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The Early Days of the S_N Method*

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From the beginning at Los Alamos, solutions to the transport equation were very important. All sorts of approximate solution techniques, including one by Feynman¹ were developed to help design nuclear weapons but are now long forgotten. Most of these methods were based on the methods of mathematical physics familiar to the project physicists and predated the use of computers, but continued research coupled with pressing need produced two new and powerful computer-based systems: Monte Carlo and the S_N Method. The healthy and long-term competition between the two Los Alamos groups responsible for these quite different approaches was both stimulating and synergistic.

By the mid-1950's, Bengt Carlson had in place all the elements of his S_N system. While the S_N method is often referred to as a discrete ordinates method, which it is because the angular variable is represented by an ensemble of discrete directions, it is more appropriate to refer to Bengt's creation as a system, that is, as an integrated whole based on a set of consistent themes. Using the multigroup formulation as a starting place, these themes included

1. insistence on particle-conserving difference schemes,
2. finite difference approximations of uniform accuracy but of the utmost simplicity for speed of evaluation,

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3. use of the same methods, regardless of geometry, and
4. minimum-angle quadratures with symmetries based on the symmetries of the geometry involved.

While Chandrasekhar developed the method of discrete ordinates suggested by Wick² in 1943, its use in curved geometries and in geometries in which two variables are required for the angular integration is awkward. By insisting that the space-angle finite difference scheme satisfy particle conservation in detail, Carlson invented a simple and elegant representation for the angular derivatives in curved geometries. This scheme worked no matter what collection of quadrature directions and weights were involved and, in addition to its practical advantages, facilitated experimentation with different quadrature schemes. This innovative use of particle conservation was partially motivated by the need of Los Alamos designers to have accurate, detailed particle balances for their iterated designs of fast, leaky assemblies. By contrast, those working with the diffusion equation did not always use this principle.

Carlson experimented at length with quadrature schemes and evolved workable recipes for reducing the required number of directions by one-half in cylindrical and higher dimensional geometries. With a clever use of the "singular" quadrature direction in which the angular derivative terms do not appear, he was able to initialize the angular variable just as readily as the spatial boundary conditions could be applied. Using a simple central difference approximation, he was then able to reduce the whole process of inverting the differential transport operator to a set of simple, stable, recursion relations.

Using source iteration, the original S_N system was nearly perfect for fast numerical evaluation. Acceleration of the source iteration was accomplished by

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enforcing whole-system particle balance (rebalance), and this was possible because of the construction of the difference equations. Unlike numerical approximations to the diffusion equation, convergence of S_n iterations did not worsen as the spatial mesh size decreased.

Not until the mid 1960's was it realized³ (at approximately the same time at Bettis and Los Alamos) that use of discrete directions in two-dimensional geometries leads to anomalous spatial variations or ray effects. This late recognition was in part because two-dimensional calculations were not widely used earlier and in part because earlier two-dimensional calculations were more often for distributed sources rather than for the isolated sources in absorbers that most clearly display anomalous results.

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