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Flow Through Three-Dimensional Arrangements of Cylinders with Alternating Streamwise Planar Tilt

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of Cylinders with Alternating Streamwise
Planar Tilt**

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SUMMARY

In this report, fluid flow through a three-dimensional model for the fibrous filters is examined. In this model, the three-dimensional Stokes equation with the appropriate periodic boundary conditions is solved using the finite volume method. In addition to the numerical solution, we attempt to model this flow analytically by using the two-dimensional extended analytic solution in each of the unit cells of the three-dimensional structure. Particle trajectories computed using the superimposed analytic solution of the flow field are closed to those computed using the numerical solution of the flow field. The numerical results show that the pressure drop is not affected significantly by the relative angle of rotation of the cylinders for the high porosity used in this study ($\epsilon = 0.8$ and $\epsilon = 0.95$). The numerical solution and the superimposed analytic solution are also compared in terms of the particle capture efficiency. The results show that the efficiency predictions using the two methods are within 10% for $St=0.01$ and 5% for $St=100$. As the the porosity decreases, the three-dimensional effect becomes more significant and a difference of 35% is obtained for $\epsilon=0.8$.

1 INTRODUCTION

In previous studies of particle capture of fibrous filters, the models mainly use two-dimensional solutions for creeping flow around cylinders. These analytical solutions are given by Kuwabara (1959), Happel (1959), Miyagi (1959) and Hasimoto (1959). The Kuwabara solution is mostly used in the study of particle capture in fibrous filters. The Kuwabara solution uses a cylindrical outer boundary which is not the correct representation of the unit cells for the periodic arrangement of cylinders. For these reason we have suggested (Marshall et al., 1993) an extended analytic solution that solves for the flow through the in-line arrangement of cylinders for the square unit-cell. This solution uses the Kuwabara solution as a basic state. Banks (1987) presented three-dimensional analytic solutions using the superposition of the solution for the axial flow obtained by Happel and the solution for the transverse flow obtained by Kuwabara.

The fluid flow through the fibrous filters is three-dimensional and the models mentioned above fail to model the three-dimensional effects. In our previous reports, we have proposed a three-dimensional model for the fibrous filters. A rendering of this model is shown in Fig. 1. In this structure alternative columns of cylinders are at an angle with each other. Below, we discuss the numerical solution of the flow field in the structure shown in Fig. 1. Some results for the pressure drop and some particle trajectories are given. In section 4, some results of particle trajectories using superposition of the two-dimensional analytic solution are presented. Also, efficiency computations are performed using the numerical and the superimposed analytic solution.

2 NUMERICAL INTEGRATION

Using periodicity of the bulk flow, the model shown in Fig. 1 for the fibrous filters can be simplified by using a periodic structure as shown in Fig. 2. In this domain, we use two unit cells (parralelepiped shape) that are at an angle with respect to each other in the plane orthogonal to the flow. In this structure we use $\ell_x/2 = \ell_y = 1$ and in the z -direction ℓ_z depends on the angle of rotation and is given by

$$\ell_z = \frac{\ell_x}{\sin \theta}. \quad (1)$$

For $\theta = 0$ the flow becomes two-dimensional and the momentum equation in the z -direction is ignored. For $\theta=90^\circ$, we have $\ell_x = \ell_y = \ell_z$ and the unit cells become cubes. In Fig. 2 the positive x -direction indicates the direction of the flow. Also, the numbering of the unit cells that we will use for our notation of velocity and pressure is shown in Fig. 2. As an example, in cell number 1 and 2 we use u_1 and u_2 to denote the x -component of the Cartesian velocity and v_{r_1} and v_{r_2} are used to denote the radial components of velocity in the cylindrical coordinates.

Figure 1. Three-dimensional structure of cylinders used to model the fibrous filters.

Figure 2. (a) Rendering of the domain decomposition used in the numerical simulation, (b) two-dimensional presentation of one cell to show the periodicity, (c) two-dimensional presentation of the two domains used in the numerical simulation.

Figure 3. Coordinate systems used in the two unit cells.

The coordinate systems used in both unit cells are shown in Fig. 3. In this system, iteration is performed in each cell and bilinear interpolation is used to obtain the x -direction boundary conditions for velocity and pressure.

In each Cartesian unit cell, we use a domain decomposition in order to map exactly the shape of the cylinder. A cylindrical grid net is used around the cylinder surface and a Cartesian grid net is used away from the cylinder. Iteration is performed alternatively in each domain and communication between them occurs by using the bilinear interpolation of the boundary conditions in the interlocking region.

Flow through fibrous filter occurs at low Reynolds number (i.e., $Re_d = \langle u \rangle \ell / \nu < 0.1$). Therefore, the Stokes equation is a good model for the flow in such structure. In the Cartesian coordinates we solve

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

$$-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (3)$$

$$-\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0 \quad (4)$$

$$-\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0. \quad (5)$$

In the cylindrical coordinates, we solve the radial, tangential and axial momentum equations given by

$$\frac{\partial r v_r}{\partial r} + \frac{\partial v_\theta}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \quad (6)$$

$$-\frac{\partial p}{\partial r} + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_r}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r} \frac{\partial v_r}{\partial \theta} \right) \right] + \frac{\partial^2 v_r}{\partial z^2} + S_r = 0 \quad (7)$$

$$-\frac{1}{r} \frac{\partial p}{\partial \theta} + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_\theta}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right) \right] + \frac{\partial^2 v_\theta}{\partial z^2} + S_\theta = 0 \quad (8)$$

$$-\frac{\partial p}{\partial z} + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right) \right] + \frac{\partial^2 w}{\partial z^2} = 0 \quad (9)$$

where the source terms S_r and S_θ are

$$S_r = \left\{ -2 \frac{v_r}{r^2} + \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_r}{\partial r} \right) + \frac{\partial}{\partial \theta} \left[\frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right] \right\} \quad (10)$$

$$S_\theta = \left(-\frac{v_\theta}{r^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right) \quad (11)$$

For the boundary conditions, we use no the slip condition at the surface of the cylinder. In the Cartesian domain we use the periodic boundary conditions for the outer boundaries of the unit cells. These boundary conditions can be found by using translation of the periodicity vectors. In the first cell, one periodicity vector is given by

$$\ell_{a_1} = \ell_z \mathbf{k}_1 \quad (12)$$

and this is called the axial (i.e., along the z -direction) periodicity vector. This is mainly used to find the boundary conditions in the axial direction. For the periodic boundary conditions in the y -direction, we use the transverse (or non-axial) periodicity vector given by

$$\ell_{t_1} = \ell_z (-\sin \theta \mathbf{j}_1 + \cos \theta \mathbf{k}_1). \quad (13)$$

For the second cell, the axial and transverse periodicity vector are given by

$$\ell_{a_2} = \ell_z \mathbf{k}_2 \quad (14)$$

$$\ell_{t_2} = \ell_z (\sin \theta \mathbf{j}_2 + \cos \theta \mathbf{k}_2). \quad (15)$$

As an example, we give the periodic boundary condition in the y - and z -direction for the u_1 component of velocity

$$u_1(x_1, -0.5, z_1) = u_1(x_1, 0.5, z_1 - \ell_z \cot \theta) \quad (\text{in the } y\text{-direction}) \quad (16)$$

$$u_1(x_1, y_1, -\frac{\ell_z}{2}) = u_1(x_1, y_1, \frac{\ell_z}{2}) \quad (\text{in the } z\text{-direction}). \quad (17)$$

Note that $\ell_x/2 = \ell_y = 1$ is used. For the x -direction we use

$$u_1(x_1 = 0, y_1, z_1) = u_2(x_2 = 1, y_2, z_2) \quad (\text{in the } x\text{-direction}) \quad (18)$$

where y_2 and z_2 are given by

$$y_2 = y_1 \cos \theta + z_1 \sin \theta \quad (19)$$

$$z_2 = -y_1 \sin \theta + z_1 \cos \theta. \quad (20)$$

When this periodic boundary condition is performed y_2 and z_2 do not necessarily fall on a grid point of the second unit cell, therefore, the bilinear interpolation of the surrounding grid points is used. In these three-dimensional numerical simulations, we use $42 \times 42 \times 42$ nodes in a typical Cartesian grid net for each of the unit cells. Numerical results for the pressure drop and the particle trajectories are presented in Section 4.

3 SUPERPOSITION OF ANALYTIC SOLUTION

For the fluid flow through the three-dimensional structure shown in Fig. 2, the numerical solutions are accurate and use the exact geometry of the fibrous filter model. However, the numerical solutions are slow for the three-dimensional, especially if greater accuracy is required and the number of points has to be increased. In addition, for particle trajectory analyses data files have to be stored for various angles of rotation and porosity. For this reason, we suggest the superposition of the extended two-dimensional analytic solution to obtain flow field for the three-dimensional structure shown in Fig. 2. In each of the unit cells, we use the stream function given by the extended solution

$$\begin{aligned} \psi(r, \theta) = & R[C_1\left(\frac{r}{R}\right)^3 + C_2\frac{r}{R} + C_3\frac{R}{r} + C_4\frac{r}{R}\ln\left(\frac{r}{R}\right)]\sin\theta \\ & + \{C_5R^2r^3[1 + 2\left(\frac{R}{r}\right)^6 - 3\left(\frac{R}{r}\right)^4] + C_6r^5[1 + 3\left(\frac{R}{r}\right)^8 - 4\left(\frac{R}{r}\right)^6]\}\sin(3\theta). \end{aligned} \quad (21)$$

The coefficients C_1 - C_4 are given by

$$C_4 = -\frac{4}{2\ln\epsilon + 3 - 4\epsilon + \epsilon^2} \quad (22)$$

$$C_1 = -\frac{\epsilon C_4}{4} \quad (23)$$

$$C_3 = C_1 + \frac{C_4}{2} \quad (24)$$

$$C_2 = -C_1 - C_3 \quad (25)$$

where ϵ is the porosity given by

$$\epsilon = 1 - \pi R^2. \quad (26)$$

The coefficients C_5 and C_6 are given by

$$C_5 = -C_7(-8R^2 + 256R^8 - 384R^{10}) - C_8(-R^2 + 256R^8 - 768R^{10}) \quad (27)$$

$$C_6 = -C_7(16R^4 - 192R^8 + 256R^{10}) - C_8(4R^4 - 192R^8 + 512R^{10}) \quad (28)$$

where C_7 and C_8 are given by

$$C_7 = \frac{4C_1 - 16R^2 + 16C_2R^2 + 64C_3R^4 + 16C_4R^2 \ln(\frac{1}{2R})}{C_9} \quad (29)$$

$$C_8 = \frac{16C_1 - 32R^2 + 32C_2R^2 + 64C_3R^4 + 32C_4R^2 \ln(\frac{1}{\sqrt{2}R})}{C_9} \quad (30)$$

and C_9 is given by

$$C_9 = 16R^6 - 1344R^{10} + 6912R^{12} - 10572R^{14} + 8192R^{18}. \quad (31)$$

In Eq. 21 the radius r stands for $r_1 = \sqrt{y_1^2 + z_1^2}$ or $r_2 = \sqrt{y_2^2 + z_2^2}$ and the angle θ stands for $\theta_1 = \tan \frac{z_1}{y_1}$ or $\theta_2 = \tan \frac{z_2}{y_2}$.

The superimposed analytic solution are used to compute trajectories passing through the geometry shown in Fig. 2. The trajectories can be calculated by using the force balance on a particle. The Lagrangian particle momentum equation is given by (Tien, 1989)

$$c_s \frac{4}{3} \pi R_p^{*3} \rho_p^* \frac{d\mathbf{u}_p^*}{dt^*} = 6\pi \mu^* R_p^* (\mathbf{u}^* - \mathbf{u}_p^*) \quad (32)$$

where R_p^* is the radius of the particle, $\mathbf{u}^*$ is the fluid velocity vector, $\mathbf{u}_p^*$ is the particle velocity vector, μ^* is the dynamic viscosity of the fluid, and c_s is the Cunningham correction factor which accounts for the velocity slip when the particle size is comparable to the mean free path of the fluid. The nondimensional form of Eq. (32) is

$$St \frac{d\mathbf{u}_p}{dt} = (\mathbf{u} - \mathbf{u}_p), \quad (33)$$

where St is the Stokes number defined by

$$St = c_s \frac{2}{9} \frac{\rho_p^* \langle u^* \rangle R_p^{*2}}{\mu^* \ell^*} \quad (34)$$

Table 1Pressure drop predictions for variable angle θ .

θ	$\epsilon=0.95$	$\epsilon=0.8$
0°	15.55	51.74
30°	16.29	55.38
45°	16.17	55.42
60°	16.15	55.52
90°	16.08	55.71

4 RESULTS

In this section, we present some typical results of the pressure drop across the unit cell and particle trajectories obtained from the numerical solution for the fluid flow through the three-dimensional structure shown in Fig. 2(a). Also, we compare particle trajectory predictions obtained using the numerical velocity field and the superimposed extended analytic solution. Further comparisons of the two methods of solution are performed in terms of the particle capture efficiency.

4.1 Particle Trajectories and Pressure Drop

Particle trajectories for low Stokes numbers (i.e., $St=0.001$) are given in Fig. 4 (a) and (b). These figures show that the trajectories of the particles that start from the same z position in Fig. 4 (a) and the same y -position in Fig. 4 (b). These results show that at the exit of the second unit cell, the particle return to the same position in the $y - z$ plane because of the periodic fluid flow. This shows that the numerical integration of the Stokes equation and the bilinear interpolation are working properly. Similar trajectories are shown in Fig. 5 (a) and (b) for $\epsilon = 0.8$. The three-dimensional effects on the particle trajectories are more apparent because the gap between the cylinders is smaller.

As mentioned above, we also have used the extended analytic solution to compute particle trajectories for comparison with the trajectories computed using the numerical solution of the flow field. The results of this particle trajectory comparison is shown in Fig. 6. The results show good agreement in the particle trajectory prediction especially in the first unit cell. As the porosity increases and the effect of the neighboring cylinders becomes more significant the difference between the numerical and the analytic results increases as shown in Fig. 6 (b) for $\epsilon = 0.8$.

(a)

(b)

Figure 4. Trajectories of particles passing through the three-dimensional model of the fibrous filters for (a) particles with the same initial z coordinate and (b) particles with the same initial y coordinate ($\epsilon = 0.95$, $St = 0.001$ and $\theta = 60^\circ$).

Figure 5. Same as Fig. 4 for $\epsilon = 0.8$.

Figure 6. Comparison of particle trajectories obtained from the analytic solution and the numerical solution for (a) $\epsilon = 0.95$ and (b) $\epsilon = 0.8$ ($St = 0.001$ and $\theta = 60^\circ$).

Table 2 Prediction of the particle capture efficiency using the superimposed analytical the numerical solution for $\theta=0^\circ$ and 45° , $St=0.01$ and 100 , $d_p=0.4$, and $\epsilon=0.95$.

θ	η			
	Numerical		Analytic	
	$St=0.01$	$St=100$	$St=0.01$	$St=100$
0°	39.3	55.0	40.0	46.5
45°	63.6	85.6	69.5	81.9

Using the three-dimensional numerical results, we examine the pressure drop as the angle varies from 0 to 90° . The results of the comparison are shown in Table 1. As the angle varies, the pressure drop does not change significantly. Most of the variation in the pressure drop occurs from 0 to 30° where transition from two- to three-dimensional flow occurs. For the three-dimensional flow, the angle variation is not significant because for this high porosity the cylinder are far apart and the fluid flow in each unit cell is not appreciably affected by the presence of the upstream or downstream cylinders.

4.2 Capture Efficiency

The particle capture efficiency is examined for the three-dimensional structure using the numerical solution of the velocity field and the superimposed two-dimensional extended analytic solution. Results obtained from the two methods of solution for a single particle diameter are shown, in Table 2 for $\epsilon=0.95$ and $\theta=45^\circ$. As expected, the particle capture efficiency increases as the tilt angle increases since the fluid flow is more tortuous and some particles that are not captured by the first fiber are captured by the following fiber. An increase of 60 % (e.g., numerical results for $St=0.01$) in the particle capture efficiency is obtained as the tilt angle changes from 0° to 45° . As the Stokes number increases the particle inertia increases, and therefore, the particles do not follow the fluid flow causing an increase in the particle capture by the fiber. Comparison of the results obtained from the numerical and the analytic flow fields for $\theta=45^\circ$ in Table 2. shows a 10 % difference which is due to the failure of the superimposed model in capturing the three-dimensional effects.

Efficiency results for $\theta=45^\circ$ presented in Fig. 7(a) for $St=0.01$, show that the particle capture efficiencies predicted using the two flow field solutions are within 10% as the particle diameter varies. For $St=100$, the efficiency results are shown in Fig. 7(b) and the difference is less than 5%. This is because for high inertia, particle paths are not strongly dependent on the local velocity field.

The particle capture efficiency is also computed for $\epsilon=0.8$ where the three-dimensional effects are more pronounced since the distance between the fibers decreases. The results for the

particle capture efficiency are presented in Fig. 8 and they show that the difference between the numerical and analytic solutions is larger due to the three-dimensional effects. A difference of 35% between the two results is obtained for $d_p=0.25$.

Figure 7. Particle capture efficiencies for (a) $St=0.01$ and (b) $St=100$ ($\epsilon=0.95$ and $\theta=45^\circ$).

Figure 8. Particle capture efficiency for $St=0.01$, $\epsilon = 0.8$, and $\theta=45^\circ$.

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