infty} = \frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R} \left[\left(\frac{E_{p_{t_2}}}{p_{t_2}} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 \right]^{1/2} \quad (133)$$

where $(R/M_\infty)(\partial M_\infty/\partial R)$ is given by the reciprocal of Eq. (65) and is tabulated in Table 4.

(3) For $R = p_\infty/p_{t_2}$, the relative error in Mach number is

$$\frac{E_{M_\infty}}{M_\infty} = \frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R} \left[\left(\frac{E_{p_\infty}}{p_\infty} \right)^2 + \left(\frac{E_{p_{t_2}}}{p_{t_2}} \right)^2 \right]^{1/2} \quad (134)$$

where $(R/M_\infty)(\partial M_\infty/\partial R)$ is given by the reciprocal of Eq. (68) and is tabulated in Table 5.

4.6 Reynolds number, Re

The free-stream Reynolds number is defined as

$$Re = \frac{\rho_\infty V_\infty l}{\mu_\infty} \quad (135)$$

where ρ_∞ and V_∞ are the free-stream static density and velocity, respectively. The reference length, l , may be an actual measurement, such as model length or diameter which has an associated error, or a defined value, such as “1 foot” which has no error. The dynamic viscosity, μ_∞ , is a function of the free-stream temperature, T_∞ . In most cases, the viscosity is adequately described by two functions. For $T < T_S$, where $T_S = 198.7^\circ\text{R}$, a linear relation can be used,

$$\mu_\infty = CT_\infty \quad (136)$$

where C is a constant. For $T \geq T_S$, Sutherland’s relation (Eq. (A3) of Ref. 11) is often used,

$$\mu_\infty = \frac{\beta T_\infty^{3/2}}{T_\infty + T_S}. \quad (137)$$

The first relation is generally used for hypersonic tunnels, while the second is used for transonic and supersonic tunnels. If a data reduction code uses both of the above viscosity relations, it may not be obvious which relation is used to reduce a particular set of data. In this case, a plot which delineates the two regimes is useful. Such a plot can be generated with the function,

$$T_t = T_S \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right). \quad (138)$$

A plot of this function is shown in Figure 10.

Since ρ_∞ , V_∞ and μ_∞ are complex functions of M_∞ and T_t , an analytical estimate of the error in Re is difficult. This is especially true if Sutherland’s relation is required. In the following sections, error estimates for Re will be developed for both viscosity relations.

4.6.1 Linear viscosity relation

Three approaches will be demonstrated for Reynolds number with the linear viscosity relation. These are: (1) analytical evaluation of the fully expanded equation with independent variables; (2) analytical evaluation of a partially expanded equation with correlated terms and (3) numerical evaluation.

4.6.1.1 Analytical evaluation–independent variables

It is always recommended that the equation for the parameter of interest be fully expanded, so that all of the variables are independent, before differentiating the parameter with respect to each of the variables. In many cases this is not practical. For the present example, Reynolds number is given by,

$$Re = \frac{\rho_\infty V_\infty l}{CT_\infty} \quad (139)$$

For a thermally perfect gas, Eqs. (114) and (115) are substituted into Eq. (139) to give,

$$\text{Re} = \sqrt{\frac{\gamma}{R_g}} \frac{l}{C} \frac{M_\infty p_\infty}{T_\infty^{3/2}}. \quad (140)$$

To express Re as a function of tunnel total conditions, p_t and T_t , substitute Eqs. (44) and (82) in Eq. (140) and simplify,

$$\text{Re} = \sqrt{\frac{\gamma}{R_g}} \frac{l}{C} \frac{p_t}{T_t^{3/2}} M_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma-3}{2(\gamma-1)}}. \quad (141)$$

To show the variation of Reynolds number with Mach number, Re is normalized by Re^* , the Reynolds number at $M_\infty=1.0$. Setting $M_\infty=1.0$ in Eq. (141) gives

$$\text{Re}^* = \sqrt{\frac{\gamma}{R_g}} \frac{l}{C} \frac{p_t}{T_t^{3/2}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma-3}{2(\gamma-1)}} \quad (142)$$

Dividing Eq. (141) by Eq. (142),

$$\frac{\text{Re}}{\text{Re}^*} = M_\infty \left[\frac{2 + (\gamma-1) M_\infty^2}{\gamma+1} \right]^{\frac{\gamma-3}{2(\gamma-1)}} \quad (143)$$

This relation is plotted in Figure 11 for $\gamma=1.40$.

Assuming that the variables in Eq. (141) are independent and there are negligible errors in γ , R_g and C , the Taylor series model for independent variables, Eq. (4), will be used to estimate the error in Re. Then,

$$E_{\text{Re}} = \left[\left(\frac{\partial \text{Re}}{\partial M_\infty} E_{M_\infty} \right)^2 + \left(\frac{\partial \text{Re}}{\partial p_t} E_{p_t} \right)^2 + \left(\frac{\partial \text{Re}}{\partial T_t} E_{T_t} \right)^2 + \left(\frac{\partial \text{Re}}{\partial l} E_l \right)^2 \right]^{1/2}. \quad (144)$$

If l is a constant, $E_l = 0$. Performing the indicated differentiation of Eq. (141) gives

$$E_{\text{Re}} = \text{Re} \left\{ \left[\frac{1 + (\gamma-2) M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2} \right]^2 \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 + \left(-\frac{3 E_{T_t}}{2 T_t} \right)^2 + \left(\frac{E_l}{l} \right)^2 \right\}^{1/2}. \quad (145)$$

The term in brackets, [...], is the relative sensitivity coefficient, $(M_\infty / \text{Re})(\partial \text{Re} / \partial M_\infty)$, for Reynolds number with respect to Mach number. Values of the coefficient are plotted in Figure 12 and tabulated in Table 10 for $\gamma=1.40$. The minus sign on the E_{T_t} term has no significance in the evaluation of E_{Re} , but is included for comparison with the numerical evaluation of Section 4.6.1.3.

4.6.1.2 Analytical evaluation—dependent variables

It is frequently much easier to evaluate an expression if it is not fully expanded to the principal variables. However, in this case the variables may be correlated and the appropriate uncertainty equation must be used. Let

$$Re = \frac{R\rho^*V^*l}{\mu_\infty} \quad (146)$$

where

$$R = \frac{\rho_\infty V_\infty}{\rho^*V^*} = f(M_\infty, \gamma), \quad (147)$$

and the function, $f(M_\infty, \gamma)$, is defined by Eqs. (73) and (69) as,

$$R = M_\infty \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right) \right]^{-\frac{\gamma+1}{2(\gamma-1)}}, \quad (148)$$

ρ^*V^* is given by Eq. (78),

$$\rho^*V^* = \sqrt{\frac{\gamma}{R_g}} \left(\frac{\gamma+1}{2} \right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \frac{p_t}{\sqrt{T_t}} \quad (78)$$

and, by use of Eqs. (136) and (82), μ_∞ can be expressed as,

$$\mu_\infty = CT_t \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{-1}. \quad (149)$$

Therefore, R and μ_∞ are correlated through M_∞ while ρ^*V^* and μ_∞ are correlated through T_t . The estimated error in Re is given by Eq. (3), with the present variables,

$$\begin{aligned} E_{Re}^2 = & \left(\frac{\partial Re}{\partial R} E_R \right)^2 + \left(\frac{\partial Re}{\partial \rho^*V^*} E_{\rho^*V^*} \right)^2 + \left(\frac{\partial Re}{\partial l} E_l \right)^2 + \left(\frac{\partial Re}{\partial \mu_\infty} E_{\mu_\infty} \right)^2 \\ & + 2 \frac{\partial Re}{\partial R} \frac{\partial Re}{\partial \mu_\infty} E'_R E'_{\mu_\infty} (M_\infty) + 2 \frac{\partial Re}{\partial \rho^*V^*} \frac{\partial Re}{\partial \mu_\infty} E'_{\rho^*V^*} E'_{\mu_\infty} (T_t) \end{aligned} \quad (150)$$

The various error terms are defined by the following equations, where the primed error terms include only the error due to the indicated measurement:

$$E_R = \frac{\partial R}{\partial M_\infty} E_{M_\infty} \quad (151)$$

$$E'_R (M_\infty) = E_R \quad (152)$$

$$E_{\rho^*V^*} = \rho^*V^* \left[\left(\frac{E_{p_t}}{p_t} \right)^2 + \left(-\frac{E_{T_t}}{2T_t} \right)^2 \right]^{1/2} \quad (153)$$

$$E'_{\rho^*V^*}(T_t) = -\frac{\rho^*V^*}{2} \frac{E_{T_t}}{T_t} \quad (154)$$

$$E_{\mu_\infty} = \mu_\infty \left\{ \left[-\frac{(\gamma-1)M_\infty^2}{1 + \frac{\gamma-1}{2}M_\infty^2} \right]^2 \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{T_t}}{T_t} \right)^2 \right\}^{1/2} \quad (155)$$

$$E'_{\mu_\infty}(M_\infty) = -\frac{(\gamma-1)M_\infty^2\mu_\infty}{\left(1 + \frac{\gamma-1}{2}M_\infty^2\right)} \frac{E_{M_\infty}}{M_\infty} \quad (156)$$

$$E'_{\mu_\infty}(T_t) = \mu_\infty \frac{E_{T_t}}{T_t} \quad (157)$$

Evaluating the partial derivatives in Eq. (150) and substituting the error terms from Eqs. (151) through (157) gives

$$\begin{aligned} \left(\frac{E_{Re}}{Re} \right)^2 &= \left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \right)^2 \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 + \left(\frac{E_{T_t}}{2T_t} \right)^2 + \left(\frac{E_l}{l} \right)^2 \\ &+ \left[\frac{(\gamma-1)M_\infty^2}{1 + \frac{\gamma-1}{2}M_\infty^2} \right]^2 \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{T_t}}{T_t} \right)^2 \\ &+ 2 \left(\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} \right) \left[\frac{(\gamma-1)M_\infty^2}{1 + \frac{\gamma-1}{2}M_\infty^2} \right] \left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{T_t}}{T_t} \right)^2 \end{aligned}$$

For $R = \rho_\infty V_\infty / \rho^* V^*$, the expression for $(M_\infty/R)(\partial R/\partial M_\infty)$ is obtained by substituting Eq. (71) into Eq. (75),

$$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty} = \frac{1 - M_\infty^2}{1 + \frac{\gamma-1}{2}M_\infty^2}.$$

Substituting this equation into the previous one and simplifying gives,

$$E_{Re} = R_e \left\{ \left[\frac{1 + (\gamma - 2) M_\infty^2}{1 + \frac{\gamma - 1}{2} M_\infty^2} \right]^2 \left(\left(\frac{E_{M_\infty}}{M_\infty} \right)^2 + \left(\frac{E_{p_t}}{p_t} \right)^2 + \left(\frac{3 E_{T_t}}{2 T_t} \right)^2 + \left(\frac{E_l}{l} \right)^2 \right) \right\}^{1/2}.$$

This result agrees with Eq. (145), which was obtained by differentiating the fully expanded Reynolds number equation with independent variables.

4.6.1.3 Numerical evaluation

The numerical evaluation code, DFDX, which is listed in the Appendix, uses Reynolds number, with linear viscosity, as its example function. For a relative interval, $\Delta x_i / x_i$, of 0.005, four test cases were evaluated ($\gamma=1.40$) and the results are listed below:

Case	Parameter	i	x_i	$\frac{\partial Re / \partial x_i}{Re / x_i}$
1	M_∞	1	5.0	-2.333
	p_t , psia	2	150.0	1.000
	T_t , °R	3	800.0	-1.500
	l , ft	4	1.0	1.000
2	M_∞	1	8.0	-2.710
	p_t , psia	2	300.0	1.000
	T_t , °R	3	1500.0	-1.500
	l , ft	4	1.0	1.000
3	M_∞	1	8.0	-2.710
	p_t , psia	2	900.0	1.000
	T_t , °R	3	1100.0	-1.500
	l , ft	4	1.5	1.000
4	M_∞	1	14.0	-2.901
	p_t , psia	2	2000.0	1.000
	T_t , °R	3	2300.0	-1.500
	l , ft	4	1.0	1.000

These examples show several important results. First, the relative sensitivity of Reynolds number to Mach number agrees with the analytical solution presented in Table 10. Second, cases 2 and 3 show that the Mach number sensitivity is independent of p_t and T_t as

it should be. Finally, the relative sensitivity of Reynolds number to p_t and l is 1.0 and to T_t is -1.5 as was shown in Eq. (145).

4.6.2 Sutherland's viscosity relation

Although the equation for Reynolds number with Sutherland's viscosity relation, Eq. (137), could be expanded to the basic variables— M_∞ , p_t , T_t and l —and then differentiated to obtain sensitivity coefficients, the calculus and algebra would be formidable. In addition, because of the sum, $T_\infty + T_S$, in the expression for μ_∞ , it is not possible to uncouple the effects of M_∞ and T_t in the error equation, as was done for the linear relation. That is, the sensitivity of Re to M_∞ is a function of T_t and the sensitivity of Re to T_t is a function of M_∞ . Therefore, simple expressions for the sensitivity coefficients cannot be obtained and numerical evaluation of the coefficients is the best method. Relative sensitivity coefficients for Reynolds number are presented in Figures 13 and 14. In Figure 13, the sensitivity of Re to Mach number is shown. For the linear viscosity relation, the curve is the same as that shown in Figure 12. Although the trend with Mach number is similar for the linear and Sutherland's relations, there is a significant effect of T_t when Sutherland's viscosity relation is used. The curve for each value of T_t intersects the linear relation curve when $T_\infty = T_S = 198.7^\circ R$. The Reynolds number sensitivity to total temperature is constant for the linear relation, as shown in Eq. (145) and Figure 14. For Sutherland's viscosity relation, the sensitivity coefficient is strongly dependent on both T_t and M_∞ . As before, the curves intersect at $T_\infty = T_S$.

Concluding Remarks

Estimation of the uncertainty in results from high-speed wind tunnel tests is hindered by the need to evaluate the sensitivity coefficients, $\partial R / \partial M_\infty$, for various aerodynamic ratios, R . The ratios can be complex functions of M_∞ and calculating the partial derivative is a demanding task. To simplify the error estimation procedure, sensitivity coefficients were evaluated for nine fundamental aerodynamic ratios. In general, these ratios relate a free-stream test condition (pressure, temperature, density or velocity) to a reference (total or sonic) condition. Methods of applying these sensitivity coefficients in error estimation were demonstrated for several nondimensional ratios and coefficients which are used in high-speed testing. In addition, the sensitivity coefficients can be applied directly to estimating the error in calculated free-stream conditions. The presence of dependent or correlated variables in a calculated test result is frequently a challenge. Examples were given which compared two analytical methods for estimating the error when variables are dependent. Also, a numerical method which approximates the partial derivatives was discussed and an example code which implements this procedure was given.

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a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R) (\partial R / \partial M_\infty)$

Figure 1. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = p_\infty / p_t$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty/R) (\partial R / \partial M_\infty)$

Figure 2. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = q_\infty / p_t$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R) (\partial R / \partial M_\infty)$

Figure 3. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = q_\infty / p_{t_2}$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R)(\partial R / \partial M_\infty)$

Figure 4. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = p_{t_2} / p_{t_1}$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R)(\partial R / \partial M_\infty)$

Figure 5 Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = p_\infty / p_{t_2}$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_{\infty}$

b. $\theta'_{M_{\infty}} = (M_{\infty}/R) (\partial R / \partial M_{\infty})$

Figure 6. Variation of R , $\partial R / \partial M_{\infty}$, and $\theta'_{M_{\infty}}$ with M_{∞} for $R = A_{\infty} / A^*$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_{\infty}$

b. $\theta'_{M_{\infty}} = (M_{\infty}/R) (\partial R / \partial M_{\infty})$

Figure 7. Variation of R , $\partial R / \partial M_{\infty}$, and $\theta'_{M_{\infty}}$ with M_{∞} for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R) (\partial R / \partial M_\infty)$

Figure 8. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = T_\infty / T_l$ and $\gamma = 1.40$

a. R and $\partial R / \partial M_\infty$

b. $\theta'_{M_\infty} = (M_\infty / R)(\partial R / \partial M_\infty)$

Figure 9. Variation of R , $\partial R / \partial M_\infty$, and θ'_{M_∞} with M_∞ for $R = V_\infty / a_t$ and $\gamma = 1.40$

Figure 10. Dynamic viscosity regimes

Figure 11. Variation of Re/Re^* with M_∞ , linear viscosity relation

Figure 12. Variation of relative sensitivity coefficient for Reynolds number with M_∞ , linear viscosity relation

Figure 13. Variation of relative sensitivity coefficient for Reynolds number with M_∞ , Sutherland's viscosity relation

Figure 14. Variation of relative sensitivity coefficient for Reynolds number with T_t , Sutherland's viscosity relation

Table 1. Sensitivity coefficients for $R = p_{\infty}/p_t$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	9.930E-1	-1.387E-1	-1.397E-2	-7.207E+0	-7.157E+1
0.2	9.725E-1	-2.701E-1	-5.556E-2	-3.702E+0	-1.800E+1
0.3	9.395E-1	-3.876E-1	-1.238E-1	-2.580E+0	-8.079E+0
0.4	8.956E-1	-4.860E-1	-2.171E-1	-2.058E+0	-4.607E+0
0.5	8.430E-1	-5.620E-1	-3.333E-1	-1.779E+0	-3.000E+0
0.6	7.840E-1	-6.143E-1	-4.701E-1	-1.628E+0	-2.127E+0
0.7	7.209E-1	-6.435E-1	-6.248E-1	-1.554E+0	-1.601E+0
0.8	6.560E-1	-6.514E-1	-7.943E-1	-1.535E+0	-1.259E+0
0.9	5.913E-1	-6.411E-1	-9.759E-1	-1.560E+0	-1.025E+0
1.0	5.283E-1	-6.163E-1	-1.167E+0	-1.623E+0	-8.571E-1
1.1	4.684E-1	-5.807E-1	-1.364E+0	-1.722E+0	-7.332E-1
1.2	4.124E-1	-5.379E-1	-1.565E+0	-1.859E+0	-6.389E-1
1.3	3.609E-1	-4.909E-1	-1.768E+0	-2.037E+0	-5.655E-1
1.4	3.142E-1	-4.425E-1	-1.971E+0	-2.260E+0	-5.073E-1
1.5	2.724E-1	-3.945E-1	-2.172E+0	-2.535E+0	-4.603E-1
1.6	2.353E-1	-3.485E-1	-2.370E+0	-2.869E+0	-4.219E-1
1.7	2.026E-1	-3.056E-1	-2.564E+0	-3.273E+0	-3.900E-1
1.8	1.740E-1	-2.661E-1	-2.752E+0	-3.758E+0	-3.633E-1
1.9	1.492E-1	-2.305E-1	-2.935E+0	-4.338E+0	-3.407E-1
2.0	1.278E-1	-1.988E-1	-3.111E+0	-5.030E+0	-3.214E-1
2.1	1.094E-1	-1.708E-1	-3.281E+0	-5.854E+0	-3.048E-1
2.2	9.352E-2	-1.464E-1	-3.443E+0	-6.832E+0	-2.904E-1
2.3	7.997E-2	-1.251E-1	-3.599E+0	-7.992E+0	-2.779E-1
2.4	6.840E-2	-1.068E-1	-3.747E+0	-9.364E+0	-2.669E-1
2.5	5.853E-2	-9.104E-2	-3.889E+0	-1.098E+1	-2.571E-1
2.6	5.012E-2	-7.756E-2	-4.024E+0	-1.289E+1	-2.485E-1
2.7	4.295E-2	-6.605E-2	-4.152E+0	-1.514E+1	-2.408E-1
2.8	3.685E-2	-5.625E-2	-4.274E+0	-1.778E+1	-2.340E-1
2.9	3.165E-2	-4.791E-2	-4.390E+0	-2.087E+1	-2.278E-1
3.0	2.722E-2	-4.084E-2	-4.500E+0	-2.449E+1	-2.222E-1
3.1	2.345E-2	-3.483E-2	-4.604E+0	-2.871E+1	-2.172E-1
3.2	2.023E-2	-2.973E-2	-4.703E+0	-3.363E+1	-2.126E-1
3.3	1.748E-2	-2.541E-2	-4.797E+0	-3.936E+1	-2.084E-1
3.4	1.512E-2	-2.174E-2	-4.886E+0	-4.600E+1	-2.046E-1

Table 1. Sensitivity coefficients for $R = p_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	1.311E-2	-1.862E-2	-4.971E+0	-5.370E+1	-2.012E-1
3.6	1.135E-2	-1.597E-2	-5.051E+0	-6.260E+1	-1.980E-1
3.7	9.903E-3	-1.372E-2	-5.127E+0	-7.287E+1	-1.950E-1
3.8	8.629E-3	-1.181E-2	-5.200E+0	-8.469E+1	-1.923E-1
3.9	7.532E-3	-1.017E-2	-5.268E+0	-9.829E+1	-1.898E-1
4.0	6.586E-3	-8.781E-3	-5.333E+0	-1.139E+2	-1.875E-1
4.1	5.769E-3	-7.591E-3	-5.395E+0	-1.317E+2	-1.853E-1
4.2	5.062E-3	-6.574E-3	-5.454E+0	-1.521E+2	-1.833E-1
4.3	4.449E-3	-5.701E-3	-5.510E+0	-1.754E+2	-1.815E-1
4.4	3.918E-3	-4.953E-3	-5.563E+0	-2.019E+2	-1.798E-1
4.5	3.455E-3	-4.311E-3	-5.614E+0	-2.320E+2	-1.781E-1
4.6	3.053E-3	-3.757E-3	-5.662E+0	-2.661E+2	-1.766E-1
4.7	2.701E-3	-3.281E-3	-5.708E+0	-3.048E+2	-1.752E-1
4.8	2.394E-3	-2.869E-3	-5.752E+0	-3.486E+2	-1.739E-1
4.9	2.126E-3	-2.513E-3	-5.794E+0	-3.979E+2	-1.726E-1
5.0	1.890E-3	-2.205E-3	-5.833E+0	-4.535E+2	-1.714E-1
5.1	1.683E-3	-1.938E-3	-5.871E+0	-5.161E+2	-1.703E-1
5.2	1.501E-3	-1.706E-3	-5.908E+0	-5.863E+2	-1.693E-1
5.3	1.341E-3	-1.504E-3	-5.942E+0	-6.651E+2	-1.683E-1
5.4	1.200E-3	-1.328E-3	-5.975E+0	-7.533E+2	-1.674E-1
5.5	1.075E-3	-1.174E-3	-6.007E+0	-8.518E+2	-1.665E-1
5.6	9.643E-4	-1.040E-3	-6.037E+0	-9.619E+2	-1.656E-1
5.7	8.663E-4	-9.220E-4	-6.066E+0	-1.085E+3	-1.648E-1
5.8	7.794E-4	-8.189E-4	-6.094E+0	-1.221E+3	-1.641E-1
5.9	7.021E-4	-7.284E-4	-6.121E+0	-1.373E+3	-1.634E-1
6.0	6.334E-4	-6.488E-4	-6.146E+0	-1.541E+3	-1.627E-1
6.1	5.721E-4	-5.787E-4	-6.171E+0	-1.728E+3	-1.621E-1
6.2	5.173E-4	-5.169E-4	-6.194E+0	-1.935E+3	-1.614E-1
6.3	4.684E-4	-4.623E-4	-6.217E+0	-2.163E+3	-1.609E-1
6.4	4.247E-4	-4.140E-4	-6.238E+0	-2.416E+3	-1.603E-1
6.5	3.855E-4	-3.712E-4	-6.259E+0	-2.694E+3	-1.598E-1
6.6	3.503E-4	-3.333E-4	-6.279E+0	-3.001E+3	-1.593E-1
6.7	3.187E-4	-2.996E-4	-6.298E+0	-3.338E+3	-1.588E-1
6.8	2.902E-4	-2.696E-4	-6.317E+0	-3.709E+3	-1.583E-1
6.9	2.646E-4	-2.430E-4	-6.335E+0	-4.116E+3	-1.579E-1

Table 1. Sensitivity coefficients for $R = p_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	2.416E-4	-2.192E-4	-6.352E+0	-4.562E+3	-1.574E-1
7.1	2.207E-4	-1.980E-4	-6.368E+0	-5.051E+3	-1.570E-1
7.2	2.019E-4	-1.790E-4	-6.384E+0	-5.586E+3	-1.566E-1
7.3	1.848E-4	-1.620E-4	-6.400E+0	-6.171E+3	-1.563E-1
7.4	1.694E-4	-1.469E-4	-6.414E+0	-6.810E+3	-1.559E-1
7.5	1.554E-4	-1.332E-4	-6.429E+0	-7.506E+3	-1.556E-1
7.6	1.427E-4	-1.210E-4	-6.442E+0	-8.265E+3	-1.552E-1
7.7	1.312E-4	-1.100E-4	-6.456E+0	-9.092E+3	-1.549E-1
7.8	1.207E-4	-1.001E-4	-6.468E+0	-9.991E+3	-1.546E-1
7.9	1.111E-4	-9.117E-5	-6.481E+0	-1.097E+4	-1.543E-1
8.0	1.024E-4	-8.313E-5	-6.493E+0	-1.203E+4	-1.540E-1
8.1	9.449E-5	-7.587E-5	-6.504E+0	-1.318E+4	-1.537E-1
8.2	8.723E-5	-6.931E-5	-6.516E+0	-1.443E+4	-1.535E-1
8.3	8.060E-5	-6.338E-5	-6.526E+0	-1.578E+4	-1.532E-1
8.4	7.454E-5	-5.800E-5	-6.537E+0	-1.724E+4	-1.530E-1
8.5	6.898E-5	-5.313E-5	-6.547E+0	-1.882E+4	-1.527E-1
8.6	6.390E-5	-4.871E-5	-6.557E+0	-2.053E+4	-1.525E-1
8.7	5.923E-5	-4.470E-5	-6.566E+0	-2.237E+4	-1.523E-1
8.8	5.494E-5	-4.105E-5	-6.575E+0	-2.436E+4	-1.521E-1
8.9	5.101E-5	-3.774E-5	-6.584E+0	-2.650E+4	-1.519E-1
9.0	4.739E-5	-3.471E-5	-6.593E+0	-2.881E+4	-1.517E-1
9.1	4.405E-5	-3.196E-5	-6.601E+0	-3.129E+4	-1.515E-1
9.2	4.099E-5	-2.945E-5	-6.610E+0	-3.396E+4	-1.513E-1
9.3	3.816E-5	-2.715E-5	-6.617E+0	-3.683E+4	-1.511E-1
9.4	3.555E-5	-2.506E-5	-6.625E+0	-3.991E+4	-1.509E-1
9.5	3.314E-5	-2.314E-5	-6.633E+0	-4.322E+4	-1.508E-1
9.6	3.092E-5	-2.138E-5	-6.640E+0	-4.677E+4	-1.506E-1
9.7	2.886E-5	-1.978E-5	-6.647E+0	-5.057E+4	-1.504E-1
9.8	2.696E-5	-1.830E-5	-6.654E+0	-5.464E+4	-1.503E-1
9.9	2.520E-5	-1.695E-5	-6.660E+0	-5.900E+4	-1.501E-1
10.0	2.356E-5	-1.571E-5	-6.667E+0	-6.366E+4	-1.500E-1
10.1	2.205E-5	-1.457E-5	-6.673E+0	-6.864E+4	-1.499E-1
10.2	2.065E-5	-1.352E-5	-6.679E+0	-7.397E+4	-1.497E-1
10.3	1.934E-5	-1.255E-5	-6.685E+0	-7.965E+4	-1.496E-1
10.4	1.813E-5	-1.167E-5	-6.691E+0	-8.572E+4	-1.495E-1

Table 1. Sensitivity coefficients for $R = p_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	1.701E-5	-1.085E-5	-6.696E+0	-9.219E+4	-1.493E-1
10.6	1.596E-5	-1.009E-5	-6.702E+0	-9.909E+4	-1.492E-1
10.7	1.499E-5	-9.395E-6	-6.707E+0	-1.064E+5	-1.491E-1
10.8	1.408E-5	-8.751E-6	-6.712E+0	-1.143E+5	-1.490E-1
10.9	1.324E-5	-8.157E-6	-6.717E+0	-1.226E+5	-1.489E-1
11.0	1.245E-5	-7.607E-6	-6.722E+0	-1.315E+5	-1.488E-1
11.1	1.171E-5	-7.099E-6	-6.727E+0	-1.409E+5	-1.487E-1
11.2	1.103E-5	-6.628E-6	-6.732E+0	-1.509E+5	-1.486E-1
11.3	1.039E-5	-6.192E-6	-6.736E+0	-1.615E+5	-1.485E-1
11.4	9.788E-6	-5.787E-6	-6.741E+0	-1.728E+5	-1.484E-1
11.5	9.228E-6	-5.412E-6	-6.745E+0	-1.848E+5	-1.483E-1
11.6	8.704E-6	-5.064E-6	-6.749E+0	-1.975E+5	-1.482E-1
11.7	8.214E-6	-4.741E-6	-6.753E+0	-2.109E+5	-1.481E-1
11.8	7.755E-6	-4.441E-6	-6.757E+0	-2.252E+5	-1.480E-1
11.9	7.325E-6	-4.162E-6	-6.761E+0	-2.403E+5	-1.479E-1
12.0	6.922E-6	-3.902E-6	-6.765E+0	-2.563E+5	-1.478E-1
12.1	6.544E-6	-3.661E-6	-6.769E+0	-2.732E+5	-1.477E-1
12.2	6.189E-6	-3.436E-6	-6.772E+0	-2.910E+5	-1.477E-1
12.3	5.856E-6	-3.226E-6	-6.776E+0	-3.100E+5	-1.476E-1
12.4	5.544E-6	-3.031E-6	-6.780E+0	-3.299E+5	-1.475E-1
12.5	5.250E-6	-2.849E-6	-6.783E+0	-3.510E+5	-1.474E-1
12.6	4.974E-6	-2.679E-6	-6.786E+0	-3.733E+5	-1.474E-1
12.7	4.714E-6	-2.520E-6	-6.790E+0	-3.968E+5	-1.473E-1
12.8	4.469E-6	-2.372E-6	-6.793E+0	-4.216E+5	-1.472E-1
12.9	4.239E-6	-2.233E-6	-6.796E+0	-4.478E+5	-1.471E-1
13.0	4.022E-6	-2.104E-6	-6.799E+0	-4.754E+5	-1.471E-1
13.1	3.818E-6	-1.982E-6	-6.802E+0	-5.044E+5	-1.470E-1
13.2	3.626E-6	-1.869E-6	-6.805E+0	-5.350E+5	-1.470E-1
13.3	3.444E-6	-1.763E-6	-6.808E+0	-5.673E+5	-1.469E-1
13.4	3.273E-6	-1.663E-6	-6.810E+0	-6.012E+5	-1.468E-1
13.5	3.111E-6	-1.570E-6	-6.813E+0	-6.369E+5	-1.468E-1
13.6	2.959E-6	-1.483E-6	-6.816E+0	-6.744E+5	-1.467E-1
13.7	2.814E-6	-1.401E-6	-6.818E+0	-7.139E+5	-1.467E-1
13.8	2.678E-6	-1.324E-6	-6.821E+0	-7.554E+5	-1.466E-1
13.9	2.550E-6	-1.252E-6	-6.823E+0	-7.990E+5	-1.466E-1

Table 1. Sensitivity coefficients for $R = p_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	2.428E-6	-1.184E-6	-6.826E+0	-8.448E+5	-1.465E-1
14.1	2.313E-6	-1.120E-6	-6.828E+0	-8.929E+5	-1.464E-1
14.2	2.204E-6	-1.060E-6	-6.831E+0	-9.434E+5	-1.464E-1
14.3	2.101E-6	-1.004E-6	-6.833E+0	-9.963E+5	-1.464E-1
14.4	2.003E-6	-9.507E-7	-6.835E+0	-1.052E+6	-1.463E-1
14.5	1.910E-6	-9.008E-7	-6.837E+0	-1.110E+6	-1.463E-1
14.6	1.823E-6	-8.538E-7	-6.840E+0	-1.171E+6	-1.462E-1
14.7	1.739E-6	-8.096E-7	-6.842E+0	-1.235E+6	-1.462E-1
14.8	1.661E-6	-7.679E-7	-6.844E+0	-1.302E+6	-1.461E-1
14.9	1.586E-6	-7.286E-7	-6.846E+0	-1.373E+6	-1.461E-1
15.0	1.515E-6	-6.915E-7	-6.848E+0	-1.446E+6	-1.460E-1

Table 2. Sensitivity coefficients for $R = q_{\infty}/p_t$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	6.951E-3	1.381E-1	1.986E+0	7.244E+0	5.035E-1
0.2	2.723E-2	2.647E-1	1.944E+0	3.777E+0	5.143E-1
0.3	5.919E-2	3.702E-1	1.876E+0	2.702E+0	5.330E-1
0.4	1.003E-1	4.471E-1	1.783E+0	2.237E+0	5.609E-1
0.5	1.475E-1	4.918E-1	1.667E+0	2.034E+0	6.000E-1
0.6	1.976E-1	5.038E-1	1.530E+0	1.985E+0	6.537E-1
0.7	2.473E-1	4.858E-1	1.375E+0	2.058E+0	7.272E-1
0.8	2.939E-1	4.429E-1	1.206E+0	2.258E+0	8.294E-1
0.9	3.352E-1	3.815E-1	1.024E+0	2.621E+0	9.765E-1
1.0	3.698E-1	3.082E-1	8.333E-1	3.245E+0	1.200E+0
1.1	3.967E-1	2.294E-1	6.361E-1	4.359E+0	1.572E+0
1.2	4.157E-1	1.506E-1	4.348E-1	6.640E+0	2.300E+0
1.3	4.270E-1	7.609E-2	2.317E-1	1.314E+1	4.316E+0
1.4	4.311E-1	8.849E-3	2.874E-2	1.130E+2	3.480E+1
1.5	4.290E-1	-4.931E-2	-1.724E-1	-2.028E+1	-5.800E+0
1.6	4.216E-1	-9.759E-2	-3.704E-1	-1.025E+1	-2.700E+0
1.7	4.098E-1	-1.360E-1	-5.640E-1	-7.354E+0	-1.773E+0
1.8	3.947E-1	-1.650E-1	-7.524E-1	-6.061E+0	-1.329E+0
1.9	3.771E-1	-1.856E-1	-9.350E-1	-5.389E+0	-1.070E+0
2.0	3.579E-1	-1.988E-1	-1.111E+0	-5.030E+0	-9.000E-1
2.1	3.376E-1	-2.058E-1	-1.281E+0	-4.858E+0	-7.809E-1
2.2	3.169E-1	-2.078E-1	-1.443E+0	-4.811E+0	-6.930E-1
2.3	2.961E-1	-2.058E-1	-1.599E+0	-4.858E+0	-6.255E-1
2.4	2.758E-1	-2.008E-1	-1.747E+0	-4.981E+0	-5.723E-1
2.5	2.561E-1	-1.935E-1	-1.889E+0	-5.169E+0	-5.294E-1
2.6	2.371E-1	-1.846E-1	-2.024E+0	-5.417E+0	-4.941E-1
2.7	2.192E-1	-1.747E-1	-2.152E+0	-5.724E+0	-4.647E-1
2.8	2.022E-1	-1.642E-1	-2.274E+0	-6.088E+0	-4.397E-1
2.9	1.863E-1	-1.536E-1	-2.390E+0	-6.512E+0	-4.184E-1
3.0	1.715E-1	-1.429E-1	-2.500E+0	-6.997E+0	-4.000E-1
3.1	1.577E-1	-1.325E-1	-2.604E+0	-7.546E+0	-3.840E-1
3.2	1.450E-1	-1.225E-1	-2.703E+0	-8.164E+0	-3.699E-1
3.3	1.332E-1	-1.129E-1	-2.797E+0	-8.855E+0	-3.575E-1
3.4	1.224E-1	-1.039E-1	-2.886E+0	-9.624E+0	-3.464E-1

Table 2. Sensitivity coefficients for $R = q_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	1.124E-1	-9.543E-2	-2.971E+0	-1.048E+1	-3.366E-1
3.6	1.033E-1	-8.754E-2	-3.051E+0	-1.142E+1	-3.277E-1
3.7	9.490E-2	-8.021E-2	-3.127E+0	-1.247E+1	-3.198E-1
3.8	8.722E-2	-7.344E-2	-3.200E+0	-1.362E+1	-3.125E-1
3.9	8.019E-2	-6.720E-2	-3.268E+0	-1.488E+1	-3.060E-1
4.0	7.376E-2	-6.147E-2	-3.333E+0	-1.627E+1	-3.000E-1
4.1	6.788E-2	-5.621E-2	-3.395E+0	-1.779E+1	-2.945E-1
4.2	6.251E-2	-5.140E-2	-3.454E+0	-1.945E+1	-2.895E-1
4.3	5.759E-2	-4.701E-2	-3.510E+0	-2.127E+1	-2.849E-1
4.4	5.309E-2	-4.299E-2	-3.563E+0	-2.326E+1	-2.806E-1
4.5	4.898E-2	-3.933E-2	-3.614E+0	-2.542E+1	-2.767E-1
4.6	4.521E-2	-3.600E-2	-3.662E+0	-2.778E+1	-2.731E-1
4.7	4.177E-2	-3.295E-2	-3.708E+0	-3.035E+1	-2.697E-1
4.8	3.861E-2	-3.018E-2	-3.752E+0	-3.313E+1	-2.665E-1
4.9	3.572E-2	-2.766E-2	-3.794E+0	-3.616E+1	-2.636E-1
5.0	3.308E-2	-2.536E-2	-3.833E+0	-3.944E+1	-2.609E-1
5.1	3.065E-2	-2.326E-2	-3.871E+0	-4.299E+1	-2.583E-1
5.2	2.842E-2	-2.135E-2	-3.908E+0	-4.683E+1	-2.559E-1
5.3	2.637E-2	-1.961E-2	-3.942E+0	-5.098E+1	-2.537E-1
5.4	2.449E-2	-1.803E-2	-3.975E+0	-5.547E+1	-2.515E-1
5.5	2.276E-2	-1.658E-2	-4.007E+0	-6.031E+1	-2.496E-1
5.6	2.117E-2	-1.526E-2	-4.037E+0	-6.552E+1	-2.477E-1
5.7	1.970E-2	-1.406E-2	-4.066E+0	-7.114E+1	-2.459E-1
5.8	1.835E-2	-1.296E-2	-4.094E+0	-7.719E+1	-2.442E-1
5.9	1.711E-2	-1.195E-2	-4.121E+0	-8.368E+1	-2.427E-1
6.0	1.596E-2	-1.103E-2	-4.146E+0	-9.066E+1	-2.412E-1
6.1	1.490E-2	-1.019E-2	-4.171E+0	-9.815E+1	-2.398E-1
6.2	1.392E-2	-9.417E-3	-4.194E+0	-1.062E+2	-2.384E-1
6.3	1.301E-2	-8.711E-3	-4.217E+0	-1.148E+2	-2.371E-1
6.4	1.218E-2	-8.064E-3	-4.238E+0	-1.240E+2	-2.359E-1
6.5	1.140E-2	-7.470E-3	-4.259E+0	-1.339E+2	-2.348E-1
6.6	1.068E-2	-6.925E-3	-4.279E+0	-1.444E+2	-2.337E-1
6.7	1.001E-2	-6.424E-3	-4.298E+0	-1.557E+2	-2.326E-1
6.8	9.395E-3	-5.964E-3	-4.317E+0	-1.677E+2	-2.316E-1
6.9	8.820E-3	-5.541E-3	-4.335E+0	-1.805E+2	-2.307E-1

Table 2. Sensitivity coefficients for $R = q_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	8.285E-3	-5.151E-3	-4.352E+0	-1.941E+2	-2.298E-1
7.1	7.788E-3	-4.792E-3	-4.368E+0	-2.087E+2	-2.289E-1
7.2	7.326E-3	-4.461E-3	-4.384E+0	-2.242E+2	-2.281E-1
7.3	6.895E-3	-4.156E-3	-4.400E+0	-2.406E+2	-2.273E-1
7.4	6.494E-3	-3.874E-3	-4.414E+0	-2.581E+2	-2.265E-1
7.5	6.120E-3	-3.614E-3	-4.429E+0	-2.767E+2	-2.258E-1
7.6	5.771E-3	-3.373E-3	-4.442E+0	-2.965E+2	-2.251E-1
7.7	5.445E-3	-3.151E-3	-4.456E+0	-3.174E+2	-2.244E-1
7.8	5.140E-3	-2.945E-3	-4.468E+0	-3.396E+2	-2.238E-1
7.9	4.855E-3	-2.754E-3	-4.481E+0	-3.631E+2	-2.232E-1
8.0	4.589E-3	-2.577E-3	-4.493E+0	-3.880E+2	-2.226E-1
8.1	4.339E-3	-2.413E-3	-4.504E+0	-4.144E+2	-2.220E-1
8.2	4.106E-3	-2.261E-3	-4.516E+0	-4.423E+2	-2.215E-1
8.3	3.887E-3	-2.120E-3	-4.526E+0	-4.718E+2	-2.209E-1
8.4	3.682E-3	-1.988E-3	-4.537E+0	-5.029E+2	-2.204E-1
8.5	3.489E-3	-1.866E-3	-4.547E+0	-5.358E+2	-2.199E-1
8.6	3.308E-3	-1.753E-3	-4.557E+0	-5.705E+2	-2.195E-1
8.7	3.138E-3	-1.647E-3	-4.566E+0	-6.072E+2	-2.190E-1
8.8	2.978E-3	-1.549E-3	-4.575E+0	-6.458E+2	-2.186E-1
8.9	2.828E-3	-1.457E-3	-4.584E+0	-6.865E+2	-2.181E-1
9.0	2.687E-3	-1.371E-3	-4.593E+0	-7.293E+2	-2.177E-1
9.1	2.554E-3	-1.291E-3	-4.601E+0	-7.744E+2	-2.173E-1
9.2	2.428E-3	-1.217E-3	-4.610E+0	-8.219E+2	-2.169E-1
9.3	2.310E-3	-1.147E-3	-4.617E+0	-8.718E+2	-2.166E-1
9.4	2.199E-3	-1.082E-3	-4.625E+0	-9.243E+2	-2.162E-1
9.5	2.094E-3	-1.021E-3	-4.633E+0	-9.795E+2	-2.159E-1
9.6	1.994E-3	-9.639E-4	-4.640E+0	-1.037E+3	-2.155E-1
9.7	1.901E-3	-9.106E-4	-4.647E+0	-1.098E+3	-2.152E-1
9.8	1.812E-3	-8.606E-4	-4.654E+0	-1.162E+3	-2.149E-1
9.9	1.729E-3	-8.137E-4	-4.660E+0	-1.229E+3	-2.146E-1
10.0	1.649E-3	-7.697E-4	-4.667E+0	-1.299E+3	-2.143E-1
10.1	1.575E-3	-7.285E-4	-4.673E+0	-1.373E+3	-2.140E-1
10.2	1.504E-3	-6.898E-4	-4.679E+0	-1.450E+3	-2.137E-1
10.3	1.436E-3	-6.534E-4	-4.685E+0	-1.530E+3	-2.134E-1
10.4	1.373E-3	-6.192E-4	-4.691E+0	-1.615E+3	-2.132E-1

Table 2. Sensitivity coefficients for $R = q_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	1.313E-3	-5.871E-4	-4.696E+0	-1.703E+3	-2.129E-1
10.6	1.255E-3	-5.569E-4	-4.702E+0	-1.796E+3	-2.127E-1
10.7	1.201E-3	-5.284E-4	-4.707E+0	-1.892E+3	-2.124E-1
10.8	1.150E-3	-5.016E-4	-4.712E+0	-1.994E+3	-2.122E-1
10.9	1.101E-3	-4.764E-4	-4.717E+0	-2.099E+3	-2.120E-1
11.0	1.054E-3	-4.526E-4	-4.722E+0	-2.209E+3	-2.118E-1
11.1	1.010E-3	-4.302E-4	-4.727E+0	-2.324E+3	-2.116E-1
11.2	9.683E-4	-4.091E-4	-4.732E+0	-2.445E+3	-2.113E-1
11.3	9.284E-4	-3.891E-4	-4.736E+0	-2.570E+3	-2.111E-1
11.4	8.904E-4	-3.703E-4	-4.741E+0	-2.701E+3	-2.109E-1
11.5	8.543E-4	-3.525E-4	-4.745E+0	-2.837E+3	-2.107E-1
11.6	8.199E-4	-3.357E-4	-4.749E+0	-2.979E+3	-2.106E-1
11.7	7.871E-4	-3.198E-4	-4.753E+0	-3.127E+3	-2.104E-1
11.8	7.559E-4	-3.047E-4	-4.757E+0	-3.281E+3	-2.102E-1
11.9	7.261E-4	-2.905E-4	-4.761E+0	-3.442E+3	-2.100E-1
12.0	6.978E-4	-2.771E-4	-4.765E+0	-3.609E+3	-2.099E-1
12.1	6.707E-4	-2.643E-4	-4.769E+0	-3.783E+3	-2.097E-1
12.2	6.449E-4	-2.523E-4	-4.772E+0	-3.964E+3	-2.095E-1
12.3	6.202E-4	-2.408E-4	-4.776E+0	-4.152E+3	-2.094E-1
12.4	5.967E-4	-2.300E-4	-4.780E+0	-4.348E+3	-2.092E-1
12.5	5.742E-4	-2.197E-4	-4.783E+0	-4.551E+3	-2.091E-1
12.6	5.527E-4	-2.100E-4	-4.786E+0	-4.763E+3	-2.089E-1
12.7	5.322E-4	-2.007E-4	-4.790E+0	-4.982E+3	-2.088E-1
12.8	5.126E-4	-1.919E-4	-4.793E+0	-5.211E+3	-2.087E-1
12.9	4.938E-4	-1.836E-4	-4.796E+0	-5.447E+3	-2.085E-1
13.0	4.758E-4	-1.757E-4	-4.799E+0	-5.693E+3	-2.084E-1
13.1	4.587E-4	-1.681E-4	-4.802E+0	-5.948E+3	-2.083E-1
13.2	4.422E-4	-1.610E-4	-4.805E+0	-6.213E+3	-2.081E-1
13.3	4.264E-4	-1.541E-4	-4.808E+0	-6.487E+3	-2.080E-1
13.4	4.114E-4	-1.477E-4	-4.810E+0	-6.772E+3	-2.079E-1
13.5	3.969E-4	-1.415E-4	-4.813E+0	-7.067E+3	-2.078E-1
13.6	3.830E-4	-1.356E-4	-4.816E+0	-7.373E+3	-2.077E-1
13.7	3.698E-4	-1.300E-4	-4.818E+0	-7.689E+3	-2.075E-1
13.8	3.570E-4	-1.247E-4	-4.821E+0	-8.018E+3	-2.074E-1
13.9	3.448E-4	-1.197E-4	-4.823E+0	-8.358E+3	-2.073E-1

Table 2. Sensitivity coefficients for $R = q_{\infty}/p_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	3.331E-4	-1.148E-4	-4.826E+0	-8.709E+3	-2.072E-1
14.1	3.218E-4	-1.102E-4	-4.828E+0	-9.074E+3	-2.071E-1
14.2	3.110E-4	-1.058E-4	-4.831E+0	-9.451E+3	-2.070E-1
14.3	3.007E-4	-1.016E-4	-4.833E+0	-9.841E+3	-2.069E-1
14.4	2.907E-4	-9.761E-5	-4.835E+0	-1.024E+4	-2.068E-1
14.5	2.811E-4	-9.379E-5	-4.837E+0	-1.066E+4	-2.067E-1
14.6	2.719E-4	-9.014E-5	-4.840E+0	-1.109E+4	-2.066E-1
14.7	2.631E-4	-8.666E-5	-4.842E+0	-1.154E+4	-2.065E-1
14.8	2.546E-4	-8.333E-5	-4.844E+0	-1.200E+4	-2.065E-1
14.9	2.464E-4	-8.015E-5	-4.846E+0	-1.248E+4	-2.064E-1
15.0	2.386E-4	-7.711E-5	-4.848E+0	-1.297E+4	-2.063E-1

Table 3. Sensitivity coefficients for $R = q_{\infty}/p_{t_2}$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	6.951E-3	1.381E-1	1.986E+0	7.244E+0	5.035E-1
0.2	2.723E-2	2.647E-1	1.944E+0	3.777E+0	5.143E-1
0.3	5.919E-2	3.702E-1	1.876E+0	2.702E+0	5.330E-1
0.4	1.003E-1	4.471E-1	1.783E+0	2.237E+0	5.609E-1
0.5	1.475E-1	4.918E-1	1.667E+0	2.034E+0	6.000E-1
0.6	1.976E-1	5.038E-1	1.530E+0	1.985E+0	6.537E-1
0.7	2.473E-1	4.858E-1	1.375E+0	2.058E+0	7.272E-1
0.8	2.939E-1	4.429E-1	1.206E+0	2.258E+0	8.294E-1
0.9	3.352E-1	3.815E-1	1.024E+0	2.621E+0	9.765E-1
1.0	3.698E-1	3.082E-1	8.333E-1	3.245E+0	1.200E+0
1.1	3.971E-1	2.416E-1	6.693E-1	4.138E+0	1.494E+0
1.2	4.187E-1	1.921E-1	5.507E-1	5.205E+0	1.816E+0
1.3	4.360E-1	1.548E-1	4.617E-1	6.459E+0	2.166E+0
1.4	4.499E-1	1.263E-1	3.931E-1	7.916E+0	2.544E+0
1.5	4.614E-1	1.043E-1	3.390E-1	9.590E+0	2.950E+0
1.6	4.710E-1	8.698E-2	2.955E-1	1.150E+1	3.384E+0
1.7	4.789E-1	7.325E-2	2.600E-1	1.365E+1	3.846E+0
1.8	4.857E-1	6.223E-2	2.306E-1	1.607E+1	4.336E+0
1.9	4.915E-1	5.329E-2	2.060E-1	1.877E+1	4.854E+0
2.0	4.964E-1	4.596E-2	1.852E-1	2.176E+1	5.400E+0
2.1	5.007E-1	3.991E-2	1.674E-1	2.506E+1	5.974E+0
2.2	5.044E-1	3.487E-2	1.521E-1	2.868E+1	6.576E+0
2.3	5.077E-1	3.063E-2	1.388E-1	3.264E+1	7.206E+0
2.4	5.106E-1	2.705E-2	1.272E-1	3.697E+1	7.864E+0
2.5	5.131E-1	2.401E-2	1.170E-1	4.166E+1	8.550E+0
2.6	5.154E-1	2.140E-2	1.079E-1	4.673E+1	9.264E+0
2.7	5.174E-1	1.915E-2	9.994E-2	5.221E+1	1.001E+1
2.8	5.192E-1	1.721E-2	9.280E-2	5.811E+1	1.078E+1
2.9	5.209E-1	1.552E-2	8.640E-2	6.444E+1	1.157E+1
3.0	5.223E-1	1.404E-2	8.065E-2	7.122E+1	1.240E+1
3.1	5.237E-1	1.275E-2	7.545E-2	7.846E+1	1.325E+1
3.2	5.249E-1	1.160E-2	7.074E-2	8.618E+1	1.414E+1
3.3	5.260E-1	1.059E-2	6.646E-2	9.439E+1	1.505E+1
3.4	5.270E-1	9.698E-3	6.256E-2	1.031E+2	1.598E+1

Table 3. Sensitivity coefficients for $R = q_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	5.280E-1	8.899E-3	5.900E-2	1.124E+2	1.695E+1
3.6	5.288E-1	8.186E-3	5.573E-2	1.222E+2	1.794E+1
3.7	5.296E-1	7.547E-3	5.273E-2	1.325E+2	1.897E+1
3.8	5.303E-1	6.972E-3	4.996E-2	1.434E+2	2.002E+1
3.9	5.310E-1	6.454E-3	4.741E-2	1.549E+2	2.109E+1
4.0	5.316E-1	5.987E-3	4.505E-2	1.670E+2	2.220E+1
4.1	5.322E-1	5.563E-3	4.286E-2	1.798E+2	2.333E+1
4.2	5.327E-1	5.178E-3	4.082E-2	1.931E+2	2.450E+1
4.3	5.332E-1	4.828E-3	3.893E-2	2.071E+2	2.569E+1
4.4	5.337E-1	4.508E-3	3.717E-2	2.218E+2	2.690E+1
4.5	5.341E-1	4.217E-3	3.552E-2	2.372E+2	2.815E+1
4.6	5.345E-1	3.949E-3	3.399E-2	2.532E+2	2.942E+1
4.7	5.349E-1	3.704E-3	3.255E-2	2.700E+2	3.073E+1
4.8	5.353E-1	3.479E-3	3.120E-2	2.875E+2	3.206E+1
4.9	5.356E-1	3.271E-3	2.993E-2	3.057E+2	3.341E+1
5.0	5.359E-1	3.080E-3	2.874E-2	3.247E+2	3.480E+1
5.1	5.362E-1	2.903E-3	2.761E-2	3.444E+2	3.621E+1
5.2	5.365E-1	2.740E-3	2.656E-2	3.650E+2	3.766E+1
5.3	5.368E-1	2.589E-3	2.556E-2	3.863E+2	3.913E+1
5.4	5.370E-1	2.448E-3	2.462E-2	4.085E+2	4.062E+1
5.5	5.373E-1	2.318E-3	2.372E-2	4.315E+2	4.215E+1
5.6	5.375E-1	2.196E-3	2.288E-2	4.553E+2	4.370E+1
5.7	5.377E-1	2.083E-3	2.208E-2	4.801E+2	4.529E+1
5.8	5.379E-1	1.978E-3	2.132E-2	5.057E+2	4.690E+1
5.9	5.381E-1	1.879E-3	2.060E-2	5.321E+2	4.853E+1
6.0	5.383E-1	1.787E-3	1.992E-2	5.596E+2	5.020E+1
6.1	5.385E-1	1.701E-3	1.927E-2	5.879E+2	5.189E+1
6.2	5.386E-1	1.620E-3	1.865E-2	6.172E+2	5.362E+1
6.3	5.388E-1	1.545E-3	1.806E-2	6.474E+2	5.537E+1
6.4	5.389E-1	1.474E-3	1.750E-2	6.786E+2	5.714E+1
6.5	5.391E-1	1.407E-3	1.696E-2	7.108E+2	5.895E+1
6.6	5.392E-1	1.344E-3	1.645E-2	7.440E+2	6.078E+1
6.7	5.393E-1	1.285E-3	1.596E-2	7.782E+2	6.265E+1
6.8	5.395E-1	1.229E-3	1.550E-2	8.135E+2	6.454E+1
6.9	5.396E-1	1.177E-3	1.505E-2	8.498E+2	6.645E+1

Table 3. Sensitivity coefficients for $R = q_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	5.397E-1	1.127E-3	1.462E-2	8.871E+2	6.840E+1
7.1	5.398E-1	1.080E-3	1.421E-2	9.256E+2	7.037E+1
7.2	5.399E-1	1.036E-3	1.382E-2	9.651E+2	7.238E+1
7.3	5.400E-1	9.942E-4	1.344E-2	1.006E+3	7.441E+1
7.4	5.401E-1	9.546E-4	1.308E-2	1.048E+3	7.646E+1
7.5	5.402E-1	9.170E-4	1.273E-2	1.091E+3	7.855E+1
7.6	5.403E-1	8.814E-4	1.240E-2	1.135E+3	8.066E+1
7.7	5.404E-1	8.475E-4	1.208E-2	1.180E+3	8.281E+1
7.8	5.405E-1	8.154E-4	1.177E-2	1.226E+3	8.498E+1
7.9	5.406E-1	7.849E-4	1.147E-2	1.274E+3	8.717E+1
8.0	5.406E-1	7.559E-4	1.119E-2	1.323E+3	8.940E+1
8.1	5.407E-1	7.283E-4	1.091E-2	1.373E+3	9.165E+1
8.2	5.408E-1	7.021E-4	1.065E-2	1.424E+3	9.394E+1
8.3	5.408E-1	6.770E-4	1.039E-2	1.477E+3	9.625E+1
8.4	5.409E-1	6.532E-4	1.014E-2	1.531E+3	9.858E+1
8.5	5.410E-1	6.305E-4	9.906E-3	1.586E+3	1.010E+2
8.6	5.410E-1	6.088E-4	9.676E-3	1.643E+3	1.033E+2
8.7	5.411E-1	5.880E-4	9.455E-3	1.701E+3	1.058E+2
8.8	5.412E-1	5.683E-4	9.241E-3	1.760E+3	1.082E+2
8.9	5.412E-1	5.494E-4	9.034E-3	1.820E+3	1.107E+2
9.0	5.413E-1	5.313E-4	8.834E-3	1.882E+3	1.132E+2
9.1	5.413E-1	5.140E-4	8.641E-3	1.946E+3	1.157E+2
9.2	5.414E-1	4.974E-4	8.453E-3	2.010E+3	1.183E+2
9.3	5.414E-1	4.816E-4	8.272E-3	2.076E+3	1.209E+2
9.4	5.415E-1	4.664E-4	8.097E-3	2.144E+3	1.235E+2
9.5	5.415E-1	4.519E-4	7.927E-3	2.213E+3	1.262E+2
9.6	5.416E-1	4.379E-4	7.763E-3	2.284E+3	1.288E+2
9.7	5.416E-1	4.245E-4	7.603E-3	2.356E+3	1.315E+2
9.8	5.416E-1	4.117E-4	7.448E-3	2.429E+3	1.343E+2
9.9	5.417E-1	3.993E-4	7.299E-3	2.504E+3	1.370E+2
10.0	5.417E-1	3.875E-4	7.153E-3	2.581E+3	1.398E+2
10.1	5.418E-1	3.761E-4	7.012E-3	2.659E+3	1.426E+2
10.2	5.418E-1	3.652E-4	6.875E-3	2.738E+3	1.455E+2
10.3	5.418E-1	3.547E-4	6.742E-3	2.820E+3	1.483E+2
10.4	5.419E-1	3.445E-4	6.613E-3	2.902E+3	1.512E+2

Table 3. Sensitivity coefficients for $R = q_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	5.419E-1	3.348E-4	6.487E-3	2.987E+3	1.542E+2
10.6	5.419E-1	3.254E-4	6.365E-3	3.073E+3	1.571E+2
10.7	5.420E-1	3.164E-4	6.247E-3	3.161E+3	1.601E+2
10.8	5.420E-1	3.077E-4	6.131E-3	3.250E+3	1.631E+2
10.9	5.420E-1	2.993E-4	6.019E-3	3.341E+3	1.661E+2
11.0	5.421E-1	2.912E-4	5.910E-3	3.434E+3	1.692E+2
11.1	5.421E-1	2.835E-4	5.804E-3	3.528E+3	1.723E+2
11.2	5.421E-1	2.759E-4	5.701E-3	3.624E+3	1.754E+2
11.3	5.421E-1	2.687E-4	5.600E-3	3.722E+3	1.786E+2
11.4	5.422E-1	2.617E-4	5.502E-3	3.821E+3	1.817E+2
11.5	5.422E-1	2.549E-4	5.407E-3	3.923E+3	1.850E+2
11.6	5.422E-1	2.484E-4	5.314E-3	4.026E+3	1.882E+2
11.7	5.422E-1	2.421E-4	5.223E-3	4.131E+3	1.914E+2
11.8	5.423E-1	2.360E-4	5.135E-3	4.238E+3	1.947E+2
11.9	5.423E-1	2.301E-4	5.049E-3	4.346E+3	1.981E+2
12.0	5.423E-1	2.244E-4	4.965E-3	4.456E+3	2.014E+2
12.1	5.423E-1	2.189E-4	4.883E-3	4.569E+3	2.048E+2
12.2	5.424E-1	2.135E-4	4.804E-3	4.683E+3	2.082E+2
12.3	5.424E-1	2.084E-4	4.726E-3	4.799E+3	2.116E+2
12.4	5.424E-1	2.034E-4	4.650E-3	4.917E+3	2.151E+2
12.5	5.424E-1	1.986E-4	4.576E-3	5.036E+3	2.186E+2
12.6	5.424E-1	1.939E-4	4.503E-3	5.158E+3	2.221E+2
12.7	5.425E-1	1.893E-4	4.433E-3	5.282E+3	2.256E+2
12.8	5.425E-1	1.849E-4	4.363E-3	5.407E+3	2.292E+2
12.9	5.425E-1	1.807E-4	4.296E-3	5.535E+3	2.328E+2
13.0	5.425E-1	1.765E-4	4.230E-3	5.665E+3	2.364E+2
13.1	5.425E-1	1.725E-4	4.166E-3	5.796E+3	2.401E+2
13.2	5.426E-1	1.686E-4	4.103E-3	5.930E+3	2.437E+2
13.3	5.426E-1	1.649E-4	4.041E-3	6.066E+3	2.474E+2
13.4	5.426E-1	1.612E-4	3.981E-3	6.203E+3	2.512E+2
13.5	5.426E-1	1.576E-4	3.922E-3	6.343E+3	2.550E+2
13.6	5.426E-1	1.542E-4	3.865E-3	6.485E+3	2.587E+2
13.7	5.426E-1	1.508E-4	3.809E-3	6.629E+3	2.626E+2
13.8	5.426E-1	1.476E-4	3.754E-3	6.775E+3	2.664E+2
13.9	5.427E-1	1.444E-4	3.700E-3	6.923E+3	2.703E+2

Table 3. Sensitivity coefficients for $R = q_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	5.427E-1	1.414E-4	3.647E-3	7.074E+3	2.742E+2
14.1	5.427E-1	1.384E-4	3.595E-3	7.226E+3	2.781E+2
14.2	5.427E-1	1.355E-4	3.545E-3	7.381E+3	2.821E+2
14.3	5.427E-1	1.327E-4	3.495E-3	7.538E+3	2.861E+2
14.4	5.427E-1	1.299E-4	3.447E-3	7.697E+3	2.901E+2
14.5	5.427E-1	1.272E-4	3.400E-3	7.859E+3	2.942E+2
14.6	5.428E-1	1.247E-4	3.353E-3	8.022E+3	2.982E+2
14.7	5.428E-1	1.221E-4	3.308E-3	8.188E+3	3.023E+2
14.8	5.428E-1	1.197E-4	3.263E-3	8.356E+3	3.065E+2
14.9	5.428E-1	1.173E-4	3.219E-3	8.527E+3	3.106E+2
15.0	5.428E-1	1.150E-4	3.177E-3	8.699E+3	3.148E+2

Table 4. Sensitivity coefficients for $R = p_{t_2}/p_{t_1}$
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
0.1	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.2	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.3	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.4	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.5	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.6	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.7	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.8	1.000E+0	0.000E+0	0.000E+0	∞	∞
0.9	1.000E+0	0.000E+0	0.000E+0	∞	∞
1.0	1.000E+0	0.000E+0	0.000E+0	∞	∞
1.1	9.989E-1	-3.022E-2	-3.327E-2	-3.309E+1	-3.005E+1
1.2	9.973E-1	-9.587E-2	-1.159E-1	-1.043E+1	-8.630E+0
1.3	9.794E-1	-1.733E-1	-2.300E-1	-5.771E+0	-4.348E+0
1.4	9.582E-1	-2.494E-1	-3.643E-1	-4.010E+0	-2.745E+0
1.5	9.298E-1	-3.170E-1	-5.114E-1	-3.155E+0	-1.955E+0
1.6	8.952E-1	-3.726E-1	-6.659E-1	-2.684E+0	-1.502E+0
1.7	8.557E-1	-4.148E-1	-8.240E-1	-2.411E+0	-1.214E+0
1.8	8.127E-1	-4.438E-1	-9.831E-1	-2.253E+0	-1.017E+0
1.9	7.674E-1	-4.608E-1	-1.141E+0	-2.170E+0	-8.764E-1
2.0	7.209E-1	-4.672E-1	-1.296E+0	-2.140E+0	-7.714E-1
2.1	6.742E-1	-4.649E-1	-1.448E+0	-2.151E+0	-6.906E-1
2.2	6.281E-1	-4.554E-1	-1.595E+0	-2.196E+0	-6.269E-1
2.3	5.833E-1	-4.406E-1	-1.737E+0	-2.270E+0	-5.756E-1
2.4	5.401E-1	-4.218E-1	-1.874E+0	-2.371E+0	-5.335E-1
2.5	4.990E-1	-4.004E-1	-2.006E+0	-2.498E+0	-4.985E-1
2.6	4.601E-1	-3.773E-1	-2.132E+0	-2.651E+0	-4.691E-1
2.7	4.236E-1	-3.533E-1	-2.252E+0	-2.830E+0	-4.440E-1
2.8	3.895E-1	-3.292E-1	-2.367E+0	-3.037E+0	-4.225E-1
2.9	3.577E-1	-3.055E-1	-2.476E+0	-3.274E+0	-4.038E-1
3.0	3.283E-1	-2.824E-1	-2.581E+0	-3.540E+0	-3.875E-1
3.1	3.012E-1	-2.604E-1	-2.680E+0	-3.840E+0	-3.732E-1
3.2	2.762E-1	-2.395E-1	-2.774E+0	-4.176E+0	-3.605E-1
3.3	2.537E-1	-2.198E-1	-2.864E+0	-4.550E+0	-3.492E-1
3.4	2.322E-1	-2.014E-1	-2.949E+0	-4.965E+0	-3.391E-1

Table 4. Sensitivity coefficients for $R = p_{t_2}/p_{t_1}$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
3.5	2.129E-1	-1.844E-1	-3.030E+0	-5.424E+0	-3.300E-1
3.6	1.953E-1	-1.686E-1	-3.107E+0	-5.933E+0	-3.219E-1
3.7	1.792E-1	-1.540E-1	-3.180E+0	-6.493E+0	-3.145E-1
3.8	1.645E-1	-1.406E-1	-3.250E+0	-7.110E+0	-3.077E-1
3.9	1.510E-1	-1.284E-1	-3.316E+0	-7.788E+0	-3.016E-1
4.0	1.388E-1	-1.172E-1	-3.378E+0	-8.533E+0	-2.960E-1
4.1	1.276E-1	-1.070E-1	-3.438E+0	-9.349E+0	-2.909E-1
4.2	1.173E-1	-9.764E-2	-3.495E+0	-1.024E+1	-2.861E-1
4.3	1.080E-1	-8.914E-2	-3.549E+0	-1.122E+1	-2.818E-1
4.4	9.948E-2	-8.140E-2	-3.600E+0	-1.228E+1	-2.777E-1
4.5	9.170E-2	-7.436E-2	-3.649E+0	-1.345E+1	-2.740E-1
4.6	8.459E-2	-6.796E-2	-3.696E+0	-1.471E+1	-2.706E-1
4.7	7.809E-2	-6.215E-2	-3.741E+0	-1.609E+1	-2.673E-1
4.8	7.214E-2	-5.685E-2	-3.783E+0	-1.759E+1	-2.643E-1
4.9	6.670E-2	-5.204E-2	-3.823E+0	-1.921E+1	-2.615E-1
5.0	6.172E-2	-4.767E-2	-3.862E+0	-2.098E+1	-2.589E-1
5.1	5.715E-2	-4.369E-2	-3.899E+0	-2.289E+1	-2.565E-1
5.2	5.297E-2	-4.007E-2	-3.934E+0	-2.495E+1	-2.542E-1
5.3	4.913E-2	-3.678E-2	-3.968E+0	-2.719E+1	-2.520E-1
5.4	4.560E-2	-3.378E-2	-4.000E+0	-2.960E+1	-2.500E-1
5.5	4.236E-2	-3.105E-2	-4.031E+0	-3.221E+1	-2.481E-1
5.6	3.938E-2	-2.855E-2	-4.060E+0	-3.502E+1	-2.463E-1
5.7	3.664E-2	-2.628E-2	-4.089E+0	-3.805E+1	-2.446E-1
5.8	3.412E-2	-2.421E-2	-4.116E+0	-4.130E+1	-2.430E-1
5.9	3.179E-2	-2.232E-2	-4.141E+0	-4.481E+1	-2.415E-1
6.0	2.965E-2	-2.059E-2	-4.166E+0	-4.857E+1	-2.400E-1
6.1	2.767E-2	-1.901E-2	-4.190E+0	-5.261E+1	-2.387E-1
6.2	2.584E-2	-1.756E-2	-4.213E+0	-5.694E+1	-2.374E-1
6.3	2.416E-2	-1.624E-2	-4.235E+0	-6.159E+1	-2.361E-1
6.4	2.259E-2	-1.502E-2	-4.256E+0	-6.656E+1	-2.350E-1
6.5	2.115E-2	-1.391E-2	-4.276E+0	-7.188E+1	-2.339E-1
6.6	1.981E-2	-1.289E-2	-4.296E+0	-7.757E+1	-2.328E-1
6.7	1.857E-2	-1.196E-2	-4.314E+0	-8.364E+1	-2.318E-1
6.8	1.741E-2	-1.110E-2	-4.332E+0	-9.013E+1	-2.308E-1
6.9	1.634E-2	-1.030E-2	-4.350E+0	-9.705E+1	-2.299E-1

Table 4. Sensitivity coefficients for $R = p_{t_2}/p_{t_1}$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
7.0	1.535E-2	-9.576E-3	-4.366E+0	-1.044E+2	-2.290E-1
7.1	1.443E-2	-8.906E-3	-4.383E+0	-1.123E+2	-2.282E-1
7.2	1.357E-2	-8.288E-3	-4.398E+0	-1.207E+2	-2.274E-1
7.3	1.277E-2	-7.719E-3	-4.413E+0	-1.296E+2	-2.266E-1
7.4	1.202E-2	-7.194E-3	-4.427E+0	-1.390E+2	-2.259E-1
7.5	1.133E-2	-6.708E-3	-4.441E+0	-1.491E+2	-2.252E-1
7.6	1.068E-2	-6.260E-3	-4.455E+0	-1.597E+2	-2.245E-1
7.7	1.008E-2	-5.846E-3	-4.468E+0	-1.711E+2	-2.238E-1
7.8	9.510E-3	-5.462E-3	-4.480E+0	-1.831E+2	-2.232E-1
7.9	8.982E-3	-5.107E-3	-4.492E+0	-1.958E+2	-2.226E-1
8.0	8.488E-3	-4.779E-3	-4.504E+0	-2.093E+2	-2.220E-1
8.1	8.025E-3	-4.474E-3	-4.515E+0	-2.235E+2	-2.215E-1
8.2	7.592E-3	-4.191E-3	-4.526E+0	-2.386E+2	-2.209E-1
8.3	7.187E-3	-3.928E-3	-4.537E+0	-2.546E+2	-2.204E-1
8.4	6.806E-3	-3.684E-3	-4.547E+0	-2.714E+2	-2.199E-1
8.5	6.449E-3	-3.457E-3	-4.557E+0	-2.892E+2	-2.195E-1
8.6	6.114E-3	-3.246E-3	-4.566E+0	-3.080E+2	-2.190E-1
8.7	5.799E-3	-3.050E-3	-4.576E+0	-3.279E+2	-2.185E-1
8.8	5.504E-3	-2.867E-3	-4.585E+0	-3.488E+2	-2.181E-1
8.9	5.226E-3	-2.697E-3	-4.593E+0	-3.708E+2	-2.177E-1
9.0	4.964E-3	-2.538E-3	-4.602E+0	-3.940E+2	-2.173E-1
9.1	4.718E-3	-2.390E-3	-4.610E+0	-4.184E+2	-2.169E-1
9.2	4.486E-3	-2.252E-3	-4.618E+0	-4.441E+2	-2.165E-1
9.3	4.267E-3	-2.122E-3	-4.626E+0	-4.712E+2	-2.162E-1
9.4	4.061E-3	-2.002E-3	-4.633E+0	-4.996E+2	-2.158E-1
9.5	3.866E-3	-1.889E-3	-4.640E+0	-5.295E+2	-2.155E-1
9.6	3.683E-3	-1.783E-3	-4.648E+0	-5.609E+2	-2.152E-1
9.7	3.510E-3	-1.684E-3	-4.654E+0	-5.938E+2	-2.149E-1
9.8	3.346E-3	-1.591E-3	-4.661E+0	-6.284E+2	-2.145E-1
9.9	3.191E-3	-1.504E-3	-4.668E+0	-6.647E+2	-2.142E-1
10.0	3.045E-3	-1.423E-3	-4.674E+0	-7.027E+2	-2.140E-1
10.1	2.906E-3	-1.347E-3	-4.680E+0	-7.426E+2	-2.137E-1
10.2	2.775E-3	-1.275E-3	-4.686E+0	-7.843E+2	-2.134E-1
10.3	2.651E-3	-1.208E-3	-4.692E+0	-8.281E+2	-2.131E-1
10.4	2.534E-3	-1.144E-3	-4.697E+0	-8.739E+2	-2.129E-1

Table 4. Sensitivity coefficients for $R = p_{t_2}/p_{t_1}$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
10.5	2.422E-3	-1.085E-3	-4.703E+0	-9.218E+2	-2.126E-1
10.6	2.317E-3	-1.029E-3	-4.708E+0	-9.719E+2	-2.124E-1
10.7	2.216E-3	-9.763E-4	-4.713E+0	-1.024E+3	-2.122E-1
10.8	2.121E-3	-9.267E-4	-4.718E+0	-1.079E+3	-2.119E-1
10.9	2.031E-3	-8.800E-4	-4.723E+0	-1.136E+3	-2.117E-1
11.0	1.945E-3	-8.360E-4	-4.728E+0	-1.196E+3	-2.115E-1
11.1	1.864E-3	-7.946E-4	-4.733E+0	-1.259E+3	-2.113E-1
11.2	1.786E-3	-7.555E-4	-4.737E+0	-1.324E+3	-2.111E-1
11.3	1.712E-3	-7.186E-4	-4.742E+0	-1.392E+3	-2.109E-1
11.4	1.642E-3	-6.837E-4	-4.746E+0	-1.463E+3	-2.107E-1
11.5	1.576E-3	-6.508E-4	-4.750E+0	-1.536E+3	-2.105E-1
11.6	1.512E-3	-6.198E-4	-4.755E+0	-1.614E+3	-2.103E-1
11.7	1.452E-3	-5.904E-4	-4.759E+0	-1.694E+3	-2.101E-1
11.8	1.394E-3	-5.626E-4	-4.762E+0	-1.777E+3	-2.100E-1
11.9	1.339E-3	-5.363E-4	-4.766E+0	-1.865E+3	-2.098E-1
12.0	1.287E-3	-5.114E-4	-4.770E+0	-1.955E+3	-2.096E-1
12.1	1.237E-3	-4.879E-4	-4.774E+0	-2.050E+3	-2.095E-1
12.2	1.189E-3	-4.656E-4	-4.777E+0	-2.148E+3	-2.093E-1
12.3	1.144E-3	-4.445E-4	-4.781E+0	-2.250E+3	-2.092E-1
12.4	1.100E-3	-4.244E-4	-4.784E+0	-2.356E+3	-2.090E-1
12.5	1.059E-3	-4.054E-4	-4.788E+0	-2.466E+3	-2.089E-1
12.6	1.019E-3	-3.874E-4	-4.791E+0	-2.581E+3	-2.087E-1
12.7	9.811E-4	-3.703E-4	-4.794E+0	-2.700E+3	-2.086E-1
12.8	9.449E-4	-3.541E-4	-4.797E+0	-2.824E+3	-2.085E-1
12.9	9.102E-4	-3.387E-4	-4.800E+0	-2.953E+3	-2.083E-1
13.0	8.771E-4	-3.241E-4	-4.803E+0	-3.086E+3	-2.082E-1
13.1	8.454E-4	-3.101E-4	-4.806E+0	-3.224E+3	-2.081E-1
13.2	8.150E-4	-2.969E-4	-4.809E+0	-3.368E+3	-2.080E-1
13.3	7.860E-4	-2.843E-4	-4.812E+0	-3.517E+3	-2.078E-1
13.4	7.581E-4	-2.724E-4	-4.814E+0	-3.671E+3	-2.077E-1
13.5	7.315E-4	-2.610E-4	-4.817E+0	-3.831E+3	-2.076E-1
13.6	7.059E-4	-2.502E-4	-4.820E+0	-3.997E+3	-2.075E-1
13.7	6.814E-4	-2.399E-4	-4.822E+0	-4.169E+3	-2.074E-1
13.8	6.579E-4	-2.300E-4	-4.825E+0	-4.347E+3	-2.073E-1
13.9	6.354E-4	-2.207E-4	-4.827E+0	-4.532E+3	-2.072E-1

Table 4. Sensitivity coefficients for $R = p_{t_2}/p_{t_1}$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
14.0	6.138E-4	-2.117E-4	-4.830E+0	-4.723E+3	-2.071E-1
14.1	5.931E-4	-2.032E-4	-4.832E+0	-4.921E+3	-2.070E-1
14.2	5.731E-4	-1.951E-4	-4.834E+0	-5.125E+3	-2.069E-1
14.3	5.540E-4	-1.874E-4	-4.836E+0	-5.337E+3	-2.068E-1
14.4	5.356E-4	-1.800E-4	-4.839E+0	-5.556E+3	-2.067E-1
14.5	5.180E-4	-1.729E-4	-4.841E+0	-5.783E+3	-2.066E-1
14.6	5.011E-4	-1.662E-4	-4.843E+0	-6.017E+3	-2.065E-1
14.7	4.848E-4	-1.598E-4	-4.845E+0	-6.259E+3	-2.064E-1
14.8	4.691E-4	-1.536E-4	-4.847E+0	-6.509E+3	-2.063E-1
14.9	4.540E-4	-1.478E-4	-4.849E+0	-6.768E+3	-2.062E-1
15.0	4.395E-4	-1.421E-4	-4.851E+0	-7.035E+3	-2.061E-1

Table 5. Sensitivity coefficients for $R = p_{\infty}/p_{t_2}$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	9.930E-1	-1.387E-1	-1.397E-2	-7.207E+0	-7.157E+1
0.2	9.725E-1	-2.701E-1	-5.556E-2	-3.702E+0	-1.800E+1
0.3	9.395E-1	-3.876E-1	-1.238E-1	-2.580E+0	-8.079E+0
0.4	8.956E-1	-4.860E-1	-2.171E-1	-2.058E+0	-4.607E+0
0.5	8.430E-1	-5.620E-1	-3.333E-1	-1.779E+0	-3.000E+0
0.6	7.840E-1	-6.143E-1	-4.701E-1	-1.628E+0	-2.127E+0
0.7	7.209E-1	-6.435E-1	-6.248E-1	-1.554E+0	-1.601E+0
0.8	6.560E-1	-6.514E-1	-7.943E-1	-1.535E+0	-1.259E+0
0.9	5.913E-1	-6.411E-1	-9.759E-1	-1.560E+0	-1.025E+0
1.0	5.283E-1	-6.163E-1	-1.167E+0	-1.623E+0	-8.571E-1
1.1	4.689E-1	-5.672E-1	-1.331E+0	-1.763E+0	-7.515E-1
1.2	4.154E-1	-5.017E-1	-1.449E+0	-1.993E+0	-6.900E-1
1.3	3.685E-1	-4.361E-1	-1.538E+0	-2.293E+0	-6.501E-1
1.4	3.280E-1	-3.764E-1	-1.607E+0	-2.657E+0	-6.223E-1
1.5	2.930E-1	-3.244E-1	-1.661E+0	-3.082E+0	-6.020E-1
1.6	2.628E-1	-2.800E-1	-1.704E+0	-3.572E+0	-5.867E-1
1.7	2.368E-1	-2.423E-1	-1.740E+0	-4.127E+0	-5.747E-1
1.8	2.142E-1	-2.105E-1	-1.769E+0	-4.750E+0	-5.652E-1
1.9	1.945E-1	-1.836E-1	-1.794E+0	-5.446E+0	-5.574E-1
2.0	1.773E-1	-1.609E-1	-1.815E+0	-6.216E+0	-5.510E-1
2.1	1.622E-1	-1.415E-1	-1.833E+0	-7.065E+0	-5.457E-1
2.2	1.489E-1	-1.251E-1	-1.848E+0	-7.996E+0	-5.411E-1
2.3	1.371E-1	-1.109E-1	-1.861E+0	-9.013E+0	-5.373E-1
2.4	1.266E-1	-9.882E-2	-1.873E+0	-1.012E+1	-5.339E-1
2.5	1.173E-1	-8.834E-2	-1.883E+0	-1.132E+1	-5.311E-1
2.6	1.089E-1	-7.926E-2	-1.892E+0	-1.262E+1	-5.285E-1
2.7	1.014E-1	-7.135E-2	-1.900E+0	-1.401E+1	-5.263E-1
2.8	9.461E-2	-6.444E-2	-1.907E+0	-1.552E+1	-5.243E-1
2.9	8.848E-2	-5.838E-2	-1.914E+0	-1.713E+1	-5.226E-1
3.0	8.291E-2	-5.305E-2	-1.919E+0	-1.885E+1	-5.210E-1
3.1	7.785E-2	-4.833E-2	-1.925E+0	-2.069E+1	-5.196E-1
3.2	7.323E-2	-4.415E-2	-1.929E+0	-2.265E+1	-5.183E-1
3.3	6.900E-2	-4.043E-2	-1.934E+0	-2.473E+1	-5.172E-1
3.4	6.513E-2	-3.711E-2	-1.937E+0	-2.694E+1	-5.161E-1

Table 5. Sensitivity coefficients for $R = p_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	6.157E-2	-3.414E-2	-1.941E+0	-2.929E+1	-5.152E-1
3.6	5.829E-2	-3.148E-2	-1.944E+0	-3.177E+1	-5.143E-1
3.7	5.526E-2	-2.908E-2	-1.947E+0	-3.438E+1	-5.135E-1
3.8	5.247E-2	-2.692E-2	-1.950E+0	-3.714E+1	-5.128E-1
3.9	4.987E-2	-2.497E-2	-1.953E+0	-4.005E+1	-5.121E-1
4.0	4.747E-2	-2.320E-2	-1.955E+0	-4.311E+1	-5.115E-1
4.1	4.523E-2	-2.159E-2	-1.957E+0	-4.632E+1	-5.109E-1
4.2	4.314E-2	-2.012E-2	-1.959E+0	-4.969E+1	-5.104E-1
4.3	4.120E-2	-1.879E-2	-1.961E+0	-5.322E+1	-5.099E-1
4.4	3.938E-2	-1.757E-2	-1.963E+0	-5.692E+1	-5.095E-1
4.5	3.768E-2	-1.645E-2	-1.964E+0	-6.079E+1	-5.090E-1
4.6	3.609E-2	-1.542E-2	-1.966E+0	-6.483E+1	-5.086E-1
4.7	3.459E-2	-1.448E-2	-1.967E+0	-6.906E+1	-5.083E-1
4.8	3.319E-2	-1.361E-2	-1.969E+0	-7.346E+1	-5.079E-1
4.9	3.187E-2	-1.281E-2	-1.970E+0	-7.805E+1	-5.076E-1
5.0	3.062E-2	-1.207E-2	-1.971E+0	-8.282E+1	-5.073E-1
5.1	2.945E-2	-1.139E-2	-1.972E+0	-8.779E+1	-5.070E-1
5.2	2.834E-2	-1.076E-2	-1.973E+0	-9.296E+1	-5.067E-1
5.3	2.730E-2	-1.017E-2	-1.974E+0	-9.833E+1	-5.063E-1
5.4	2.631E-2	-9.624E-3	-1.975E+0	-1.039E+2	-5.062E-1
5.5	2.537E-2	-9.117E-3	-1.976E+0	-1.097E+2	-5.060E-1
5.6	2.448E-2	-8.645E-3	-1.977E+0	-1.157E+2	-5.058E-1
5.7	2.364E-2	-8.204E-3	-1.978E+0	-1.219E+2	-5.056E-1
5.8	2.284E-2	-7.793E-3	-1.979E+0	-1.283E+2	-5.054E-1
5.9	2.208E-2	-7.409E-3	-1.979E+0	-1.350E+2	-5.052E-1
6.0	2.136E-2	-7.049E-3	-1.980E+0	-1.419E+2	-5.050E-1
6.1	2.067E-2	-6.713E-3	-1.981E+0	-1.490E+2	-5.049E-1
6.2	2.002E-2	-6.397E-3	-1.981E+0	-1.563E+2	-5.047E-1
6.3	1.939E-2	-6.101E-3	-1.982E+0	-1.639E+2	-5.046E-1
6.4	1.880E-2	-5.823E-3	-1.983E+0	-1.717E+2	-5.044E-1
6.5	1.823E-2	-5.561E-3	-1.983E+0	-1.798E+2	-5.043E-1
6.6	1.768E-2	-5.315E-3	-1.984E+0	-1.882E+2	-5.041E-1
6.7	1.716E-2	-5.083E-3	-1.984E+0	-1.967E+2	-5.040E-1
6.8	1.667E-2	-4.864E-3	-1.985E+0	-2.056E+2	-5.039E-1
6.9	1.619E-2	-4.658E-3	-1.985E+0	-2.147E+2	-5.038E-1

Table 5. Sensitivity coefficients for $R = p_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	1.573E-2	-4.463E-3	-1.985E+0	-2.241E+2	-5.037E-1
7.1	1.530E-2	-4.279E-3	-1.986E+0	-2.337E+2	-5.036E-1
7.2	1.488E-2	-4.104E-3	-1.986E+0	-2.436E+2	-5.035E-1
7.3	1.448E-2	-3.940E-3	-1.987E+0	-2.538E+2	-5.034E-1
7.4	1.409E-2	-3.783E-3	-1.987E+0	-2.643E+2	-5.033E-1
7.5	1.372E-2	-3.635E-3	-1.987E+0	-2.751E+2	-5.032E-1
7.6	1.336E-2	-3.495E-3	-1.988E+0	-2.861E+2	-5.031E-1
7.7	1.302E-2	-3.362E-3	-1.988E+0	-2.975E+2	-5.030E-1
7.8	1.269E-2	-3.235E-3	-1.988E+0	-3.091E+2	-5.030E-1
7.9	1.237E-2	-3.115E-3	-1.989E+0	-3.211E+2	-5.029E-1
8.0	1.207E-2	-3.000E-3	-1.989E+0	-3.333E+2	-5.028E-1
8.1	1.177E-2	-2.891E-3	-1.989E+0	-3.459E+2	-5.027E-1
8.2	1.149E-2	-2.787E-3	-1.989E+0	-3.588E+2	-5.027E-1
8.3	1.122E-2	-2.689E-3	-1.990E+0	-3.720E+2	-5.026E-1
8.4	1.095E-2	-2.594E-3	-1.990E+0	-3.855E+2	-5.025E-1
8.5	1.070E-2	-2.504E-3	-1.990E+0	-3.993E+2	-5.025E-1
8.6	1.045E-2	-2.419E-3	-1.990E+0	-4.135E+2	-5.024E-1
8.7	1.021E-2	-2.337E-3	-1.991E+0	-4.280E+2	-5.024E-1
8.8	9.983E-3	-2.258E-3	-1.991E+0	-4.428E+2	-5.023E-1
8.9	9.761E-3	-2.184E-3	-1.991E+0	-4.580E+2	-5.023E-1
9.0	9.546E-3	-2.112E-3	-1.991E+0	-4.735E+2	-5.022E-1
9.1	9.338E-3	-2.044E-3	-1.991E+0	-4.893E+2	-5.022E-1
9.2	9.137E-3	-1.978E-3	-1.992E+0	-5.056E+2	-5.021E-1
9.3	8.943E-3	-1.915E-3	-1.992E+0	-5.221E+2	-5.021E-1
9.4	8.754E-3	-1.855E-3	-1.992E+0	-5.391E+2	-5.020E-1
9.5	8.572E-3	-1.797E-3	-1.992E+0	-5.564E+2	-5.020E-1
9.6	8.395E-3	-1.742E-3	-1.992E+0	-5.740E+2	-5.019E-1
9.7	8.223E-3	-1.689E-3	-1.992E+0	-5.920E+2	-5.019E-1
9.8	8.057E-3	-1.638E-3	-1.993E+0	-6.105E+2	-5.019E-1
9.9	7.895E-3	-1.589E-3	-1.993E+0	-6.292E+2	-5.018E-1
10.0	7.739E-3	-1.542E-3	-1.993E+0	-6.484E+2	-5.018E-1
10.1	7.587E-3	-1.497E-3	-1.993E+0	-6.680E+2	-5.018E-1
10.2	7.439E-3	-1.454E-3	-1.993E+0	-6.879E+2	-5.017E-1
10.3	7.296E-3	-1.412E-3	-1.993E+0	-7.082E+2	-5.017E-1
10.4	7.157E-3	-1.372E-3	-1.993E+0	-7.290E+2	-5.017E-1

Table 5. Sensitivity coefficients for $R = p_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	7.022E-3	-1.333E-3	-1.994E+0	-7.501E+2	-5.016E-1
10.6	6.890E-3	-1.296E-3	-1.994E+0	-7.717E+2	-5.016E-1
10.7	6.763E-3	-1.260E-3	-1.994E+0	-7.936E+2	-5.016E-1
10.8	6.638E-3	-1.226E-3	-1.994E+0	-8.160E+2	-5.015E-1
10.9	6.517E-3	-1.192E-3	-1.994E+0	-8.387E+2	-5.015E-1
11.0	6.400E-3	-1.160E-3	-1.994E+0	-8.620E+2	-5.015E-1
11.1	6.285E-3	-1.129E-3	-1.994E+0	-8.856E+2	-5.015E-1
11.2	6.174E-3	-1.099E-3	-1.994E+0	-9.096E+2	-5.014E-1
11.3	6.065E-3	-1.071E-3	-1.994E+0	-9.341E+2	-5.014E-1
11.4	5.960E-3	-1.043E-3	-1.994E+0	-9.591E+2	-5.014E-1
11.5	5.857E-3	-1.016E-3	-1.995E+0	-9.844E+2	-5.014E-1
11.6	5.757E-3	-9.899E-4	-1.995E+0	-1.010E+3	-5.013E-1
11.7	5.659E-3	-9.648E-4	-1.995E+0	-1.036E+3	-5.013E-1
11.8	5.564E-3	-9.406E-4	-1.995E+0	-1.063E+3	-5.013E-1
11.9	5.471E-3	-9.171E-4	-1.995E+0	-1.090E+3	-5.013E-1
12.0	5.380E-3	-8.945E-4	-1.995E+0	-1.118E+3	-5.012E-1
12.1	5.292E-3	-8.725E-4	-1.995E+0	-1.146E+3	-5.012E-1
12.2	5.206E-3	-8.513E-4	-1.995E+0	-1.175E+3	-5.012E-1
12.3	5.121E-3	-8.308E-4	-1.995E+0	-1.204E+3	-5.012E-1
12.4	5.039E-3	-8.109E-4	-1.995E+0	-1.233E+3	-5.012E-1
12.5	4.959E-3	-7.917E-4	-1.995E+0	-1.263E+3	-5.011E-1
12.6	4.881E-3	-7.730E-4	-1.995E+0	-1.294E+3	-5.011E-1
12.7	4.805E-3	-7.550E-4	-1.996E+0	-1.325E+3	-5.011E-1
12.8	4.730E-3	-7.375E-4	-1.996E+0	-1.356E+3	-5.011E-1
12.9	4.657E-3	-7.205E-4	-1.996E+0	-1.388E+3	-5.011E-1
13.0	4.586E-3	-7.040E-4	-1.996E+0	-1.420E+3	-5.011E-1
13.1	4.516E-3	-6.881E-4	-1.996E+0	-1.453E+3	-5.010E-1
13.2	4.448E-3	-6.726E-4	-1.996E+0	-1.487E+3	-5.010E-1
13.3	4.382E-3	-6.576E-4	-1.996E+0	-1.521E+3	-5.010E-1
13.4	4.317E-3	-6.430E-4	-1.996E+0	-1.555E+3	-5.010E-1
13.5	4.253E-3	-6.289E-4	-1.996E+0	-1.590E+3	-5.010E-1
13.6	4.191E-3	-6.151E-4	-1.996E+0	-1.626E+3	-5.010E-1
13.7	4.130E-3	-6.018E-4	-1.996E+0	-1.662E+3	-5.010E-1
13.8	4.071E-3	-5.888E-4	-1.996E+0	-1.698E+3	-5.009E-1
13.9	4.012E-3	-5.762E-4	-1.996E+0	-1.735E+3	-5.009E-1

Table 5. Sensitivity coefficients for $R = p_{\infty}/p_{t_2}$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	3.955E-3	-5.640E-4	-1.996E+0	-1.773E+3	-5.009E-1
14.1	3.900E-3	-5.521E-4	-1.996E+0	-1.811E+3	-5.009E-1
14.2	3.845E-3	-5.406E-4	-1.996E+0	-1.850E+3	-5.009E-1
14.3	3.791E-3	-5.293E-4	-1.997E+0	-1.889E+3	-5.009E-1
14.4	3.739E-3	-5.184E-4	-1.997E+0	-1.929E+3	-5.009E-1
14.5	3.688E-3	-5.078E-4	-1.997E+0	-1.969E+3	-5.009E-1
14.6	3.637E-3	-4.974E-4	-1.997E+0	-2.010E+3	-5.008E-1
14.7	3.588E-3	-4.874E-4	-1.997E+0	-2.052E+3	-5.008E-1
14.8	3.540E-3	-4.776E-4	-1.997E+0	-2.094E+3	-5.008E-1
14.9	3.493E-3	-4.681E-4	-1.997E+0	-2.136E+3	-5.008E-1
15.0	3.446E-3	-4.588E-4	-1.997E+0	-2.180E+3	-5.008E-1

Table 6. Sensitivity coefficients for $R = A_{\infty}/A^*$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	5.822E+0	-5.752E+1	-9.880E-1	-1.738E-2	-1.012E+0
0.2	2.964E+0	-1.411E+1	-9.524E-1	-7.086E-2	-1.050E+0
0.3	2.035E+0	-6.064E+0	-8.939E-1	-1.649E-1	-1.119E+0
0.4	1.590E+0	-3.236E+0	-8.140E-1	-3.090E-1	-1.229E+0
0.5	1.340E+0	-1.914E+0	-7.143E-1	-5.224E-1	-1.400E+0
0.6	1.188E+0	-1.182E+0	-5.970E-1	-8.458E-1	-1.675E+0
0.7	1.094E+0	-7.262E-1	-4.645E-1	-1.377E+0	-2.153E+0
0.8	1.038E+0	-4.142E-1	-3.191E-1	-2.414E+0	-3.133E+0
0.9	1.009E+0	-1.833E-1	-1.635E-1	-5.456E+0	-6.116E+0
1.0	1.000E+0	0.000E+0	0.000E+0	∞	∞
1.1	1.008E+0	1.549E-1	1.691E-1	6.455E+0	5.914E+0
1.2	1.030E+0	2.933E-1	3.416E-1	3.409E+0	2.927E+0
1.3	1.066E+0	4.230E-1	5.157E-1	2.364E+0	1.939E+0
1.4	1.115E+0	5.492E-1	6.897E-1	1.821E+0	1.450E+0
1.5	1.176E+0	6.760E-1	8.621E-1	1.479E+0	1.160E+0
1.6	1.250E+0	8.062E-1	1.032E+0	1.240E+0	9.692E-1
1.7	1.338E+0	9.424E-1	1.198E+0	1.061E+0	8.349E-1
1.8	1.439E+0	1.087E+0	1.359E+0	9.203E-1	7.357E-1
1.9	1.555E+0	1.241E+0	1.516E+0	8.060E-1	6.598E-1
2.0	1.687E+0	1.406E+0	1.667E+0	7.111E-1	6.000E-1
2.1	1.837E+0	1.585E+0	1.812E+0	6.309E-1	5.519E-1
2.2	2.005E+0	1.778E+0	1.951E+0	5.624E-1	5.125E-1
2.3	2.193E+0	1.988E+0	2.085E+0	5.031E-1	4.797E-1
2.4	2.403E+0	2.215E+0	2.212E+0	4.515E-1	4.521E-1
2.5	2.637E+0	2.461E+0	2.333E+0	4.063E-1	4.286E-1
2.6	2.896E+0	2.728E+0	2.449E+0	3.666E-1	4.083E-1
2.7	3.183E+0	3.017E+0	2.559E+0	3.315E-1	3.908E-1
2.8	3.500E+0	3.330E+0	2.664E+0	3.003E-1	3.754E-1
2.9	3.850E+0	3.668E+0	2.763E+0	2.726E-1	3.619E-1
3.0	4.235E+0	4.033E+0	2.857E+0	2.480E-1	3.500E-1
3.1	4.657E+0	4.427E+0	2.947E+0	2.259E-1	3.394E-1
3.2	5.121E+0	4.851E+0	3.031E+0	2.061E-1	3.299E-1
3.3	5.629E+0	5.308E+0	3.112E+0	1.884E-1	3.213E-1
3.4	6.184E+0	5.799E+0	3.188E+0	1.724E-1	3.136E-1

Table 6. Sensitivity coefficients for $R = A_{\infty}/A^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	6.790E+0	6.326E+0	3.261E+0	1.581E-1	3.067E-1
3.6	7.450E+0	6.891E+0	3.330E+0	1.451E-1	3.003E-1
3.7	8.169E+0	7.495E+0	3.395E+0	1.334E-1	2.946E-1
3.8	8.951E+0	8.142E+0	3.457E+0	1.228E-1	2.893E-1
3.9	9.799E+0	8.833E+0	3.516E+0	1.132E-1	2.844E-1
4.0	1.072E+1	9.570E+0	3.571E+0	1.045E-1	2.800E-1
4.1	1.171E+1	1.036E+1	3.624E+0	9.656E-2	2.759E-1
4.2	1.279E+1	1.119E+1	3.675E+0	8.935E-2	2.721E-1
4.3	1.395E+1	1.208E+1	3.723E+0	8.277E-2	2.686E-1
4.4	1.521E+1	1.303E+1	3.768E+0	7.676E-2	2.654E-1
4.5	1.656E+1	1.403E+1	3.812E+0	7.128E-2	2.623E-1
4.6	1.802E+1	1.509E+1	3.853E+0	6.626E-2	2.595E-1
4.7	1.958E+1	1.622E+1	3.893E+0	6.166E-2	2.569E-1
4.8	2.126E+1	1.741E+1	3.930E+0	5.744E-2	2.544E-1
4.9	2.307E+1	1.867E+1	3.966E+0	5.356E-2	2.522E-1
5.0	2.500E+1	2.000E+1	4.000E+0	5.000E-2	2.500E-1
5.1	2.707E+1	2.140E+1	4.033E+0	4.672E-2	2.480E-1
5.2	2.928E+1	2.288E+1	4.064E+0	4.370E-2	2.461E-1
5.3	3.165E+1	2.444E+1	4.093E+0	4.091E-2	2.443E-1
5.4	3.417E+1	2.609E+1	4.122E+0	3.834E-2	2.426E-1
5.5	3.687E+1	2.781E+1	4.149E+0	3.596E-2	2.410E-1
5.6	3.974E+1	2.963E+1	4.175E+0	3.375E-2	2.395E-1
5.7	4.280E+1	3.153E+1	4.200E+0	3.171E-2	2.381E-1
5.8	4.605E+1	3.353E+1	4.224E+0	2.982E-2	2.368E-1
5.9	4.951E+1	3.563E+1	4.246E+0	2.806E-2	2.355E-1
6.0	5.318E+1	3.783E+1	4.268E+0	2.643E-2	2.343E-1
6.1	5.708E+1	4.013E+1	4.289E+0	2.492E-2	2.331E-1
6.2	6.121E+1	4.255E+1	4.309E+0	2.350E-2	2.321E-1
6.3	6.559E+1	4.507E+1	4.329E+0	2.219E-2	2.310E-1
6.4	7.023E+1	4.770E+1	4.347E+0	2.096E-2	2.300E-1
6.5	7.513E+1	5.046E+1	4.365E+0	1.982E-2	2.291E-1
6.6	8.032E+1	5.333E+1	4.382E+0	1.875E-2	2.282E-1
6.7	8.580E+1	5.633E+1	4.399E+0	1.775E-2	2.273E-1
6.8	9.159E+1	5.946E+1	4.415E+0	1.682E-2	2.265E-1
6.9	9.770E+1	6.272E+1	4.430E+0	1.594E-2	2.257E-1

Table 6. Sensitivity coefficients for $R = A_{\infty}/A^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	1.041E+2	6.612E+1	4.444E+0	1.512E-2	2.250E-1
7.1	1.109E+2	6.966E+1	4.459E+0	1.436E-2	2.243E-1
7.2	1.181E+2	7.334E+1	4.472E+0	1.363E-2	2.236E-1
7.3	1.256E+2	7.718E+1	4.485E+0	1.296E-2	2.229E-1
7.4	1.335E+2	8.116E+1	4.498E+0	1.232E-2	2.223E-1
7.5	1.418E+2	8.530E+1	4.510E+0	1.172E-2	2.217E-1
7.6	1.506E+2	8.960E+1	4.522E+0	1.116E-2	2.211E-1
7.7	1.598E+2	9.406E+1	4.533E+0	1.063E-2	2.206E-1
7.8	1.694E+2	9.870E+1	4.544E+0	1.013E-2	2.201E-1
7.9	1.795E+2	1.035E+2	4.555E+0	9.662E-3	2.195E-1
8.0	1.901E+2	1.085E+2	4.565E+0	9.218E-3	2.190E-1
8.1	2.012E+2	1.137E+2	4.575E+0	8.799E-3	2.186E-1
8.2	2.128E+2	1.190E+2	4.585E+0	8.403E-3	2.181E-1
8.3	2.250E+2	1.245E+2	4.594E+0	8.029E-3	2.177E-1
8.4	2.378E+2	1.303E+2	4.603E+0	7.675E-3	2.173E-1
8.5	2.511E+2	1.362E+2	4.612E+0	7.341E-3	2.168E-1
8.6	2.650E+2	1.424E+2	4.620E+0	7.024E-3	2.164E-1
8.7	2.796E+2	1.487E+2	4.628E+0	6.724E-3	2.161E-1
8.8	2.948E+2	1.553E+2	4.636E+0	6.440E-3	2.157E-1
8.9	3.106E+2	1.621E+2	4.644E+0	6.170E-3	2.153E-1
9.0	3.272E+2	1.691E+2	4.651E+0	5.914E-3	2.150E-1
9.1	3.445E+2	1.763E+2	4.658E+0	5.671E-3	2.147E-1
9.2	3.625E+2	1.838E+2	4.665E+0	5.441E-3	2.143E-1
9.3	3.812E+2	1.915E+2	4.672E+0	5.221E-3	2.140E-1
9.4	4.008E+2	1.995E+2	4.679E+0	5.013E-3	2.137E-1
9.5	4.211E+2	2.077E+2	4.685E+0	4.815E-3	2.134E-1
9.6	4.423E+2	2.161E+2	4.691E+0	4.626E-3	2.132E-1
9.7	4.644E+2	2.249E+2	4.697E+0	4.447E-3	2.129E-1
9.8	4.873E+2	2.339E+2	4.703E+0	4.276E-3	2.126E-1
9.9	5.112E+2	2.431E+2	4.709E+0	4.113E-3	2.124E-1
10.0	5.359E+2	2.527E+2	4.714E+0	3.958E-3	2.121E-1
10.1	5.617E+2	2.625E+2	4.720E+0	3.810E-3	2.119E-1
10.2	5.884E+2	2.726E+2	4.725E+0	3.669E-3	2.116E-1
10.3	6.162E+2	2.830E+2	4.730E+0	3.534E-3	2.114E-1
10.4	6.450E+2	2.937E+2	4.735E+0	3.405E-3	2.112E-1

Table 6. Sensitivity coefficients for $R = A_{\infty}/A^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	6.750E+2	3.047E+2	4.740E+0	3.282E-3	2.110E-1
10.6	7.060E+2	3.160E+2	4.744E+0	3.165E-3	2.108E-1
10.7	7.382E+2	3.276E+2	4.749E+0	3.052E-3	2.106E-1
10.8	7.715E+2	3.396E+2	4.753E+0	2.945E-3	2.104E-1
10.9	8.061E+2	3.518E+2	4.758E+0	2.842E-3	2.102E-1
11.0	8.419E+2	3.645E+2	4.762E+0	2.744E-3	2.100E-1
11.1	8.790E+2	3.774E+2	4.766E+0	2.650E-3	2.098E-1
11.2	9.174E+2	3.907E+2	4.770E+0	2.559E-3	2.096E-1
11.3	9.572E+2	4.044E+2	4.774E+0	2.473E-3	2.095E-1
11.4	9.983E+2	4.184E+2	4.778E+0	2.390E-3	2.093E-1
11.5	1.041E+3	4.328E+2	4.781E+0	2.311E-3	2.091E-1
11.6	1.085E+3	4.475E+2	4.785E+0	2.235E-3	2.090E-1
11.7	1.130E+3	4.626E+2	4.789E+0	2.162E-3	2.088E-1
11.8	1.177E+3	4.781E+2	4.792E+0	2.091E-3	2.087E-1
11.9	1.226E+3	4.940E+2	4.795E+0	2.024E-3	2.085E-1
12.0	1.276E+3	5.103E+2	4.799E+0	1.959E-3	2.084E-1
12.1	1.328E+3	5.270E+2	4.802E+0	1.897E-3	2.083E-1
12.2	1.382E+3	5.442E+2	4.805E+0	1.838E-3	2.081E-1
12.3	1.437E+3	5.617E+2	4.808E+0	1.780E-3	2.080E-1
12.4	1.494E+3	5.796E+2	4.811E+0	1.725E-3	2.079E-1
12.5	1.553E+3	5.980E+2	4.814E+0	1.672E-3	2.077E-1
12.6	1.614E+3	6.169E+2	4.817E+0	1.621E-3	2.076E-1
12.7	1.676E+3	6.361E+2	4.820E+0	1.572E-3	2.075E-1
12.8	1.741E+3	6.559E+2	4.822E+0	1.525E-3	2.074E-1
12.9	1.807E+3	6.760E+2	4.825E+0	1.479E-3	2.073E-1
13.0	1.876E+3	6.967E+2	4.828E+0	1.435E-3	2.071E-1
13.1	1.947E+3	7.178E+2	4.830E+0	1.393E-3	2.070E-1
13.2	2.020E+3	7.394E+2	4.833E+0	1.352E-3	2.069E-1
13.3	2.095E+3	7.615E+2	4.835E+0	1.313E-3	2.068E-1
13.4	2.172E+3	7.841E+2	4.837E+0	1.275E-3	2.067E-1
13.5	2.252E+3	8.072E+2	4.840E+0	1.239E-3	2.066E-1
13.6	2.333E+3	8.308E+2	4.842E+0	1.204E-3	2.065E-1
13.7	2.418E+3	8.549E+2	4.844E+0	1.170E-3	2.064E-1
13.8	2.504E+3	8.795E+2	4.847E+0	1.137E-3	2.063E-1
13.9	2.594E+3	9.047E+2	4.849E+0	1.105E-3	2.062E-1

Table 6. Sensitivity coefficients for $R = A_{\infty}/A^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	2.685E+3	9.304E+2	4.851E+0	1.075E-3	2.062E-1
14.1	2.780E+3	9.567E+2	4.853E+0	1.045E-3	2.061E-1
14.2	2.877E+3	9.835E+2	4.855E+0	1.017E-3	2.060E-1
14.3	2.976E+3	1.011E+3	4.857E+0	9.892E-4	2.059E-1
14.4	3.079E+3	1.039E+3	4.859E+0	9.626E-4	2.058E-1
14.5	3.184E+3	1.067E+3	4.861E+0	9.368E-4	2.057E-1
14.6	3.292E+3	1.097E+3	4.862E+0	9.120E-4	2.057E-1
14.7	3.404E+3	1.126E+3	4.864E+0	8.879E-4	2.056E-1
14.8	3.518E+3	1.157E+3	4.866E+0	8.646E-4	2.055E-1
14.9	3.635E+3	1.188E+3	4.868E+0	8.421E-4	2.054E-1
15.0	3.755E+3	1.219E+3	4.870E+0	8.203E-4	2.054E-1

Table 7. Sensitivity coefficients for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	1.718E-1	1.697E+0	9.880E-1	5.892E-1	1.012E+0
0.2	3.374E-1	1.607E+0	9.524E-1	6.223E-1	1.050E+0
0.3	4.914E-1	1.464E+0	8.939E-1	6.830E-1	1.119E+0
0.4	6.289E-1	1.280E+0	8.140E-1	7.814E-1	1.229E+0
0.5	7.464E-1	1.066E+0	7.143E-1	9.379E-1	1.400E+0
0.6	8.416E-1	8.374E-1	5.970E-1	1.194E+0	1.675E+0
0.7	9.138E-1	6.063E-1	4.645E-1	1.649E+0	2.153E+0
0.8	9.632E-1	3.842E-1	3.191E-1	2.602E+0	3.133E+0
0.9	9.912E-1	1.801E-1	1.635E-1	5.553E+0	6.116E+0
1.0	1.000E+0	0.000E+0	0.000E+0	∞	∞
1.1	9.921E-1	-1.525E-1	-1.691E-1	-6.557E+0	-5.914E+0
1.2	9.705E-1	-2.763E-1	-3.416E-1	-3.620E+0	-2.927E+0
1.3	9.378E-1	-3.720E-1	-5.157E-1	-2.688E+0	-1.939E+0
1.4	8.969E-1	-4.418E-1	-6.897E-1	-2.263E+0	-1.450E+0
1.5	8.502E-1	-4.886E-1	-8.621E-1	-2.047E+0	-1.160E+0
1.6	7.998E-1	-5.158E-1	-1.032E+0	-1.939E+0	-9.692E-1
1.7	7.476E-1	-5.267E-1	-1.198E+0	-1.899E+0	-8.349E-1
1.8	6.949E-1	-5.248E-1	-1.359E+0	-1.906E+0	-7.357E-1
1.9	6.430E-1	-5.129E-1	-1.516E+0	-1.950E+0	-6.598E-1
2.0	5.926E-1	-4.938E-1	-1.667E+0	-2.025E+0	-6.000E-1
2.1	5.444E-1	-4.697E-1	-1.812E+0	-2.129E+0	-5.519E-1
2.2	4.988E-1	-4.424E-1	-1.951E+0	-2.261E+0	-5.125E-1
2.3	4.560E-1	-4.133E-1	-2.085E+0	-2.420E+0	-4.797E-1
2.4	4.161E-1	-3.835E-1	-2.212E+0	-2.607E+0	-4.521E-1
2.5	3.793E-1	-3.540E-1	-2.333E+0	-2.825E+0	-4.286E-1
2.6	3.453E-1	-3.252E-1	-2.449E+0	-3.075E+0	-4.083E-1
2.7	3.142E-1	-2.978E-1	-2.559E+0	-3.358E+0	-3.908E-1
2.8	2.857E-1	-2.718E-1	-2.664E+0	-3.679E+0	-3.754E-1
2.9	2.598E-1	-2.475E-1	-2.763E+0	-4.041E+0	-3.619E-1
3.0	2.362E-1	-2.249E-1	-2.857E+0	-4.446E+0	-3.500E-1
3.1	2.147E-1	-2.041E-1	-2.947E+0	-4.900E+0	-3.394E-1
3.2	1.953E-1	-1.850E-1	-3.031E+0	-5.406E+0	-3.299E-1
3.3	1.777E-1	-1.675E-1	-3.112E+0	-5.969E+0	-3.213E-1
3.4	1.617E-1	-1.517E-1	-3.188E+0	-6.594E+0	-3.136E-1

Table 7. Sensitivity coefficients for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	1.473E-1	-1.372E-1	-3.261E+0	-7.288E+0	-3.067E-1
3.6	1.342E-1	-1.241E-1	-3.330E+0	-8.055E+0	-3.003E-1
3.7	1.224E-1	-1.123E-1	-3.395E+0	-8.903E+0	-2.946E-1
3.8	1.117E-1	-1.016E-1	-3.457E+0	-9.839E+0	-2.893E-1
3.9	1.021E-1	-9.199E-2	-3.516E+0	-1.087E+1	-2.844E-1
4.0	9.329E-2	-8.330E-2	-3.571E+0	-1.201E+1	-2.800E-1
4.1	8.536E-2	-7.546E-2	-3.624E+0	-1.325E+1	-2.759E-1
4.2	7.818E-2	-6.840E-2	-3.675E+0	-1.462E+1	-2.721E-1
4.3	7.166E-2	-6.204E-2	-3.723E+0	-1.612E+1	-2.686E-1
4.4	6.575E-2	-5.631E-2	-3.768E+0	-1.776E+1	-2.654E-1
4.5	6.038E-2	-5.115E-2	-3.812E+0	-1.955E+1	-2.623E-1
4.6	5.550E-2	-4.649E-2	-3.853E+0	-2.151E+1	-2.595E-1
4.7	5.107E-2	-4.229E-2	-3.893E+0	-2.364E+1	-2.569E-1
4.8	4.703E-2	-3.851E-2	-3.930E+0	-2.597E+1	-2.544E-1
4.9	4.335E-2	-3.509E-2	-3.966E+0	-2.850E+1	-2.522E-1
5.0	4.000E-2	-3.200E-2	-4.000E+0	-3.125E+1	-2.500E-1
5.1	3.694E-2	-2.921E-2	-4.033E+0	-3.423E+1	-2.480E-1
5.2	3.415E-2	-2.669E-2	-4.064E+0	-3.747E+1	-2.461E-1
5.3	3.160E-2	-2.440E-2	-4.093E+0	-4.098E+1	-2.443E-1
5.4	2.926E-2	-2.233E-2	-4.122E+0	-4.477E+1	-2.426E-1
5.5	2.712E-2	-2.046E-2	-4.149E+0	-4.888E+1	-2.410E-1
5.6	2.516E-2	-1.876E-2	-4.175E+0	-5.331E+1	-2.395E-1
5.7	2.337E-2	-1.722E-2	-4.200E+0	-5.809E+1	-2.381E-1
5.8	2.172E-2	-1.581E-2	-4.224E+0	-6.324E+1	-2.368E-1
5.9	2.020E-2	-1.454E-2	-4.246E+0	-6.879E+1	-2.355E-1
6.0	1.880E-2	-1.338E-2	-4.268E+0	-7.476E+1	-2.343E-1
6.1	1.752E-2	-1.232E-2	-4.289E+0	-8.117E+1	-2.331E-1
6.2	1.634E-2	-1.136E-2	-4.309E+0	-8.806E+1	-2.321E-1
6.3	1.525E-2	-1.048E-2	-4.329E+0	-9.546E+1	-2.310E-1
6.4	1.424E-2	-9.672E-3	-4.347E+0	-1.034E+2	-2.300E-1
6.5	1.331E-2	-8.938E-3	-4.365E+0	-1.119E+2	-2.291E-1
6.6	1.245E-2	-8.266E-3	-4.382E+0	-1.210E+2	-2.282E-1
6.7	1.165E-2	-7.651E-3	-4.399E+0	-1.307E+2	-2.273E-1
6.8	1.092E-2	-7.088E-3	-4.415E+0	-1.411E+2	-2.265E-1
6.9	1.024E-2	-6.571E-3	-4.430E+0	-1.522E+2	-2.257E-1

Table 7. Sensitivity coefficients for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	9.602E-3	-6.097E-3	-4.444E+0	-1.640E+2	-2.250E-1
7.1	9.015E-3	-5.661E-3	-4.459E+0	-1.767E+2	-2.243E-1
7.2	8.469E-3	-5.260E-3	-4.472E+0	-1.901E+2	-2.236E-1
7.3	7.961E-3	-4.892E-3	-4.485E+0	-2.044E+2	-2.229E-1
7.4	7.490E-3	-4.552E-3	-4.498E+0	-2.197E+2	-2.223E-1
7.5	7.050E-3	-4.240E-3	-4.510E+0	-2.359E+2	-2.217E-1
7.6	6.641E-3	-3.951E-3	-4.522E+0	-2.531E+2	-2.211E-1
7.7	6.259E-3	-3.685E-3	-4.533E+0	-2.714E+2	-2.206E-1
7.8	5.903E-3	-3.439E-3	-4.544E+0	-2.908E+2	-2.201E-1
7.9	5.571E-3	-3.212E-3	-4.555E+0	-3.113E+2	-2.195E-1
8.0	5.260E-3	-3.002E-3	-4.565E+0	-3.331E+2	-2.190E-1
8.1	4.970E-3	-2.807E-3	-4.575E+0	-3.562E+2	-2.186E-1
8.2	4.698E-3	-2.627E-3	-4.585E+0	-3.807E+2	-2.181E-1
8.3	4.444E-3	-2.460E-3	-4.594E+0	-4.065E+2	-2.177E-1
8.4	4.206E-3	-2.305E-3	-4.603E+0	-4.339E+2	-2.173E-1
8.5	3.983E-3	-2.161E-3	-4.612E+0	-4.628E+2	-2.168E-1
8.6	3.773E-3	-2.027E-3	-4.620E+0	-4.933E+2	-2.164E-1
8.7	3.577E-3	-1.903E-3	-4.628E+0	-5.255E+2	-2.161E-1
8.8	3.393E-3	-1.787E-3	-4.636E+0	-5.595E+2	-2.157E-1
8.9	3.219E-3	-1.680E-3	-4.644E+0	-5.953E+2	-2.153E-1
9.0	3.056E-3	-1.579E-3	-4.651E+0	-6.331E+2	-2.150E-1
9.1	2.903E-3	-1.486E-3	-4.658E+0	-6.729E+2	-2.147E-1
9.2	2.759E-3	-1.399E-3	-4.665E+0	-7.148E+2	-2.143E-1
9.3	2.623E-3	-1.318E-3	-4.672E+0	-7.588E+2	-2.140E-1
9.4	2.495E-3	-1.242E-3	-4.679E+0	-8.052E+2	-2.137E-1
9.5	2.375E-3	-1.171E-3	-4.685E+0	-8.539E+2	-2.134E-1
9.6	2.261E-3	-1.105E-3	-4.691E+0	-9.052E+2	-2.132E-1
9.7	2.153E-3	-1.043E-3	-4.697E+0	-9.589E+2	-2.129E-1
9.8	2.052E-3	-9.848E-4	-4.703E+0	-1.015E+3	-2.126E-1
9.9	1.956E-3	-9.305E-4	-4.709E+0	-1.075E+3	-2.124E-1
10.0	1.866E-3	-8.796E-4	-4.714E+0	-1.137E+3	-2.121E-1
10.1	1.780E-3	-8.319E-4	-4.720E+0	-1.202E+3	-2.119E-1
10.2	1.699E-3	-7.872E-4	-4.725E+0	-1.270E+3	-2.116E-1
10.3	1.623E-3	-7.452E-4	-4.730E+0	-1.342E+3	-2.114E-1
10.4	1.550E-3	-7.058E-4	-4.735E+0	-1.417E+3	-2.112E-1

Table 7. Sensitivity coefficients for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	1.482E-3	-6.688E-4	-4.740E+0	-1.495E+3	-2.110E-1
10.6	1.416E-3	-6.340E-4	-4.744E+0	-1.577E+3	-2.108E-1
10.7	1.355E-3	-6.012E-4	-4.749E+0	-1.663E+3	-2.106E-1
10.8	1.296E-3	-5.705E-4	-4.753E+0	-1.753E+3	-2.104E-1
10.9	1.241E-3	-5.415E-4	-4.758E+0	-1.847E+3	-2.102E-1
11.0	1.188E-3	-5.142E-4	-4.762E+0	-1.945E+3	-2.100E-1
11.1	1.138E-3	-4.885E-4	-4.766E+0	-2.047E+3	-2.098E-1
11.2	1.090E-3	-4.642E-4	-4.770E+0	-2.154E+3	-2.096E-1
11.3	1.045E-3	-4.414E-4	-4.774E+0	-2.266E+3	-2.095E-1
11.4	1.002E-3	-4.198E-4	-4.778E+0	-2.382E+3	-2.093E-1
11.5	9.608E-4	-3.995E-4	-4.781E+0	-2.503E+3	-2.091E-1
11.6	9.218E-4	-3.802E-4	-4.785E+0	-2.630E+3	-2.090E-1
11.7	8.847E-4	-3.621E-4	-4.789E+0	-2.762E+3	-2.088E-1
11.8	8.493E-4	-3.449E-4	-4.792E+0	-2.899E+3	-2.087E-1
11.9	8.157E-4	-3.287E-4	-4.795E+0	-3.042E+3	-2.085E-1
12.0	7.836E-4	-3.133E-4	-4.799E+0	-3.191E+3	-2.084E-1
12.1	7.530E-4	-2.988E-4	-4.802E+0	-3.347E+3	-2.083E-1
12.2	7.238E-4	-2.851E-4	-4.805E+0	-3.508E+3	-2.081E-1
12.3	6.959E-4	-2.720E-4	-4.808E+0	-3.676E+3	-2.080E-1
12.4	6.693E-4	-2.597E-4	-4.811E+0	-3.851E+3	-2.079E-1
12.5	6.440E-4	-2.480E-4	-4.814E+0	-4.032E+3	-2.077E-1
12.6	6.197E-4	-2.369E-4	-4.817E+0	-4.221E+3	-2.076E-1
12.7	5.966E-4	-2.264E-4	-4.820E+0	-4.417E+3	-2.075E-1
12.8	5.744E-4	-2.164E-4	-4.822E+0	-4.621E+3	-2.074E-1
12.9	5.533E-4	-2.069E-4	-4.825E+0	-4.832E+3	-2.073E-1
13.0	5.330E-4	-1.979E-4	-4.828E+0	-5.052E+3	-2.071E-1
13.1	5.137E-4	-1.894E-4	-4.830E+0	-5.280E+3	-2.070E-1
13.2	4.951E-4	-1.813E-4	-4.833E+0	-5.517E+3	-2.069E-1
13.3	4.774E-4	-1.736E-4	-4.835E+0	-5.762E+3	-2.068E-1
13.4	4.604E-4	-1.662E-4	-4.837E+0	-6.016E+3	-2.067E-1
13.5	4.441E-4	-1.592E-4	-4.840E+0	-6.280E+3	-2.066E-1
13.6	4.286E-4	-1.526E-4	-4.842E+0	-6.554E+3	-2.065E-1
13.7	4.136E-4	-1.463E-4	-4.844E+0	-6.837E+3	-2.064E-1
13.8	3.993E-4	-1.402E-4	-4.847E+0	-7.131E+3	-2.063E-1
13.9	3.856E-4	-1.345E-4	-4.849E+0	-7.435E+3	-2.062E-1

Table 7. Sensitivity coefficients for $R = \rho_{\infty} V_{\infty} / \rho^* V^*$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	3.724E-4	-1.290E-4	-4.851E+0	-7.750E+3	-2.062E-1
14.1	3.597E-4	-1.238E-4	-4.853E+0	-8.077E+3	-2.061E-1
14.2	3.476E-4	-1.188E-4	-4.855E+0	-8.414E+3	-2.060E-1
14.3	3.360E-4	-1.141E-4	-4.857E+0	-8.764E+3	-2.059E-1
14.4	3.248E-4	-1.096E-4	-4.859E+0	-9.125E+3	-2.058E-1
14.5	3.140E-4	-1.053E-4	-4.861E+0	-9.499E+3	-2.057E-1
14.6	3.037E-4	-1.012E-4	-4.862E+0	-9.886E+3	-2.057E-1
14.7	2.938E-4	-9.722E-5	-4.864E+0	-1.029E+4	-2.056E-1
14.8	2.843E-4	-9.347E-5	-4.866E+0	-1.070E+4	-2.055E-1
14.9	2.751E-4	-8.988E-5	-4.868E+0	-1.113E+4	-2.054E-1
15.0	2.663E-4	-8.645E-5	-4.870E+0	-1.157E+4	-2.054E-1

Table 8. Sensitivity coefficients for $R = T_{\infty}/T_t$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	9.980E-1	-3.984E-2	-3.992E-3	-2.510E+1	-2.505E+2
0.2	9.921E-1	-7.874E-2	-1.587E-2	-1.270E+1	-6.300E+1
0.3	9.823E-1	-1.158E-1	-3.536E-2	-8.636E+0	-2.828E+1
0.4	9.690E-1	-1.502E-1	-6.202E-2	-6.656E+0	-1.612E+1
0.5	9.524E-1	-1.814E-1	-9.524E-2	-5.512E+0	-1.050E+1
0.6	9.328E-1	-2.088E-1	-1.343E-1	-4.788E+0	-7.444E+0
0.7	9.107E-1	-2.322E-1	-1.785E-1	-4.306E+0	-5.602E+0
0.8	8.865E-1	-2.515E-1	-2.270E-1	-3.976E+0	-4.406E+0
0.9	8.606E-1	-2.666E-1	-2.788E-1	-3.751E+0	-3.586E+0
1.0	8.333E-1	-2.778E-1	-3.333E-1	-3.600E+0	-3.000E+0
1.1	8.052E-1	-2.852E-1	-3.897E-1	-3.506E+0	-2.566E+0
1.2	7.764E-1	-2.893E-1	-4.472E-1	-3.456E+0	-2.236E+0
1.3	7.474E-1	-2.905E-1	-5.052E-1	-3.443E+0	-1.979E+0
1.4	7.184E-1	-2.890E-1	-5.632E-1	-3.460E+0	-1.776E+0
1.5	6.897E-1	-2.854E-1	-6.207E-1	-3.504E+0	-1.611E+0
1.6	6.614E-1	-2.799E-1	-6.772E-1	-3.572E+0	-1.477E+0
1.7	6.337E-1	-2.731E-1	-7.326E-1	-3.662E+0	-1.365E+0
1.8	6.068E-1	-2.651E-1	-7.864E-1	-3.772E+0	-1.272E+0
1.9	5.807E-1	-2.563E-1	-8.386E-1	-3.902E+0	-1.193E+0
2.0	5.556E-1	-2.469E-1	-8.889E-1	-4.050E+0	-1.125E+0
2.1	5.313E-1	-2.372E-1	-9.373E-1	-4.217E+0	-1.067E+0
2.2	5.081E-1	-2.272E-1	-9.837E-1	-4.401E+0	-1.017E+0
2.3	4.859E-1	-2.172E-1	-1.028E+0	-4.604E+0	-9.726E-1
2.4	4.647E-1	-2.073E-1	-1.071E+0	-4.824E+0	-9.340E-1
2.5	4.444E-1	-1.975E-1	-1.111E+0	-5.063E+0	-9.000E-1
2.6	4.252E-1	-1.880E-1	-1.150E+0	-5.319E+0	-8.698E-1
2.7	4.068E-1	-1.788E-1	-1.186E+0	-5.594E+0	-8.429E-1
2.8	3.894E-1	-1.698E-1	-1.221E+0	-5.888E+0	-8.189E-1
2.9	3.729E-1	-1.613E-1	-1.254E+0	-6.201E+0	-7.973E-1
3.0	3.571E-1	-1.531E-1	-1.286E+0	-6.533E+0	-7.778E-1
3.1	3.422E-1	-1.452E-1	-1.316E+0	-6.886E+0	-7.601E-1
3.2	3.281E-1	-1.378E-1	-1.344E+0	-7.258E+0	-7.441E-1
3.3	3.147E-1	-1.307E-1	-1.371E+0	-7.651E+0	-7.296E-1
3.4	3.019E-1	-1.240E-1	-1.396E+0	-8.066E+0	-7.163E-1

Table 8. Sensitivity coefficients for $R = T_\infty/T_t$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
3.5	2.899E-1	-1.176E-1	-1.420E+0	-8.502E+0	-7.041E-1
3.6	2.784E-1	-1.116E-1	-1.443E+0	-8.960E+0	-6.929E-1
3.7	2.675E-1	-1.059E-1	-1.465E+0	-9.441E+0	-6.826E-1
3.8	2.572E-1	-1.006E-1	-1.486E+0	-9.945E+0	-6.731E-1
3.9	2.474E-1	-9.548E-2	-1.505E+0	-1.047E+1	-6.644E-1
4.0	2.381E-1	-9.070E-2	-1.524E+0	-1.102E+1	-6.562E-1
4.1	2.293E-1	-8.619E-2	-1.541E+0	-1.160E+1	-6.487E-1
4.2	2.208E-1	-8.194E-2	-1.558E+0	-1.220E+1	-6.417E-1
4.3	2.129E-1	-7.793E-2	-1.574E+0	-1.283E+1	-6.352E-1
4.4	2.053E-1	-7.415E-2	-1.589E+0	-1.349E+1	-6.291E-1
4.5	1.980E-1	-7.058E-2	-1.604E+0	-1.417E+1	-6.235E-1
4.6	1.911E-1	-6.722E-2	-1.618E+0	-1.488E+1	-6.181E-1
4.7	1.846E-1	-6.404E-2	-1.631E+0	-1.561E+1	-6.132E-1
4.8	1.783E-1	-6.105E-2	-1.643E+0	-1.638E+1	-6.085E-1
4.9	1.724E-1	-5.822E-2	-1.655E+0	-1.718E+1	-6.041E-1
5.0	1.667E-1	-5.556E-2	-1.667E+0	-1.800E+1	-6.000E-1
5.1	1.612E-1	-5.304E-2	-1.678E+0	-1.886E+1	-5.961E-1
5.2	1.561E-1	-5.065E-2	-1.688E+0	-1.974E+1	-5.925E-1
5.3	1.511E-1	-4.840E-2	-1.698E+0	-2.066E+1	-5.890E-1
5.4	1.464E-1	-4.628E-2	-1.707E+0	-2.161E+1	-5.857E-1
5.5	1.418E-1	-4.426E-2	-1.716E+0	-2.259E+1	-5.826E-1
5.6	1.375E-1	-4.236E-2	-1.725E+0	-2.361E+1	-5.797E-1
5.7	1.334E-1	-4.055E-2	-1.733E+0	-2.466E+1	-5.769E-1
5.8	1.294E-1	-3.885E-2	-1.741E+0	-2.574E+1	-5.743E-1
5.9	1.256E-1	-3.723E-2	-1.749E+0	-2.686E+1	-5.718E-1
6.0	1.220E-1	-3.569E-2	-1.756E+0	-2.802E+1	-5.694E-1
6.1	1.185E-1	-3.424E-2	-1.763E+0	-2.921E+1	-5.672E-1
6.2	1.151E-1	-3.286E-2	-1.770E+0	-3.044E+1	-5.650E-1
6.3	1.119E-1	-3.154E-2	-1.776E+0	-3.170E+1	-5.630E-1
6.4	1.088E-1	-3.030E-2	-1.782E+0	-3.301E+1	-5.610E-1
6.5	1.058E-1	-2.911E-2	-1.788E+0	-3.435E+1	-5.592E-1
6.6	1.030E-1	-2.799E-2	-1.794E+0	-3.573E+1	-5.574E-1
6.7	1.002E-1	-2.692E-2	-1.800E+0	-3.715E+1	-5.557E-1
6.8	9.758E-2	-2.590E-2	-1.805E+0	-3.861E+1	-5.541E-1
6.9	9.504E-2	-2.493E-2	-1.810E+0	-4.011E+1	-5.525E-1

Table 8. Sensitivity coefficients for $R = T_\infty/T_i$ (Continued)
 $\gamma = 1.40$

M_∞	R	$\frac{\partial R}{\partial M_\infty}$	$\frac{M_\infty}{R} \frac{\partial R}{\partial M_\infty}$	$\frac{\partial M_\infty}{\partial R}$	$\frac{R}{M_\infty} \frac{\partial M_\infty}{\partial R}$
7.0	9.259E-2	-2.401E-2	-1.815E+0	-4.166E+1	-5.510E-1
7.1	9.024E-2	-2.313E-2	-1.820E+0	-4.324E+1	-5.496E-1
7.2	8.797E-2	-2.229E-2	-1.824E+0	-4.487E+1	-5.482E-1
7.3	8.578E-2	-2.148E-2	-1.828E+0	-4.654E+1	-5.469E-1
7.4	8.367E-2	-2.072E-2	-1.833E+0	-4.826E+1	-5.457E-1
7.5	8.163E-2	-1.999E-2	-1.837E+0	-5.002E+1	-5.444E-1
7.6	7.967E-2	-1.930E-2	-1.841E+0	-5.183E+1	-5.433E-1
7.7	7.777E-2	-1.863E-2	-1.844E+0	-5.368E+1	-5.422E-1
7.8	7.594E-2	-1.799E-2	-1.848E+0	-5.558E+1	-5.411E-1
7.9	7.417E-2	-1.739E-2	-1.852E+0	-5.752E+1	-5.401E-1
8.0	7.246E-2	-1.680E-2	-1.855E+0	-5.951E+1	-5.391E-1
8.1	7.081E-2	-1.625E-2	-1.858E+0	-6.155E+1	-5.381E-1
8.2	6.921E-2	-1.571E-2	-1.862E+0	-6.364E+1	-5.372E-1
8.3	6.767E-2	-1.520E-2	-1.865E+0	-6.578E+1	-5.363E-1
8.4	6.617E-2	-1.471E-2	-1.868E+0	-6.797E+1	-5.354E-1
8.5	6.472E-2	-1.424E-2	-1.871E+0	-7.021E+1	-5.346E-1
8.6	6.332E-2	-1.379E-2	-1.873E+0	-7.250E+1	-5.338E-1
8.7	6.197E-2	-1.336E-2	-1.876E+0	-7.484E+1	-5.330E-1
8.8	6.065E-2	-1.295E-2	-1.879E+0	-7.723E+1	-5.323E-1
8.9	5.938E-2	-1.255E-2	-1.881E+0	-7.968E+1	-5.316E-1
9.0	5.814E-2	-1.217E-2	-1.884E+0	-8.218E+1	-5.309E-1
9.1	5.694E-2	-1.180E-2	-1.886E+0	-8.473E+1	-5.302E-1
9.2	5.578E-2	-1.145E-2	-1.888E+0	-8.734E+1	-5.295E-1
9.3	5.465E-2	-1.111E-2	-1.891E+0	-9.000E+1	-5.289E-1
9.4	5.356E-2	-1.078E-2	-1.893E+0	-9.272E+1	-5.283E-1
9.5	5.249E-2	-1.047E-2	-1.895E+0	-9.550E+1	-5.277E-1
9.6	5.146E-2	-1.017E-2	-1.897E+0	-9.833E+1	-5.271E-1
9.7	5.046E-2	-9.879E-3	-1.899E+0	-1.012E+2	-5.266E-1
9.8	4.949E-2	-9.599E-3	-1.901E+0	-1.042E+2	-5.260E-1
9.9	4.854E-2	-9.330E-3	-1.903E+0	-1.072E+2	-5.255E-1
10.0	4.762E-2	-9.070E-3	-1.905E+0	-1.103E+2	-5.250E-1
10.1	4.672E-2	-8.820E-3	-1.907E+0	-1.134E+2	-5.245E-1
10.2	4.585E-2	-8.579E-3	-1.908E+0	-1.166E+2	-5.240E-1
10.3	4.501E-2	-8.346E-3	-1.910E+0	-1.198E+2	-5.236E-1
10.4	4.419E-2	-8.122E-3	-1.912E+0	-1.231E+2	-5.231E-1

Table 8. Sensitivity coefficients for $R = T_{\infty}/T_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	4.338E-2	-7.905E-3	-1.913E+0	-1.265E+2	-5.227E-1
10.6	4.260E-2	-7.696E-3	-1.915E+0	-1.299E+2	-5.222E-1
10.7	4.184E-2	-7.494E-3	-1.916E+0	-1.334E+2	-5.218E-1
10.8	4.110E-2	-7.299E-3	-1.918E+0	-1.370E+2	-5.214E-1
10.9	4.038E-2	-7.111E-3	-1.919E+0	-1.406E+2	-5.210E-1
11.0	3.968E-2	-6.929E-3	-1.921E+0	-1.443E+2	-5.207E-1
11.1	3.900E-2	-6.753E-3	-1.922E+0	-1.481E+2	-5.203E-1
11.2	3.833E-2	-6.583E-3	-1.923E+0	-1.519E+2	-5.199E-1
11.3	3.768E-2	-6.418E-3	-1.925E+0	-1.558E+2	-5.196E-1
11.4	3.705E-2	-6.259E-3	-1.926E+0	-1.598E+2	-5.192E-1
11.5	3.643E-2	-6.105E-3	-1.927E+0	-1.638E+2	-5.189E-1
11.6	3.583E-2	-5.956E-3	-1.928E+0	-1.679E+2	-5.186E-1
11.7	3.524E-2	-5.811E-3	-1.930E+0	-1.721E+2	-5.183E-1
11.8	3.466E-2	-5.672E-3	-1.931E+0	-1.763E+2	-5.180E-1
11.9	3.410E-2	-5.536E-3	-1.932E+0	-1.806E+2	-5.177E-1
12.0	3.356E-2	-5.405E-3	-1.933E+0	-1.850E+2	-5.174E-1
12.1	3.302E-2	-5.278E-3	-1.934E+0	-1.895E+2	-5.171E-1
12.2	3.250E-2	-5.155E-3	-1.935E+0	-1.940E+2	-5.168E-1
12.3	3.199E-2	-5.036E-3	-1.936E+0	-1.986E+2	-5.165E-1
12.4	3.149E-2	-4.920E-3	-1.937E+0	-2.033E+2	-5.163E-1
12.5	3.101E-2	-4.807E-3	-1.938E+0	-2.080E+2	-5.160E-1
12.6	3.053E-2	-4.698E-3	-1.939E+0	-2.128E+2	-5.157E-1
12.7	3.007E-2	-4.593E-3	-1.940E+0	-2.177E+2	-5.155E-1
12.8	2.961E-2	-4.490E-3	-1.941E+0	-2.227E+2	-5.153E-1
12.9	2.917E-2	-4.391E-3	-1.942E+0	-2.278E+2	-5.150E-1
13.0	2.874E-2	-4.294E-3	-1.943E+0	-2.329E+2	-5.148E-1
13.1	2.831E-2	-4.200E-3	-1.943E+0	-2.381E+2	-5.146E-1
13.2	2.790E-2	-4.109E-3	-1.944E+0	-2.434E+2	-5.143E-1
13.3	2.749E-2	-4.020E-3	-1.945E+0	-2.488E+2	-5.141E-1
13.4	2.709E-2	-3.934E-3	-1.946E+0	-2.542E+2	-5.139E-1
13.5	2.670E-2	-3.850E-3	-1.947E+0	-2.597E+2	-5.137E-1
13.6	2.632E-2	-3.769E-3	-1.947E+0	-2.653E+2	-5.135E-1
13.7	2.595E-2	-3.690E-3	-1.948E+0	-2.710E+2	-5.133E-1
13.8	2.558E-2	-3.613E-3	-1.949E+0	-2.768E+2	-5.131E-1
13.9	2.523E-2	-3.538E-3	-1.950E+0	-2.826E+2	-5.129E-1

Table 8. Sensitivity coefficients for $R = T_{\infty}/T_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	2.488E-2	-3.465E-3	-1.950E+0	-2.886E+2	-5.128E-1
14.1	2.453E-2	-3.394E-3	-1.951E+0	-2.946E+2	-5.126E-1
14.2	2.420E-2	-3.326E-3	-1.952E+0	-3.007E+2	-5.124E-1
14.3	2.387E-2	-3.258E-3	-1.952E+0	-3.069E+2	-5.122E-1
14.4	2.354E-2	-3.193E-3	-1.953E+0	-3.132E+2	-5.121E-1
14.5	2.323E-2	-3.130E-3	-1.954E+0	-3.195E+2	-5.119E-1
14.6	2.292E-2	-3.068E-3	-1.954E+0	-3.260E+2	-5.117E-1
14.7	2.262E-2	-3.007E-3	-1.955E+0	-3.325E+2	-5.116E-1
14.8	2.232E-2	-2.949E-3	-1.955E+0	-3.391E+2	-5.114E-1
14.9	2.203E-2	-2.891E-3	-1.956E+0	-3.459E+2	-5.113E-1
15.0	2.174E-2	-2.836E-3	-1.957E+0	-3.527E+2	-5.111E-1

Table 9. Sensitivity coefficients for $R = V_{\infty}/a_t$
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
0.1	9.990E-2	9.970E-1	9.980E-1	1.003E+0	1.002E+0
0.2	1.992E-1	9.881E-1	9.921E-1	1.012E+0	1.008E+0
0.3	2.973E-1	9.736E-1	9.823E-1	1.027E+0	1.018E+0
0.4	3.937E-1	9.539E-1	9.690E-1	1.048E+0	1.032E+0
0.5	4.880E-1	9.294E-1	9.524E-1	1.076E+0	1.050E+0
0.6	5.795E-1	9.010E-1	9.328E-1	1.110E+0	1.072E+0
0.7	6.680E-1	8.692E-1	9.107E-1	1.151E+0	1.098E+0
0.8	7.532E-1	8.347E-1	8.865E-1	1.198E+0	1.128E+0
0.9	8.349E-1	7.983E-1	8.606E-1	1.253E+0	1.162E+0
1.0	9.129E-1	7.607E-1	8.333E-1	1.315E+0	1.200E+0
1.1	9.870E-1	7.225E-1	8.052E-1	1.384E+0	1.242E+0
1.2	1.057E+0	6.841E-1	7.764E-1	1.462E+0	1.288E+0
1.3	1.124E+0	6.461E-1	7.474E-1	1.548E+0	1.338E+0
1.4	1.187E+0	6.089E-1	7.184E-1	1.642E+0	1.392E+0
1.5	1.246E+0	5.727E-1	6.897E-1	1.746E+0	1.450E+0
1.6	1.301E+0	5.379E-1	6.614E-1	1.859E+0	1.512E+0
1.7	1.353E+0	5.045E-1	6.337E-1	1.982E+0	1.578E+0
1.8	1.402E+0	4.727E-1	6.068E-1	2.116E+0	1.648E+0
1.9	1.448E+0	4.425E-1	5.807E-1	2.260E+0	1.722E+0
2.0	1.491E+0	4.141E-1	5.556E-1	2.415E+0	1.800E+0
2.1	1.531E+0	3.873E-1	5.313E-1	2.582E+0	1.882E+0
2.2	1.568E+0	3.622E-1	5.081E-1	2.761E+0	1.968E+0
2.3	1.603E+0	3.387E-1	4.859E-1	2.952E+0	2.058E+0
2.4	1.636E+0	3.168E-1	4.647E-1	3.157E+0	2.152E+0
2.5	1.667E+0	2.963E-1	4.444E-1	3.375E+0	2.250E+0
2.6	1.695E+0	2.772E-1	4.252E-1	3.607E+0	2.352E+0
2.7	1.722E+0	2.595E-1	4.068E-1	3.854E+0	2.458E+0
2.8	1.747E+0	2.430E-1	3.894E-1	4.115E+0	2.568E+0
2.9	1.771E+0	2.277E-1	3.729E-1	4.392E+0	2.682E+0
3.0	1.793E+0	2.134E-1	3.571E-1	4.685E+0	2.800E+0
3.1	1.814E+0	2.002E-1	3.422E-1	4.995E+0	2.922E+0
3.2	1.833E+0	1.879E-1	3.281E-1	5.321E+0	3.048E+0
3.3	1.851E+0	1.765E-1	3.147E-1	5.665E+0	3.178E+0
3.4	1.868E+0	1.659E-1	3.019E-1	6.027E+0	3.312E+0

Table 9. Sensitivity coefficients for $R = V_{\infty}/a_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
3.5	1.884E+0	1.561E-1	2.899E-1	6.408E+0	3.450E+0
3.6	1.899E+0	1.469E-1	2.784E-1	6.808E+0	3.592E+0
3.7	1.914E+0	1.384E-1	2.675E-1	7.227E+0	3.738E+0
3.8	1.927E+0	1.304E-1	2.572E-1	7.666E+0	3.888E+0
3.9	1.940E+0	1.231E-1	2.474E-1	8.126E+0	4.042E+0
4.0	1.952E+0	1.162E-1	2.381E-1	8.607E+0	4.200E+0
4.1	1.963E+0	1.098E-1	2.293E-1	9.110E+0	4.362E+0
4.2	1.974E+0	1.038E-1	2.208E-1	9.635E+0	4.528E+0
4.3	1.984E+0	9.820E-2	2.129E-1	1.018E+1	4.698E+0
4.4	1.993E+0	9.299E-2	2.053E-1	1.075E+1	4.872E+0
4.5	2.002E+0	8.812E-2	1.980E-1	1.135E+1	5.050E+0
4.6	2.011E+0	8.356E-2	1.911E-1	1.197E+1	5.232E+0
4.7	2.019E+0	7.929E-2	1.846E-1	1.261E+1	5.418E+0
4.8	2.027E+0	7.530E-2	1.783E-1	1.328E+1	5.608E+0
4.9	2.034E+0	7.155E-2	1.724E-1	1.398E+1	5.802E+0
5.0	2.041E+0	6.804E-2	1.667E-1	1.470E+1	6.000E+0
5.1	2.048E+0	6.474E-2	1.612E-1	1.545E+1	6.202E+0
5.2	2.054E+0	6.165E-2	1.561E-1	1.622E+1	6.408E+0
5.3	2.060E+0	5.874E-2	1.511E-1	1.703E+1	6.618E+0
5.4	2.066E+0	5.600E-2	1.464E-1	1.786E+1	6.832E+0
5.5	2.071E+0	5.342E-2	1.418E-1	1.872E+1	7.050E+0
5.6	2.077E+0	5.099E-2	1.375E-1	1.961E+1	7.272E+0
5.7	2.082E+0	4.871E-2	1.334E-1	2.053E+1	7.498E+0
5.8	2.086E+0	4.655E-2	1.294E-1	2.148E+1	7.728E+0
5.9	2.091E+0	4.451E-2	1.256E-1	2.247E+1	7.962E+0
6.0	2.095E+0	4.259E-2	1.220E-1	2.348E+1	8.200E+0
6.1	2.099E+0	4.077E-2	1.185E-1	2.453E+1	8.442E+0
6.2	2.103E+0	3.905E-2	1.151E-1	2.561E+1	8.688E+0
6.3	2.107E+0	3.742E-2	1.119E-1	2.672E+1	8.938E+0
6.4	2.111E+0	3.588E-2	1.088E-1	2.787E+1	9.192E+0
6.5	2.114E+0	3.442E-2	1.058E-1	2.905E+1	9.450E+0
6.6	2.118E+0	3.304E-2	1.030E-1	3.027E+1	9.712E+0
6.7	2.121E+0	3.173E-2	1.002E-1	3.152E+1	9.978E+0
6.8	2.124E+0	3.048E-2	9.758E-2	3.281E+1	1.025E+1
6.9	2.127E+0	2.930E-2	9.504E-2	3.413E+1	1.052E+1

Table 9. Sensitivity coefficients for $R = V_{\infty}/a_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
7.0	2.130E+0	2.818E-2	9.259E-2	3.549E+1	1.080E+1
7.1	2.133E+0	2.711E-2	9.024E-2	3.689E+1	1.108E+1
7.2	2.135E+0	2.609E-2	8.797E-2	3.833E+1	1.137E+1
7.3	2.138E+0	2.512E-2	8.578E-2	3.980E+1	1.166E+1
7.4	2.140E+0	2.420E-2	8.367E-2	4.132E+1	1.195E+1
7.5	2.143E+0	2.332E-2	8.163E-2	4.287E+1	1.225E+1
7.6	2.145E+0	2.249E-2	7.967E-2	4.447E+1	1.255E+1
7.7	2.147E+0	2.169E-2	7.777E-2	4.611E+1	1.286E+1
7.8	2.149E+0	2.093E-2	7.594E-2	4.778E+1	1.317E+1
7.9	2.152E+0	2.020E-2	7.417E-2	4.950E+1	1.348E+1
8.0	2.154E+0	1.951E-2	7.246E-2	5.126E+1	1.380E+1
8.1	2.155E+0	1.884E-2	7.081E-2	5.307E+1	1.412E+1
8.2	2.157E+0	1.821E-2	6.921E-2	5.492E+1	1.445E+1
8.3	2.159E+0	1.760E-2	6.767E-2	5.681E+1	1.478E+1
8.4	2.161E+0	1.702E-2	6.617E-2	5.875E+1	1.511E+1
8.5	2.162E+0	1.647E-2	6.472E-2	6.073E+1	1.545E+1
8.6	2.164E+0	1.593E-2	6.332E-2	6.276E+1	1.579E+1
8.7	2.166E+0	1.543E-2	6.197E-2	6.483E+1	1.614E+1
8.8	2.167E+0	1.494E-2	6.065E-2	6.695E+1	1.649E+1
8.9	2.169E+0	1.447E-2	5.938E-2	6.912E+1	1.684E+1
9.0	2.170E+0	1.402E-2	5.814E-2	7.133E+1	1.720E+1
9.1	2.171E+0	1.359E-2	5.694E-2	7.360E+1	1.756E+1
9.2	2.173E+0	1.317E-2	5.578E-2	7.591E+1	1.793E+1
9.3	2.174E+0	1.278E-2	5.465E-2	7.827E+1	1.830E+1
9.4	2.175E+0	1.239E-2	5.356E-2	8.068E+1	1.867E+1
9.5	2.177E+0	1.203E-2	5.249E-2	8.315E+1	1.905E+1
9.6	2.178E+0	1.167E-2	5.146E-2	8.566E+1	1.943E+1
9.7	2.179E+0	1.133E-2	5.046E-2	8.822E+1	1.982E+1
9.8	2.180E+0	1.101E-2	4.949E-2	9.084E+1	2.021E+1
9.9	2.181E+0	1.069E-2	4.854E-2	9.351E+1	2.060E+1
10.0	2.182E+0	1.039E-2	4.762E-2	9.623E+1	2.100E+1
10.1	2.183E+0	1.010E-2	4.672E-2	9.901E+1	2.140E+1
10.2	2.184E+0	9.819E-3	4.585E-2	1.018E+2	2.181E+1
10.3	2.185E+0	9.549E-3	4.501E-2	1.047E+2	2.222E+1
10.4	2.186E+0	9.288E-3	4.419E-2	1.077E+2	2.263E+1

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Table 9. Sensitivity coefficients for $R = V_{\infty}/a_t$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
10.5	2.187E+0	9.036E-3	4.338E-2	1.107E+2	2.305E+1
10.6	2.188E+0	8.794E-3	4.260E-2	1.137E+2	2.347E+1
10.7	2.189E+0	8.560E-3	4.184E-2	1.168E+2	2.390E+1
10.8	2.190E+0	8.334E-3	4.110E-2	1.200E+2	2.433E+1
10.9	2.190E+0	8.116E-3	4.038E-2	1.232E+2	2.476E+1
11.0	2.191E+0	7.905E-3	3.968E-2	1.265E+2	2.520E+1
11.1	2.192E+0	7.701E-3	3.900E-2	1.298E+2	2.564E+1
11.2	2.193E+0	7.505E-3	3.833E-2	1.332E+2	2.609E+1
11.3	2.194E+0	7.315E-3	3.768E-2	1.367E+2	2.654E+1
11.4	2.194E+0	7.131E-3	3.705E-2	1.402E+2	2.699E+1
11.5	2.195E+0	6.953E-3	3.643E-2	1.438E+2	2.745E+1
11.6	2.196E+0	6.781E-3	3.583E-2	1.475E+2	2.791E+1
11.7	2.196E+0	6.615E-3	3.524E-2	1.512E+2	2.838E+1
11.8	2.197E+0	6.454E-3	3.466E-2	1.549E+2	2.885E+1
11.9	2.198E+0	6.298E-3	3.410E-2	1.588E+2	2.932E+1
12.0	2.198E+0	6.147E-3	3.356E-2	1.627E+2	2.980E+1
12.1	2.199E+0	6.001E-3	3.302E-2	1.666E+2	3.028E+1
12.2	2.199E+0	5.859E-3	3.250E-2	1.707E+2	3.077E+1
12.3	2.200E+0	5.722E-3	3.199E-2	1.748E+2	3.126E+1
12.4	2.201E+0	5.589E-3	3.149E-2	1.789E+2	3.175E+1
12.5	2.201E+0	5.460E-3	3.101E-2	1.831E+2	3.225E+1
12.6	2.202E+0	5.335E-3	3.053E-2	1.874E+2	3.275E+1
12.7	2.202E+0	5.214E-3	3.007E-2	1.918E+2	3.326E+1
12.8	2.203E+0	5.096E-3	2.961E-2	1.962E+2	3.377E+1
12.9	2.203E+0	4.982E-3	2.917E-2	2.007E+2	3.428E+1
13.0	2.204E+0	4.871E-3	2.874E-2	2.053E+2	3.480E+1
13.1	2.204E+0	4.764E-3	2.831E-2	2.099E+2	3.532E+1
13.2	2.205E+0	4.659E-3	2.790E-2	2.146E+2	3.585E+1
13.3	2.205E+0	4.558E-3	2.749E-2	2.194E+2	3.638E+1
13.4	2.206E+0	4.459E-3	2.709E-2	2.243E+2	3.691E+1
13.5	2.206E+0	4.363E-3	2.670E-2	2.292E+2	3.745E+1
13.6	2.206E+0	4.270E-3	2.632E-2	2.342E+2	3.799E+1
13.7	2.207E+0	4.180E-3	2.595E-2	2.392E+2	3.854E+1
13.8	2.207E+0	4.092E-3	2.558E-2	2.444E+2	3.909E+1
13.9	2.208E+0	4.007E-3	2.523E-2	2.496E+2	3.964E+1

Table 9. Sensitivity coefficients for $R = V_{\infty}/a_1$ (Continued)
 $\gamma = 1.40$

M_{∞}	R	$\frac{\partial R}{\partial M_{\infty}}$	$\frac{M_{\infty}}{R} \frac{\partial R}{\partial M_{\infty}}$	$\frac{\partial M_{\infty}}{\partial R}$	$\frac{R}{M_{\infty}} \frac{\partial M_{\infty}}{\partial R}$
14.0	2.208E+0	3.923E-3	2.488E-2	2.549E+2	4.020E+1
14.1	2.208E+0	3.843E-3	2.453E-2	2.602E+2	4.076E+1
14.2	2.209E+0	3.764E-3	2.420E-2	2.657E+2	4.133E+1
14.3	2.209E+0	3.687E-3	2.387E-2	2.712E+2	4.190E+1
14.4	2.210E+0	3.613E-3	2.354E-2	2.768E+2	4.247E+1
14.5	2.210E+0	3.540E-3	2.323E-2	2.825E+2	4.305E+1
14.6	2.210E+0	3.470E-3	2.292E-2	2.882E+2	4.363E+1
14.7	2.211E+0	3.401E-3	2.262E-2	2.940E+2	4.422E+1
14.8	2.211E+0	3.334E-3	2.232E-2	2.999E+2	4.481E+1
14.9	2.211E+0	3.269E-3	2.203E-2	3.059E+2	4.540E+1
15.0	2.212E+0	3.205E-3	2.174E-2	3.120E+2	4.600E+1

Table 10. Sensitivity coefficients for Re, linear viscosity relation
 $\gamma = 1.40$

M_∞	Re/Re*	$\frac{M_\infty}{Re} \frac{\partial Re}{\partial M_\infty}$
0.1	1.434E-1	9.920E-1
0.2	2.834E-1	9.683E-1
0.3	4.169E-1	9.293E-1
0.4	5.408E-1	8.760E-1
0.5	6.531E-1	8.095E-1
0.6	7.518E-1	7.313E-1
0.7	8.361E-1	6.430E-1
0.8	9.054E-1	5.461E-1
0.9	9.598E-1	4.423E-1
1.0	1.000E+0	3.333E-1
1.1	1.027E+0	2.206E-1
1.2	1.042E+0	1.056E-1
1.3	1.046E+0	-1.046E-2
1.4	1.040E+0	-1.264E-1
1.5	1.027E+0	-2.414E-1
1.6	1.008E+0	-3.545E-1
1.7	9.831E-1	-4.651E-1
1.8	9.544E-1	-5.728E-1
1.9	9.227E-1	-6.771E-1
2.0	8.889E-1	-7.778E-1
2.1	8.538E-1	-8.746E-1
2.2	8.180E-1	-9.675E-1
2.3	7.820E-1	-1.056E+0
2.4	7.463E-1	-1.141E+0
2.5	7.111E-1	-1.222E+0
2.6	6.768E-1	-1.299E+0
2.7	6.435E-1	-1.373E+0
2.8	6.114E-1	-1.442E+0
2.9	5.806E-1	-1.509E+0
3.0	5.510E-1	-1.571E+0
3.1	5.228E-1	-1.631E+0
3.2	4.960E-1	-1.688E+0
3.3	4.705E-1	-1.741E+0
3.4	4.463E-1	-1.792E+0

Table 10. Sensitivity coefficients for Re, linear viscosity relation
 $\gamma = 1.40$ (Continued)

M_∞	Re/Re^*	$\frac{M_\infty}{Re} \frac{\partial Re}{\partial M_\infty}$
3.5	4.234E-1	-1.841E+0
3.6	4.018E-1	-1.886E+0
3.7	3.813E-1	-1.930E+0
3.8	3.620E-1	-1.971E+0
3.9	3.437E-1	-2.010E+0
4.0	3.265E-1	-2.048E+0
4.1	3.103E-1	-2.083E+0
4.2	2.950E-1	-2.117E+0
4.3	2.805E-1	-2.149E+0
4.4	2.669E-1	-2.179E+0
4.5	2.541E-1	-2.208E+0
4.6	2.420E-1	-2.235E+0
4.7	2.306E-1	-2.262E+0
4.8	2.198E-1	-2.287E+0
4.9	2.096E-1	-2.311E+0
5.0	2.000E-1	-2.333E+0
5.1	1.909E-1	-2.355E+0
5.2	1.824E-1	-2.376E+0
5.3	1.743E-1	-2.396E+0
5.4	1.666E-1	-2.415E+0
5.5	1.593E-1	-2.433E+0
5.6	1.525E-1	-2.450E+0
5.7	1.460E-1	-2.467E+0
5.8	1.398E-1	-2.482E+0
5.9	1.340E-1	-2.498E+0
6.0	1.285E-1	-2.512E+0
6.1	1.233E-1	-2.526E+0
6.2	1.183E-1	-2.540E+0
6.3	1.136E-1	-2.552E+0
6.4	1.091E-1	-2.565E+0
6.5	1.048E-1	-2.577E+0
6.6	1.008E-1	-2.588E+0
6.7	9.691E-2	-2.599E+0
6.8	9.324E-2	-2.610E+0
6.9	8.975E-2	-2.620E+0

Table 10. Sensitivity coefficients for Re, linear viscosity relation
 $\gamma = 1.40$ (Continued)

M_∞	Re/Re*	$\frac{M_\infty}{Re} \frac{\partial Re}{\partial M_\infty}$
7.0	8.642E-2	-2.630E+0
7.1	8.325E-2	-2.639E+0
7.2	8.023E-2	-2.648E+0
7.3	7.735E-2	-2.657E+0
7.4	7.460E-2	-2.665E+0
7.5	7.197E-2	-2.673E+0
7.6	6.946E-2	-2.681E+0
7.7	6.707E-2	-2.689E+0
7.8	6.478E-2	-2.696E+0
7.9	6.259E-2	-2.703E+0
8.0	6.049E-2	-2.710E+0
8.1	5.849E-2	-2.717E+0
8.2	5.657E-2	-2.723E+0
8.3	5.473E-2	-2.729E+0
8.4	5.297E-2	-2.735E+0
8.5	5.128E-2	-2.741E+0
8.6	4.966E-2	-2.747E+0
8.7	4.810E-2	-2.752E+0
8.8	4.661E-2	-2.757E+0
8.9	4.518E-2	-2.762E+0
9.0	4.381E-2	-2.767E+0
9.1	4.249E-2	-2.772E+0
9.2	4.122E-2	-2.777E+0
9.3	4.000E-2	-2.781E+0
9.4	3.882E-2	-2.786E+0
9.5	3.770E-2	-2.790E+0
9.6	3.661E-2	-2.794E+0
9.7	3.556E-2	-2.798E+0
9.8	3.456E-2	-2.802E+0
9.9	3.359E-2	-2.806E+0
10.0	3.265E-2	-2.810E+0
10.1	3.175E-2	-2.813E+0
10.2	3.088E-2	-2.817E+0
10.3	3.005E-2	-2.820E+0
10.4	2.924E-2	-2.823E+0

Table 10. Sensitivity coefficients for Re, linear viscosity relation
 $\gamma = 1.40$ (Continued)

M_∞	Re/Re^*	$\frac{M_\infty}{Re} \frac{\partial Re}{\partial M_\infty}$
10.5	2.846E-2	-2.826E+0
10.6	2.771E-2	-2.830E+0
10.7	2.698E-2	-2.833E+0
10.8	2.628E-2	-2.836E+0
10.9	2.560E-2	-2.838E+0
11.0	2.494E-2	-2.841E+0
11.1	2.431E-2	-2.844E+0
11.2	2.370E-2	-2.847E+0
11.3	2.310E-2	-2.849E+0
11.4	2.253E-2	-2.852E+0
11.5	2.198E-2	-2.854E+0
11.6	2.144E-2	-2.857E+0
11.7	2.092E-2	-2.859E+0
11.8	2.042E-2	-2.861E+0
11.9	1.993E-2	-2.864E+0
12.0	1.946E-2	-2.866E+0
12.1	1.900E-2	-2.868E+0
12.2	1.856E-2	-2.870E+0
12.3	1.813E-2	-2.872E+0
12.4	1.771E-2	-2.874E+0
12.5	1.731E-2	-2.876E+0
12.6	1.691E-2	-2.878E+0
12.7	1.653E-2	-2.880E+0
12.8	1.616E-2	-2.882E+0
12.9	1.581E-2	-2.883E+0
13.0	1.546E-2	-2.885E+0
13.1	1.512E-2	-2.887E+0
13.2	1.479E-2	-2.888E+0
13.3	1.447E-2	-2.890E+0
13.4	1.416E-2	-2.892E+0
13.5	1.386E-2	-2.893E+0
13.6	1.357E-2	-2.895E+0
13.7	1.328E-2	-2.896E+0
13.8	1.301E-2	-2.898E+0
13.9	1.274E-2	-2.899E+0

Table 10. Sensitivity coefficients for Re, linear viscosity relation
 $\gamma = 1.40$ (Continued)

M_∞	Re/Re^*	$\frac{M_\infty}{Re} \frac{\partial Re}{\partial M_\infty}$
14.0	1.247E-2	-2.900E+0
14.1	1.222E-2	-2.902E+0
14.2	1.197E-2	-2.903E+0
14.3	1.173E-2	-2.905E+0
14.4	1.150E-2	-2.906E+0
14.5	1.127E-2	-2.907E+0
14.6	1.104E-2	-2.908E+0
14.7	1.083E-2	-2.910E+0
14.8	1.061E-2	-2.911E+0
14.9	1.041E-2	-2.912E+0
15.0	1.021E-2	-2.913E+0

Appendix

Program DFDX, Sensitivity Coefficient Estimation by Sequential Perturbation

```
C  PROGRAM DFDX

C  This code uses a finite differencing technique with a central difference to
C  estimate sensitivity coefficients for uncertainty analyses. The function to be
C  evaluated, F, is defined in FUNCTION F(X) and has NX (up to 10) variables,
C  denoted as X(I). The increment for the difference is RDX which is a relative
C  value, i.e., the actual increment is given by, INC = RDX*X(I). If desired, an
C  absolute increment can be used as indicated in the comments. In this case, the
C  value of the increment, DX, will be a function of the particular variable and
C  should be input as an array of NX values. The user must provide the function
C  subprogram. For this example, F = Reynolds number with Minf, Pt, Tt and
C  L as the four variables, X(1) thru X(4). A linear viscosity relation is assumed.
C  Program output is the absolute sensitivity coefficient, DFDX (= dF/dX), and the
C  relative sensitivity coefficient, RDFDX [= (X/F) (dF/dX)], for each variable.

C
C  DIMENSION X(10),DFDX(10),RDFDX(10)
C...  define relative interval for finite difference
      RDX=0.005
C...  input number of variables and their nominal values
      NX=0
      DO WHILE (NX.LT.1 .OR. NX.GT.10)
          WRITE(6,/'No. of variables in function, F, is: ",$"')
          READ(5,*) NX
      ENDDO
      WRITE(6,/'Nominal values for X(1) thru X("I1,"):"') NX
      READ(5,*) (X(I),I=1,NX)
C...  calculate value of function for nominal values of variables
```

```

      FNOM=F(X)
C... calculate derivative, DFDX, for each variable
      DO I=1,NX
          XSV=X(I) ! save nominal value of X(I)
          XN=XSV*(1.0-RDX/2.0) ! for absolute dx, XN=XSV-DX/2.0
          X(I)=XN
          FN=F(X)
          XP=XSV*(1.0+RDX/2.0) ! for absolute dx, XN=XSV+DX/2.0
          X(I)=XP
          FP=F(X)
          DFDX(I)=(FP-FN)/(XP-XN) ! absolute sensitivity coefficient
          X(I)=XSV ! restore nominal value of X(I)
          RDFDX(I)=DFDX(I)/(FNOM/X(I)) ! relative sensitivity coefficient
      ENDDO
C... print results
      WRITE(50,'(5X,"I",9X,"X",11X,"dF/dX",5X,"(dF/dX)/(F/X)")')
      DO I=1,NX
          WRITE(50,'(I6,1P3E14.4E1)') I,X(I),DFDX(I),RDFDX(I)
      ENDDO
      WRITE(50,'(/" F(nominal) =",1PE11.4E1/)') FNOM
      END
      FUNCTION F(X)
      DIMENSION X(*)
      REAL MUINF
C      F = Reynolds number = RHOinf * Vinf * L / MUinf
C      The linear viscosity relation, MUinf = C * Tinf is used.
C      X(1) = Minf
C      X(2) = Pt, lbf / in2
C      X(3) = Tt, deg R
C      X(4) = L, ft
      G=1.400 ! gamma, ratio of specific heats
      RG=1716.55 ! gas constant, ft2 / sec2-deg R
      C=8.05E-10 ! viscosity constant, lbf-sec / ft2
      FIP=1.0 + (G-1.0) / 2.0 * X(1)**2 ! isentropic flow function
      PINF=X(2) / FIP**(G / (G-1.0)) ! Pinf, Eq. (44)
      TINF=X(3) / FIP ! Tinf, Eq. (82)
      RHOINF=PINF * 144.0 / (RG * TINF) ! RHOinf, Eq. (114)
      VINF=X(1) * SQRT(G * RG * TINF) ! Vinf, Eq. (115)

```

```

MUINF=C * TINF                                ! MUinf, Eq. (136)
F=RHOINF * VINP * X(4) / MUINF                ! Re, Eq. (135)
RETURN
END

```

Typical input:

No. of variables in function, F, is: 4

Nominal values of X(1) thru X(4):

5.0,150.0,800.0,1.0

Typical output:

I	X	dF/dX	(X/F)*(dF/dX)	
1	5.0000E+0	-2.1951E+6	-2.3334E+0	[X(1) = M_∞]
2	1.5000E+2	3.1357E+4	1.0000E+0	[X(2) = p_t]
3	8.0000E+2	-8.8190E+3	-1.5000E+0	[X(3) = T_t]
4	1.0000E+0	4.7035E+6	1.0000E+0	[X(4) = l]
F(nominal) =		4.7036E+6	[F = Re]	

From the output, it is seen that the Reynolds number relative sensitivity coefficient, $(X/F)*(dF/dX)$, for M_∞ is -2.3334 which agrees with the value in Table 10 for $M_\infty = 5$. The relative sensitivity coefficients for p_t , T_t and l , are 1.0, -1.5 and 1.0, respectively, which agree with the values in Eq. (145).

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