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**EXPERIMENTAL AND THEORETICAL STUDIES OF A HIGH
TEMPERATURE CESIUM-BARIUM TACITRON, WITH
APPLICATION TO LOW VOLTAGE-HIGH CURRENT
INVERSION**

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EXPERIMENTAL AND THEORETICAL STUDIES OF A HIGH TEMPERATURE CESIUM-BARIUM TACITRON, WITH APPLICATION TO LOW VOLTAGE-HIGH CURRENT INVERSION

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A low voltage/high current switch referred as "Cs-Ba tacitron" is studied for use as a dc to ac inverter in high temperature and/or ionizing radiation environments. The operational characteristics of the Cs-Ba tacitron as a switch were investigated experimentally in three modes: (a) breakdown mode, (b) I-V mode, and (c) current modulation mode. Operation parameters measured include switching frequencies up to 20 kHz, hold-off voltages up to 200 V, current densities in excess of 15 A/cm², switch power density of 1 kW/cm², and a switching efficiency in excess of 90 % at collector voltages greater than 30 V. Also, if the discharge current is circuit limited to a value below the maximum thermal emission current density, the voltage drop is constant and below 3 V.

Results also show that discharge extinguishing during current modulation occurs as a result of the decrease in the heavy component density in the gap during the discharge. Therefore, a definite correlation exists between the discharge current density, Cs vapor pressure, off-time, and on-time. The off-time required to replenish Cs atoms in the gap for subsequent ignition depends on the amplitude of the positive grid pulse, while the ignition delay-time depends on both the off-time and the magnitude of the positive grid potential for ignition. In general, stable

current modulation occurs at a device duty cycle $\geq 45\text{-}55\%$.

Preliminary results of the use of two Cs-Ba tacitrons operating in a "push-pull" configuration to invert a dc signal to a square wave ac signal was demonstrated. The inverter operated successfully at a discharge current of up to 13.5 A, collector voltage of 25 V, conduction voltage drop of ~ 4 V, and modulation frequencies up to 20 kHz.

A model of a low pressure Cs-Ba discharge was developed, which calculates the plasma properties for uniform conditions across the discharge gap. A Maxwellian electron distribution is assumed in determining the ion generation processes. The model predictions were in good agreement with experimental results and theoretical calculations of other investigators for low pressure Cs discharges. Comparing the model to experimental data of a Cs-Ba diode, reinforced the validity of the system of equations and confirmed that barium insignificantly affects the transport characteristics of the discharge. It was also shown that radiation losses play a significant role in the ion generation processes of a low pressure Cs discharge.

The steady-state Cs-Ba triode model is developed, which used the diode model as building blocks, that calculates the discharge parameters in each of the three discharge regions of the device. The triode model is used to determine the effect of the grid design parameters (hole size, thickness and transparency) on the forward voltage drop in the device. Results showed that for the same gap spacing, the voltage drop in the plasma of the triode is almost identical to that in the diode. Also, it was shown that increasing or decreasing the grid thickness has very little effect on the forward voltage. It was shown that the smaller the grid hole, the lower the forward voltage drop. Results also showed that the grid transparency has the most significant effect on the forward voltage drop in the triode device. The calculated voltage drop increased from 1.29 V to 2.58 V as the grid transparency

was decreased from 55 % to 15 %, respectively. It was also shown that 70 % of the voltage drop increase in the device was due to an increase in the voltage drop across the grid.

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1. INTRODUCTION

Space nuclear power systems are being designed to supply electrical power in the range of a few tens to hundreds of kilowatts to support a host of civilian and military missions. A block diagram showing the main components of a generic space nuclear power system (SNPS) is shown in Fig. 1.1(a) and Fig. 1.1(b) illustrates a typical configuration for a SNPS. Typically included in SNPS's is a shadow shield, heat rejection radiator, transmission line boom power conditioning unit (PCU), and the payload (see Fig. 1.1(b)). The nuclear energy heat source can either be a radioisotope or a nuclear reactor. The radioisotope source generates heat due to the decay of a radioactive material such as plutonium-238, where a typical fuel would be plutonium oxide (PuO_2). The heat in a nuclear reactor is generated due to fission in fuels such as uranium oxide (UO_2), uranium nitride (UN), or uranium carbide (UC). The thermal output power of the nuclear reactor can be converted to electricity by using either dynamic or static conversion devices. Dynamic conversion devices such as Brayton and Rankine cycle generators and Stirling engines could be used. Static conversion devices include thermoelectric generators and thermionic converters. The dynamic conversion systems produce alternating current (ac) power and the static conversion devices produce direct current (dc) output. The reliability of the SNPS is one of the main drivers when choosing the energy conversion system, since maintenance of such a system is impossible. Moving parts, associated with the dynamic conversion devices, can cause unwanted vibrations in the SNPS, which may affect the performance of other satellite subsystems (i.e. payload). Also, since dynamic systems are so large only 2 or 3 are used to generate the necessary electricity. If one of the devices happens to fail, the output power of the system drops dramatically which might cause a total failure of the system. Therefore, to

Figure 1.1 Block Diagram of Generic Space Nuclear Power System.

increase the reliability of the SNPS, static energy conversion devices can be used which have no moving parts. Also, thousands of devices are used to generate electricity resulting in increased reliability of the system because a failure of a couple of static conversion devices has no affect on the operation of the overall system. However, there is a penalty when using static conversion devices which is due to the fact that they generate low voltage/high current dc power. The low voltage signal needs to be conditioned to a higher voltage signal in order to reduce the transmission line losses incurred while transmitting the power to the payload. This power conditioning unit (PCU) can either be a dc to dc converter or a dc to ac inverter.

In determining whether a conversion or an inversion process should be used for the PCU, it must be decided whether the distributed power system should use a dc-bus or an ac-bus. This debate continues today but some authors (Tabisz et al. 1992, Tsai and Lee 1987, and Hansen & Sundburg 1986) discuss some of the advantages of using an ac-bus power distribution system (PDS). For an ac-bus PDS only one inversion (dc-ac) process is necessary; whereas, for the dc-bus one inversion (dc-ac) and one rectification (ac-dc) process are needed. Since the same switch will be used for each process the potential losses for the dc-bus could be higher because two processes are necessary instead of only one.

Almost all of the PCU's being considered today use semiconductor switching elements; such as thyristors for the conversion/inversion process. Although thyristors have low forward voltage drops, are capable of high repetition rates, and are relatively light weight and compact, they are vulnerable to high temperatures ($>250\text{ }^{\circ}\text{C}$) and to nuclear radiation. Both shielding and separation distance from nuclear reactors are used to provide a tolerable radiation environment for the thyristor at a mass penalty. Schwarze et. al. (1993) reported a dramatic increase in the forward voltage drop of a mos-controlled thyristor (MCT)

from 1.5 V to greater than 8 V at neutron fluences above 10^{13} neutrons/cm²s; on the other hand, no significant increases in the forward voltage drop were seen at gamma ray doses up to 5.1 Mrads. It was also reported that the switching characteristics, i.e. rise-time, fall-time, etc., are affected by both neutron and gamma radiation. These semiconductor devices are also susceptible to high temperature operation (Frasca and Schwarze 1992 and Schwarze et al. 1993). Therefore, it would be more advantageous to use switches that are compatible with high temperature and nuclear radiation environments for the conversion/inversion. The switch should also have a relatively low mass, high efficiency, high repetition rate, and long operation lifetime.

To avoid the high temperature and radiation shortcomings of semiconductor switches, tube switches could be used. Tube switches either operate in a vacuum or employ a gas for the conduction medium. Vacuum tube devices are limited to low currents and are more suited for high voltage applications because the forward voltage drop is typically greater than 1 kV (King 1923). There have been many different types of gas switches over the years; such as, the thyatron, plasmatron, and tacitron. The most common of the gas switches is the thyatron and the gas of preference is hydrogen. Although the thyatron can handle very high currents at very high repetition rates, it is more suited for high voltage applications because the forward voltage drop is high (100-500 V). The plasmatron is limited to very low anode voltages (< 20 V) and requires an auxiliary discharge for controllability which increases its complexity and reliability (Johnson et al. 1952).

A Cesium(Cs)-Barium(Ba) vapor thermionic switch called "tacitron" shows tremendous promise for satisfying all the requirements listed above. The tacitron is a triode device that by applying a positive voltage pulse to the grid is turned on and by applying a negative voltage pulse to the grid is turned off. The extinguishing mechanism for Cs-Ba tacitrons has been explained by a number of

theories. Experiments are conducted at the University of New Mexico's Institute for Space Nuclear Power Studies (ISNPS) to investigate the operational characteristics of a single triode device. The tacitron employs a low pressure cesium and barium vapor mixture (pressure $\approx 10^{-3}$ Torr) for the conduction medium and uses refractory metal (molybdenum) for the electrodes. The Cs is used to increase the conductivity of the switch (reduced forward voltage drop) and the Ba is used to obtain high conduction currents. It can conduct relatively high currents at high repetition rates with a forward voltage drop as low as 3 V. Although this is higher than the thyristor, the tacitron can operate at high temperatures (> 1200 °C) and in an ionizing radiation environment, unlike the semiconductor thyristor.

Other possible industries, besides power conditioning in space nuclear power systems (aerospace), that have been identified for Cs-Ba tacitrons are (Piper 1991): 1) high temperature batteries, 2) circuit breakers, 3) electroplating, and 4) metallurgical extrusion. The market survey by Piper (1991) mentioned that the most interest was generated by companies in the aerospace and high temperature battery industries.

2. OBJECTIVE

The objectives of this research are: 1) experimentally investigate the operation characteristics of Cs-Ba diode and triode devices installed at the ISNPS, (2) develop a point mass plasma model for the Cs-Ba diode to understand the discharge processes which affect its operation, and (3) Incorporate the diode steady-state model as subsections for a point mass model that includes the three different regions in the triode: a) emitter-grid gap, b) grid aperture, and c) grid-collector gap.

(1) The operation characteristics of Cs-Ba diode and triode (tacitron) devices are investigated and the use of two Cs-Ba tacitrons operating in an inverter circuit are presented. The operation of the Cs-Ba tacitron (single triode) is investigated in three modes: (a) breakdown mode, (b) I-V mode, and (c) extinguishing mode. The breakdown mode is used to test the voltage hold-off capabilities of the tacitron. The I-V characteristics are measured and used to determine the minimum forward voltage drop for given operating parameters (i.e. Cs and Ba pressures, cathode temperature, etc.) In the extinguishing mode, the capability of the tacitron to generate pulses at high repetition rates is investigated. The diode characteristics are used to measure the effect of the grid on the forward voltage drop. The measurements are being made with two objectives in mind: first, to gain enough data on the Cs-Ba tacitron to develop a characterization of its operation for comparison with other switching elements, and to determine its usefulness as a switch for power conditioning in high temperature and radiation harsh environments and; secondly, to study the microscopic behavior of the discharge of the device in order to formulate a theory on the extinguishing mechanism.

(2) Develop a point mass plasma model for the Cs-Ba diode to understand the discharge processes which affect its operation. The model calculates the particle densities (i.e. Cs atom ground state, Cs atom excited states, plasma density and electron temperature) and results are compared with experimental measurements from the Cs-Ba diode. The steady-state diode model is used to predict the forward voltage drop as a function of the emitter temperature, Cs pressure, and Ba pressure. Also the steady-state model is used to understand the complex ion generation process in a low pressure Cs discharge including the effects of the number of excited states and radiation emission and absorption. A transient diode model is developed to verify the steady-state results.

(3) Incorporate the diode steady-state model as subsections for a point mass model that includes the three different regions in the triode: a) emitter-grid gap, b) grid aperture, and c) grid-collector gap. Appropriate boundary conditions are used to couple the three regions to one another. Current-voltage characteristics from the triode are compared to calculations from the model to test its accuracy and verify the system of equations in the model. Also, by using the grid as a probe, the measured ion current to the grid is compared with that calculated by the model. The triode model is used to predict the trends of the forward voltage drop as a function of the grid transparency, Cs pressure, emitter temperature, Ba pressure, and discharge current density.

To better understand why the Cs-Ba tacitron is the best available switch in meeting the power conditioning needs of space power nuclear systems, the thyristor and some hot-cathode switching elements that fall short of meeting these needs (low voltage, high current, temperature and radiation hard) are outlined in Chapter 3. Chapter 4 describes the Cs-Ba tacitron test facility, experimental procedure, and operational characteristics. Chapter 5 describes the Cs-Ba diode model with calculations compared to the experimental data. Both steady-state and

transient results are discussed. Chapter 6 describes the Cs-Ba triode model and compares steady-state results to the experimental data obtained from the tacitron. Summary and conclusions are presented in Chapter 7 and Chapter 8 offers recommendations for future work, both experimental and theoretical.

3. LITERATURE SURVEY - SWITCHING ELEMENTS

3.1 Introduction

Hot-cathode switching elements have been around since the early 1900's. Work on these switches began in an effort to develop devices that could be used in communications, consisting mostly of developing amplifiers. Switches have also been developed as rectifiers, which can convert ac signals to dc signals for power conditioning. These same circuits, by reversing the input and the output, can be used as inverters to convert a dc signal to an ac signal. Before the mid to late 1950's, the rectifiers or inverters used tube switches, either in a vacuum or with a gas. After the invention of semiconductor devices, low voltage applications were addressed by transistors and later by thyristors. The transistor can be eliminated as a choice for space power applications because it is typically a low current device which does not meet the requirements of space nuclear power systems.

3.2 Thyristor

The thyristor is a high power device and can meet the power conditioning needs of space power systems. The thyristor is discussed in some detail because it has similar turn-on and turn-off characteristics to the gas tacitron switch. The thyristor is a three terminal, three junction, four-layered semiconductor device made of alternating p-type and n-type semiconductor materials such as silicon. An illustration of the layers of the thyristor is shown in Fig. 3.1a (Ramshaw 1973). Figure 3.1b is the equivalent circuit diagram if transistors are used and Fig. 3.1c shows the symbol that is used in circuits utilizing thyristors.

Figure 3.1. Schematic of a thyristor; a) semiconductor layers, b) transistor equivalent circuit, and c) circuit symbol.

The four layer configuration of the thyristor prevents the flow of current independent of the bias voltage that is applied between the cathode and anode because there is always at least one reversed bias junction. As can be seen in Fig. 3.1a, junction one, J_1 , prevents the flow of holes or positive charge from the p-type to the n-type. Junction three, J_3 , prevents electrons from flowing from the n-type to the p-type. With junctions one and three reversed biased the device remains in the off-state independent of the polarity of the voltage applied between the cathode and anode. Junction two is naturally forward biased because holes can flow from the n-type to the p-type and electrons from the p-type to the n-type. Two other terms that are used quite frequently when discussing semiconductor devices are majority and minority carriers. In a p-type material the majority carriers are the holes and the minority carriers are the electrons. The opposite is true for the n-type material. In the following sections the turn-on, turn-off, forward voltage drop, and applications of the thyristor are outlined.

3.2.1 Turn-on Characteristics

Ramshaw (1973) describes four ways that a thyristor can be triggered to the conducting state: 1) electrical signal at the gate, 2) light activation at the gate, 3) large applied voltage between the cathode and anode, and 4) rapid rise of the applied voltage to the anode. Sugandhi and Sugandhi (1984) mentioned a fifth triggering method called thermal triggering. The most common methods are those that use gate signals.

Electrical Signal Gate Turn-on

If the minority carrier density can be increased in the p-type material between J_2 and J_3 (see Fig. 3.1a) to a sufficient level, triggering will occur and the

device will turn on. This can be accomplished by providing a source of electrons from an external circuit to the gate. The turn-on can be understood best by studying the transistor equivalent circuit of a thyristor shown in Fig. 3.1b. As current is injected into the base of the npn transistor (thyristor gate), this section of the thyristor turns on, which provides a base current to the pnp transistor. When the pnp transistor section turns on, feedback is provided to the base of the npn locking the device in the on-state. During this process, the anode current rapidly rises to the saturation value.

Light Activated Gate Turn-on

This is similar to the above method in that the minority carrier density in the p-type is increased. The increase comes from the photon energy contained in a beam of light impinging on the gate which breaks the electron bonds in the semiconductor material. This additional electron density triggers the thyristor to the conducting state.

Large Applied Voltage

If a large enough voltage is applied between the cathode and anode, the accelerating voltage at J_2 is increased. This increased acceleration gives the electrons sufficient energy to remove additional electrons from the atoms in the semiconductor material and an avalanche in the minority density occurs, resulting in turn-on.

Rapid Rise of Applied Voltage

There is an intrinsic capacitance associated with the semiconductor materials between the anode/gate and gate/cathode regions. If the time rate of change of the voltage is sufficiently high, a current transient, $I_C = C \, dV/dt$, can

occur that increases the electron density to the value necessary for turn-on.

Thermal Triggering

For a given operating temperature and applied voltage, the accelerating voltage at J_2 can be low enough to maintain the device in the off-state because the electrons can not make it past the reversed biased junctions. However, as the temperature is increased, the accelerating voltage is increased and eventually the device will turn-on in a manner similar to applying too large of an applied voltage. This is one reason the thyristor must operate in low temperature environments.

Turn-on Time

When discussing switching elements and which applications a certain device is most suited for, the time it takes to turn the device on can be of critical importance. The turn-on of a thyristor is controlled by the increasing area of the semiconductor that contributes to the anode current. This spreading of area can take between 1 μs and 3 μs for commercially available devices and can be as low as 300 ns for specially built devices. With these low turn-on times, the turn-on should not be a limitation for use in space nuclear power systems.

3.2.2 Turn-off Characteristics

There are essentially three ways to turn-off a thyristor: 1) natural commutation, 2) reverse bias turn-off, and 3) gate turn-off. Complete turn-off or commutation of a thyristor is achieved when the anode current has decreased to zero and breakdown does not occur after application of the anode voltage.

Natural Commutation

When the anode current is reduced below the holding current, which is the minimum current necessary in order to maintain the thyristor in the on-state, the thyristor will turn-off. The anode current can be decreased by opening the switch leads, increasing the load impedance or shunting some of the anode current through a circuit parallel to the thyristor (short circuiting the device).

Reverse Bias Turn-off

If the polarity of the voltage that is being applied to the anode is reversed (negative instead of positive), the thyristor will turn off. If the reversal of the voltage is achieved by discharging a capacitor that is in parallel with the thyristor, the turn-off is called forced commutation. If an ac signal is being supplied to the anode, the voltage will be negative every half-cycle and the thyristor will turn-off during this period. This is called phase commutation or ac line commutation.

Gate Turn-off

In some thyristors, it is possible to increase the holding current above the anode current by applying a negative gate current. Anode currents for these devices are usually limited to below 10 A. This type of gate turn-off can be very advantageous because the signal to the anode can remain unchanged, which is necessary for use in space nuclear reactor systems. Also, there is no need for extra circuitry for forced commutation which would have the same temperature and radiation limitations as the thyristor, if not worse.

Turn-off Time

The turn-off time in a thyristor is determined by the time it takes for the charge that has built up in the semiconductor materials to decay to a value close to

the equilibrium off-state level. Turn-off times depend on the conduction current, applied voltage, and temperature; and range anywhere from 10 μ s to 100 μ s.

3.2.3 Forward Voltage Drop

In thyristors, the forward voltage drop is almost independent of the conduction current if the increased heat can be dissipated to maintain the temperature at the rated value (Ramshaw 1973). But as the applied voltage increases, the thickness of the semiconductor materials must also increase, which increases the forward voltage drop. The amount of increase in the voltage drop would be from 1.2 V to 2.0 V for an applied voltage increase from 1 kV to 10 kV. If more voltage or current is needed the thyristors can be connected in series or parallel which would not affect the forward voltage drop.

3.2.4 Applications

Presently, semiconductor devices (thyristors included) are used in a variety of areas; such as, well logging, automotive, aircraft, and space industries (Thome and King 1992). The use of the thyristor in space applications will be focused on herein. Thyristors employ a number of different materials which include silicon, gallium arsenide, aluminum gallium arsenide, silicon carbide, and diamond (Thome and King 1992). Only silicon and gallium arsenide have been used commercially in thyristors, but the other materials are being studied because they have better characteristics in harsh temperature and radiation environments.

A typical output characteristic of a thyristor that could fit comfortably in ones hand and easily meet the power conditioning needs of space nuclear reactor systems has currents as high as 200 A with a forward voltage drop of only 1.5 - 2.0

V [Schwarze et al. 1993]. These devices can be used for conversion of a low voltage dc signal to a higher voltage dc signal or for inversion of a dc signal to an ac signal. Two different configurations of utilizing thyristors as inverters are shown in Fig. 3.2. Figure 3.2a shows a scheme of using thyristors that can operate in the gated-off mode. For this scheme while a positive voltage is applied to TH1 to turn the device on, a negative voltage is applied to TH2 to turn the device off. Operation in this mode is limited to small currents (< 10 A) (Ramshaw 1973). A more convenient and more often used scheme is forced commutation, shown in Fig. 3.2b, where a capacitor is used to reverse bias the thyristor and turn the device off. Initially both thyristors are in the off-state until a positive voltage is applied to the gate of TH1 and it turns on. During the conduction of TH1 the capacitor, C, charges up to two times the source voltage, V, due to induction of the transformer primary. TH2 must be able to withstand this voltage so that it does not turn on before the gate signal is supplied. A positive voltage is applied to the gate of TH2, at which time the capacitor discharges, reverse biasing TH1 and turning it off. This is repeated to generate an alternating current signal at the load. There are many other configurations that thyristors can be used in to invert a dc signal to an ac signal. For a discussion of these methods refer to the book by Ramshaw (1973).

3.2.5 Summary

The operating characteristics of a thyristor are quite useful for the conditioning of low voltage/high current D.C. signals. A typical thyristor can conduct up to 200 A with a forward voltage drop of only 1.5 V. The turn-on and turn-off times of these devices are on the order of microseconds, so repetition rates in the ten's of kilohertz range should not be a problem. However, as mentioned

1 : Primary of Transformer
 2 : Secondary of Transformer
 V : DC Voltage Source
 C : Capacitor
 G : Gate
 TH1 : Thyristor 1
 TH2 : Thyristor 2

Figure 3.2. Thyristor inverter circuit; a) complete grid control mode, and b) forced commutation mode.

earlier, the thyristor (and other semiconductor devices) are susceptible to high temperatures and harsh radiation environments. In addition, if a thyristor is operating at 200 A with a 1.5 V drop, there is a problem with removing the 300 W of power dissipated in the device. In space, the excess heat generated in the PCU, is removed solely by radiating into space environment. Since heat removal by radiation is proportional to the temperature of the device to the fourth power, the lower the temperature, the lower the heat removal power density, which translates directly to larger size and more mass for the radiator. In the last 5 - 10 years a lot of research has gone into developing high temperature and radiation hardened electronics. A good summary of the state of the technology can be found in the work by Thome and King (1992). To avoid the problem associated with operating in a high temperature, radiation harsh environment, tube switches could be used that either operate in a vacuum or gas.

3.3 Thermionic Vacuum Tubes

The development of the first two-electrode thermionic vacuum tube dates back to the work done by Edison. The conduction current is obtained by thermally emitting electrons from a hot cathode. These earlier devices have predominantly been used as amplifiers and oscillators. They were also used as rectifiers to convert an ac signal to a dc signal. In 1906 DeForest developed the three electrode vacuum tube that used a grid to control the flow of current. This three electrode device has gone by several names in the early 1900's, some of which are audion, vacuum tube, triode, and pliotron (King 1923). The control electrode made it possible to stop the flow of electrons and turn the device off.

3.3.1 Conduction Current

The conduction current in a thermionic vacuum tube is limited by either space charge, thermal emission, or the circuit. The space charge limit is due to the electrostatic repulsion forces that each electron experiences as it leaves the cathode surface. The amount of current that can be transferred across a distance d is given by the "3/2 Law" (Child & Langmuir 1913 and Fry 1921) in which the electron current density in A/cm² is:

$$j = 2.33 \times 10^{-6} \frac{V^{3/2}}{d^2}, \quad (3.1)$$

where V is the voltage between the cathode and anode. For an anode voltage of 500 V and a spacing of ≈ 0.86 cm, the space charge limited current is only 35.2 mA/cm². At a typical filament area of only 1 cm², this equates to a conduction current of only 35.2 mA. This current can be increased above that given by Eqn. (3.1) by applying positive potentials to the grid. The positive voltage gives the electrons an initial velocity as they enter the grid/anode region. The expression shown in Eqn. (3.1) is derived assuming the electrons have no initial velocity; however, the current density obtained from Eqn. (3.1) is a good estimate. For example, at the same conditions mentioned above, but with a grid voltage of +20 V, the anode current increased to 120 mA (Hull 1929). For more details refer to the work done by Langmuir (1913) and Fry (1921).

The thermionic emission current is a function of the electrode temperature and work function, which is defined as the amount of energy it takes to remove an electron from the surface. The electron current density for a material at a given temperature is represented by the Richardson-Dushman formula (Hemenway

1962):

$$j = A T^2 \exp\left(\frac{-\phi}{k_B T}\right), \quad (3.2)$$

where A is a physical constant equal to $120 \text{ A/cm}^2\text{K}^2$, T is the electrode temperature in K, and ϕ is the electrode material work function in joules. Oxide coatings are deposited on the cathode surface to reduce the work function and increase the emission current density. For a cathode (filament) temperature of 1725°C and a tungsten filament that has a work function of 5 eV ($8 \times 10^{-19} \text{ J}$), the thermionic emission current would be 0.12 mA/cm^2 . If these were the conditions for the anode voltage and spacing mentioned above, the tube would have reached temperature saturation. If the work function of the filament is decreased to 3.5 eV , the emission current would increase to 740 mA/cm^2 , and the tube would have reached voltage saturation. Therefore, the amount of current is determined by the hold-off voltage, because this is the maximum anode voltage that can be applied and by the temperature and work function of the filament. The limitation on the filament temperature is due to lifetime considerations, the higher the temperature the shorter the tube operating life.

3.3.2 Turn-on Characteristics

The pliotron could be transferred to the conducting state by removing the negative voltage that is applied to the grid. However, the amount of anode current has a strong dependence on the value of the grid voltage. Therefore, the limit on the turn-on time of a pliotron is determined by how quickly the grid drive can be changed from the negative voltage to the positive voltage that is necessary to

obtain the desired anode current. This is a very simple turn-on mechanism and takes only tenths of microseconds, which is the reason these tubes were developed and used for so many years.

3.3.3 Turn-off Characteristics

Like the turn-on of the vacuum tube, the turn-off is just as straight forward. In order to turn the device off a large negative potential is applied to the grid which retards the flow of electrons from the cathode and prevents them from reaching the anode; hence shutting the device off. The turn-off time is also fairly rapid (also tenths of microseconds) and is not a limitation of the device.

3.3.4 Forward Voltage Drop

The forward voltage drop for a vacuum tube can be a major disadvantage. It is a strong function of the conduction current and puts major limitations on the amount of current that one tube can conduct. The voltage drop can be upwards of 1 kV because of the large space charge associated with conducting electrons in a vacuum. This large voltage drop is the reason the vacuum tube is limited to high voltage applications (> 10 kV) or very low current applications (< 1 mA).

3.3.5 Applications

Some of the earlier vacuum tube devices were used as rectifiers to convert an ac signal to a dc signal at the output of a rotating generator. At the time the possibility of "inverting" this dc signal back to an ac signal without the use of rotating machines, prompted Prince (1925) to use two pliotron switches to perform

this task. A schematic of a plotron switch is shown in Fig. 3.3. It should be noted that most of the vacuum tubes developed between 1910 and 1930 looked similar and only had slight design modifications.

Vacuum tubes use a filament cathode, plate anode, and a grid that consists of a wire bent in a zigzag shape placed between the cathode and anode (see Fig. 3.3). The filament voltage shown in Fig. 3.3 supplies the necessary power to heat the filament and supply the desired emission current. The grid in these devices was used to keep the device off by applying a large negative voltage and retarding the electron emission from the filament. By removing the negative voltage at the grid, the anode current would increase and the switch would transition to the on-state.

In 1925, D.C. Prince inverted his rectifier circuit so that a dc signal could be converted to an ac signal; hence the name "inverter". In this early work, Prince (1925) utilized two plotrons in an inverter configuration to generate an ac signal to drive an ac motor from a dc input. The output from the motor was used to drive the grid. A schematic of the inverter circuit that Prince (1925) used is shown in Fig. 3.4. The input voltage to the plotrons was about 15 kV at a power of ≈ 10 kW ($I \sim 0.6$ A) and had a switching efficiency of ≈ 90 % (forward voltage drop ≈ 1.5 kV). The signals to the grids of the plotrons to control the conduction were ac voltages approximately 180° out of phase from each other, where the negative portion of the signal retarded the electron emission and maintained the device in the non-conducting state. This ac signal to the grids was feedback from the output of the ac motor at the load.

When the motor at the load began to rotate, an alternating voltage was established across the secondary of the transformer, 2. This alternating voltage would add and oppose the source voltage, V . Therefore during one cycle, the anode voltage on plotron one, P_1 , would be decreasing while the anode voltage on

Figure 3.3. Schematic of a Vacuum Tube.

- 1 : Primary of the transformer
- 2 : Secondary of the transformer
- V : DC voltage source
- L : Inductor
- M : AC Motor

Figure 3.4. Inverter Circuit Using Two Plotrons Developed by D.C. Prince.

pliotron two, P_2 , would be increasing. In this situation, P_1 is on and P_2 is off. When the anode voltage on P_2 became slightly more positive than the anode voltage on P_1 , a positive potential was applied to the grid of P_2 and negative potential was applied to the grid of P_1 which would transfer the current to P_2 . Then P_2 was in the on-state and P_1 was in the off-state. For a small period of time both switches were in the conducting state. The inductor shown in Fig. 3.4 was used to insure the current never went completely to zero since the grid excitation and alternating anode voltages were supplied by the rotating motor at the load.

3.3.6 Summary

Although the pliotron inverter circuit was able to invert a dc signal to an ac signal, the on-state forward voltage drop for these types of switches is very high, >1 kV; therefore, these switches are limited to high voltage applications. Also, the voltage drop is a strong function of the current being drawn by the anode which limits its operation to low currents. The pliotron vacuum tube is a very simple device; however, the limitations on the power loss and conduction current make it unsuitable for environments that exist in space nuclear power systems.

In the 1920's there was a movement to decrease the forward voltage drop associated with the vacuum tubes and this was accomplished by operating the tube in a mercury vapor environment. One of the first devices developed was the mercury vapor thyatron.

3.4 Thyratrons

If a gas (such as Hg, Xe, etc.) is added to a thermionic vacuum tube, ions will be created in the gap between the cathode and anode and at the hot electrode

surfaces. These ions neutralize the space charge associated with the flow of electrons in a vacuum. Investigation into the mercury vapor thyatron began in the mid to late 1920's. This was proceeded by an extensive investigation by Langmuir and other researches into the electrical characteristics of low pressure mercury vapor arcs (Langmuir and Mott-Smith 1924). In the arc, the mercury vapor is ionized, neutralizing the space charge; thus allowing higher conduction currents. At the same time, the forward voltage drop decreases because of the high conductivity of gas discharges, unlike the low conductivity of electron discharges associated with vacuum tubes. The conduction current for these devices are no longer subject to space charge limits (voltage saturation), and is determined by the thermionic emission current density (Eqn. 3.2).

3.4.1 Hold-off Voltage

The hold-off voltage for a device that uses a gas for the conduction medium is dependent on the gas pressure in the tube and the electrode separation distance. This relationship is called Paschen's law. Breakdown in a discharge occurs after enough secondary electrons are created from ionization, causing an avalanche in the electron population. The multiplication coefficient, m , is derived by balancing the electron current in the discharge and has the following form (Papoular 1965):

$$m = \frac{e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)}, \quad (3.3)$$

where d is the interelectrode gap, α is the first Townsend coefficient which depends on the gas pressure, anode voltage, and type of gas used (Hemenway et al. 1962), and γ is the second Townsend coefficient which depends on the processes at

the cathode surface (i.e. thermal emission, secondary electron emission, etc.). As $\gamma(e^{\alpha d} - 1)$ approaches one, m goes to infinity and breakdown will occur, this relationship is called the Townsend criterion and is written as:

$$\gamma(e^{\alpha d} - 1) = 1. \quad (3.4)$$

A more convenient form can be obtained by replacing d with the breakdown voltage, V_B , divided by the electric field, E . Also the ratio α/E is called the primary ionization coefficient and is known by the symbol η (Garamoon and Surplice 1973). Substituting η into Eqn. (3.4) and solving for V_B yields:

$$V_B = \frac{1}{\eta} \ln\left(\frac{1}{\gamma} + 1\right). \quad (3.5)$$

Therefore, if the 1st and 2nd ionization coefficients are known for a given cathode material, electrode spacing, gas pressure, and gas type, the breakdown potential can be calculated. Because γ is a weak function of gas pressure and electrode spacing and Eqn. (3.5) can be written as:

$$V_B = \frac{\text{Constant}}{\eta}. \quad (3.6)$$

Since,

$$\eta = \frac{\alpha}{E} = \frac{\alpha}{P_{\text{gap}}} \frac{P_{\text{gap}}}{E} = F\left(\frac{P_{\text{gap}}}{E}\right) = G\left(\frac{P_{\text{gap}} d}{V_B}\right), \quad (3.7)$$

where P_{gap} is the pressure in the interelectrode gap. Substituting Eqn. (3.7) into

Eqn. (3.6) gives:

$$V_B = f (P_{\text{gap}} d). \quad (3.8)$$

From Eqn. (3.8) it can be seen that the breakdown voltage is directly a function of the product of the interelectrode width and the gas pressure.

At typical operating pressures of tens of mTorr, a thyatron can effectively hold-off voltages greater than 100 V. The Paschen law can be altered slightly by placing a grid between the cathode and anode, which can have both positive and negative effects on the maximum value of the anode voltage. An advantage of the grid is that it increases the dielectric strength of the gap by adding a barrier between the cathode and anode. Furthermore, if a negative potential is applied to the grid, the hold-off voltage is increased further. However, a disadvantage of the grid comes from the fact that if the cathode temperature increases, so does the grid temperature. If the grid temperature gets high enough ($< 725^\circ\text{C}$), it begins to emit electrons (refer to Eqn. 3.2) and the thyatron will breakdown prematurely. The source of electrons from the grid increases γ in Eqn. (3.5) and lowers the breakdown voltage.

3.4.2 Turn-on Characteristics

As with the pliotron, a small negative potential applied to the control grid maintains the thyatron in the off-state, and when this potential is removed, the device turns on. However, once the arc is created, the grid voltage has no effect on the anode current. The only limits on the anode current are those imposed by the load impedance and the emission limits of the cathode. The increase in the anode current to the limited value occurs in a fraction of a microsecond.

3.4.3 Turn-off Characteristics

Unlike the pliotron, the grid in a thyatron, after ignition, has no further control over the arc, nor can it be used to modulate, limit or extinguish the current (Prince 1928 and Hull 1929). A typical gas thyatron is shown in Fig. 3.5. The grid surrounds the cathode and the anode surrounds the grid. By surrounding the cathode with the grid, better control characteristics for the grid are obtained. In the thyatron, the arc creates a plasma of negative and positive charges that encompasses the inside of the tube. The grid, which is immersed in the middle of the plasma, has a positive ion sheath that surrounds it (Langmuir and Mott-Smith 1924 and Hull 1929). This sheath shields any voltage that is applied to the grid from affecting the arc characteristics. Figure 3.6 shows an illustration of the "shielding" effect on the grid wires. When a negative voltage is applied to the grid, the sheath may grow slightly, but not enough to perturb the bulk plasma.

The extinguishing of the device is accomplished by removing the anode potential; hence, turning the discharge off. If the anode voltage is reapplied before the plasma has sufficiently de-ionized (ions recombining with electrons to form neutral gas atoms), the discharge can be ignited. The repetition rate at which a thyatron can operate is determined by this de-ionization time, which is essentially the turn-off time. Hull (1929) reported the empirical results for the de-ionization time in seconds, which has the following form:

$$\tau_d = \frac{1.6 P I_C^{0.7}}{V_g^{1.5}} \quad (3.9)$$

where P is the gas pressure in the tube in Torr, I_C the anode current in amps, and V_g the grid voltage in volts. Using this formula with the following inputs, gas

Figure 3.5. Typical Electrode Arrangement for a Mercury Thyatron.

Figure 3.6. Illustration of Positive Ion Sheath "Shielding" of a Grid Wire.

pressure of 1 mTorr, current of 10 A, and grid voltage of 20 V, the deionization time, τ_d , is 90 μ s. This would limit modulation frequencies to about 4 kHz. Johnson (1952) using helium as the fill gas measured similar recovery times with values ranging from 30 μ s to 160 μ s, depending on the grid voltage, helium pressure, and conduction current. Generally speaking, the de-ionization times in a hydrogen discharge are smaller than in most other gas discharges because of the increased mobility of the hydrogen gas.

3.4.4 Forward Voltage Drop

The forward voltage drop for the mercury thyratron is about 12 V and independent of the anode current up to about 60 A, after which the voltage drop increases linearly to a value of about 16 V at 200 A (Hull 1929). The minimum voltage drop in a tube is usually close to the first ionization potential of the gas for moderate currents. The first ionization potential is the amount of energy that must be expended in order to create an ion to neutralize the space charge.

In the more recently developed hydrogen thyratrons that conduct tens of kiloamperes of current, the forward voltage drop increased up to 150 - 200 V (Pirrie et al. 1988). This is a satisfactory voltage drop because of the high anode voltages (> 40 kV).

3.4.5 Applications

Over the years thyratrons have used many gases for the conduction medium; such as mercury, xenon, krypton, argon, and hydrogen (Swenne 1960). Table 3.1 lists the first ionization potentials for some of these gases. As can be seen the first ionization potential for these gases, except for argon; are comparable;

therefore, when choosing the gas to use, other considerations must be taken into account. Some of these are toxicity of the gas, material compatibility, availability of the gas, and available method for obtaining the desired vapor pressure of the gas. Today the gas of use in thyratrons is predominantly hydrogen due to its low cost, easy fabrication of a vapor source, and it has the highest mobility which allows rapid turn-on times. A schematic of a typical hydrogen thyatron that is still in use today is shown in Fig. 3.7(a) and the symbol used in circuit diagrams for the thyatron is shown in Fig. 3.7(b) (Swenne 1960). If a comparison is made to the old mercury thyratrons, several major differences can be seen. First of all the size of the cathode is much larger and no longer a filament type, but a plate type similar to the anodes. Also, the grid does not surround the cathode and has very few holes which tends to increase the dielectric strength of the device; hence increasing the hold-off voltage. The developmental process of the thyatron was not directed towards obtaining complete grid control and reducing the forward voltage drop, but to conducting high currents (> 40 kA) at high anode voltages (> 60 kV) (Pirrie et al. 1988). The type of grid structure shown in Fig. 3.7a allowed much higher anode voltages. Also, this type of thyatron operates very similar to the mercury vapor thyatron, except triggering of the device is initiated by applying a positive potential to the control grid.

3.4.6 Summary

In summary, the direction of development of the thyatron makes it unsuitable for space nuclear power systems and more suited for high power systems. Nevertheless, the desire to develop a switch that is completely grid controllable continued. In the early 1950's a switch called the plasmatron was

Figure 3.7. Hydrogen Thyatron; a) schematic of a high voltage, high current thyatron, and b) circuit symbol.

constructed that was ignited via a grid and extinguished when a negative voltage was applied to the grid.

Table 3.1 First Ionization Potentials of Gases Used in Thyratrons.

<u>Gas</u>	<u>First Ionization Potential (eV)</u>
Mercury, Hg	10.437
Argon, Ar	15.759
Krypton, Kr	13.999
Xenon, Xe	12.130
Hydrogen, H	13.598

3.5 Plasmatron

The plasmatron, developed in the early 1950's by E.O. Johnson (E.O. Johnson 1951 & 1952), was a grid-controlled hot-cathode electron tube. In this device the grid interactively controls the conduction of the discharge current through the use of an auxiliary discharge. A schematic of the plasmatron is shown in Fig. 3.8. Unlike the thyratrons, the auxiliary discharge controls the plasma concentration (i.e. electron and ion densities), hence allowing the grid to control the conduction of the main discharge current

3.5.1 Hold-off Voltage

The hold-off voltage capabilities of the plasmatron are not as good as the thyatron. Because of the auxiliary discharge creating a plasma in the main

Figure 3.8. Schematic of a Plasmatron Gas-Discharge Switch.

cathode-anode gap, if the anode voltage exceeds the first ionization potential of the gas being used, ignition can occur without a decrease in the grid voltage. The gas used in the plasmatron was helium, which has a first ionization potential of 24.6 eV and the maximum anode potential applied was 24 V. Therefore, one limitation of the plasmatron is that the modulation voltage is limited to the first ionization potential of the gas that is being used for the conduction medium.

3.5.2 Turn-on Characteristics

In the three electrode plasmatron, an auxiliary discharge is developed between an auxiliary cathode and passed through a small aperture that surrounds the auxiliary cathode which operates continuously (see Fig. 3.8). This aperture (constriction) increases the energy of the auxiliary electron beam such that ionization can take place in the gap between the main cathode and anode. As the negative potential on the grid is decreased, the electron current to the anode begins to increase, and around -2 V, ignition of the discharge takes place. This ignition process is the same as in the thyatron.

3.5.3 Turn-off Characteristics

For extinguishing the discharge in the plasmatron, a negative potential is applied to the grid which causes the sheath around the grid wires to expand and overlap. This closure of the grid hole repels the electrons at the grid and interrupts the anode current. In order for the grid sheaths to overlap the grid apertures have to be very small. Also there are restrictions on the plasma density, which is why there is the need for the auxiliary discharge. This discharge allows the proper plasma density necessary to be able to extinguish the discharge. An illustration of

this extinguishing mechanism is shown in Figure 3.9. It was shown by Johnson 1952 that currents up to hundreds of milliamperes could be modulated at frequencies up to 100 kHz.

3.5.4 Forward Voltage Drop

Since the plasma in the plasmatron is created by an auxiliary discharge, the voltage drop during current conduction is as low as tenths of volts. However, it must be kept in mind that additional power is required to operate the auxiliary discharge and this could affect the overall efficiency of the switch. Nevertheless, the switching efficiency for the plasmatron is very high and exceeds 90 %.

3.5.5 Applications

As with any low voltage device there are many applications which the switch might be used. Some of these include motor drive, direct loudspeaker drive, and high-efficiency rectification and inversion. But the switch had the added complexity of controlling an auxiliary discharge and it would be much simpler and convenient to have a grid-controlled device without an auxiliary discharge. Therefore, work continued to improve devices that existed at that time and develop a completely grid controllable switch.

3.5.6 Summary

Although the forward voltage drop of the plasmatron was very low (<0.3 V), it could only conduct currents of hundreds of milliamperes and had hold-off voltages less than 25 V. Also, because of the need of an auxiliary discharge to

Figure 3.9. Illustration of Extinguishing Mechanism of a Plasmatron.

control the plasma density, the device was more complex. Therefore, the need still existed for a moderately high current device that maintained a low forward voltage drop that was completely grid controlled. Some of these limitations were addressed by the grid-controlled hydrogen tacitron.

3.6 Grid-Controlled Hydrogen Tacitron

A few years after Johnson and Webster (1951 and 1952) developed the plasmatron, a device was developed in 1954 by Johnson, Olmstead, and Webster that utilized hydrogen gas as the ionizing medium and was capable of current interruption by grid action without the use of an auxiliary discharge. They called this device a tacitron. This was the first time the term tacitron was attached to a switching device. The construction of the device was very similar to hydrogen thyratrons that were developed at that time but was capable of grid interruption due to the ability to limit the ionization in the tube to the grid-anode region only. Also, the grid structure was slightly different with respect to the diameter of the grid holes which needed to be quite small. The turn-on mechanism for the tacitron is the same as the thyatron and plasmatron.

3.6.1 Hold-off Voltage

An advantage of removing the auxiliary discharge, besides increased simplicity, is that the hold-off voltage (applied anode voltage with no breakdown) of the device is increased. With no pre-ionization of the cathode-anode gap, anode potentials far above the first ionization potential of the gas can be applied (> 100 V). Also, by increasing the negative potential applied to the grid, the anode potential can be increased even further. With the size of the grid being very small

for these devices, adequate cooling of the grid must be supplied in order to keep the emission from the grid to tolerable levels and maintain the high hold-off voltages. This effect is the same as was seen in mercury thyratrons.

3.6.2 Turn-on Characteristics

As with the pliotron and thyatron, a small negative potential applied to the control grid maintains the hydrogen tacitron in the off-state, and when this potential is removed, the device turns on.

3.6.3 Turn-off Characteristics

The method of current interruption with grid control was accomplished by the same method as with the plasmatron (see Fig. 3.9). Because the dimensions of the grid wires are so important for current interruption by this method, a little more detail is given. The thickness of the positive ion sheath that forms around an electrode that is immersed in a plasma can be calculated from the "3/2 Law" in which the ion current density, j_i , is:

$$j_i = \frac{2.33 \times 10^{-6}}{\sqrt{M_i / m_e}} \frac{V^{3/2}}{d^2} , \quad (3.10)$$

where V is the magnitude of the grid voltage, d is the sheath thickness, and M_i and m_e are the ion and electron masses, respectively. The ion current density, j_i , can be calculated from the particle density, n_i , and ion velocity, v_i , by:

$$j_i = q n_i v_i , \quad (3.11)$$

where q is the electron charge (1.6×10^{-19} coulombs) and v_i is:

$$v_i = \sqrt{\frac{k_B T_e}{M_i}}, \quad (3.12)$$

where k_B is Boltzmann's constant and T_e is the electron temperature in the proximity of the grid. By combining Eqns. (3.10) - (3.12), the following is obtained for the sheath thickness, d , in meters:

$$d = \sqrt{\frac{3.74 \times 10^9 V^{3/2}}{n_i T_e^{1/2}}}, \quad (3.13)$$

For a hydrogen operating pressure of 100 mTorr, the ion density would be $\approx 10^{17}$ - 10^{18} ions/m³. Therefore, for a grid voltage of 10 V and an electron temperature of 5000 K and an ion density of 10^{18} ion/m³, the sheath thickness would be ≈ 40 μ m. This small sheath thickness is why grids that are useful for this type of current interruption are very small, fragile, and require a lot of cooling, which is one limitation of the device. Finally, since the ion density in the plasma is directly proportional to the anode current and since the sheath thickness decreases with increasing ion density, there is a limit to the maximum current that was obtainable of ≈ 0.5 A. The low current problem was solved by scientists from the Atomic Energy Institute in Moscow, Russia when hydrogen tacitron devices were developed that could conduct currents up to 500 A.

3.6.4 Forward Voltage Drop

The forward voltage drop for the hydrogen tacitron is very similar to that of the mercury thyratrons (≈ 15 - 30 V), which is still high for low voltage

applications. It will be shown in great detail in Chapter 4 that the Cs-Ba tacitron has a forward voltage drop as low as 2 - 3 V.

3.6.5 Applications

The issue of low conduction currents was resolved in the former Soviet Union where several commercially available high current hydrogen tacitrons were developed in the 1970's (Dvornikov et al. 1972 & 1974). The increased current was possible by decreasing the size of the grid aperture while maintaining its structural integrity. The hydrogen operating pressures for these devices ranged from 50 mTorr to 300 mTorr. The spacing is such that the grid aperture is on the order of the thickness of the ion sheath at the grid. This small grid is very fragile and requires a lot of water cooling. Depending on the grid design and the amount of cooling to the grid, currents of up to 500 A at an anode voltage of 20 kV have been interrupted with grid control at modulation frequencies up to 200 kHz. The grid voltage necessary to turn these devices off is higher than those for the plasmatron and are typically -150 to -200 V.

Like the hydrogen thyatron, the hydrogen tacitron could be used in many high power applications; such as, pulse forming networks for lasers, high power radars, etc.. However, these devices are not suitable for low voltage applications (power conditioning in space nuclear power systems) because of the large forward voltage drop.

3.6.6 Summary

Switch technology, by the early 1970's, had evolved to a point that a gas discharge tube could conduct hundred's of amps of current, hold-off voltages of

ten's of kilovolts, and modulate at hundreds of kilohertz with a moderately low forward voltage of less than 100 V. Remembering that the output voltage of a typical thermionic space nuclear power system is only tens of volts, the hydrogen tacitron is unsuitable for conditioning the SNPS's output power. However, with the knowledge gained in developing the earlier switches, it was determined that by changing the fill gas to one with a very low first ionization potential, it might be possible to decrease the forward voltage drop to below 5 V. The gas that was decided upon was Cesium.

3.7 Cs Tacitron

During the space race in the 1960's, thermionic converters that utilized Cesium vapor as the conduction medium were developed. These converters were used for energy conversion and not inversion, but had attributes that could possibly be used in grid-controlled devices to lower the forward voltage drop. This lower voltage drop is due to the fact that the first ionization potential of Cs is only 3.89 eV (see Table 3.1 for comparison of other gases) and the ionization of Cs is a multi-step process. This stepwise ionization occurs through a resonant state of the Cs atom resulting in a voltage drop of less than 1 V. With the development of the grid-controlled switches and the low voltage drop Cs thermionic converter, the idea of combining the two concepts was developed that addressed the problem of high forward voltage drop.

The use of Cs vapor as the conduction medium for grid-controlled triodes was studied by several authors in the Soviet Union in the mid to late 1970's (Kaplan et al. 1977a, Baksht et al. 1978a and Baksht et al. 1978b). Although these devices had very low forward voltage drops, they still had a few problems, one of which was low conduction currents. This is discussed in the next sub-section

followed by a description of the other operating parameters; such as, hold-off voltage, turn-on and turn-off characteristics, etc.

3.7.1 Conduction Current

With the introduction of Cs vapor in the interelectrode gap, a layer of Cs forms on the surface of the cathode. This layer might not be uniform which tends to create small fields at the surface and increase the emission current. Also, with ionization of Cs taking place in the gap, there is an ion current that flows to the surface that can increase the emission current even more. Finally, the sheath potential that forms in front of the cathode surface can also increase the amount of emission current due to field effects. All of these factors are combined to determine an effective electric field at the cathode surface. This increase in current due to this effective electric field is called the Schottky effect (Hemenway et al. 1962 and Baksht 1978c) and is written as:

$$j_{th} = j_s \exp \left(\frac{-q^{3/2} E^{1/2}}{(4\pi\epsilon_0)^{1/2} k_B T} \right), \quad (3.14)$$

where j_s is the thermal saturation current given by Eqn. 3.2, ϵ_0 is the permittivity of free space (8.85×10^{-12} F/m), and E is the electric field at the cathode surface. Since it is difficult to measure the electric field, it is more convenient to express it in terms of the ion current at the cathode surface and the cathode sheath potential, ΔV_E . The electric field, which is found by solving Poisson's equation with the appropriate boundary conditions, is (Hemenway 1962):

$$E = \left(\frac{4}{\epsilon_0} \right)^{1/2} \left(\frac{M_i}{2q} \right)^{1/4} \Delta V_E^{1/4} j_{iE}^{1/2} \quad , \quad (3.15)$$

where ΔV_E is the cathode sheath potential, and j_{iE} is the ion current density at the cathode. Substituting Eqn. (3.15) into Eqn. (3.14) with the appropriate physical constants, yields the following expression for the Schottky emission current density:

$$j_{th} = j_s \exp \left(\frac{612.7 j_{iE}^{1/4} \Delta V_E^{1/8}}{T} \right). \quad (3.16)$$

At a cathode temperature of 1225 °C and a Cs pressure of 10^{-1} Torr (Cs reservoir temperature of 210 °C), the work function of a tungsten cathode is approximately 2.6 eV. Refer to Appendix A for the cesiated tungsten work function curves. This work function yields a saturation emission current density, j_s , of only 0.51 A/cm². The current density can be increased to about 2 A/cm² if special cathodes are used (e.g. platinum) or if an oxide layer is formed on the tungsten electrode surface that reduces the work function a few tenths of a volt. It should be noted that the anode current density is approximately equal to j_s when operation of the device is at the minimum forward voltage drop.

3.7.2 Hold-off Voltage

As discussed earlier, the temperature of the grid in three electrode switching elements must be maintained at a sufficiently low value to prevent premature breakdown of the device due to the emission of electrons into the grid-anode region. One of the disadvantages of using Cs vapor is the reduced dielectric strength of the switch due to increased emission from the grid. The maximum tolerable emission current from the grid is between 0.5 and 1 mA/cm² (Kaplan

1977a) to prevent breakdown at a grid temperature of $\approx 350^\circ\text{C}$ and a work function of 1.5 eV.

The maximum anode potential shown by Kaplan et al. (1977a) in a Cs tacitron was 50 V. It is shown that the maximum anode potential is a strong function of the Cs pressure and can be increased by supplying a small negative potential to the grid (-1 to -2 V) during current conduction. For example, with a grid voltage of -0.1 V, the ignition of a Cs tacitron at a Cs pressure of 1.33×10^{-2} Torr occurred at only 3 V; whereas, at a grid voltage of -0.8 V, the ignition voltage was ≈ 25 V (Kaplan et al. 1977a).

3.7.3 Turn-on Characteristics

Ignition of the Cs tacitron is slightly different than that of the mercury thyratron, but just as simple to achieve. Instead of removing a negative potential from the grid to initiate breakdown, a positive grid potential pulse of several volts for only microsecond duration is sufficient to cause breakdown.

3.7.4 Extinguishing Mechanism

In extinguishing or quenching a low pressure Cs discharge, Kaplan (1977b) observed three different mechanisms: 1) potential quenching, 2) fast quenching, and 3) slow quenching.

Potential quenching

Potential quenching is the same as the extinguishing mechanism in the hydrogen tacitron where the sheaths of the grids overlap and a potential barrier repels the electrons, interrupting the current. Also the decrease in the ion density

around the grid during the negative voltage pulse can not be replenished before current interruption occurs. When the current is cut off there are no longer any electrons to cause ionization and excitation in the grid/anode region. Therefore, the plasma density drops very rapidly and the device remains off even as the anode voltage increases. In potential quenching, turn-off occurs in less than a microsecond. Sometimes the grid sheaths do not completely overlap, but the discharge is still extinguished, which is called fast quenching.

Fast quenching

In fast quenching, there are two steps associated with current interruption. The first step is similar to potential quenching in that the grid sheaths expand, reducing the amount of current reaching the anode. Since the sheaths do not completely overlap, electrons still exist in the grid/anode region and cause excitation and ionization. As the grid sheaths grow, the impedance to the flow of electrons increases, and the anode voltage increases which is the second step of the process. The increase in the anode voltage increases the energy of the electrons entering the grid/anode region, thereby increasing the rate at which ions are being created. Ultimately, if the rate of ion loss at the grid surface is greater than the ion production rate in the grid/anode region, the discharge will extinguish; if not the discharge will reignite. It takes $\approx 1 \mu\text{s}$ for the discharge to extinguish under these conditions.

Slow quenching

Slow quenching of the discharge is seen when a small negative voltage is applied to the grid. In this case there is no sudden increase in the ion current drawn by the grid, but a monotonically increasing ion current. After about $5 \mu\text{s}$, there is a rapid decrease in the anode current and the discharge is quenched. Again

the energy loss in the plasma due to the loss of ions eventually extinguishes the discharge.

3.7.5 Forward Voltage Drop

It was shown by Kaplan et al. (1977a) that the minimum forward voltage drop in a Cs triode ranged from 1.75 V to 2.75 V, depending on the placement of the grid, grid thickness and the size of the grid aperture. The closer the grid is to the anode, the lower the forward voltage drop because the ionization predominantly takes place close to the anode surface (the higher the plasma density, the lower the voltage drop). If the grid is moved closer to the cathode surface, the forward voltage drop increases. Also discussed by Kaplan et al. (1977a) is the effect of the thickness of the grid (more surface area). Since the forward voltage drop is a function of the plasma density, as the surface area of the grid is increased the ion leakage to the grid is also increased; hence decreasing the plasma density and increasing the forward voltage drop. The easiest to visualize is the decrease of the grid aperture diameter. The smaller the grid hole (decreased transparency), the more constrictive the grid is to the flow of electrons and the higher the forward voltage drop. Since all of these parameters affect the forward voltage drop some combination will result in a minimum voltage drop.

3.7.6 Applications

The Cs tacitron is most suited for power conditioning in thermionic space nuclear reactor systems. Its low forward voltage drop would allow high switching efficiencies, even at input voltages of only 30 V, unlike the other gas-discharge switching elements outlined previously. It is also a high temperature, radiation

hard device that can be placed either in the reactor core or in close proximity, unlike the semiconductor thyristor. Switching frequencies ranging from 0.1 kHz to 1 kHz have been reported (Kaplan 1977a and 1977b).

3.7.7 Disadvantages of Cs Tacitron

Due to Cs low ionization potential and relatively high pre-discharge plasma concentration, the Cs vapor pressure in the tacitron must be maintained below 10^{-2} Torr (Kaibyshev et al. 1972). Otherwise, the grid apertures would need to be very small (10^{-4} to 10^{-3} cm) in order to extinguish the device. The small grid apertures cause overheating of the grid, thus increasing grid emission and inducing premature breakdown between the grid and anode. Another limitation of the Cs tacitron is the low emission current from the cathode when low Cs vapor pressures are used. The maximum current density cited was 1 A/cm² (Baksht 1978a).

3.7.8 Summary

The Cs tacitron has a low forward voltage drop (1.75 - 2.75 V), has the capability of grid control (at anode voltages of up to 50 V), and modulation frequencies of up to 1 kHz. Although it overcomes many of the disadvantages of the semiconductor and other gas-discharge switching elements, the Cs tacitron still has limitations. One of the major limitations is its low cathode emission current. The cathode temperature is limited to values around 825 °C in order to minimize grid emission. Therefore, an effort should be made to increase the cathode emission current and decrease the grid emission current while maintaining low forward voltage drop and grid controllability. When low pressure Ba vapor is introduced into the Cs tacitron, these problems are alleviated, as discussed in the

next section.

3.8 Cs-Ba Tacitron

As mentioned in the earlier section, the Cs tacitron has two main problems: 1) low emission currents at the low Cs pressures of interest, and 2) low dielectric strength due to emission from the grid. By adding Ba vapor to the interelectrode gap, both of these areas are improved upon. From 1972-1978 a large amount of research in the former Soviet Union was devoted to developing triode devices that used Cs and Ba vapor mixtures and could be used as a switch for power conditioning in low voltage applications. A combination of Cs and Ba vapor mixtures was chosen because of the limitations of the Cs devices discussed earlier. Although some work has been done on Cs-Ba tacitrons in the 1970's, further understanding of the operation characteristics is needed.

Before discussing the most recent experimental data that has been obtained on the Cs-Ba tacitron, a summary of the results of past experiments is presented in the following sections. As done previously, the summary is divided into sections on conduction current, hold-off voltage, turn-on and turn-off characteristics, forward voltage drop, and a summary focusing on additional work that needs to be completed. Many of the operational characteristics of the Cs-Ba tacitron are the same as those discussed for the Cs tacitron, except for a few differences due to the addition of Ba vapor into the interelectrode gap.

3.8.1 Conduction Current

The maximum thermal emission current density is increased by adding Ba vapor as opposed to Cs alone, because the heat of adsorption energy of Ba is

higher than that of Cs (4.5 eV compared to 2.5 eV, Baksht 1978c) resulting in higher Ba coverage on the electrode surface. The higher coverage results in a reduced work function and hence a larger emission current for a given electrode temperature. For comparisons of the work functions of refractory metals coated with Cs and coated with Ba refer to Appendix A.

As mentioned earlier in the a device using only Cs vapor, the maximum thermal current density is approximately 0.49 A/cm^2 at a cathode temperature of $1225 \text{ }^\circ\text{C}$ and Cs pressure of 10^{-1} Torr for a tungsten cathode (work function of 2.6 eV, see Fig. A.1a). However, by adding Ba at a reservoir temperature of $600 \text{ }^\circ\text{C}$ (0.4 mTorr), the cathode work function could be as low as 2.28 eV (see Fig. A.2) and at the same conditions as above, the maximum thermal emission current density increases to $\approx 5.84 \text{ A/cm}^2$. Also, the limit on the maximum emitter temperature can be increased to at least $1300 \text{ }^\circ\text{C}$ which results in a maximum thermal emission current density of 11.12 A/cm^2 (recall that the limit on the emitter temperature is determined by the emission from the grid). The following section addresses the issue of increased emitter temperature and reduced grid emission. Therefore, by adding Ba to the system, the emission current density can be increased by an order of magnitude compared to devices that use only Cs vapor. Several authors reported current densities in excess of 15 A/cm^2 (Kaibyshev et al. 1972 and Kaplan et al. 1979).

3.8.2 Hold-off Voltage

Another limitation of the Cs tacitron, that is improved in a Cs-Ba tacitron, is premature breakdown. At grid temperatures of $500 \text{ }^\circ\text{C}$ and Cs reservoir temperature of $210 \text{ }^\circ\text{C}$, the work function in a Cs environment for a Molybdenum grid would be 1.6 eV ($j = 2.71 \text{ mA/cm}^2$). When Ba is added to the device, the

work function of the grid increases from 1.6 eV to approximately 2.2 to 2.3 eV (see Fig. A.2). This decreases the grid emission current density from 2.71 A/cm² to 0.34 μ A/cm². Since this is well below the tolerable grid emission, the grid temperature can be increased, hence the emitter temperature can be increased. Figure 3.10 shows the emission current density versus electrode temperature for different electrode work functions. As can be seen the amount of emission current is a strong function of the electrode work function. For the case of a Cs tacitron with a work function of approximately 1.6 eV, in order to limit the grid emission current density to below 1 mA/cm², the grid temperature must remain below 475 °C. However, if the work function was increased to 2.2 eV (for the case with a Ba coating), the emission current density from the grid would be only 10⁻⁴ mA/cm². If the grid emission current density were allowed to remain at 1 mA/cm², the grid temperature could be increased from 475 °C to 725 °C in a Ba environment (see Fig. 3.10). This increase in the grid temperature allows the cathode temperature to be increased, resulting in higher emission currents from the cathode. Hold-off voltages of greater than 200 V were reported by Kaibyshev (1972) and Kaibyshev and Kuzin (1975). It was stated by Kaibyshev and Kuzin (1975) and Kaplan et al. (1977a) that the hold-off voltage might be increased further by applying small negative voltages to the grid (like the thyatron and Cs tacitron).

3.8.3 Turn-on Characteristics

Ignition of the Cs-Ba tacitron is the same as the Cs tacitron. A positive voltage pulse of several volts for only microsecond duration is applied to the grid to initiate breakdown.

Figure 3.10. Emission Current Density Versus Electrode Temperature for Various Electrode Work Functions.

3.8.4 Turn-off Characteristics

Several theories on the extinguishing of Cs-Ba devices have been put forth depending on the Cs pressure and grid design. Kaplan (1977c) operated a Cs-Ba tacitron at a Cs pressure of 6×10^{-3} Torr, Ba pressure of 7.3×10^{-3} Torr, and cathode temperature of 1327 °C. The extinguishing voltage pulse to the grid was -20 V. The grid had a wire diameter of ≈ 30 μm and an aperture diameter of 150 μm . The extinguishing mechanism as described by Kaplan was due to the overlap of the grid sheaths just like the hydrogen tacitron (see Fig. 2.9). Substituting the parameters given above with an ion density of 10^{12} ions/cm³ and an electron temperature of 5000 K into Eqn. (2.13) yields a sheath thickness of 69 μm . Therefore, the sheaths on either side of the grid aperture could indeed overlap. With this extinguishing mechanism, modulation frequencies of up to 1.65 kHz were reported (Kaplan 1977c).

Work on another Cs-Ba tacitron that had grid aperture diameters greater than 500 μm , described a different extinguishing mechanism. It was suggested by Kaibyshev and Kuzin (1975) that the mechanism for quenching the discharge is a decrease in the density of ions at the grid aperture as a result of the ionization rate decreasing below the rate at which ions are removed at the electrode surfaces, especially the grid surface. This is somewhat similar to the potential and fast quenching mechanisms described by Kaplan (1977b), where the rate of ion loss to the grid is greater than the rate of ion production in the grid/anode region.

Finally, Kaplan et al. (1979) describe the spontaneous current cut-off in a Cs-Ba triode device operated at a repetition rate of ≈ 6 kHz. The current cutoff occurs when the current density approaches the maximum value for a fully ionized plasma. When the current exceeds this value, nonlinear oscillations cause the negative potential to extend into the grid region which short-circuits the grid

conductors and cuts the current off.

The geometry of the grid (size and thickness) in the Cs-Ba tacitron located at the University of New Mexico more closely resembles those described in the work by Kaibyshev and Kuzin (1975) and Kaplan (1977c).

3.8.5 Forward Voltage Drop

The forward voltage drop in the Cs-Ba tacitron is approximately the same as in the Cs tacitron and has the same dependence on the gas pressure, current density, and grid design. Values of forward voltage drop between 1 V and 2.5 V have been reported in the literature. Also the same dependencies of the forward voltage drop on the grid geometry and location exist in the Cs-Ba tacitron as with the Cs tacitron. The closer the grid is to the anode, the lower the forward voltage drop. As the surface area of the grid increases the ion leakage to the grid is increases, which lowers the plasma density and increases the forward voltage drop. Finally, the smaller the grid hole (decreased transparency), the more constrictive the grid is to the flow of electrons and the higher the forward voltage drop.

3.8.6 Summary

The Cs-Ba tacitron devices developed in the 1970's showed both increased emission current densities and enhanced grid controllability (decreased grid emission) due to the addition of Ba vapor to a Cs tacitron. Conduction currents greater than 15 A/cm² at forward voltage drops less than 2.5 V were reported. However, a quantitative understanding of the effect of the grid size and thickness on the extinguishing of Cs-Ba tacitrons is not yet well understood. Also, the modulation characteristics of the device as a function of the input parameters

(cathode temperature, Cs and Ba pressures, etc.) need further investigation.

For the remainder of this work, the name that is used for the cathode of the Cs-Ba tacitron will be emitter and for the anode of the Cs-Ba tacitron will be collector. Previously cathode and anode have been used consistently in the literature. The change to emitter and collector is now being made to be consistent with the recently published papers. Keep in mind that cathode/emitter and anode/collector, are interchangeable.

4. CESIUM-BARIUM TACITRON

This chapter is divided into four main sections: 1) description of the test facility, 2) hold-off voltage and I-V characteristics, 3) current modulation characteristics, and 4) inverter characteristics using two Cs-Ba tacitrons.

4.1 Description of Test Facility

The Cs-Ba tacitron test facility at the University of New Mexico's Institute for Space Nuclear Power Studies consists of a vacuum system, Cs and Ba reservoirs, a power rack, a control rack, PC-based data acquisition equipment (CAMAC), and the tacitron. A photograph showing each component of the test facility is shown in Fig. 4.1. A detailed description of each component follows.

Vacuum System

The vacuum system consists of a vacuum chamber, isolation swing gate valve, two absorption pumps, an ion pump, and vacuum gauges. The vacuum chamber consists of a vacuum housing with a device mounting plate and electrical feed-throughs and is shown in Fig. 4.2. There are 32 thermocouples channels, 12 high current insulated feed-throughs, 14 medium current channels for heaters, and 32 signal channels for control and measurement. The entire vacuum chamber is connected to the swing gate valve and pumps and is enclosed in a water cooled jacket. The total volume of the system is ≈ 16 liters.

The swing gate valve is made out of stainless steel and has a Viton seal. The valve is used to isolate the main vacuum chamber from the ion pump during activation of the pump. The initial pumping of the system is performed by two Varian VacSorb roughing pumps. Using these pumps, the system can maintain a

Figure 4.1. Photograph of Cs-Ba Tacitron Test Facility.

Figure 4.2. Photograph of Cs-Ba Tacitron Vacuum Chamber.

pressure of tenths of mTorr. Liquid nitrogen is used to cool the VacSorbs. The ion pump is a Varian StarCell VacIon pump with a pumping speed of 230 liter/sec. This pump is engaged when the pressure becomes less than 10 mTorr and has a base pressure of 10^{-10} Torr.

A Varion 843 ratiomatic ionization gauge controller is used to monitor the pressure in the vacuum chamber. The controller can monitor both the roughing pressure and the high-vacuum pressure. The roughing pressure is monitored with a type 531 thermocouple gauge tube. The controller can monitor pressures between 10^{-3} Torr and 2 Torr with this tube. The high vacuum pressure is monitored with a 563 series Bayard-Albert ionization gauge tube. The operating range for the tube is 2×10^{-10} Torr to 10^{-3} Torr.

Cs and Ba Reservoirs

The Cs and Ba reservoirs are independently controlled. The Cs reservoir is supplied by distilling Cs from an ampoule. The distillation process takes about 2 hours. There is separate water cooling for both the Cs reservoir and the Cs ampoule. The heater power has a maximum voltage and current rating of 42 V and 4 A, respectively. The box that surrounds the reservoir (see Fig. 4.2), is used to maintain the ambient temperature surrounding the reservoir at 250 °C, which decreases the necessary heater power to the reservoir. The temperature in the reservoir can be maintained between 20 °C and 200 °C with a combination of heater power and water cooling. The thermocouple connected to the Cs reservoir senses the temperature and adjusts the heater power in order to maintain the temperature at the desired level.

The barium reservoir is located inside the vacuum chamber. The voltage and current ratings for the heater for this reservoir are 21 V and 6.5 A, respectively. As with the Cs reservoir, the Ba reservoir contains thermocouples

which sense the temperature and adjust the heater power appropriately.

Power Rack

The power rack contains all the heater power units for the system. The power is supplied to the system for two different functions: 1) to outgas the system after it has been exposed to air and 2) to supply power to the tacitron device for operation.

The outgassing procedure takes approximately one week and is completed in two phases. Initially, the VacSorb pumps are used to outgas the chamber, vacuum pipeline and Cs system. After approximately 24 hours, the chamber and vacuum pipeline heaters are turned off and the tacitron is heated (i.e. emitter, collector, grid, etc.). There are two main heaters for the device: 1) the emitter and 2) flange tube heater (basis flange). The emitter heater supplies ≈ 800 W of electrical power to the device and the flange tube heater ≈ 275 W to the basis flange. During this outgassing phase, the ion pump is used until the vacuum is at least 2×10^{-6} Torr while the emitter temperature is 1500°C . At this point the system is opened and Ba is added to its reservoir. The initial outgassing is necessary in order to prevent the Ba from becoming contaminated. Adding the Ba to the system only takes approximately 30 minutes; therefore, the system is not exposed to air for long. Again the outgassing procedure is performed. Once the emitter temperature reaches its maximum and the vacuum is below 2×10^{-6} Torr, the Ba is melted (melting temperature for Ba is $\approx 620^\circ\text{C}$) by increasing the Ba reservoir temperature up to 700°C . For emitter temperatures above 1300°C , additional heat is supplied by electron bombardment. The bombardment voltage is 300 V which is applied between the emitter heater and the emitter.

Control Rack

The control rack houses the feedback for the Cs and Ba reservoir heaters, and controls the amount of power supplied to all of the heaters except the emitter and flange tube heaters. The control rack also houses the data acquisition equipment.

Data Acquisition Equipment

The diagnostic equipment consists of a CAMAC based multi-channel waveform digitizer (LeCroy 6810) for electrical measurements and an A/D converter (Kinetic Systems 3581) for temperature measurements. In addition, a D/A converter (BiRa 5408) is used to send test signals for calibration of the current and voltage and to locate system errors. The system is capable of computer or manual control. The computer control is performed by an IBM compatible 386-PC which uses a GPIB interface to connect to the CAMAC.

Cs-Ba Tacitron

The Cs-Ba tacitron device consists of two triode sections and one diode section. This configuration allows testing of not only the triode sections, but also the diode section, under similar operating conditions. Figure 4.3 presents a schematic cross section and Fig. 4.4 is a photograph of the device. Table 4.1 lists its characteristic dimensions. The emitter, grid and collector are constructed of molybdenum and the emitter heater is a 2 mm diameter tungsten rod surrounded by a tantalum tube. The tube returns the current from the tungsten heater to the power supply and prevents magnetic fields from developing due to the high heater current. Heating of the emitter is accomplished by thermal radiation from the tantalum tube or by electron bombardment, if a higher temperature is desired ($>1300\text{ }^{\circ}\text{C}$). Each triode section has a 1.0 mm thick grid, placed approximately

1 - Cylindrical Molybdenum Emitter, 2 - Tungsten Heater,
 3 - Insulator, 4 - Grid, triode 1, 5 - Collector, triode 1,
 6 - Collector, diode, 7 - Grid, triode 2, 8 - Collector, triode 2,
 9 - Barium reservoir connecting tube, 10 - Cesium reservoir,
 11 - Throttle aperture

Figure 4.3. A Schematic of the Cs-Ba Tacitron Device.

Figure 4.4. Photograph of Cs-Ba Tacitron Device.

1.0 mm from the emitter surface. The temperature of the emitter is measured using three W-Re thermocouples mounted at different depths within the emitter (12 mm, 21 mm, and 37 mm, from the top of the emitter section), which correspond to the locations of the collectors in the three sections. The temperatures of the collectors, grid, Cs reservoir, and Ba reservoir are measured using type-K thermocouples. Type-K thermocouples are also used to monitor the temperatures in the different sections of the test facility.

Table 4.1 Tacitron Physical Dimensions

<u>EMITTER - MOLYBDENUM</u>	
	Length: 44 mm
	Inside Diameter: 8 mm
	Outside Diameter: 16.5 mm
<u>TRIODES</u>	
Collector - Molybdenum	
	Working Length: 7 mm
	Inside Diameter: 25 mm
	Outside Diameter: 70 mm
Grid - Molybdenum	
	Thickness: 1 mm
	Aperture Diameter: 0.8 mm - 0.9 mm
	Interelectrode Gap: 4.25 mm
	Geometric Transparency: $\approx 34\%$
	Emitter-Grid Gap: ≈ 1 mm
<u>DIODE</u>	
Collector - Molybdenum	
	Working Length: 5 mm
	Inside Diameter: 17.5 mm
	Outside Diameter: 70 mm
	Interelectrode Gap: 0.5 mm

4.2 Operational Characteristics

As mentioned earlier, the performance of a switch is determined from its hold-off voltage, forward voltage drop, and current modulation characteristics. Before discussing the operation characteristics of the tacitron, it is important to define the nomenclature associated with the different processes during current modulation. A typical timing diagram for stable modulation of the device is shown in Fig. 4.5. The upper part of the figure describes the collector voltage and current characteristics and the lower part of the figure describes the grid voltage and current characteristics. In each part, the darker (thicker) line represents the voltage and the lighter (thinner) line represents the current. Stable current modulation in the Cs-Ba tacitron occurs when the modulation frequency of the tacitron, f_m , equals the applied modulation frequency to the grid, f_g . Another term used quite frequently when describing the operation of a switch is the duty cycle, δ , which is defined as $(\tau_g - \tau_{off})/\tau_g$.

The legend for all of the terms shown on Fig. 4.5 are listed with the timing diagram under nomenclature, but a few of the more often used terms will be elaborated on. The hold-off voltage is labeled V_{CE} and the forward voltage drop is labeled V_d . Both of these are important in determining the switching efficiency. The switching efficiency, η , is defined herein as *the ratio of the difference between the hold-off voltage and the forward voltage drop, $(V_{CE} - V_d)$ to the hold-off voltage, V_{CE}* . Another term that is used often is the collector delay time labeled τ_{ad} which is defined herein as *the time between when the positive grid potential is applied and ignition occurs* (see Fig. 4.5). Before showing the results of the hold-off voltage, forward voltage drop, and modulation characteristics of the Cs-Ba tacitron, the experimental procedure is described in the following section.

Nomenclature

—	: Voltage
—	: Current
τ_m	: Modulation period of tacitron
τ_{off}	: Off-time of discharge
τ_{rise}	: Rise-time of discharge
τ_{on}	: On-time of discharge
τ_{fall}	: Fall-time of discharge
τ_{ad}	: Arode delay-time
τ_p	: Pulse width of negative grid voltage
τ_{pmax}	: Pulse width of maximum negative grid voltage
τ_g	: Applied grid period
V_{ce}	: Applied voltage between collector and emitter
I_c	: Discharge current to collector
V_d	: Voltage drop between collector and emitter
V_{r+}	: Applied positive grid voltage
V_{r-}	: Applied negative grid voltage
I_g	: Grid Current
$f_m (1/\tau_m)$	: Modulation frequency of tacitron
$f_g (1/\tau_g)$	: Applied modulation frequency to grid
Stable Modulation Occurs when $f_g = f_m$	

Figure 4.5. Current Modulation Timing Diagram for the Cs-Ba Tacitron.

4.2.1 Experimental Procedure

In the experiments, measurements are always made after the Cs and Ba pressures and the temperatures of the device have reached steady-state. When the device is first turned on, a waiting period of one to two hours is required for the device to reach steady-state. Subsequently, a change of one of the operating parameters (for example, emitter temperature, Cs pressure, or Ba pressure) would require a much shorter waiting period, on the order of ten to fifteen minutes, for the device to reach steady-state.

In the breakdown experiments, a half-sinusoidal voltage pulse is applied to the collector to determine the collector voltage when ignition occurs (see Appendix B). In these experiments, the amplitude of the voltage pulse to the collector is kept constant at 170 V and the collector voltage for ignition in the breakdown mode is determined as a function of the applied positive potential to the grid, V_{g+} .

In the I-V mode the applied voltage is increased to 20 V during the first millisecond, and then is maintained constant for the next 5 ms. During the last millisecond the voltage is decreased to zero. To initiate discharge a constant positive voltage is applied to the grid (see Appendix B).

In the experiments investigating the current modulation mode of operation, the collector voltage is kept constant at 150 V. At the selected values of the Cs pressure, Ba pressure and emitter temperature, the off-times for stable and unstable modulation of the tacitron are measured as functions of the applied voltage pulse to the grid for ignition.

4.2.2 Hold-off Voltage

Figure 4.6 presents a typical response of the Cs-Ba tacitron operating in the breakdown mode; the operating conditions are shown at the bottom of the graph. As shown in Fig. 4.6, ignition of the tacitron occurs approximately 7 ms after the application of the breakdown pulse to the collector, at which time the collector voltage drops from 160 V to 10 V and the collector current rises simultaneously from 0 A to 60 A. At the point of ignition the grid potential, V_{g+} , also drops from its pre-ignition value of 5.5 V to zero, while the grid current, I_g , increases from approximately 250 mA to 4.0 A. After 9 ms (or approximately 2 ms following ignition) the collector current and voltage and the grid voltage reach steady values of 30 A, 12 V and -0.5 V, respectively, while the grid current remains unchanged at 4 A.

The measured values of the collector voltage required for ignition in the breakdown mode, at the same operating conditions as in Fig. 4.6, are plotted in Fig. 4.7 versus the applied positive grid potential, V_{g+} , for different Cs reservoir temperatures (or Cs pressure). Figure 4.7 shows that at a given collector voltage, the grid potential required for ignition increases as the Cs reservoir temperature (or Cs pressure) decreases. For example, at a collector voltage of 150 V, the grid potential necessary for ignition increases from 5 V to as much as 24 V as the Cs pressure decreases from 16.4 mTorr ($T_{Cs} = 164\text{ }^{\circ}\text{C}$) to 3.8 mTorr ($T_{Cs} = 134\text{ }^{\circ}\text{C}$). Results in Fig. 4.7 also show that while the grid potential required for ignition is strongly dependent on the Cs pressure in the gap it is almost independent of the collector voltage, V_{CE} .

The data in Fig. 4.7 can be used to generate a Paschen curve for the Cs-Ba tacitron. As was shown in the section 3.4.1 (see Eqn. 3.8), the breakdown voltage is a function of the Cs pressure and interelectrode spacing. The spacing for the

Figure 4.6. Typical Operation Characteristics of the Cs-Ba Tacitron in the Breakdown Mode.

Figure 4.7. Effect of Cs Pressure and Collector Voltage on the Grid Potential for Ignition in the Breakdown Mode.

tacitron Paschen curve is the emitter-grid gap. This is justified by the data shown in Fig. 4.8. As seen in the figure, the discharge is initially ignited between the emitter-grid gap which is shown by the increase in the grid current. After a 2-3 μ s lag, the collector current begins to increase. This 2-3 ms lag is the approximate time it takes for an ion to travel from the grid to the collector.

Additional experiments are performed to measure the effects of changing the Ba pressure and/or the emitter temperature on the grid potential required for ignition in the breakdown mode. The results are presented in Figs. 4.9 and 4.10. In these figures the grid voltage for ignition is plotted versus the $P_{\text{gap}}d$ parameter, where d is the distance between the emitter and grid (1 mm), and P_{gap} is the Cs pressure in the gap (in Torr). Again, these figures demonstrate that the collector voltage insignificantly affects the grid potential for ignition, while the effects of the emitter temperature and Ba pressure are small. Figures 4.9 and 4.10 indicate that when the Ba reservoir temperature is increased from 500 °C ($P_{\text{Ba}} = 0.02$ mTorr) to 530 °C ($P_{\text{Ba}} = 0.07$ mTorr) the effect of the emitter temperature on the grid potential for ignition is reversed, such that increasing the emitter temperature from 1200 °C to 1300 °C increases the required grid voltage for ignition. This reversal will be examined in more detail in the section 4.6 from the calculated Cs density required for ignition via the measured off-time during stable current modulation. For example, at a $P_{\text{gap}}d$ value of 0.01 mm-Torr, the grid potential for ignition increases from 13 V to 14 V as the emitter temperature is increased from 1200 °C to 1300 °C. Figure 4.10 also shows that the effect of emitter temperature on the difference in V_{g+} for ignition at T_E of 1200 °C and 1300 °C is approximately the same for all $P_{\text{gap}}d$ values (≈ 1 V). A comparison of the data in Figs. 4.9 and 4.10 shows that at $T_E = 1200$ °C the grid potential for ignition is almost independent of T_{Ba} ; however, at the higher emitter temperature of 1300 °C,

Figure 4.8. Grid and Collector Currents as Functions of Time Showing that Breakdown is Initially Between the Emitter and Grid.

Figure 4.9. Effect of Emitter Temperature on the Grid Potential for Ignition in the Breakdown Mode at $T_{Ba} = 500^\circ C$.

Figure 4.10. Effect of Emitter Temperature on the Grid Potential for Ignition in the Breakdown Mode at $T_{Ba} = 530^\circ\text{C}$.

the Ba vapor pressure affects the grid potential for ignition. For example, at a P_{gapd} of 0.01 mm-Torr and $T_E = 1200^\circ\text{C}$, the grid potential of 13 V in Fig. 4.10 is identical to that shown in Fig. 4.9 at the same emitter temperature but lower P_{Ba} ($T_{\text{Ba}} = 500^\circ\text{C}$). On the other hand, at an emitter temperature of 1300°C , the grid potential increases from 11.5 V to 14 V $\approx 40\%$ as the Ba reservoir temperature increases from 500°C to 530°C . It is worth noting that at P_{gapd} values higher than 0.025 mm-Torr, the grid potential for ignition becomes almost independent of P_{gapd} .

4.2.3 Forward Voltage Drop

The forward conduction voltage drop is an important parameter to future applications of the tacitron because it dictates the switching efficiency of the device and the range of input voltages that the device could be beneficial for. In order to maintain a switching efficiency $> 90\%$ for power systems having an output voltage as low as 50 V DC, the maximum voltage drop in the device should not exceed 4-5 V. Results to date show that at a given discharge current, the voltage drop in the triode sections decreases with increasing the Cs pressure in the gap and emitter temperature. Because the Cs vapor pressure in the gap during current modulation is less than in the first moment of discharge ignition, the voltage drop, in the I-V mode, at the same discharge current, will be somewhat lower than that during current modulation at the same initial Cs pressure at the moment of the discharge. Therefore, it is important to analyze the change in the voltage drop with time during discharge ignition, I-V mode, as a function of emission conditions (T_E and T_{Ba}) and Cs pressure (T_{Cs}).

4.2.3.1 Effect of Cs Pressure

As shown in Fig. 4.11, the forward voltage drop is a strong function of the Cs vapor pressure. At a Cs reservoir temperature of 127 °C (Cs vapor pressure of ≈ 2.6 mTorr), Ba reservoir temperature of 575 °C (Ba vapor pressure of ≈ 0.6 mTorr), and an emitter temperature of 1310 °C, the voltage drop in the tacitron is 2.6 V. When the Cs reservoir temperature is increased to 176 °C (Cs pressure of 27.9 mTorr) the voltage drop is as low as 1.3 V. This strong decrease in the forward voltage drop is due to an increase in the plasma density and a decrease in the electron temperature.

4.2.3.2 Effect of Ba Pressure

The Ba vapor pressure only affects the emission current and does not affect the transport characteristics of the device and hence the forward voltage drop. As can be seen in Fig. 4.12, increasing the Ba reservoir temperature from 498 °C to 580 °C increases the emission current from 14 A to 27 A, respectively, but has no affect on the forward voltage drop which remains constant at 2.0 V. This voltage drop is consistent with the data shown in Fig. 4.11 at the same emitter temperature and Cs pressure.

4.2.3.3 Effect of Emitter Temperature

The emitter temperature has a slight affect on the electron temperature and therefore on the forward voltage drop, but mostly changes the emission current. As can be seen in Fig. 4.13, at an emitter temperature of 1150 °C, the forward voltage drop is ≈ 1.8 V and the emission current is 10 A. However, when the

Figure 4.11. Effect of Cs Pressure on the Forward Voltage Drop in the Triode.

Figure 4.12. Effect of Ba Pressure on the Forward Voltage Drop in the Triode.

Figure 4.13. Effect of Emitter Temperature on the Forward Voltage Drop in the Triode.

emitter temperature is increased to 1300 °C, the forward voltage drop decreased to ≈ 1.3 V and the emission current increased to 24 A. Higher emitter temperature slightly increases the energy of the electrons at the emitter surface, which increases the plasma density; hence decreasing the forward voltage drop.

4.2.3.4 Effect of the Grid

The effect of the grid can be seen by comparing the I-V characteristics for the diode with those of the triode. Figure 4.14 shows the I-V curves for Cs reservoir temperatures of 150 °C (8.5 mTorr), 160 °C (13.6 mTorr), and 170 °C (21.4 mTorr). At a Cs reservoir temperature of 150 °C the forward voltage drop is ≈ 1.2 V; whereas in the triode (see Fig. 4.11) the forward voltage drop is ≈ 1.7 V at the same Cs reservoir temperature. Therefore, the increase in the voltage drop due to the grid sheaths is ≈ 0.5 V. At the higher Cs reservoir temperature (170 °C), the influence of the grid is about the same, causing a ≈ 0.55 V increase in the voltage drop.

4.2.3.5 Time Dependent Forward Voltage Drop

The changes in the discharge current and voltage drop with time after ignition in the I-V mode are shown in Figs. 4.15(a)-(e), 4.16(a)-(e), and 4.17(a)-(e), for different emitter temperatures and Ba and Cs vapor pressures. The source voltage used to generate the results in these figures is gradually increased to 20 V during the first millisecond, then maintained constant for the next 5 ms. During the subsequent millisecond the source voltage is gradually decreased from 20 V to 0 V.

Figure 4.14. Effect of Cs Pressure on the Forward Voltage in the Diode.

Figure 4.15. Effect of Increasing Emitter Temperature on Discharge Current and Forward Voltage Drop in the Breakdown Mode.

In Figs. 4.15 - 4.17 the changes in the discharge current and voltage drop during the 5 ms period (starting at 6 ms on the time scale), when the source voltage is constant, is due to the decreasing Cs vapor pressure as a result of heavy components leaving the gap (i.e. ions, atoms). As shown from the figures, the current rises initially at a practically constant voltage drop. At the end of the constant voltage drop region, the point where the voltage drop begins to increase, the discharge current is practically equal to the maximum thermal emission current from the emitter (j_s from Eqn. (3.2)). From the figures it is seen that the maximum thermal emission current increases with rising emitter temperature and/or Ba pressure. Further increase in the discharge current, causes the voltage drop to rise in order to increase the discharge current partially by Schottky effect (refer to Eqn. (3.16)) and partially by the increased ion flux to the emitter surface. After the discharge current reaches a maximum value, it begins to decrease in spite of the increase in voltage drop. Such increase in the voltage drop occurs as a result of the decrease in the voltage drop on the external load as the discharge current decreases. The duration of decreasing discharge current, at the end of which equilibrium occurs, is related to non-stationary processes associated with heavy components leaving the discharge gap. As indicated earlier, such equilibrium is characterized by the equality of ion flux from the gap and the Cs atom flux into the gap. Results in Fig. 4.15 show that increasing the maximum thermal emission current, by increasing the emitter temperature, not only reduces the voltage drop, but also increases the discharge duration for reaching equilibrium (see Figs. (4.15a) and (4.15e)). For example, when the emitter temperature is increased from 1200 °C to 1400 °C, the voltage drop decreases from ≈ 3.5 V to 2.5 V and the duration for reaching equilibrium increases from 3 ms to 4.5 ms. Figure 4.15 also shows that the duration of the regions during which equilibrium is established increases as the emitter temperature is increased. As it is seen in Figs. 4.15 - 4.17,

Figure 4.16. Effect of increasing Ba Pressure on Discharge Current and Forward Voltage Drop in the Breakdown Mode.

Figure 4.17. Effect of Increasing Cs Pressure on Discharge Current and Forward Voltage Drop in the Breakdown Mode.

the duration of this equilibrium regime depends not only on the emitter emission but also on the Cs vapor pressure. Increasing the emitter emission (i.e. high T_E and/or P_{Ba}) and/or Cs vapor pressure increases the discharge time before equilibrium conditions in the gap and lowers the corresponding voltage drop in the discharge region.

The dependencies on the emitter temperature and Ba pressure of the discharge current and forward voltage drop in the discharge region at equilibrium conditions are shown in Figs. 4.18(a) and 4.18(b). The data delineated in Fig. 4.18 is taken at the point corresponding to 10 ms on the time scale in Figs. 4.15 - 4.17. The forward voltage drop values shown in Fig. 4.18(a), correspond to the discharge current in Fig. 4.18(b) at the same T_E/T_{Ba} . In Fig. 4.18(a) the forward voltage drop is not the lowest at the given emitter temperature, and Ba and Cs vapor pressures, but rather that measured for a fixed external load and source voltage. It is possible to obtain a lower voltage drop at given T_E , P_{Cs} , P_{Ba} , and I_C by changing the external load. For example, at the beginning of the curves in Figs. 4.15 - 4.17 when the discharge current is below the maximum thermal emission current, the voltage drop is almost constant. Nevertheless, it is possible to evaluate the influence of emission current and Cs vapor pressure on the voltage drop utilizing the data in Figs. 4.18(a) and 4.18(b) and the data is based on Figs. 4.15 - 4.17. As these figures show the voltage drop decreases as the Cs vapor pressure increases.

Although the data in Fig. 4.18 is not directly applicable to current modulation, the effects of Cs pressure and emission conditions (T_E and P_{Ba}) on the voltage drop in the tacitron are expected to be similar. It is important to recognize that for a given grid geometry, stable current modulation is possible at definite combinations of the discharge current and Cs vapor pressure. In order to receive the lowest voltage drop, the discharge current, which can be controlled by the

Figure 4.18. Effect of T_E/T_{Ba} Ratio on the; a) forward voltage drop, and b) discharge current.

external load should be kept below the maximum thermal emission current; this is the region in Figs. 4.15 - 4.17 where the voltage drop is independent of the discharge current. The appropriate Cs vapor pressure is that resulting in highly ionized plasma in the discharge region at a given discharge current. For example, it is possible to reduce the voltage drop in the tacitron during stable current modulation from 6 - 8 V to 3 - 3.5 V by fulfilling the aforementioned two conditions, namely: (a) operating at a discharge current that is lower than the emission current, and (b) maintaining a Cs pressure that is high enough for ignition to occur when a positive pulse is applied to the grid. This Cs pressure, however, should also be low enough to induce close to a fully ionized plasma in the discharge region so that the discharge will extinguish when a negative pulse is applied to the grid. While the first condition reduces the voltage drop during discharge, the second condition should be fulfilled in order to maintain stable current modulation.

4.3 Current Modulation

This section describes the current modulation characteristics of the Cs-Ba tacitron. It includes a description of past extinguishing mechanisms that have been described for low pressure Cs discharges, along with a general hypothesis of the extinguishing mechanism. The present device extinguishes under two conditions; self-extinguishing and with the application of a negative voltage. In this section an analysis is done on the conditions in the discharge leading to equilibrium (steady-state). The experimental results are compared to the calculations that are described in Chapters 5 and 6.

4.3.1 Extinguishing

In most of the devices described in Chapter 3 current interruption was accomplished by the removal of the collector voltage or overlapping of the grid sheaths (see Fig. 3.9). In the Cs-Ba tacitron, for typical values of the ion density (10^{13} ions/cm³) and electron temperature (8×10^3 K), the sheath thickness given by Eqn. (3.13) is ≈ 0.02 mm, suggesting that the mechanism for extinguishing the Cs-Ba tacitron is different.

The spontaneous current interruption of grid-controlled switching devices filled with Cs-Ba vapor mixtures has been previously studied by Kaibyshev et al. in 1972 and Kaibyshev and Kuzin in 1975. It was shown that the discharge current can be interrupted spontaneously when it reaches a critical value. Several hypothesis have been suggested in the past to explain this phenomena in mercury discharges, which include: (1) a reduction in the neutral gas density at high current densities, which leads to the formation of a charged double sheath at constrictions in the discharge channel, such as a grid aperture (Andrews and Allen 1971 and Lutsenko et al. 1975); (2) a pronounced inhomogeneity within the grid control elements in the discharge volume (Korendo et al. 1974); and (3) a depletion of neutrals due to the removal of ions when the plasma is highly ionized (Langmuir and Mott-Smith 1924, Hull and Elder 1942, and Standgeby and Allen 1971).

At high emission current densities, the plasma in the Cs-Ba tacitron could be highly ionized; therefore, most of the heavy components are ions. During the conduction state of the device the leakage of ions to the surrounding reduces the concentration of heavy components in the gap. Under certain conditions ion leakage should cause the discharge process to become unstable and self-extinguish. If the ionization fraction is high, an instability can be elicited by applying a negative potential to a grid with relatively large apertures. In this case,

in order to successfully extinguish the discharge the on-time should be sufficiently long, which affects the switching frequency of the tacitron. Another factor which affects the switching frequency of the Cs-Ba tacitron during stable current modulation is the off-time needed to replenish the Cs atoms in the interelectrode gap after the discharge has been extinguished. If Cs atoms are not replenished during the off-time to the value necessary to ignite the discharge when a positive pulse is applied to the grid, subsequent discharge ignition would not occur.

4.3.2 Establishment of Discharge Equilibrium Conditions

Following ignition of the discharge, the time required for the concentration of heavy components in the interelectrode gap to reach equilibrium depends on: (a) the flux of Cs ions leaving the gap, (b) the flux of Cs atoms entering the gap, and (c) the amount of Cs atoms in the gap. The latter depends on the Cs vapor pressure in the gap immediately before ignition and the net desorption of Cs atoms from the surfaces of the grid and collector (cold surfaces) during discharge. The initial equilibrium density of Cs atoms in the gap, n_{a0} , may be derived from conditions of equality of fluxes at low Cs vapor pressure, and is given by:

$$n_{a0} = n_{Cs} \sqrt{\frac{T_{Cs}}{T_{gap}}}, \quad (4.1)$$

where T_{Cs} is the cesium reservoir temperature. The cesium vapor density at the reservoir, n_{Cs} , and the average temperature in the interelectrode gap, T_{gap} , are respectively given by:

$$n_{Cs} = \frac{133 P_{Cs}}{k_B T_{Cs}} \text{ and } T_{gap} = \frac{T_E + T_C}{2}, \quad (4.2)$$

where P_{Cs} is the cesium vapor pressure in the reservoir, T_E is the emitter temperature, and T_C is the collector temperature. Also, the flux of Cs atoms entering the gap from the surrounding, Γ_{a0} , is given by:

$$\Gamma_{a0} = \frac{1}{4} n_{a0} \overline{v_a} = \frac{1}{4} n_{Cs} \sqrt{\frac{T_{Cs}}{T_{gap}}} \sqrt{\frac{8 k_B T_{gap}}{\pi M_a}} = \frac{1}{4} n_{Cs} \sqrt{\frac{8 k_B T_{Cs}}{\pi M_a}}, \quad (4.3)$$

where $\overline{v_a}$ and M_a are the thermal velocity and mass of the Cs atoms, respectively.

During discharge, the Cs atom flux from the surrounding into the discharge region remains the same as when the discharge is absent (Eqn. 4.3). The ion flux from the discharge region to the surrounding, Γ_i , can be expressed as:

$$\Gamma_i = 0.61 n_i \sqrt{\frac{k_B T_e}{M_i}}, \quad (4.4)$$

where n_i is the Cs ion density, T_e is the electron temperature, and M_i is the mass of the Cs ion. At equilibrium conditions for a highly ionized plasma the fluxes in Eqns. (4.3) and (4.4) are equal. Through the use of this equality, the equilibrium ion density in the discharge volume, n_i^{eq} , can be calculated as follows:

$$n_i^{eq} = 0.654 n_{a0} \sqrt{\frac{T_{gap}}{T_e}}. \quad (4.5)$$

The ion density in the gap at any time during discharge can also be determined from the measured ion current, I_i , to the grid when a short negative

pulse is applied to the grid as:

$$n_i = \frac{1.64 \times 10^{-2} I_i}{q A_g} \sqrt{\frac{M_i}{k_B T_e}}, \quad (4.6)$$

where q is the electron charge and A_g is the surface area of the grid. In this mode, the grid serves as a probe for determining the plasma conditions in the gap.

The time for reaching equilibrium conditions during discharge, τ_{eq} , depends on the fluxes in Eqns. (4.3) and (4.4) and on the total amount of Cs atoms leaving the gap during the discharge, ΔN_a . This time can be represented as:

$$\tau_{eq} = \frac{\Delta N_a}{(\Gamma_i^m - \Gamma_{a0}) S}, \quad (4.7)$$

where Γ_i^m is the mean ion flux leaving the discharge region and S is the cross-sectional area between the emitter and grid through which the ions leave the discharge region.

The time dependent discharge currents and forward conduction voltage are shown in Fig. 4.18. The negative voltage pulses to the grid are utilized to probe the plasma and measure the ion current in the discharge gap, I_i , which is used to determine the mean plasma concentration in the discharge volume. The voltage pulses to the grid are applied continuously during the entire measurement time (≈ 20 ms), although the pulses are plotted only during the first 2 ms in order to view the ion current. The clipped or constant portion of the grid ion current at the beginning of operation ($t=0$) is due to the limitation of the current by the grid pulse generator. The fact that the grid voltage during the constant portion of the grid current is almost zero, suggests that the ion current is being limited by the circuit.

Assuming an electron temperature, $T_e \approx 1.5 \times 10^4$ K, the equilibrium ion density obtained from Eqn. (4.5) is $n_i = 2.2 \times 10^{13} \text{ cm}^{-3}$. As shown in Fig. 4.18, the measured ion current to the grid at the time of reaching equilibrium is about 3 A, for which $n_i = 2.45 \times 10^{13} \text{ cm}^{-3}$ according to Eqn. (4.6). This ion current should have been close to equilibrium as indicated by the very slow increase in the grid ion current with time (see Fig. 4.19). Indeed, the ion concentration calculated from the measured ion current to the grid at the time of extinguishing ($2.45 \times 10^{13} \text{ cm}^{-3}$) is very close to that calculated above assuming equilibrium and a highly ionized plasma in the interelectrode gap. However, this concentration is more than 3 times lower than that of the Cs atom density in the gap (10^{14} cm^{-3}), before discharge is ignited at the initial Cs pressure of 10^{-2} Torr. As shown in Fig. 4.19 the initial ion current at ignition is about 12 A, which corresponds to an ion density of $9.8 \times 10^{13} \text{ cm}^{-3}$ using Eqn. (4.6). This value coincides with the initial Cs atom density of 10^{14} cm^{-3} , suggesting that the plasma becomes highly ionized very quickly.

It is also possible from the measured ion current to the grid to calculate the approximate time to reach equilibrium during discharge. In Fig. 4.19, I_C , I_g , V_{CE} , and V_g are the collector discharge current, grid current, applied collector voltage, and applied grid voltage, respectively. As shown in Fig. 4.19, after the discharge is ignited, the ion flux to the grid decreases gradually with time. As Fig. 4.19 shows, the ion current at the time the discharge reaches equilibrium is about 3 A, which corresponds to an ion flux of $1.44 \times 10^{18} \text{ ion/cm}^2\text{s}$. This ion current could have been as much as 12 A at the time of ignition, which corresponds to an ion flux of $5.77 \times 10^{18} \text{ ions/cm}^2\text{s}$. These values give a mean ion flux during discharge, $\Gamma_i^m = 3.51 \times 10^{18} \text{ ions/cm}^2\text{s}$. Because this value is much higher than the Cs atom flux from the surrounding region into the gap, the average rate at which the heavy components in the gap decrease during the discharge period or on-time, τ_{on} , is

Figure 4.19. Discharge Current, Forward Voltage Drop, and Ion Current (to the grid) Illustrating the Time to Reach Equilibrium During the Discharge.

equal to $(\Gamma_i^m - \Gamma_{a0})$; which is on the order of 2.26×10^{18} particles/cm²s. Assuming that ΔN_a in Eqn. (4.7) is the difference between the amount of heavy components in the discharge gap before the ignition of the discharge, n_{a0} , and when the discharge is at equilibrium, n_i^{eq} , yields:

$$\begin{aligned}\Delta N_a &= (n_{a0} - n_i^{eq})V \cong (10^{14} - 2.45 \times 10^{13} \text{ cm}^{-3})1.5 \text{ cm}^3 \\ &= 1.16 \times 10^{14} \text{ atoms,}\end{aligned}\tag{4.8}$$

where V is the volume of the discharge region between the emitter and grid. Substituting this value of ΔN_a into Eqn. (4.7) yields a characteristic time $\tau_{eq} = 100$ μ s. As indicated in Fig. 4.19 (also see Fig. 4.6), this time is an order of magnitude lower than measured ($\approx 2 - 3$ ms). Therefore, the amount of atoms in the gap, immediately following discharge ignition, must have been significantly greater. The additional Cs atoms in the gap could have been provided by the desorption of Cs atoms from the electrode surfaces.

For a monolayer coating, the surface concentration of Cs atoms is $\approx 2 \times 10^{14}$ atoms/cm². Because of its low adsorption capability to Cs in the presence of a barium film it could be assumed that no Cs atoms are present on the hot emitter surface. The concentration of Cs atoms on the comparatively colder surfaces of the collector and grid decreases proportional to the heavy component density reduction in the gap during discharge. These assumptions give:

$$\begin{aligned}\Delta N_a &\cong (10^{14} - 2.45 \times 10^{13} \text{ cm}^{-3})1.5 \text{ cm}^3 \\ &+ (19 \text{ cm}^2 \times 1.25 \times 10^{14} \text{ atoms/cm}^2) = 2.5 \times 10^{15} \text{ atoms}\end{aligned}\tag{4.9},$$

In Eqn. (4.9) the first term accounts for the net change in the heavy components in

the discharge volume, in the absence of Cs desorption. The second term represents the contribution due to desorption of Cs atoms from the surface of the grid and collector during discharge. Substituting this value of ΔN_a in Eqn. (4.7) increases the time to reach equilibrium during discharge, τ_{eq} , to ≈ 2 ms, which is very close to the measured time of 2 - 3 ms in Fig. 4.19. Therefore, it is concluded that the source of Cs atoms in the gap, immediately following the initial ignition of the discharge ($t=0$), is not only the volume of the gap, but also desorption of atoms from the grid and collector surfaces.

4.3.3 Influence of the Voltage Drop on Equilibrium Conditions

Low voltage drop is indicative of lower energy dissipation in the plasma and therefore, lower electron temperature, resulting in a lower ion flux from the gap (Eqn. 4.4). This lower ion flux, increases the time for establishing equilibrium (Eqn. 4.7). In addition, increasing the Cs pressure, increases the amount of Cs atoms into the gap from the surrounding regions, hence increasing the time to reach equilibrium in the gap. This effect is shown well in Figs. 4.15 - 4.18. The comparison of discharge current and corresponding voltage drop as a function of time at different emission conditions (T_E and/or T_{Ba}) testifies to the significant role the electron temperature has on the amount of heavy components leaving the gap during discharge. At a given Cs pressure (or T_{Cs}), the lower the emission and therefore discharge current, the larger the voltage drop and hence, the electron temperature, which affects the heavy component density in the gap, (refer to Eqn. (4.5)). As Eq. (4.5) indicates, the heavy component density in the gap at equilibrium decreases as the electron temperature increases and/or the Cs vapor pressure decreases. The current and voltage oscillations in Figs. 4.15 - 4.18 at low emission (low emitter temperature and/or low Ba pressure) is indicative of the low

heavy component density in the gap at high electron temperatures. When the Cs vapor pressure in the surrounding space is increased, according to Eqn. (4.5), the equilibrium ion density in the gap increases, causing the plasma oscillations in the discharge region to disappear (see Figs. 4.16(e) and 4.17).

As mentioned earlier, for a given grid geometry, stable current modulation is possible at definite combinations of discharge current and Cs vapor pressure. In order to receive the lowest voltage drop, the discharge current, which can be controlled by the external load should be kept below the maximum thermal emission current. This is the region in Figs. 4.15 - 4.17 where the voltage drop is independent of the discharge current.

4.3.4 Stable and Unstable Current Modulation Regimes

Typical extinguishing and ignition characteristics at a frequency of about 5 kHz are shown in Fig. 4.20; the operating conditions are listed in the figure caption. The five regions of interest in Fig. 4.20 are labeled as: (a) off-state, (b) positive grid potential for ignition, (c) ignition of discharge, (d) on-state, and (e) negative grid potential for extinguishing. The non-square shape of the collector potential (see Fig. 4.20) is due to a 100 Ω resistor at the output of the collector voltage supply. This resistor is installed in order to determine the value of the collector current during the off-state from the measured voltage drop across the resistor. At 40 μ s after applying the positive grid potential, a discharge develops between the emitter and grid. This time is known as the collector-delay-time, τ_{ad} , which is the time necessary for the ignition of the discharge to occur after the application of the positive grid potential. Shortly thereafter the main discharge ignites, refer to point (b'), causing the collector and grid currents (I_C and I_g) to increase, collector potential to decrease (voltage drop in the gap), and grid

Figure 4.20. Typical Extinguishing and Ignition Characteristics During Stable Current Modulation at 5 kHz ($T_E = 1233\text{ }^{\circ}\text{C}$, $T_C = 583\text{ }^{\circ}\text{C}$, $T_g = 585\text{ }^{\circ}\text{C}$, $T_{Cs} = 151\text{ }^{\circ}\text{C}$, and $T_{Ba} = 530\text{ }^{\circ}\text{C}$).

potential to drop to almost zero. As demonstrated in Fig. 4.20, full discharge is achieved after a 15 μs rise-time (τ_{rise}), point (c). Full discharge, point (d), remains on until a negative potential, $V_{g-} = 28 \text{ V}$, is applied to the grid for a time duration of $\tau_{g-} = 5 \mu\text{s}$, point (e). At this point, the induced instabilities interrupt the discharge. The maximum duration of the negative grid potential for this device is 50 μs ($\tau_{g\text{max}}$). Eventually, the discharge is extinguished after a 5 μs fall-time (τ_{fall}), resulting in $I_C = 0$, $V_{CE} = 140 \text{ V}$ and I_g going to zero.

Although operating at a switching frequency of 5 kHz for a single triode is shown to be possible (see Fig. 4.20), the plasma behavior in the tacitron is not well understood. Sometimes, the main discharge is not ignited after applying a positive potential to the grid nor extinguished when a negative potential is applied to the grid (unstable modulation). Representative results of unstable modulation are shown in Figs. 4.21 and 4.22. In Fig. 4.21, the time after extinguishing (off-time, τ_{off}) is not long enough to replenish the Cs atoms. As a result, the discharge does not ignite when the positive grid voltage pulse is applied, but ignites when a second pulse is applied at a later time, point (a). After the second time, the discharge is extinguished successfully, point (b), the discharge ignites at the end of the positive grid pulse, point (c). Shortly thereafter, a negative potential is applied to the grid, but because the discharge has not been on long enough to reduce the amount of heavy components (mostly ions) in the discharge volume, the discharge did not extinguish. It is also seen in Fig. 4.18 that when the on-time is not long enough, the discharge fails to extinguish, point (a), but it extinguishes when a second negative pulse is applied 2-4 ms later, point (b). Figure 4.22 also shows that there is always a delay time for the ignition to occur after applying a positive pulse to the grid, τ_{ad} .

It is seen from Figs. 4.21 and 4.22 that in order to maintain stable modulation the discharge should be kept on for a definite time (on-time, τ_{on}) in

Figure 4.21. Typical Extinguishing and Ignition Characteristics During Unstable Current Modulation Showing Failure to Ignite.

Figure 4.22. Typical Extinguishing and Ignition Characteristics During Unstable Current Modulation Showing Failure to Extinguish.

order to successfully extinguish with a negative pulse to the grid. Also, ignition occurs only with a definite delay time after applying the positive pulse on the grid (τ_{ad}). The on-time to extinguish depends on the discharge current, I_C , electron temperature, T_e , and Cs vapor pressure. The collector delay-time, τ_{ad} , depends on the rate of decrease in the heavy component density in the gap during discharge, the magnitude of the positive voltage pulse to the grid and the Cs vapor pressure. The Cs pressure in the gap decreases after ignition due to loss of heavy components to the surrounding, and is restored after extinguishing due to the diffusion of Cs atoms from the surrounding into the gap. Therefore, the Cs vapor pressure in the discharge gap oscillates between two definite values at a frequency coinciding with the modulation frequency. Extinguishing and ignition are possible when these definite values of the Cs pressure are reached.

The relationship between the on-time and off-time for stable modulation can be expressed in terms of the duty cycle, δ . As previously mentioned, stable modulation of the Cs-Ba tacitron is possible when ignition and extinguishing of the discharge successfully occur when a positive and a negative potential is applied to the grid, respectively. However, during stable modulation, results show that the on-time should be approximately equal to or greater than the off-time of the device, which results in a duty cycle for the tacitron in excess of 50 %. For a given modulation frequency of the grid, f_g , the longer the off-time for ignition the shorter are the on-time and the duty cycle of the device. However, if the on-time is not long enough to lower the ion density in the gap to successfully extinguish the discharge when a negative potential is applied, the device is operating in the unstable modulation mode.

As shown in Figs. 4.23 - 4.26, stable modulation of the tacitron occurs at a duty cycle $\delta \geq 40\text{-}50\%$. At a lower duty cycle, the modulation of the device becomes unstable because it fails to extinguish when a negative potential is applied

Figure 4.23. Effect of Ignition Grid Potential on the Duty Cycle for $T_E = 1200^\circ\text{C}$ and $f_g = 2\text{ kHz}$.

Figure 4.24. Effect of Ignition Grid Potential on the Duty Cycle for $T_E = 1250^\circ\text{C}$ and $f_g = 2\text{ kHz}$.

Figure 4.25. Effect of Ignition Grid Potential on the Duty Cycle for $T_E = 1300^\circ\text{C}$ and $f_g = 2\text{ kHz}$.

Figure 4.26. Effect of Ignition Grid Potential on the Duty Cycle for $T_E = 1250\text{ }^{\circ}\text{C}$ and $f_g = 3\text{ kHz}$.

to the grid. The results in Figs. 4.23 - 4.26 indicate that increasing the Cs reservoir temperature beyond 150 °C lowers the grid potential required for ignition, but insignificantly affects the duty cycle threshold value for stable modulation. Also, increasing the emitter temperature increases V_{g+} for ignition and slightly increases the duty cycle threshold for stable modulation ($\approx 60\%$). For $T_E \leq 1250$ °C, the duty cycle for stable modulation is almost independent of the emitter temperature. For example, increasing the emitter temperature from 1200 °C in Fig. 4.23 to 1300 °C in Fig. 4.25 increases the duty cycle threshold from 50 % to 60 %.

As indicated earlier, increasing the Cs reservoir temperature decreases τ_{off} and hence increases the duty cycle for a given V_{g+} , or decreases V_{g+} for ignition at a given duty cycle. Figures 4.23 - 4.26 also show that not only the Cs pressure but also the modulation time affects the values of V_{g+} corresponding to the duty cycle threshold for stable modulation. For example, the data shown in Fig. 4.23 indicates that increasing the Cs temperature from 150 °C to 160 °C decreases V_{g+} that corresponds to the duty cycle threshold for stable modulation from 20 V to as low as 12 V. Such a decrease in V_{g+} is due to the higher Cs atom flux into the gap from the reservoir. A comparison of the data in Figs. 4.24 and 4.26 at $T_E = 1250$ °C show that increasing the applied grid frequency from 2 kHz to 3 kHz slightly lowers the duty cycle threshold for stable current modulation, but increases the corresponding values of V_{g+} by $\sim 20\%$. For example at $T_{Cs} = 150$ °C, when the grid modulation frequency is increased from 2 kHz to 3 kHz, the duty cycle threshold remains almost the same at about 48 %. However, the corresponding value of V_{g+} increases from approximately 20 V to 25 V, respectively. This increase in V_{g+} with f_g is due to the fact that increasing the modulation frequency proportionally decreases the off-time required for ignition, lowering the Cs atom density in the gap at the time of ignition and increasing the value of V_{g+} required for ignition.

In summary, Figs. 4.23 - 4.26 show that increasing either the grid voltage for ignition or the initial Cs pressure, decreases the delay-time, τ_{ad} , and increases both the discharge time and the duty cycle. These results are slightly different at other modulation frequencies and discharge currents, but the general trend is that stable modulation for most regimes is seen at $\delta \geq 50\%$. This similarity in behavior might be due to equal variations in the values of the Cs vapor pressure in the gap corresponding to discharge ignition and extinguishing processes.

4.3.5 Discharge Ignition During Current Modulation

During stable current modulation because the Cs atom concentration in the gap at the moment of applying the ignition pulse to the grid is lower than that needed for breakdown, discharge ignition occurs after a certain delay time (see Fig. 4.21). This delay-time is needed to replenish the Cs atom density in the gap. In Fig. 4.21, the operation in the modulation regime lowers the Cs atom concentration in the gap to the point where a positive grid voltage as high as 22 V would not cause instantaneous breakdown. When a high voltage pulse of 22 V is applied to the grid, small currents can be seen at the collector and the grid, which is an indication of ionization strengthening. The flux of Cs atoms entering the gap does not change whether the discharge is on or off since it is solely controlled by the temperature of the cesium reservoir (Eqn. (4.3)). Conversely, the heavy particle density in the interelectrode gap drops during discharge due to ion leakage to the surrounding region. Therefore, after the discharge is extinguished, the Cs atom density in the gap must recover to a definite value determined by the applied grid potential, otherwise ignition will not occur. The off-time, τ_{off} , required for the Cs atom density in the gap to reach the value required for ignition, n_{ai} , may be expressed as:

$$\tau_{\text{off}} = \frac{\Delta N_a}{(\Gamma_{a0} - \Gamma_a^m) S}, \quad (4.10)$$

where Γ_a^m is the mean atom flux leaving the gap during the discharge and ΔN_a is the increase in the amount of Cs atoms in the gap during τ_{off} , which is given by:

$$\Delta N_a = (n_{ai} - n_{ae})V_{\text{gap}} + \Delta\sigma_a A_{gc}. \quad (4.11)$$

In Eqn. (4.11), n_{ai} is the Cs atom density needed for ignition during modulation, n_{ae} is the Cs atom density in the gap after extinguishing, A_{gc} is the sum of the surface areas of the grid and collector, and $\Delta\sigma_a$ is the change in the Cs surface atom density on the grid and collector surfaces. The density of Cs atoms on the electrode surfaces is a function of the net adsorption/desorption of Cs atoms onto these surfaces during the off-time. If it is assumed that the surface density at the operating temperature of the grid and collector is equivalent to a monolayer coating of cesium (2×10^{14} atoms/cm²), then $\Delta\sigma_a$ can be given by:

$$\Delta\sigma_a \approx 2 \times 10^{14} \left(\frac{n_{ai} - n_{ae}}{n_{a0}} \right) \text{ atoms/cm}^2. \quad (4.12)$$

Also, Γ_a^m is given by:

$$\Gamma_a^m = \frac{1}{4} v_a \left(\frac{n_{ai} + n_{ae}}{2} \right) \approx \frac{1}{4} v_a n_{ai}. \quad (4.13)$$

Substituting Eqns. (4.11) through (4.13) into Eqn. (4.10) gives:

$$\tau_{\text{off}} = (n_{\text{ai}} - n_{\text{ae}}) \left\{ \frac{V_{\text{gap}} + \frac{2 \times 10^{14}}{n_{\text{a0}}} A_{\text{gC}}}{\frac{S v_a}{4} (n_{\text{a0}} - n_{\text{ai}})} \right\}. \quad (4.14)$$

The first term in the numerator is the net change in the amount of Cs atoms in the gap during the off-time, and the second term accounts for the net adsorption/desorption of Cs atoms onto/from the surface of the grid and collector during the off-time. From Eqn. (4.14), n_{ai} can be expressed as:

$$n_{\text{ai}} = \frac{S n_{\text{a0}} \tau_{\text{off}} + 4 n_{\text{ae}} \left(V_{\text{gap}} + \frac{2 \times 10^{14}}{n_{\text{a0}}} A_{\text{gC}} \right)}{S \tau_{\text{off}} v_a + 4 V_{\text{gap}} + \frac{8 \times 10^{14}}{n_{\text{a0}}} A_{\text{gC}}}. \quad (4.15)$$

If the net adsorption of Cs atoms onto the electrode surfaces during the off-time is negligible, then $\Delta\sigma_a$ could be set equal to zero, and Eqns. (4.14) and (4.15) are reduced, respectively, to:

$$\tau_{\text{off}} = \frac{(n_{\text{ai}} - n_{\text{ae}}) V_{\text{gap}}}{\frac{S v_a}{4} (n_{\text{a0}} - n_{\text{ai}})}, \quad (4.16)$$

and

$$n_{\text{ai}} = \frac{S n_{\text{a0}} \tau_{\text{off}} v_a + 4 n_{\text{ae}} V_{\text{gap}}}{S \tau_{\text{off}} v_a + 4 V_{\text{gap}}}. \quad (4.17)$$

Equations (4.14) and (4.16) give the off-time required for successful ignition at a certain Cs reservoir temperature and applied grid potential, with and without the effects of Cs adsorption, respectively. Equations (4.15) and (4.17) give the

required Cs density in the gap for successful ignition as a function of the measured operating conditions (P_{Cs} and T_{gap}) and the off-time, with and without the effect of Cs adsorption onto the cold electrode surfaces, respectively. To ascertain the effect of the Cs adsorption on the off-time during stable current modulation, the calculated value of n_{ai} is compared with the values determined from the measured Cs pressure in the gap during the breakdown mode of operation of the device.

The P_{gapd} values in Figs. 4.9 and 4.10 directly correspond to the densities of Cs atoms, n_{a0} , required to successfully ignite the Cs-Ba tacitron in the breakdown mode, at a given grid voltage. To determine the effect of the net Cs adsorption occurring during the off-time of stable current modulation, the value of n_{ai} calculated from Eq. (4.15) and (4.17) are compared to these values corresponding to the Cs pressure for ignition in the breakdown mode from Fig. 4.9. To achieve this, the off-time during current modulation is measured, and the results are shown in Fig 4.27. As can be seen in Fig. 4.27, the off-time required for ignition during stable current modulation decreases as the Cs pressure is increased. For V_{g+} less than 20 V ($\tau_{off} \approx 200 \mu s$), a large jump in the off-time occurs because current modulation becomes unstable; whereas $f_m < f_g$. During unstable current modulation, ignition of the discharge occurs intermittently when a positive potential is applied to the grid, but the discharge fails to extinguish when a negative potential is applied to the grid, resulting in a longer off-time (see Fig. 4.21). Based on the results presented in Fig. 4.27, it is concluded that a necessary condition for the calculation of n_{ai} at $f_g = 3$ kHz is that $\tau_{off} \leq 200 \mu s$.

Figure 4.28 shows the calculated and measured P_{gapd} values for ignition in the stable current modulation and the breakdown mode, as a function of V_{g+} for $T_{Cs} = 155^\circ C$, $f_g = 3$ kHz, $T_E = 1200^\circ C$, and $T_{Ba} = 500^\circ C$. The calculated values are obtained from the stable current modulation off-times measurements (see Fig. 4.26). As can be seen in Fig. 4.28, the calculated values of P_{gapd} for ignition

Figure 4.27. Effect of Cs Pressure on the Off-Time During Current Modulation.

Figure 4.28. Comparison of P_{gapd} Values for Ignition in the Breakdown and Current Modulation Modes for $T_{Ba} = 500^\circ\text{C}$.

during stable current modulation, with Cs adsorption (Eqn. (4.15)), are approximately 0.003 mm-Torr and almost independent of V_{g+} . Also it can be seen that these P_{gapd} values are far from the measured P_{gapd} values for ignition. On the other hand, using the formulation without a net Cs adsorption during the off-time (Eqn. (4.17)), the calculated P_{gapd} values lie close to those from the breakdown mode. Such close agreement suggests that the effects of Cs adsorption, if any, insignificantly influences the off-time required to restore the Cs density in the gap in order for ignition to occur during stable current modulation. Similar results are shown in Fig. 4.29 where adsorption of Cs during the off-time does not change by increasing the Ba reservoir temperature up to 530 °C.

In summary, the operation characteristics of the Cs-Ba tacitron as a switch are investigated experimentally in three modes: (a) breakdown mode, (b) I-V mode, and (c) current modulation mode. The switching frequency, grid potentials for ignition and extinguishing of discharge, and the Cs pressure and emission conditions (Ba pressure and emitter temperature) for stable current modulation are determined. The experimental data are also used to determine the off-time required for successful ignition, and the effects of the aforementioned operational parameters on the ignition duty cycle threshold for stable modulation. Based on the agreement between analysis and measurements it is concluded that at the onset of instability, leading to discharge extinguishing, the plasma in the discharge gap is highly ionized and equilibrium exists between the ion flux leaving the gap and the Cs atom flux entering the gap. Experimental results demonstrate that in the initial discharge ignition, equilibrium occurs within several milliseconds, depending on the leakage of the ions from the discharge gap and the desorption of Cs atoms from the electrode surfaces. The time to reach equilibrium depends not only on the current density and Cs vapor pressure, but also on the voltage drop during discharge (higher voltage drop increases the electron temperature resulting in a

Figure 4.29. Comparison of P_{gapd} Values for Ignition in the Breakdown and Current Modulation Modes for $T_{Ba} = 530^\circ\text{C}$.

lower time for reaching equilibrium).

Because the density of heavy component in the discharge gap during current modulation is lower than at the moment of initial discharge ignition, the measured voltage drop in the I-V mode is lower than that during current modulation, at the same Cs reservoir temperature. The voltage drop strongly depends on the Cs pressure and to a lesser extent on the emission conditions. Increasing the Cs pressure and/or the emission current lowers the voltage drop. However, if the discharge current is kept lower than the emission current, the voltage drop during stable current modulation is constant and could be as low as 3 V.

Results show that discharge extinguishing during current modulation occurs as a result of the decrease in the heavy component density in the gap during the discharge. Therefore, a definite correlation exists between the discharge current density, Cs vapor pressure, off-time, and on-time. The off-time required to replenish Cs atoms in the gap for subsequent ignition depends on the amplitude of the positive grid pulse, while the ignition delay-time depends on both the off-time and the magnitude of the positive grid potential for ignition. In general, stable current modulation occurs at a device duty cycle $\geq 45-55\%$. Such a duty cycle is almost independent of the modulation frequency of the grid, but increases as the grid potential for ignition or the Cs pressure is increased. Experimental results show that the net effect of Cs adsorption/desorption on the off-time for ignition during stable current modulation or ignition in the breakdown mode is insignificant. These findings suggest that during discharge, the ion flux to the surface of the electrodes does not practically influence their coverage by Cs atoms. Results also indicate that the off-time required for ignition is strongly dependent on the Cs pressure, and to a lesser extent on the emitter temperature, but it is almost independent of the Ba vapor pressure.

4.4. Inverter Using Two Cs-Ba Tacitrons

The inverter assembly consists of two Cs-Ba tacitrons operating in a push-pull configuration, a grid modulation circuit, and a step-up transformer circuit as shown in Fig. 4.30. The ratio of the primary to secondary windings is 1:10. This allows the voltage to be stepped up and the current stepped down. During modulation, discharge is ignited in one tacitron by applying a short duration, positive potential pulse to its grid. At the same time, discharge is extinguished in the second tacitron by applying a negative potential pulse to the grid, or by rapidly lowering the collector potential below that necessary for sustaining the discharge. This process is repeated in a reversed order generating an alternating low voltage/high current signal of square pulses. This signal is then converted to a high voltage/low current ac signal via the use of a step-up transformer. The performance parameters of the inverter are directly related to the current modulation characteristics of the tacitrons (i.e. efficiency, frequency, duty cycle, etc.) as well as the current interruption method used to extinguish the discharge. Depending on the latter, the inverter can be operated in one of two modes: the thyatron (forced commutation) mode or the tacitron (self commutation) mode. In the first mode, the discharge in the Cs-Ba switches is ignited by applying a positive potential pulse to the grids, but the current is interrupted by lowering the collector potential below that necessary for sustaining the discharge through the use of reactive components in the inverter circuit (i.e. capacitors or inductors, see Fig. 4.31). The semiconductor thyristors and gas discharge thyatrons operate in this mode.

The disadvantages of this type of inverter circuit are the large power dissipation in the reactive element and the relatively low modulation frequency. However, the advantage of operating in the thyatron mode is that stable

Figure 4.30. Inverter Circuit Using Two Cs-Ba Tacitrons.

modulation can occur at high Cs vapor pressures, resulting in lower voltage drop in the tacitron switch ($\sim 2 - 2.5$ V). Another advantage of operating in the thyatron mode is that the operation characteristics of the inverter are insensitive to changes in the Cs vapor pressure in the tacitron (5-20 mTorr). Fig. 4.31 shows the circuit used in the present work in the thyatron mode. The value of the capacitance is defined by the time necessary to interrupt the current in the switch that was initially on. In order to maintain the time it takes to discharge the capacitor, but use a smaller capacitance, an inductor is placed between the voltage source and the emitters of the triodes (see Fig. 4.31(d)). Also, to reduce the power required to recharge the capacitor, diodes are placed in parallel with the tacitrons, which prevent the reverse current from flowing through the voltage source and the leads of the transformer. When the inverter is operated in the tacitron mode, discharge extinguishing is induced by applying a negative pulse to the grid. The semiconductor transistor and gas discharge tacitron operate in this mode. The inverter circuit used for operating in the tacitron mode is the same as that for operating in the thyatron mode, but without the capacitor (see Fig. 4.31). In this mode of operation, the absence of high power reactive components in the inverter circuit enables operation at higher modulation frequencies, and the slightly larger voltage drop in the tacitron can be compensated for by the absence of power dissipation in reactive components, as in the thyatron mode.

4.4.1 Cs-Ba Tacitron Inverter Operating in the Thyatron Mode

In the thyatron mode inverter, stable modulation was observed for Cs pressures ranging from 5×10^{-3} Torr to 2×10^{-2} Torr. Typical plots of the current and voltage characteristics are shown in Figs. 4.32(a) and 4.32(b). These figures show the collector and grid voltages and currents for the top triode section (Fig.

Figure 4.31. Cs-Ba Tacitron Inverter Circuit for the Thyatron Mode Operation.

4.32(a)) and for the bottom triode section (Fig. 4.32(b)). At almost the same time the discharge in the bottom tacitron was ignited, which is indicated by the increase in the grid and collector currents, the collector voltage for the top section increased from 3 to 40 volts, causing the discharge to extinguish. As the discharge was ignited in the bottom triode, (Fig. 4.32(b)) the collector current initially increased to a value (> 36 A) that is greater than the steady-state current, which is determined by the inverter circuit (~ 15 A). This high current spike is the sum of the current passing through the primary lead of the transformer and the current for recharging the capacitor. These figures also show that at the time of discharge extinguishing in the top section, a voltage spike occurs simultaneously on the collector of the top section as a result of the self-induction of the transformer. These overshoots, in the collector current and voltage, are typical of the transient response of inverter circuits that utilize capacitors or inductors. After the voltage and current spikes subside, the collector current and voltage characteristics become similar to those seen in a single tacitron when operated as a switch, without the inverter circuit connected. The demonstrated operating characteristics of the Cs-Ba tacitron inverter in the thyatron mode are discharge currents up to 20 A, collector voltage of up to 32 V, voltage drop of about 2.5 V, and a modulation frequency up to 10 kHz. Typical results of the voltage and current to the load (secondary of the transformer) in the thyatron mode of operation are shown in Figs. 4.33(a) and 4.33(b), at a modulation frequency of 2.7 kHz and 10 kHz, respectively. It can be seen that the shapes of the voltage and current curves are close to a square wave even at frequencies as high as 10 kHz.

Figure 4.32. Measured Voltage and Current Signals for the Tacitron While the Inverter is Operating in The Thyatron Mode: (a) top tacitron, (b) bottom tacitron.

Figure 4.33. Measured Voltage and Current Signals at the Output of the Step-Up Pulse Transformer While the Inverter is Operating in the Thyatron Mode: (a) $f_g = 2.7$ kHz, and (b) $f_g = 10.6$ kHz.

4.4.2 Cs-Ba Tacitron Inverter Operating in the Tacitron Mode

In the tacitron mode of inverter operation, the frequency characteristics are better than in the thyatron mode, which can be seen by comparing the rise-times and fall-times for each mode of operation. For tacitron mode of operation, a comparison is made between tacitrons with large (0.8 mm diameter, 1.0 mm thickness) and small (0.4 mm diameter, 0.5 mm thickness) aperture grids. However, in the tacitron mode, stable modulation is observed at low Cs vapor pressures resulting in a higher voltage drop than in the thyatron mode.

Large Aperture Grid

As shown in Figs. 4.34(a) and 4.34(b), at a modulation frequency of about 4 kHz, the voltage drop in each tacitron is about 3 - 3.5 V. Fig. 4.35 also demonstrates that at a modulation frequency of 10 kHz, the shapes of the current and voltage wave forms at the load have the same square wave appearance as in the thyatron mode (see Fig. 4.33), but with much steeper rise and fall characteristics. This reduction in the rise and fall times, when operating in the tacitron mode, is more apparent when operating at higher frequencies. The highest frequency measured in the tacitron inverter mode was 20 kHz; this maximum was due to limitations of the grid pulse generator, not in the tacitrons. As seen in Fig. 4.35(b), even at this high frequency, the shape of the current and voltage pulses on the load is close to square waves.

In the test device, because the Cs reservoir is closer to the bottom tacitron, there is a difference in the Cs vapor pressure in the discharge gaps of the top and bottom tacitrons, which makes it difficult to find a Cs reservoir temperature that allowed stable current modulation in both devices. This difference in the Cs vapor pressure in each section, at the same reservoir temperature, could be caused by a

Figure 4.34. Measured Voltage and Current Signals for the Tacitron While the Inverter is Operating in The Tacitron Mode: (a) top tacitron, (b) bottom tacitron.

Figure 4.35. Measured Voltage and Current Signals at the Output of the Step-Up Pulse Transformer While the Inverter is Operating in the Tacitron Mode: (a) $f_g = 10.4$ kHz, and (b) $f_g = 20$ kHz.

strong Cs adsorption by the construction materials. Therefore, depending on the temperature in the different tacitron sections and the time during which current modulation was maintained, the Cs pressure in the inter-electrode gaps changed with time and was usually larger in the bottom section.

Small Aperture Grid

Results showing the inverter operating at current modulation frequencies of 6 kHz and 20 kHz are shown in Figs. 4.36 - 39. Figure 4.36 shows the collector voltage and current signals while operating the inverter at a modulation frequency of 6 kHz. As can be seen in the figure, the discharge current is ≈ 13.5 A, the off-state collector voltage is 24 V and the on-state forward voltage drop is 4 V. Figure 4.37 illustrates the voltage and current wave forms at the output of the transformer obtained with an oscillograph. It can be seen that the voltage is approximately 80 V and the current is approximately 1.2 A. Figures 4.38 and 4.39 show the inverter operating at 20 kHz. As can be seen the maximum discharge has been reduced to 7 A in order to modulate at 20 kHz. The off-state and on-state voltages remained about the same as the 6 kHz modulation case. The decrease in the tacitron discharge current causes a similar decrease in the current at the output of the transformer from 1.2 A to 0.8 A as shown in Fig. 4.39.

In summary, experiments were conducted which demonstrated that an inverter can be configured by connecting and operating two Cs-Ba tacitrons in a push-pull mode with alternating grids voltage control. Two modes of operation were investigated: 1) thyatron mode (forced commutation) and 2) tacitron mode (self commutation). In the thyatron mode, the voltage drop was slightly lower because stable modulation occurs at higher Cs vapor pressures. However, the overall efficiency of the inverter may not be lower because of the increased power loss in the reactive elements.

Figure 4.36. Collector Voltage and Current Wave Forms at $f_g = 6$ kHz.

Figure 4.37. Current and Voltage Waveforms at the Output of the Step-Up Transformer at $f_g = 6 \text{ kHz}$.

In the tacitron mode of operation two different grid designs were investigated. It was shown that decreasing the diameter of the grid hole from 0.8 mm to 0.4 mm, increased the primary current from 3.5 A to 13.5 A (increased power to the load). Also, the rise-times and fall-times in tacitron mode of operation are much steeper compared to thyatron mode of operation and the output signal more closely resembles a square wave. The inverter was operated successfully at a collector voltage of 25 V, conduction voltage drop of ~ 4 V, and modulation frequencies up to 20 kHz.

Figure 4.38. Collector Voltage and Current Waveforms at $f_g = 20$ kHz.

Figure 4.39. Current and Voltage Waveforms at the Output of the Step-Up Transformer at $f_g = 20\ \text{kHz}$.

5. A LOW PRESSURE CESIUM-BARIUM DIODE MODEL

In the preceding chapter, the operational characteristics of a Cs-Ba tacitron were outlined and demonstrated. It was shown that the tacitron could operate at high current densities (up to 15 A/cm^2), low forward voltage drops (less than 3 V) and modulation frequencies of up to 20 kHz. Chapter 4 also contained a few calculations predicting the plasma density during the discharge and the time it takes for the discharge to reach equilibrium. Both of these parameters affect the operation of the device. The former determines the forward voltage drop and the latter determines the modulation characteristics of the device. However, these simplified calculations do not include many of the physical processes that occur in a low Cs pressure discharge; most importantly ion generation. Ion generation determines the plasma density and electron temperature; and hence the forward voltage drop in the device. Therefore, in order to understand the processes occurring in the Cs-Ba tacitron, two models are developed. The first is a model of a Cs-Ba diode and the second, which is described in Chapter 6, is a model of a Cs-Ba triode.

The diode model is developed first in order to understand the complex ion generation process of a low pressure Cs discharge. The model is benchmarked by comparisons with experiments conducted on Cs and Cs-Ba diodes, both past and present. Once the physical processes are modeled and understood, the model will be in a triode configuration (grid placed between the emitter and collector) to calculate the conditions in the discharge of the triode under steady-state conditions. The triode model concentrates on investigating the effects of operating conditions and grid transparency on the forward voltage drop in the device, and identify possible near term improvements in the design of the device.

As discussed earlier, in the diode or triode hot cathode emitters at low Cs pressure there is a possibility of the discharge self-extinguishing due to current oscillations in the frequency range of a few kHz to tens of kHz. The appearance and frequency of these oscillations depend on the Cs vapor pressure, current density, and applied voltage between the electrodes. The study of this phenomena is important in understanding the nature of the extinguishing process of the discharge in the Cs-Ba tacitron that utilizes a grid with large holes (> 0.4 mm in diameter). Earlier it was mentioned that self-extinguishing in the Cs-Ba tacitron is linked to leakage of heavy components, mostly ions, out of the discharge volume. This phenomena was also seen by Kaibyshev and Kuzin (1975) in a Cs-Ba device. Furthermore, current interruption in mercury gas discharge devices was seen by a number of investigators (Andrews & Allen 1971, Langmuir & Mott-Smith 1924, and Hull & Elder 1942) and was explained by the "blowing" of neutral atoms from the discharge gap by the electron flux or by an excessive electron energy corresponding to the maximum ionization cross-section.

Electrons "blowing" neutral gas atoms from the discharge is likely at high Cs pressures when elastic scattering is frequent and energy transfer from electrons to neutral atoms is large. However, at low Cs pressure, when the electron mean free path is larger or close to the interelectrode spacing, the above mechanism can not cause a noticeable change in the Cs atom density because there are too few scattering collisions. Furthermore, the electron energy corresponding to the maximum ionization cross-section also can not be achieved in this discharge because the electron temperature is significantly lower than that at the maximum cross-section (< 1 eV compared to 10 eV). Therefore, the processes leading to current interruption in the low pressure Cs-Ba discharge may be different, depending on the operating conditions and discharge medium.

Another process that may could a significant role in current interruption in the low voltage, low pressure Cs discharge, is Cs adsorption/desorption on the electrode surfaces. Since the gap spacing (small discharge volume) is small and the number of Cs atoms which are adsorbed on the electrode surfaces is significantly larger than the number in the gap. Shifting the adsorption equilibrium, either up (more coverage) or down (less coverage), may have a substantial influence on the amount of Cs atoms in the discharge gap, and therefore on the discharge conditions. A process which contributes to the decrease of the Cs atom density in these discharges is the leakage of ions from the gap. Although the effect of this process is still unclear, it could impact future designs. For example, Cs ions can leave the discharge gap and return to the gap as Cs atoms, but the atom flux from the surrounding is constant and controlled by the temperature of the Cs reservoir. As a result of the difference between the Cs ion flux leaving the gap and the Cs atom flux entering the gap, the net amount of heavy components in the gap would decrease. Also, some of the ion flux at the electrode surfaces can recombine to form atoms, which then return to the discharge volume.

In stationary or steady-state discharge conditions, the total atom and ion fluxes from the discharge volume to the electrode surfaces and those to the gap from the electrode surfaces are equal. Any influence Cs adsorption/desorption could have on the amount of Cs atoms in the gap depends on the relationship between the gap and surrounding. In one case, the total atom and ion fluxes to the surface during stationary conditions will be equal to the initial atom flux to the surface before discharge ignition; therefore, the flux from the surrounding region will determine the degree of Cs coverage. For this case, the degree of surface coverage by Cs atoms is constant and this process does not influence the heavy component density in the gap.

In the opposite case, when the discharge gap is not connected with the

surrounding region, the total atom and ion fluxes in the stationary regime will be in excess of the initial atom flux. As a result, the degree of surface coverage by Cs atoms will be larger and the amount of heavy components in the gap will decrease. However, if only the atom flux at the surface contributes to the degree of Cs coverage, the coverage can decrease during the discharge. The analysis of the experimentally measured time to reach equilibrium, described in section 4.4.1, has shown that adsorbed/desorbed atoms play an important role in determining the Cs atom density in the gap and that a substantial amount of Cs atoms must desorb from the surface for such long equilibrium times. Therefore, it is beneficial to develop a model of a low pressure Cs discharge and investigate the phenomenon of heavy particle leakage and Cs atom adsorption/desorption from the electrode surfaces. The model is used to determine the particle densities and electron temperature (forward voltage drop) as a function of input conditions.

A model of a low pressure Cs-Ba diode discharge is developed to calculate the I-V characteristics and discharge parameters in order to understand the physical processes involved. The diode model is developed first in order to understand the complex ion generation process of a low pressure Cs discharge. The model is benchmarked by comparisons with experiments conducted on Cs and Cs-Ba diodes, both past and present. Once the physical processes are modeled and understood, the model will be is in a triode configuration (grid placed between the emitter and collector) to calculate the conditions in the discharge of the triode under steady-state conditions. The triode model concentrates on investigating the effects of operating conditions and grid transparency on the forward voltage drop in the device, and identify possible near term improvements in the design of the device.

5.1 System of Equations

In the low pressure Cs vapor discharge at a plasma concentration of $\approx 10^{13}$ cm^{-3} , the ionization length is on the order of several mm, which leads to a uniform ion generation over the gap (Baksht et al. 1977 and Dyuzhev et al. 1972). The electron temperature and plasma density may also be assumed uniform in the gap because of negligible electron scattering with Cs atoms and ions. As a result it is possible, without solving the transport equations, to assume that the plasma parameters in the gap are uniform. The balance of charged and neutral particles and electron energy are described by the boundary conditions of the discharge volume, namely: near the emitter and collector, and at the plasma-surrounding interface. Figure 5.1 is a schematic of the diode configuration and the assumed potential distribution in the discharge region. As can be seen in this figure, the potential is assumed uniform over the discharge volume between the emitter and collector sheaths. Also shown in Fig. 5.1 are the leakage of ions and electrons from the discharge volume and to the electrode surfaces. It is also assumed that the ion density, n_i , equals the electron density, n_e , and both will be represented by the symbol n throughout the remainder of this work.

The system of equations include ion generation, Cs atom density balance, heavy particle flux balance, electron energy balance, current continuity, potential balance, and Cs adsorption/desorption balance. Each term is discussed in detail in the following sections.

Ion Generation

The ion balance is a function of the rate of ion generation and the rate at which ions are lost from the discharge volume to the surrounding and to the electrode surfaces. The ion generation for the low pressure Cs discharge has the

Figure 5.1. A Schematic of Cs-Ba Diode.

following form:

$$\frac{dn}{dt} = \sum_{k=0}^4 n N_k \langle \sigma v_e \rangle_{k5} - \frac{j_i}{q} \left(\frac{S_E + S_C + S_S}{V} \right), \quad (5.1)$$

where S_E , S_C , S_S are the emitter, collector, and leakage areas, and V is the discharge volume. The first term on the right hand side of Eqn. (5.1) is the rate of ion generation and the second term is the rate of ion leakage. The ion current density, j_i , is:

$$j_i = 0.61 q n \sqrt{\frac{k_B T_e}{M_i}}, \quad (5.2)$$

As can be seen in Eq. (5.2), the ion current density is a function of the electron temperature not the ion temperature. Equation (5.2) is derived from the Bohm sheath criterion at the boundary of the sheaths in the discharge (Chen 1984). The constant in front of the equation (0.61) comes from the assumption that the ion has the Bohm velocity, $\sqrt{\frac{k_B T_e}{M_i}}$, as it enters the electrode sheath and the electron distribution is Maxwellian. Both assumptions are valid for the discharge conditions in the Cs-Ba diode and triode (Chen 1984).

5.1.1 Cesium Atom Ionization

Five states (or energy levels) of the Cs atom from which ions can be created are included in the model, namely: the $6s^2$ (ground state), $6p^2$ (1st excited state), $5d^2$ (2nd excited state), $7s^2$ (3rd excited state) and $7p^2$ (4th excited state). These correspond to $k = 0, 1, 2, 3$, and 4 , respectively. A diagram of the Cs atom energy levels included in this model is shown in Fig. 5.2 (Moore 1958). Figure

Figure 5.2. Energy Level Diagram for the Cs Excited States Included in the Model.

5.2(a) shows the energy of the level, the wavelength of the optically allowed transitions, λ_t , and the higher lying states which are excluded from the ion generation process. Also shown in this figure is the energy level for the Cs ion, which corresponds to $k=5$. Later it is shown that very little information is lost by excluding the upper states of the Cs atom from the ion generation process. Since the states with different magnetic spins are very close to each other with respect to energy, they are collapsed into one state as shown in Fig. 5.2. This makes the calculation simpler because only five different Cs level densities need to be calculated instead of eight.

The assumption of a Maxwellian electron distribution allows for electron impact excitation, superelastic de-excitation, and ionization rate factors to be calculated which are a function of the electron temperature only. Rate factors are the mean value between the cross-section, σ , and the velocity of the electron, v_e . In general, collision cross-sections are dependent on the type of gas, type of interaction, and energy of the electron. Since a Maxwellian electron distribution is assumed in the model, some discussion is presented in the next section.

5.1.2 Maxwellian Electron Distribution and Rate Factors

A Maxwellian electron distribution, $f(\epsilon)$, is a function of energy and is written as (Chen 1984):

$$f(\epsilon) = \frac{2}{\sqrt{\pi}} \left(\frac{1}{k_B T_e} \right)^{3/2} \epsilon^{1/2} \exp \left(\frac{-\epsilon}{k_B T_e} \right), \quad (5.3)$$

where ϵ is the energy of the electron in joules. Figure 5.3 is a plot of Eqn. (5.3) which shows the distribution reaches a peak (the peak depends on the plasma

Figure 5.3. Plot of the Maxwellian Electron Distribution Function.

density), and then falls off exponentially. The peak of the electron distribution occurs at $0.5k_B T_e$ as shown in the Fig. 5.3. This distribution is important because the peak of the collision cross-sections for Cs occurs at $\approx 8-10$ eV (see Fig. 5.4), which is much greater than $0.5k_B T_e$ (typically 0.3 - 0.5 eV for the low pressure Cs discharge). There has been both experimental and theoretical work done to determine the cross-sections for the transitions of the Cs atom shown in Fig. 5.2. The cross-section is different depending on type of interaction (excitation, superelastic de-excitation, or ionization collision) and on the energy difference between the levels of the transition (level j to level k). In total there are a total of 21 different cross-sections that are included in the model. They include 10 excitation, 6 superelastic de-excitation, and 5 ionization cross-sections. There are fewer de-excitation cross-sections because only the transitions that are optically allowed are considered for superelastic de-excitation. Superelastic de-excitation can occur from the other four transitions ($5d^2$ to $6s^2$, $7s^2$ to $6s^2$, $7s^2$ to $5d^2$ and $7p^2$ to $6p^2$); however, the probability is much lower and the effect on the model results negligible. Figure 5.4 shows the cross-section for electron impact excitation from the ground state ($6s^2$, $k=0$) to the 1st excited state ($6p^2$, $k=1$) and Fig. 5.5 shows the cross-section for electron impact excitation from the ground state ($6s^2$, $k=0$) to the ionization state ($k=5$) from several different sources.

As can be seen in Figs. 5.4 and 5.5, there is a large discrepancy in the cross-section data depending on the source and whether it was experimentally measured or theoretically predicted. Since experimental data is difficult to obtain for all 21 cross-sections, the cross-sections are calculated using the Gryzinski formula (Gryzinski 1958 and Nygaard 1968):

$$\sigma_{kj}(\epsilon) = \frac{q^2}{16\pi \epsilon_0^2 (\epsilon_j - \epsilon_k)^2} \frac{1}{x} \left(\frac{x-1}{x+1} \right)^{3/2} \left\{ 1 + \frac{2}{3} \left(1 - \frac{1}{2x} \right) \ln[2.7 + \sqrt{x-1}] \right\} \quad (5.4)$$

Figure 5.4. Experimental and Theoretical Data of the Direct Electron Impact Cross-Section for Excitation of the 1st Excited State from the Ground State.

Figure 5.5. Experimental and Theoretical Data of the Direct Electron Impact Cross-Section for Ionization from the Ground State.

where $x = \epsilon/(\epsilon_j - \epsilon_k)$, ϵ_j and ϵ_k are the energies of the j^{th} and k^{th} levels of the Cs atom, and ϵ_0 is the permittivity of free space (8.85×10^{-12} F/m). The data using the above formula is shown by the filled squares on Fig. 5.4 and by triangles on Fig. 5.5.

By combining Eqn. (5.3) with Eqn. (5.4), the rate factor can be calculated, which is simply the area under the intersection of the cross-section and distribution function curves as shown in Fig. 5.6. Since the correlation for the cross-sections (Eqn. 5.4) is not readily integratable, a piecewise linear fit is used to obtain a relationship for the cross-section as a function of energy that is used in the following relationship to obtain the rate factors:

$$\langle \sigma v_e \rangle_{kj} = \int_{\epsilon_j - \epsilon_k}^{\infty} \sigma_{kj}(\epsilon) \sqrt{\frac{2\epsilon}{m_e}} f(\epsilon) d\epsilon . \quad (5.5)$$

Each rate factor for electron impact excitation or ionization is calculated from Eqn. (5.5). The electron impact superelastic de-excitation rate factors are calculated from the excitation rate factors assuming a Maxwell-Boltzmann distribution (Verdeyen 1982) as:

$$\langle \sigma v_e \rangle_{jk} = \frac{g_k}{g_j} \langle \sigma v_e \rangle_{kj} \exp\left(\frac{\epsilon_j - \epsilon_k}{k_B T_e}\right) \quad (5.6)$$

where g_k and g_j are the statistical weights (or degeneracy's) for the k^{th} and j^{th} levels. The expressions for all 21 rate factors using Eqns. (5.5) and (5.6) can be found in Appendix D. The only term that remains to be defined in Eqn. (5.1) is the Cs atom densities (N_k 's) which are found by solving the Cs atom density balance for the collision processes just outlined.

Figure 5.6. Intersection of Maxwellian Distribution Function and Cross-Section.

Cs Atom Density Balance

The Cs atom density balance calculates the densities of the Cs ground state and the excited states taking into account electron impact excitation, superelastic de-excitation, ionization and radiation emission and absorption. Recombination is not included because the electron temperature in the Cs-Ba low pressure discharge is on the order of 1 eV (11594 K) and the contribution of recombination is negligible (Gurevich and Pitaevskii 1964). The Cs atom density balance equations are written, respectively as:

(A) Ground State

$$\begin{aligned} \frac{dN_0}{dt} = & n \sum_{j=1}^4 N_j \langle \sigma v_e \rangle_{j0} + \sum_{j=1}^4 N_j A_{j0} - n N_0 \sum_{j=1}^5 \langle \sigma v_e \rangle_{0j} - \\ & \frac{N_0 v_a S_S}{4 V} + \frac{N_{a0} v_a S_S}{4 V} - \frac{N_{mc} S_C}{V} \frac{d\theta_C}{dt} - \frac{N_{mc} S_E}{V} \frac{d\theta_E}{dt} \end{aligned} \quad (5.7a)$$

(B) Excited States

$$\begin{aligned} \frac{dN_k}{dt} = & n \sum_{j=0, j \neq k}^4 N_j \langle \sigma v_e \rangle_{jk} + \sum_{j=1, j > k}^4 N_j A_{jk} - n N_k \sum_{j=0, j \neq k}^5 \langle \sigma v_e \rangle_{kj} - \\ & N_k \sum_{j=0, j < k}^4 A_{kj} - \frac{N_k v_a}{4} \left(\frac{S_E + S_C + S_S}{V} \right). \end{aligned} \quad (5.7b)$$

In these equations, $k = 0, 1, 2, 3, 4$, $\langle \sigma v_e \rangle_{jk}$ and $\langle \sigma v_e \rangle_{kj}$ are the rate factors for electron impact excitation and superelastic de-excitation, A_{jk} and A_{kj} are the probabilities for optical transitions between level j (k) and level k (j), N_0 is the Cs ground state density and N_k 's are the Cs excited state densities.

The probabilities for optical transitions, A_{jk} , are calculated as follows

(Baksht 1978c):

$$A_{jk} = \frac{2 q^2}{4\pi m_e c^3 \epsilon_0} \frac{g_k}{g_j} f_{kj} w_{jk}^2, \quad (5.8)$$

where c is the speed of light (3×10^8 m/s), f_{kj} are the Cs oscillator strengths, and w_{jk} are the frequencies of the transition from level j to level k . The Cs oscillator strengths were measured by Stone (1962) and the frequencies can be calculated from the allowed optical transitions (Moore 1958). Table 5.1 lists the values of A_{jk} for the optically allowed transitions used in the model. Now that the rate factors for each transition are a function of the electron temperature only, it is necessary to balance the amount of energy being deposited into the discharge and being removed from the discharge by both ions and electrons. This balance determines the electron temperature and is discussed next.

Table 5.1. Optical Transitions and Their Corresponding Probabilities for Cs Atoms

Optical Transition, k_{th} to j_{th} level	Transition Probability, A_{kj} (s^{-1})
1 to 0	0.370×10^8
2 to 1	0.001×10^8
3 to 1	0.062×10^8
4 to 0	0.008×10^8
4 to 2	0.077×10^8
4 to 3	0.040×10^8

Electron Energy Balance

The electron energy balance equation includes the energy transfer from the discharge volume to the surrounding region by ion and electron leakage, the energy loss due to radiation, R_r , and the energy loss due to diffusion of excited atoms, R_{ex} . These quantities are the last three terms, respectively, on the right hand side of the following equation:

$$\begin{aligned} \frac{3}{2} k_B V \frac{d(nT_e)}{dt} = & j_{E0} \left(\phi_E + \frac{2k_B T_E}{q} \right) S_E - j_r \left(\phi_E + \frac{2k_B T_e}{q} \right) S_E \exp \left[\frac{-q\Delta V_E}{k_B T_e} \right] \\ & - j_r \left(\phi_E - \Delta V_E + \Delta V_C + \frac{2k_B T_e}{q} \right) S_C \exp \left[\frac{-q\Delta V_C}{k_B T_e} \right] - j_i (\epsilon_i - \phi_E + \Delta V_E) S_E \\ & - j_i (\epsilon_i - \phi_E + \Delta V_E) S_C - j_i \left(\epsilon_i + \Delta V_S + \frac{2k_B T_e}{q} \right) S_S - R_r - R_{ex} \end{aligned} \quad (5.9a)$$

where R_r , R_{ex} , and the random electron current density in the discharge volume, j_r , are given, respectively, as:

$$R_r = q V \sum_{k=1}^4 \sum_{j=0}^{k-1} N_k A_{kj} \epsilon_{kj} \quad (5.9b)$$

$$R_{ex} = \frac{q v_a}{4} (S_E + S_C + S_S) \sum_{k=1}^4 N_k \epsilon_{k0} \quad (5.9c)$$

$$j_r = \frac{q n v_e}{4} \quad (5.9d)$$

In Eqn. (5.9a), T_E is the emitter temperature, ϕ_E and ϕ_C are the emitter and collector work functions, and ΔV_E and ΔV_C are the emitter and collector sheath

potentials. The terms ϵ_{kj} stands for the energy difference between level k and level j .

Current Continuity

The current continuity balances the electron/ion currents at the emitter-plasma boundary, collector-plasma boundary, and the surrounding-plasma boundary (see Fig. 5.1) and is written as:

$$(j_{e0} - j_{eE} + j_{iE})S_E = (j_{eC} - j_{iC})S_C + (j_{eS} - j_{iS})S_S . \quad (5.10a)$$

Substituting the electron and ion current densities in Eqn. (5.10a) yields:

$$\left(j_{E0} - j_r \exp\left[\frac{-q\Delta V_E}{k_B T_e}\right] + j_i \right) S_E = \left(j_r \exp\left[\frac{-q\Delta V_C}{k_B T_e}\right] - j_i \right) S_C + \left(j_r \exp\left[\frac{-q\Delta V_S}{k_B T_e}\right] - j_i \right) S_S \quad (5.10b)$$

Potential Balance

The potential balance determines the applied voltage, V_{CE} , necessary to sustain the discharge (see Fig. 5.1).

$$V_{CE} = \phi_C - \phi_E + \Delta V_E - \Delta V_C. \quad (5.11)$$

Cs Atom Adsorption/Desorption

Adsorption/desorption of Cs atoms can affect the atom density in the gap and hence the discharge conditions. For a monolayer coating of Cs on the electrodes, the coverage, θ , is unity. The following equations show a balance between the Cs atom flux from the collector and emitter surfaces and the Cs atom

and ion fluxes to the collector and emitter surfaces, respectively, as:

$$\frac{d\theta_C}{dt} = \frac{(1 - \theta_C)}{4 N_{mc}} v_a \sum_{k=0}^4 N_k + \frac{j_i(1 - \theta_C)}{q N_{mc}} - \frac{\theta_C}{\gamma_C} \exp\left(\frac{-q H_C}{k_B T_C}\right), \quad (5.12a)$$

$$\frac{d\theta_E}{dt} = \frac{(1 - \theta_E)}{4 N_{mc}} v_a \sum_{k=0}^4 N_k + \frac{j_i(1 - \theta_E)}{q N_{mc}} - \frac{\theta_E}{\gamma_E} \exp\left(\frac{-q H_E}{k_B T_E}\right). \quad (5.12b)$$

In these equations, H_C and H_E are the heat of adsorption for the collector and emitter, and γ_C and γ_E are the average vibration periods of adsorbed Cs particles on the collector and emitter surfaces.

Discharge Volume/Surrounding Boundary Conditions

There are two boundary conditions connecting the discharge volume with the surrounding region (Cs reservoir). The first is a balance between the ions and atoms leaving the discharge volume and the atoms entering the gap from the surrounding. The second is the condition describing the flux of ions and electrons leaving the gap during the discharge.

The heavy particle balance is determined at the boundary between the discharge volume and the surrounding regions. The Cs reservoir temperature controls the initial Cs atom density, N_{a0} , in the gap. The following equation balances the Cs atom source into the discharge gap from the surrounding regions and the electrode surfaces, and the loss of Cs atoms (including excited states) and ions by leakage from the discharge volume to the surrounding regions.

$$\begin{aligned}
\frac{V}{S_S} \left(\sum_{k=0}^4 \frac{dN_k}{dt} \right) + \frac{V}{S_S} \left(\frac{dn}{dt} \right) + \left(\frac{N_{mc} S_C}{S_S} \right) \frac{d\theta_C}{dt} + \left(\frac{N_{mc} S_E}{S_S} \right) \frac{d\theta_E}{dt} \\
= \frac{N_{a0} v_a}{4} - \frac{v_a}{4} \sum_{k=0}^4 N_k - \frac{j_i}{q}
\end{aligned} \tag{5.13}$$

The first and second terms on the left-hand side of Eqn. (5.13) are the atom and ion fluxes leaving the discharge volume to the surrounding, respectively, and the third and fourth terms are the atom fluxes entering the discharge volume by desorption from the collector and emitter surfaces.

It is assumed that the surrounding region is an isolated boundary, where ambipolar diffusion (equal ion and electron currents leaving the discharge) occurs. With this assumption, it is possible to calculate the voltage drop between the discharge and the surrounding region in the following manner:

$$j_{eS} = j_{iS} \tag{5.14a}$$

$$\frac{q n}{4} \sqrt{\frac{8 k_B T_e}{\pi m_e}} \exp\left(\frac{-q \Delta V_S}{k_B T_e}\right) = 0.61 q n \sqrt{\frac{k_B T_e}{M_i}} \tag{5.14b}$$

Solving Eqn. (5.14b) for ΔV_S yields:

$$\Delta V_S = \frac{5.77 k_B T_e}{q} \tag{5.14c}$$

This value of ΔV_S is then substituted into the electron energy balance (Eqn. (5.9a))

This completes the system of equations that are used to model a low pressure Cs-Ba diode. Both steady-state and transient solutions are generated, with the latter being performed as a check for the former. Therefore, the brunt of the analysis for the diode model is the steady-state solutions for the I-V

characteristics and not the transient effects in the diode discharge.

5.2 Steady-State Analysis

An analysis is performed using the steady-state system of equations discussed above. The solution procedure and comparison of the model calculations with data from Cs and Cs-Ba discharges is presented. Also, the ion generation processes involved in the low pressure Cs discharge, and the effect of ion leakage on the discharge conditions leading to self-extinguishing of the discharge are discussed.

5.2.1 Solution Procedure

The steady-state solution is generated by setting the transient terms in Eqns. (5.1) through (5.14) to zero. After eliminating the transient terms and performing a few simple algebraic manipulations, five steady-state expressions are obtained. From the ion generation equation (Eqn. (5.1)) and the heavy particle balance at the discharge volume/surrounding region boundary (Eqn. (5.13)), the following relationships are obtained:

$$\sum_{k=0}^4 N_k \langle \sigma v_e \rangle_{k5} = 0.61 \sqrt{\frac{k_B T_e}{M_i}} \left(\frac{S_E + S_C + S_S}{V} \right), \quad (5.15)$$

and,

$$\sum_{k=0}^4 N_k = N_{a0} - 8\pi(0.61) n \sqrt{\frac{\pi T_e}{T_a}}. \quad (5.16)$$

Combining the current continuity equation (Eqn. (5.14)) with the electron energy balance (Eqn. (5.10)), yields the following expression for the emitter sheath potential:

$$\eta_E = -\ln \left[\frac{\eta_E I_{E0} + B \left\{ \ln \left[\frac{S_E}{S_C} (B - e^{-\eta_E}) \right] - 2 \right\} + 2 I_{E0} \frac{T_E}{T_e} - 0.61(7.77) \frac{S_S}{S_E} \sqrt{\frac{2\pi m_e}{M_i}} - A \frac{q \epsilon_i}{k T_e}}{\eta_E + \ln \left[\frac{S_E}{S_C} (B - e^{-\eta_E}) \right]} \right], \quad (5.17a)$$

where

$$A = 0.61 \sqrt{\frac{2\pi m_e}{M_i}} \left(1 + \frac{S_C}{S_E} + \frac{S_S}{S_E} \right), \quad (5.17b)$$

$$B = I_{E0} + 0.61 \sqrt{\frac{2\pi m_e}{M_i}} \left(1 + \frac{S_C}{S_E} \right), \quad (5.17c)$$

$$I_{E0} = \frac{j_{E0}}{q n v_e / 4}, \quad (5.17d)$$

$$\eta_{E,C,S} = \frac{q \Delta V_{E,C,S}}{k T_e}. \quad (5.17e)$$

Finally, the current continuity and potential balance equations for steady-state conditions are as follows:

$$\eta_C = -\ln \left[\frac{S_E}{S_C} (B - e^{-\eta_E}) \right], \quad (5.18)$$

and,

$$V_{CE} = \phi_C - \phi_E + \Delta V_E - \Delta V_C. \quad (5.19)$$

In these five expressions (Eqns. 5.15-5.19), there are 10 unknowns, consisting of n , T_e , N_0 , N_1 , N_2 , N_3 , N_4 , ΔV_E , ΔV_C , and V_{CE} . However, 4 of these unknowns can be eliminated by using Eqns. (5.7a and 5.7b) to develop relationships between N_0 and N_1 through N_4 . Appendix E outlines the procedure used to develop these

relationships between N_0 and N_1 through N_4 . Using these relationships, the number of unknowns reduces to six since the left-hand sides of Eqns. (5.15) and (5.16) can be rewritten as:

$$\sum_{k=0}^4 N_k \langle \sigma v_e \rangle_{k5} = N_0 \langle \sigma v_e \rangle_{05} + N_1 \langle \sigma v_e \rangle_{15} + N_2 \langle \sigma v_e \rangle_{25} + N_3 \langle \sigma v_e \rangle_{35} + N_4 \langle \sigma v_e \rangle_{45} = (X_{18} + X_{19}) N_0 \quad (5.20)$$

$$\sum_{k=0}^4 N_k = N_0 + N_1 + N_2 + N_3 + N_4 = X_{20} N_0 \quad (5.21)$$

However, there are still 6 unknowns and only 5 equations, the sixth unknown being I_{E0} . Substituting Eqns. (5.20) and (5.21) into Eqns. (5.15) and (5.16), respectively, and replacing n with the expression for I_{E0} (Eqn. 5.17d), yields the following two expressions for N_0 :

$$N_0 = \frac{0.61 \sqrt{\frac{k T_e}{M_i}} \left(\frac{S_E + S_C + S_S}{V} \right)}{\langle \sigma v_e \rangle_i^{\text{eff}}} \quad (5.22)$$

$$N_0 = \frac{1}{X_{20}} \left(N_{a0} - \frac{2\pi (0.61) j_{E0}}{q I_{E0}} \sqrt{\frac{m_e}{k T_a}} \right) \quad (5.23)$$

where the effective ionization rate factor, $\langle \sigma v_e \rangle_i^{\text{eff}}$, replaces $(X_{18} + X_{19})$ only due to its more physical meaning. As seen from Eqns. (5.22) and (5.23), if a value of I_{E0} is chosen, Eqns. (5.22) and (5.23) could be used to solve for the electron temperature. The initial value I_{E0} is not chosen arbitrarily, but is determined from the heavy particle balance (Eqn. (5.23)). If Eqn. (5.23) is expressed as an inequality such that the Cs atom ground state density must always be greater than

zero, it is possible to obtain an inequality for I_{E0} , which has the following form:

$$I_{E0} > \frac{2\pi (0.61) j_{E0}}{q N_{a0}} \sqrt{\frac{m_e}{k T_a}} \quad (5.24)$$

As can be seen in Eqn. (5.24), the minimum value for I_{E0} is a function of the initial Cs atom density in the gap, N_{a0} , and the emission current density, j_{E0} , both of which are inputs to the model. Therefore, to determine the first point of the I-V characteristic, the minimum I_{E0} is calculated and a bisectional iteration is performed on Eqns. (5.22) and (5.23), until the electron temperature that satisfies both equations is found. At this point, the Cs ground state density, the Cs excited state densities, and the plasma density are known. The relationships relating N_0 and N_1 through N_4 , are (refer to Appendix E for the details of the derivation):

$$\begin{aligned} N_1 &= X_{17} N_0, \\ N_2 &= \frac{X_7 + X_8}{X_3} N_0 + \frac{X_{10}}{X_3} N_1, \\ N_3 &= \frac{X_{13}}{X_2} N_0 + \frac{X_{14}}{X_2} N_1, \\ N_4 &= \frac{X_{15}}{X_1} N_0 + \frac{X_{16}}{X_1} N_1. \end{aligned} \quad (5.25)$$

After the electron temperature is determined, a bisectional iteration is performed on Eqns. (5.17) and (5.18) until the value of ΔV_E that satisfies both the electron energy balance and the current continuity is found. Then the collector sheath potential, ΔV_C , is calculated from Eqn. (5.18), the applied voltage, V_{CE} , is determined from Eqn. (5.19). Finally, the discharge current is calculated from either side of Eqn. (5.14). To generate the remaining points of the I-V characteristics, a small value (0.001) is added to the expression for I_{E0} minimum

and the procedure is repeated until ΔV_C is equal to zero. Once ΔV_C reaches zero and an accelerating field for electrons is formed at the collector, the system of equations is no longer valid and another approach must be used to generate the I-V characteristics below this point.

5.2.2 Results and Discussion

Comparisons of the present model are made with experimental data from a low pressure Cs discharge, with no barium, [Baksht et al. 1977] and those from the Cs-Ba device described in Chapters 3 and 4. The model is compared to Baksht et al. (1977) for two reasons: 1) to investigate the assumption that Ba vapor has no or little affect on the transport of the electrons, and 2) more extensive plasma diagnostics were performed which allows for the comparison of other discharge parameters other than just the I-V characteristics.

5.2.2.1 Low Pressure Cs Diode

Results of the diode model are compared in Figs. 5.7 - 5.11 with the theoretical and experimental results obtained by Baksht et al. (1977). The experiments of Baksht et al. 1977 utilized a closed volume, planar diode; therefore, a slight modification is made to the system of equations. The heavy particle balance (Eqn. (5.20)), is replaced with a pressure balance in the discharge volume, which can be expressed as follows:

$$N_{a0} k_B T_a = n k_B T_e + n k_B T_a + \sum_{k=0}^4 N_k k_B T_a \quad (5.26)$$

Also the dimensions of the system had to be changed since the tacitron has a cylindrical geometry and a larger emitter area than the emitter electrode described by Baksht et al. (1977). The inputs to the model obtained from Baksht et al. are emitter temperature, T_E , of 794 °C, collector temperature, T_C , of 226 °C, Cs reservoir pressure, P_{Cs} , of 1.33×10^{-2} Torr (Cs reservoir temperature, T_{Cs} , of 159 °C), interelectrode gap, d , of 1.3 mm, and emission current density, j_{E0} , of 1.0 A/cm². In Figures 5.7-5.11, the solid lines are the model predictions and the symbols are the theoretical and experimental data from Baksht et al. (1977). Since they reported the current density instead of current, j-V characteristics are compared.

The j-V characteristics generated using the present model are shown in Fig. 5.7. For these curves, the collector work function, ϕ_C , by Baksht et al. (1977) and in the present model is assumed to be 1.7 eV. As can be seen, the agreement between the present model and both the model and experimental data of Baksht et al. (1977) is good (within ≈ 3 %). Besides the j-V characteristics, other parameters, such as plasma density, atom density, sheath potentials, and minimum voltage drops, are compared in Figs. 5.8 - 5.11, respectively. Figure 5.8 shows that the present model is actually closer to the experimental results than that of Baksht et al. (1977).

The densities for the five states of the Cs atom considered in this work are shown in Fig. 5.9, with the ground state density calculated by Baksht et al. (1977). Again, there is good agreement with Baksht et al. (1977) calculations. The sheath potentials of the emitter and collector, which determine the voltage drop across the plasma in the diode, are shown in Fig. 5.10. As this figure shows, the present model agrees well with Baksht et al. (1977) calculations; however, both models are generally lower than the experimental data. Finally, the effect of Cs pressure on the minimum voltage drop (voltage at the knee of the j-V curve, see Fig. 5.7) is

Figure 5.7. J-V Characteristics of Present Model Compared to a Low Pressure Cs Discharge [Baksht et al. 1977].

Figure 5.8. Comparison of the Plasma Density.

Figure 5.9. Comparison of the Cs Atom Densities.

shown in Fig. 5.11. At low Cs pressures, the present model agrees more closely with experimental data, but deviates slightly at higher Cs pressures. Fig. 5.11 is important because it shows the minimum voltage drop as a function of Cs pressure. Since the tacitron (triode) is being developed as a switch for power conditioning, the forward voltage drop is an important parameter. As shown in figure 5.11, as the Cs pressure is increased the voltage drop decreases, which is consistent with the increased conductivity of a discharge at higher operating pressures.

The good agreement of the present model with both the theoretical calculations and experimental data of Baksht et al. (1977) validates the soundness of the system of equations used for low Cs pressure, closed volume diodes. Next comparisons are made with the cylindrical Cs-Ba diode.

5.2.2.2 Low Pressure Cs-Ba Diode

In this section, the present model is compared with the experimental data for an open volume, cylindrical Cs-Ba diode. Since the Cs-Ba device has very limited diagnostic capabilities at present, two cases are compared to the experimental results investigating: 1) the effect of changing the Cs pressure, and 2) the effect of changing the Ba pressure (emission current) on the output voltage.

Figure 5.12 compares the calculated and experimental I-V characteristics for the Cs-Ba diode for various Cs pressures. As the Cs pressure increases, the conductivity of the plasma increases, and therefore the forward voltage drop decreases. As can be seen, the model predicts the minimum voltage drop quite well, but there is a large discrepancy at discharge currents above the knee current. The model incorporates normal Schottky effect, which is an increase in the emission current due to the ion flux at the emitter surface and the emitter sheath

Figure 5.10. Comparison of the Emitter and Collector Sheath Potentials.

Figure 5.11. Effect of Cs Pressure on the Minimum Voltage Drop.

Figure 5.12. I-V Characteristics for Cs-Ba Diode for Various Cs Pressures.

potential (see Eqn. 2.16). It can be seen in Fig. 5.12 that the increase in discharge current due to normal Schottky effect is only several amperes; whereas the experimental data has a steep increase in current for small changes in the applied voltage. This large increase in current is due to the anomalous Schottky effect [Baksht 1978] which is not included in the present model.

Changes in the Ba vapor pressure only affect the thermal emission current density of the Cs-Ba diode as shown by the I-V characteristics in Fig. 5.13. Increasing the Ba pressure, insignificantly affects the minimum voltage drop. This result strongly suggests that Ba has little or no effect on the transport properties of the discharge. Again the model deviates from the experiment at currents above the knee (maximum thermal emission current) because anomalous Schottky is not incorporated in the model.

As discussed in section 5.2.1.1, the minimum voltage drop is a very important parameter when defining the uses for the device. One way to reduce the minimum voltage drop is to increase the Cs pressure. Figure 5.14 shows the minimum voltage drop, V_{CEmin} , versus the Cs reservoir temperature for the Cs-Ba diode. As can be seen, the minimum voltage drop begins to reach an asymptote at a Cs reservoir temperature of approximately 200 °C. However, as the Cs reservoir temperature is decreased, the voltage drop increases quite rapidly. This trend is very important because as discussed in Chapter 4, section 2.5, the tacitron (triode) only exhibits stable modulation at Cs reservoir temperatures below 140 °C. The increase in the minimum voltage drop for the triode is discussed in detail in Chapter 6.

As can be seen in Figs. 5.12 and 5.13, there is a point on the I-V curve where the slope changes, this corresponds to the maximum thermal emission current density (knee of the I-V curve). When the discharge current decreases further below the knee, the applied voltage is almost independent of the value of

Figure 5.13. I-V Characteristics for Cs-Ba Diode for Various Ba Pressures.

Figure 5.14. Minimum Voltage Drop versus Cs Reservoir Temperature for the Cs-Ba Diode.

the current. In all three figures, the gap resistance for currents below the knee either has a positive, negative, or constant slope. This condition can occur only with the existence of a "virtual" emitter. With a "virtual" emitter, the voltage during the discharge for currents below the knee on the I-V characteristics is independent of the emitter work function determined by the Ba pressure. Instead it depends on the effective work function ("virtual emitter") and the voltage drop in the plasma at a given discharge current density. Therefore, the slope of the resistance below the knee depends on what has the strongest influence on the voltage drop. On one hand, the applied voltage decreases because the contact potential increases during the growth of the virtual emitter. On the other hand, the applied voltage increases because increasing electron temperature increases the voltage drop in the plasma. If both effects balance, the slope will be constant (see Fig. 4.11).

For the other case with contraction of the discharge (decrease of emitter area), the applied voltage depends only on the emitter-collector work function difference. Since the plasma parameters remain constant during the contraction of the discharge below the knee, the I-V characteristics must always be vertical and not overlap for different emitter work functions. Therefore, the experimental results show that for the Cs-Ba tacitron, the formation of a "virtual" emitter exists at discharge currents below the knee.

In order to determine the effect of anomalous Schottky on decreasing the effective work function, the model is used to determine the effective work function needed for the model calculations to match experimental results. The effective emitter work function is shown on one of the vertical axis in Fig. 5.15. The other axis shows the relative change in the work function from the value at the knee current density. As can be seen, only very small changes in the effective work function are needed for the model calculations to match the experimental results.

Figure 5.15. Effective Emitter Work Function.

Figure 5.16. I-V Characteristics with Effective Emitter Work Function for the Same Conditions as in Fig. 5.12.

The model calculations with the adjusted emitter work functions is shown in Fig. 5.16 for the Cs pressures and conditions shown in Fig. 5.12.

5.2.2.3 Ion Generation Process

As mentioned earlier, in modeling a low pressure Cs discharge the mechanism for ion creation and loss is very important in calculating the discharge conditions, and hence understanding the operation of the device. Therefore, two areas are investigated to gain an understanding of their affect on the ion generation process: 1) the number of excited states included in the model and 2) the effect of radiation emission and absorption.

Effect of Excited States

The first area investigated is the effect of the number of excited states included in the ion generation process. If there are no excited states included in the model, the only mechanism for ion creation is direct electron impact ionization from the ground state. Since the energy difference between the ground state and ion state is 3.89 eV, the cross-section for this interaction is small, which would result in a large forward voltage drop. However, if ions can be created from excited Cs atoms (multi-step ionization) the energy difference between the excited states and ion state diminishes, resulting in a lower forward voltage drop. By comparing the model results with different numbers of excited states included, two questions are investigated: 1) is single-step or multi-step ionization the dominant process and 2) how many excited states must be included to gain sufficient accuracy?

Figure 5.17 shows the I-V curves with several different excited states included in the model. One curve includes the ground state only, the next curve

Figure 5.17. Effect of Number of Excited States on the I-V Characteristics.

includes the ground state and 1st excited state, and so on up to four excited states. The experimental results for the given input conditions are also shown in the figure. As can be seen in Fig. 5.17, the forward voltage drop with single-step ionization is much larger ($V_{CEmin} = 4$ V) than the experimental value ($V_{CEmin} = 1.2$ V). Therefore, at these discharge conditions, single-step ionization can not be the dominant process. The next curve adds the 1st excited state to the ion generation process and as can be seen the improvement on the prediction of the minimum forward voltage drop is quite significant with V_{CEmin} decreasing to 2.3 V. As more states are included in the ion generation process (up to 4), the model calculation approaches the experimental result. Therefore, two conclusions can be made: 1) multi-step ionization dominates over single-step ionization for these discharge conditions, and 2) four excited states are sufficient to accurately model the ionization process for a low Cs pressure discharge and no additional states are necessary.

Effect of Radiation Emission and Absorption

There are many sources of radiation in a discharge; such as, black-body, spontaneous, stimulated, Bremsstrahlung, and so on (Bekefi 1966). The only type that is included in the model is spontaneous emission. This section outlines the probability that the spontaneous emission radiation is lost or absorbed. In order to quantitatively determine the amount of energy that is removed from the discharge, the energy of the photon must be known. However, the emitted radiation is not monochromatic (one frequency), but falls within a finite energy bandwidth. The bandwidth of the emitted radiation can be broadened by a number of mechanisms: 1) natural broadening, 2) resonant broadening, 3) Stark broadening, and 4) Doppler broadening (Baksht 1978 and Yariv 1985). Each type of broadening mechanism is discussed below.

(A) Natural Broadening

In quantum theory it is stated that the properties (i.e. momentum, position, energy) of a particle can not be known exactly. The accuracy with which these properties are known is dictated by the Heisenberg uncertainty principle (Eisberg & Resnick 1985). There are two parts to the uncertainty principle, the first says that the measurement of the position and momentum of a particle can not both be known, but are governed by the following relationship:

$$\Delta p \Delta x \geq \frac{h}{2\pi} \quad (5.27)$$

where h is Planck's constant, and Δp and Δx are the uncertainties in the momentum and position, respectively. The second part of the uncertainty principle deals with the measurement of the energy and the time required for the measurements. For the emission of a photon, the time interval, Δt , that the photon with an energy spread, ΔE , is emitted from an atom is governed by the following relationship:

$$\Delta E \Delta t \geq \frac{h}{2\pi} \quad (5.28)$$

Since the energy of a photon is equal to $h\nu$ (ν is the frequency), and the characteristic lifetime of an excited atom is given by the A-coefficient, the natural broadening linewidth of the radiation spectrum is:

$$\Delta \nu_n = \frac{A_{lk}}{4\pi} \quad (5.29)$$

Even natural broadening can have the affect of increasing the amount of radiation lost by decreasing the absorption coefficient.

(B) Resonant Broadening

Resonant broadening is the broadening of the linewidth due to interaction of like atoms. This is important in a low pressure Cs discharge because the 1st excited state (resonance state) has the highest density and significantly contributes to the ion generation process as was shown in the previous section. It turns out for resonant broadening that the linewidth is a function of the density of excited atoms and is given by (Baksht 1978, Gregory 1942, and Foley 1946):

$$\Delta\nu_r = 2\pi\eta N_k \quad (5.30)$$

where $2\pi\eta$ is a proportionality constant. The values for η were measured by Niemax and Pichler (1974) and are 0.84×10^{-7} and 1.45×10^{-7} , respectively, for the $6s-6p_{1/2}$ and $6s-6p_{3/2}$ transitions.

(C) Stark Broadening

The broadening of the linewidth due to the interaction of excited atoms and charged particles (ions and electrons) is called Stark Broadening. Other than resonant broadening, Stark broadening is the most significant because the plasmas in a low pressure Cs discharge are so highly ionized. It is stated by Griem (1962) that the electrons contribute the most to the broadening of the linewidth due to the Stark effect and the ions contribute only about 20 %. For details of the contributions of each species to increasing the linewidth, refer to the work by Griem (1962). The expression derived by Griem for the linewidth taking into account both ion and electrons is:

$$\Delta\nu_s = \left[1 + 1.75\alpha \left(\frac{n}{n'} \right)^{1/4} \right] \left(\frac{n}{n'} \right) \Delta\nu_e . \quad (5.31)$$

In Eqn. (5.31), $\Delta\nu_e$ is the linewidth due to broadening by electrons, n' is the electron density for the calculations (10^{16} cm^{-3}), and α is the ion broadening parameter. Table 5.1 lists the values of α and $\Delta\nu_e$ for three different electron temperatures. In the model, a linear fit of α and $\Delta\nu_e$ as a function of the electron temperature was incorporated.

Table 5.1. Parameters for Stark Broadening (Griem 1962).

Optical Transition k^{th} to j^{th} level	Electron Temperature, T_e (K)	Ion Broadening Parameter, α	Electron Broadening Linewidth, $\Delta\nu_e$ (GHz)
1 to 0	2500	0.1	1.5
	5000	0.09	1.9
	10000	0.08	2.3
3 to 1	2500	0.16	8.5
	5000	0.14	10.6
	10000	0.12	13.9
4 to 0	2500	0.21	3.5
	5000	0.17	4.5
	10000	0.15	5.3

(D) Doppler Broadening

Doppler broadening arises from the thermal motion of the atoms and the linewidth is given by:

$$\Delta\nu_D = 2\nu_0 \sqrt{\frac{2k_B T_a \ln(2)}{c^2 M_a}} \quad (5.32)$$

where ν_0 is the transition frequency.

The four broadening mechanisms described above can be divided into two groups of lineshapes: 1) Lorentzian, and 2) Gaussian. Natural, resonant, and Stark broadening have Lorentzian lineshapes, and Doppler broadening is characteristic of a Gaussian lineshape. The difference between the two lineshapes is Gaussian

tends to be slightly wider at the wings, which would result in more of the emitted radiation being lost. The absorption coefficient, which determines how much of the radiation is lost, is given by Baksht et al. (1977) for Lorentzian lineshapes as:

$$k_{jk}^{(L)} = \lambda^2 \frac{g_j}{g_k} N_T \frac{A_{jk}}{2\pi \Delta\nu}. \quad (5.33)$$

In Eqn. (5.33), λ is the wavelength of the emitted photon, and $\Delta\nu$ is the linewidth for the given broadening mechanism (natural, resonant, or Stark). Once the absorption coefficient is known, the effective optical deexcitation probability can be found by the following relationship:

$$A_{jk}^{\text{eff}} = x_{jk} A_{jk} \quad (5.34)$$

where x_{jk} is:

$$x_{jk}^{(L)} = \frac{4}{3\sqrt{\pi k_{jk}^{(L)} d}} \quad (5.35)$$

For the case of Doppler broadening (Gaussian lineshape), the absorption coefficient is:

$$k_{jk}^{(D)} = \frac{\lambda^3}{8\pi} \frac{g_j}{g_k} N_T A_{jk} \sqrt{\frac{M_a}{2\pi k_B T_a}} \quad (5.36)$$

and x_{jk} is:

$$x_{jk}^{(D)} = \frac{1}{\sqrt{\pi}} \frac{\sqrt{\ln(k_{jk}^{(D)} d)}}{k_{jk}^{(L)} d} \quad (5.37)$$

The effect of radiation on the I-V characteristic is best understood by plotting two additional cases along with the data shown in Fig. 5.13: 1) no radiation loss (all absorbed), and 2) all radiation lost (none absorbed) which is shown in Fig. 5.18. As can be seen from this figure, most of the radiation is reabsorbed, especially at high Cs pressures; however, at low Cs pressures the voltage drop begins to deviate quite significantly from the curve with no radiation loss. This is due to the decrease in the radiation absorption coefficient. As can be seen from the above equations, as the Cs atom density is increased, the absorption coefficient increases except for the case of resonant broadening. To better understand this effect, the absorption coefficient is plotted versus the Cs reservoir temperature in Fig. 5.19 for the same conditions shown in Fig. 5.18. As shown, the absorption coefficient decreases quite significantly as the Cs pressure is decreased.

Another way of understanding the effect radiation loss has on the discharge conditions is to investigate the effective ionization cross-section. As discussed briefly earlier, the effective ionization cross-section includes all of the processes involved creating ions (excitation, superelastic de-excitation, radiation emission and radiation absorption). Figure 5.20 shows effective ionization cross-section versus the log of the ion fraction which is the ratio of the plasma density, n , and the Cs atom density, N_T , for several Cs pressures at a constant electron temperature ($T_e = 0.6$ eV). The ion fraction is denoted by the symbol β . Also shown in Fig. 5.20 is the effective ionization cross-section without radiation included in the ion generation. At low values of β , the plasma density is very low

Figure 5.18. Effect of Radiation Loss on the Minimum Voltage Drop.

Figure 5.19. Photon Mean Free Path versus the Cs Reservoir Temperature.

Figure 5.20. Effective Ionization Cross-Section versus the Ion Fraction for Several Cs Pressures.

and the atom density high, therefore the dominant broadening mechanism is resonant broadening because the absorption coefficient is independent of the atom density. Recall from Eqn. (5.30) that the linewidth for resonant broadening is proportional to the atom density ($\Delta\nu_r \propto N_T$). Also, since the plasma density is low, the population of the excited states is small and the effective ionization cross-section is close to the direct electron impact cross-section. As β is increased (increased plasma density) Stark and Doppler broadening begin to dominate; however, since the role of radiative transitions diminishes and the effective ionization cross-section increases. A further increase in β beyond one ($\log(\beta) = 0$), again has radiative transitions becoming important because the atom density decreases. The decreasing atom density results in less of the emitted radiation being absorbed and hence the effective ionization cross-section decreases. This same phenomena was described in the work by Baksht et al. (1977). Figure 5.21 shows the effective ionization cross-section for several electron temperatures for a Cs pressure of 10^{-2} Torr ($T_{Cs} = 154$ °C). As would be expected as the electron temperature is increased the peak of the curve increases (recall the peak of the cross-sections occurs between 8 and 10 eV). During the discharge, at a constant Cs pressure, both the ion fraction and the electron temperature change and the radiation significantly affects the voltage drop as was shown in Fig. 5.18.

Equilibrium Ion Density

The equilibrium ion density has implications on both the ignition and extinguishing of the discharge while operating in current modulation mode. Although the ion density at equilibrium will be different for the diode than for the triode, it can be used to study qualitative effects. As discussed in Chapter 4, section 4.1, because the ion fraction is quite high for low Cs pressure discharges, it is assumed that the atom density in the gap after extinguishing of the discharge is

Figure 5.21. Effective Ionization Cross-Section versus the Ion Fraction for Several Electron Temperatures.

approximately equal to the equilibrium ion density. Therefore, the number of atoms (and more importantly delay-time) needed to reignite the discharge, is a strong function of the ion and atom densities following extinguishing.

In section 4.2.2, the argument was made that the equilibrium ion density can simply be calculated by equating the flux of atoms from the Cs reservoir to the discharge gap and the flux of ions leaving the discharge gap. Since the hypothesis to date on the extinguishing mechanism for the Cs-Ba tacitron is strongly linked to the amount of ion leakage during the discharge and therefore the ion density close to equilibrium, a more detailed calculation needs to be made for this density. Using Eqn. (4.5), it is straight forward to plot the equilibrium ion density versus the electron temperature in the discharge gap, which is shown in Fig. 5.22 for several different Cs pressures (darkened circles). As would be expected as the Cs pressure increases, the equilibrium plasma density increases (large atom flux from Cs reservoir). However, from Eqn. (4.5), the electron temperature decreases, (for a given Cs pressure) as the plasma density increases, which contradicts what takes place in a discharge. In a discharge, as the electron temperature decreases, the collision cross-sections decrease quite rapidly (see Figs. 5.4 and 5.5), resulting in a lower rate factor. This lower rate factor would tend to decrease the plasma density.

To further explain this process, the diode model is run for the same Cs pressures and the results are also shown in Fig. 5.22 (lines). As expected, the ion density falls off quite rapidly as the electron temperature decreases resulting in an increase in the Cs atom density. Since the discharge during current modulation operates at electron temperatures around 8000 K ($V_{CE} \approx 2$ V), using the particle balance to determine the plasma density can result in a factor of two error.

Figure 5.22. Equilibrium Ion Density Improvement with Model Calculation.

5.2.2.4. Effect of Ion Leakage on Discharge Self-Extinguishing

From Eqns. (5.22) and (5.23) it is possible to derive an expression that describes the limitations on the maximum value of the random electron flux, which has the following form:

$$\left(\frac{n \bar{v}_e}{4} \right) < \frac{1}{2\pi(0.61)} \sqrt{\frac{k T_a}{m_e}} \left[N_{a0} - 0.61 \sqrt{\frac{k T_e}{M_i}} \left(\frac{S_E + S_C + S_S}{V} \right) \frac{1}{< \sigma v_e >_{i,max}^{eff}} \right] \quad (5.38)$$

As seen by the inequality in Eqn. (5.38), the maximum value of the random electron flux (left-hand side), and therefore the maximum value of the discharge current, depends on the Cs vapor pressure (atom density in surrounding region, N_{a0}) and the maximum value of the effective ionization rate factor $< \sigma v_e >_{i,max}^{eff}$ (see Fig. 5.20). This is because the decrease in the heavy component density in the discharge depends on the difference between the ion flux out of the discharge volume and the atom flux to the discharge volume from the surrounding. Since the ion flux, $0.61 n \sqrt{\frac{k T_e}{M_i}}$, is proportional to the random electron current, j_r , decreasing the Cs pressure causes the maximum value of the discharge current to decrease. The maximum value of the effective ionization cross-section also decreases with decreasing Cs vapor pressure, which can reduce the maximum value of the current even more (Baksht et al. 1977).

It is necessary to note that in the absence of ion leakage from the discharge (e.g. enclosed volume discharge) the limitation on the maximum random electron flux depends solely on the plasma density. Instead of Eqn. (5.23), the ground state atom density, N_0 , would be equal to $\frac{1}{X_{20}}[N_{a0} - n]$ which when substituted into Eqn. (5.22), it is possible to obtain the limitation on the plasma density, n .

In the presence of ion leakage, which is the case in the Cs-Ba diode, the discharge may self-extinguish, even at a constant plasma density. This is because an increase in the applied voltage due to an increase in the electron temperature results in a higher random electron current.

From current continuity equation (5.10), another limitation on the value of the random electron current is derived as follows:

$$\frac{j_{E0}}{e^{-\eta_E} - 0.61 \sqrt{\frac{2\pi m_e}{M_i}} (1 + \frac{S_C}{S_E})} > \frac{q n \bar{v}_e}{4} > \frac{j_{E0}}{\frac{S_C}{S_E} + e^{-\eta_E} - 0.61 \sqrt{\frac{2\pi m_e}{M_i}} (1 + \frac{S_C}{S_E})} \quad (5.39)$$

The left-hand side of the inequality in Eqn. (5.39) is the upper limit for non-zero electron current to the collector. In reality, the limitation shown in the inequality in Eqn. (5.38) is more rigid than that represented by the left-hand side of Eqn. (5.39). The right-hand side of Eqn. (5.39) represents the lower limit of random electron current for a collector sheath potential equal to zero ($\Delta V_C = 0$). Should an accelerating field develop at the collector, the current continuity equation yields the following:

$$\eta_C \frac{T_e}{T_a} = -\ln \left[\frac{\frac{S_C}{S_E} + e^{-\eta_E} - I_{E0} - 0.61 \sqrt{\frac{2\pi m_e}{M_i}}}{\frac{S_C}{S_E} \sqrt{\frac{T_a m_e}{T_e M_i}}} \right] \quad (5.40)$$

From Eqn. (5.40), the following limitation on the random electron current for an accelerating field at the collector is obtained:

$$\frac{j_{E0}}{\frac{S_C}{S_E} + e^{-\eta_E} - 0.61 \sqrt{\frac{2\pi m_e}{M_i}}} < \frac{q n \bar{v}_e}{4} \leq \frac{j_{E0}}{\frac{S_C}{S_E} + e^{-\eta_E} - 0.61 \sqrt{\frac{2\pi m_e}{M_i}} - \frac{S_C}{S_E} \sqrt{\frac{T_a m_e}{T_e M_i}}} \quad (5.41)$$

The left-hand side of the inequality in Eqn. (5.41) is the lower limit of the random electron current for non-zero ion current to the collector. The right-hand side is the upper limit for the random electron current for the existence of an accelerating voltage drop near the collector. As seen from the inequality in Eqn. (5.41), the possible range of random electron currents for an accelerating voltage drop near the collector is very narrow and is practically the value for zero voltage drop at the collector. Therefore, the regime with an accelerating field near the collector is not realized when electron emission from the collector is absent.

The lower limit of Eqn. (5.39), which is for $\Delta V_C = 0$, defines the minimum value of electron random current possible for a given value of emission current and applied voltage. Also, when $\Delta V_C = 0$, the thermal emission, random electron, and discharge currents are practically equal. The system of equations that are being utilized are not valid for discharge currents below this minimum value (herein defined as the thermal emission current). Therefore another method must be used to calculate the current below this minimum value.

In the analysis in Baksht et al. (1977), ion leakage from the discharge was not taken into account and the limitation of the random electron current has the following form:

$$\left(\frac{n \bar{v}_e}{4} \right) < \sqrt{\frac{k T_e}{2\pi m_e}} \left[N_{a0} - 0.61 \sqrt{\frac{k T_e}{M_i}} \left(\frac{S_E + S_C}{V} \right) \frac{1}{< \sigma v_e >_{i,max}^{eff}} \right] \quad (5.42)$$

It is seen as compared with expression (5.38) that the limit value of electron random flux, without taking into account ion leakage from the gap, is significantly larger than that when ion leakage is taken into account. Therefore, the discharge mode at high plasma density may not be realized under normal

operating conditions due to self-extinguishing of the discharge. Finally, by taking the ratio of the maximum random electron current with and without ion leakage, the following expression can be derived:

$$\frac{qn v_e / 4 \big|_{\text{no ion leakage}}}{qn v_e / 4 \big|_{\text{ion leakage}}} \cong 0.61 F \sqrt{\frac{2\pi T_e}{T_a}} \quad (5.43)$$

where

$$F = \frac{N_{a0} - 0.61 \sqrt{\frac{k T_e}{M_i}} \left(\frac{S_E + S_C}{V} \right) \frac{1}{\langle \sigma v_e \rangle_i^{\text{eff}}}}{N_{a0} - 0.61 \sqrt{\frac{k T_e}{M_i}} \left(\frac{S_E + S_C + S_S}{V} \right) \frac{1}{\langle \sigma v_e \rangle_i^{\text{eff}}}} \quad (5.44)$$

The term in the numerator of F is the difference between the initial Cs atom density and the Cs atom density in the discharge with no ion leakage and the term in the denominator is the difference of the initial Cs atom density and the Cs atom density in the discharge with ion leakage. This ratio is never lower than unity. For typical electron temperatures (≈ 8000 K), the ratio of the random electron currents is approximately 4 for F equal to 1.

Figure 5.23 compares the calculated I-V characteristics with ion leakage (open volume) and without ion leakage (closed volume). According to Fig. 5.23, with ion leakage taken into account, self extinguishing of the discharge would occur at significantly lower applied voltages than in the case without ion leakage. The corresponding dependencies of the plasma densities with and without ion leakage are shown in Fig. 5.24. As can be seen the ratio of the plasma densities (random electron currents) is approximately 5. From Eqn. (11), this corresponds to an F value of 1.25 which means the Cs atom density in

Figure 5.23. Effect of Ion Leakage on the Calculated I-V Characteristics.

Figure 5.24. Effect of Ion Leakage on the Plasma Density.

the discharge volume without ion leakage is higher than the Cs atom density with ion leakage.

Since the full system of equations contain transient terms that were set equal to zero for the steady-state analysis, in the next section, a transient solution is generated to check the steady-state solution.

5.3 Transient Diode Model Analysis

A transient solution for the Cs-Ba diode model is generated to verify the steady-state solution and any additional analysis of the transient processes in the discharge is left for future work.

5.3.1 Solution Procedure

Whereas in the steady-state solution the adsorption/desorption of Cs atoms from the emitter and collector surfaces is not important to the solution, it is very important to the transient solution. For the transient model there are 12 unknowns (n , T_e , N_0 , N_1 , N_2 , N_3 , N_4 , ΔV_E , ΔV_C , V_{CE} , θ_E , and θ_C) and only 11 equations. The relationships derived in the steady state solution can not be used to relate N_0 to N_1 through N_4 , instead each rate equation must be solved separately (Eqns. 5.6a-e). Again one of the variables must be assumed in order to generate a solution. Since the transient is being generated to check the steady-state solution, it is easiest to assume the voltage, V_{CE} , and calculate the other discharge parameters. The results of the steady-state at the given V_{CE} can then be compared to the values at equilibrium in the transient. Also, since the emitter is much hotter than the collector, the emitter coverage remains fairly constant; therefore, the equation for θ_E is not included in the calculation. That leaves 10 unknowns and 10

equations for which a solution can be generated.

The solution begins by selecting the initial conditions for n , T_e , N_0 , N_1 , N_2 , N_3 , and N_4 . The initial values for ΔV_E and ΔV_C are calculated after V_{CE} is chosen. Since the model does not predict the ignition of the discharge, a technique had to be chosen to simulate the increase in current as with discharge ignition. It is decided to emulate a "virtual" emitter as discussed in the previous section; however, in the transient solution j_{E0} is set equal to j_r until $j_r > j_{E0}$ at which time j_{E0} is calculated from Schottky emission. Table 5.2 lists the initial conditions used for the transient solution. The solution is not very sensitive to the initial conditions. The following equations are used to calculate the transient solution.

Table 5.2. Initial Conditions for Transient Solution

T_e (K)	n (cm ⁻³)	N_0 (cm ⁻³)	N_1 (cm ⁻³)	N_2 (cm ⁻³)	N_3 (cm ⁻³)	N_4 (cm ⁻³)
11594	10^{11}	N_{a0}	0	0	0	0

$$\eta_E = -\ln \left[\frac{1 + \frac{S_C}{S_E} \exp\left(\frac{-q(\phi_C - \phi_E - V_{CE})}{k_B T_e}\right)}{\frac{4 j_{E0}}{q n v_e} + 0.61 \left(1 + \frac{S_C}{S_E}\right) \sqrt{\frac{2\pi m_e}{M_i}}} \right] \quad (5.45)$$

$$\eta_c = \eta_E + \frac{q(\phi_C - \phi_E - V_{CE})}{k_B T_e} \quad (5.46)$$

$$\frac{dT_e}{dt} = -F_1 T_e + F_2 \quad (5.47)$$

$$\frac{dn}{dt} = -F_3 n \quad (5.48)$$

$$\frac{dN_0}{dt} = -F_4 N_0 + F_5 \quad (5.49)$$

$$\frac{dN_1}{dt} = -F_6 N_1 + F_7 \quad (5.50)$$

$$\frac{dN_2}{dt} = -F_8 N_2 + F_9 \quad (5.51)$$

$$\frac{dN_3}{dt} = -F_{10} N_3 + F_{11} \quad (5.52)$$

$$\frac{dN_4}{dt} = -F_{12} N_4 + F_{13} \quad (5.53)$$

$$\frac{d\theta_C}{dt} = -F_{14} \theta_C + F_{15} \quad (5.54)$$

Refer to Appendix F for the values and description of F_1 through F_{15} . It should be noted that F_1 through F_{15} are not constants, but are highly non-linear functions of combinations of all of the discharge parameters (i.e. n , T_e , etc.). Therefore, F_1 through F_{15} are calculated from the values of the discharge parameters from the previous time step.

5.3.2 Results and Discussion

As a reminder, the objective of the transient solution is to verify the steady-state solution. Comparisons between the two solutions are performed for several cases with different numbers of excited states of the Cs atom included for comparison to the steady-state case shown in Fig. 5.17. The transient solution is

shown in Fig. 5.25 for the same input conditions shown in Fig. 5.17 with an applied voltage of 5 V. Table 5.2 lists several of the discharge parameter values for the steady-state and transient solutions. As can be seen in the table, the steady-state values of the transient solution match the steady-state model quite well for all cases of the number of excited states.

Table 5.3. Comparison of Steady-State and Transient Output Parameters.

	# of Excited States	I (A)	T _e (K)	n (cm ⁻³) x10 ¹³	N ₀ (cm ⁻³) x10 ¹³	ΔV _E (V)	ΔV _C (V)
SS	0	36.07	27338	0.59	7.08	7.47	2.36
Trans	0	36.49	27326	0.59	7.10	7.43	2.33
SS	1	39.50	25757	1.33	1.15	8.85	3.74
Trans	1	39.80	25742	1.35	1.15	8.84	3.74
SS	2	39.80	25835	1.41	0.42	8.99	3.88
Trans	2	40.14	25788	1.46	0.39	9.00	3.91
SS	3	40.05	25727	1.50	0.26	9.07	3.96
Trans	3	40.47	25639	1.56	0.25	9.10	4.01
SS	4	40.14	25675	1.52	0.25	9.09	3.98
Trans	4	40.62	25543	1.61	0.21	9.14	4.04

Figure 5.25. Discharge Current as a Function of Time with Different Cs Excited States Included in the Model at the Same Conditions in Fig. 5.17.

6. A LOW PRESSURE CESIUM-BARIUM TRIODE MODEL

Understanding the operation of the Cs-Ba diode and modeling gave important insight into the physical processes, which is quite useful when modeling the more complicated arrangement of the triode. The triode includes a control electrode (grid) placed between the emitter and collector. Experimental data for both Cs and Cs-Ba triodes has been obtained both past and present as discussed in Chapter 3. The experimental data of the present device was discussed extensively in Chapter 4. Although an abundance of data exists for the Cs-Ba triode for a wide range of operating conditions, a good understanding of the physical processes does not yet exist. Therefore, the development of analytical models is needed to compare with experimental data to help in understanding the discharge processes.

In 1978, Baksht et al. described both experimental and analytical results for a Cs triode device. However, the device described by Baksht differs considerably from the Cs-Ba device being investigated in this work. First and foremost is the difference in the grid designs. As mentioned earlier, the present devices employ a grid with a thickness of 1 mm and hole diameter of 0.8 mm (grids with thickness of 0.5 mm and hole diameter of 0.4 mm were also used). In the experimental device of Baksht et al. (1978a and 1978b), the grid was a wire mesh with a cell size of $180\text{ }\mu\text{m} \times 180\text{ }\mu\text{m}$ and the grid thickness was only $30\text{ }\mu\text{m}$. Also, the device used only Cs vapor and therefore operated at low currents $< 1\text{ A/cm}^2$; whereas, the present device is capable of operating at 15 A/cm^2 . This difference in emission current can result in large differences in the plasma conditions in the discharge regions in the triode, which is discussed later. The difference in the grid design has the most impact on the operation of the two devices and therefore on any modeling that is performed.

The present Cs-Ba triode model calculates the electron temperature, plasma density, Cs atom densities, and sheath potentials for all three regions: 1) emitter-grid gap, 2) grid aperture, and 3) grid-collector gap. Figure 6.1 shows the triode configuration and the assumed potential distribution. As can be seen in the figure, the reservoir is coupled to the triode only through the emitter-grid gap. Therefore, the amount of heavy particles (ions and atoms) that enter regions 2 and 3 are also determined by the Cs atom flux from the reservoir. In the potential diagram, it is assumed that the sheath potential, ΔV_{g1} , accelerates electrons and decelerates ions from region 1 to region 2 and the sheath potential, ΔV_{g3} , accelerates electrons and decelerates ions from region 2 to region 3. As with the diode model, ion generation, electron energy, current continuity, Cs atom density, and potential balances are considered for each region with the appropriate boundary conditions and coupling with adjacent regions.

6.1 System of Equations and Boundary Conditions

The triode system of equations uses the same lumped approach as the diode model with the appropriate coupling between the three regions. The following system of equations are for Region 1 (R_1) between the emitter and grid, Region 2 (R_2) within the grid holes and Region 3 (R_3) between the grid and the collector.

Ion Generation in Region 1

As with the ion generation term for the diode model, the following expression consists of the rate of ion creation, ion leakage; however, there is an additional term of ion flux from Region 2 (R_2) to Region 1 (R_1).

Figure 6.1. Schematic of Triode Configuration and Potential Distribution.

$$\frac{dn_1}{dt} = \sum_{k=0}^4 n_1 N_{k1} \langle \sigma v_{e1} \rangle_{k5} - \frac{j_{ir1}}{q} \left(\frac{S_E + S_g + S_S}{V_1} \right) + \frac{j_{ir2}}{q} \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right) \frac{S_h}{V_1}, \quad (6.1a)$$

where j_{ir1} and j_{ir2} are the random ion fluxes in R_1 and R_2 , respectively and are represented as:

$$j_{ir1} = \frac{q n_1}{4} \sqrt{\frac{8k_B T_{a1}}{\pi M_i}}, \text{ and } j_{ir2} = \frac{q n_2}{4} \sqrt{\frac{8k_B T_{a2}}{\pi M_i}}. \quad (6.1b)$$

The last term on the right hand side of Eqn. (6.1a) is the ion flux entering R_1 from R_2 through the grid holes.

Cs Atom Density Balance in Region 1

Recall from the diode model description that the Cs atom density balance serves two main purposes: 1) to couple the discharge volume in R_1 with the Cs reservoir and 2) to obtain a relationship between the ground state Cs atom, N_0 , and the excited state Cs atoms, N_k . For the transient solution, which is beyond the scope of this work, the adsorption/desorption terms ($d\theta_{g1}/dt$ and $d\theta_E/dt$) and the emitter and grid surfaces could play an important role; however, in the steady-state solution those terms go to zero.

$$\begin{aligned} \frac{dN_{01}}{dt} = & n_1 \sum_{j=1}^4 N_{j1} \langle \sigma v_{e1} \rangle_{j0} + \sum_{j=1}^4 N_{j1} A_{j0} - n_1 N_{01} \sum_{j=1}^5 \langle \sigma v_{e1} \rangle_{0j} - \\ & - \frac{N_{01} v_{a1}}{4} \frac{S_{S1}}{V_1} + \frac{N_{a0} v_{a1}}{4} \frac{S_{S1}}{V_1} - \frac{N_{mc} S_{g1}}{V_1} \frac{d\theta_{g1}}{dt} - \frac{N_{mc} S_E}{V_1} \frac{d\theta_E}{dt} + \frac{N_{02} v_{a2}}{4} \frac{S_h}{V_1} \end{aligned} \quad (6.2a)$$

$$\begin{aligned} \frac{dN_{k_1}}{dt} = & n_1 \sum_{j=0, j \neq k}^4 N_{j_1} \langle \sigma v_{e_1} \rangle_{jk} + \sum_{j=1, j > k}^4 N_{j_1} A_{jk} - n_1 N_{k_1} \sum_{j=0, j \neq k}^5 \langle \sigma v_{e_1} \rangle_{kj} - \\ & - N_{k_1} \sum_{j=0, j < k}^4 A_{kj} - \frac{N_{k_1} v_{a_1}}{4} \left(\frac{S_E + S_{g_1} + S_{S_1}}{V_1} \right) + \frac{N_{k_2} v_{a_2}}{4} \frac{S_h}{V_1}. \end{aligned} \quad (6.2b)$$

The last terms on the right hand sides of Eqns. (6.2a) and (6.2b) represent the ground and excited state Cs atoms entering R_1 from R_2 .

Electron Energy Balance in Region 1

The electron energy balance in R_1 of the triode has several differences from the expression for the diode. Since the grid has a finite transparency, the voltage drop between the plasmas of R_1 and R_2 and between the plasma of R_1 and the grid surface are different. Therefore, there are four energy terms associated with the electron and ion currents from R_1 instead of two as in the diode model (there is only one sheath in front of the collector of the diode). The other difference is in the energy of the electrons and ions being transferred from R_2 to R_1 , which are the last two terms on the right hand side of Eqn. (6.3a).

$$\begin{aligned} \frac{3}{2} k_B V_1 \frac{d(n_1 T_{e_1})}{dt} = & j_{E0} \left(\phi_E + \frac{2k_B T_E}{q} \right) S_E - j_{r_1} \exp \left(\frac{-q \Delta V_E}{k_B T_{e_1}} \right) \left(\phi_E + \frac{2k_B T_{e_1}}{q} \right) S_E \\ & - j_{r_1} \exp \left(\frac{-q(\Delta V_g + \Delta V_g)}{k_B T_{e_1}} \right) \left(\phi_g - V_g + \frac{2k_B T_{e_1}}{q} \right) (S_{g_1} - S_h) - j_{i_1} (\epsilon_i - \phi_E + \Delta V_E) S_{gs} \\ & - j_{i_1} (\epsilon_i - \phi_E + \Delta V_E) S_h - j_{r_1} \exp \left(\frac{-q \Delta V_{g_1}}{k_B T_{e_1}} \right) \left(\phi_E - \Delta V_E + \Delta V_{g_1} + \frac{2k_B T_{e_1}}{q} \right) S_h \\ & - j_{i_1} \left(\epsilon_i + \Delta V_S + \frac{2k_B T_e}{q} \right) S_S - R_{r_1} - R_{ex_1} + j_{r_2} \left(\phi_g - \Delta V_g - V_g + \frac{2k_B T_{e_2}}{q} \right) S_h \\ & + j_{i_2} (\epsilon_i - \phi_E + \Delta V_E) \exp \left(\frac{-q \Delta V_{g_1}}{k_B T_{a_2}} \right) S_h, \end{aligned} \quad (6.3a)$$

where R_{r1} , R_{ex1} , j_{r1} , and j_{r2} are given by:

$$R_{r1} = q V_1 \sum_{k=1}^4 \sum_{j=0}^{k-1} N_{k1} A_{kj} \varepsilon_{kj} \quad (6.3b)$$

$$R_{ex1} = \frac{q v_{a1}}{4} (S_E + S_{g1} + S_{S1}) \sum_{k=1}^4 N_{k1} \varepsilon_{k0} \quad (6.3c)$$

$$j_{r1} = \frac{q n_1 v_{e1}}{4} \quad \text{and} \quad j_{r2} = \frac{q n_2 v_{e2}}{4} \quad (6.3d)$$

Current Continuity in Region 1

The current continuity equation in R_1 balances the electron/ion currents at the boundaries of the emitter-plasma R_1 , plasma R_1 -plasma R_2 , plasma R_1 -grid, and the surrounding region-plasma R_1 and is written as:

$$(j_{E0} - j_{eE} + j_{iE})S_E = (j_{eg} - j_{ig})(S_{g1} - S_h) + (j_{eh1} - j_{ih1} - j_{eh2} + j_{ih2})S_h \quad (6.4a)$$

Substituting for the electron and ion current densities in Eqn. (6.4a) yields:

$$\begin{aligned} \left(j_{E0} - j_{r1} \exp\left[\frac{-q\Delta V_E}{k_B T_{e1}}\right] + j_{i1} \right) S_E &= \left(j_{r1} \exp\left[\frac{-q(\Delta V_{g1} + \Delta V_g)}{k_B T_{e1}}\right] - j_{i1} \right) (S_{g1} - S_h) \\ &+ \left(j_{r1} \exp\left[\frac{-q\Delta V_{g1}}{k_B T_{e1}}\right] - j_{ir1} + j_{ir2} \exp\left[\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right] - j_{r2} \right) S_h, \end{aligned} \quad (6.4b)$$

Potential Balance in Region 1

The potential balance for R_1 is obtained by looping from the emitter to the

grid surface as depicted by Fig. 6.1, which gives:

$$\phi_E - \Delta V_E + \Delta V_{g1} + \Delta V_g - \phi_g + V_g = 0 \quad (6.5)$$

Cs Adsorption/Desorption in Region 1

The Cs adsorption/desorption can occur from the emitter surface and the grid surface facing the emitter and are determined from the following expressions:

$$\frac{d\theta_{g1}}{dt} = \frac{(1 - \theta_{g1})}{4 N_{mc}} v_{a1} \sum_{k=0}^4 N_{k1} - \frac{\theta_{g1}}{\gamma_g} \exp\left(\frac{-q H_g}{k_B T_g}\right), \quad (6.6a)$$

and,

$$\frac{d\theta_E}{dt} = \frac{(1 - \theta_E)}{4 N_{mc}} v_{a1} \sum_{k=0}^4 N_{k1} - \frac{\theta_E}{\gamma_E} \exp\left(\frac{-q H_E}{k_B T_E}\right). \quad (6.6b)$$

However, as indicated earlier in the present steady-state formulation of the triode device, the left-hand sides of Eqns. (6.6a) and (6.6b) will be equated to zero.

Region 1/Surrounding Boundary Conditions

There are two boundary conditions connecting the discharge volume of region 1 with the surrounding region (Cs reservoir) similar to the diode model. The first boundary condition is a balance between the ions and atoms leaving R_1 to the surrounding and to R_2 , the ions and atoms entering R_1 from R_2 , and the atoms entering the R_1 from the surrounding. The second boundary condition describes the fluxes of ions and electrons leaving R_1 to the surrounding assuming ambipolar diffusion.

The heavy particle balance in the volume of region 1 is determined by the boundary conditions between R_1 and R_2 , R_1 and the emitter surface, R_1 and the

grid surface, and R_1 and the surrounding region. The Cs reservoir temperature controls the initial Cs atom density, N_{a0} , in the gap. The following equation is obtained by equating the flux of ions and atoms (heavy particles) at each boundary of region 1. It is shown in this equation that the Cs atom source into the R_1 is from the surrounding region, region 2, and the electrode surfaces, and the loss of Cs atoms (including excited states) and ions is by leakage from the R_1 to the surrounding region and region 2.

$$\begin{aligned} \frac{V_1}{S_S} \left(\sum_{k=0}^4 \frac{dN_{k1}}{dt} + \frac{dn_1}{dt} \right) + \left(\frac{N_{mc} S_{g1}}{S_{S1}} \right) \frac{d\theta_{g1}}{dt} + \left(\frac{N_{mc} S_E}{S_{S1}} \right) \frac{d\theta_E}{dt} = \frac{N_{a0} v_{a1}}{4} \\ - \frac{v_{a1}}{4} \left(1 + \frac{S_h}{S_{S1}} \right) \sum_{k=0}^4 N_{k1} - \frac{j_{i1}}{q} - \frac{j_{ir1}}{q} \frac{S_h}{S_{S1}} + \frac{v_{a2}}{4} \frac{S_h}{S_{S1}} \sum_{k=0}^4 N_{k2} + \frac{j_{ir2}}{q} \exp \left(\frac{-q\Delta V_{g1}}{k_B T_{a2}} \right) \frac{S_h}{S_{S1}}. \end{aligned} \quad (6.7)$$

The first term on the left-hand side of Eqn. (6.7) is the atom and ion fluxes leaving the region R_1 to the surrounding, respectively, and the second and third terms are the atom fluxes entering/leaving R_1 due to desorption/adsorption from the grid and emitter surface, respectively. The last two terms on the right-hand side are the atoms and ions entering R_1 from R_2 .

In the same manner as the diode model, the expression for ambipolar diffusion is:

$$\Delta V_S = \frac{5.77 k_B T_{e1}}{q} \quad (6.8)$$

The value of ΔV_S from Eqn. (6.8) is substituted into the electron energy balance for R_1 (Eqn. (6.3a)).

Equations (6.1) through (6.8) represent the system of equations that are used to describe the discharge parameters for R_1 of the Cs-Ba triode. The following set of equations are for the discharge region R_2 .

Ion Generation in Region 2

As with the ion generation term for R_1 , the following expression consists of the rate of ion creation, and ion leakage to R_1 and R_3 from R_2 and ion leakage to R_2 from R_1 and R_3 .

$$\begin{aligned} \frac{dn_2}{dt} = & \sum_{k=0}^4 n_2 N_{k2} \langle \sigma v_{e2} \rangle_{k5} - \frac{j_{ir2}}{q} \frac{S_h}{V_2} \left\{ \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right) + \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{a2}}\right) \right\} \\ & - j_{i2} \frac{S_{hs}}{V_2} + j_{ir1} \frac{S_h}{V_2} + j_{i3} \frac{S_h}{V_2}, \end{aligned} \quad (6.9a)$$

where j_{i2} and j_{i3} are the ion fluxes to the inner grid surface and to R_2 from R_3 , respectively. These values are calculated from the following expressions:

$$j_{i2} = 0.61q n_2 \sqrt{\frac{k_B T_{e2}}{M_i}} \text{ and } j_{i3} = 0.61q n_3 \sqrt{\frac{k_B T_{e3}}{M_i}} \quad (6.9b)$$

Cs Atom Density Balance in Region 2

The Cs atom density balance for R_2 has the same form as that for region 1 and is given as:

$$\begin{aligned} \frac{dN_{02}}{dt} = & n_2 \sum_{j=1}^4 N_{j2} \langle \sigma v_{e2} \rangle_{j0} + \sum_{j=1}^4 N_{j2} A_{j0} - n_2 N_{02} \sum_{j=1}^5 \langle \sigma v_{e2} \rangle_{0j} - \\ & - \frac{N_{02} v_{a1}}{4} \frac{2S_h}{V_2} + \frac{N_{01} v_{a1}}{4} \frac{S_{S1}}{V_2} + \frac{N_{03} v_{a3}}{4} \frac{S_h}{V_2} - \frac{N_{mc} S_g}{V_2} \frac{d\theta_g}{dt}, \end{aligned} \quad (6.10a)$$

and,

$$\begin{aligned} \frac{dN_{k2}}{dt} = & n_2 \sum_{j=0, j \neq k}^4 N_{j2} \langle \sigma v_{e2} \rangle_{jk} + \sum_{j=1, j > k}^4 N_{j2} A_{jk} - n_2 N_{k2} \sum_{j=0, j \neq k}^5 \langle \sigma v_{e2} \rangle_{kj} - \\ & - N_{k2} \sum_{j=0, j < k}^4 A_{kj} - \frac{N_{k2} v_{a2}}{4} \frac{2S_h}{V_2} + \frac{N_{k1} v_{a1}}{4} \frac{S_h}{V_2} + \frac{N_{k3} v_{a3}}{4} \frac{S_h}{V_2}. \end{aligned} \quad (6.10b)$$

Electron Energy Balance in Region 2

$$\begin{aligned} \frac{3}{2} k_B V_2 \frac{d(n_2 T_{e2})}{dt} = & j_{r1} \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{e1}}\right) \left(\phi_g - V_g - \Delta V_g + \frac{2k_B T_{e1}}{q}\right) S_h \\ & + j_{ir1} (\epsilon_i - \phi_g + \Delta V_g + \Delta V_{g1} + V_g) S_h - j_{r2} \left(\phi_g - V_g - \Delta V_g + \frac{2k_B T_{e1}}{q}\right) S_h \\ & - j_{ir2} \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right) (\epsilon_i - \phi_g + V_g + \Delta V_g + \Delta V_{g1}) S_h - j_{ir2} \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right) \\ & (\epsilon_i - \phi_g + V_g + \Delta V_g + \Delta V_{g1}) S_h - j_{i2} (\epsilon_i - \phi_g + V_g + \Delta V_g) S_{hs} - R_{r2} - R_{ex2} \\ & - j_{r2} \exp\left(\frac{-q\Delta V_g}{k_B T_{e2}}\right) \left(\phi_g - V_g + \frac{2k_B T_{e2}}{q}\right) S_{hs} + j_{r3} \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{e3}}\right) \\ & \left(\phi_g - V_g - \Delta V_g + \frac{2k_B T_{e2}}{q}\right) S_h + j_{i3} (\epsilon_i - \phi_g + V_g + \Delta V_g + \Delta V_{g3}) S_h, \end{aligned} \quad (6.11a)$$

where R_{r2} , R_{ex2} , and j_{r3} are given by:

$$R_{r_2} = q V_2 \sum_{k=1}^4 \sum_{j=0}^{k-1} N_{k_2} A_{kj} \varepsilon_{kj}, \quad (6.11b)$$

$$R_{ex_2} = \frac{q v_{a_2}}{4} (2 + S_{hs}) \sum_{k=1}^4 N_{k_2} \varepsilon_{k0}, \quad (6.11c)$$

and,

$$j_{r_3} = \frac{q n_3 v_{e_3}}{4}. \quad (6.11d)$$

Current Continuity in Region 2

The current continuity equation for R_2 balances the electron/ion currents at the boundaries between regions R_1 and R_2 , regions R_2 and R_3 , and region R_2 and the inner grid surface and is written as:

$$(j_{eh_1} - j_{ih_1} - j_{eh_2} + j_{ih_2})S_h = (j_{eg_2} - j_{ig_2})S_{hs} + (j_{eh_2} - j_{ih_2} - j_{eh_3} + j_{ih_3})S_h. \quad (6.12a)$$

Substituting for the electron and ion current densities in Eqn. (6.12a) yields:

$$\begin{aligned} & \left(j_{r_1} \exp\left[\frac{-q\Delta V_{g_1}}{k_B T_{e_1}}\right] - j_{ir_1} - j_{r_2} + j_{ir_2} \exp\left[\frac{-q\Delta V_{g_1}}{k_B T_{a_2}}\right] \right) S_h = \left(j_{r_2} \exp\left[\frac{-q\Delta V_g}{k_B T_{e_2}}\right] - j_{i_2} \right) S_{hs} \\ & + \left(j_{r_2} - j_{ir_2} \exp\left[\frac{-q\Delta V_{g_3}}{k_B T_{a_1}}\right] - j_{r_3} \exp\left[\frac{-q\Delta V_{g_3}}{k_B T_{e_3}}\right] + j_{i_3} \right) S_h. \end{aligned} \quad (6.12b)$$

Potential Balance in Region 2

The potential balance for R_2 is obtained by looping from the grid to the collector surface as depicted by Fig. 6.1 which gives:

$$V_{CE} - \phi_C + \phi_g - \Delta V_g + \Delta V_C - \Delta V_{g_3} - V_g = 0 \quad (6.13)$$

Cs Adsorption/Desorption in Region 2

The Cs adsorption/desorption for region 2 can occur from the grid surface within the grid holes and is determined from the following expression:

$$\frac{d\theta_g}{dt} = \frac{(1 - \theta_g)}{4 N_{mc}} v_{a1} \sum_{k=0}^4 N_{k2} - \frac{\theta_g}{\gamma_g} \exp\left(\frac{-q H_g}{k_B T_g}\right) \quad (6.14)$$

Region 2 Boundary Condition

The only boundary conditions for R_2 are the couplings with R_1 and R_3 . The heavy particle balance in the volume in region 2 is determined by equating the ion and atom fluxes at the boundaries between $R_2 \backslash R_1$ and $R_2 \backslash R_3$ and between R_2 and the inner grid surface. The following equation shows that the Cs atom source into region 2 is from region 1, region 3, and the inner grid surface, and the loss of Cs atoms (including excited states) and ions is by leakage from the R_2 to R_1 , R_3 , and the inner grid surface.

$$\begin{aligned} \frac{V_2}{2S_h} \left(\sum_{k=0}^4 \frac{dN_{k2}}{dt} + \frac{dn_2}{dt} \right) + \left(\frac{N_{mc} S_g}{S_h} \right) \frac{d\theta_g}{dt} = \frac{v_{a1}}{4} \sum_{k=0}^4 N_{k1} + \frac{j_{i1}}{q} + \frac{v_{a3}}{4} \sum_{k=0}^4 N_{k3} \\ + \frac{j_{i3}}{q} - \frac{v_{a2}}{4} \sum_{k=0}^4 N_{k2} - \frac{j_{ir2}}{q} \exp\left(\frac{-q\Delta V_{g1}}{k_B T_{a2}}\right) - \frac{j_{ir2}}{q} \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{a2}}\right) - \frac{j_{i2} S_{hs}}{q S_h}. \end{aligned} \quad (6.15)$$

Equations (6.9) through (6.15) represent the system of equations that are used to describe the discharge parameters for R_2 of the Cs-Ba triode. The following set of equations are representative of R_3 .

Ion Generation in Region 3

As with the ion generation term for R_1 and R_2 , the following expression consists of the rate of ion creation, and ion leakage to R_2 from R_3 .

$$\frac{dn_3}{dt} = \sum_{k=0}^4 n_3 N_{k3} \langle \sigma v_{e3} \rangle_{k5} - \frac{j_{i3}}{q} \left(\frac{S_C + S_{g3} + S_{S3}}{V_3} \right) + \frac{j_{ir2}}{q} \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{a2}}\right) \frac{S_h}{V_3}. \quad (6.16)$$

Cs Atom Density Balance in Region 3

$$\begin{aligned} \frac{dN_{03}}{dt} = & n_3 \sum_{j=1}^4 N_{j3} \langle \sigma v_{e3} \rangle_{j0} + \sum_{j=1}^4 N_{j3} A_{j0} - n_3 N_{03} \sum_{j=1}^5 \langle \sigma v_{e3} \rangle_{0j} - \\ & + \frac{N_{02} v_{a2}}{4} \frac{S_h}{V_3} - \frac{N_{03} v_{a3}}{4} \frac{S_h}{V_3} - \frac{N_{mc} S_{g3}}{V_3} \frac{d\theta_{g3}}{dt}, \end{aligned} \quad (6.17a)$$

and,

$$\begin{aligned} \frac{dN_{k3}}{dt} = & n_3 \sum_{j=0, j \neq k}^4 N_{j3} \langle \sigma v_{e3} \rangle_{jk} + \sum_{j=1, j > k}^4 N_{j3} A_{jk} - n_3 N_{k3} \sum_{j=0, j \neq k}^5 \langle \sigma v_{e3} \rangle_{kj} - \\ & - N_{k3} \sum_{j=0, j < k}^4 A_{kj} - \frac{N_{k3} v_{a3}}{4} \frac{S_h}{V_3} + \frac{N_{k2} v_{a2}}{4} \frac{S_h}{V_3}. \end{aligned} \quad (6.17b)$$

Electron Energy Balance in Region 3

$$\begin{aligned} \frac{3}{2} k_B V_3 \frac{d(n_3 T_{e3})}{dt} = & j_{r2} \left(\phi_g - V_g - \Delta V_g + \frac{2k_B T_{e2}}{q} \right) S_h + j_{ir2} \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{a2}}\right) (\epsilon_i - \phi_g + \Delta V_g + \Delta V_{g3} + V_g) S_h \\ & - j_{r3} \exp\left(\frac{-q\Delta V_{g3}}{k_B T_{e3}}\right) \left(\phi_g - V_g - \Delta V_g + \frac{2k_B T_{e3}}{q} \right) S_h - j_{i3} (\epsilon_i - \phi_g + V_g + \Delta V_g + \Delta V_{g3}) \left(\frac{S_C + S_{g3} + S_{S3}}{S_h} \right) \\ & - R_{r3} - R_{ex3} - j_{r3} \exp\left(\frac{-q(\Delta V_g + \Delta V_{g3})}{k_B T_{e3}}\right) \left(\phi_g - V_g + \frac{2k_B T_{e2}}{q} \right) \left(\frac{S_{g3} - S_h + S_{S3}}{S_h} \right) - j_{r3} \exp\left(\frac{-q\Delta V_C}{k_B T_{e3}}\right) \\ & \left(\phi_g - V_g - \Delta V_g + V_C - \Delta V_{g3} + \frac{2k_B T_{e2}}{q} \right) \frac{S_C}{S_h}, \end{aligned} \quad (6.18a)$$

where R_{r3} and R_{ex3} are given by:

$$R_{r3} = q V_3 \sum_{k=1}^4 \sum_{j=0}^{k-1} N_{k3} A_{kj} \varepsilon_{kj}, \quad (6.18b)$$

and

$$R_{ex3} = \frac{q v_{a3}}{4} (S_C + S_{g3} + S_{S3}) \sum_{k=1}^4 N_{k3} \varepsilon_{k0}. \quad (6.18c)$$

Current Continuity in Region 3

The current continuity equation for R_3 balances the electron/ion currents at the boundaries of R_2 and R_3 , R_3 and the grid, and R_3 and the collector and is written as:

$$(j_{eh2} - j_{ih2} - j_{eh3} + j_{ih3}) S_h = (j_{eg3} - j_{ig3}) (S_{g3} - S_h) + (j_{eC} - j_{iC}) S_C. \quad (6.19a)$$

Substituting for the electron and ion current densities in Eqn. (6.19a) yields:

$$\begin{aligned} \left(j_{r2} - j_{ir2} \exp \left[\frac{-q \Delta V_{g3}}{k_B T_{a1}} \right] - j_{r3} \exp \left[\frac{-q \Delta V_{g3}}{k_B T_{e3}} \right] + j_{i3} \right) S_h &= \left(j_{r3} \exp \left[\frac{-q (\Delta V_{g1} + \Delta V_{g3})}{k_B T_{e3}} \right] - j_{i3} \right) (S_{g3} - S_h) \\ &+ \left(j_{r3} \exp \left[\frac{-q \Delta V_C}{k_B T_{e3}} \right] - j_{i3} \right) S_C. \end{aligned} \quad (6.19b)$$

Potential Balance in Region 3

The potential balance for R_3 is obtained by looping from the emitter to the collector (see Fig. 6.1), which gives:

$$V_{CE} - \phi_C + \phi_E - \Delta V_E + \Delta V_C - \Delta V_{g3} + V_{g1} = 0 \quad (6.20)$$

Cs Adsorption/Desorption in Region 3

The Cs adsorption/desorption for R_3 can occur from the grid surface facing the collector and the collector surface and is determined from the following expressions:

$$\frac{d\theta_{g3}}{dt} = \frac{(1 - \theta_{g3})}{4 N_{mc}} v_{a3} \sum_{k=0}^4 N_{k3} - \frac{\theta_{g3}}{\gamma_g} \exp\left(\frac{-q H_g}{k_B T_g}\right), \quad (6.21a)$$

and

$$\frac{d\theta_C}{dt} = \frac{(1 - \theta_C)}{4 N_{mc}} v_{a3} \sum_{k=0}^4 N_{k3} - \frac{\theta_C}{\gamma_C} \exp\left(\frac{-q H_C}{k_B T_C}\right). \quad (6.21b)$$

Region 3 Boundary Condition

The heavy particle balance in the volume of region 3 is determined by the boundary conditions between R_2 and R_3 , R_3 and the collector surface, and R_3 and the grid surface. At each boundary the ion and atom fluxes are equated. The following equation shows that the Cs atom source into the R_3 is from R_2 , and desorption from the grid surface facing the collector, and the collector surface. The loss of Cs atoms (including excited states) and ions is by leakage from the R_3 to R_2 , and adsorption on the grid and collector surfaces.

$$\begin{aligned} \frac{V_3}{S_h} \left(\sum_{k=0}^4 \frac{dN_{k3}}{dt} + \frac{dn_3}{dt} \right) + \left(\frac{N_{mc} S_{g3}}{S_h} \right) \frac{d\theta_{g3}}{dt} + \left(\frac{N_{mc} S_C}{S_h} \right) \frac{d\theta_C}{dt} = \frac{v_{a2}}{4} \sum_{k=0}^4 N_{k2} \\ + \frac{j_{ir2}}{q} \exp\left(\frac{-q \Delta V_{g3}}{k_B T_{a2}}\right) - \frac{j_{i3}}{q}. \end{aligned} \quad (6.22)$$

6.2 Steady-State Analysis

Equations (6.1) through (6.22) represent the system of equations that describe the discharge processes (atom, ion, energy, current, and potential) that occur in the three regions of the Cs-Ba triode. This system of equations solves for volumetrically average discharge parameters in each region. The solution procedure and comparison of the model calculations with the experimental data is presented in the following subsections.

6.2.1 Solution Procedure

In order to generate a steady-state solution using the system of equations outlined above, the transient terms are set equal to zero. Also before actually developing a source code to solve the system of equations, a few algebraic manipulations must be performed. Once this has been completed, five algebraic equations are obtained for each region which are shown in the following expressions.

Region 1

$$N_{01} = \frac{0.61 \sqrt{\frac{k_B T_{e1}}{M_i}} \left(\frac{S_E + S_{S1} + S_{g1} - S_h}{V_1} \right) + \frac{v_{a1}}{4} \frac{S_h}{V_1} - \frac{v_{a2}}{4} \frac{S_h}{V_1} \frac{n_2}{n_1} e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}}}{(X_{18} + X_{19})_1}, \quad (6.23)$$

$$N_{01} = \frac{N_{a0} - 2.44 n_1 \sqrt{\frac{\pi T_{e1}}{8 T_{a1}}}}{(X_{20})_1}, \quad (6.24)$$

$$\eta_E = -\ln \left[\frac{I_{E0} - e^{-\eta_{g1}} \frac{S_h}{S_E} + C_1 + \frac{j_{r2}}{j_{r1}} \left(e^{\eta_{g1} \frac{T_{e1}}{T_{e2}}} - \sqrt{\frac{m_e T_{a2}}{M_i T_{e1}}} \frac{n_2}{n_1} e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}} \frac{S_h}{S_E} \right)}{1 + e^{\frac{-q(\phi_g - V_g - \phi_E)}{k_B T_{e1}}} \left(\frac{S_{g1} - S_h}{S_E} \right)} \right], \quad (6.25)$$

$$\frac{j_{r2}}{j_{r1}} = \frac{-2I_{E0} \frac{T_E}{T_{e1}} + 2e^{-\eta_E} + e^{-\eta_{g1}} (\eta_{g1} - \eta_E + 2) \frac{S_h}{S_E} + e^{-(\eta_g + \eta_{g1})} (\eta_g + \eta_{g1} - \eta_E + 2) \left(\frac{S_{g1} - S_h}{S_E} \right) + A_1 + B_1}{e^{\eta_{g1} \frac{T_{e1}}{T_{e2}}} (\eta_{g1} - \eta_E + 2 \frac{T_{e2}}{T_{e1}}) \frac{S_h}{S_E} + \sqrt{\frac{m_e T_{a2}}{M_i T_{e1}}} e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}}} \quad (6.26)$$

and

$$\phi_E - \Delta V_E + \Delta V_{g1} + \Delta V_g - \phi_g + V_g = 0, \quad (6.27)$$

where

$$A_1 = \sqrt{\frac{2\pi m_e}{M_i}} \left[(U_i + 7.77) \frac{S_{S1}}{S_E} + (U_i + 7.77) \left(1 + \frac{S_{gs}}{S_h} \right) + \sqrt{\frac{T_{a1}}{2\pi T_{e1}}} (U_i + \eta_E) \frac{S_h}{S_E} \right],$$

$$B_1 = \frac{q}{k_B T_{e1}} \left[\frac{R_{ex1} (S_E + S_{g1} + S_{S1}) + R_{r1}}{j_{r1} S_E} \right],$$

$$C_1 = \sqrt{\frac{2\pi m_e}{M_i}} \left(\frac{S_E + S_{gs}}{S_E} \right) + \sqrt{\frac{m_e T_{a1}}{M_i T_{e1}}} \frac{S_h}{S_E},$$

and

$$\eta_{E,g1,g} = \frac{q\Delta V_{E,g1,g}}{k_B T_{e1}}, \quad U_i = \frac{q\epsilon_i}{k_B T_{e1}} \quad \text{and} \quad I_{E0} = \frac{j_{E0}}{j_{r1}}.$$

Region 2

$$N_{02} = \frac{0.61 \sqrt{\frac{k_B T_{e2}}{M_i}} \frac{S_{hs}}{V_2} + \frac{v_{a2}}{4} \left(e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}} + e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \right) \frac{S_h}{V_2} - \left(\frac{v_{a1}}{4} \frac{n_1}{n_2} + 0.61 \sqrt{\frac{k_B T_{e3}}{M_i}} \frac{n_3}{n_2} \right) \frac{S_h}{V_2}}{(X_{18} + X_{19})_2}, \quad (6.28)$$

$$N_{02} = \frac{\frac{1}{2} \sqrt{\frac{T_{a1}}{T_{a2}}} (N_{T1} + n_1) + \frac{1}{2} \sqrt{\frac{T_{a3}}{T_{a2}}} N_{T3} + 1.22 n_3 \sqrt{\frac{\pi T_{e3}}{8 T_{a2}}} - \frac{1}{2} n_2 \left(e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}} + e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \right)}{(X_{20})_2}, \quad (6.29)$$

$$\frac{j_{r3}}{j_{r2}} = \frac{-\frac{j_{r1}}{j_{r2}} \frac{T_{e1}}{T_{e2}} \left[e^{-\eta_{g1}(-\eta_g + 2)} + \sqrt{\frac{m_e T_{a1}}{M_i T_{e1}}} \right] + 2 e^{-\eta_g \frac{T_{e1}}{T_{e2}}} \frac{S_{hs}}{S_h} + \left(1 + e^{\eta_{g1} \frac{T_{e1}}{T_{e2}}} \right) \left(-\eta_g \frac{T_{e1}}{T_{e2}} + 2 \right) \frac{S_h}{S_E} + A_2 + B_2}{e^{-\eta_{g3}} \left(\eta_g \frac{T_{e1}}{T_{e2}} + 2 \frac{T_{e3}}{T_{e2}} \right) + \sqrt{\frac{2\pi m_e}{M_i}} \left(\eta_{g3} \frac{T_{e3}}{T_{e2}} + \frac{T_{e1}}{T_{e2}} \{ \eta_g + U_i \} \right)} \quad (6.30)$$

$$\frac{j_{r3}}{j_{r2}} = \frac{\frac{j_{r1}}{j_{r2}} \left(\sqrt{\frac{m_e T_{a1}}{M_i T_{e1}}} - e^{-\eta_{g1}} \right) + 1 + e^{\eta_{g1} \frac{T_{e1}}{T_{e2}}} + e^{-\eta_g \frac{T_{e1}}{T_{e2}}} \frac{S_{hs}}{S_h} - C_2}{e^{-\eta_{g3}} - \sqrt{\frac{2\pi m_e}{M_i}}} \quad (6.31)$$

and

$$V_{CE} - \phi_C + \phi_g - \Delta V_g + \Delta V_C - \Delta V_{g3} - V_g = 0, \quad (6.32)$$

where

$$A_2 = \sqrt{\frac{2\pi m_e}{M_i}} \left[\frac{T_{e1}}{T_{e2}} (U_i + \eta_g) \frac{S_{hs}}{S_h} + \sqrt{\frac{T_{a2}}{2\pi T_{e2}}} \left\{ e^{-\eta_{g1} \frac{T_{e1}}{T_{e2}}} (\eta_{g1} + \eta_g + U_i) \frac{T_{e1}}{T_{e2}} + e^{-\eta_{g1} \frac{T_{e1}}{T_{e2}}} \left(\eta_{g3} \frac{T_{e3}}{T_{e2}} + (\eta_g + U_i) \frac{T_{e1}}{T_{e2}} \right) \right\} \right]$$

$$B_2 = \frac{q}{k_B T_{e2}} \left[\frac{R_{ex2} (2S_h + S_{hs}) + R_{r2}}{j_{r2} S_h} \right],$$

$$C_2 = \sqrt{\frac{m_e}{M_i} \frac{T_{a1}}{T_{e1}}} \left(e^{-\eta_{g1} \frac{T_{e1}}{T_{a2}}} + e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \right) + \sqrt{\frac{2\pi m_e}{M_i}} \frac{S_{hs}}{S_h},$$

and

$$\eta_{g3} = \frac{q \Delta V_{g3}}{k_B T_{e3}}.$$

Region 3

$$N_{03} = \frac{0.61 \sqrt{\frac{k_B T_{e3}}{M_i}} \left(\frac{S_C + S_{g3} + S_{S3}}{V_3} \right) - \frac{v_{a2}}{4} \frac{n_2}{n_3} e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \frac{S_h}{V_3}}{(X_{18} + X_{19})_3}, \quad (6.33)$$

$$N_{03} = \frac{\sqrt{\frac{T_{a2}}{T_{a3}}} \left(N_{T_2} + n_2 e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \right) - 2.44 n_3 \sqrt{\frac{\pi}{8} \frac{T_{e3}}{T_{a2}}}}{(X_{20})_3}, \quad (6.34)$$

$$\eta_C = -\ln \left[\frac{S_h}{S_C} \left\{ \frac{j_{r2}}{j_{r3}} \left(1 - \sqrt{\frac{m_e}{M_i} \frac{T_{a2}}{T_{e1}}} e^{-\eta_{g1} \frac{T_{e3}}{T_{a2}}} \right) - e^{-\eta_{g3}} + \sqrt{\frac{2\pi m_e}{M_i}} \left(\frac{S_C + S_{g3} + S_{S3}}{S_h} \right) \right\} \right] \quad (6.35)$$

$$\frac{j_{r2}}{j_{r3}} = \frac{-e^{-\eta_{g3}} \left(-\eta_g \frac{T_{e1}}{T_{e3}} + 2 \right) - e^{-\eta_C} \left(-\eta_g \frac{T_{e1}}{T_{e3}} - \eta_{g3} + \eta_C + 2 \right) \frac{S_C}{S_h} - 2e^{-\left(\eta_{g3} + \eta_g \frac{T_{e1}}{T_{e2}} \right)} \left(\frac{S_{S3} + S_{g3} - S_h}{S_h} \right) + A_3 + B_3}{-\eta_g \frac{T_{e1}}{T_{e3}} + 2 \frac{T_{e2}}{T_{e3}} + \sqrt{\frac{m_e}{M_i}} \frac{T_{a2}}{T_2} e^{-\eta_{g3} \frac{T_{e3}}{T_{a2}}} \left(\eta_{g3} + \frac{T_{e1}}{T_{e3}} \{ \eta_g + U_i \} \right)} \quad (6.36)$$

and

$$V_{CE} - \phi_C + \phi_E - \Delta V_E + \Delta V_C - \Delta V_{g3} + V_{g1} = 0, \quad (6.37)$$

where

$$A_3 = \sqrt{\frac{2\pi m_e}{M_i}} \left(\eta_{g3} + U_i \frac{T_{e1}}{T_{e3}} \right) \left(\frac{S_C + S_{g3} + S_{S3}}{S_h} \right),$$

$$B_3 = \frac{q}{k_B T_{e3}} \left[\frac{R_{ex3} (S_C + S_{g3} + S_{S3}) + R_{r3}}{j_{r3} S_h} \right],$$

and

$$\eta_C = \frac{q \Delta V_C}{k_B T_{e3}}.$$

In Eqns. (6.23) through (6.37) there are 28 unknowns for all three regions which are listed in Table 6.1.

Table 6.1. Triode Model Unknowns in Each Discharge Region.

Region 1	Region 2	Region 3
$T_{e1}, n_1, N_{01}, N_{11}, N_{21},$ $N_{31}, N_{41}, \Delta V_E, \text{ and } \Delta V_{g1}$	$T_{e2}, n_2, N_{02}, N_{12}, N_{22},$ $N_{32}, N_{42}, \Delta V_g, \text{ and } V_g$	$T_{e3}, n_3, N_{03}, N_{13}, N_{23},$ $N_{33}, N_{43}, \Delta V_{g3}, \Delta V_C,$ $\text{and } V_{CE}$

However, as with the diode model, relationships exist between the ground state density, N_{0k} , and excited state densities (N_{1k} through N_{4k}). These relationships

reduce the number of unknowns to 16, which are shown in Table 6.2.

Table 6.2. Triode Model Unknowns in Each Discharge Region After Cs Excited State Balance.

Region 1	Region 2	Region 3
T_{e1} , n_1 , N_{01} , ΔV_E , and ΔV_{g1}	T_{e2} , n_2 , N_{02} , ΔV_g , and V_g	T_{e3} , n_3 , N_{03} , ΔV_{g3} , ΔV_C , and V_{CE}

After the discharge ignites due to the positive grid pulse, the grid is floated and the current through the grid is limited to 2 A by the pulse generator circuitry. By equating the total current drawn to the grid from R_1 , R_2 , and R_3 , the floating grid potential can be calculated using the following equation:

$$I_g = \left[j_{r1} \exp\left(\frac{-q(\Delta V_g + \Delta V_{g1})}{k_B T_{e1}}\right) - j_{i1} \right] S_{gs} + \left[j_{r2} \exp\left(\frac{-q\Delta V_g}{k_B T_{e1}}\right) - j_{i2} \right] S_{hs} + \left[j_{r3} \exp\left(\frac{-q(\Delta V_g + \Delta V_{g3})}{k_B T_{e3}}\right) - j_{i3} \right] (S_{g3} - S_h + S_{s3}) \quad (6.38)$$

Now there are 15 unknowns and 15 equations; however, further inspection of Eqns. (6.27), (6.32) and (6.37) reveals that there is no new information contained in Eqn. (6.37). Therefore, there are 15 unknowns and only 14 equations. Similar to the solution procedure for the diode, the 15th equation needed is the non-dimensional parameter, I_{E0} , which is determined from the flux balance between region 1 and the Cs reservoir (see Eqn. (5.24)).

A flow diagram illustrating the solution procedure is shown in Fig. 6.2. The model is initiated by reading the input file which consists of the following parameters: T_E , T_C , T_{Cs} , j_{E0} , ϕ_C , ϕ_g , and τ , the grid transparency. As shown in Fig. 6.2, the next step is to determine the initial value of I_{E0} which is done from the same expression used for the diode model (see Eqn. (5.24)). At this point it is

Figure 6.2. Flow Diagram of Solution Procedure for Cs-Ba Triode Model.

possible to begin the calculation of the discharge parameters in each region. By examining Eqns. (6.23) and (6.24), it becomes obvious that if the plasma density in region 2, n_2 , is set equal to zero, the only unknown is T_{e1} which is found by using a bisectional iteration from initial upper and lower limits. Once T_{e1} is determined, it is possible to calculate ΔV_{g1} from Eqns. (6.25) and (6.26). This is accomplished by first assuming a value for ΔV_{g1} and solving for ΔV_E (Eqn. (6.25)). Since n_2 is equal to zero ($j_{r2}=0$) for the first iteration, the left-hand side of Eqn. (6.26) is also equal to zero and ΔV_{g1} can be determined, also by a bisectional iteration from upper and lower limits. After using the relationships in Appendix E for the Cs excited state densities, the variables listed in Table 6.1 for region 1 are all known. However, since n_2 is assumed to be equal to zero, the calculated values for region 1 need to be updated.

The solution of the parameters in region 2 is completed in a similar fashion as was just described for region 1. However, in this case, the plasma density in region 3, n_3 , is set equal to zero. Equations (6.28) and (6.29) determine the electron temperature in region 2, T_{e2} , and Eqns. (6.30) and (6.31) determine the plasma density in region 2, n_2 . At this point, instead of calculating the parameters in region 3, a check on the current balance at the boundary between regions 1 and 2 is performed. If the difference in the current leaving region 1 and the current entering region 2 is greater than 10^{-6} , the model recalculates T_{e1} . During this calculation the plasma density for region 2 has a non-zero value. Once the current difference decreases below 10^{-6} , the parameters in region 3 are calculated. It usually takes about 20 iterations before the model converges on this current check.

Before beginning the calculation of the region 3 parameters, the plasma density in region 3 must be determined. This should have been accomplished from the electron energy balance equation in region 2 (Eqn. (6.32)), however, when this equation is used the model becomes unstable after only 5 iterations. Therefore, the

initial value of n_3 is determined from Eqn. (6.32, but subsequent values are determined from a linear relationship between the plasma density in region 1 and the plasma density in region 3. The endpoint of the n_3/n_1 ratio is arbitrarily chosen between 0.7 and 1.0. This value was measured using Langmuir probes for similar discharge conditions by Baksht et al. (1978a and 1978b) and Kaplan et al. (1977a). Before calculating the electron temperature in region 3, an arbitrary initial value for ΔV_{g3} must be chosen. A value of one is acceptable, but the model solution is insensitive to this initial value.

Now it is possible to calculate the electron temperature in region 3 from Eqns. (6.33) and (6.34). In the same manner that ΔV_E and ΔV_{g1} are calculated, ΔV_C and ΔV_{g3} are determined from Eqns. (6.35) and (6.36). At this point initial values for all the discharge parameters outlined in Table 6.1 are known, but an overall current balance between the emitter and collector has not been obtained. The second current check is now performed between the amount of current that leaves the emitter surface and the sum of the currents reaching the grid and collector surfaces, respectively. The constraint on determining if suitable accuracy has been obtained is for the current to be balanced within a tolerance of 10^{-4} . Once this has been achieved, the first point of the I-V characteristic is determined and the data is written to the output files. A step increment is added to I_{E0} and the process is repeated as shown in the flow diagram (see Fig. 6.2). This continues until one of two things occurs: 1) I_{E0} reaches a value of 1.2 or 2) ΔV_{g3} becomes equal to zero. The first condition is set arbitrarily in order to have a common reference for comparison of the data for changes in the input conditions; however, the end of the diode model runs always occurred around $I_{E0} = 1.2$. The second condition is necessary because the system of equations is not valid for ΔV_{g3} less than zero. Modifying the equations to allow ΔV_{g3} to go negative is left for future work.

Once either one of these conditions is reached the upper portion (above the knee) of the I-V characteristic is complete. The portion of the I-V curve below the knee is calculated just like in the diode model by assuming that a virtual emitter exists at the emitter surface. The solution of the equations is the same for a virtual emitter, except the thermal emission current, j_{E0} , is decremented instead of incrementing I_{E0} . This continues until the output current becomes less than 5 A at which time the complete I-V characteristic is obtained. The 5 A lower limit is chosen because at most combinations of input conditions, the critical current for ignition of the tacitron is below this value (typically 1 or 2 A, see Figure 4.11). Now that the procedure for obtaining model results has been outlined, model results are discussed in the next section.

6.2.2 Results and Discussion

The first results consist of a parametric analysis of the effect changing the Cs and Ba vapor pressures by comparing model results with experimental data. The most abundant experimental data available for comparison with model calculations is I-V characteristics. From comparison of the I-V characteristics between the diode and triode, the effect of the grid on the forward voltage drop is investigated. Also, an analysis is done to investigate whether the grid can be used as a probe to obtain additional information on the plasma conditions. These are the only diagnostics presently available for comparing the calculated and measured results. However, no additional analysis of the ion generation processes or the physics leading to the system of equations is performed since Chapter 5, which was on the diode, covered these areas.

6.2.2.1 I-V Characteristics

Comparisons between the measured and calculated I-V characteristics is made for changes in the Cs reservoir temperature and the thermal emission current density (Ba reservoir temperature). During the calculation of the I-V characteristic, I_{E0} varies from about 0.1 at the upper discharge current to 1.2 at the knee of the curve. Using Eqn. (6.32), the initial value the plasma density in region 3, n_3 , is calculated to be about 0.3. Assuming an end value of 0.9 for the n_3/n_1 ratio, the following linear relationship is used to calculate n_3 :

$$\frac{n_3}{n_1} = 0.55I_{E0} + 0.24 \quad (6.39)$$

Effect of Cs Pressure

Figure 6.3 is a comparison of the experimental measurement and model calculation for both the diode and triode devices at a Cs reservoir temperature of 160 °C. As was discussed in Chapter 5, the diode model calculations compare quite well with the experimental measurements. However, in the much more complicated triode configuration, the model calculations do not predict the experimental data as well as the diode model. As seen in Fig. 6.3, there is similar deviation between calculations and the experimental I-V curve above the knee, as was seen in the diode case, because anomalous Schottky emission is not included in either the triode or diode models. In the triode model, as the discharge current approaches the knee from above, both the current and voltage begin to increase again and a loop forms close to the thermal emission current. The cause of this loop will be explained in a later section.

Figure 6.3. Comparison of Model Calculations and Experimental Results for Cs-Ba Diode and Triode with Virtual Emitter ($T_{Cs} = 160\text{ }^{\circ}\text{C}$ and $j_{E0} = 6.34\text{ A/cm}^2$).

Also, as seen in Fig. 6.3, below the knee there is a negative resistance region indicated by the model calculation due to the assumption of the formation of a virtual emitter. However, unlike the diode (both experimental and calculation), the slope of the negative resistance region is much steeper and at lower currents it drastically deviates from the measured values. Figure 6.4 is a comparison of model calculations for the assumption of a virtual emitter and a contracting discharge area. As can be seen, the contracting area case more closely resembles the experimental results for discharge currents below the knee as shown in Figs. 6.5 and 6.6. Fig. 6.5 is for the same conditions shown in Fig. 6.3 except with the assumption of contracting discharge area for the triode. Figure 6.6 is at a Cs reservoir temperature of 180 °C and maximum thermal emission current density of 8.27 A/cm².

The effect on the minimum forward voltage drop of changes in the Cs reservoir temperature are very important in determining the efficiency of the tacitron. Figure 6.7 shows the minimum forward voltage drop versus Cs reservoir temperature for both measured (symbols) and calculated (dashed lines) values. The model prediction is not very good at lower thermal emission current densities because of the large negative slope calculated below the knee. But at higher emission currents, the model prediction is closer to the measured values. As for the experimental data, at Cs reservoir temperatures above 145 °C, a positive resistance region exists which is shown by the fact of that there is an increase in the forward voltage drop as the thermal emission current is increased. This trend suggests that a virtual emitter is forming at the emitter surface, but maybe the conditions of the plasma have changed (i.e. a Non-Maxwellian distribution exists), resulting in the poor prediction of the model with experimental data at low currents.

Figure 6.4. Model Calculations Comparison of Virtual Emitter and Contracting Discharge Area for Currents Below the Knee.

Figure 6.5. Comparison of Model Calculations and Experimental Results for Cs-Ba Diode and Triode with Contracting Discharge Area ($T_{Cs} = 160\text{ }^{\circ}\text{C}$ and $j_{E0} = 6.34\text{ A/cm}^2$).

Figure 6.6. Comparison of Model Calculations and Experimental Results for Cs-Ba Diode and Triode with Contracting Discharge Area ($T_{Cs} = 180\text{ }^{\circ}\text{C}$ and $j_{E0} = 8.27\text{ A/cm}^2$).

Figure 6.7. Minimum Forward Voltage Drop as a Function of Cs Reservoir Temperature.

Effect of Ba Pressure

Changes in the Ba vapor pressure result in changes in the thermal emission current density only as was established in the analysis in Chapter 5. From the experimental I-V curve in Chapter 4 (Fig. 4.12), it was shown that changes in the Ba pressure resulted in increases in the knee current but with only slight changes in the minimum voltage drop (see Figure 6.7). Figure 6.8 shows the calculated I-V curve for five different emission current densities. This figure shows that as the thermal emission current increases the voltage drop remains practically constant with only a slight decrease in the minimum voltage drop. The slight decrease in the minimum forward voltage drop for increasing j_{E0} contradicts the trend seen in the experiment (see Fig. 4.12). Therefore, with this contradiction and the fairly good agreement with respect to the slope of the negative resistance region for the assumption of a contracting discharge area, it is concluded that a combination of these two affects might be taking place. This area will be further investigated in future work.

Effect of the Grid

Determining the effect the grid has on the discharge parameters is most easily accomplished by comparing results of the I-V characteristics from the diode and triode, both experimentally and theoretically, at the minimum forward voltage drop. However, before presenting this data, the potential distribution across the emitter-collector gap, needs to be revisited. The potential distribution as shown in Fig. 6.1(b), is for a decelerating potential for electrons that flow from region 1 into region 2. Remember that the sign of this potential difference is determined from the electron energy balance and current continuity equations. If the plasma potential of region 2 is more negative than that of region 1 (positive ΔV_{g1} , see Fig. 6.1(b)) than electrons are decelerated and in the opposite case, electrons are

Figure 6.8. Effect of Ba Reservoir Temperature on I-V Characteristics.

accelerated from region 1 into region 2. The calculated potential distributions for a cesium reservoir temperature of 160 °C and maximum thermal emission current density of 6.34 A/cm² (same conditions as shown in Fig. 6.3) are shown in Figs. 6.9(a) and 6.9(b) for the maximum and minimum forward voltage drops, respectively. As shown in Fig. 6.9(a), at the maximum forward voltage drop, the potential difference between the plasmas in region 1 and region 2 (ΔV_{g1}) decelerates electrons and as shown in Fig. 6.9(b), at the minimum forward voltage drop, the potential difference accelerates electrons. Further analysis of how this change in the electron energy as it enters the grid hole affects the extinguishing mechanism will be performed in the future. The remaining calculation results are shown for the minimum forward voltage drop; therefore, the potential difference between the plasmas in regions 1 and 2 is negative (electrons are accelerated from region 1 into region 2).

The most straight forward comparison is the forward voltage drop for various Cs reservoir temperatures and emission current densities. Figure 6.10 shows experimental data for the minimum forward voltage drop for the diode as a function of the Cs reservoir temperature at j_{E0} equal to 4.9 A/cm², 6.3 A/cm² and 8.5 A/cm². The interelectrode gap spacing was 1.0 mm with the grid of the triode being used as the collector for the diode measurements (see Fig. 6.1). As expected, when the Cs reservoir temperature (Cs pressure) is increased, the plasma becomes more conductive and the forward voltage drop decreases. The minimum forward voltage drop for the triode under the same conditions as the diode is shown in Fig. 6.11, with the measured trend also as expected. Finally, in order to investigate the increase in the forward voltage drop due to the presence of the grid, V_{dg} , the difference between the data shown in Figs. 6.10 and 6.11 is shown in Fig. 6.12. The increase in the forward voltage drop due to the presence of the grid in the triode device is approximately 0.65 V for Cs reservoir temperatures between

(a) maximum voltage

(b) minimum voltage

Figure 6.9. Calculated Potential Distributions for the Triode.

Figure 6.10. Experimental Results of Minimum Forward Voltage Drop for the Diode.

Figure 6.11. Experimental Results of Minimum Forward Voltage Drop for the Triode.

Figure 6.12. Experimental Results of Minimum Forward Voltage Drop Difference Between the Triode and Diode.

165 °C and 200 °C. However, at T_{Cs} below 165 °C, V_{dg} decreases and eventually becomes equal to zero at $T_{Cs} \approx 125$ °C for the two lowest emission current densities. For $j_{E0} = 8.5$ A/cm², V_{dg} does not become zero until $T_{Cs} \approx 115$ °C. The shift in the Cs reservoir temperature where $V_{dg} = 0$ is due to the fact that it is more difficult to flow higher currents through the grid holes, resulting in a higher voltage drop across the triode. The fact that the difference in the forward voltage drop between the diode and triode eventually reaches zero can also be explained.

In any gas discharge, there is a certain probability that an electron will have an ionizing collision with an atom. As more collisions take place, more ions are created and the conductivity of the plasma increases. Although the model does not take into account actual spatial dependencies, it can determine the effect of changing the volume of the discharge, which for a constant electrode surface, is a pseudo spatial dependency, at least on the interelectrode gap, d . Also, the probability of ionizing collisions is a function of the neutral atom density. Therefore, the product of the Cs pressure and the interelectrode gap, gives a fairly good estimate of the forward voltage drop for different combinations of P_{Cs} and d .

Figure 6.13 shows a calculation of the minimum forward voltage drop in the diode as a function of $P_{Cs}d$ for three Cs reservoir temperatures (130 °C, 140 °C, and 150 °C) and $j_{E0} = 3$ A/cm². There are two points of significance labeled one and two on the plot. One corresponds to a gap spacing of 1.0 mm and two corresponds to a gap spacing of 4.25 mm (emitter-collector gap in the triode) for a Cs reservoir temperature of 150 °C. As can be seen in the figure, the voltage drop decreases from 1.43 V to 0.99 V as the interelectrode gap is increased from 1 to 2. The conclusion that can be made from this result is simply that if the voltage drop across the grid is the same as the decrease due to a larger gap, then the diode and triode can have the same forward voltage drop. Unfortunately, the model does not converge at Cs reservoir temperatures less than 140 °C, so this point can not be

Figure 6.13. Model Calculation of the Minimum Forward Voltage Drop in the Diode as a Function of $P_{Cs,d}$.

shown analytically. However, an analysis can be performed on the discharge parameters contributing to the value of the forward voltage drop, i.e. the sheath potentials, at higher Cs pressures to investigate this conclusion.

Table 6.3 summarizes the results of the calculations at $T_{Cs} = 150^\circ\text{C}$ and $j_{E0} = 5 \text{ A/cm}^2$. The table contains data for the diode at $d = 1.0 \text{ mm}$ and 4.25 mm and the triode at $d = 4.25 \text{ mm}$. As the spacing in the diode is increased, at a constant T_{Cs} , the voltage drop in the plasma, $V_{dp} = \Delta V_E - \Delta V_C$, decreases. For example, at $T_{Cs} = 140^\circ\text{C}$, the plasma drop decreases from 1.802 V to 1.223 V as d is increased from 1.0 mm to 4.25 mm , respectively. The data for the triode shows that the voltage drop in the plasma is very close to the value for the diode with $d = 4.25 \text{ mm}$. This means that the increase in the forward voltage drop for the triode is almost solely equal to the voltage drop across the grid for diodes and triodes with the same interelectrode gap. Therefore, by gaining an understanding of how the voltage drop across the grid changes with input parameters (i.e. T_{Cs} , j_{E0} , grid hole diameter, grid thickness, and grid transparency), a fairly good estimate of the forward voltage drop can be gained. Furthermore, by having a correlation describing a relationship between the above parameters, the much simpler diode model could be run to gain an estimate of the forward voltage drop. Unfortunately, an accurate correlation can not be developed without comparisons to experimental data, that does not presently exist.

The voltage drop across the grid is a function of the grid diameter, grid thickness, grid transparency, Cs pressure, and discharge current. The easiest trends to show are for changes in the Cs pressure and emission current density. The data listed in Table 6.3 for the triode is plotted in Fig. 6.14 with the voltage drop across the grid on the y-axis and the Cs reservoir temperature on the x-axis. Calculation results for $j_{E0} = 5 \text{ A/cm}^2$ are also included in Fig. 6.14. Recall that the relationship between the Cs reservoir temperature and the Cs pressures in both

Table 6.3. Comparison Between Diode and Triode Sheath Potentials.

Configuration	T_{Cs}	ΔV_E	ΔV_C	V_{dp}	ΔV_{g1}	ΔV_{g3}	V_{dg}	V_{CE}
Diode, d=1mm	140	1.812	0.010	1.802	-	-	-	1.594
	150	1.521	0.009	1.512	-	-	-	1.304
	160	1.338	0.009	1.329	-	-	-	1.121
	170	1.218	0.006	1.212	-	-	-	1.003
	180	1.138	0.006	1.132	-	-	-	0.924
Diode, d=4.25mm	140	1.229	0.006	1.223	-	-	-	1.015
	150	1.117	0.006	1.111	-	-	-	0.903
	160	1.037	0.005	1.032	-	-	-	0.824
	170	0.990	0.006	0.984	-	-	-	0.776
	180	0.960	0.004	0.956	-	-	-	0.747
Triode, d=4.25mm *Data in this row has been obtained by extrapolation	140*	1.408	0.220	1.188	-0.878	0.005	0.883	1.854
	150	1.288	0.194	1.094	-0.761	0.087	0.848	1.751
	160	1.169	0.168	1.001	-0.644	0.194	0.838	1.647
	170	1.090	0.149	0.941	-0.565	0.268	0.833	1.581
	180	1.038	0.134	0.904	-0.508	0.333	0.841	1.552

Figure 6.14. Calculated Voltage Drop Across the Grid as a Function of the Cs Reservoir Temperature.

the reservoir and the gap can be found in Appendix C. As can be seen in this figure, for the higher current case the voltage drop across the grid is almost constant with an average value of ≈ 0.84 V. However, the low current case seems to have a fairly well defined minimum occurring at $T_{Cs} = 160$ °C. The voltage drop at the low discharge current begins to increase because the voltage drop across the plasma has begun to increase again. As seen in Fig. 6.13, the minimum voltage drop occurs at a $P_{Cs,d}$ of ≈ 0.055 mm-Torr. At 180 °C, the $P_{Cs,d}$ is 0.142 mm-Torr which as can be seen in Fig. 6.13 has a slightly higher voltage drop. At the higher emission current density, the increased ionization rate compensates slightly for this fact, and the minimum value increases more slowly. The difference between the two current cases arises from the rate of ion generation. At the lower current it is more difficult to ionize the Cs atoms (lower source of projectile electrons) which results in a lower ion density and a higher voltage drop.

To further confirm the fact that the diode model can be used to predict the plasma drop in the triode, the data in Fig. 6.15 is added to the plasma drop for the diode with a spacing of 4.25 mm. This is then subtracted from the output voltage for the diode with a spacing of 1.0 mm. This value is equivalent to the experimental data shown in Fig. 6.12 and is reproduced in Fig. 6.15. As can be seen, for a current density of 5 A/cm², there is very good agreement between the experimental data and the model prediction that uses a combination of diode and triode calculations. Lastly, as the Cs pressure is increased, the voltage drop due to the presence of the grid begins to approach an asymptote. This is because for a given discharge current, a certain ion density is required to carry the current from the emitter to the collector, and the electron temperature necessary to reach this density decreases and approaches an asymptote as the neutral atom density increases (increased T_{Cs}). The electron temperatures and plasma densities in each

Figure 6.15. Model Calculation and Experimental Data for Voltage Drop Increases Due to the Presence of the Grid.

region directly determines each sheath potential which in turn determines the output voltage for both the diode and triode.

The next area investigated is the effect of the grid transparency on the forward voltage drop. It would be expected that as the transparency of the grid is decreased, the voltage drop across the device would increase and the most significant increase being the drop across the grid. The model calculation confirms this fact as shown in Fig. 6.16. The voltage drop across the plasma and grid for grid transparencies ranging from 15 % to 55 % are plotted in this figure. Recall that the transparency of the Cs-Ba tacitron is ≈ 40 %. As the transparency is decreased from 55 % to 15 %, the voltage drop across the grid increases from 0.62 V to 1.48 V; whereas, the increase in the voltage drop across the plasma is only from 0.97 V to 1.29 V. Furthermore, the increase in V_{dg} is almost solely due to an increase in ΔV_{g1} as shown in Fig. 6.17. This makes a certain amount of physical sense because as the grid holes become smaller, the electrons need more energy to be able to pass through the holes. Also, as the volume of the hole gets smaller, the plasma density in region 2 must increase to maintain the discharge current at the same value. The plasma density in region 2 increases from $0.5 \times 10^{-13} \text{ cm}^{-3}$ to $1.07 \times 10^{-13} \text{ cm}^{-3}$ as the transparency is decreased from 55 % to 15 %.

The next parameter investigated is the effect of the grid thickness. For these cases, when the thickness of the grid is increased, the radius of the collector was not changed; therefore, the volumes of both region 2 and 3 changed. Also, with a thicker grid, the ion leakage to the inner surface within the grid holes would increase which should result in a larger voltage drop. Although this is the result shown in Fig. 6.18, the change is insignificant. This is an important result, because if the grid thickness needs to be increased for strength and cooling requirements, the penalty of increased forward voltage drop is negligible. On the other hand, if the grid thickness must be decreased because of size or weight

Figure 6.16. Calculated Grid Transparency Effect on the Forward Voltage Drop.

Figure 6.18. Calculated Effect of Grid Thickness on the Forward Voltage Drop.

constraints, there is no savings in the voltage drop incurred. These results however would need experimental verification before being introduced into future designs of Cs-Ba tacitron prototypes.

The last grid parameter investigated is the grid aperture. The trend calculated by the model for changes in the grid aperture diameter is slightly surprising. One might think that as the grid diameter gets smaller, at a constant transparency and thickness, that the forward voltage drop would increase. But just the opposite occurs as shown in Fig. 6.19. For two different Cs reservoir temperatures the trend is such that the forward voltage drop decreases slightly as the grid diameter decreases. As the grid diameter changes, the only geometric dimension that changes is the number of holes, and the inner grid surface; which decreases for increasing diameter. However, as the area of the inner surface increases (decreasing a_d), the grid floating potential, V_g , increases in order to retard more electrons and maintain the current at the circuit limited value of 2 A. The increase in V_g causes a subsequent decrease in ΔV_{g3} (ΔV_E , ΔV_C and ΔV_{g1} remain almost constant) and therefore a decrease in the forward voltage drop. The sheath potential between region 2 and region 3, ΔV_{g3} , decreases because as V_g increases, the energy of the electrons between the grid and collector surfaces increase which means not as much energy needs to be transferred to the electrons as they pass from the plasma in R_2 to the plasma in R_3 . The decrease in ΔV_{g3} offsets any increase due to more ion leakage (ΔV_{g1} does increase slightly as a_d decreases) and the net effect is a decrease in the forward voltage drop.

6.2.2.2 Probe Measurements Using the Grid

Since the diagnostics of the Cs-Ba tacitron are limited to the I-V characteristics only, an attempt is made to use the grid as a probe to obtain data on

Figure 6.19. Calculated Effect of Grid Aperture Diameter on the Forward Voltage Drop.

the other discharge parameters. For a little background in probe theory, refer to Appendix G. In order to have confidence in the ion current value for comparison to the steady-state calculation, only experimental data was used for which the discharge stayed in the conducting state when a negative voltage pulse is applied to the grid (see Fig. 4.22). Figure 6.20 shows a comparison of the measured and calculated ion current to the grid for several different Cs pressures. As can be seen there is not a good prediction by the model. It should be noted that the calculated ion current is at the floating grid potential (for $I_g = 2$ A) and not at a specific applied grid voltage. Also, all of the calculation results are at a constant applied voltage, V_{CE} , of 2.0 V. The experimental data is at different Cs pressures and applied voltages for which the values corresponding to the numbers on Fig. 6.20 can be found in Table 6.4. As can be seen in Fig. 6.20, the experimental data is above the calculated values for all discharge conditions investigated. However, at the applied grid potential of -30 V, the current being drawn by the grid is most likely all ions because the electrons are being repelled. This ion saturation current is higher than the ion current being drawn by the grid when the net grid current is 2 A of electron current (grid floating). Therefore, the ion saturation current must be calculated and compared to the measured values. This is accomplished by using the model to plot an I-V probe characteristic by inputting values of the grid voltage from large negative values to small positive values (see Appendix G).

Figure 6.21 is a plot of both the net current and total ion current being drawn by the grid as a function of the applied grid voltage. At grid voltages below -5 V, the net current and ion current are almost equal with the saturation ion current obtained at $V_g = -30$ V. At grid voltages above -5 V, a strong deviation takes place as the current to the grid becomes dominated by electron current. Unfortunately, the maximum voltage that can be obtained from the model at grid voltages of -30 V is 2 V. Therefore, there are only two data points (numbers 1 and

Figure 6.20. Measured and Calculated Ion Current Drawn by the Grid.

Table 6.4. Measured Ion Current to the Grid for Various Input Parameters.

Number	T_{Cs}	I_C	V_{CE}	I_g
1	156	20	2	1.2
2	156	20	3	1.5
3	156	29	3	1.45
4	148	30	4	1.5
5	156	31	3	1.45
6	171	40	2	2.25
7	161	40	5	2.5
8	171	40	3	2.63
9	154	52	10	2.5
10	155	54	10	2.95
11	155	55	10	2.75

6) in Table 6.4 for which a fair comparison between the model and calculation can be made. The results of the comparison are shown in Table 6.5.

Table 6.5. Comparison Between Measured and Calculated Ion Saturation Currents.

Number	T_{Cs}	V_{CE}	I_C	I_g	I_g
				Experiment	Calculation
1	156	2	20	1.2	1.196
6	171	2	40	2.25	2.267

For these two cases, the model seems to do a fairly decent job of predicting the ion saturation current. However, in order to use the probe plot to predict the electron

Figure 6.21. Calculated Ion and Net Current to the Grid from All Three Regions as a Function of the Applied Grid Voltage.

temperatures and plasma densities for each region, the current drawn to the grid from each region must be known separately. This is not possible in the experimental device, but is quite easy to do using the model. For the same input conditions as in Fig. 6.21, the three components of the total ion saturation current are 0.39 A from region 1, 0.25 A from region 2, and 0.29 A from region 3 from the total of 0.93 A. From the probe theory in Appendix G, the electron temperature in Kelvin can be found from:

$$T_e = \frac{q}{k_B} \frac{(V_1 - V_2)}{\ln\left(\frac{j_1}{j_2}\right)} \quad (6.40)$$

This relationship is valid on the linear portion of Fig 6.21 when the net current to the grid is dominated by electron current. Using Eqn. (6.40) to determine an electron temperature from the total net current to the grid results in $T_e = 6971$ K. Figure 6.22 shows the calculated probe plot for the current drawn to the grid from region 1 only. Using this curve, the electron temperature in region 1 would be $T_{e1} = 7385$ K. From similar plots for regions 2 and 3, the electron temperatures are $T_{e2} = 7785$ K and $T_{e3} = 6661$ K, respectively. From the calculation results with the grid floating ($I_g = 2$ A), the electron temperatures are $T_{e1} = 8107$ K, $T_{e2} = 8261$ K, and $T_{e3} = 7435$ K. Although the temperatures relative to each other follow the same trend, in all cases the electron temperature using the probe plot is lower than the electron temperature under operating conditions. This is due to the fact that, unlike the statement in Appendix G, the grid can not act as a probe because it perturbs the plasma, even at low applied grid voltages, and alters the discharge conditions.

The other relationship derived in Appendix G is the calculation of the plasma density from the ion saturation current, once the electron temperature is

known. The expression is as follows:

$$n = \frac{I_g}{0.61qA_g} \sqrt{\frac{M_i}{k_B T_e}} \quad (6.41)$$

Again referring to Fig. 6.21, and using the total area of the grid (16.68 cm²), the plasma density would be 0.87x10¹³ cm⁻³. Using the grid surfaces facing each region, respectively, and the electron temperatures from the probe plots, the calculated plasma densities are $n_1 = 2.44 \times 10^{13}$ cm⁻³, $n_2 = 0.45 \times 10^{13}$ cm⁻³, and $n_3 = 0.77 \times 10^{13}$ cm⁻³. Again the trend is correct, but quantitatively the numbers are quite different. The results are summarized in Table 6.6. Finally, referring back to Fig. 6.21, the electron saturation current is never obtained as V_g is made more and more positive which should occur as discussed in Appendix G. Again, this goes to the heart of the reason the grid does not meet the probe criteria. As V_g is increased the electron current to the grid increases very rapidly (see Figs. 6.21 and 6.22) resulting in a large drain of the discharge current that should be flowing to the collector. The decrease in current alters the discharge parameters and makes any grid current measurements very suspect for calculation of the plasma conditions.

In conclusion, it may be possible to gain an order of magnitude estimate of an electron temperature and plasma density that would represent an average of the conditions in all three regions. However, no quantitative results or information on the difference in T_e 's and n 's in each region can be gained. Lastly, since at positive V_g (and in some cases very small negative voltages) the grid begins to behave as the collector, drawing a large portion of the discharge current, there is no way to gain an estimate on the plasma potential.

Figure 6.22. Calculated Ion Current to the Grid from Region 1 only as a Function of Applied Grid Voltage.

Table 6.6. Probe Calculation Results.

Case	T_{e1} (K)	T_{e2} (K)	T_{e3} (K)	T_e average	n_1 (10^{13} cm^{-3})	n_2 (10^{13} cm^{-3})	n_3 (10^{13} cm^{-3})	n average
1	7385	7785	6661	7277	2.44	0.45	0.77	1.22
2	8107	8261	7435	7934	1.57	0.29	0.53	0.80
		T_e				n		
3		6971				0.87		
<p>1 : Data from probe plots for grid to the grid from each region.</p> <p>2 : Data from calculation with grid floating, $I_g = 2 \text{ A}$.</p> <p>3 : Data from probe plots for total current to the grid.</p>								

7. SUMMARY AND CONCLUSIONS

The work contained in this dissertation focused on four main areas: 1) experimentally investigating the operational characteristics of the Cs-Ba tacitron, 2) Developing a Cs-Ba diode model and use it to investigate the physical processes in a low pressure Cs discharge, 3) Developing a Cs-Ba triode model to investigate the role of the grid in the operation of the device, and 4) Demonstrate the operation of two Cs-Ba tacitrons in the inverter mode.

The operational characteristics of the Cs-Ba tacitron as a switch were investigated experimentally in three modes: (a) breakdown mode, (b) I-V mode, and (c) current modulation mode. It was shown that the hold-off voltage for the Cs-Ba tacitron is as high as 200 V and that during current modulation the discharge could be initiated for a wide range of input conditions at a positive grid pulse of only 30 V. The forward voltage drop of the device strongly depends on the Cs vapor pressure and to a lesser extent on the Ba vapor pressure. Changes in the emitter temperature increase the thermal emission current density and at the same time decreases the forward voltage drop. The forward voltage drop has a linear dependence on the emitter temperature of -0.002 V/K. Also, if the discharge current is circuited limited to a value below the maximum thermal emission current density, the voltage drop is constant.

The experimental data was used to determine the off-time required for successful ignition and the effects of the input conditions on the ignition duty cycle threshold for stable modulation. Based on the agreement between analysis and measurements it is concluded that at the onset of instability, leading to discharge extinguishing, the plasma in the discharge gap is highly ionized and equilibrium exists between the ion flux leaving the gap and the Cs atom flux entering the gap.

Experimental results demonstrate that in the initial discharge ignition, equilibrium occurs within several milliseconds, depending on the leakage of the ions from the discharge gap and the desorption of Cs atoms from the electrode surfaces. The time to reach equilibrium depends not only on the current density and Cs vapor pressure, but also on the voltage drop during discharge (higher voltage drop increases the electron temperature resulting in a lower time for reaching equilibrium).

Results also show that discharge extinguishing during current modulation occurs as a result of the decrease in the heavy component density in the gap during the discharge. Therefore, a definite correlation exists between the discharge current density, Cs vapor pressure, off-time, and on-time. The off-time required to replenish Cs atoms in the gap for subsequent ignition depends on the amplitude of the positive grid pulse, while the ignition delay-time depends on both the off-time and the magnitude of the positive grid potential for ignition. In general, stable current modulation occurs at a device duty cycle $\geq 45\text{-}55\%$. Such a duty cycle is almost independent of the modulation frequency of the grid, but increases as the grid potential for ignition or the Cs pressure is increased. Experimental results show that the net effect of Cs adsorption/desorption on the off-time for ignition during stable current modulation or ignition in the breakdown mode is insignificant. These findings suggest that during discharge, the ion flux to the surface of the electrodes does not practically influence their coverage by Cs atoms. The off-time required for ignition was found to be strongly dependent on the Cs pressure, and to a lesser extent on the emitter temperature, but it is almost independent of the Ba vapor pressure.

Preliminary results of the use of two Cs-Ba tacitrons operating in a "push-pull" configuration to invert a dc signal to a square wave ac signal was also shown. The inverter was operated successfully at a discharge current of up to 13.5 A,

collector voltage of 25 V, conduction voltage drop of ~ 4 V, and modulation frequencies up to 20 kHz.

A model of a low pressure Cs-Ba discharge was developed, which calculates the plasma properties (n , N_k 's, T_e , ΔV_E , ΔV_C , and V_{CE}) for uniform conditions across the discharge gap. The model predictions were in good agreement with experimental results and theoretical calculations of Baksht et al. (1977) for low pressure Cs discharges. Comparing the model to experimental data from a Cs-Ba diode reinforced the validity of the system of equations and also confirmed the assumption that barium has practically no effect on the transport characteristics of the discharge.

It was also shown that including five states of the Cs atom, the $6s^2$ (ground state), $6p^2$ (1st excited state), $5d^2$ (2nd excited state), $7s^2$ (3rd excited state) and $7p^2$ (4th excited state) in the ion generation process improved the accuracy of the model results. Another interesting finding of the model is the characteristics of the mechanism for ion generation. Electron impact excitation, electron impact de-excitation, radiation and electron impact ionization were included in the ion generation model. It was shown that the effective cross-section for creating ions is not only a function of energy, like a typical collision cross-section, but also a function of the Cs pressure and the ratio of the plasma density to neutral Cs atom density. This trend arose because of radiation losses. The higher the Cs pressure the closer the peak effective cross-section is to the value when no radiation is included because the number of target particles for the emitted photon increases and the radiation loss decreases. For a given Cs pressure, if the plasma density is low it is difficult to create the excited atoms and the effective cross-section approaches the value for single-step ionization only (ground state to ion state). As the plasma density becomes too high, the atom density becomes small and the radiation loss increases, which reduces the effective ionization cross-section. The

peak of the curve occurs when the ratio of the plasma density to atom density is approximately equal to one. Once the ion generation process was understood, the diode model was used as a building block for the triode model. The diode model was used in each region of the triode with the appropriate boundary conditions between each region to model the Cs-Ba triode.

The steady-state Cs-Ba triode model is developed that calculates the discharge parameters in each region. Although the steady-state triode model does not do as good a job as the diode model in predicting the experimental results, some very interesting results were found. The model predictions as a function of Cs pressure are fairly accurate but there are some glaring differences with experimental data, especially at low discharge currents. Since the formation of a "virtual" emitter during the decreasing current portion of the I-V curve was confirmed by the diode model, the same assumption is used in the triode model. However, the slope of the negative resistance was much greater in the triode model calculation than in the experimental measurements. It was shown that a virtual emitter might not be forming at the emitter surface, but that the area of the discharge is contracting.

One of the reasons for the developing the triode model is to determine the effect of the grid on the forward voltage drop in the device. When comparing the diode and triode with the same interelectrode gap it was shown that the increase in the voltage drop is due solely to the voltage drop across the grid. For the same gap spacing, the voltage drop in the plasma of the triode is almost identical to that in the diode. Also, it was shown that increasing or decreasing the grid thickness has very little effect on the forward voltage. The model predictions of the effect of the diameter of the grid hole were a little surprising. It was shown that the smaller the grid hole, the lower the forward voltage drop. As the grid diameter was increased, the voltage drop increased, reaching an asymptotic value. The last parameter of

the grid that was investigated was the effect of the grid transparency, which was shown to have the most important effect on the forward voltage drop in the triode device. The results show that the voltage drop increased from 1.29 V to 2.58 V as the grid transparency was decreased from 55 % to 15 %, respectively. It was also shown 70 % of the voltage drop increase in the device was due to an increase in the voltage drop across the grid. Furthermore, the increase in the voltage drop across the grid was almost solely due to an increase in the sheath potential between regions 1 and 2 (ΔV_{g1}).

8. FUTURE WORK

The brunt of the future work consists of developing a theory for the actual extinguishing mechanism. In order to accomplish this task, further experimental and theoretical investigations need to be performed. Experimentally, the work consists of mainly two areas: 1) improved plasma diagnostics and 2) characterization of the affects of the grid geometry on the forward voltage drop. Model calculations that could contribute to a better understanding of the operation of the device include: 3) conversion of the model from the present cylindrical geometry to planar geometry with the appropriate boundary conditions and coupling between regions, 4) investigate other electron distribution functions; such as, shifted Maxwellian, two electron energy groups (bump-on-tail), etc., 5) determine whether at the emitter surface a virtual emitter forms, the discharge area contracts or a combination of both exists for discharge currents below the maximum thermal emission current, and 6) develop a transient triode model to investigate the extinguishing mechanism. Explanations of the above areas are as follows:

1) The need exists for further plasma diagnostics on the Cs-Ba tacitrons, because presently they are limited to I-V characteristics. Using the I-V characteristics it is possible to obtain the trends for changes in the Cs and Ba vapor pressures, the emitter temperature, grid geometry, etc. on the forward voltage drop. However, since there are at least 4 unknowns (ΔV_E , ΔV_C , ΔV_{g1} , and ΔV_{g3}) that contribute to the forward voltage drop, V_{CE} , no detailed information can be gained unless these parameters can be measured. Therefore, some form of plasma diagnostics should be performed; such as, microwaves, lasers, particle beams, spectroscopy, or probes. Langmuir probe measurements are planned to be performed in the near

future on the test device at the ISNPS.

2) Also, detailed experimental results for different grid geometry's must be obtained to verify the calculation results outlined in Chapter 6. This includes measurements for changes in the grid aperture diameter, thickness and transparency for different Cs and Ba vapor pressures. It was shown by the model calculations for the cylindrical geometry that the grid aperture diameter and thickness affect the forward voltage drop very little and the most significant changes occur due to changes in the grid transparencies. These trends need to be verified by experimental results.

3) In order to properly compare model results and experimental measurements, it is important to include the proper geometry. All of the experimental data and model calculations are for a cylindrical geometry triode. The present data is obtained from a planar device. This geometry needs to be incorporated into the steady-state model for comparison to experimental data with careful attention paid to the coupling between regions and the surroundings.

4) The driver for determining the discharge parameters in both the diode and triode models is the ion generation process. The most important parameter affecting this process is the assumption of a Maxwellian distribution function. It would be interesting to investigate the changes in the model results with different distribution functions such as a shifted Maxwellian or a distribution function that includes both a thermalized and beam component. The need for such an exercise will become more apparent after the probe measurements are made.

5) The comparison between the experimental I-V curves and the calculated I-V

curves were inconclusive in determining whether a virtual emitter or discharge contraction takes place at the emitter surface for discharge currents below the maximum thermal emission current. It seems it might be a combination of both processes. Also, the possibility exists that as the discharge current decreases, the distribution function becomes less and less Maxwellian. These areas need to be examined.

6) Finally, a model needs to be developed that has the capability to investigate the extinguishing process in the Cs-Ba triode. Extinguishing of the device occurs over a very short time, less than 5 μs and in some cases less than 1 μs . For these short times, the point mass model is probably not valid and it is necessary to develop a model that can investigate electron-wave interactions. This can be accomplished by developing a lumped transient triode model coupled to a second model that takes into account spatial dependence in the grid aperture.

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APPENDIX A - WORK FUNCTIONS OF REFRACTORY METALS IN CESIUM AND BARIUM VAPORS

The work function of tungsten and molybdenum in a Cs environment as a function of the ratio of the electrode surface temperature to Cs reservoir temperature is shown in Figs. A.1(a) and A.1(b), respectively (Houston 1962). Also, shown on the figures are empirical relationships that fit the experimental data fairly well.

The work function of molybdenum for several different Ba reservoir temperatures is shown in Fig. A.2 (Baksht 1978c).

Figure A.1. Work Function in Cesium Vapor of: (a) tungsten and (b) molybdenum.

Figure A.2. Work Function of Molybdenum in Barium Vapor.

APPENDIX B - CIRCUIT DIAGRAM AND DESCRIPTION

The device is tested in three different modes: 1) breakdown, 2) I-V characteristics and 3) current modulation. In order to test in these three modes without the use of a 200 V/100 A dc power source, two power supplies are used that are connected in parallel. The first source is rated at 200 V/0.2 A and the second is rated at 20 V/100 A. The high voltage/low current source is shown in Fig. B.1 and the low voltage/high current source is shown in Fig. B.2.

The circuit shown in Fig. B.1 is used for investigating the breakdown and current modulation modes only. For operation in the breakdown mode, the relay is positioned such that 1 is connected to 2, which supplies a half-sinusoidal pulse to the collector with an amplitude of 170 V. The diode open circuits during the negative portion of the line voltage. If breakdown occurs, such that the current increases above 0.2 A, the transistor, T_1 , is opened and the current is drawn from the high current source shown in Fig. B.2. The decrease in the source voltage (200 V to 20 V) is not a problem because the forward voltage drop in the conducting state is usually less than 20 V. In current modulation mode, the relay is positioned such that 1 is connected to 3. A full wave bridge is used to rectify the line voltage and charge a capacitor to source an ≈ 200 V pulse to the collector. Again if the current increases above 0.2 A, the high current source is used.

In I-V mode only the high current source is used (see Fig. B.2). A control signal from the computer turns the transistor, T_2 , on which supplies 20 V to the collector. If breakdown occurs, the current is supplied from the capacitor C_2 (1 F, 25 V). The resistor R_2 is used to limit the peak current to any value between 5 A and 100 A.

Figure B.1. High Voltage, Low Current Power Source Used in Breakdown and Current Modulation Modes.

Figure B.2. Low Voltage, High Current Source Used in I-V Mode and During Ignition of the Discharge.

APPENDIX C - CESIUM AND BARIUM VAPOR PRESSURES: RESERVOIR AND GAP

As a convenience to the reader, since throughout the text the Cs and Ba reservoir temperatures and Cs and Ba pressures are used, correlations that relate the Cs reservoir temperature to the Cs pressure and the Ba reservoir temperature to the Ba pressure are given. The relationship for Cs is given as (Hatsopoulos and Gyftopoulos 1973):

$$P_{Cs} = \frac{2.45 \times 10^8 \exp\left(\frac{-8910}{T_{Cs}}\right)}{\sqrt{T_{Cs}}} \quad (C.1)$$

However, since there is a temperature gradient that exists between the Cs reservoir and the interelectrode gap, the Cs pressure in the gap is not equal to the Cs pressure in the reservoir. The relationship between the flux leaving the Cs reservoir and the flux entering the interelectrode gap assuming molecular flow is:

$$\frac{1}{4} N_{gap} \bar{v}_{gap} = \frac{1}{4} N_{Cs} \bar{v}_{Cs} \quad (C.2)$$

$$\frac{1}{4} N_{gap} \sqrt{\frac{8 k_B T_{gap}}{\pi M_i}} = \frac{1}{4} N_{Cs} \sqrt{\frac{8 k_B T_{Cs}}{\pi M_i}} \quad (C.3)$$

Since the pressure is equal to:

$$p = N k_B T, \quad (C.4)$$

the densities in Eqn. (C.3) can be replaced by pressures, which results in the

following relationship:

$$P_{\text{gap}} = P_{\text{Cs}} \sqrt{\frac{T_{\text{gap}}}{T_{\text{Cs}}}} \quad (\text{C.5})$$

Recall from Chapter 4 that $T_{\text{gap}} = (T_{\text{E}} + T_{\text{C}})/2$. For typical emitter and collector temperatures of 1290 °C and 600 °C, respectively, the Cs pressure in the reservoir and interelectrode gap as a function of the Cs reservoir temperature are plotted in Fig. C.1.

The equation that relates the Ba pressure in the reservoir to the Ba reservoir temperature is given as (Liempt 1936):

$$\log(P_{\text{Ba}}) = 7.83 - \frac{9730}{T_{\text{Ba}}} \quad (\text{C.6})$$

Using Eqn. (C.5) to relate the Ba reservoir and interelectrode gap pressures, the curves shown in Fig. C.2 are obtained.

Figure C.1. Cs Pressure in the Reservoir and Gap as a Function of Cs Reservoir Temperature.

Figure C.2. Ba Pressure in the Reservoir and Gap as a Function of Ba Reservoir Temperature.

APPENDIX D - RATE FACTORS

When describing the conditions of a low pressure discharge or any discharge for that manner, the generation of ions is of prime importance. The rate at which ions are created affects both the stationary and transient conditions of a discharge and in general has the following form:

$$R_i = n N_a \sigma(\epsilon) v_e \quad (D.1)$$

where n is the plasma density (ions and electrons), N_a the atom density, $\sigma(\epsilon)$ is the cross-section (probability) of the collision process, which is a function of the electron energy, and v_e is the velocity of the electrons. Since the electrons in a typical plasma are not monoenergetic, but consist of a distribution of velocities, it is typical when modeling discharges to assume the electron distribution is Maxwellian. This is the assumption in the modeling of a low pressure Cs discharge in this work. The following relationship is the Maxwellian electron distribution function. A Maxwellian electron distribution, $f(\epsilon)$, is a function of energy and is written as (Chen 1984):

$$f(\epsilon) = \frac{2}{\sqrt{\pi}} \left(\frac{1}{k_B T_e} \right)^{3/2} \epsilon^{1/2} \exp \left(\frac{-\epsilon}{k_B T_e} \right) \quad (D.2)$$

where ϵ is the energy of the electron in joules (see Fig. 5.3). As mentioned in Chapter 5, the rate factors, which are a measure of the likelihood of a collision, are found by integrating the product of the distribution function and cross-section from the threshold energy to infinity. The threshold energy is determined by the

collision process. The following equation is used to calculate the rate factors for excitation and ionization collision processes:

$$\langle \sigma v_e \rangle_{kj} = \int_{\epsilon_j - \epsilon_k}^{\infty} \sigma_{kj}(\epsilon) \sqrt{\frac{2\epsilon}{m_e}} f(\epsilon) d\epsilon \quad (D.3)$$

The electron impact superelastic de-excitation rate factors are calculated from the excitation rate factors assuming a Maxwell-Boltzmann distribution (Verdeyen 1982) as:

$$\langle \sigma v_e \rangle_{jk} = \frac{g_k}{g_j} \langle \sigma v_e \rangle_{kj} \exp\left(\frac{\epsilon_j - \epsilon_k}{k_B T_e}\right) \quad (D.4)$$

where g_k and g_j are the statistical weights (or degeneracy's) for the k_{th} and j_{th} levels. The expressions for all 21 rate factors used in this model are listed below. In the following expressions, the variables Y and Z are equal to:

$$Y = k_B T_e \text{ and } Z = 10^6 \sqrt{\frac{8}{\pi k_B T_e m_e}} \quad (D.5)$$

The rate factor for excitation from the ground state to the 1st excited state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{01} = & Z \{ (4.69Y^2 + 1.56 \times 10^{-18}Y) \exp(-2.22 \times 10^{-19}/Y) \\ & + (-3.95Y^2 - 1.14 \times 10^{-18}Y) \exp(-5.75 \times 10^{-19}/Y) \\ & + (-0.62Y^2 - 2.69 \times 10^{-19}Y) \exp(-8.65 \times 10^{-19}/Y) \\ & + (-0.30Y^2 - 1.94 \times 10^{-19}Y) \exp(-1.30 \times 10^{-18}/Y) \\ & + (0.18Y^2 - 6.41 \times 10^{-19}Y - 2.11 \times 10^{-36}) \exp(-2.46 \times 10^{-18}/Y) \}. \end{aligned} \quad (D.6)$$

The rate factor for excitation from the ground state to the 2nd excited state is:

$$\begin{aligned}
 \langle \sigma v_e \rangle_{02} = & Z \{ (-2.71Y^2 + 3.92 \times 10^{-19}Y) (-2.89 \times 10^{-19}/Y - 1) + 1.13 \times 10^{-37} \\
 & \exp(-2.89 \times 10^{-19}/Y) + ((1.49Y^2 - 3.7 \times 10^{-19}Y) (-4.97 \times 10^{-19}/Y - 1) - 1.84 \times 10^{-37} \\
 & \exp(-4.97 \times 10^{-19}/Y) + ((0.81Y^2 - 2.56 \times 10^{-19}Y) (-6.41 \times 10^{-19}/Y - 1) - 1.66 \times 10^{-37} \\
 & \exp(-6.41 \times 10^{-19}/Y) + ((0.23Y^2 - 1.05 \times 10^{-19}Y) (-8.81 \times 10^{-19}/Y - 1) - 8.88 \times 10^{-38} \\
 & \exp(-8.81 \times 10^{-19}/Y) + ((0.22Y^2 - 1.23 \times 10^{-19}Y) (-1.1 \times 10^{-18}/Y - 1) - 1.35 \times 10^{-37} \\
 & \exp(-1.1 \times 10^{-18}/Y) + ((0.1Y^2 - 6.46 \times 10^{-20}Y) (-1.65 \times 10^{-18}/Y - 1) - 1.39 \times 10^{-37} \\
 & \exp(-1.65 \times 10^{-18}/Y) + ((-0.06Y^2 + 9.7 \times 10^{-20}Y) (-3.28 \times 10^{-18}/Y - 1) + 3.18 \times 10^{-37} \\
 & \exp(-3.28 \times 10^{-18}/Y) + ((-0.05Y^2 + 1.31 \times 10^{-19}Y) (-5.61 \times 10^{-18}/Y - 1) + 7.36 \times 10^{-37} \\
 & \exp(-5.61 \times 10^{-18}/Y) + ((-0.04Y^2 + 2.98 \times 10^{-19}Y) (-9.61 \times 10^{-18}/Y - 1) + 1.62 \times 10^{-36} \\
 & \exp(-9.61 \times 10^{-18}/Y) \}. \quad (D.7)
 \end{aligned}$$

The rate factor for excitation from the ground state to the 3rd excited state is:

$$\begin{aligned}
 \langle \sigma v_e \rangle_{03} = & Z \{ ((-1.21Y^2 + 2.23 \times 10^{-19}Y) (-3.69 \times 10^{-19}/Y - 1) + 8.23 \times 10^{-38} \\
 & \exp(-3.69 \times 10^{-19}/Y) + ((0.59Y^2 - 1.88 \times 10^{-19}Y) (-6.41 \times 10^{-19}/Y - 1) - 1.21 \times 10^{-37} \\
 & \exp(-6.41 \times 10^{-19}/Y) + ((0.46Y^2 - 2.04 \times 10^{-19}Y) (-8.97 \times 10^{-19}/Y - 1) - 1.83 \times 10^{-37} \\
 & \exp(-8.97 \times 10^{-19}/Y) + ((0.18Y^2 - 1.01 \times 10^{-19}Y) (-1.19 \times 10^{-18}/Y - 1) - 1.3 \times 10^{-37} \\
 & \exp(-1.19 \times 10^{-18}/Y) + ((0.05Y^2 - 4.14 \times 10^{-20}Y) (-1.6 \times 10^{-18}/Y - 1) - 6.65 \times 10^{-38} \\
 & \exp(-1.06 \times 10^{-18}/Y) + ((-0.05Y^2 + 1.2 \times 10^{-19}Y) (-5.22 \times 10^{-18}/Y - 1) + 6.27 \times 10^{-37} \\
 & \exp(-5.22 \times 10^{-18}/Y) + ((-0.02Y^2 + 1.92 \times 10^{-19}Y) (-9.61 \times 10^{-18}/Y - 1) + 8.87 \times 10^{-37} \\
 & \exp(-9.61 \times 10^{-18}/Y) \}. \quad (D.8)
 \end{aligned}$$

The rate factor for excitation from the ground state to the 4th excited state is:

$$\begin{aligned}
\langle \sigma v_e \rangle_{04} = & Z \{ ((-0.69Y^2 + 1.54 \times 10^{-19}Y)(-4.34 \times 10^{-19}/Y - 1) + 6.68 \times 10^{-38}) \\
& \exp(-4.34 \times 10^{-19}/Y) + ((0.37Y^2 - 1.54 \times 10^{-19}Y)(-8.01 \times 10^{-19}/Y - 1) - 1.23 \times 10^{-37}) \\
& \exp(-8.01 \times 10^{-19}/Y) + ((0.30Y^2 - 1.81 \times 10^{-19}Y)(-1.20 \times 10^{-18}/Y - 1) - 2.18 \times 10^{-37}) \\
& \exp(-1.20 \times 10^{-18}/Y) + ((0.07Y^2 - 5.38 \times 10^{-20}Y)(-1.63 \times 10^{-18}/Y - 1) - 8.80 \times 10^{-38}) \\
& \exp(-1.63 \times 10^{-18}/Y) + ((0.02Y^2 + 6.16 \times 10^{-20}Y)(-5.29 \times 10^{-18}/Y - 1) + 3.27 \times 10^{-37}) \\
& \exp(-5.29 \times 10^{-18}/Y) + ((-0.02Y^2 + 1.73 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 8.96 \times 10^{-37}) \\
& \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.9}
\end{aligned}$$

The rate factor for excitation from the ground state to the ion state is:

$$\begin{aligned}
\langle \sigma v_e \rangle_{05} = & Z \{ (0.26Y^2 + 8.01 \times 10^{-20}Y) \exp(-6.23 \times 10^{-19}/Y) \\
& + (-0.21Y^2 - 1.19 \times 10^{-19}Y) \exp(-1.13 \times 10^{-18}/Y) + (-0.05Y^2 - 3.32 \times 10^{-20}Y) \\
& \exp(-1.46 \times 10^{-18}/Y) + (+0.1Y^2 + 9.92 \times 10^{-20}Y) \exp(-1.95 \times 10^{-18}/Y) \\
& + (-0.14Y^2 - 1.7 \times 10^{-19}Y) \exp(-2.45 \times 10^{-18}/Y) + (0.06Y^2 + 1.02 \times 10^{-19}Y) \\
& \exp(-3.23 \times 10^{-18}/Y) + (-0.03Y^2 - 1.6 \times 10^{-19}Y - 4.65 \times 10^{-37}) \exp(-4.62 \times 10^{-18}/Y) \} \tag{D.10}
\end{aligned}$$

The rate factor for deexcitation from the 1st excited state to the ground state is:

$$\langle \sigma v_e \rangle_{10} = (g_0/g_1) \exp(qE_{10}/Y) \langle \sigma v_e \rangle_{01}. \tag{D.11}$$

The rate factor for excitation from the 1st excited state to the 2nd excited state is:

$$\begin{aligned}
\langle \sigma v_e \rangle_{12} = & Z \{ ((-137.45Y^2 + 4.4 \times 10^{-18}Y)(-6.41 \times 10^{-20}/Y - 1) + 2.82 \times 10^{-37}) \\
& \exp(-6.41 \times 10^{-20}/Y) + ((118.93Y^2 - 1.11 \times 10^{-17}Y)(-1.87 \times 10^{-19}/Y - 1) - 2.08 \times 10^{-36}) \\
& \exp(-1.87 \times 10^{-19}/Y) + ((22.68Y^2 - 2.57 \times 10^{-18}Y)(-2.27 \times 10^{-19}/Y - 1) - 5.84 \times 10^{-37}) \\
& \exp(-2.27 \times 10^{-19}/Y) + ((9.25Y^2 - 1.35 \times 10^{-18}Y)(-2.91 \times 10^{-19}/Y - 1) - 3.92 \times 10^{-37})
\end{aligned}$$

$$\begin{aligned}
& \exp(-2.91 \times 10^{-19}/Y) + ((-4.66Y^2 + 1.57 \times 10^{-18}Y)(-6.74 \times 10^{-19}/Y - 1) + 1.06 \times 10^{-36}) \\
& \exp(-6.74 \times 10^{-19}/Y) + ((-3.14Y^2 + 1.45 \times 10^{-18}Y)(-9.28 \times 10^{-19}/Y - 1) + 1.35 \times 10^{-36}) \\
& \exp(-9.2 \times 10^{-19}/Y) + ((-3.23Y^2 + 2.28 \times 10^{-18}Y)(-1.42 \times 10^{-18}/Y - 1) + 3.23 \times 10^{-36}) \\
& \exp(-1.42 \times 10^{-18}/Y) + ((-1.54Y^2 + 1.84 \times 10^{-18}Y)(-2.40 \times 10^{-18}/Y - 1) + 4.43 \times 10^{-36}) \\
& \exp(-2.4 \times 10^{-18}/Y) + ((-0.62Y^2 + 1.57 \times 10^{-18}Y)(-5.06 \times 10^{-18}/Y - 1) + 7.94 \times 10^{-36}) \\
& \exp(-5.06 \times 10^{-18}/Y) + ((-0.24Y^2 + 1.91 \times 10^{-18}Y)(-9.61 \times 10^{-18}/Y - 1) + 1.13 \times 10^{-35}) \\
& \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.12}
\end{aligned}$$

The rate factor for excitation from the 1st excited state to the 3rd excited state is:

$$\begin{aligned}
\langle \sigma v_e \rangle_{13} = Z \{ & ((-18.87Y^2 + 1.36 \times 10^{-18}Y)(-1.44 \times 10^{-19}/Y - 1) + 1.96 \times 10^{-37}) \\
& \exp(-1.44 \times 10^{-19}/Y) + ((12.12Y^2 - 1.66 \times 10^{-18}Y)(-2.74 \times 10^{-19}/Y - 1) - 4.54 \times 10^{-37}) \\
& \exp(-2.74 \times 10^{-19}/Y) + ((7.74Y^2 - 1.65 \times 10^{-18}Y)(-4.27 \times 10^{-19}/Y - 1) - 7.06 \times 10^{-37}) \\
& \exp(-4.27 \times 10^{-19}/Y) + ((-0.47Y^2 + 3.82 \times 10^{-19}Y)(-1.63 \times 10^{-18}/Y - 1) + 6.25 \times 10^{-37}) \\
& \exp(-1.63 \times 10^{-18}/Y) + ((-0.43Y^2 + 8.85 \times 10^{-19}Y)(-4.09 \times 10^{-18}/Y - 1) + 3.58 \times 10^{-36}) \\
& \exp(-4.09 \times 10^{-18}/Y) + ((-0.08Y^2 + 6.83 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 3.89 \times 10^{-36}) \\
& \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.13}
\end{aligned}$$

The rate factor for excitation from the 1st excited state to the 4th excited state is:

$$\begin{aligned}
\langle \sigma v_e \rangle_{14} = Z \{ & ((-6.15Y^2 + 6.41 \times 10^{-19}Y)(-2.08 \times 10^{-19}/Y - 1) + 1.33 \times 10^{-37}) \\
& \exp(-2.08 \times 10^{-19}/Y) + ((3.68Y^2 - 8.89 \times 10^{-19}Y)(-4.01 \times 10^{-19}/Y - 1) - 2.96 \times 10^{-37}) \\
& \exp(-4.01 \times 10^{-19}/Y) + ((2.81Y^2 - 7.49 \times 10^{-19}Y)(-6.41 \times 10^{-19}/Y - 1) - 5.76 \times 10^{-37}) \\
& \exp(-6.41 \times 10^{-19}/Y) + ((-0.09Y^2 + 1.08 \times 10^{-19}Y)(-2.4 \times 10^{-18}/Y - 1) + 2.59 \times 10^{-37}) \\
& \exp(-2.4 \times 10^{-18}/Y) + ((-0.2Y^2 + 4.82 \times 10^{-19}Y)(-4.81 \times 10^{-18}/Y - 1) + 5.34 \times 10^{-37}) \\
& \exp(-4.81 \times 10^{-18}/Y) + ((-0.05Y^2 + 4.07 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 2.14 \times 10^{-36}) \\
& \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.14}
\end{aligned}$$

The rate factor for excitation from the 1st excited state to the ion state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{15} = Z \{ & (1.42Y^2 + 2.85 \times 10^{-19}Y) \exp(-4.01 \times 10^{-19}/Y) + (-0.95Y^2 - 3.8 \times 10^{-19}Y) \\ & \exp(-8.01 \times 10^{-19}/Y) + (-0.27Y^2 - 1.63 \times 10^{-19}Y) \exp(-1.2 \times 10^{-18}/Y) \\ & + (0.5Y^2 + 4.66 \times 10^{-19}Y) \exp(-1.85 \times 10^{-18}/Y) + (-0.93Y^2 - 1.04 \times 10^{-18}Y) \\ & \exp(-2.22 \times 10^{-18}/Y) + (0.38Y^2 + 5.75 \times 10^{-19}Y) \exp(-3.0 \times 10^{-18}/Y) \\ & + (-0.11Y^2 - 2.30 \times 10^{-19}Y) \exp(-4.18 \times 10^{-18}/Y) + (-0.05Y^2 - 7.12 \times 10^{-19}Y \\ & - 2.75 \times 10^{-36}) \exp(-4.61 \times 10^{-18}/Y) \}. \end{aligned} \quad (D.15)$$

The rate factor for deexcitation from the 2nd excited state to the 1st excited state is:

$$\langle \sigma v_e \rangle_{21} = (g_1/g_2) \exp(qE_{21}/Y) \langle \sigma v_e \rangle_{12}. \quad (D.16)$$

The rate factor for excitation from the 2nd excited state to the 3rd excited state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{23} = Z \{ & ((-87.15Y^2 + 3.49 \times 10^{-18}Y) (-8.01 \times 10^{-20}/Y - 1) + 2.80 \times 10^{-37}) \\ & \exp(-8.01 \times 10^{-20}/Y) + ((59.42Y^2 - 5.75 \times 10^{-18}Y) (-1.94 \times 10^{-19}/Y - 1) - 1.11 \times 10^{-36}) \\ & \exp(-1.94 \times 10^{-19}/Y) + ((22.89Y^2 - 2.67 \times 10^{-18}Y) (-2.34 \times 10^{-19}/Y - 1) - 6.24 \times 10^{-37}) \\ & \exp(-2.34 \times 10^{-19}/Y) + ((6.28Y^2 - 1.29 \times 10^{-18}Y) (-2.94 \times 10^{-19}/Y - 1) - 2.7 \times 10^{-37}) \\ & \exp(-2.94 \times 10^{-19}/Y) + ((6.0Y^2 - 7.21 \times 10^{-19}Y) (-3.6 \times 10^{-19}/Y - 1) - 3.9 \times 10^{-37}) \\ & \exp(-3.6 \times 10^{-19}/Y) + ((-2.3Y^2 + 8.45 \times 10^{-19}Y) (-7.34 \times 10^{-19}/Y - 1) + 6.21 \times 10^{-37}) \\ & \exp(-7.34 \times 10^{-19}/Y) + ((-1.73Y^2 + 8.92 \times 10^{-19}Y) (-1.03 \times 10^{-18}/Y - 1) + 9.16 \times 10^{-37}) \\ & \exp(-1.03 \times 10^{-18}/Y) + ((-1.58Y^2 + 1.17 \times 10^{-18}Y) (-1.48 \times 10^{-18}/Y - 1) + 1.73 \times 10^{-36}) \\ & \exp(-1.48 \times 10^{-18}/Y) + ((-1.08Y^2 + 1.3 \times 10^{-18}Y) (-2.4 \times 10^{-18}/Y - 1) + 3.12 \times 10^{-36}) \\ & \exp(-2.4 \times 10^{-18}/Y) + ((-0.57Y^2 + 1.33 \times 10^{-18}Y) (-4.65 \times 10^{-18}/Y - 1) + 6.19 \times 10^{-36}) \\ & \exp(-4.65 \times 10^{-18}/Y) + ((-0.18Y^2 + 1.41 \times 10^{-18}Y) (-9.61 \times 10^{-18}/Y - 1) + 8.1 \times 10^{-36}) \} \end{aligned}$$

$$\exp(-9.61 \times 10^{-18}/Y)\}. \quad (D.17)$$

The rate factor for excitation from the 2nd excited state to the 4th excited state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{24} = & Z \{ ((-17.35Y^2 + 1.25 \times 10^{-18}Y)(-1.44 \times 10^{-19}/Y - 1) + 1.8 \times 10^{-37}) \\ & \exp(-1.44 \times 10^{-19}/Y) + ((9.92Y^2 - 1.42 \times 10^{-18}Y)(-2.87 \times 10^{-19}/Y - 1) - 4.08 \times 10^{-37}) \\ & \exp(-2.87 \times 10^{-19}/Y) + ((5.66Y^2 - 1.12 \times 10^{-18}Y)(-3.94 \times 10^{-19}/Y - 1) - 4.39 \times 10^{-37}) \\ & \exp(-3.94 \times 10^{-19}/Y) + ((2.98Y^2 - 8.16 \times 10^{-19}Y)(-5.47 \times 10^{-19}/Y - 1) - 4.47 \times 10^{-37}) \\ & \exp(-5.47 \times 10^{-19}/Y) + ((-0.7Y^2 + 5.59 \times 10^{-19}Y)(-1.6 \times 10^{-18}/Y - 1) + 8.95 \times 10^{-37}) \\ & \exp(-1.6 \times 10^{-18}/Y) + ((-0.4Y^2 + 7.56 \times 10^{-19}Y)(-3.74 \times 10^{-18}/Y - 1) + 2.83 \times 10^{-36}) \\ & \exp(-3.74 \times 10^{-18}/Y) + ((-0.11Y^2 + 7.88 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 4.85 \times 10^{-36}) \\ & \exp(-9.61 \times 10^{-18}/Y) \}. \end{aligned} \quad (D.18)$$

The rate factor for excitation from the 2nd excited state to the ion state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{25} = & Z \{ ((-1.96Y^2 + 3.28 \times 10^{-19}Y)(-3.35 \times 10^{-19}/Y - 1) + 1.1 \times 10^{-37}) \\ & \exp(-3.35 \times 10^{-19}/Y) + ((1.15Y^2 - 2.87 \times 10^{-19}Y)(-4.97 \times 10^{-19}/Y - 1) - 1.43 \times 10^{-37}) \\ & \exp(-4.97 \times 10^{-19}/Y) + ((0.48Y^2 - 1.85 \times 10^{-19}Y)(-7.69 \times 10^{-19}/Y - 1) - 1.42 \times 10^{-37}) \\ & \exp(-7.69 \times 10^{-19}/Y) + ((0.33Y^2 - 1.76 \times 10^{-19}Y)(-1.07 \times 10^{-18}/Y - 1) - 1.89 \times 10^{-37}) \\ & \exp(-1.07 \times 10^{-18}/Y) + ((0.1Y^2 - 8.4 \times 10^{-20}Y)(-1.65 \times 10^{-18}/Y - 1) - 1.39 \times 10^{-37}) \\ & \exp(-1.65 \times 10^{-18}/Y) + ((-0.08Y^2 + 1.54 \times 10^{-19}Y)(-4.01 \times 10^{-18}/Y - 1) + 6.19 \times 10^{-37}) \\ & \exp(-4.01 \times 10^{-18}/Y) + ((-0.03Y^2 + 2.50 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 1.16 \times 10^{-36}) \\ & \exp(-9.61 \times 10^{-18}/Y) \}. \end{aligned} \quad (D.19)$$

The rate factor for deexcitation from the 3rd excited state to the 1st excited state is:

$$\langle \sigma v_e \rangle_{31} = (g_1/g_3) \exp(qE_{31}/Y) \langle \sigma v_e \rangle_{13}. \quad (D.20)$$

The rate factor for excitation from the 3rd excited state to the 4th excited state is:

$$\begin{aligned}
 \langle \sigma v_e \rangle_{34} = & Z \{ ((-155.33Y^2 + 4.98 \times 10^{-18}Y)(-6.41 \times 10^{-20}/Y - 1) + 3.19 \times 10^{-37}) \\
 & \exp(-6.41 \times 10^{-20}/Y) + ((140.71Y^2 - 1.2 \times 10^{-17}Y)(-1.71 \times 10^{-19}/Y - 1) - 2.06 \times 10^{-36}) \\
 & \exp(-1.71 \times 10^{-19}/Y) + ((29.2Y^2 - 3.75 \times 10^{-18}Y)(-2.57 \times 10^{-19}/Y - 1) - 9.64 \times 10^{-37}) \\
 & \exp(-2.57 \times 10^{-19}/Y) + ((-2.25Y^2 + 5.4 \times 10^{-19}Y)(-4.81 \times 10^{-19}/Y - 1) + 2.56 \times 10^{-37}) \\
 & \exp(-4.81 \times 10^{-19}/Y) + ((-5.52Y^2 + 2.12 \times 10^{-18}Y)(-7.69 \times 10^{-19}/Y - 1) + 1.63 \times 10^{-36}) \\
 & \exp(-7.69 \times 10^{-19}/Y) + ((-2.67Y^2 + 1.51 \times 10^{-18}Y)(-1.13 \times 10^{-18}/Y - 1) + 1.7 \times 10^{-36}) \\
 & \exp(-1.13 \times 10^{-18}/Y) + ((-2.0Y^2 + 1.6 \times 10^{-18}Y)(-1.6 \times 10^{-18}/Y - 1) + 2.57 \times 10^{-36}) \\
 & \exp(-1.6 \times 10^{-18}/Y) + ((-1.17Y^2 + 1.49 \times 10^{-18}Y)(-2.55 \times 10^{-18}/Y - 1) + 3.8 \times 10^{-36}) \\
 & \exp(-2.55 \times 10^{-18}/Y) + ((-0.78Y^2 + 1.85 \times 10^{-18}Y)(-4.76 \times 10^{-18}/Y - 1) + 8.79 \times 10^{-36}) \\
 & \exp(-4.76 \times 10^{-18}/Y) + ((-0.2Y^2 + 1.69 \times 10^{-18}Y)(-9.61 \times 10^{-18}/Y - 1) + 9.16 \times 10^{-36}) \\
 & \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.21}
 \end{aligned}$$

The rate factor for excitation from the 3rd excited state to the ion state is:

$$\begin{aligned}
 \langle \sigma v_e \rangle_{35} = & Z \{ ((-4.3Y^2 + 5.4 \times 10^{-19}Y)(-2.55 \times 10^{-19}/Y - 1) + 1.4 \times 10^{-37}) \\
 & \exp(-2.55 \times 10^{-19}/Y) + ((3.12Y^2 - 6.88 \times 10^{-19}Y)(-4.41 \times 10^{-19}/Y - 1) - 3.03 \times 10^{-37}) \\
 & \exp(-4.41 \times 10^{-19}/Y) + ((0.58Y^2 - 1.81 \times 10^{-19}Y)(-6.17 \times 10^{-19}/Y - 1) - 1.11 \times 10^{-37}) \\
 & \exp(-6.17 \times 10^{-19}/Y) + ((0.6Y^2 - 2.39 \times 10^{-19}Y)(-8.01 \times 10^{-19}/Y - 1) - 1.92 \times 10^{-37}) \\
 & \exp(-8.01 \times 10^{-19}/Y) + ((0.22Y^2 - 1.21 \times 10^{-19}Y)(-1.09 \times 10^{-18}/Y - 1) - 1.32 \times 10^{-37}) \\
 & \exp(-1.09 \times 10^{-18}/Y) + ((-0.1Y^2 + 1.28 \times 10^{-19}Y)(-2.44 \times 10^{-18}/Y - 1) + 3.1 \times 10^{-37}) \\
 & \exp(-2.44 \times 10^{-18}/Y) + ((-0.06Y^2 + 1.54 \times 10^{-19}Y)(-4.81 \times 10^{-18}/Y - 1) + 1.36 \times 10^{-36}) \\
 & \exp(-4.81 \times 10^{-18}/Y) + ((-0.05Y^2 + 4.0 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 2.5 \times 10^{-38}) \\
 & \exp(-9.61 \times 10^{-18}/Y) \}. \tag{D.22}
 \end{aligned}$$

The rate factor for deexcitation from the 4th excited state to the ground state is:

$$\langle \sigma v_e \rangle_{40} = (g_0/g_4) \exp(qE_{40}/Y) \langle \sigma v_e \rangle_{04}. \quad (D.23)$$

The rate factor for deexcitation from the 4th excited state to the 2nd excited state is:

$$\langle \sigma v_e \rangle_{42} = (g_2/g_4) \exp(qE_{42}/Y) \langle \sigma v_e \rangle_{24}. \quad (D.24)$$

The rate factor for deexcitation from the 4th excited state to the 3rd excited state is:

$$\langle \sigma v_e \rangle_{43} = (g_3/g_4) \exp(qE_{43}/Y) \langle \sigma v_e \rangle_{34}. \quad (D.25)$$

The rate factor for excitation from the 4th excited state to the ion state is:

$$\begin{aligned} \langle \sigma v_e \rangle_{45} = & Z \{ ((-8.07Y^2 + 7.56 \times 10^{-19}Y)(-1.89 \times 10^{-19}/Y - 1) + 1.44 \times 10^{-37}) \\ & \exp(-1.89 \times 10^{-19}/Y) + ((5.68Y^2 - 1.05 \times 10^{-18}Y)(-3.69 \times 10^{-19}/Y - 1) - 5.48 \times 10^{-37}) \\ & \exp(-3.69 \times 10^{-19}/Y) + ((1.45Y^2 - 3.94 \times 10^{-19}Y)(-5.45 \times 10^{-19}/Y - 1) - 2.15 \times 10^{-37}) \\ & \exp(-5.45 \times 10^{-19}/Y) + ((0.94Y^2 - 3.15 \times 10^{-19}Y)(-6.73 \times 10^{-19}/Y - 1) - 2.12 \times 10^{-37}) \\ & \exp(-6.73 \times 10^{-19}/Y) + ((0.545Y^2 - 2.31 \times 10^{-19}Y)(-8.33 \times 10^{-19}/Y - 1) - 1.93 \times 10^{-37}) \\ & \exp(-8.33 \times 10^{-19}/Y) + ((-0.18Y^2 + 1.49 \times 10^{-19}Y)(-1.63 \times 10^{-18}/Y - 1) + 2.4 \times 10^{-37}) \\ & \exp(-1.63 \times 10^{-18}/Y) + ((-0.17Y^2 + 2.07 \times 10^{-19}Y)(-2.48 \times 10^{-18}/Y - 1) + 5.14 \times 10^{-37}) \\ & \exp(-2.48 \times 10^{-18}/Y) + ((-0.12Y^2 + 2.97 \times 10^{-19}Y)(-4.81 \times 10^{-18}/Y - 1) + 1.43 \times 10^{-36}) \\ & \exp(-4.81 \times 10^{-18}/Y) + ((-0.08Y^2 + 5.8 \times 10^{-19}Y)(-9.61 \times 10^{-18}/Y - 1) + 3.84 \times 10^{-36}) \\ & \exp(-9.61 \times 10^{-18}/Y) \}. \quad (D.26) \end{aligned}$$

APPENDIX E -DERIVATION OF COEFFICIENTS DESCRIBING CESIUM ATOM DENSITY BALANCE

When determining the density of each excited state of the Cs atom it is necessary to develop rate equations for each state with the appropriate source and sink terms. These rate equations are shown in Eqns. (5.7a and 5.7b) in a condensed form. The following are the long forms of the excited state rate equations.

$$\begin{aligned} \frac{dN_0}{dt} + \left\{ n(<\sigma v_e>_{01} + <\sigma v_e>_{02} + <\sigma v_e>_{03} + <\sigma v_e>_{04} + <\sigma v_e>_{05}) + \frac{v_a S_S}{4 V} \right\} N_0 \\ = (n<\sigma v_e>_{10} + A_{10})N_1 + (n<\sigma v_e>_{40} + A_{40})N_4 + \frac{v_a S_S}{4 V} N_{a0} \end{aligned} \quad (E.1)$$

$$\begin{aligned} \frac{dN_1}{dt} + \left\{ n(<\sigma v_e>_{10} + <\sigma v_e>_{12} + <\sigma v_e>_{13} + <\sigma v_e>_{14} + <\sigma v_e>_{15}) + A_{10} + \frac{v_a S_S}{4 V} \right\} N_1 \\ = (n<\sigma v_e>_{01})N_0 + (n<\sigma v_e>_{21} + A_{21})N_2 + (n<\sigma v_e>_{31} + A_{31})N_3 \end{aligned} \quad (E.2)$$

$$\begin{aligned} \frac{dN_2}{dt} + \left\{ n(<\sigma v_e>_{21} + <\sigma v_e>_{23} + <\sigma v_e>_{24} + <\sigma v_e>_{25}) + A_{21} + \frac{v_a S_S}{4 V} \right\} N_2 \\ = (n<\sigma v_e>_{02})N_0 + (n<\sigma v_e>_{12})N_1 + (n<\sigma v_e>_{42} + A_{42})N_4 \end{aligned} \quad (E.3)$$

$$\begin{aligned} \frac{dN_3}{dt} + \left\{ n(<\sigma v_e>_{31} + <\sigma v_e>_{34} + <\sigma v_e>_{35}) + A_{31} + \frac{v_a S_S}{4 V} \right\} N_3 \\ = (n<\sigma v_e>_{03})N_0 + (n<\sigma v_e>_{13})N_1 + (n<\sigma v_e>_{23})N_2 + (n<\sigma v_e>_{43} + A_{43})N_4 \end{aligned} \quad (E.4)$$

$$\begin{aligned} \frac{dN_4}{dt} + \left\{ n(<\sigma v_e>_{40} + <\sigma v_e>_{42} + <\sigma v_e>_{43} + <\sigma v_e>_{45}) + A_{40} + A_{42} + A_{43} + \frac{v_a S_S}{4 V} \right\} N_4 \\ = (n <\sigma v_e>_{04}) N_0 + (n <\sigma v_e>_{14}) N_1 + (n <\sigma v_e>_{24}) N_2 + (n <\sigma v_e>_{34}) N_3 \end{aligned} \quad (E.5)$$

In developing the relationships for the X-coefficients, the density of the 4th excited state, N_4 , is solved for first by setting the transient term to zero. As can be seen the expression obtained for N_4 is:

$$N_4 = \frac{<\sigma v_e>_{04} N_0 + <\sigma v_e>_{14} N_1 + <\sigma v_e>_{24} N_2 + <\sigma v_e>_{34} N_3}{<\sigma v_e>_{40} + <\sigma v_e>_{42} + <\sigma v_e>_{43} + <\sigma v_e>_{45} + \frac{A_{40} + A_{42} + A_{43}}{n} + \frac{v_a S_S}{4 V n}} \quad (E.6)$$

The denominator of Eqn. (E.6) is the first of the X-coefficients, X_1 . The remaining X-coefficients were determined in a similar fashion by solving for N_3 in Eqn. (E.4) and substituting for N_4 .

$$X_1 = <\sigma v_e>_{40} + <\sigma v_e>_{42} + <\sigma v_e>_{43} + <\sigma v_e>_{45} + \frac{A_{40} + A_{42} + A_{43}}{n} + \frac{v_a S_S}{4 n} \quad (E.7)$$

$$X_2 = <\sigma v_e>_{31} + <\sigma v_e>_{35} - \left(\frac{A_{43}}{n} + <\sigma v_e>_{43} \right) \frac{<\sigma v_e>_{34}}{X_1} + \frac{A_{31}}{n} + \frac{v_a S_S}{4 n} \quad (E.8)$$

$$\begin{aligned}
X_3 = & \langle \sigma v_e \rangle_{21} + \langle \sigma v_e \rangle_{25} + \frac{A_{21}}{n} - \frac{A_{42}}{n X_1} \left(\langle \sigma v_e \rangle_{24} + \frac{\langle \sigma v_e \rangle_{23} \langle \sigma v_e \rangle_{34}}{X_2} + \right. \\
& + \frac{A_{43} \langle \sigma v_e \rangle_{23} \langle \sigma v_e \rangle_{34}}{n X_1 X_2} + \left. \frac{\langle \sigma v_e \rangle_{24} \langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{43}}{X_1 X_2} \right) - \\
& - \left(\frac{\langle \sigma v_e \rangle_{32}}{X_2} + \frac{\langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{42}}{X_1 X_2} \right) \left(\langle \sigma v_e \rangle_{23} + \frac{A_{43} \langle \sigma v_e \rangle_{32}}{n X_1} + \right. \\
& + \left. \frac{\langle \sigma v_e \rangle_{24} \langle \sigma v_e \rangle_{43}}{X_1} \right) + \frac{v_a S_v}{4 n} \quad (E.9)
\end{aligned}$$

$$X_4 = \frac{\langle \sigma v_e \rangle_{32}}{X_2} + \frac{\langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{42}}{X_1 X_2} \quad (E.10)$$

$$X_5 = \langle \sigma v_e \rangle_{31} + \frac{A_{31}}{n} + \frac{\langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{41}}{X_1 X_2} \quad (E.11)$$

$$\begin{aligned}
X_6 = & \langle \sigma v_e \rangle_{21} + \frac{A_{21}}{n} + \frac{\langle \sigma v_e \rangle_{24} \langle \sigma v_e \rangle_{41}}{X_1} + \\
& + \frac{X_5}{X_2} \left(\langle \sigma v_e \rangle_{23} + \frac{\langle \sigma v_e \rangle_{24}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \right) \quad (E.12)
\end{aligned}$$

$$\begin{aligned}
X_7 = & \langle \sigma v_e \rangle_{02} + \frac{A_{42}}{n X_1} \left(\langle \sigma v_e \rangle_{04} + \frac{\langle \sigma v_e \rangle_{03} \langle \sigma v_e \rangle_{34}}{X_2} + \right. \\
& + \frac{A_{43}}{n} \left(\frac{\langle \sigma v_e \rangle_{04} \langle \sigma v_e \rangle_{34}}{X_1 X_2} \right) + \left. \left(\frac{\langle \sigma v_e \rangle_{04} \langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{43}}{X_1 X_2} \right) \right) \quad (E.13)
\end{aligned}$$

$$X_8 = X_4 \left(\langle \sigma v_e \rangle_{03} + \frac{A_{43} \langle \sigma v_e \rangle_{04}}{n X_1} + \frac{\langle \sigma v_e \rangle_{04} \langle \sigma v_e \rangle_{43}}{X_1} \right) \quad (E.14)$$

$$X_9 = \langle \sigma v_e \rangle_{12} + \frac{A_{42}}{n X_1} \left(\langle \sigma v_e \rangle_{14} + \frac{\langle \sigma v_e \rangle_{13} \langle \sigma v_e \rangle_{34}}{X_2} + \right. \\ \left. + \frac{A_{43} \langle \sigma v_e \rangle_{14} \langle \sigma v_e \rangle_{34}}{n X_1 X_2} + \frac{\langle \sigma v_e \rangle_{14} \langle \sigma v_e \rangle_{34} \langle \sigma v_e \rangle_{43}}{X_1 X_2} \right) \quad (E.15)$$

$$X_{10} = X_4 \left(\langle \sigma v_e \rangle_{13} + \frac{A_{43} \langle \sigma v_e \rangle_{14}}{n X_1} + \frac{\langle \sigma v_e \rangle_{14} \langle \sigma v_e \rangle_{43}}{X_1} \right) \quad (E.16)$$

$$X_{11} = \langle \sigma v_e \rangle_{01} + \frac{\langle \sigma v_e \rangle_{04} \langle \sigma v_e \rangle_{41}}{X_1} + \frac{X_5}{X_2} \left(\langle \sigma v_e \rangle_{03} + \frac{\langle \sigma v_e \rangle_{04}}{X_1} \right. \\ \left. \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \right) \quad (E.17)$$

$$X_{12} = \frac{\langle \sigma v_e \rangle_{14} \langle \sigma v_e \rangle_{41}}{X_1} + \frac{X_5}{X_2} \left(\langle \sigma v_e \rangle_{13} + \frac{\langle \sigma v_e \rangle_{14}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \right) \quad (E.18)$$

$$X_{13} = \frac{X_7 + X_8}{X_3} \left(\langle \sigma v_e \rangle_{23} + \frac{\langle \sigma v_e \rangle_{24}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \right) \\ + \langle \sigma v_e \rangle_{03} + \frac{\langle \sigma v_e \rangle_{04}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \quad (E.19)$$

$$X_{14} = \frac{X_9 + X_{10}}{X_3} \left(\langle \sigma v_e \rangle_{23} + \frac{\langle \sigma v_e \rangle_{24}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \right) \\ + \langle \sigma v_e \rangle_{13} + \frac{\langle \sigma v_e \rangle_{14}}{X_1} \left(\frac{A_{43}}{n} + \langle \sigma v_e \rangle_{43} \right) \quad (E.20)$$

$$X_{15} = \langle \sigma v_e \rangle_{04} + \frac{X_7 + X_8}{X_3} \langle \sigma v_e \rangle_{24} + \frac{X_{13} \langle \sigma v_e \rangle_{34}}{X_2} \quad (E.21)$$

$$X_{16} = <\sigma v_e>_{14} + \frac{X_9 + X_{10}}{X_3} <\sigma v_e>_{24} + \frac{X_{14} <\sigma v_e>_{34}}{X_2} \quad (E.22)$$

$$X_{17} = \frac{\frac{X_6(X_7 + X_8)}{X_3} + X_{11}}{<\sigma v_e>_{10} + <\sigma v_e>_{12} + <\sigma v_e>_{13} + <\sigma v_e>_{14} + <\sigma v_e>_{15} + \frac{A_{10}}{n} + \frac{v_a S_v}{4n} - \frac{X_6(X_9 + X_{10})}{X_3} - X_{12}} \quad (E.23)$$

$$X_{18} = <\sigma v_e>_{05} + \frac{X_7 + X_8}{X_3} <\sigma v_e>_{25} + \frac{X_{13} <\sigma v_e>_{35}}{X_2} + \frac{X_{15} <\sigma v_e>_{45}}{X_1} \quad (E.24)$$

$$X_{19} = X_{17} \left(<\sigma v_e>_{15} + \frac{X_9 + X_{10}}{X_3} <\sigma v_e>_{25} + \frac{X_{14} <\sigma v_e>_{35}}{X_2} + \frac{X_{16} <\sigma v_e>_{45}}{X_1} \right) \quad (E.25)$$

$$X_{20} = 1 + \frac{X_7 + X_8}{X_3} + \frac{X_{13}}{X_2} + \frac{X_{15}}{X_1} + X_{17} \left(1 + \frac{X_9 + X_{10}}{X_3} + \frac{X_{14}}{X_2} + \frac{X_{16}}{X_1} \right) \quad (E.26)$$

where S_v for each region is:

$$S_v = \begin{cases} \frac{S_E + S_{g1} + S_{S1}}{V_1} & \text{for Region 1} \\ \frac{2S_h + S_{hs}}{V_2} & \text{for Region 2} \\ \frac{S_C + S_{g3} + S_{S3}}{V_3} & \text{for Region 3} \end{cases} \quad (E.27)$$

**APPENDIX F - COEFFICIENTS IN GOVERNING EQUATIONS OF
TRANSIENT DIODE MODEL**

$$F_1 = (X_{18} + X_{19})N_0 - \frac{AS_v v_e}{4} - \frac{\eta_E S_E v_e}{6V} \left(-AB + \frac{S_C}{S_E} \exp(-\eta_C) \right) + \frac{S_C v_e (2 + \eta_C) \exp(-\eta_C)}{6V} + \frac{7.77 AS_s v_e}{6V} + \frac{2S_E v_e \exp(-\eta_E)}{6V} \quad (F.1)$$

$$F_2 = \frac{8j_{E0} S_E T_E}{6q n V} - \frac{q \epsilon_i S_E A v_e \left(B + \frac{S_S}{S_E} \right)}{6k_B V} - \frac{2R_{ex} S_v}{3n k_B} - \frac{2R_r}{3n k_B} \quad (F.2)$$

$$F_3 = 0.61 S_v v_i - (X_{18} + X_{19})N_0 \quad (F.3)$$

$$F_4 = n \sum_{k=1}^5 \langle \sigma v_e \rangle_{0k} + \frac{v_a S_S}{4V} \quad (F.4)$$

$$F_5 = \frac{N_{a0} v_a S_S}{4V} - \frac{N_{mc} S_C (-F_{14} \theta_C + F_{15})}{V} + \frac{0.61 n v_i (S_C + S_E)}{V} + N_1 (n \langle \sigma v_e \rangle_{10} + A_{10}) + N_4 (n \langle \sigma v_e \rangle_{40} + A_{40}) \quad (F.5)$$

$$F_6 = n \sum_{k=0, k \neq 1}^5 \langle \sigma v_e \rangle_{1k} + A_{10} + \frac{v_a S_S}{4V} \quad (F.6)$$

$$F_7 = N_0 n \langle \sigma v_e \rangle_{01} + N_2 (n \langle \sigma v_e \rangle_{21} + A_{21}) + N_3 (n \langle \sigma v_e \rangle_{31} + A_{31}) \quad (F.7)$$

$$F_8 = n \sum_{k=1, k \neq 2}^5 \langle \sigma v_e \rangle_{2k} + A_{21} + \frac{v_a S_S}{4V} \quad (F.8)$$

$$F_9 = N_0 n \langle \sigma v_e \rangle_{02} + N_1 n \langle \sigma v_e \rangle_{12} + N_4 (n \langle \sigma v_e \rangle_{42} + A_{42}) \quad (F.9)$$

$$F_{10} = n \sum_{k=1, k \neq 2, 3}^5 \langle \sigma v_e \rangle_{3k} + A_{31} + \frac{v_a S_S}{4V} \quad (F.10)$$

$$F_{11} = N_0 n \langle \sigma v_e \rangle_{03} + N_1 n \langle \sigma v_e \rangle_{13} + N_2 n \langle \sigma v_e \rangle_{23} + N_4 (n \langle \sigma v_e \rangle_{43} + A_{43}) \quad (F.11)$$

$$F_{12} = n \sum_{k=0, k \neq 1, 4}^5 \langle \sigma v_e \rangle_{4k} + A_{40} + A_{42} + A_{43} + \frac{v_a S_S}{4V} \quad (F.12)$$

$$F_{13} = N_0 n \langle \sigma v_e \rangle_{04} + N_1 n \langle \sigma v_e \rangle_{14} + N_2 n \langle \sigma v_e \rangle_{24} + N_3 n \langle \sigma v_e \rangle_{34} \quad (F.13)$$

$$F_{14} = \frac{(N_0 + N_1 + N_2 + N_3 + N_4) v_a}{4 N_{mc}} + \frac{\exp\left(\frac{-q H_C}{k_B T_C}\right)}{\gamma_C} \quad (F.14)$$

$$F_{15} = \frac{(N_0 + N_1 + N_2 + N_3 + N_4) v_a}{4 N_{mc}} \quad (F.15)$$

APPENDIX G - PROBE THEORY

There are several ways to investigate the plasma conditions of gas discharges; such as, microwaves, lasers, particle beams, spectroscopy, and probes (Chen 1965 and Baksht 1984). The easiest and earliest developed method is that using a probe, i.e. a small metal electrode that is submersed in the plasma. By varying the voltage applied to the probe from negative values to positive values and measuring the current, an I-V characteristic for the plasma is obtained. A typical probe characteristic is shown in Fig. G.1. At very large negative voltages, the electrons are repelled and the current being drawn by the probe is the ion saturation current, I_{is} . As the probe is made more positive, a combination of electrons and ions are flowing to the probe; until only electrons reach the probe at sufficiently high positive probe voltages. From this probe characteristic several of the plasma parameters can be determined. However, for the data to be valid, several criteria must be met, which are as follows:

- 1) dimensions of the probe $\ll$ dimensions of the system,
- 2) probe currents $\ll$ system currents,
- 3) diameter of probe $\gg$ debye length, λ_d ,
- 4) mean free path of ions and electrons $\gg \lambda_d$, so there are no collisions in the probe sheath.

The debye length of a plasma is defined as the radius of a sphere that a charged particle at the center of the sphere is not influenced by any charged particles outside of the sphere. Therefore, it describes an effective shielding distance. The debye length is given by the following expression:

$$\lambda_d = \sqrt{\frac{\epsilon_0 k_B T_e}{q^2 n}} \quad (\text{G.1})$$

Assuming a Maxwellian electron density distribution, the current being drawn by the probe at voltages less than the plasma potential, V_p , is given by:

$$j = j_0 \exp\left(\frac{-q(V_p - V)}{k_B T_e}\right) \quad (\text{G.2})$$

where V is the applied probe voltage and j_0 is the electron saturation current density. If two points for $V \leq V_p$ are considered, such as:

$$j_1 = j_0 \exp\left(\frac{-q(V_p - V_1)}{k_B T_e}\right) \quad (\text{G.3})$$

$$j_2 = j_0 \exp\left(\frac{-q(V_p - V_2)}{k_B T_e}\right). \quad (\text{G.4})$$

If Eqn. (G.3) is divided by Eqn. (G.4), the following relationship is obtained:

$$\frac{j_1}{j_2} = \exp\left(\frac{-q(V_2 - V_1)}{k_B T_e}\right) \quad (\text{G.5})$$

and solving for T_e yields:

$$T_e = \frac{q}{k_B} \frac{(V_1 - V_2)}{\ln\left(\frac{j_1}{j_2}\right)}. \quad (\text{G.6})$$

Therefore, by choosing two points along the linear portion of the probe

characteristic for $V \leq V_p$, the electron temperature of the plasma can be determined.

Also, the plasma density can be calculated from the ion saturation current under the same assumption that were outlined in section 5.1 (see Eqn. (5.2)) as:

$$n = \frac{I_{is}}{0.61qA_g} \sqrt{\frac{M_i}{k_B T_e}} \quad (G.7)$$

So from the ion saturation current, I_{is} , the plasma density can be determined. The last plasma parameter that can be found from the probe characteristics is the plasma potential, V_p . To find this one needs only to find the intersection of the slope of the electron saturation current and the linear portion of the curve as shown in Fig. G.1. In conclusion, by using a small metal probe that is submersed in the plasma and negatively and positively biased, a great deal of information can be learned as to the conditions of the plasma in a gas discharge.

Figure G.1 Typical Probe I-V Characteristic.

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