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Relativistic Interactions and Realistic Applications*

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Abstract

A four-fermion-coupling Lagrangian (relativistic Skyrme-type) interaction has been proposed for relativistic nuclear structure calculations. This interaction, which has the merit of simplicity, is from the outset tailored as an effective interaction for relativistic Hartree-Fock calculations. Various extensions of such a model are discussed and compared with Walecka's meson-nucleon mean field approach. We also present results of the calculation of nuclear ground state properties with an extended (density dependent) version of the four fermion interaction in a relativistic Hartree-Fock approximation.

*Contribution presented at the Conference "Realistic Nuclear Structure", May 28-30, 1992 at Stony Brook, to honour T.T.S. Kuo at the occasion of his 60th birthday

1 Introduction

Thanks to Tom Kuo's pioneering work and its later extensions, the derivation of two body effective interactions for nucleons in nuclei starting from the nucleon-nucleon force, has attained a high level of accuracy.

On the other hand an accurate calculation of the mean field for nucleons in nuclei is more difficult and still leaves room for phenomenological input. Relativistic mean field models inspired by Lorentz invariant Lagrangian densities, where a Dirac field ψ for the nucleons is coupled to scalar and vector meson fields became popular [1] They provided on the level of Hartree calculations a successful scenario for the description of ground state properties of finite nuclei and nuclear matter. These successful models roughly correspond to tree diagram approximations or in some cases to special subclasses of one loop insertions in the language of perturbative field theory. However, the attempt to take such Lagrangians literally as a basis for a systematic loopwise expansion of a renormalizable field theory has revealed [2, 3] serious deficiencies already at the one loop level. This should not be too surprising because a theory treating nucleons as elementary Dirac fields can be expected to be accurate at most for observables that do not depend on the internal structure of nucleons.

To avoid these difficulties we proposed [4, 5] a more pragmatic approach of using a relativistic Hartree Fock model with manifestly non-renormalizable

four fermion couplings with a Lagrangian of the form

$$\begin{aligned}\mathcal{L}(x) &= \bar{\psi}(x)(i\gamma_\nu\partial^\nu - M)\psi(x) + \sum_K \mathcal{L}^{(K)}(x) \\ \mathcal{L}^{(K)}(x) &= -\frac{1}{2}G^{(K)}\left(\bar{\psi}(x)\mathcal{O}^{(K)}\psi(x)\right)\left(\bar{\psi}(x)\mathcal{O}^{(K)}\psi(x)\right)\end{aligned}\quad (1)$$

Here $\mathcal{O}^{(K)} \otimes \mathcal{O}^{(K)}$ denote the well known invariant combinations of scalar, vector, pseudoscalar, axial vector, tensor Dirac matrices

$$1 \otimes 1, \gamma^\mu \otimes \gamma_\mu, \gamma_5 \otimes \gamma_5, \gamma^\mu \gamma_5 \otimes \gamma_\mu \gamma_5, \sigma_{\mu\nu} \otimes \sigma^{\mu\nu} \quad (2)$$

as well as those arising from the previous ones by multiplication with $\vec{\tau} \otimes \vec{\tau}$ in isotopic space.

In the simplest model corresponding to the $\sigma - \omega$ model only the isoscalar scalar and isoscalar vector coupling constants $G^{(S)}$ and $G^{(V)}$ are different from zero. In Hartree approximation we obtain for infinite nuclear matter the same results as the $\sigma - \omega$ mean field theory when the parameter values are

$$G^{(V)} = \left(\frac{g_V}{m_V}\right)^2 \text{ and } G^{(S)} = -\left(\frac{g_S}{m_S}\right)^2 \quad (3)$$

Obviously we define by the Lagrangian (1) an effective interaction model where mesonic degrees of freedom do not appear explicitly. Furthermore, the effective interaction is tailored for use in relativistic Hartree Fock calculations and not for other approximation schemes. This is an approach much in the spirit of the nonrelativistic Hartree Fock calculations with Skyrme interactions for nuclei.

Unlike the $\sigma - \omega$ field theory the Fock contribution can easily be calculated in the present model. The four fermion coupling Lagrangian leads in

the space of two fermion states to an interaction without retardation, which is hence necessarily of Skyrme type due to relativistic invariance. For this reason the exchange terms may be absorbed in the direct terms by exploiting Fierz-relations. This is conveniently achieved by replacing the original coupling parameters $G^{(K)}$ by $\tilde{G}^{(K)}$, which are uniquely defined by the condition

$$\sum_K \tilde{G}^{(K)} \mathcal{O}^{(K)} \otimes \mathcal{O}^{(K)} = \sum_K G^{(K)} \mathcal{O}^{(K)} \otimes \mathcal{O}^{(K)} (1 - P_\tau P_D) \quad (4)$$

where

$$P_\tau = \frac{1}{2} (1 \otimes 1 + \vec{\tau} \otimes \vec{\tau}) \quad (5)$$

is the isospin exchange operator and

$$P_D = \frac{1}{4} \left(1 \otimes 1 + \gamma_5 \otimes \gamma_5 + \gamma_\mu \otimes \gamma^\mu + \frac{1}{2} \sigma_{\mu\nu} \otimes \sigma^{\mu\nu} - \gamma_5 \gamma_\mu \otimes \gamma_5 \gamma^\mu \right) \quad (6)$$

is the exchange operator for Dirac spinors.

Due to this feature the present four-fermion coupling model lends itself as an instrument for the study of the role of exchange terms. This can be illustrated by an application to nuclear matter. The study of exchange terms for finite nuclei involves additional features as compared to nuclear matter. This is nicely demonstrated in H. Mütter's contribution [6] to this meeting. For the application to nuclear matter we take all coupling parameters with the exception of $G^{(S)}$ and $G^{(V)}$ in the Lagrangian equal to zero, so as to keep contact with the $\sigma - \omega$ model. The result of the Hartree-Fock approximation is immediately obtained by replacing the parameters $G^{(S)}$ and $G^{(V)}$ in the Hartree approximation by the corresponding parameters $\tilde{G}^{(S)}$, $\tilde{G}^{(V)}$ obtained from them by Fierz relation (4). As shown in Fig.1 the results of the Hartree

and Hartree-Fock approximation are very different. This provides a clue to understanding the large modifications arising by exchange terms in the application of the $\sigma - \omega$ model.

2 Applications to Finite Nuclei

For nuclear matter our relativistic four-fermion point coupling model yields on the Hartree-Fock level, similarly as the $\sigma - \omega$ mean field model, a successful scenario to fit the nuclear binding energy and avoid the Coester band. It is therefore interesting to apply the model systematically for the calculation of ground states of finite nuclei.

From the well known successful nonrelativistic Hartree-Fock calculations with Skyrme type interactions, one may expect that some density dependence of the interaction strength and furthermore derivative terms might be needed to fit finite nuclei. Obviously if the four fermion coupling is viewed as a zero range approximation to a finite range effective interaction, then derivative terms will appear in the higher order correction terms. For the application to finite nuclei we have therefore used an extended version of the model with the following Lagrangian as starting point.

$$\mathcal{L} = \bar{\psi} (i\gamma_\nu \partial^\nu - M) \psi + \mathcal{L}_{4f} + \mathcal{L}_{ho} + \mathcal{L}_{der} \quad (7)$$

where

$$\mathcal{L}_{4f} = -\frac{1}{2}A_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}A_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

$$-\frac{1}{2}A_{TS}(\bar{\psi}\vec{\tau}\psi) \cdot (\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}A_{TV}(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \cdot (\bar{\psi}\vec{\tau}\gamma^\mu\psi) \quad (8)$$

$$\mathcal{L}_{ho} = -\frac{1}{3}B_S(\bar{\psi}\psi)^3 - \frac{1}{4}C_S(\bar{\psi}\psi)^4 - \frac{1}{4}C_V((\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi))^2 \quad (9)$$

$$\mathcal{L}_{der} = -\frac{1}{2}D_S(\partial_\nu(\bar{\psi}\psi))(\partial^\nu(\bar{\psi}\psi)) - \frac{1}{2}D_V(\partial_\nu\bar{\psi}\gamma_\mu\psi)(\partial^\nu\bar{\psi}\gamma^\mu\psi) \quad (10)$$

Furthermore as the exchange terms of the basic four fermion coupling may be absorbed in a redefinition of parameters, we shall confine ourselves to a relativistic Hartree approximation in the following. The Dirac-Hartree equation for the field operator is

$$(i\gamma_\mu\partial^\mu - M - U_{sc})\psi(x) = 0 \quad (11)$$

where U_{sc} is the static selfconsistent potential given by

$$\begin{aligned} U_{sc} = & A_S\rho_S + A_V\rho_V\gamma^0 + A_{TS}\rho_{TS}\tau_3 + A_{TV}\rho_{TV}\tau_3\gamma^0 + B_S\rho_S^2 \\ & + C_S\rho_S^3 + C_V\rho_V^3\gamma^0 + D_S\Delta\rho_S + D_V\Delta\rho_V\gamma^0 \end{aligned} \quad (12)$$

where

$$\rho_S = \langle \bar{\psi}\psi \rangle, \rho_V = \langle \bar{\psi}\gamma^0\psi \rangle, \quad (13)$$

$$\rho_{TS} = \langle \bar{\psi}\tau_3\psi \rangle, \rho_{TV} = \langle \bar{\psi}\gamma^0\tau_3\psi \rangle \quad (14)$$

are expectation values calculated with the Slater determinant of the occupied orbitals, the negative energy orbitals being excluded.

A self-consistent procedure has been developed by Nikolaus, Hoch and Madland [7] to solve the model equations for several nuclei simultaneously by use of a generalized nonlinear least-squares adjustment algorithm. With this procedure the nine coupling constants of our model are determined so as to reproduce measured ground state binding energies, E_B , rms charge radii, $\langle r^2 \rangle_{charge}^{1/2}$, and spin-orbit energy splittings $\Delta E_{s.o.}^{(n)}$ and $\Delta E_{s.o.}^{(p)}$, for the least bound neutron and proton spin orbit pair of selected closed major shell and closed subshell nuclei in spherical regions. The values of the parameters obtained in this way are given in Table 1.

TABLE 1. Optimized Coupling Constants for the Relativistic Point

Coupling Model			
Potential	Coupling Constant	Magnitude	Dimension
V_S	A_S	-4.50765×10^{-4}	MeV^{-2}
V_S	B_S	1.10951×10^{-11}	MeV^{-5}
V_S	C_S	5.73522×10^{-17}	MeV^{-8}
V_S	D_S	-4.23944×10^{-10}	MeV^{-4}
V_{TS}	A_{TS}	7.40291×10^{-7}	MeV^{-2}
V_V	A_V	3.42665×10^{-4}	MeV^{-2}
V_V	C_V	-4.38858×10^{-17}	MeV^{-8}
V_V	D_V	-1.14365×10^{-10}	MeV^{-4}
V_{TV}	A_{TV}	3.25677×10^{-5}	MeV^{-2}

The model with these coupling parameters predicts then these same observables for a much larger set of closed shell spherical nuclei to good accuracy and also predicts these quantities for similar nuclei far outside the valley of beta stability. The results in testing the predictive power of the model for 32 nuclei are summarized in Table 2.

Table 2. Average Absolute Deviations of Predicted Observables from Measured Observables

Observable	Average Absolute Deviation
E_B	2.783 MeV
$\langle r^2 \rangle_{charge}^{1/2}$	0.021 fm
$\Delta E_{s.o.}^{(n)}$	1.409 MeV
$\Delta E_{s.o.}^{(p)}$	0.844 MeV

Let us mention here that the results for binding energies and particularly mean square radii are very good. These results which are better than hoped, indicate that the model may be valuable on the phenomenological level and give us incentive for further work.

3 Chiral extensions of the model

The model in the previous sections may be viewed as an effective theory below some appropriate cut off scale without explicit mesonic degrees of freedom. The purpose of the present section is to extend our model to include chiral symmetry and chiral symmetry breaking. In this way we may recover the light degrees of freedom, since we choose to implement chiral symmetry by using a nonlinear (CCWZ) realization [8, 9]. By using the conventional ideas on chiral symmetry breaking we recover the pions; the pion self coupling and the form of the pion-nucleon coupling is completely determined by chiral symmetry

Going one step further we may even use the alternative realization of chiral symmetry, called vector symmetry [10] or hidden local symmetry. In

this way we may in addition recover the coupling of the nucleon to the ρ -meson, which is then again determined solely by vector symmetry. This extension shall not be discussed here.

The inclusion of chiral symmetry is a completion of our previous work ,since the effects of light degrees of freedom can not be fully included in an appropriate redefinition of the four fermion couplings.

For the present purpose the chiral symmetry group is $SU(2)_L \otimes SU(2)_R$ and the diagonal subgroup $SU(2)_{L+R}$ is the familiar isospin $SU(2)$. To apply the recipe of CCWZ one defines an $SU(2)$ -field

$$U = \exp \left(\frac{2i}{f_\pi} \Pi(x) \right) \quad (15)$$

where $\Pi(x)$ is given in terms of the three pion fields

$$\Pi(x) = T^a \pi^a(x) \quad (16)$$

and T^a are the generators of $SU(2)$ with the normalization

$$\text{Tr} \{ T^a T^b \} = \frac{1}{2} \delta^{ab}$$

and $f_\pi \approx 93$ MeV is the pion decay constant.

The chiral group acts on the field U in the following way:

$$U \rightarrow LUR^\dagger \text{ with } L \in SU(2)_L, R \in SU(2)_R \quad (17)$$

The diagonal subgroup $L = R$ acts linearly on the pion fields and is thus identified with the isospin group. In the next step we introduce the “square

root" of the field U

$$\Sigma = \exp \left(\frac{i}{f_\pi} \Pi(x) \right) \quad (18)$$

to define on it the nonlinear representation of the chiral group. The transformation of the Σ field under the chiral group is

$$\Sigma \rightarrow L\Sigma W^\dagger = W\Sigma R^\dagger \quad (19)$$

Eqn.(19) defines a matrix W depending on the pion fields which encodes the chiral transformations in a nonlinear way. The nucleon isospinors

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

transform under the chiral group by the mapping W defined by (19).

$$\Psi \rightarrow W\Psi \quad (20)$$

The remaining task is now to construct a Lagrangian which is invariant under the transformation (19) and (20). Starting from our four fermion coupling model (1) we see that isospin invariance already guarantees invariance for the terms of the Lagrangian not containing derivatives. The derivative terms have to be considered extra, since the nonlinear representation matrix W is space time dependent through its dependence on the pion fields. To make the derivative terms also invariant under (20) we define a vector field V_μ by

$$V_\mu = \frac{1}{2} (\Sigma^\dagger \partial_\mu \Sigma + \Sigma \partial_\mu \Sigma^\dagger) \quad (21)$$

which transforms under the chiral group like

$$V_\mu \rightarrow W V_\mu W^\dagger + W \partial_\mu W^\dagger \quad (22)$$

Thus V_μ is a gauge field for the local transformation W and one may then make the terms of the Lagrangian containing derivatives invariant under (20) by replacing the normal derivatives by covariant ones:

$$\partial_\mu \Psi \rightarrow D_\mu \Psi = \partial_\mu \Psi + V_\mu \Psi \quad (23)$$

In this way we may find a version of our model which includes chiral symmetry by just replacing the ordinary derivatives by covariant ones. In addition we may add further coupling terms by forming all possible invariants from Ψ and Σ . One important term is the coupling of the nucleon to the axial current

$$A_\mu = \frac{i}{2} (\Sigma^\dagger \partial_\mu \Sigma - \Sigma \partial_\mu \Sigma^\dagger) \quad (24)$$

transforming linearly under the W transformations

$$A_\mu \rightarrow W A_\mu W^\dagger. \quad (25)$$

The invariant coupling term is then given by

$$\mathcal{L}_{Axial} = g_A \bar{\Psi} \gamma_\mu \gamma_5 A^\mu \Psi \quad (26)$$

Following the usual procedure of chiral perturbation theory further coupling terms may be constructed [11]. These are, however, expected to be suppressed by higher powers of $1/\Lambda_{\chi sb}$, where $\Lambda_{\chi sb}$ is the chiral symmetry breaking scale which is assumed to lie somewhere around 2 GeV.

The total Lagrangian of the model is given by the sum of all the terms discussed above, the four fermion coupling terms and the kinetic term of the matrix field U .

$$\begin{aligned}
\mathcal{L} = & \frac{f_\pi}{4} \text{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) \\
& + \bar{\Psi} (i \not{D} - M) \Psi + g_A \bar{\Psi} \gamma_\mu \gamma_5 A^\mu \Psi \\
& + \sum_k G_k \left(\bar{\Psi} \mathcal{O}_k \Psi \right) \left(\bar{\Psi} \mathcal{O}_k \Psi \right)
\end{aligned} \tag{27}$$

where the the sum over k runs over all possible combinations of gamma and isospin matrices yielding an invariant Lagrangian.

The pions in (27) are massless Goldstone bosons of the spontaneous breaking of $SU(2)_L \otimes SU(2)_R$. To give them a finite mass some explicit breaking term has to be added. Following the procedure in [11] we introduce

$$\mathcal{L}_{m_\pi} = \frac{1}{2} m_\pi^2 f_\pi^2 \text{Tr} \left(U + U^\dagger \right) \tag{28}$$

The present model reproduces the results of the static Hartree Fock calculations with the original four fermion coupling and furthermore allows one to calculate pion production, absorption and scattering.

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