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**Analytical Solutions of Minimum Ionization Particle Induced
Current Shapes of Silicon Detectors and Simulation of Charge
Collection Properties***

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Analytical Solutions of Minimum Ionization Particle induced Current Shapes of Silicon Detectors and Simulation of Charge Collection Properties

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Abstract

A new analytical, one dimensional method to obtain the induced current shapes and simulation of charge shapes for p^+-n-n^+ silicon detectors in the case of minimum ionization particle has been developed here. Exact solutions have been found for both electron and hole current shapes. Simulations of induced charge shapes of detectors have also been given. The results of this work are consistent with the earlier work where a semi-analytical method had been used.

I. INTRODUCTION

Charge collection properties of silicon detector under high fluence of neutron radiation in the proposed large colliders, such as the SSC and LHC, is an important effect of radiation damage in detectors. There have been some modeling and simulation works done semi-analytically in the past about induced current shapes for alpha particle as well as minimum ionization particle (MIP), and trapping effects^[1]. In this work we report a new analytical, one dimensional method to obtain the induced current shapes and simulation of charge shapes for p^+-n-n^+ silicon detector in the case of MIP. Here the diffusion and de-trapping effects are not taken into account. The trapping effect will be considered in the paper.

II. ANALYTICAL SOLUTIONS

The electron and hole densities are denoted as $n(t)$ and $p(t)$ respectively, with initial values of $n(0)$ and $p(0)$. In order to have field distribution in the detector not being disturbed by $n(t)$ and $p(t)$, the signal level must be low, i.e. $n(t) \ll N_{eff}$, $p(t) \ll N_{eff}$. In the calculations, the bias used is a constant, the space charge density is defined as $\rho = eN_{eff}$ and detector thickness is d .

A. Calculation of electron pulse shape

The field distribution in the detector is shown in Fig. 1 where the shaded area is the layer of moving electrons. The boundaries of the charge layers $\xi(t)$ are assumed to move with velocity $\mu_n E(x)$ for electrons and $\mu_p E(x)$ for holes. The

track length for the boundary layer of charges is defined as $l(t)$ which is related to $\xi(t)$ as following:

$$l(t) = d - \xi(t) \quad (1)$$

Fig. 1. Field distribution in detector. Shaded area is the layer of moving electrons

The field distribution is:

$$E(x) = E_0 - \frac{\rho}{\epsilon\epsilon_0} x \quad (2)$$

where $\rho = eN_{eff}$ and E_0 is defined as:

$$E_0 = \frac{V}{d} + \frac{d\rho}{2\epsilon\epsilon_0} \quad (3)$$

The current density of electrons $i_e(x, t)$ at $x \geq \xi(t)$ is

$$i_e(x, t) = \mu_n e n(x, t) E(x) + \epsilon\epsilon_0 \frac{\partial E(x, t)}{\partial t} \quad (4)$$

($E(x,t)$ is taken into account only in the case of displacement current). The current $i_e(x,t)$ is a constant at each cross-section of the detector according the Kizhgoft law. Integrating the current of equation (4) along the detector's thickness we have

$$i_e = \frac{1}{d} \mu_e e \int_{\xi(t)}^d n(x,t) E(x) dx \quad (5)$$

For each separate point (layer), we can write the following formula for $n(x,t)$:

$$\frac{dn(x,t)}{dt} = \frac{\partial n}{\partial t} + \mu E \frac{\partial n}{\partial x} \quad (6)$$

From continuation law we have:

$$\frac{\partial i}{\partial x} = -e \frac{\partial n}{\partial t} \quad (7)$$

Combining equation (4), (7) and using Poisson equation

$$\frac{\partial E}{\partial x} = -\frac{\rho}{\epsilon \epsilon_0}, \text{ we get}$$

$$\frac{\partial n}{\partial t} + \mu E \frac{\partial n}{\partial x} = \frac{\mu n \rho}{\epsilon \epsilon_0} \quad (8)$$

Substitute (8) into (6) we have

$$\frac{dn}{dt} = \frac{\mu n \rho}{\epsilon \epsilon_0} \quad (9)$$

Solution of (9) is

$$n(t) = n_0 e^{\frac{\mu \rho}{\epsilon \epsilon_0} t} \quad (10)$$

It means that concentration in each layer of moving track increases in time with rate $\frac{\mu \rho}{\epsilon \epsilon_0}$, which does not depend on coordinate x . So the homogeneous concentration in the track is conserved with time. From (5) and (10) we get

$$i_e(t) = \frac{1}{d} \mu_e e n_0 e^{\frac{\mu \rho}{\epsilon \epsilon_0} t} [E_0(d - \xi(t)) - \frac{\rho}{2 \epsilon \epsilon_0} (d^2 - \xi^2(t))] \quad (11)$$

where $\xi(t)$ is the solution of the following equation:

$$\frac{\partial \xi}{\partial t} = \mu E(\xi) = \mu E_0 \left(1 - \frac{\xi \rho}{\epsilon \epsilon_0 E_0}\right) \quad (12)$$

and is found:

$$\xi(t) = E_0 \frac{\epsilon \epsilon_0}{\rho} \left(1 - e^{-\frac{\mu \rho}{\epsilon \epsilon_0} t}\right) \quad (13)$$

We note here that t should be limited by the value of collection time t_c . According to equation (13) we know that

at $t = t_c$, $\xi = d$, we get

$$t_c^e = \frac{\epsilon \epsilon_0}{\mu_e \rho} \ln \frac{\frac{2V \epsilon \epsilon_0}{\rho d^2} + 1}{\frac{2V \epsilon \epsilon_0}{\rho d^2} - 1} \quad (14)$$

The current shape for electrons calculated from equation (11) and equation (13) is shown in Fig. 2. The values used for electron and hole mobilities in this paper are 1450 cm²/v/s and 450 cm²/v/s, respectively.

Fig. 2. The electron current shape simulation result for MIP

B. Calculation of hole pulse shape

Similarly, we can calculate the hole current shape using the same method described above. The field distribution in the detector is shown in Fig. 3. Again, the hole current can be written as:

$$i_p(t) = \frac{1}{d} \mu_p e \int_0^{\xi(t)} p(x, t) E(x) dx \quad (15)$$

Fig. 3. Field distribution in detector. Shaded area is the layer of moving holes.

Similar to the solution of equation (6) to equation (9), we obtain the following expression for $p(x, t)$:

$$p(t) = p_0 e^{-\frac{\mu_p \rho}{\epsilon \epsilon_0} t} \quad (16)$$

So for holes, the concentration decreases with time because holes move in the direction of increasing of field. In this case the front layer of holes has greater velocity than the back layer. From (15) and (16) we get the hole current shape:

$$i_p(t) = \frac{\mu_p e p_0}{d} e^{-\frac{\mu_p \rho}{\epsilon \epsilon_0} t} \left[E_0 \xi_p(t) - \frac{\rho}{2 \epsilon \epsilon_0} \xi_p^2(t) \right] \quad (17)$$

Where $\xi_p(t)$ is:

$$\xi_p(t) = d e^{\frac{\mu_p \rho}{\epsilon \epsilon_0} t} + \frac{E_0 \epsilon \epsilon_0}{\rho} (1 - e^{\frac{\mu_p \rho}{\epsilon \epsilon_0} t}) \quad (18)$$

The collection time of holes t_c^p can be found from equation (18) using $\xi_p(t_c^p) = 0$:

$$t_c^p = \frac{\epsilon \epsilon_0}{\mu_p \rho} \ln \left[n \frac{\frac{2V \epsilon \epsilon_0}{\rho d^2} + 1}{\frac{2V \epsilon \epsilon_0}{\rho d^2} - 1} \right] \quad (19)$$

We note that the expression for collection time for holes t_c^p is similar to that for electrons (equation (14)), with the only difference in the carrier mobilities: μ_p for holes and μ_e for electrons.

The hole current shape calculated from equation (17) to equation (19) is shown in Fig. 4.

Fig. 4. The hole current shape simulation result for MIP

The total induced current is:

$$i(t) = i_e(t) + i_p(t) \quad (20)$$

Fig. 5. shows the total induced current by MIP in a p+n-n+ silicon detector.

Comparison between the MIP current shape calculated by the analytical solutions in this paper and that by the semi-analytical method in Ref. [1] is shown in Fig. 6. It is clear that current shapes are similar and the difference is caused by the fact that in Ref. [1], carrier saturation velocities in the high field was considered, resulting in longer charge collection time.

Fig. 5. The total current shape of simulation result for MIP

Fig. 6 Comparison between the analytical solution of this study and the semi-analytical one of Ref.[1].

C. Calculation of charge shape

By integrating equation (20), one can get the induced charge shape by MIP for the detector:

$$Q(t) = \int_0^t i(t') dt' \quad (21)$$

$Q(t)$ can be calculated numerically and the computation result is shown in Fig. 7.

Fig. 7. The total charge simulation result for MIP

III. TRAPPING EFFECTS

Deep levels in silicon detectors trap free carriers, which cause charge collection loss. This charge trapping effect is worse for detectors exposed to high fluence ($\Phi_n > 10^{13}$ n/cm²) of neutron radiation that induce high concentrations of deep levels[3]. The trapping of a deep with an energy level E_t in the band gap is characterized by a trapping time constant τ_t [1] and eq. (6) can be modified as the following:

$$\frac{dn(x,t)}{dt} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial x} \frac{\partial x}{\partial t} - \frac{n}{\tau_t} \quad (22)$$

This leads to:

$$\frac{dn}{dt} = \frac{\mu p \rho}{\varepsilon \varepsilon_0} - \frac{n}{\tau_t} \quad (23)$$

and

$$n(t) = n_0 e^{\frac{\mu p \rho}{\varepsilon \varepsilon_0} t} e^{-\frac{t}{\tau_t}} \quad (24)$$

Eqs. (11) and (17) can be rewritten as

$$i_e(t) = \frac{1}{d} \mu_e e n_0 e^{\frac{\mu_e \rho}{\varepsilon \varepsilon_0} t} e^{-\frac{t}{\tau_t^e}} * \left[E_0(d - \xi(t)) - \frac{\rho}{2\varepsilon \varepsilon_0} (d^2 - \xi^2(t)) \right] \quad (25)$$

$$i_p(t) = \frac{\mu_p e p_0}{d} e^{-\frac{\mu_p \rho}{\varepsilon \varepsilon_0} t} e^{-\frac{t}{\tau_t^p}} * \left[E_0 \xi_p(t) - \frac{\rho}{2\varepsilon \varepsilon_0} \xi_p^2(t) \right] \quad (26)$$

where τ_t^e and τ_t^p are trapping constants for electrons and holes, respectively; and $\xi(t)$ and $\xi_p(t)$ are still defined by eqs. (13) and (18), respectively.

Fig. 8 shows the trapping effect on the current shapes for MIP. Obviously, more trapping (smaller τ_t) means less area under the current curve and therefore less charge collection. This point is illustrated clearly in Fig. 9, where charge shapes are plotted which various trapping time constants τ_t . It has been known that τ_t is proportional inversely to the neutron fluence^[1,3], which leads to smaller τ_t at higher neutron fluence and therefore less charge collection.

SIMULATED CURRENT SHAPES WITH TRAPPING

$d=300 \text{ um}$, $A=1.0 \text{ cm}^2$, $N_d=1E12/\text{cm}^3$

Fig. 8 Effect of deep level trapping on the MIP current shapes

SIMULATED CHARGE SHAPES WITH TRAPPING

$d=300 \text{ um}$, $A=1.0 \text{ cm}^2$, $N_d=1E12/\text{cm}^3$

Fig. 9 Charge shapes at a given bias with various trapping constants

For a detector irradiated to a certain neutron fluence with given τ_t and doping concentration (N_d or N_{eff}), higher operating voltages would give shorter charge collection times and less trapping. This voltage or field effect is shown in Fig. 10. The full depletion voltage for this detector is about 68 volts and τ_t is set at 0.01 μs . The charge collection at $V=70$ volts is about 60%, while it is 71% at 100 volts and 80% at 150 volts.

Fig. 10 Charge shapes for the same trapping time constant but various biases ($V > V_d$)

As it is shown in Fig. 11, the charge collection of the detector has little dependence on the effective doping concentration (N_{eff}) in the space charge region (SCR). In other words, as long as the detector is biased well over the full depletion voltage, the field distribution in the SCR has little effect on the charge collection efficiency. This is a very important point given the fact that N_{eff} has a strong dependence on the neutron fluence [1].

SIMULATED CHARGE SHAPES WITH TRAPPING

d=300 μm , A=1.0 cm²

Fig. 11 Charge shape dependence on the effective doping concentration (N_{eff}) in the space charge region.

IV. SUMMARY

It has been shown that, for MIP, one can get analytical solutions for the electron and hole current shapes in a one-dimensional problem. The results in this work are similar to those in reference [1], where a semi-analytical method was used. The trapping effects by deep levels introduced by displacement radiation has also been considered, and it does not affect analytical nature of the solutions. The effect of carrier saturation velocity in the high electrical field can also be incorporated in the calculations, but one may only get numerical solutions.

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