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On the scattering of atmospheric muons
in the rock above Soudan 2*

HANS-JOCHEN TROST

High Energy Physics Division - 362 -

Argonne National Laboratory

9700 S. Cass Ave., Argonne, IL 60439-4815, USA

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ABSTRACT

I have investigated the effect of scattering in the rock overburden on the position and direction of atmospheric muons as observed at the Soudan 2 detector relative to the surface. The dominant contributions to the fluctuations in those observables arise from multiple Coulomb scattering and delta ray production. The contribution of the other energy loss mechanisms, bremsstrahlung, pair production and nuclear interactions, is small, with slowly increasing importance at very large deviations from the means of the distributions.

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1. Introduction

In a previous note^[1] I have studied the total energy of atmospheric muons passing through the rock overburden into (and through) the Soudan 2 cavern. The simulation program I have used is based on the GEANT^[2] package, allowing to also investigate the position and direction of particles at any point in space. In this note, I make use of this feature and extend the previous calculations by including the effects of explicit delta ray production (which was implicit in the ionization loss calculation before) and multiple Coulomb scattering to determine the fluctuations in the position and direction of atmospheric muons caused by the passage through the rock overburden.

2. The simulation of delta rays and multiple Coulomb scattering

The simulation program and its good performance have been described in the previous note.^[1] Here I want to discuss only the additional elements used in the simulation, the explicit production of delta rays and multiple Coulomb scattering.

The production of delta rays as a part of the energy loss is fairly well understood. For muons, the production cross section as used in GEANT is

$$\frac{d\sigma}{dT} = 2\pi Z r_e^2 m_e \frac{1}{\beta_\mu^2 T^2} \left\{ 1 - \beta_\mu^2 \frac{T}{T_{max}} + \frac{T^2}{2E_\mu^2} \right\}$$

where, with $c = 1$, Z is the atomic number of the medium (*i.e.* rock) through which the muon passes, E_μ is the total energy and β_μ the velocity of the muon, r_e is the classical electron radius, m_e the electron mass, T the *kinetic* energy of the emitted delta-electron, and T_{max} the maximum possible delta electron energy,

$$T_{max} = \frac{2m_e(\gamma_\mu^2 - 1)}{1 + 2\gamma_\mu(m_e/M_\mu) + (m_e/M_\mu)^2}$$

(Here, the Lorentz factor γ_μ and rest mass M_μ of the muon come into the game.) This cross section is used above a user-settable threshold $DCUTM$ to explic-

itly generate delta electrons. The angular distribution is taken uniform in the azimuthal angle, and the polar angle is determined by energy and momentum conservation.^[2] Thus, the delta rays can contribute in the simulation deflections of the muon from its original direction. The cross section below the cut is kept with the continuous ionization energy loss with Landau fluctuations superimposed, observing appropriate restrictions to avoid double counting. The lowest cut value allowed by GEANT is $DCUTM = 100 \text{ keV}$.

The effect on the position and direction of atmospheric muons after passing through 700 *m* of rock is demonstrated in the comparison of Figures 1 and 2 for muons of 1 *TeV* energy at the surface. (Throughout this note, I use the spatial angle, labelled θ , not the one projected on a scattering plane, to quantify angular deflections.) The small tails brought about by the discrete bremsstrahlung, e^+e^- pair production and nuclear interactions are exceeded in size by far by the delta ray production. Moving the cut on the kinetic energy of the delta electrons up to $DCUTM = 10 \text{ MeV}$ reduces both the radial translation and angular deflection by about 10% (Fig.3). If this inaccuracy is irrelevant for an application, it becomes important for the computational economics: The delta ray production is the limiting process for the step size that the program allows, at least with this range of kinetic energies of the delta electrons considered. Therefore the kinetic energy cut $DCUTM$ determines the amount of cpu time needed for simulating the propagation of a muon. The effect on the total path length is negligible: Without the delta rays, the average pathlength through 700 *m* of rock exceeds the depth by only 1 *mm*; the delta rays add up to another 3 *mm* to this number. This is still within the range of numerical inaccuracy of the whole simulation, so the net result at this point is that there is almost no effect. Thus, the explicit production of delta rays needs only be included to the extent to which the delta electrons are needed or the effect on the position and direction of the muon has to be taken care of. In the present study, I stick to the expensive, lowest possible cut in order to assure the best possible accuracy for the position and direction, avoiding a lengthy optimization of the cutoff value.

The multiple Coulomb scattering has been discussed extensively in the literature.^[5-10] For inclusion in a simulation program like GEANT, a specific problem arises: The net effect of multiple scattering is the sum of the squares of the deviations calculated step by step when propagating particles through matter. To give a correct result, this means that

$$\theta_{total}^2 = \sum_{steps} \theta_{step}^2$$

(θ denoting the mean angle in space of deviation created by multiple scattering) should hold. This happens to be violated by the current standard formula

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \sqrt{2z} \sqrt{x/X_0} \{1 + 0.038 \cdot \ln(x/X_0)\}$$

(p , βc , and z are the momentum, velocity and charge number of the incident particle, x/X_0 is the thickness of material passed in radiation lengths, the $\sqrt{2}$ converts from a projected planar angle to a space angle) due to the logarithmic term. In fact, the sum comes out *smaller* than the total if the latter is calculated directly from the total thickness traversed and a Gaussian approximation is made for the deflection angle (that is what the above formula is fitted for).^[9,11]

For this reason, I force GEANT to apply the Molière theory^[6,7] throughout as recommended by G.R.Lynch^[11] which means effectively using a more elaborate distribution than a Gaussian to randomly generate the multiple scattering angles. The resulting r.m.s angle, determined by fitting a Gaussian to the observed distribution to be comparable to the standard formula, comes out as $\theta_0 = 6.93 \pm 0.14 \text{ mrad}$ (Fig.4b), close to the formula value of $\theta_0 = 8.0 \text{ mrad}$. (For this value, I have averaged $1/p^2$ with the mean energy loss and put the resulting $1/p = \sqrt{\langle 1/p^2 \rangle}$ into the formula.) The formula is determined "only" for depths up to $100 X_0$ and quoted to be good to within 11%.^[10] For comparison, the overburden at Soudan 2 has a thickness of about $8000 X_0$. Forcing GEANT to use much smaller step sizes for the muon propagation (by allowing only very

small energy losses per step due to ionization, $DEEMAX = 0.000005$ for the largest fraction) changes the resulting r.m.s. angle very little, to $6.91 \pm 0.18 \text{ mrad}$.

For comparison, I have repeated these calculations using a Gaussian approximation available in GEANT, *i.e.* for each step, the multiple scattering angle is determined from the positive half of a Gaussian distribution with a mean of zero and variance of θ_{LD}^2 taken from the work of G.R.Lynch and O.I.Dahl.^[2,9] Their angle θ_{LD} does not scale with the square root of the step size either, so despite being more accurate it suffers from the same problem for implementation in a track-following simulation as the standard formula. In the "normal" running mode, *i.e.* allowing as large step sizes as reasonably possible, I measure $\theta_0 = 5.66 \pm 0.12 \text{ mrad}$ (Fig.4d). Restricting the step size again, this value reduces to $\theta_0 = 4.92 \pm 0.09 \text{ mrad}$ (Fig.4e), in agreement with the observation reported by G.R.Lynch.^[11]

To summarize the discussion on multiple Coulomb scattering, I find the implementation of the Molière approach well compatible with the extrapolation of the present knowledge. The apparent independence of this approach from the step sizes used in propagating muons shows the method to be internally consistent. Thus I can expect the predictions from the simulation for the positional and angular spread of muons after passing 700 *m* of rock to be reasonably credible. In Fig.5 the effect of adding the multiple scattering to the basic four energy loss processes (no explicit delta rays) is shown. The spreading effect is even stronger than the one from the delta electrons (Fig.2).

3. Predictions for the smearing of position and direction

Adding the multiple Coulomb scattering and the explicit treatment of delta rays to the previous^[1] simulation of the energy loss, I arrive at the desired predictions for the spreads of the positions and directions of atmospheric muons in the Soudan 2 cavern (Fig.6). The r.m.s. radius, of the spread-out muon beam including the non-Gaussian tails, is 1.65 *m* for an energy of 1 *TeV* at the surface (1.45 *m* for the Gaussian core), and the r.m.s. angle with respect to the incident direction, also including the non-Gaussian tail, is 9.8 *mrad*. A Gaussian fit gives 6.74 ± 0.22 *mrad*, within errors (and with lesser statistics) the same as the previous result using only multiple scattering but not explicit delta ray production. The increase in path length remains still small with about 7 *mm* at 1 *TeV* incident energy. Thus, the results of the previous study^[1] on transmission and energy loss remain complete valid.

The effects of multiple scattering and delta rays are strongly energy dependent (Fig.7). For example, at 10 *TeV* the radius is reduced to 52 *cm* and the r.m.s. angle to 2.4 *mrad* (12.4 *cm* and 1.1 *mrad* for the Gaussian cores respectively). Here, the factor $1/p$ in the standard formula comes in stronger than just with the ratio of initial energies because with increasing energies the *fraction* of energy surviving after 700 *m* of rock increases as well.^[1] This raises the average of $1/p$ by more than the ratio of incident energies.

The energy dependence can be translated into an effective smearing of the position and direction for muons with a spectrum of energies coming from a fixed, pointlike source. I have run my simulation program for two cases, a standard energy spectrum of atmospheric muons falling as $dN/dE \sim E^{-3.71}$, and one falling as $dN/dE \sim E^{-2}$ as has been suggested^[12] for muons out of the direction of Cygnus X-3. The resulting distributions are shown in Fig.8 together with fitted curves corresponding to Gaussian distributions centered at $r = 0.0$ *cm* and $\theta = 0.0$ *mrad* respectively. From these data I obtain the following r.m.s. widths of vertically incident muon beams for the given spectrum slopes:

		$\sqrt{\langle r^2 \rangle}$ [cm]	$\sqrt{\langle \theta^2 \rangle}$ [mrad]
$\gamma = 2.0$	Gaussian fit	52 ± 4	3.9 ± 0.2
	full histogram	113 ± 1	7.9 ± 0.2
$\gamma = 3.71$	Gaussian fit	83 ± 0.8	5.5 ± 0.2
	full histogram	146 ± 1	14.1 ± 0.2

The shallower spectrum gives smaller values as has to be expected due to the larger contributions from higher energies. The distributions do not show very strongly outstanding cores of Gaussian shape, making the fit results more uncertain than the quoted errors from the fits indicate. Earlier analyses for this work with lesser statistics gave results differencing from the present ones by about 5% while reporting more similar errors from the fits to those above. This sets the uncertainty of the present results to about 5% or perhaps a bit more. The tails are likely to be caused by the low energy end of the spectrum (I have not introduced any cutoff that would guarantee a muon to still move more or less straight locally, *e.g.* through the thickness of the Soudan detector) and hard stochastic processes (bremsstrahlung and nuclear interactions).

4. Summary

I have used a simulation program to predict the spreads in the position and direction of muons vertically incident onto the rock overburden of the Soudan 2 experiment. The position resolution introduced by scattering processes in the rock is about 1.5 *m* for the regular cosmic ray energy spectrum with a Gaussian core of 0.8 *m* resolution. For a hard source spectrum ($\gamma = 2.0$), this number reduces to about 1.1 *m*, (0.5 *m* for the core). The angular accuracies overall for the two cases are 0.8° and 0.5° respectively. The largest contribution to the resolution is the multiple Coulomb scattering which gives rise to most of the Gaussian cores and a part of the tails of the distributions. The cores are badly defined for the full spectra, so any signal from a point source will likely exhibit

shapes of the radial and angular distribution quite different from the standard Gaussian form.

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FIGURE CAPTIONS

1. Lateral displacement (a), one projection of the same (b), square of the transverse radius relative to the undeflected beam position (c) and square of the angle with the original beam direction (d) for muons of vertical incidence into 700 *m* of rock at 1 *TeV* energy, without explicit production of delta rays and without multiple Coulomb scattering included.
2. Displacement, radius and angle as in Fig.1, with explicit production of delta rays down to $T_e \geq 100$ *keV*.
3. Same as Fig.2 but with explicit delta ray production only beyond $T_e \geq 10$ *MeV*.
4. Square of angle, in $mrad^2$, of muon direction after passing 700 *m* of rock with respect to the incident direction (no delta rays in all cases) and distribution of step sizes used in the muon propagation: a) without any multiple Coulomb scattering, b) with multiple scattering using the Molière approach, c) as b) but with strongly reduced step size for muon propagation, d) with multiple scattering using the Gaussian approximation, e) as d) but with strongly reduced step size for muon propagation.
5. Same as Fig.1 but with multiple Coulomb scattering using the Molière approach included
6. Same as Fig.1 but with both explicit delta rays and multiple Coulomb scattering included
7. Dependence on the incident energy of a) the r.m.s. radius of beam spread and b) the r.m.s. angle w.r.t. the incident direction of atmospheric muons reaching the Soudan cavern

8. Squared radii and space angles for muons from atmospheric spectra with $\gamma = 2.0$ (Cygnus model⁽¹²⁾) and $\gamma = 3.71$ ("normal" atmospheric spectrum); the fits are for functions of the form $\exp(p_1 + p_2 \cdot x)$ with $x = r^2$ and $x = \theta^2$ respectively.

1 TeV muons, no mult.scatt., no delta rays

Fig.1

1 TeV muons, no mult.scatt., delta rays $T_e \geq 100$ keV

Fig.2

1 TeV muons, no mult.scatt., delta rays $t_e \geq 10$ MeV

Fig.3

1 TeV muons, angular deviations and step sizes

a) no mult.scatt.

b) Moliere

c) Moliere small steps

Fig.4 (1 of 2)

1 TeV muons, angular deviations and step sizes

d) Gauss

e) Gauss small steps

Fig.4 (2 of 2)

1 TeV muons, Moliere mult.scatt., no delta rays

Fig.5

1 TeV muons, Moliere mult.scatt., delta rays $T_e \geq 100$ keV

Fig.6

Fig.7

$\langle r^2 \rangle$ and $\langle \Theta^2 \rangle$ for different spectra

$\gamma=2.0$

$\gamma=3.71$

Fig.8

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