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**VALIDITY OF THE RULE-OF-FRACTIONS FOR
ASSURING CRITICALITY SAFETY MARGINS (U)**

October, 1991

C.E. Agnew
Derivative Classifier

**Westinghouse Savannah River Company
Savannah River Laboratory
Aiken, SC 29808**

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**VALIDITY OF THE RULE-OF-FRACTIONS FOR
ASSURING CRITICALITY SAFETY MARGINS (U)**

by

Donald R. Finch
and
Thomas G. Williamson

Issued: October, 1991

Donald R. Finch, Senior Fellow Scientist

Thomas G. Williamson, Senior Consulting Scientist

SRL SAVANNAH RIVER LABORATORY, AIKEN, SC 29808
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APPROVALS

M. R. Buckner, Manager
Scientific Computations Section
Date: 10-22-91

C. E. Apperson, Manager
Reactor Physics Group
Date: 10-15-91

J. F. Mincey, Technical Reviewer
Date: 10/14/91

ABSTRACT

A computational study was performed to determine the validity of the Rule-Of Fractions for assuring criticality safety margins. The form of the Rule-Of-Fractions examined was the equation:

$$\sum_i \frac{a_i}{A_i} \leq 1$$

where a_i is the quantity of fissile/fissionable nuclide present, and A_i is the quantity of the same nuclide i that corresponds to a known margin in K_{eff} . The study considered binary mixtures of two nuclides that exactly satisfied the equality condition of the equation. K_{eff} values of homogeneous and heterogeneous binary mixtures of fissile/fissionable nuclides in metal and solution form were considered in spherical geometry. Results were compared to Rule-Of-Fraction predicted K_{eff} values using an assumed linear relationship. Calculations were performed with the ANISN code with Hansen-Roach and ENDF/B-IV cross sections. The results of this study demonstrated that the Rule-Of-Fractions in the above equation does not always maintain criticality safety margins in K_{eff} for mixtures of nuclides.

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I. INTRODUCTION

The Rule-Of-Fractions has been used in many applications to assure a minimum margin in K_{eff} for various homogeneous and heterogeneous mixtures of fissile and fissionable isotopes. In its simplest form the rule of fractions may be stated as follows:

$$\sum_i \frac{a_i}{A_i} \leq 1 \quad (1)$$

where a_i is the actual amount of quantity i present, and A_i is an amount of quantity i that corresponds to a specified margin in K_{eff} . The implication of equation 1 is that if the K_{eff} margin is known for each quantity A_i , then a margin at least equal to the smallest K_{eff} margin for any A_i is maintained when arbitrary amounts of quantities i are intermixed, and the conditions of equation 1 are met.

The relationship in equation 1 is quoted in Section 5.2 of the ANSI/ANS 8.15 standard¹ for use with fissile isotopes. The statement is not a justification for the use of equation 1. Rather, it is a statement taken from some document in the public domain, although no reference is given in the standard.

Assurance of safe margins in K_{eff} based on equation 1 has been used in several applications at the Savannah River Site^{2,3}, although a full justification for equation 1 cannot now be found. The justification for a method (called Method "B") derived from equation 1 was questioned during a Technical Safety Appraisal, and led to this study which was originally intended to provide the required justification.

II. EXAMPLES OF THE RULE-OF-FRACTIONS

Three examples of the use of the Rule-Of-Fractions to assure a margin in K_{eff} will be shown below. Each example is a documented use of the Rule-Of-Fractions at the Savannah River Site to assure a justified margin of safety in K_{eff} .

A. Mixtures of Fissile Containers In Shipping Package

A SRS shipping package² is used to ship three types of fissile materials (referred to as Type A, Type B, and Type C respectively). The package contains up to five bottles of fissile material in a vertical column as shown in Figure 1.

The Rule-Of-Fractions was used as a quick method to determine the K_{eff} for mixtures of bottles of the three fissile materials. This was done by weighting the infinite array reactivity of each type of material by itself by the fractional

number of bottles of that kind present. The calculated K_{eff} for each infinite array was as follows:

<u>PACKAGE FISSILE CONTENT</u>	<u>INFINITE ARRAY K_{eff}</u>
5 bottles Type A	0.770±0.004
5 bottles Type B	0.583±0.004
5 bottles Type C	0.517±0.004

The Rule-Of-Fractions was checked by calculation for loadings of five bottles of two types of fissile material (Type A and Type B). For this case the Rule-Of-Fractions K_{eff} is given by:

$$K_{eff} = \frac{N_A}{5} 0.770 + \frac{N_B}{5} 0.583 \quad (2)$$

where N_A and N_B are the number of bottles of Type A and Type B, and the calculated values of K_{eff} are shown in Table I (the derivation of equation 2 will be shown later). A comparison between calculated K_{eff} values and equation 2 is shown in Figure 2. These results show excellent agreement between the two methods, and provides reasonable assurance that a specified margin in K_{eff} is preserved. This is an example of the use of the Rule-Of-Fractions for heterogeneous mixtures of fissile materials in geometrically same containers. The good agreement between calculation and Rule-Of-Fractions appears to be as much dependent on the geometry as on the fissile material content.

B. Savannah River Laboratory Method "B"

This particular method appeared in the SRL Criticality Control Procedure Manual³ as a method for maintaining criticality safety in mixtures of fissile/fissionable isotopes. The method starts by rewriting equation 1 in terms of a quantity Q_i defined by:

$$Q_i = 1/A_i \quad (3)$$

equation 1 then becomes:

$$\sum_i a_i Q_i \leq 1 \quad (1A)$$

A table of values of Q_i are provided for each isotope i , hence satisfying equation 1A assures criticality safety. The Q_i are defined primarily from the ANSI/ANS 8.1⁴ and 8.15¹ standards by taking the mass limits from the standards as the A_i and rounding the resulting Q_i upward. Table II shows the Q_i defined in the Procedure Manual.

TABLE I

**CALCULATIONS OF MIXTURES OF TYPE A AND TYPE B FISSILE MATERIAL
BOTTLES IN INFINITE ARRAY OF SHIPPING PACKAGES
(See Figure 1)**

<u>Position Of Type B Bottles</u>	<u>Calculated K_{eff}</u>
1 Bottle Type B - 4 Bottles Type A	
1	0.734±0.004
2	0.736±0.004
3	0.730±0.004
Average	0.733±0.005
Equation 2	0.733
2 Bottles Type B - 3 Bottles Type A	
1,5	0.694±0.004
2,4	0.701±0.004
2,3	0.698±0.004
1,2	0.695±0.004
1,3	0.686±0.004
Average	0.695±0.005
Equation 2	0.695
3 Bottles Type B - 2 Bottles Type A	
2,3,4	0.655±0.004
1,4,5	0.662±0.004
1,3,5	0.656±0.004
3,4,5	0.663±0.004
2,4,5	0.663±0.004
Average	0.660±0.005
Equation 2	0.658
4 Bottles Type B - 1 Bottle Type A	
1,2,4,5	0.615±0.004
1,3,4,5	0.617±0.004
2,3,4,5	0.618±0.004
Average	0.617±0.005
Equation 2	0.620

SHIPPING PACKAGE ARRANGEMENT

FIGURE 1

Rule-Of-Fractions
For Mixtures Of Type A And B Bottles
In Infinite Shipping Package Array

FIGURE 2

C. ISOTOPIC EQUIVALENTS

A variety of equivalency expressions abound in the literature⁵ that express quantities of fissile/fissionable isotopes that are critically equivalent to another fissile/fissionable isotope. Such expressions are very subject to the external conditions (eg., water reflection, concentration in solution, or bare metal) and many times are stated specifically for one condition only.

To express the combined mass of several fissile isotopes as an equivalent amount of ^{235}U one might proceed as follows. First write equation 1 as an equality because exact equivalency that assures a specified margin in K_{eff} is the desired result. Define the a_i as the actual quantity of isotope i present and the A_i as the limiting quantities of isotope i that assures a known margin in K_{eff} . Now write equation 1 as:

$$\sum_i' \frac{a_i}{A_i} = 1 - \frac{a_{235}}{A_{235}} = \frac{A_{235} - a_{235}}{A_{235}} \quad (4)$$

where the prime on the summation indicates the ^{235}U term was removed. Multiplying by A_{235} then yields the form:

$$A_{235} - a_{235} = \sum_i' a_i \frac{A_{235}}{A_i} \quad (4A)$$

The ratios of A_{235}/A_i can now be defined from accepted mass quantities that assure a specified K_{eff} margin, or simply taken from ANSI/ANS 8.1⁴ or 8.15¹. The statement that equation 4A makes is that any quantity of ^{235}U mass in a mixture not explicitly present is equivalent to the sum of masses of isotopes weighted by the terms on the right side of equation 4A. Another way to state this is that the equivalent mass of ^{235}U represented by mass a_i is defined by:

$$\text{equivalent } ^{235}\text{U mass} = a_i \frac{A_{235}}{A_i} \quad (5)$$

Equation 5 more often appears with the summation from the right side of equation 4A on its right side so that several isotopes may appear in the isotopic mixture.

The limitations of equivalency relationships in the form of equations 4A or 5 are the following:

- All A_i must be defined for the same environmental conditions.
- These equations assume that all isotopes behave the same no matter how the spectra is changed by the presence of other isotopes.

TABLE II
Quality Factors for Method "B"

"Q" FACTOR	MATERIAL	1/Q (grams)	REF.
7.5×10^{-8}	Normal uranium (<0.72% Non- ^{238}U)	1.33×10^7	
1.5×10^{-3}	Enriched Uranium (excluding ^{238}U)	667	4
2.3×10^{-3}	Plutonium ($^{240}\text{Pu} > ^{241}\text{Pu}$)	435	4
1.0×10^{-3}	Americium (< 10% $^{242}\text{Am} + ^{242\text{m}}\text{Am}$)	1000	1
1.4×10^{-3}	Curium (< 5% $^{243}\text{Cm} + ^{245}\text{Cm} + ^{247}\text{Cm}$)	714	1
5.3×10^{-5}	Neptunium, any isotopic	18868	1
2.1×10^{-1}	Californium, any isotopic	4.75	1
1.7×10^{-5}	Protactinium, any isotopic	58824	
2.1×10^{-3}	^{233}U	476	4
7.0×10^{-5}	^{234}U and ^{236}U	14275	
1.5×10^{-3}	^{235}U and ^{232}U	667	4
3.5×10^{-4}	^{238}Pu	2857	1
2.3×10^{-3}	^{239}Pu	435	4
7.0×10^{-5}	^{240}Pu	14275	1
5.3×10^{-3}	^{241}Pu	189	4
2.6×10^{-5}	^{242}Pu	38462	1
6.6×10^{-5}	^{241}Am	15152	1
8.1×10^{-2}	^{242}Am and $^{242\text{m}}\text{Am}$	12.35	1
4.2×10^{-5}	^{243}Am	23810	1
1.2×10^{-2}	^{243}Cm	83.3	1
3.5×10^{-4}	^{244}Cm	2857	1
3.5×10^{-2}	^{245}Cm	28.6	1
1.2×10^{-3}	^{247}Cm	833	1

Berkelium, Einsteinium, Fermium, etc.: No "Q" Factors are assigned for isotopes of these elements. The handling of these materials will require special evaluation and approval.

III. SPECULATION ON THE K_{eff} MARGIN PREDICTED BY R-O-F

The major unknown in specifying a mass limit or limiting condition using equation 1 is knowledge of the K_{eff} value, hence K_{eff} margin the equation predicts. A literature search has revealed very little of meaning in this regard. What appears here is the authors' speculation on the K_{eff} margin produced from the relation.

The shipping package in Section II will provide a first look at the question. In this example the following information was available:

- The K_{eff} value of the empty package was 0.0.
- The K_{eff} value of the package with 5 Type A bottles was 0.77.
- The K_{eff} value of the package with 5 Type B bottles was 0.583.

It would be natural to approximate the K_{eff} behavior assuming a linear relationship with the number of bottles as the independent variable. This yields:

$$K_{eff} \text{ (Type A)} = \frac{N_A}{5} 0.77 \quad (6)$$

$$K_{eff} \text{ (Type B)} = \frac{N_B}{5} 0.583$$

Since the number of bottles ($N_A + N_B$) always equals 5, then:

$$N_A = 5 - N_B \quad (7)$$

and a superposition of equations 6 yields:

$$K_{eff} \text{ (Type A + Type B)} = \frac{N_A}{5} 0.77 + \frac{N_B}{5} 0.583 \quad (8)$$

which is identical to equation 2. Using the relationship between N_A and N_B in equation 7 yields:

$$K_{eff} \text{ (Type A + Type B)} = 0.77 - \frac{N_B}{5} 0.187 \quad (9)$$

which is the straight line shown in Figure 2. The validity of equation 9 depends completely on equations 6. Calculations were performed with one to five Type A bottles in the package and the results are shown in Figure 3 along with the first of equations 6. The straight line approximation in equations 6 appears to be a reasonable assumption, although none of the calculated points agree perfectly with equation 6.

Keff vs Number of Type A Bottles
In Infinite Shipping Package Array

FIGURE 3

At this point it is convenient to define a term called the Single Unit Mass as follows:

Single Unit Mass - A mass of fissile/fissionable material of known geometry that provides a known margin $(1-K_{eff})$ for specified conditions.

Using this single unit mass (SUM), look now at the relationship between K_{eff} and SUM fraction for a homogeneous bare sphere of fissile material. Figure 4 shows this relationship for ^{235}U , ^{239}Pu , and ^{240}Pu derived from $S_{16}P_1$ ANISN⁶ calculations using Hansen-Roach cross sections⁷. The SUM was chosen to produce a K_{eff} of 0.95 and gave calculated masses of 38.97 Kg., 8.28 Kg., and 29.56 Kg. for ^{235}U , ^{239}Pu , and ^{240}Pu respectively. Note how the curves for different isotopes almost reproduce each other. Also shown in Figure 4 is a straight line which is analogous to the linear assumption used for the shipping package above, however it does not represent the calculated results well.

Next, examine the changes in K_{eff} for a homogeneous binary mixture of ^{235}U and ^{240}Pu as the $^{235}\text{U}/^{240}\text{Pu}$ ratio changes. The two isotopes are mixed so that the Rule-Of-Fractions at equality is always satisfied at a K_{eff} of 0.95 (0.05 margin). That is:

$$\frac{a_{235}}{A_{235}} + \frac{a_{240}}{A_{240}} = 1 \quad (10)$$

The two ratios in equation 10 are simply the fractions of a Single Unit Mass in both cases so that it is more easily written as:

$$F_{235} + F_{240} = 1$$

or:

$$F_{240} = 1 - F_{235} \quad (11)$$

Using these Single Unit Masses, K_{eff} vs. $^{235}\text{U}/^{240}\text{Pu}$ mixture for a bare spherical homogeneous geometry is shown in Figure 5. The isotopic mixtures vary from a ^{240}Pu SUM at the left side to a ^{235}U SUM at the right side. Also shown in Figure 5 are curves and superpositions of the curves shown in Figure 4 for the two isotopes, and superposition of linear assumption curves for the two isotopes. Because the K_{eff} values for the two single unit mass curves give 0.95 at the end points, it is reasonable to assume that the Rule-Of-Fractions would predict that a margin of 0.05 is maintained through the entire range of mixtures. However, for this to be true requires use of the linear assumption curves from Figure 4. Superposition of the individual reactivity curves gives a K_{eff} exceeding 1.0 by a large amount. Results of actual K_{eff} calculations using ANISN ($S_{16}P_1$) are also shown in Figure 5 and do not agree with either of the superposition curves.

Keff vs. Fraction of Single Unit Mass
For 235U, 239Pu, and 240Pu
S16P1 ANISN --- Hansen-Roach Xsecs

FIGURE 4

Bare Homogeneous Sphere Of 235U & 240Pu
 Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 5

The result of the above exercise is confusion. It is logical to assume that K_{eff} (and its associated margin) is defined by a straight line connecting the 0.95 values at the end of the curves for the two isotopes, but this is not borne out by calculation. A more detailed picture of the calculations is shown in Figure 6, and shows significant deviation from the straight line. The calculations demonstrate that a K_{eff} margin is not maintained, at least not one that is determined a priori.

These two examples, one that appears to confirm the Rule-Of-Fractions and another that does not, illustrate the problem of assuring a margin in K_{eff} with limits placed on physical entities like mass, geometry, and spacing. A well-defined relationship between K_{eff} and the controlled entities must exist to be assured that proper margins are maintained over a range of conditions. For the Rule-Of-Fractions the question is whether a relationship exists, or whether agreement is fortuitous.

IV. Basis Of Computational Study

Because of the mixed results shown in Section III, a computational study of homogeneous and heterogeneous mixtures of fissile/fissionable isotopes was performed to find the general conditions under which the Rule-Of-Fractions might be a valid method for assuring criticality safety margins. The geometry for this study was chosen to be one-dimensional spheres surrounded by different reflectors. Within this geometry restriction examination of interaction effects between isotopes were accomplished, but geometrical effects like those in the shipping package discussed above could not be examined.

Any investigation of interactions between isotopes involves detailed consideration of neutron spectrum and the factors that influence neutron spectrum. Multiple cross section sets should be used for each problem to elicit spectrum changes from that source. Where spectrum effects are important different cross sections always produce different answers.

The study was performed for two isotope (binary) mixtures of isotopes ^{235}U , ^{238}Pu , ^{239}Pu , and ^{240}Pu in metal form and solution form. Each mixture of isotopes was defined to meet the equality condition of the Rule-of-Fractions as was done in Equations 10 and 11. The isotopes were selected because good cross sections were available from the Hansen-Roach⁷ and ENDF/B-IV⁸ libraries, and permitted examination of fissile-fissile, fissionable-fissionable, and fissile-fissionable mixtures of the four isotopes. S_n transport calculations were performed using the ANISN⁹ module in the KOKO System⁹. Mixture cross sections were prepared using the HRXN¹⁰ and GLASS¹¹ modules in the KOKO System. All K_{eff} values were calculated for S_4 , S_8 , and S_{16} quadrature and extrapolated to S_∞ .

The question raised in Section III - What is the K_{eff} predicted by the Rule-Of-Fractions? - is still unanswered. For

Bare Homogeneous Sphere Of ^{235}U & ^{240}Pu
Frac. ^{240}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 6

this study the logical approach was used; that is, a straight line between the K_{eff} values of the two Single Unit Masses plotted as in Figure 6. Implied in this is the acceptance of the linear assumption curves shown in Figure 5 which are obviously incorrect. The justification for using this linear relationship between K_{eff} and fractions of Single Unit Masses is that it appears closer to the presumed intent of the Rule-Of-Fractions (i.e., maintaining a known margin in K_{eff}) than any other assumption that might be made.

The study had four major divisions:

- Definition of Single Unit Masses (SUMs) for each isotope, cross section set, metal and solution form, and reflector (bare, water, and steel).
- Calculations of binary Fissile-Fissile mixtures of isotopes.
- Calculations of binary Fissionable-Fissionable mixtures of isotopes.
- Calculations of binary Fissile-Fissionable mixtures of isotopes.

In each section of the study, all computations were performed by the ANISN module of the KOKO system using Hansen-Roach and ENDF/B-IV cross sections. Because the Single Unit Mass K_{eff} values used to define the Rule-Of-Fractions K_{eff} values were derived from calculation using the same codes and cross sections, no correlated bias corrections were needed to enable comparison of results. The calculations were internally consistent so that results without bias correction were proper for comparison.

V. Definition Of Single Unit Masses (SUMs)

The Single Unit Mass for each isotope under each different condition was defined and the K_{eff} value corresponding to this mass and condition defined as the starting point for the study. Selection of the Single Unit Masses was done as follows:

- Where a single parameter mass limit for an isotope existed in the ANSI/ANS 8.1 or 8.15 Standard it was used.
- Where single parameter mass limits did not exist in standards the masses were defined from a S_{0P1} ANISN calculation using Hansen-Roach cross sections for K_{eff} equal to 0.95.

Single Unit Masses for metal spheres with different reflectors defined according to the above rules are shown in Table III. These units define the end points of the Rule-Of-Fractions values of K_{eff} as described above. Between these end points the variation of K_{eff} is assumed to be linear.

TABLE III
SINGLE UNIT MASSES FOR METAL SPHERES

^{235}U (Density = 18.8 gm/cc)				
<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	38.97	7.910	0.950	0.961
Water	17.74	6.085	0.950	1.133
Steel	16.39	5.926	0.950	0.996

^{238}Pu (Density = 19.74 gm/cc)				
<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	8.35	4.657	0.950	1.015
Water	4.00	3.644	0.806	0.911
Steel	3.00	3.311	0.829	0.905

^{239}Pu (Density = 19.82 gm/cc)				
<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	8.28	4.637	0.950	0.943
Water	4.566	3.803	0.950	1.059
Steel	4.26	3.716	0.950	0.964

^{240}Pu (Density = 19.90 gm/cc)				
<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	29.58	7.080	0.950	0.913
Water	20.00	6.214	0.878	0.876
Steel	15.00	5.646	0.921	0.890

Water reflector thickness - 30.48 Cm.

Steel reflector thickness - 25.4 Cm.

H-R - Hansen-Roach Cross Sections

E-IV - ENDF/B-IV Cross Sections

The steel reflector used in Table III was 25.4 cm. This value is in line with the footnote to Table I in ANSI/ANS 8.15 standard which suggests that 20 Cm. may be considered an infinite reflector. Steel reflector thickness was examined when leakage from the reflected spheres was too high in the Table III calculations. Results of this examination are shown in Figure 7 which suggests that a larger thickness of steel is necessary for some isotopes, and that ENDF/B-IV cross sections are more susceptible to the reflector thickness. Many of the calculations that appear in Sections VI, VII, and VIII for steel reflected systems were completed before the thickness effects in Figure 7 were observed. The results of these calculations were included but must be considered somewhat suspect.

Single Unit Masses for solutions of fissile and fissionable isotopes were created by mixing nuclides in metal form uniformly with water in the proper spherical size to form a unit of acceptable K_{eff} . The masses of fissiles were taken directly from the ANSI/ANS 8.1 Standard; however, different concentrations were used for bare and reflected spheres to achieve a specified K_{eff} with minimum mass. A side study examined the concentration of fissiles that produced the minimum mass at $K_{eff}=0.95$ for bare, water reflected, and steel reflected spheres separately. These results are incorporated into the final Single Unit Masses for fissile isotopes in solution shown in Table IV. Another side study showed that the two fissionable nuclides achieve maximum K_{eff} when in metal form, and the presence of any quantity of water uniformly mixed will decrease the reactivity to almost zero. Figure 8 shows the reactivity results for mixtures of fissionable nuclides in water, with the upper limit of concentration of each fissionable nuclide the theoretical metal density limit. The final Single Unit Masses for fissile/fissionable solutions are shown in Table IV.

VI. Fissile-Fissile Mixtures

The first group of tests of the Rule-of-Fraction were for homogeneous fissile-fissile binary mixtures of ^{235}U and ^{239}Pu . For convenience the Single Unit Masses were divided into quarters so that mixtures consisting of 0.25 A + 0.75 B, 0.5 A + 0.5 B, and 0.75 A + 0.25 B, where A and B are two fissile/fissionable isotope SUM values from Tables III and IV. The masses in proper proportion were mixed stoichiometrically to form homogeneous mixtures for both metal and solution. Results of these calculations are shown in Table V and Figures 9, 10, and 11. These results confirm that the Rule-Of-Fractions, with the linear assumption, assures a minimum margin in K_{eff} for all mixtures and reflectors. The displacement of the results from calculations with ENDF/B-IV cross sections occurs because the Single Unit Masses calculated using Hansen-Roach were used in the calculations. A second set of Single Unit Masses could have been determined for ENDF/B-IV cross sections, but no advantage was gained by doing so. The important thing is the results of calculation using both ENDF/B-IV and Hansen-Roach cross sections

Steel Reflector Thickness Effect S16P1 ANISN - Metal Spheres

FIGURE 7

Fissionable Mass vs. Concentration
 $K_{eff} = 0.95$

FIGURE 8

TABLE IV

SINGLE UNIT MASSES FOR SOLUTION SPHERES

 ^{235}U (Density = 18.8 gm/cc)

<u>Reflector</u>	<u>Mass (Gm)</u>	<u>Radius (Cm)</u>	<u>Gram/L</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	1248	19.5325	40	0.950	0.984
Water	760	15.3669	50	0.974	1.007
Steel	568	13.9426	50	0.950	1.006

 ^{238}Pu (Density = 19.74 gm/cc)
Metal Unit (Same as Table III)

<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	<u>Kg/L</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	8.35	4.657	19.74	0.950	1.015
Water	4.00	3.644	19.74	0.806	0.911
Steel	3.00	3.311	19.74	0.829	0.905

 ^{239}Pu (Density = 19.82 gm/cc)

<u>Reflector</u>	<u>Mass (Gm)</u>	<u>Radius (Cm)</u>	<u>Gram/L</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	768	19.4249	25	0.950	0.984
Water	480	15.6318	30	0.976	1.009
Steel	335	13.8683	30	0.950	1.005

 ^{240}Pu (Density = 19.90 gm/cc)
Metal Unit (Same as Table III)

<u>Reflector</u>	<u>Mass (Kg)</u>	<u>Radius (Cm)</u>	<u>Kg/L</u>	$\frac{S_{\infty}P_1}{\text{Keff H-R}}$	$\frac{S_{\infty}P_1}{\text{Keff E-IV}}$
Bare	29.58	7.080	19.90	0.950	0.913
Water	20.00	6.214	19.90	0.878	0.876
Steel	15.00	5.646	19.90	0.921	0.890

TABLE V
CALCULATIONS OF HOMOGENEOUS MIXTURES OF
 ^{235}U AND ^{239}Pu Metal

0.25 ^{235}U SUM + 0.75 ^{239}Pu SUM

Reflector	Density (gm/cc)	Wt.% ^{235}U	Wt.% ^{239}Pu	Radius (Cm)
Bare	19.184	61.072	38.928	5.834
Water	19.231	56.429	43.571	4.604
Steel	19.234	56.188	43.812	4.490

Reflector	Volume (L)	Mass (Kg)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	0.8317	15.95	0.895	0.898
Water	0.4088	7.86	0.913	1.080
Steel	0.3792	7.29	0.907	0.941

0.5 ^{235}U SUM + 0.5 ^{239}Pu SUM

Reflector	Density (gm/cc)	Wt.% ^{235}U	Wt.% ^{239}Pu	Radius (Cm)
Bare	18.971	82.476	17.524	6.674
Water	19.000	79.530	20.470	5.194
Steel	19.002	79.370	20.630	5.062

Reflector	Volume (L)	Mass (Kg)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	1.2453	23.62	0.903	0.911
Water	0.5870	11.15	0.917	1.080
Steel	0.5434	10.33	0.912	0.951

0.75 ^{235}U SUM + 0.25 ^{239}Pu SUM

Reflector	Density (gm/cc)	Wt.% ^{235}U	Wt.% ^{239}Pu	Radius (Cm)
Bare	18.864	93.386	6.614	7.344
Water	18.877	92.098	7.902	5.674
Steel	18.877	92.027	7.973	5.528

Reflector	Volume (L)	Mass (Kg)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	1.6591	31.29	0.925	0.935
Water	0.7653	14.45	0.932	1.113
Steel	0.7076	13.36	0.929	0.941

Water reflector thickness - 30.48 Cm.
Steel reflector thickness - 25.4 Cm.
H-R - Hansen-Roach Cross Sections
E-IV - ENDF/B-IV Cross Sections

Homogeneous ^{235}U - ^{239}Pu Metal Mixtures
 Bare Spheres
 Frac. ^{239}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 9

Homogeneous ^{235}U - ^{239}Pu Metal Mixtures
 Water Reflected Spheres
 Frac. ^{239}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 10

Homogeneous ^{235}U - ^{239}Pu Metal Mixtures
 Steel Reflected Spheres
 $\text{Frac. } ^{239}\text{Pu SUM} = 1 - \text{Frac. } ^{235}\text{U SUM}$

FIGURE 11

give the same behavior relative to the straight line for each individual set of cross sections.

The second set of calculations were for heterogeneous fissile-fissile binary mixtures of ^{235}U and ^{239}Pu . The mixtures were combined as for the homogeneous cases in quarters of Single Unit Masses, but the material was arranged with the two fissile materials segregated into a ball and shell. Calculations were done for both ^{235}U in the ball (^{239}Pu in the shell) and ^{239}Pu in the ball (^{235}U in the shell). Results of these calculations are shown in Table VI and Figures 12, 13, and 14. These results deserve comment.

The extremely high K_{eff} values for the bare spheres are a direct outcome of the boundary condition. Table III shows that the ^{235}U Single Unit Mass for the steel reflector is less than half the ^{235}U Single Unit Mass for the bare case. It comes as no surprise that a shell of ^{239}Pu , which is a better reflector than steel and fissions too, will produce an enormous K_{eff} value. Exactly the same condition is true of the ^{239}Pu ball geometry.

Complex spectral effects enter the picture for the water reflected spheres. The water reflector provides a more thermalized spectrum in the outer parts of the sphere, the overall effect being a thermal to intermediate average spectrum. The ^{239}Pu mass is only about one fourth that of ^{235}U which provides only a very thin shell permitting spectral effects to extend a significant distance into the ball. The ^{235}U shell is very thick in comparison and permits virtually no spectral changes in the ball. Both fissile metals are good reflectors, but ^{235}U is significantly better than ^{239}Pu . The overall effect is to give results that show the Rule-Of-Fractions is valid when ^{239}Pu is in the shell, and that the Rule-Of-Fractions is invalid when ^{235}U is in the shell. The spectral effects combine in a very complicated manner to produce these results. The major spectral effects are summarized in the following table.

SPECTRAL EFFECTS IN WATER REFLECTED
HETEROGENEOUS ^{235}U - ^{239}Pu SPHERES

GEOMETRY	SPECTRUM IN BALL	SPECTRUM IN SHELL
^{235}U Ball ^{239}Pu Shell	Intermediate Energy in outer region - fewer fissions	Intermediate Energy - fewer fissions Lower reflection
^{239}Pu Ball ^{235}U Shell	High energy spectrum - more fissions	Intermediate Energy - slightly fewer fission. Higher reflection

The overall effect is to produce more fissions when ^{239}Pu is in the ball, and less when ^{235}U is in the ball. An increase or decrease of fissions was determined relative to a weighted

TABLE VI
CALCULATIONS OF HETEROGENEOUS BALL-SHELLS OF
 ^{235}U AND ^{239}Pu

0.25 ^{235}U SUM + 0.75 ^{239}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.9828	5.8335	2.551	2.554
Water	3.8331	4.6036	0.877	1.040
Steel	3.7334	4.4900	0.946	1.093

^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.2137	5.8335	2.561	2.567
Water	3.4552	4.6036	0.987	1.131
Steel	3.3762	4.4900	1.063	1.191

0.5 ^{235}U SUM + 0.5 ^{239}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	6.2780	6.6742	2.408	2.402
Water	4.8294	5.1942	0.891	1.063
Steel	4.7036	5.0622	0.972	1.145

^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	3.6807	6.6742	2.419	2.415
Water	3.0184	5.1942	0.987	1.141
Steel	2.9494	5.0622	1.073	1.222

0.75 ^{235}U SUM + 0.25 ^{239}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	7.1865	7.3439	2.327	2.317
Water	5.5283	5.6743	0.918	1.091
Steel	5.3843	5.5280	1.012	1.194

^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	2.9214	7.3439	2.337	2.329
Water	2.3957	5.6743	0.971	1.137
Steel	2.3409	5.5280	1.066	1.233

Heterogeneous ^{235}U - ^{239}Pu Metal Mixtures
Bare Spheres
Frac. ^{239}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 12

Heterogeneous 235U-239Pu Metal Mixtures Water Reflected Spheres Frac. 239Pu SUM = 1 - Frac. 235U SUM

FIGURE 13

Heterogeneous 235U-239Pu Metal Mixtures
Steel Reflected Spheres
Frac. 239Pu SUM = 1 - Frac. 235U SUM

FIGURE 14

average of the fissions in the calculations of the Single Unit Masses that define the straight line

The steel reflected heterogeneous ^{235}U - ^{239}Pu spheres are somewhat simpler than the water reflected in that the spectrum is very high in energy throughout the sphere. The results shown in Figure 14 indicate that the ENDF/B-IV cross sections predict a bigger effect than Hansen-Roach, a result that appears to arise from differences in cross sections for the constituents of steel. Basically ENDF/B-IV cross sections provide more neutron reflection (less leakage) for a given thickness of steel reflector than Hansen-Roach cross sections. The reasons why K_{eff} increases for all heterogeneous ball-shell arrangements is based in the relative differences in fissions and scattering. The similarity in shape of the Hansen-Roach and ENDF/B-IV curves indicates the same mechanisms at work. The fissions peak is in group 2 of the sixteen group Hansen-Roach cross sections, and for that group the differences in fission and scattering cross sections are shown in the following table.

CROSS SECTION RATIOS IN HANSEN-ROACH GROUP 2

	<u>^{235}U</u>	<u>^{239}Pu</u>
	Increasing Neutron Production	
$\nu\Sigma_f/\Sigma_a$	2.46	3.01
Σ_s/Σ_a	2.46	1.25
	Increasing Scatter	

Different mechanisms change the spectrum depending on whether ^{235}U or ^{239}Pu is in the ball. For a small ^{235}U ball the effects arise from slightly increased fissions in the shell, higher reflection from the heavy metal shell, higher reflection from the ball surface, and slightly reduced fissions in the ball. The result is dependent on the magnitude of the changes and gives different results for the two cross section sets. As the ^{235}U ball size increases, the reflection in both directions from the surface increases, yielding more total fissions overall. For the ^{239}Pu ball case the increased fissions occur primarily in the ball, and arise from adding a better reflector around the ball.

The last case to be considered was for mixtures of ^{235}U - ^{239}Pu Single Unit Masses for solutions. Here only homogeneous mixtures were considered because heterogeneous geometries appeared very unlikely. The SUMs were combined in quarter units as in the previous cases. Results of these calculations are shown in Table VII and Figures 15, 16, and 17. These results show similar curve shapes for K_{eff} for both Hansen-Roach and ENDF/B-IV with K_{eff} values that exceed the Rule-Of-Fractions linear values. ENDF/B-IV shows a larger effect than does Hansen-Roach which probably accrues from better treatment of the thermal and resonance region in ENDF/B-IV cross sections.

TABLE VII

**CALCULATIONS OF HOMOGENEOUS MIXTURES OF
²³⁵U AND ²³⁹Pu SOLUTION SINGLE UNIT MASSES**

0.25 ²³⁵U SUM + 0.75 ²³⁹Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²³⁹Pu</u>	<u>Radius (Cm)</u>
Bare	1.0256	0.987	1.821	19.4540
Water	1.0312	1.166	2.209	15.5664
Steel	1.0315	1.228	2.172	13.8858

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	30.84	31.63	0.951	1.009
Water	15.80	16.29	0.977	1.213
Steel	11.22	11.57	0.951	1.241

0.5 ²³⁵U SUM + 0.5 ²³⁹Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²³⁹Pu</u>	<u>Radius (Cm)</u>
Bare	1.0291	1.959	1.205	19.4792
Water	1.0359	2.351	1.485	15.5005
Steel	1.0362	2.433	1.435	13.9057

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	30.96	31.86	0.952	1.035
Water	15.60	16.16	0.977	1.323
Steel	11.26	11.67	0.952	1.338

0.75 ²³⁵U SUM + 0.25 ²³⁹Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²³⁹Pu</u>	<u>Radius (Cm)</u>
Bare	1.0326	2.916	0.598	19.5043
Water	1.0407	3.557	0.749	15.4340
Steel	1.0409	3.618	0.711	13.9256

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	31.08	32.09	0.951	1.038
Water	15.40	16.03	0.976	1.320
Steel	11.31	11.77	0.951	1.340

Water reflector thickness - 30.48 Cm.
 Steel reflector thickness - 25.4 Cm.
 H-R - Hansen-Roach Cross Sections
 E-IV - ENDF/B-IV Cross Sections

Homogeneous ^{235}U - ^{239}Pu Solutions
 Bare Spheres
 $\text{Frac. } ^{239}\text{Pu SUM} = 1 - \text{Frac. } ^{235}\text{U SUM}$

FIGURE 15

Homogeneous ^{235}U - ^{239}Pu Solutions
 Water Reflected Spheres
 Frac. ^{239}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 16

Homogeneous ^{235}U - ^{239}Pu Solutions
Steel Reflected Spheres
Frac. ^{239}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 17

Two effects are occurring that affect the fissions in these very thermal spectrum cases. First, the $H/^{235}\text{U}$ and $H/^{239}\text{Pu}$ values are no longer at optimum values in the mixtures. Second, the volume of the sphere changes which affects the size of the asymptotic spectrum region in the center of the sphere. This change in size has the effect of increasing the fissions in the ^{239}Pu more than decreasing the fissions in ^{235}U . Total fissions are increased slightly for Hansen-Roach cross sections and more significantly for ENDF/B-IV cross sections.

The conclusions to be drawn from these homogeneous and heterogeneous ^{235}U - ^{239}Pu metal and solution mixtures is that the Rule-Of-Fractions works when the spectrum changes result in fewer fissions than for a weighted average of the SUMs that define the straight line, and does not work when the spectrum changes result in greater fissions than for the weighted average of the SUMs that define the straight line. All of the homogeneous metal mixtures fall in the first category, and all of the heterogeneous metal mixtures and homogeneous solution mixtures fall in the second category. It would be prudent to check any real-world mixture by analysis before using the Rule-Of-Fractions.

VII. FISSIONABLE-FISSIONABLE MIXTURES

The two fissionable isotopes considered were ^{238}Pu and ^{240}Pu which are both fast fissioning isotopes. The Hansen-Roach cross sections for ^{238}Pu were not part of the original Hansen-Roach cross sections. These cross sections (in Hansen-Roach form) were derived from Persimmon Bomb Test Data¹² by LANL and originally sent to H. K. Clark at SRS. The calculated critical mass of ^{238}Pu from these cross sections does not reproduce values in standards¹ and other papers¹³. For the purposes of this study the normalization is not important so long as it is consistent. The Single Unit Masses for ^{238}Pu and ^{240}Pu were described in Section V and define the normalization for these calculations.

Only homogeneous and heterogeneous mixtures of metal Single Unit Masses were considered in this study because both isotopes show maximum reactivity in the metal form. The presence of water literally drives reactivity to very small values because few fissions occur in the thermal region for these isotopes.

Results for homogeneous mixtures of ^{238}Pu and ^{240}Pu metal are shown in Table VIII and Figures 18, 19, and 20. In all of these cases the Rule-Of-Fractions appears to work well. The ENDF/B-IV results showed a greater variation in K_{eff} values than Hansen-Roach results. The results stem primarily from the volumetric changes produced in the metal mixes. The ^{238}Pu SUM is about 25% of the ^{240}Pu SUM. This results from the fission cross section for ^{238}Pu being significantly bigger than for ^{240}Pu at high energy. Mixing a large volume of ^{240}Pu with the ^{238}Pu dilutes the mixture so that the ^{238}Pu is less effective as a fissioning nuclide. The result in all cases is a decrease in K_{eff} that confirms the Rule-Of-Fractions.

TABLE VIII
CALCULATIONS OF HOMOGENEOUS MIXTURES OF
 ^{238}Pu AND ^{240}Pu Metal

0.25 ^{238}Pu SUM + 0.75 ^{240}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ^{238}Pu</u>	<u>Wt.% ^{240}Pu</u>	<u>Radius (Cm)</u>
Bare	19.886	8.600	91.400	6.630
Water	19.890	6.250	93.750	5.769
Steel	19.890	6.250	93.750	5.242

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	1.2206	24.27	0.944	0.926
Water	0.8044	16.00	0.858	0.871
Steel	0.6033	12.00	0.900	0.815

0.5 ^{238}Pu SUM + 0.5 ^{240}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ^{238}Pu</u>	<u>Wt.% ^{240}Pu</u>	<u>Radius (Cm)</u>
Bare	19.864	22.014	77.986	6.108
Water	19.873	16.667	83.333	5.243
Steel	19.873	16.667	83.333	4.764

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	0.9547	18.965	0.941	0.943
Water	0.6038	12.00	0.837	0.869
Steel	0.4529	9.00	0.875	0.806

0.75 ^{238}Pu SUM + 0.25 ^{240}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ^{238}Pu</u>	<u>Wt.% ^{240}Pu</u>	<u>Radius (Cm)</u>
Bare	19.826	45.854	54.146	5.479
Water	19.840	37.500	62.500	4.583
Steel	19.840	37.500	62.500	4.164

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	0.6889	13.66	0.938	0.967
Water	0.4032	8.00	0.815	0.874
Steel	0.3024	6.00	0.849	0.802

Water reflector thickness - 30.48 Cm.

Steel reflector thickness - 25.4 Cm.

H-R - Hansen-Roach Cross Sections

E-IV - ENDF/B-IV Cross Sections

Homogeneous ^{238}Pu - ^{240}Pu Metal Mixtures

Bare Spheres

Frac. ^{240}Pu SUM = $1 - \text{Frac. } ^{238}\text{Pu SUM}$

FIGURE 18

Homogeneous ^{238}Pu - ^{240}Pu Metal Mixtures
 Water Reflected Spheres
 $\text{Frac. } ^{240}\text{Pu SUM} = 1 - \text{Frac. } ^{238}\text{Pu SUM}$

FIGURE 19

Homogeneous ^{238}Pu - ^{240}Pu Metal Mixtures
 Steel Reflected Spheres
 Frac. ^{240}Pu SUM = 1 - Frac. ^{238}Pu SUM

FIGURE 20

Calculated results for heterogeneous ball-shell arrangements of ^{238}Pu and ^{240}Pu metal are shown in Table IX and Figures 21, 22, and 23. These results are characterized by satisfying the Rule-Of-Fractions when ^{240}Pu is in the ball, and not satisfying the Rule-Of-Fractions when ^{238}Pu is in the ball. These effects arise because ^{238}Pu has a higher fast fission cross section than ^{240}Pu , and ^{240}Pu has a higher fast scattering cross section than ^{238}Pu . When ^{240}Pu is in the ball it is surrounded by much less mass of a poorer reflector than in single unit form, even though that reflector has a very high fission cross section. When ^{238}Pu is in the ball it is surrounded by a much bigger mass of a better reflector than in single unit form, and that reflector has a moderate fission cross section. The reflector surrounding the unit does not seem to play much role, as indicated by the shapes of all the curves being almost identical.

The conclusions to be drawn from the above results appear to be the following:

- the Rule-Of-Fractions appears to hold for homogeneous mixtures of two pure fissionable nuclides.
- the Rule-Of-Fractions appears to be valid for heterogeneous arrangements of two fissionable nuclides when the nuclide having the lower fission cross section is in the ball region of a ball-shell arrangement.

These conditions are not only hard to apply for any general application, but are restricted to two-nuclide mixtures.

VIII. FISSILE-FISSIONABLE MIXTURES

There are four possible combinations of the two fissile and two fissionable isotopes under consideration. For this study only two combinations were examined, and were chosen to prevent too great a disparity between masses of fissile and fissionable Single Unit Masses. A large disparity in Single Unit Masses seemed to play a very large role in the ^{238}Pu - ^{240}Pu mixtures in the previous section. It is obvious that a combination of ^{235}U and ^{238}Pu would represent a very large mass difference between single units, as also would be the case for combinations of ^{239}Pu and ^{240}Pu . Homogeneous and heterogeneous mixtures of ^{235}U and ^{240}Pu , and of ^{238}Pu and ^{239}Pu were studied.

The mixtures were combined in quarter unit increments as in the previous cases. Calculated results for homogeneous mixtures of ^{235}U and ^{240}Pu metal units are shown in Table X and Figures 24, 25, and 26. These results indicate that the Rule-Of-Fractions is not valid for any mixture or reflector. The spectral effects that lead to reduced margins are more complex than in previous cases when homogeneous mixtures increased the margins. The spectral effects arise from differences in $\nu\Sigma_f/\Sigma_a$, Σ_s/Σ_a , and reflector effects between the two isotopes. These differences are summarized in Table XI.

TABLE IX
CALCULATIONS OF HETEROGENEOUS BALL-SHELLS OF
 ^{238}Pu AND ^{240}Pu

<u>0.25 ^{238}Pu SUM + 0.75 ^{240}Pu SUM</u>				
^{238}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	2.9336	6.6297	1.016	1.090
Water	2.2954	5.7690	0.905	1.007
Steel	2.0855	5.2420	0.935	1.009
^{240}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	6.4324	6.6297	0.912	0.876
Water	5.6457	5.7690	0.841	0.847
Steel	5.1294	5.2420	0.882	0.854
<u>0.5 ^{238}Pu SUM + 0.5 ^{240}Pu SUM</u>				
^{238}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	3.6961	6.1084	1.037	1.113
Water	2.8920	5.2430	0.910	1.013
Steel	2.6276	4.7640	0.934	1.010
^{240}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	5.6192	6.1084	0.877	0.842
Water	4.9320	5.2430	0.798	0.804
Steel	4.4810	4.7640	0.841	0.817
<u>0.75 ^{238}Pu SUM + 0.25 ^{240}Pu SUM</u>				
^{238}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.2309	5.4787	1.020	1.094
Water	3.3105	4.5830	0.886	0.990
Steel	3.0078	4.1640	0.908	0.986
^{240}Pu Ball				
Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.4600	5.4784	0.858	0.827
Water	3.9145	4.5830	0.759	0.774
Steel	3.5566	4.1640	0.802	0.788

Heterogeneous 238Pu-240Pu Metal Mixtures

Bare Spheres

Frac. 240Pu SUM = 1 - Frac. 238Pu SUM

FIGURE 21

Heterogeneous 238Pu-240Pu Metal Mixtures
 Water Reflected Spheres
 Frac. 240Pu SUM = 1 - Frac. 238Pu SUM

FIGURE 22

Heterogeneous 238Pu-240Pu Metal Mixtures Steel Reflected Spheres Frac. 240Pu SUM = 1 - Frac. 238Pu SUM

FIGURE 23

TABLE X

**CALCULATIONS OF HOMOGENEOUS MIXTURES OF
²³⁵U AND ²⁴⁰Pu Metal**

0.25 ²³⁵U SUM + 0.75 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Radius (Cm)</u>
Bare	19.551	30.514	69.486	7.305
Water	19.638	22.820	77.180	6.182
Steel	19.594	26.698	73.302	5.718

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	1.6330	31.93	0.970	0.942
Water	0.9897	19.44	0.969	0.954
Steel	0.7833	15.35	0.961	0.960

0.5 ²³⁵U SUM + 0.5 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Radius (Cm)</u>
Bare	19.259	56.849	43.151	7.518
Water	19.367	47.006	52.994	6.150
Steel	19.310	52.214	47.786	5.789

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	1.7797	34.275	0.969	0.954
Water	0.9743	18.870	0.958	1.084
Steel	0.8128	15.695	0.967	0.979

0.75 ²³⁵U SUM + 0.25 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Radius (Cm)</u>
Bare	19.012	79.807	20.193	7.719
Water	19.088	72.685	27.315	6.118
Steel	19.046	76.625	23.375	5.859

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁ Keff H-R</u>	<u>S_∞P₁ Keff E-IV</u>
Bare	1.9263	36.623	0.961	0.960
Water	0.9590	18.305	0.957	1.117
Steel	0.8423	16.043	0.962	0.992

Water reflector thickness - 30.48 Cm.

Steel reflector thickness - 25.4 Cm.

H-R - Hansen-Roach Cross Sections

E-IV - ENDF/B-IV Cross Sections

Homogeneous ^{235}U - ^{240}Pu Metal Mixtures

Bare Spheres

Frac. ^{240}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 24

Homogeneous ^{235}U - ^{240}Pu Metal Mixtures
 Water Reflected Spheres
 Frac. ^{240}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 25

Homogeneous ^{235}U - ^{240}Pu Metal Mixtures
Steel Reflected Spheres
Frac. ^{240}Pu SUM = 1 - Frac. ^{235}U SUM

FIGURE 26

TABLE XI
HANSEN-ROACH CROSS SECTION RATIOS FOR ^{235}U AND ^{240}Pu

Energy Region	$\nu\Sigma_f/\Sigma_a$		Σ_s/Σ_a	
	^{235}U	^{240}Pu	^{235}U	^{240}Pu
High: > 1 Mev	2.32	3.25	2.52	1.78
Intermediate: 10 ev - 1 Mev	2.0	0.21	0.2	0.3
Thermal: < 10 ev	2.1	<0.001	0.1	0.3

For the bare spheres, the spectrum peaks in the high energy region. Here the admixed ^{235}U provides a better reflector than ^{240}Pu , and thus increasing the net fissions in ^{240}Pu in the center of the sphere. The effect essentially increases the size of the asymptotic spectrum region in the center, increasing its importance, hence yielding more net fissions.

For the water reflected spheres, the major effect comes from a combination of increased intermediate and thermal fissions in ^{235}U , and increased net fissions in the center of the sphere as for the bare spheres. The ENDF/B-IV data shows a markedly larger effect here than Hansen-Roach data. This is probably due to the better treatment of water scattering in ENDF/B-IV.

For the steel reflected spheres, the major effects that increase fission arise from the same source as for the bare spheres, and the magnitude of the K_{eff} change is larger for both sets of cross sections.

Calculated results for homogeneous mixtures of ^{238}Pu and ^{239}Pu metal units are shown in Table XII and Figures 27, 28, and 29. Here maintaining Rule-Of-Fractions margins seems to depend on the reflector.

The bare sphere mixtures display increased margin for both sets of cross sections thus satisfying the Rule-Of-Fractions.

The water reflected spheres show a net increase in fissions that result primarily from increased thermal fissions in ^{239}Pu and intermediate fissions in ^{238}Pu .

The steel reflected spheres show a smaller increase in margin than the bare spheres, and an almost exact preservation of the Rule-Of-Fractions margins with Hansen-Roach cross sections. Figure 29 shows results that almost exactly agree with the Rule-Of-Fractions, certainly better than any other case considered.

Overall the homogeneous metal mixture cases do not confirm the margins of the Rule-Of-Fractions.

TABLE XII
CALCULATIONS OF HOMOGENEOUS MIXTURES OF
 ^{238}Pu AND ^{239}Pu Metal

0.25 ^{239}Pu SUM + 0.75 ^{238}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt. % ^{238}Pu</u>	<u>Wt. % ^{239}Pu</u>	<u>Radius (Cm)</u>
Bare	19.760	75.158	24.842	4.652
Water	19.762	72.438	27.562	3.685
Steel	19.765	67.873	32.127	3.421

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	0.4217	8.333	0.947	0.990
Water	0.2096	4.142	0.856	0.984
Steel	0.1677	3.315	0.860	0.911

0.5 ^{239}Pu SUM + 0.5 ^{238}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt. % ^{238}Pu</u>	<u>Wt. % ^{239}Pu</u>	<u>Radius (Cm)</u>
Bare	19.780	50.210	49.790	4.647
Water	19.783	46.696	53.304	3.725
Steel	19.787	41.322	58.678	3.525

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	0.4204	8.315	0.947	0.969
Water	0.2165	4.283	0.890	1.015
Steel	0.1835	3.630	0.890	0.925

0.75 ^{239}Pu SUM + 0.25 ^{238}Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt. % ^{238}Pu</u>	<u>Wt. % ^{239}Pu</u>	<u>Radius (Cm)</u>
Bare	19.800	25.158	74.842	4.642
Water	19.802	22.601	77.397	3.764
Steel	19.805	19.011	80.989	3.623

<u>Reflector</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	$S_{\infty}P_1$ <u>Keff H-R</u>	$S_{\infty}P_1$ <u>Keff E-IV</u>
Bare	0.4191	8.2975	0.947	0.953
Water	0.2234	4.4245	0.920	1.038
Steel	0.1992	3.9450	0.920	0.943

Water reflector thickness - 30.48 Cm.

Steel reflector thickness - 25.4 Cm.

H-R - Hansen-Roach Cross Sections

E-IV - ENDF/B-IV Cross Sections

Homogeneous ^{238}Pu - ^{239}Pu Metal Mixtures

Bare Spheres

Frac. ^{238}Pu SUM = 1 - Frac. ^{239}Pu SUM

FIGURE 27

Homogeneous ^{238}Pu - ^{239}Pu Metal Mixtures
 Water Reflected Spheres
 $\text{Frac. } ^{238}\text{Pu SUM} = 1 - \text{Frac. } ^{239}\text{Pu SUM}$

FIGURE 28

Homogeneous ^{238}Pu - ^{239}Pu Metal Mixtures
 Steel Reflected Spheres
 Frac. ^{238}Pu SUM = 1 - Frac. ^{239}Pu SUM

FIGURE 29

The next cases considered were heterogeneous ball-shell geometries using the fissile-fissionable metals.

Calculated results for heterogeneous mixtures of ^{235}U and ^{240}Pu metal are shown in Table XIII and Figures 30, 31, and 32. Calculations were performed with ^{235}U in the ball and ^{240}Pu in the shell, and also with the isotopes reversed in the same geometry. The results of these calculations showed a wide variation.

Results for bare spheres are shown in Figure 30. The Hansen-Roach cross sections show Rule-Of-Fraction margins are not preserved for the geometry with a ^{240}Pu ball and ^{235}U shell, but ENDF/B-IV cross sections show that the Rule-Of-Fractions does maintain the margin. The cross section ratios in Table XI show that the Hansen-Roach result arises from having a higher fissioning metal in the ball and a better reflecting material in the shell. The ENDF/B-IV result arises because those cross sections show a much lower Σ_s/Σ_a ratio for ^{235}U and a slightly lower $\nu\Sigma_f/\Sigma_a$ for ^{240}Pu giving on balance a reduction in total fissions.

Results for water reflected spheres are shown in Figure 31, and demonstrate that the Rule-Of-Fractions margins are not preserved for the geometry with a ^{240}Pu ball and ^{235}U shell. The Hansen-Roach results for the bare spheres is preserved, but the magnitude is larger because of significantly more fissions in the outer part of the shell from thermalized neutrons. The ENDF/B-IV results now also show the Rule-Of-Fractions margins are not preserved which results from increased fissions in the shell from thermalized neutrons. The magnitude of the shift in ENDF/B-IV results from the bare sphere results is surprisingly large, and probably results from the better treatment of water scattering.

Results for steel reflected spheres are shown in Figure 32, and demonstrate that Rule-Of-Fractions margins are not preserved irrespective of which isotope is in the ball or shell. The spectrum change that produces this arise from the scattered neutrons from the steel reflector producing a very large increase in fissions in the ^{235}U shell. Second, fewer neutrons are removed from the high energy region by scattering or leakage before fissioning, hence the overall magnitude of change is much larger.

The second set of heterogeneous ball-shell spheres were for ^{238}Pu and ^{239}Pu metal arrangements. Results of these calculations are shown in Table XIV and Figures 33, 34 and 35. The results for these cases appear to be strongly affected by the quality of the ^{238}Pu cross sections which for the Hansen-Roach set are not the best.

Results for the bare sphere arrangements are shown in Figure 33. Here ENDF/B-IV shows the Rule-Of-Fractions margins are not preserved for ^{238}Pu in the ball and ^{239}Pu in the shell. The

TABLE XIII
CALCULATIONS OF HETEROGENEOUS BALL-SHELLS OF
 ^{235}U AND ^{240}Pu

0.25 ^{235}U SUM + 0.75 ^{240}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.9828	7.3053	0.939	0.923
Water	3.8331	6.1821	0.866	0.934
Steel	3.7333	5.7180	0.935	1.024

^{240}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	6.4324	7.3053	0.969	0.931
Water	5.6457	6.1821	0.992	1.121
Steel	5.1294	5.7180	0.941	1.020

0.5 ^{235}U SUM + 0.5 ^{240}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	6.2780	7.5176	0.935	0.937
Water	4.8294	6.1499	0.855	0.956
Steel	4.7036	5.7894	0.942	1.071

^{240}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	5.6192	7.5176	0.983	0.951
Water	4.9320	6.1499	0.999	1.140
Steel	4.4810	5.7894	0.962	1.074

0.75 ^{235}U SUM + 0.25 ^{240}Pu SUM
 ^{235}U Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	7.1865	7.7187	0.939	0.948
Water	5.5283	6.1175	0.849	0.967
Steel	5.3843	5.8586	0.945	1.100

^{240}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.4600	7.7187	0.985	0.965
Water	3.9145	6.1175	0.988	1.144
Steel	3.5566	5.8586	0.968	1.107

Heterogeneous 235U-240Pu Metal Mixtures

Bare Spheres

Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 30

Heterogeneous 235U-240Pu Metal Mixtures
 Water Reflected Spheres
 Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 31

Heterogeneous 235U-240Pu Metal Mixtures Steel Reflected Spheres Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 32

TABLE XIV
CALCULATIONS OF HETEROGENEOUS BALL-SHELLS OF
 ^{238}Pu AND ^{239}Pu

0.25 ^{238}Pu SUM + 0.75 ^{239}Pu SUM
 ^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	2.9214	4.6519	0.953	0.944
Water	2.3957	3.6848	0.827	0.879
Steel	2.3409	3.4211	0.863	0.877

^{238}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.2309	4.6519	0.946	1.013
Water	3.3105	3.6848	0.888	1.035
Steel	3.0078	3.4211	0.858	0.930

0.5 ^{238}Pu SUM + 0.5 ^{239}Pu SUM
 ^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	3.6807	4.6471	0.953	0.942
Water	3.0184	3.7250	0.846	0.899
Steel	2.9494	3.5249	0.940	0.909

^{238}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	3.6961	4.6471	0.943	1.007
Water	2.8920	3.7250	0.909	1.046
Steel	2.6276	3.5249	0.887	0.952

0.75 ^{238}Pu SUM + 0.25 ^{239}Pu SUM
 ^{239}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	4.2134	4.6423	0.952	0.942
Water	3.4552	3.7643	0.866	0.921
Steel	3.3762	3.6230	0.923	0.936

^{238}Pu Ball

Reflector	Ball Rad. (Cm)	Shell Rad. (Cm)	$S_{\infty}P_1$ Keff H-R	$S_{\infty}P_1$ Keff E-IV
Bare	2.9336	4.6423	0.943	0.994
Water	2.2954	3.7643	0.928	1.051
Steel	2.0855	3.6230	0.917	0.970

Heterogeneous 238Pu-239Pu Metal Mixtures
Bare Spheres
Frac. 238Pu SUM = 1 - Frac. 239Pu SUM

FIGURE 33

Heterogeneous 238Pu-239Pu Metal Mixtures
 Water Reflected Spheres
 Frac. 238Pu SUM = 1 - Frac. 239Pu SUM

FIGURE 34

Heterogeneous 238Pu-239Pu Metal Mixtures Steel Reflected Spheres Frac. 238Pu SUM = 1 - Frac. 239Pu SUM

FIGURE 35

cross section mechanism for this is exactly the same as for the ^{235}U shell and ^{240}Pu ball substituting fissile for fissionable and fissionable for fissionable. The Hansen-Roach cross sections for ^{238}Pu display a somewhat different set of ratios in the high energy region. Table XV shows the comparative data for the two isotopes.

TABLE XV
Hansen-Roach Cross Section Ratios for ^{238}Pu and ^{239}Pu

Energy Region	$\nu\Sigma_f/\Sigma_a$		Σ_s/Σ_a	
	^{238}Pu	^{239}Pu	^{238}Pu	^{239}Pu
High: > 1 Mev	3.0	3.1	2.1	1.25
Intermediate: 10 ev - 1 Mev	0.5	2.2	0.2	0.3
Thermal: < 10 ev	<0.002	2.7	0.1	0.15

Here ^{238}Pu is the best scatterer, but there is no essential difference in fissions between the nuclides. These cross sections make the geometry with a ^{239}Pu ball produce more net fissions. What is most surprising is the relatively large increase in K_{eff} for the ENDF/B-IV cross sections.

The calculated results for water reflected heterogeneous ^{238}Pu - ^{239}Pu metal ball and shell arrangements are shown in Figure 34. These results return to those observed previously that a net increase in fissions occurs for the fissionable isotope in the ball. Here both Hansen-Roach and ENDF/B-IV confirm the same result. The spectrum change that produces this result arises from the increase in thermal fissions in the shell outer regions. In addition the water reflects a significant number of the fast neutrons providing possible additional fissions.

The results for the steel reflected ^{238}Pu - ^{239}Pu metal ball shell arrangements are shown in Figure 35. These results are similar to the bare sphere results except that the magnitude of the changes in K_{eff} are significantly lower. The difference from the bare sphere case appears to be that the steel reflector enhances the fissions in the ^{238}Pu shell for the ^{239}Pu ball case which raises the lower curve without changing the ^{238}Pu ball case very much.

The overall result of these heterogeneous fissile-fissionable metal ball-shell cases is that little confirmation of the Rule-Of-Fractions safety margins may be found.

The last cases examined were for homogeneous fissile-fissionable solution spheres, except that in this case the most reactive fissionable solution is a metal sphere. A few calculations were done to assure that reactivity does decrease

when a ^{235}U solution SUM is homogenized with a ^{240}Pu metal SUM. Results for these calculations are shown in Table XVI and Figures 36, 37, and 38. Calculations were only performed with Hansen-Roach cross sections.

The results of these calculations showed that in every case the loss of fissions in fast fissioning ^{240}Pu decreased reactivity below a K_{eff} of 0.25. This results from the presence of water producing a dominant thermal spectrum.

The calculated results from all of the fissile-fissionable mixtures examined can be summarized as follows:

- Homogeneous metal mixtures of fissionable-fissile nuclides do not confirm the safety margins of the Rule-Of-Fractions except for a few cases.
- Heterogeneous metal mixtures of fissionable-fissile nuclides do not confirm the safety margins of the Rule-Of-Fractions.
- Homogeneous solution mixtures of fissionable-fissile nuclides confirm the safety margins of the Rule-Of-Fractions.

Overall there is no compelling reason to use the Rule-Of-Fractions with fissionable-fissile mixtures of nuclides.

IX. CONCLUSIONS

The conclusion from the calculated examples in this study is that the Rule-Of-Fractions cannot be relied on to preserve an acceptable margin of safety for any arbitrary combination of fissile and fissionable nuclides. However, for certain specific conditions the Rule-Of-Fractions margin was preserved. The conditions under which it appears valid are:

- Homogeneous mixtures of fissile-fissile metals.
- Homogeneous mixtures of fissionable-fissionable solutions.
- Homogeneous mixtures of fissile-fissionable solutions.

Even these conditions must be viewed with suspicion because this study examined only binary mixtures (two nuclides), and most circumstances of application of the Rule-Of-Fractions can have more nuclides than two.

It should be noted that the calculated examples in this study represent pathological cases because such combinations of nuclides are not experienced in real world processing of fissile/fissionable mixtures of nuclides. These nuclides were chosen because of the quality of their cross section data and enabling various combinations of nuclides to be examined. In addition ball-shell arrangements of nuclides or stoichiometric mixing of nuclides are rare events in real world handling of fissile/fissionable nuclides. These geometries and assumptions were made to enable well defined cases and ease in calculation.

TABLE XVI

**CALCULATIONS OF HOMOGENEOUS MIXTURES OF
²³⁵U AND ²⁴⁰Pu SOLUTION SINGLE UNIT MASSES**

0.25 ²³⁵U SUM + 0.75 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Wt.% O</u>	<u>Wt.% H</u>
Bare	3.3951	1.031	73.299	22.800	2.870
Water	3.9487	1.001	70.059	17.709	2.231
Steel	4.1757	0.999	79.117	17.659	2.225

<u>Reflector</u>	<u>Radius (Cm)</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁</u> <u>Keff H-R</u>
Bare	12.863	8.915	30.267	0.189
Water	10.468	4.805	18.973	0.216
Steel	9.333	3.405	14.220	0.228

0.5 ²³⁵U SUM + 0.5 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Wt.% O</u>	<u>Wt.% H</u>
Bare	1.8805	2.016	47.782	44.589	5.613
Water	2.2149	2.118	55.721	37.444	4.718
Steel	2.2188	2.219	55.808	37.370	4.709

<u>Reflector</u>	<u>Radius (Cm)</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁</u> <u>Keff H-R</u>
Bare	15.743	16.343	30.953	0.124
Water	12.460	8.103	17.947	0.130
Steel	11.308	6.057	13.439	0.140

0.75 ²³⁵U SUM + 0.25 ²⁴⁰Pu SUM

<u>Reflector</u>	<u>Density (gm/cc)</u>	<u>Wt.% ²³⁵U</u>	<u>Wt.% ²⁴⁰Pu</u>	<u>Wt.% O</u>	<u>Wt.% H</u>
Bare	1.3310	2.959	23.372	65.433	8.236
Water	1.4522	3.369	29.551	59.574	7.506
Steel	1.4536	3.365	29.724	59.512	7.499

<u>Reflector</u>	<u>Radius (Cm)</u>	<u>Volume (L)</u>	<u>Mass (Kg)</u>	<u>S_∞P₁</u> <u>Keff H-R</u>
Bare	17.837	23.772	31.640	0.193
Water	14.064	11.651	16.920	0.191
Steel	12.763	8.708	12.659	0.184

Water reflector thickness - 50.48 Cm.
 Steel reflector thickness - 25.4 Cm.
 H-R - Hansen-Roach Cross Sections

Homogeneous 235U-240Pu Solution Mixtures
Bare Spheres
Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 36

Homogeneous 235U-240Pu Solution Mixtures
Water Reflected Spheres
Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 37

Homogeneous 235U-240Pu Solution Mixtures Steel Reflected Spheres Frac. 240Pu SUM = 1 - Frac. 235U SUM

FIGURE 38

An application of the Rule-Of-Fractions that was not examined in this study is illustrated in the shipping package in Section II. Here geometry effects are important as well as mixtures of nuclides. The addition of the second parameter may well make the Rule-Of-Fractions valid over a wide range of applications. This requires justification which was not provided by this study.

The conclusion of this study is that the Rule-Of-Fractions does not provide a generally applicable way of assuring a specified or desired margin of safety in K_{eff} . Its use must be justified by computational or experimental results that demonstrate its applicability to the specific condition of use, and establish the range of parameters over which the rule may be applied.

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