AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

O

Photonics for Accelerator Instrumentation*

Arnold Stillman

Brookhaven National Laboratory, Upton, NY 11973

RECEIVED

MAY 23 1994

OSTI

Abstract

Nonlinear optical effects in certain types of crystals have the capacity to improve the performance of sampling circuits by a large margin. The theory of nonlinear optics and the associated devices is rather straightforward. I will present some of the relevant ideas and illustrate how these principles can result in order of magnitude improvements in sampling circuits. As an example of the photonic technique, I will illustrate how to design a beam position monitor that uses a nonlinear crystal.

*Work performed under the auspices of the U.S. Department of Energy, Contract No. DE-AC02-76CH00016.

Introduction

When electromagnetic fields act on the charge carriers in a crystal, they cause the microscopic rearrangement of these charge carriers. When only electric fields cause this rearrangement, it becomes a net polarization, since the electric field causes the charges to migrate from their equilibrium positions. The greater the electric field strength, the greater the induced polarization.

The direction of the polarization vector is not necessarily collinear with either the electric displacement vector or the electric field vector. It is the direction of the polarization vector, however, that determines the optical properties of a crystal. There are usually only two possible linearly polarized modes for any given direction in a crystal, and each of these modes has its own index of refraction and polarization direction. The easiest method of determining the directions and indices of refraction of these modes is by using the index ellipsoid and performing a principal axis transformation.

The Index Ellipsoid

The indices of refraction in a crystal are not necessarily equal in all three propagation axes. In fact, their values directly reflect the symmetry properties of the space group of the crystal. Since the index of refraction of an optically active crystal depends on the induced polarization caused by the application of an external electric field, the index ellipsoid will depend on $\vec{D}$, the electric displacement vector in the crystal. Since there is no requirement that $\vec{D}$ be parallel to $\vec{E}$, the relationship between the two fields is a tensor relationship,

$$D_k = \epsilon_{kl} E_l. \quad (1)$$

The energy density associated to the fields is

$$\mathcal{E} = \frac{1}{2} \vec{E} \cdot \vec{D}, \quad (2)$$

or,

$$\mathcal{E} = \frac{1}{2} E_k \epsilon_{kl} E_l. \quad (3)$$

Expanding this equation in the most general way yields,

$$2\mathcal{E} = \epsilon_{xx} E_x^2 + \epsilon_{yy} E_y^2 + \epsilon_{zz} E_z^2 + 2\epsilon_{xy} E_x E_y + 2\epsilon_{xz} E_x E_z + 2\epsilon_{yz} E_y E_z. \quad (4)$$

This equation is diagonalizable, since it is the equation of an ellipse which can rotate freely to the diagonal coordinate frame. In the diagonal frame,

$$2\mathcal{E} = \epsilon_x E_x^2 + \epsilon_y E_y^2 + \epsilon_z E_z^2; \quad (5)$$

simply the equation for an ellipse lying along on the coordinate axes. The purpose of the principal axis transformation is to generate the diagonal components of the tensor ϵ , which depend on the crystal properties. Since ϵ is diagonal, then

$$\mathbf{D} = \epsilon \cdot \mathbf{E}, \quad (6)$$

where ϵ now has only three components, ϵ_x, ϵ_y , and ϵ_z . Finally, plugging Eq. 6 into Eq. 5 yields

$$2\mathcal{E} = \frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z} \quad (7)$$

which is the equation for an ellipsoid of constant energy density in the space D_x, D_y, D_z . Now, let the relative dielectric constant along a direction i be $\epsilon'_i = \epsilon_i/\epsilon_0$. Also, define a vector $\mathbf{r}$ by $\mathbf{D}/\sqrt{2\mathcal{E}_e\epsilon_0}$. Then the index ellipsoid, Eq. 7, becomes

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1, \quad (8)$$

where $n_i^2 = \epsilon'_i$. This is the most general form for the index ellipsoid, in which the principal axes lie along the two directions of $\mathbf{D}$ and the optical axis of the crystal. The length of each semi-axis is equal to the index of refraction along that axis.

The Electro-Optic Effect

At this point, it is apparent that the response of a crystal to an applied electric field is to modulate the light that passes through it, since its indices of refraction are changing. Although non-linear effects can arise due to the dependence of the refractive index change on higher powers of the electric field, the main effect, for electric field detection, is linear in the incident field. Several devices now exist that sample electric fields with picosecond time resolution [1, 2]. These devices detect the rather weak fields from integrated circuit traces by integration over many pulses. Sensing the field of a particle beam should be easier, given its greater field strength.

When a crystal reacts to applied fields, the linear change in refractive index is, by definition,

$$\Delta\left(\frac{1}{n^2}\right)_i = r_{ij}E_j. \quad (9)$$

This definition applies for a crystal whose optic axis lies along the path of an incident light ray. The most general form for the index ellipsoid in this orientation is one for which

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 yz + \left(\frac{1}{n^2}\right)_5 xz + \left(\frac{1}{n^2}\right)_6 xy = 1, \quad (10)$$

where the subscript notation follows a crystallographic convention. A moments inspection shows that the tensor r_{ij} , then, is a 6×3 tensor; 6 indices coming from Eq .10 and 3 coming from the electric fields in Eq. 9. The symmetry properties of the crystal completely determine this tensor, known as the electro-optic tensor. Diagonalizing r_{ij} will yield new principal axes of the crystal, which were absent in the absence of $\vec{E}$. This is the basic, linear electro-optic effect. Not only do the indices of refraction change, but the optical axis of the crystal changes as well.

A BPM Calculation

The next step, of course, is to estimate the magnitude of this effect for the electric field of a particle beam and some actual crystal parameters. The electric space charge field of a Gaussian particle beam, in cylindrical coordinates, is

$$\vec{E}(r) = \frac{i_0 \sigma^2}{2\pi R f} \sigma^2 \frac{1 - e^{-r^2/2\sigma^2}}{r} \hat{r}, \quad (11)$$

where i_0 is the normalization constant imposed by the condition that $\int \vec{E} \cdot d\vec{a} = 4\pi Q$, with Q the total charge in the beam, f the revolution frequency in a circular machine, and R the radius of the machine. This can be of the order of 1kV/m at a few centimeters.

For a BaTiO_3 (barium titanate) crystal whose optic axis lies in the radial ($\hat{r}$) direction, the non-zero components of the electro-optic tensor are $r_{33} = 23 \times 10^{-12} \text{ m/V}$, $r_{13} = 8.0 \times 10^{-12} \text{ m/V}$, and $r_{42} = 820 \times 10^{-12} \text{ m/V}$. The values for the indices of refraction are $n_e = 2.365$ and $n_o = 2.437$. The o and e subscripts refer to *ordinary* and *extarordinary* modes of propagation. The ordinary mode is transverse, the extraordinary is along the optic axis.

Rewriting Eq. 10 by inserting all non-zero components of r and the components of the electric field gives a specific ellipsoid,

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{13}x^2 E_z + 2r_{42}yz E_y + 2r_{33}E_z z^2 = 1, \quad (12)$$

or,

$$x^2 \left(1/n_o^2 + 2r_{13}E_z\right) + y^2/n_o^2 + z^2 \left(1/n_e^2 + 2r_{33}E_z\right) = 1, \quad (13)$$

since, in this orientation, E_y is zero. The index ellipsoid is now squashed, since the semi-minor axis in the y direction is unchanged, but the new one along the x axis is given by,

$$\frac{1}{n_x^2} = \frac{1}{n_0^2} + 2r_{13}E_z. \quad (14)$$

For the good approximation that $1/n_0^2$ is much greater than $2r_{13}E_z$,

$$n_0 \approx n_0 - n_0^3 r_{13} E_z. \quad (15)$$

A light ray propagating along the optic axis then suffers a retardation in the phase of the x -component, but none in the y -component. The total phase difference for $.5\mu\text{m}$ light in a crystal $l = 5\text{mm}$ long is

$$\Gamma = (n_0 - n_0^3 r_{13} E_z - n_0) l \omega / c = -n_0^3 r_{13} E_z l / \lambda, \quad (16)$$

or about .015 rad, a clearly detectable signal. The advantage of the photonic approach, however, is that its frequency response extends well into the high GHz range, and in some autocorrelation schemes, can give time resolutions better than a picosecond. Not only is it practical, but it has the potential to be at least an order of magnitude faster than fast oscilloscopes, and the crystal detection occurs in a single shot, rather than in an average of samples.

References

- [1] J. A. Valdmanis and S. S. Pei, in Picosecond Electronics and Optoelectronics, Springer-Verlag, 1987.
- [2] K. J. Weingarten, M. J. W. Rodwell, and D. M. Bloom, in Picosecond Electronics and Optoelectronics, Springer-Verlag, 1987.

**DATE
FILMED**

6/23/94

END