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PROGRESS REPORT

STUDY OF EFFECTS OF SMALL PERTURBATIONS ON CHAOTIC SYSTEMS

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INTRODUCTION

The following report summarizes our research activities under the U.S. Department of Energy, Scientific Computing Staff Office of Energy Research, Contract No. DE-FG05-87ER25036. We have organized the report under the following categories:

1. Work done in the period January 1992 through December 1992.
 - (a) Summary of the research done.
 - (b) Papers published in refereed journals or refereed proceedings.
2. Appendix of a few recent reprints.

I. WORK DONE IN THE PERIOD JANUARY 1992 THROUGH DECEMBER 1992

A. Summary of the Research Done

1. "Prevalence"

We introduced a measure-theoretic condition for a property to hold "almost everywhere" on an infinite dimensional vector space, with particular emphasis on function spaces such as C^k and L^p . Like the concept of "Lebesgue almost every" on finite dimensional spaces, our notion of "prevalence" is translation invariant. Instead of using a particular measure on the entire space, we define prevalence in terms of the class of all probability measures with compact support. Prevalence is a more appropriate condition than the topological concepts of "open and dense" or "generic" when one desires a probabilistic result on the likelihood of a given property on a function space. We give several examples of properties which hold "almost everywhere" in the sense of prevalence. For instance, we prove that almost every C^1 map on R^n has the property that all of its periodic orbits are hyperbolic.

2. "Border Crossing Bifurcations"

We investigate the phenomenon of border bifurcation. These occur in piecewise smooth systems, which in turn occur in a variety of situations involving optimization and/or constraints. We have presented qualitatively different types of border bifurcations for piecewise smooth maps. Degree theory, which is a useful tool analyzing bifurcation problems involving periodic orbits only, cannot always be applied as the examples show. Therefore, new methods were developed to attack the problem of border bifurcation phenomenon.

3. "Numerical Procedure for Computing the Dimension"

Based on our experience with the development of saddle straddle trajectories and the equality for Hausdorff, box-counting and uncertainty dimension, we developed a numerical procedure for computing the dimension of one dimensionally unstable attractor. We show that the method is guaranteed to work for a class of

one-dimensionally unstable hyperbolic attractors.

4. “Generalized Saddle Straddle Trajectories Methods”

In many systems one observes transient chaos. This is due to the presence of chaotic saddles in the system. So far we have a numerical method (SST method) that generates a trajectory on a chaotic saddle (that is, a chaotic set that is not attracting) that has been implemented in the Yorke’s DYNAMICS problem. We have shown that the SST method is guaranteed to work for a class of one-dimensionally unstable saddle-hyperbolic systems.

We also developed numerical methods for finding trajectories on chaotic saddles which are either at least two-dimensionally unstable saddle-hyperbolic sets or include points that have a different unstable dimension. A problem related to shadowing and our SST method is the following. Assume that for a given system, the SST method generates a numerical trajectory that can be computed as long as one wishes. Is it true that the numerical trajectory can be shadowed by a true trajectory?

5. “Finding Typical Trajectories Numerically”

Finding typical numerical trajectories is a central problem of simulating of chaotic systems. A fundamental question is whether or not the behavior of a typical trajectory is compatible with the behavior of some numerically generated trajectory. For a variety of problems, we are not interested in tracking a particular trajectory but rather we need information about the behavior of typical trajectories. In such cases, the problem becomes first of showing that numerical trajectories are shadowed by true trajectories, and second, that these true trajectories are typical. We believe these true trajectories are usually in fact “typical”. Examples are common where the specific initial condition is not important, but the long term behavior is important. There are examples where one wants to know the statistical behavior.

We have investigated a theoretical foundation for methods producing typical numerical trajectories. The different methods for different systems might vary slightly, depending on some global properties of the system that is considered. Our objective was to find a numerical procedure which produces a numerical trajectory which has

the properties (1) the trajectory is shadowable, and (2) the trajectory is distributed throughout the attractor in accord with the natural measure. Of course, the latter statement needed to be made precise, since only a finite number of iterates will be available. We believe we have a procedure that will work for low dimensional chaotic systems, but proving that it will work will be difficult. Our objective will be first to prove that it works for uniformly hyperbolic attractors. Some progress has been made on investigating some particular systems, such as maps near the map $x_{n+1} = 3x_n \bmod 1$. We developed methods that are guaranteed to generate (nearly) typical numerical trajectories in a variety of dynamical systems. Furthermore we are able to obtain useful estimates for how close our trajectories are to being typical.

B. Papers Published in Refereed Journals or Refereed Proceedings during the Current Proposal

1. Controlling Chaos and Targeting

We have introduced the concept whereby one converts chaotic motion to one of the infinite number of time-periodic motions embedded in the chaotic set by making only small time dependent perturbations in an available system parameter. We have also introduced the concept of targeting in which one rapidly takes the system state from an initial point to some specified targeting region in phase space by applying small controls. Those ideas and basic methods for accompanying that have been published in the following papers:

1. "Controlling Chaos," E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196-1199 (1990).
2. "Using Chaos to Direct Trajectories to Targets," T. Shinbrot, E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **65**, 3215-3218 (1990).
3. "Controlling Chaotic Dynamical Systems," F. J. Romeiras, E. Ott, C. Grebogi, and W. P. Dayawansa, IEEE Proc. of the 1991 American Control Conference (American Automatic Control Council, IEEE Service Center, Piscataway, NJ, 1991), pp. 1112-1119.

4. "Using the Sensitive Dependence of Chaos (the "Butterfly Effect") to Direct Orbits to Targets in an Experimental Chaotic System," T. Shinbrot, W. Ditto, C. Grebogi, E. Ott, M. Spano, and J. A. Yorke, *Phys. Rev. Lett.* **68**, 2863-2866 (1992).
5. "Controlling Chaotic Dynamical Systems," F. J. Romeiras, C. Grebogi, E. Ott, and W. P. Dayawansa, *Physics* **58D**, 165-192 (1992).
6. "Controlling Chaotic Dynamical Systems," E. Ott, C. Grebogi, and J. A. Yorke, in *CHAOS: Soviet-American Perspective on Nonlinear Science* (American Institute of Physics, 1990), pp. 153-172.
7. "Feedback Control of Chaotic Systems," R. J. Romeiras, C. Grebogi, E. Ott, and W. P. Dayawansa, in *Chaotic Dynamics: Theory and Practice*, Ed. T. Bountis (Plenum, London, 1992), pp. 177-193.

2. Crises and Chaotic Transients

Crises - sudden macroscopic changes in chaotic attractors due to their collision with unstable orbits - and the associated chaotic transients have been extensively studied by us in a series of papers listed below.

8. "Scaling Law for Characteristic Times of Noise-Induced Crises," J. C. Sommerer, E. Ott, and C. Grebogi, *Phys. Rev. A* **43**, 1754-1769 (1991).
9. "Experimental Confirmation of the Theory for Critical Exponent of Crises," J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. Spano, *Phys. Rev. Lett.* **153A**, 105-109 (1991).
10. "Experimental Confirmation of the Scaling Theory for Noise-Induced Crises," J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. L. Spano, *Phys. Rev. Lett.* **66**, 1947-1950 (1991).
11. "Determination of Crisis Parameter Values by Direct Observation of Manifold Tangencies," J. C. Sommerer and C. Grebogi, *Int. J. Bif. and Chaos* **2**, 383-396 (1992).

3. The Dimension of Chaotic Attractors

In this area we have investigated the various definitions of attractor dimension, their possible relationship to Lyapunov numbers, their implications for the effect of noise on attractors, and numerical techniques for their calculation. Papers on this topic are listed below.

12. "Multifractal Properties of Snapshot Attractors of Random Maps," F. J. Romeiras, C. Grebogi, and E. Ott, Phys. Rev. A **41**, 784-799 (1990).
13. "The Spectrum of Fractal Dimensions of Passively Convected Scalar Gradients in Chaotic Fluid Flows," F. Varosi, T. M. Antonsen, and E. Ott, Phys. Fluids A **3**, 1017-1028 (1991).

4. Other Research

Other research papers not falling in the above three categories are the following. These cover work in quasiperiodicity, Hamiltonian systems, windows and their scaling, fat fractals, shadowing, etc.

14. "Shadowing of Physical Trajectories in Chaotic Dynamics: Containment and Refinement," C. Grebogi, S. M. Hammel, J. A. Yorke, and Tim Sauer, Phys. Rev. Lett. **65**, 1527-1530 (1990).
15. "Bifurcation to Chaotic Scattering," S. Bleher, C. Grebogi, and E. Ott, Physica **46D**, 87-121 (1990).
16. "Transition to Chaotic Scattering," M. Ding, C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. A **42**, 7025-7040 (1990).
17. "Analyzing Chaos: A Visual Essay in Nonlinear Dynamics," C. Grebogi, E. Ott, F. Varosi, and J. A. Yorke, in Energy Sciences Supercomputing, U.S. DOE National Energy Research Supercomputer Center (1990), p. 30-33.
18. "Noise Reduction: Finding the Simplest Dynamical System Consistent with the Data," E. Kostelich and J. A. Yorke, Physica D **41**, 183-196 (1990).

19. "Modelling Non-Monogamous Heterosexual Transmission of AIDS," I. Kramer, J. A. Yorke, and E. D. Yorke, *Math. Comput. Modelling* **13**, 99-107 (1990).
20. "Sudden Change in the Size of Chaotic Attractors: How Does It Occur?," Y. Lai, C. Grebogi, and J. A. Yorke, in *Applications of Chaos*, Electric Power Research Institute Workshop on the Applications of Chaos (EPRI, 1990).
21. "Embedology," T. Sauer, J. A. Yorke, and M. Casdagli, *J. Stat. Phys.* **65**, 579-616 (1991).
22. "Fractal Distribution of Floaters on a Fluid Surface and the Transition to Chaos for Random Maps," L. Yu, E. Ott, and Q. Chen, *Physica* **53D** 102-124 (1991).
23. "Calculating Topological Entropies of Chaotic Dynamical Systems," Q. Chen, E. Ott, and L. P. Hurd, *Phys. Lett.* **156A**, 48-52 (1991).
24. "Massive Bifurcation of Chaotic Scattering," M. Ding, C. Grebogi, E. Ott, and J. A. Yorke, *Phys. Lett.* **153A**, 21-26 (1991).
25. "Cubic Maps as Models of Two Dimensional Antimonotonicity," S. P. Dawson and C. Grebogi, *Chaos, Solitons and Fractals* **1**, 137-144 (1991).
26. "Chaotic Scattering in Time-Dependent Hamiltonian Systems," Y. C. Lai and C. Grebogi, *Int. J. Bif. and Chaos* **1**, 667-679 (1991).
27. "Pointwise Dimension and Unstable Periodic Orbits," C. Grebogi, E. Ott, and J. A. Yorke, in *Essays on Classical and Quantum Dynamics*, Ed. H. Uberall (Gordon and Breach, 1991), p. 57-62.
28. "Using Chaos to Direct Orbits to Targets in Systems Describable by a One-Dimensional Map," T. Shinbrot, E. Ott, C. Grebogi, and J. A. Yorke, *Phys. Rev. A* **45**, 4165-4168 (1992).
29. "Antimonotonicity: Inevitable Reversals of Period-Doubling Cascades," S. P. Dawson, C. Grebogi, I. Kan, H. Koçak, and J. A. Yorke, *Phys. Lett.* **162A**, 249-254 (1992).

30. "Chaos in a Double Pendulum," T. Shinbrot, C. Grebogi, J. Wisdom, and J. A. Yorke, Am. J. Phys. **60**, 491-499 (1992).
31. "Antimonotonicity: Concurrent Creation and Annihilation of Periodic Orbits," I. Kan, H. Koçak, and J. A. Yorke, Ann. Amer. Math. Soc. **136**, 219-252 (1992).
32. "Border Collision Bifurcations Including "Period Two to Period Three" Bifurcation for Piecewise Smooth Systems," H. E. Nusse and J. A. Yorke, Phys. D. **57**, 39-57 (1992).
33. "Prevalence: A Translation-Invariant "Almost Every" on Infinite Dimensional Spaces," B. Hunt, T. Sauer, and J. A. Yorke, Bull. Amer. Math. Soc. **27**, 217-238 (1992).
34. "Algebraic Decay and Phase-Space Metamorphoses in Microwave Ionization of Hydrogen Rydberg Atoms," Y. C. Lai, C. Grebogi, R. Blümel, M. Z. Ding, Phys. Rev. A **45**, 8284-8287 (1992).
35. "Quantum Manifestations of Chaotic Scattering," Y. C. Lai, R. Blümel, E. Ott, and C. Grebogi, Phys. Rev. Lett. **68**, 3491-3494 (1992).

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