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USE OF CONTROL CHARTS FOR ANALYSIS OF
PLANT RUPTURE EXPERIENCE

AUTHOR

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USE OF CONTROL CHARTS FOR ANALYSIS OF
PLANT RUPTURE EXPERIENCE

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USE OF CONTROL CHARTS FOR ANALYSIS OF
PLANT RUPTURE EXPERIENCE

INTRODUCTION

Goal exposure and power level planning and administration on either a short-term or long-term basis, is heavily dependent upon assessment of the level of metal performance. It is the purpose of this document to present a method of monitoring the metal performance level employing the concept of control charts and to propose adoption of this method as the basis for detecting changes in the metal performance level of the three main types of production fuel elements, i.e., C-II-N, O-II-N, and K-III-N material. Supplemented with predetermined decision rules for evaluating the extent to which the metal performance level has improved or declined, these control charts become valuable tools for use in making IPD operating decisions.

SUMMARY AND CONCLUSIONS

Assessment of the level of metal performance, essentially defining the rupture rate of fuel elements under irradiation, is pivotal to the economic operation of the IPD reactors. It is known that chance variations in the "observed" rupture rates are of the same magnitude as the differences one wishes to detect for purposes of adjusting reactor operating conditions. Consequently, in order to minimize the frequency of incorrect decisions -- i.e., those leading to adjustments which result in unnecessary economic loss over both the short and long run -- objective methods of analyzing rupture experience should be employed whenever possible; this means before-the-fact analytical procedures and pre-determined courses of action to be followed, when warranted by the data.

This document deals wholly with the development of such methods. It is earnestly hoped that the reader might acquire a better intuitive grasp of the nature of the problem. The recommendations that are made are not purported to solve "all the problems," nor to obviate the application of engineering or administrative judgment in all cases. They represent, however, what is believed to be maximum technical utilization of knowledge that has been accumulated over the past several years for purposes of evaluating changes in the metal performance level.

It is concluded that:

1. The metal performance levels of C-II-N, O-II-N, and K-III-N materials should be monitored using control charts of the time between rupture events, after being adjusted for known and measured changes in the rupture producing climate of the reactor(s).
2. Adjustments to the metal performance level indices (CR values) of O-II-N material should be based on the combined experience of the B, D, DR, F and H reactors; and for K-III-N material, on the combined experience of the KE and KW reactors. Adjustments to CR values of C-II-N material will, of necessity, have to be based strictly on C reactor data. Criteria for CR adjustments should be as follows:

- a. When a (w) point (as defined in the body of this report) falls outside the ninety-five per cent control limits, adjust CR by a factor of 1.2 upward or downward as indicated by the data.
 - b. When a (w') point (as defined in the body of this report) falls outside the ninety-five per cent control limits adjust CR by a factor of 1.5, in the direction indicated by the data.
3. The Reactor Process Engineers should maintain control charts of the experience at the individual reactors, for historical purposes. Individual reactor's experience, except for C Reactor, should not be used as a basis for adjusting CR values.

DISCUSSION

A. Concept of Control Charts

Variations in observed or measured characteristics are usually due to a large variety of causes. It is generally accepted that measurements of a given characteristic, even under carefully controlled experimental conditions, will vary to some degree. One cannot expect even the most carefully controlled process to produce all material with identical properties. Without expanding too far, it is clear that some sources of variation can be identified and removed; these can be said to emanate from assignable causes. The variations that remain stem from chance causes. There can be no sharp dividing line between these sources; as some causes are identified and removed, others may become assignable, until finally nothing remains but purely chance causes. In most practical situations, it is not possible to economically identify or remove all assignable causes.

A major part of the variation in rupture frequency has been shown⁽¹⁾ to be assignable to the readily measureable reactor parameters of average power, average temperature, and exposure. One can take advantage of this fact to prepare a control chart, the use of which will facilitate the detection and identification of other assignable causes of variations in rupture frequency. One can establish from past measurements, by statistical methods, the limits within which repeated measurements (or observations) of a given characteristic should almost always fall, if no assignable causes are present. These limits are called control limits, and variations beyond the control limits are considered to indicate the presence of an assignable cause which should be eliminated. The choice of limits evolves finally to a matter of economics. The wider the limits are, the more likely the presence of an assignable cause if an out-of-control point is detected, and also the greater the possibility that an assignable cause will be present in a point within limits. The limits become meaningless, however, if they are so narrow that complete and detailed investigation of points that are out-of-control is not practical. The benefits of the establishment of control limits are in the identification of assignable causes and in actions based on points being out-of-control. A control chart,

(1) HW-55377, Bloomstrand, RR and WI Neef, "Analyses and Correlations of HAPO Rupture Experience With Natural Uranium Material," 4/23/58, (Secret).

finally, is a graphical picture of the concept of control limits, In this problem, i.e., the analysis of rupture experience, based on our current knowledge, an out-of-control point is most likely assignable to (1) changes in axial flux distribution or other reactor parameters excluding power, temperature, and exposure, and/or (2) changes in intrinsic metal quality.

B. Control Chart Application to Rupture Analysis

To improve our ability to objectively assess the significance of the plant rupture experience, a control chart approach to monitoring metal performance has been developed. The control chart scheme which has been selected is one wherein the time between rupture events, after being adjusted for known and measured changes in the rupture producing climate of the reactor(s),* is the statistic being monitored. Intuitively, if the time lapse between ruptures is shorter than expected, it follows that this could be indicative of a higher-than-expected rupture rate. Conversely, if the time lapse between ruptures is longer than expected, this could be indicative of a lower-than-expected rupture rate. Since an increase in the rupture rate, is, by definition, synonymous with a decline in the metal performance level, and a decrease in the rupture rate represents an improved metal performance level, the distribution of the statistic calculated from the rupture experience can be utilized to make probabilistic statements or inferences about changes in the underlying metal performance level. These inferences are directly translatable to changes in the metal performance indices (CR values) which influence, to a large extent, the choice of reactor power level, temperature and exposure conditions.

To apply control charts of the type discussed above to monitor rupture performance it is necessary to make three basic decisions:

- (1) How many time periods shall constitute a point on the control chart?
- (2) What fraction of the time is one willing to decide that the metal performance level has changed when indeed it has not?
- (3) What action will be taken when a point is out-of-control?

The answers to these questions are interrelated; the particular answer to one will affect the other two. In the following sections, the conclusions that have been reached regarding each of the above questions are summarized; the details, including the technical calculations and, where necessary, the philosophy, are appended.

C. Basic Decisions

Two statistics have been selected for use in monitoring metal performance in an attempt to achieve balanced ability in detecting real improvements and/or declines in metal performance over both the long and short range. The first statistic is the "adjusted time between four consecutive ruptures." This statistic, designated(w), is computed after every three ruptures, since the fourth rupture of the preceeding group becomes the first rupture in the current group. The second statistic is the average value of five consecutive

* Using the rupture model $RR = k p^{3.3} T^{8.7} M^5$.

three-rupture points and is designated (w') . This statistic is more efficient than (w) in detecting changes in metal performance, since each point is based on about five times as much data as (w) . In practice (w') will prove useful mainly for detecting a decline in the metal performance level. A "high" rupture rate gives low values of (w) or (w') , and a "low" rupture rate gives high values of (w) or (w') . For both statistics, i.e., the "three-rupture" points (w) and the "average of five consecutive three-rupture points" (w') , the calculated quantity, if the underlying metal performance level is known, will have an average value of 3, over the long run. Because the distribution of these statistics is skewed, the median value will be 2.67, over the long run.

Ninety-five per cent (95%) control limits have been selected for use in this problem. This means that if the metal performance level used in the calculations is correct, 95% of the (w) points will fall within the upper and lower control limits, which are respectively 7.22 and 0.61. The 95% control limits on (w') are 4.70 and 1.67. A single point that is out-of-control implies behavior that is considered to be inconsistent with the metal performance level used in calculating the point, and an adjustment to the metal performance level is warranted. The reader is referred to Appendix V where examples of the chance variations in (w) that occur in situations of constant metal performance level are graphically illustrated.

While the selection of 95% control limits is somewhat arbitrary, it is not as "all-protective" or "all-conservative" as it might intuitively appear to be. To illustrate, when 95% limits are chosen such that the probability that a single point being out-of-control due to chance alone is 0.05, then if ten (10) points are plotted (such as might be the case for six months rupture experience at the five old reactors) and, further, if the underlying metal performance level is in fact constant, the probability is only $(0.95)^{10} = .60$ that all ten points will be in control. If an out-of-control point requires action, and this, of course, is the purpose of the control chart, then the probability is 0.40 that unwarranted action will be taken for this group of 10 points. If 99% control limits were used, the probability of unwarranted action for this group of ten points would be reduced to 0.095.

In order to choose a decision rule (or set of decision rules) one must be guided by a knowledge of how the metal performance level has varied in the past. Experience has indicated that, except where changes in fuel type have been made, e.g., from solid to I & E, the metal performance level has been quite stable over the long run. Cycling to some degree undoubtedly exists but it is probably not too extreme nor of too long a duration. In this situation it can be shown (Appendix III) that a conservative rule, specifying relatively small changes in reactor conditions to be made only when one is quite sure that an out-of-control point really represents a true change, will be the most economic rule in the long run.

Preliminary investigations lead to the following recommendations for revising metal performance level estimates:

- (1) When a "three-rupture" point (w) falls outside the 95% control limits, adjust the metal performance level by a factor of 1.2, either upward or downward as indicated by the data.

- (2) When the average value of five consecutive "three-rupture" points (w) falls outside the 95% control limits, notwithstanding that any or all ~~all~~ of the (w) points are in control, adjust the metal performance level by a factor of 1.5.

The following examples will serve to illustrate how the statistic (w) would be calculated. Suppose that a reactor or group of reactors is operated in a manner so as to produce an expected (long run) rupture rate of 0.15 ruptures per calendar day. Ruptures, from an index point, occur respectively on days 5, 21, and 23. The statistic (w) would be the number of days until the third rupture occurred multiplied by the expected rupture rate per day or (23) (0.15) = 3.45. Since this "point" is within the control limits, there is no evidence of a shift in the metal performance level. There is no basis whatever for even "suspecting" that the performance level has "improved slightly" just because the value of the statistic was greater than three (3).

An alternate situation might be that the reactor(s) operate(s) so as to produce an expected rupture rate of 0.10 for ten days, 0.15 for the next ten days, and 0.19 for the next ten days. The ruptures again occur respectively on days 5, 21, and 23. In this case (w) would be computed as follows:

$$(10)(.10) + (10)(.15) + (3)(.19) = 3.07$$

The point is in control.

D. Base Metal Performance Levels

To calculate the expected rupture rate (ruptures per calendar day) it is necessary to establish a base metal performance level. Rupture experience in CY-1959 was selected for this purpose, and the performance level was established by evaluating the quantity $q = R/\Delta RI$ for C-II-N, K-III-N,* and O-II-N material from the data, where:

q is the metal performance level multiplier.

R is the number of ruptures observed.

ΔRI is the change in "Rupture Index."

Rupture Index is a measure of the severity of reactor irradiation conditions, i.e., a measure of the rupture producing climate of the reactor(s). The rupture index calculations are routinely performed on the IBM 709 computer on a monthly basis by application of the current rupture model to the irradiation conditions of every tube charge of production type fuel elements using the following equation:

$$(1) RI = P^{3.3} T^{8.7} M^5$$

Where: P is the average tube power in kw.

T is the average outlet temperature in °C.

M is the tube exposure in MWD per tube.

* Both K-II-N and K-III-N experience were used in this case.

Average power, average outlet temperature and accumulated exposure status are calculated at the end of each calendar month and at discharge for each tube charge. The exposures are the official values calculated by IPD - Production, and the average powers and temperatures are the arithmetic averages of the total data (from reactor temperature maps and individual tube flow rate determinations) accumulated for each tube since its charge date. This is usually from 10 to 20 sets of data per charge over the course of its irradiation. The change in rupture index for a particular tube over a given time period is the difference between the RI values calculated at the end and beginning of the time period. The change in rupture index (ΔRI) for a given reactor over a given time period is simply the summation of the tube ΔRI 's. The change in rupture index (ΔRI) is directly proportional to the expected rupture rate during the subject time period. For example, if the ΔRI for a reactor during time period θ_1 is 1.0 (arbitrary units) and for time period θ_2 , is 2.0, the expected number of ruptures in θ_2 is twice as large as in θ_1 .

From the data in Appendix II the average values of q during CY-1959 were determined as follows:

Type Material	Average q	95% Confidence Limits on q	
		Lower	Upper
C-II-N	0.675×10^{-39}	0.30×10^{-39}	1.28×10^{-39}
K-III-N	0.136×10^{-39}	0.08×10^{-39}	0.22×10^{-39}
O-II-N	0.317×10^{-39}	0.22×10^{-39}	0.43×10^{-39}

For the control chart calculations, the q values above are treated as constants, i.e., the uncertainty is neglected. The control limits while called 95% limits, are, therefore, only approximate 95% limits. The confidence level they actually represent depends upon how close the estimates really are to the true value, and this, of course, is not known. For O-II-N, the material, it is seen that the uncertainty in q is relatively small; about plus or minus a factor of 1.35. For C-II-N and K-III-N material this uncertainty approaches plus or minus a factor of two. Treating the q estimates as constants, which is necessary in order to apply the control chart concept to monitoring the metal performance level, results in some sacrifice in precision. This is a small price to pay, however, for objectivity.

The estimates of the metal performance level multipliers (q) in effect at the date of issue of this report, and the detailed procedure for making the control chart calculations, are in Appendix IV.

E. Combining Data

Theoretically, the control chart scheme discussed in this report can be applied to single reactors or to groups of reactors. In practice, for a variety of reasons, it is believed best to combine the experience for several reactors whenever possible. An important consideration is to maximize the number of observations per unit time insofar as is possible, for the control chart to have any practical value. This is best accomplished, in the author's opinion, by examining the data "by type metal." This means combining the data from the older reactors for monitoring the performance level of O-II-N material and

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combining the experience from the K Reactors to monitor the performance level of K-III-N material. Unfortunately, the performance level of C-II-N material can only be examined on a single reactor basis.

When, on the basis of past experience or sound theory, a decision can be reached before-hand to combine sets of data, it is advantageous to do so because statistical precision will be improved.

Consider the situation where a single reactor is operated such that the expected number of ruptures in one year is twelve (12). One might observe, in fact, as few as six or as many as nineteen ruptures during the year and the experience would be consistent, at the 5% level, with the hypothesized value. In this case the number of control chart points in one year could be as few as two or as many as six. It is intuitively clear from the above that the ability to detect small changes in the true underlying metal performance level is not very great when one is limited to one reactor. This can be further illustrated by an example. Suppose that twelve ruptures were sustained (four points on the control chart), and further suppose that the metal performance level was, in fact, a factor of 1.5 worse than assumed. It can be shown, in this case, that the probability of one or more of the "four" points falling out-of-control is only 0.28.

Now consider the alternate case where five reactors are operating, each with an expectation of twelve ruptures per year. The total expected number is sixty (60). By deciding beforehand to examine the total experience, the expected number of ruptures for all five reactors, with 95% confidence, will fall between forty-six and seventy-four. The number of control chart points would be at least fifteen. If the metal performance level was, as before, really a factor of 1.5 worse than assumed, the minimum number of control chart points would probably exceed fifteen. It can be shown, in this case, that the probability that one or more of a group of fifteen points would be out-of-control is 0.72. The combined approach is without a doubt more desirable for detecting declines in metal performance. The combined approach is superior to the single reactor approach for the detection of improvements as well.

An important factor, which perhaps is not adequately understood by the casual observer, concerns the significance of the fact that for a given set of reactor operating conditions the probability of incurring a rupture has been fairly well established. This means that when ruptures occur in an essentially predictable fashion, there is no more reason to suspect poor quality in one group of material than in another. That is, if say forty groups or "lots" of homogeneous material, were irradiated under conditions that predict four ruptures will be sustained, the fact that four given lots sustained these ruptures, while the other thirty-six did not, cannot be considered unusual, and quality of the rupture lots cannot be considered suspect.* An after-the-fact analysis, which is completely subjective even

* There are occurrences, specifically multiple failure lots, which experience has shown are probably typical. For these lots, if it is determined that unusual reactor conditions were not causative in a given case (e.g. charging damage, twisted tube, etc.), there is reason to suspect that the ruptures occurred because of unusually poor quality in some respect.

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when dignified by the erroneous application of statistical analysis to the data, would be performed in about this manner: There are two groups of material; one consisting of four lots and the other of thirty-six; reactor treatment of the two groups was identical. Is the fact that four ruptures occurred in the small group and one in the large group significant? The answer is yes; the rupture rates are significantly different at the .01% level. Granted, it is theoretically possible that there is some attribute, reactor other otherwise, which if known could "explain" the observed behavior more fully. The objective approach, (the "scientific method"), if such an attribute were discovered from investigation of the irradiated fuel elements, (both ruptured and non-ruptured elements would have to be examined because one must know that the attribute was unique to the ruptured elements) would be to test the hypothesis formed with another set of data, i.e., perform another experiment.

By similar reasoning as above, caution should be taken in breaking down individual reactor's experience. Such splits among five old reactors as 3-0-0-0 would occur due to chance alone 4% of the time, and it is not indicative that something is wrong with the one reactor having three ruptures. Also, the occurrence of six ruptures in only two of the five reactors is not too significant, since such a thing will also occur due to chance about 4% of the time. Breaking down a single reactor's experience by zones, after the fact, probably has even more serious implications.

Returning to the "by type metal" decision, this is really a mixture of "by reactor" and "by type" which, on the basis of current knowledge, is pretty much indistinguishable. It is recognized that differences between reactors irradiating the same type metal, e.g., axial flux shapes, probably do exist. Individual reactors can then be expected to perform somewhat differently. From the available data and analyses of past experience, we believe that the effects of differences between reactors irradiating the same fuel type, specifically those effects that are not now accounted for by the rupture model, are small enough most of the time to justify their being disregarded for this control chart application. Combining data "by-type metal" is believed to offer the best method for monitoring changes in the performance level. As currently not-accounted-for factors become adequately defined and monitored, their inclusion in a revised rupture model can be easily accomplished.

Combining data from all fuel types and all reactors could well be the "best" method for monitoring changes in intrinsic metal quality; but, at this time, such a procedure is not believed to be technically justified. Estimates of the ratios of metal performance levels between metal types and/or "theoretical correction factors" which could be used for normalizing the data between-metal-types-between-reactors, are not precise enough to be used for this purpose.

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APPENDIX I

THEORETICAL BASIS FOR CONTROL CHARTS

In order to set up control charts useful in the interpretation of rupture data, it is necessary to assume some mathematical model which describes the results expected if indeed the underlying rupture potential were constant. For example, if we wished to test the hypothesis that, under a given set of conditions, 4 ruptures would result on the average, we would intuitively be unwilling to reject this hypothesis if, say, 5 ruptures were observed, because we know that exactly 4 ruptures would not occur during each time period. The same might be said for the occurrence of 6 ruptures, or 7, but there comes a point when the number of ruptures actually observed would be inconsistent with the number expected under the hypothesis. This point is determined by the mathematical model which specifies the numbers of ruptures expected with given probabilities. Thus, if we want to reject the above hypothesis if ≥ 7 ruptures are observed, we will want to know from the model what is the probability of observing ≥ 7 ruptures if indeed the average were 4, i.e., what is the probability of rejecting the hypothesis when it is true?

In determining the model for a reactor or a group of reactors, consider first an individual tube: Given the time interval $[0, t]$, we wish to find the model describing the number of ruptures incurred in this tube during the interval. Divide the time interval into increments Δt such that $n\Delta t = t$, and the probability that exactly one rupture occurs in a given increment is

$$\lambda_1 \Delta t \text{ or } \frac{\lambda_1 t}{n},$$

where λ_1 is some constant for tube 1. Then we want to find the probability of observing exactly k ruptures in this interval. This follows the binomial distribution given by:

$$\text{Prob (k ruptures)} = \frac{n!}{k! (n-k)!} \left(\frac{\lambda_1 t}{n} \right)^k \left(1 - \frac{\lambda_1 t}{n} \right)^{n-k}$$

We want the limit of this expression as $n \rightarrow \infty$, i.e., as the size of the intervals approaches zero. Write the probability:

$$\begin{aligned} \text{Prob} &= \frac{n(n-1)(n-2)\dots(n-k+1)}{k! n^k} (\lambda_1 t)^k \left[1 - (n-k) \frac{\lambda_1 t}{n} + \frac{(n-k)(n-k-1)(\lambda_1 t)^2}{n^2 2!} - \dots \right] \\ &= \frac{[n^k + c_1 n^{k-1} + c_2 n^{k-2} + \dots + c_k]}{k! n^k} (\lambda_1 t)^k \left[1 - \lambda_1 t + \frac{(\lambda_1 t)^2}{2!} - \dots + \frac{k}{n} \lambda_1 t + \frac{k^2}{n^2} \frac{(\lambda_1 t)^2}{2!} \right] \end{aligned}$$

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$$\text{and } \lim_{n \rightarrow \infty} \text{Prob} = \frac{1}{k!} (\lambda_1 t)^k e^{-\lambda_1 t}$$

which is the Poisson distribution with parameter $(\lambda_1 t)$. Thus, the number of ruptures occurring in tube 1 is given by the Poisson distribution with parameter $\lambda_1 t$.

For a group of tubes, we want the probability of observing exactly k ruptures. Since the tubes are statistically independent of one another, it is well known that the probability of observing exactly k ruptures is given by the Poisson distribution with parameter

$$\sum_1 \lambda_1 t = t \sum \lambda_1 = t \lambda, \text{ where } \lambda = \sum \lambda_1.$$

Note that in practice the individual λ_1 's are so small that for a reasonable time period the probability of observing more than one rupture in a given tube is extremely small. However, there is nothing in the derivation that excludes large λ_1 's; the λ_1 's may differ in size considerably from tube to tube, but the over-all distribution will still be Poisson.

There is, however, one step in the derivation that requires further comment. Note that in the time period from 0 to t , it is assumed that each λ_1 , and hence λ , is constant, i.e., the rupture potential is constant. Although one can theoretically satisfy this assumption by frequently correcting for changes in power, temperature, and exposure, it is clearly impossible from a practical standpoint (and is not even warranted) to do this too frequently, say more than once a month. Hence, the individual λ_1 , and the over-all λ , are not constant in the interval.

To determine the effects of this on the resulting distribution, we consider an example. Say that over a specified time period, 3 ruptures are expected in a group of tubes, so that $\lambda = 3$ on the average. However, λ is not constant but has itself a distribution specified by

$$f(\lambda) d\lambda = \frac{a^b}{\Gamma(b)} e^{-a\lambda} \lambda^{b-1} d\lambda,$$

such that the probability that λ , at any time, lies in the interval $d\lambda$ is given by this expression. If we choose an example where $a = 3$ and $b = 9$, we get the distribution of λ sketched in figure (1), page 15.

Now, if λ were constant during this time period, the probabilities of observing k ruptures are given by the Poisson distribution as discussed above, viz.,

$$\text{Prob} (k \text{ ruptures}) = e^{-3} \frac{3^k}{k!}.$$

If λ varies as sketched in figure (1), then it can be shown that the probabilities of observing k ruptures are given by the negative binomial distributions, viz.,

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$$\text{Prob (k ruptures)} = \left(\frac{a}{a+1}\right)^b \frac{(b+k-1)!}{(a+1)^k (b-1)! k!}$$

$$= \frac{(.75)^9 (8+k)!}{4^k 8! k!}$$

The probabilities specified by these two distributions are given below, and the resulting distributions are sketched in figure (2), page 15.

Table I - Probabilities of Observing k Ruptures

<u>k</u>	<u>λ constant (=3)</u>	<u>λ varies (=3 on the average)</u>
0	.050	.075
1	.149	.169
2	.224	.211
3	.224	.194
4	.168	.146
5	.101	.095
6	.050	.055
7	.022	.029
8	.008	.015
9	.003	.007
≥ 10	<u>.001</u>	<u>.004</u>
	1.000	1.000

Observe from Table I and figure (2) that the probabilities do not differ appreciably (from a practical standpoint) based on the two models. With varying λ , the distribution is a bit more spread out, as one would expect. The practical consequences of this is that by assuming λ constant, one would be slightly more inclined to reject a hypothesis $\lambda = \lambda_0$ when indeed it is true. However, unless λ varies appreciably during a time period, it appears that assuming λ constant is certainly adequate.

With this basic model, we are in a position to set up a control chart based on the number of ruptures observed during a given time period. However, rather than using this statistic, we choose to use the statistic consisting of the length of time required to accumulate a fixed number of ruptures. This statistic is chosen because:

- (1) The hypothetical λ will not be constant from time period to time period due to changing reactor conditions. If the statistic used is the number of ruptures per time period, the fact that the hypothetical λ shifts would necessitate plotting different sets of control limits for each time period. Such charts are difficult to maintain and interpret. This difficulty can be avoided using time as the statistic.

- (2) Unless λ is large, i.e., the time period is long, it will be impossible to detect an improvement in λ with just one point by observing the number of ruptures since, for λ as large as 3 or 4, the occurrence of 0 ruptures is not unusual.
- (3) Strict adherence to a chart employing the number of ruptures per time period would preclude the possibility of taking action as quickly as it may be warranted.
- (4) We are used to interpreting rupture data by observing sequences of ruptures, i.e., by observing how long it takes to accumulate R ruptures. We may as well formalize this approach and interpret such data in a more objective manner.

Mathematically, we want to find the distribution of the time between ruptures given that the number of ruptures observed for a given time period follows the Poisson distribution. Consider the length of time to accumulate R ruptures.

Let t_R = time in days to accumulate R additional ruptures. [i.e., (R+1) ruptures total]

λ = average number of ruptures per day.

Now, the probability that t_R is greater than some value t_0 is the same as the probability that there are (R-1) or less ruptures prior to t_0 . That is,

$$\text{Prob}(t_R > t_0) = \text{Prob}[\leq (R-1) \text{ ruptures up to } t_0]$$

$$= \sum_{j=0}^{(R-1)} \frac{e^{-\lambda t_0} (\lambda t_0)^j}{j!}$$

Thus, for R=1, i.e., time to accumulate 1 rupture, we have

$$\text{Prob}(t_1 > t_0) = e^{-\lambda t_0}$$

for R=2:

$$\text{Prob}(t_2 > t_0) = e^{-\lambda t_0} + \lambda t_0 e^{-\lambda t_0}$$

and, for R=3:

$$\text{Prob}(t_3 > t_0) = e^{-\lambda t_0} + \lambda t_0 e^{-\lambda t_0} + \frac{(\lambda t_0)^2}{2} e^{-\lambda t_0}$$

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In general, letting $F_R(t) = \text{Prob}(t_R < t)$, we have

$$F_R(t) = 1 - \sum_{j=0}^{(R-1)} \frac{e^{-\lambda t} (\lambda t)^j}{j!}.$$

These probabilities can be read directly from a table of the Poisson distribution. Or, we can also write:

$$F_R(t) = \int_0^t \frac{(\lambda t)^{R-1}}{(R-1)!} e^{-\lambda t} d(\lambda t),$$

which is the incomplete gamma function and is tabulated.

Note that if we make the transformation $u = \lambda t$, then

$$F_R(u) = \int_0^u \frac{u^{R-1}}{(R-1)!} e^{-u} du.$$

In the discussion above, we have assumed that λ is constant during the time interval. Suppose now that the interval from 0 to t is broken up into d intervals within each of which λ is constant, but such that λ may differ from interval to interval. Say that λ_j is the parameter within interval j , i.e., the expected number of ruptures per day within interval j is λ_j . Then it can be shown that if one considers the statistic

$$w = \sum_{j=1}^d \lambda_j t_j,$$

$$F_R(w) = \int_0^w \frac{w^{R-1}}{(R-1)!} e^{-w} dw$$

gives the probability that this statistic is less than some value w for given R .

The average value of this statistic is shown to be R , and the median is found by solving for w

$$\int_0^w \frac{w^{R-1}}{(R-1)!} e^{-w} dw = .50 .$$

For $R=3$, for example, the median value is $w = 2.67$.

We consider also the situation in which λ is some function of t , say $\lambda(t)$, but we do not account for it. This is really the situation in practice for a single reactor especially, where over a given operating period λ varies with time. The use of different λ 's for different time periods discussed in the above paragraph covers only the situation where appreciable changes in λ occur. We ask what are the consequences of using a constant λ over a shorter time period.

It can be shown that if λ is a function of t , $\lambda(t)$, then, for three time periods, for example,

$$\text{Prob } (t < t_0) = e^{-\int_0^{t_0} \lambda(t) dt} \left\{ \frac{\left[\int_0^{t_0} \lambda(t) dt \right]^2}{2!} + \int_0^{t_0} \lambda(t) dt + 1 \right\} .$$

Note that if $\lambda(t) = \lambda$, i.e., λ is constant, then

$$\text{Prob } (t < t_0) = e^{-\lambda t_0} \left\{ \frac{[\lambda t_0]^2}{2} + \lambda t_0 + 1 \right\} ,$$

consistent with the previous result.

Suppose now that λ is a linear function of t , in the interval $[0, t_0]$, viz.,

$$\lambda(t) = \lambda_0 + bt, \quad 0 \leq t \leq t_0,$$

then,

$$\text{Prob } (t < t_0) = e^{-\int_0^{t_0} (\lambda_0 + bt) dt} \left\{ \frac{\left[\int_0^{t_0} (\lambda_0 + bt) dt \right]^2}{2} + \int_0^{t_0} (\lambda_0 + bt) dt + 1 \right\}$$

$$= e^{-(\lambda_0 t_0 + \frac{bt_0^2}{2})} \left\{ \frac{\left[\lambda_0 t_0 + \frac{bt_0^2}{2} \right]^2}{2} + \left(\lambda_0 t_0 + \frac{bt_0^2}{2} \right) + 1 \right\}.$$

Further suppose that the average value of λ had been used in this interval. This average value is

$$\frac{\lambda_0 + (\lambda_0 + bt_0)}{2} = \lambda_0 + \frac{bt_0}{2}.$$

Then, by using this constant value for λ , we would get

$$\text{Prob } (t < t_0) = e^{-(\lambda_0 + \frac{bt_0}{2})t_0} \left\{ \frac{\left[(\lambda_0 + \frac{bt_0}{2})t_0 \right]^2}{2} + \left(\lambda_0 + \frac{bt_0}{2} \right)t_0 + 1 \right\}$$

which is seen to be equivalent to the exact result. Thus, the average λ in the case where λ is a linear function of time gives results equivalent to the exact result, if the average is computed over the given interval. This, of course, does not say that the use of an average λ over a long time period will give the correct reactor correction for points at one end or another of the interval. This is to say that for points close together, the λ used may be biased slightly. If a point goes out of control on the low side, one should at least consider this as a possibility. For a group of reactors, this danger is not great, but for a single reactor, this danger exists. It is impossible from a practical viewpoint to stay right on top of the situation, and the input data are not sufficiently accurate to even warrant this.

The above formulation permits us to set up the control charts once a value for R is chosen, and once it is determined what percent of the time in the long run we are willing to say that "metal performance" has shifted when it has not. Suppose we choose $R=3$ and we decide that 5% of the time we are willing to call a shift in "metal performance" real when it is not. Then, to find the control limits, we solve for w_1 and w_2 :

$$\int_0^{w_1} \frac{w^2}{2!} e^{-w} dw = \int_{w_2}^{\infty} \frac{w^2}{2!} e^{-w} dw = .025.$$

From a table of the gamma function, these limits are found to be

$$w_1 = 0.61$$

$$w_2 = 7.22$$

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Suppose now that we observe a rupture on day 10, one on day 35, and one on day 49, and that the following values hold for λ due to changing reactor conditions

<u>Day</u>	<u>λ</u>
1-30	.09
31-60	.12

In this case, the statistic w is $(30)(.09) + (19)(.12) = 4.98$. Since 4.98 lies within the interval $[0.61 - 7.22]$, we conclude that metal performance has not shifted. If, on the other hand, ruptures had occurred on days 3, 4, and 4, then $w = (4)(.09) = 0.36$ which is less than 0.61, indicative of a worsening of "metal performance." Note that it doesn't matter on which days in the interval the ruptures occur; only the last day counts.

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APPENDIX II

EVALUATION OF BASE METAL PERFORMANCE LEVELS

Rupture experience during CY-1959 for C-II-N, K-II-N, K-III-N, and O-II-N materials, both C-64 and X-8001 clad,* were selected for evaluating base metal performance levels to be used in making control chart calculations subsequent to December, 1959. Average values of the quantity $q = R/\Delta RI$, called the metal performance level multiplier, were estimated from the data in Table I.

The "Rupture Index" data from July through December were determined from IBM 709 machine calculations applied to the irradiation conditions experienced for each tube charge and summed over all tubes of a given type, e.g., O-II-N, during the periods in question. The basic equation used was

$$RI = P^{3.3} T^{8.7} M^5$$

Where: P is average tube power in kw.

T is average tube outlet temperature in °C.

M is exposure in MWD/tube.

The "Rupture Index" data from January through July were estimated from comparison of data calculated by the IBM 709 program and data calculated from the previously used IBM 702 program. "Rupture Index" calculations were performed using both methods for the time period July through November, resulting in five sets of data per reactor. The factors used to convert the 702 results to equivalent 709 results were determined for each reactor based on those data.

Average values of q were estimated using the equation

$$q = \frac{R}{\Delta RI} = \frac{\sum_i R}{\sum_j \Delta RI}$$

Subscript i refers to reactor.

Subscript j refers to month.

For O-II-N Material

$$q = \frac{27}{(85.14)(1039)} = 0.317 \times 10^{-39}; (q - O-II-N)$$

The true value of q, with 95% confidence, lies between 0.22×10^{-39} and 0.43×10^{-39} .

For C-II-N Material

$$q = \frac{9}{(13.34)(1039)} = 0.675 \times 10^{-39}; (q - C-II-N)$$

The true value of q, with 95% confidence, lies between 0.30×10^{-39} and 1.28×10^{-39} .

*Other studies not included in this report concluded there was no significant difference in the rupture performance of the two types during the first eleven months of CY-1959.

For K-III-N Material

Data for both K-III-N and K-II-N material were used in estimating the base performance level of K-III-N material. From July, 1959, through January, 1960, (January data are not tabulated in this report) the Δ RI data from KW Reactor were essentially 100 per cent K-III-N material; for the same period the Δ RI data for KE Reactor were essentially 100 per cent K-II-N material. Theoretically, using "best estimate" slug-by-slug rupture potential calculations, the values of q should differ from each other by only 20-30 per cent, in favor of the K-III-N material. The q estimates, based on the July through January data were, respectively:

K-II-N Material (July 1959 through January 1960)

$$q = \frac{4}{(55.6)(10^{39})} = .072 \times 10^{-39}$$

K-III-N Material (July 1959 through January 1960)

$$q = \frac{4}{(53.8)(10^{39})} = .076 \times 10^{-39}$$

These estimates of q are obviously not different from each other. On this basis it was decided to combine the K-II-N and K-III-N experience for CY-1959 in estimating the base metal performance level multiplier as follows:

$$q = \frac{17}{(125.15)(10^{39})} = 0.136 \times 10^{-39}; (q - K-III-N)$$

The true value of q , with 95% confidence, lies between 0.078×10^{-39} and 0.216×10^{-39} .

The tube C_R values corresponding to the above average values of q at the stated base conditions of flow rate, inlet temperature, and loss per rupture are as follows:

Type Material	Base Conditions			Average Tube C_R In CY-1959
	Inlet ($^{\circ}$ C)	Flow Rate (gpm)	Loss/Rupture (hrs.)	
C-II-N	0	50	24	2.7×10^{-18}
O-II-N	0	45	24	4.4×10^{-18}
K-III-N	0	61	19	1.7×10^{-19}

Current estimates of q and C_R (at time of issue of this report) are included in Section F of Appendix IV.

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TABLE I

CY-1959 RUPTURE INDEX AND RUPTURE DATA BY REACTOR

MONTH	B		D		DR		F		H		C		KE		KW	
	Δ RI	R	Δ RI	R	Δ RI	R	Δ RI	R	Δ RI	R	Δ RI	R	Δ RI	R	Δ RI	R
January	0.26	0	0.15	0	0.16	0	0.15	0	0.14	0	0.24	0	1.81	0	0.73	3
February	1.11	0	0.47	0	1.55	0	0.91	0	0.51	0	0.49	0	5.32	3	1.94	0
March	0.58	1	1.03	1	2.12	2	1.33	0	0.65	0	1.20	0	5.07	1	4.28	1
April	1.07	0	1.56	0	1.18	1	0.53	0	1.02	0	2.05	1	1.04	0	4.47	2
May	0.78	0	0.58	0	1.27	0	0.22	0	0.73	2	2.09	2	0.80	0	1.23	1
June	1.78	1	1.03	1	2.67	0	0.45	1	0.31	1	1.02	2	3.22	0	0.99	1
July	2.66	1	1.69	1	1.34	1	2.99	1	1.22	0	1.14	1	3.64	0	4.08	0
August	2.02	0	2.71	0	1.91	0	1.13	1	1.44	1	0.96	0	7.62	1	6.60	0
Sept.	3.27	0	2.39	0	1.66	1	1.43	0	2.62	0	0.59	0	5.89	1	8.56	0
October	2.53	0	2.34	1	2.24	1	2.00	0	0.46	1	1.35	1	7.06	0	10.41	1
November	5.81	0	1.51	0	1.59	0	0.34	0	1.27	1	1.26	0	11.44	0	8.32	0
December	1.31	0	2.21	1	1.62	1	1.10	2	2.03	1	0.95	2	13.79	1	6.84	1

Δ RI - Change in Rupture Index $\times 10^{-39}$

R - Number of ruptures (excludes second hole failures in multiple failure lots)

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APPENDIX IIIDECISION RULES IN CONNECTION WITH CONTROL CHARTS

There are basically three choices one must make in connection with using control charts of the type discussed to monitor rupture performance. The questions are:

- (1) How many time periods shall constitute a point on the control chart?
- (2) What percent of the time will we be willing to decide that "metal performance" has changed when indeed it has not? That is, how far apart shall the limits on the control chart be placed?
- (3) What action will be taken when a point is out of control?

These questions are all related in that the particular answer to any one of them will affect the answers to the other two. Two general considerations to keep in mind when making these choices are:

- (1) Provisions should be made for taking action rather quickly when warranted.
- (2) The consequences of making the wrong decisions must be evaluated and compared with the consequences of making correct decisions.

It must also be kept in mind that there is no one set of answers to these questions. There are any numbers of sets of decision rules which are comparable. However, some sets are clearly better than others in the light of present knowledge of fuel element performance based on years of experience. It is our intent to select a set of decision rules that is "better" than others. It may be that as operating experience builds up, changes of one kind or another will be made, but it is not anticipated that these will be major.

First, consider the number of time periods involved in each point. With too few points, the "power" of the test will be small, i.e., it would be extremely difficult to detect a real difference in "metal performance" with one time period per point since the random variation associated with a single point is so large. On the other hand, if too many time periods constitute a point, even though it will be easier to detect a true shift in "metal performance," the disadvantage comes in because of the length of time required to accumulate enough information to make a change that is indeed warranted. For example, if "metal performance" has indeed improved, resulting in longer time periods between ruptures, and if a large number of time periods constituted each point, then it would take an unreasonably long time to accumulate the number of ruptures, or time periods, required to reach a decision regarding changing reactor conditions to benefit from the improved metal quality. Thus, the number of time periods or ruptures per point should not be too small nor too large. Based on these considerations, the decision has been made to use three time periods per point.

The selection of the significance level and of the action to take when a point is out of control depends on the consequences of taking specified actions when not

warranted, and of failing to take action when warranted. With control charts, repetitive tests of hypotheses are being made, since the plotting of each point constitutes a test of a hypothesis. This is an important consideration to keep in mind. It states, for example, that if 95% limits are chosen such that the probability that a point is out of control due to chance alone is .05, then, if, say 10 points are plotted, and further, if the underlying metal quality is unchanged, the probability is only $(.95)^{10} = .60$ that all points will be in control. Stated differently, the probability is .40 that action will be taken when not warranted for this group of 10 points.

An investigation of the characteristics of a given decision rule for use with control charts can become quite complicated. Although a start has been made in this direction, further investigation is needed, and will be undertaken in the near future. Until this time, results found in an investigation of decision rules used in connection with a single hypothesis will be used as a guide in choosing significance levels and courses of action. Preliminary investigation of such rules in connection with control-chart usage points out that the types of result found for a single hypothesis will probably carry over in general to the multiple-hypothesis control chart use. Again, as experience accumulates, changes in the decision rules may logically be made, if these changes are made before-the-fact, of course.

Let us consider the evaluation of decision rules based on a single point, or a single test of hypothesis. The quantity used as a measure of productivity is net return. These are other measures that could perhaps be used, but it is important to remember that: (1) all these measurements depend strongly on the rupture experience, and (2) what really matters is the difference in productivity under different sets of conditions. Unless the conditions change appreciably, any measure of productivity will give like results.

The first step in the evaluation was to compute net returns under a given set of conditions for various combinations of power and exposure and different C_R values, where C_R is a measure of "metal performance." This resulted in the table below.

Table I

NET RETURNS UNDER DIFFERENT CONDITIONS*

Operate at Optimum conditions for $C_R =$	True C_R							
	2	4	6	8	10	12	14	16
2	1376	1181	1004	841	692	554	428	312
4	1354	1232	1127	1028	934	846	761	678
6	1298	1217	1144	1072	1004	939	876	815
8	1245	1189	1135	1083	1032	982	933	885
10	1203	1160	1117	1075	1034	994	955	916
12	1168	1134	1100	1066	1032	998	964	930
14	1136	1108	1081	1052	1023	996	969	940
16	1106	1082	1058	1035	1012	988	965	941

* Expressed in tens of dollars per day and coded by subtracting a constant.

From this table, we see, for example, that if C_R is truly 4, but if we believe it is 8 and operate accordingly, the net return is 1189. The maximum net return had we correctly assessed the situation is 1232.

Next, the hypothesis was set up that $C_R=8$, and four rules were evaluated. (These were based on observing the number of ruptures for a fixed time period rather than counting the number of days to accumulate a fixed number of ruptures, but the results with respect to the significance levels and actions to take would be the same.) It was assumed that the time period was such that a C_R of 8 would correspond to 4 ruptures. The following rules were then evaluated. Note that they cover the range of possibilities in that rule (1) essentially assumes there is no such thing as random variation, and rule (2) is the "stand-pat" rule which says to ignore rupture experience. Rules (3) and (4) are intermediate. With rule (3), we are more inclined to conclude greater changes than with rule (4).

Table II

# of ruptures observed	Rule (1)	Rule (2)	Rule (3)	Rule (4)		
0	}	Conclude	Conclude $C_R=8$	}	Conclude $C_R=6$	
1		$C_R = 2$	"		Conclude $C_R=4$	
2	}	$C_R = 4$	"	}	Conclude $C_R=8$	
3		$C_R = 6$	"			Conclude
4		$C_R = 8$	"			$C_R = 8$
5		$C_R = 10$	"			
6		$C_R = 12$	"			
7		$C_R = 14$	"			
8	}	$C_R = 16$	}	}		
≥ 9						

These rules are then evaluated using the concept of expected net return, which must be calculated for each C_R . A sample calculation will indicate how this is done.

Consider rule (3) at a true C_R of 4. For a true C_R of 4, the expected number of ruptures is 2, since at a true C_R of 8, as mentioned above, the expected number of ruptures is 4. For a true expected number of 2, we get the following probabilities of observing x ruptures:

# of ruptures observed	Probability	By rule (3) Conclude $C_R =$
0,1	.406	4
2-7	.593	8
≥ 8	.001	16

Thus, we would conclude that $C_R = 4$ with a probability of .406, etc. Now, from Table I, the net return in this situation is 1232. If we conclude $C_R=8$, the net return is 1189, and if we conclude $C_R=16$, the net return is 1082. Therefore, the expected net return is

$$(.406)(1232) + (.593)(1189) + (.001)(1082) = 1206.$$

This calculation was performed for various C_R values and for each decision rule. The results are tabulated in Table III, and depicted graphically in figure (1).

Table III

EXPECTED NET RETURNS

<u>True C_R</u>	<u>Rule 1</u>	<u>Rule 2</u>	<u>Rule 3</u>	<u>Rule 4</u>
2	1365	1245	1325	1265
4	1200	1189	1206	1193
6	1103	1135	1132	1135
8	1043	1083	1075	1083
10	1003	1032	1025	1032
12	973	982	981	984
14	948	933	945	939
16	926	885	915	898

It is clear from Table III and figure (1) that rule (1), which ignores the concept of random variation and is quick to take action, results in a poor expected net return curve. It gains only when fairly large changes in metal quality occur, and even in this event, it is almost matched by rule (3), (the "5%" rule), which even in the region near the hypothesis gives an expected net return curve close to the maximum possible. Rule (2), the "stand-pat" rule, is also not acceptable because it is practically identical with rule (4) near the hypothesis, and fails to detect large changes as will rule (4). All in all, one would be inclined to choose rule (4), or a similar rule, if it were known that changes in true metal performance are not very great (say less than a factor of 2), and rule (3), or a similar rule, if the cycles in the metal performance level are, on occasion, larger than a factor of 2.

It is evident that the important factor in assessing the various rules is a knowledge of what has been the experience with regards to cycles and trends in metal performance level. Interpretation of rupture data is after the fact. If cycles are short range, then it does us no good to detect them because by the time action is taken, it is too late. Control charts on ruptures can only be useful as a control tool, or as an action taking device if there exist trends or longer term cycles in metal quality. Otherwise they are useful primarily from a historical standpoint which, in itself, has a certain amount of merit from a presentation standpoint. The fact remains that if cycles in metal quality are short range, the only way to take appropriate action is on the basis of pre-irradiation data. This, of course, is the primary aim of the Quality Certification Program. This is not to say, of course, that short term cycles in metal quality have no effect on reactor performance or productivity. Even short term improvements pay off in a reduced number of ruptures.

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As mentioned previously, some preliminary investigation has been made with regards to the decision rules when repeated tests of hypotheses have been made. More work needs to be done in this area. Investigation thus far has been restricted to a simple alternative in which we believe C_R to be some value, but it is truly different by a given factor. Even in this simple case, the calculations involved become laborious. One must calculate, for each point on the control chart, the probability that a certain decision is reached. These probabilities are conditional in that they depend on the decision reached at the previous stage.

To indicate what sort of results are found, two decisions rules were evaluated in an example. The first rule states that 95% control limits are used, and when a point is out of control, a change of 20% in rupture potential is made. In the second rule, 80% limits are used, and a change of 50% is made in rupture potential when a point is out of control. The alternative considered was that C_R is truly larger than hypothesized by 50%.

Consider this second rule for the situation when the hypothesis is true, i.e., C_R is correctly assessed. Application of this rule would indicate, for example, that after 6 ruptures have been sustained (2 points plotted), the probability is .694 that we would be operating under the hypothesized C_R during the next time interval. This may arise from:

- (1) Operation at this level for the 2 time periods, with probability $(.80)^2 = .640$.
- (2) Operation at this level; then 50% higher; then back again with a probability of $(.100)(.312) = .031$.
- (3) Operation at this level; then 50% lower; then back again with a probability of $(.100)(.229) = .023$.

Therefore, the over-all probability is $(.640 + .031 + .023) = .694$ that in the interval from 6 to 9 ruptures, operation would be at the hypothetically correct C_R value.

This calculation was performed for each of the two rules for the situations when the hypothesis was true and when C_R was higher by 50%. Table IV results:

Table IV

EXPECTED NET RETURNS

Interval	Rule(1) $\Rightarrow$ 95% Limits and 20% Shift		Rule(2) = 80% Limits and 50% Shift	
	$CR = CR_0$	$CR = 1.5 CR_0$	$CR = CR_0$	$CR = 1.5 CR_0$
0-3 Ruptures	1083	982	1083	982
3-6	1082.7	982.6	1079.5	984.8
6-9	1082.4	983.2	1077.4	986.4
9-12	1082.1	983.8	1076.0	987.2
12-15	1081.9	984.3	1075.3	987.7
15-18	1081.6	984.9	1074.9	987.9
18-21	1081.4	985.4	1074.6	988.0
21-24	1081.2	985.8	1074.5	988.1

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Notice the similarity between these results and those in figure (1). The more conservative rule (1) is preferable near the hypothetical value, whereas the less conservative rule (2) has the advantage for values removed from the hypothetical value. Against an alternative that consists of cycling C_p values, or metal performance, with time, the conservative rule will no doubt have the advantage if cycles are not severe or of short duration, whereas less conservative rules are preferred if cycles are long range and of appreciable magnitude. The specific evaluation of this has not yet been attempted.

In any event, in order to choose a set of decision rules at which to operate, one must be guided by a knowledge of how metal quality has varied in the past. Since experience has indicated that such variations are apparently not extreme, one is inclined to use a more conservative rule consisting of small changes in reactor conditions, and of making these changes only when one is pretty sure that an out of control point really represents a true change in metal quality. Specific recommendations are contained elsewhere in this report.

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APPENDIX IV

PROCEDURE FOR CONTROL CHART CALCULATIONS

The statistics that will be used for monitoring changes in the metal performance level are designated (w) and (w'). The first statistic, (w), is defined as the "adjusted time between four consecutive ruptures." This statistic is computed after every three ruptures, since the fourth rupture of the preceeding group becomes the first rupture in the second group, hence, the application of the term "three-rupture point" to (w). The second statistic (w') is the arithmetic average value of five consecutive three-rupture points.

A. Calculation of (w) and (w')

Given λ_1 (the method for determining λ_1 is outlined in Section B of this appendix) the statistic (w) is calculated from the following general equation:

$$(1) W = \sum_{i=1}^3 \lambda_i t_i$$

Where: λ_i = expected rupture rate for the i th time interval, in ruptures per calender (or operating) day.

t_i = time interval between consecutive ruptures in calender (or operating) days.

For example, given the data

- 1) From an index point, t_0 , ruptures occur on $t_1 = 4$, $t_2 = 35$, and $t_3 = 38$.
- 2) $\lambda = .035$, from t_0 through $t = 19$.
 $\lambda = .020$, from $t = 19$ through $t = 38$.

Compute w as follows:

$$w = (.035)(19-4) + (.020)(35-19) + (.020)(38-35)$$

which is equivalent to

$$w = (.035)(19-0) + (.020)(38-19) = 0.67 + .38 = 1.05$$

Note that it does not matter on which days in the interval that the ruptures occur, only the last day counts. Also note that once the interval is defined, that is, the time to accumulate three ruptures from the index point, that changes (shifts) in λ define break-points in the calculational procedure. If λ had, for example, been constant over the entire 38 day interval, say at .027, then $w = (38)(.027) = 1.03$. Suppose λ had shifted twice during the interval; say $\lambda = .035$ from t_0 through $t = 19$, $\lambda = 0.020$ from $t = 20$ through $t = 29$, and $\lambda = .029$ from $t = 30$ through $t = 38$. In this case $w = (.035)(19) + (.020)(10) + (.029)(9) = 1.13$.

The control chart is prepared by plotting the consecutive values of (w) on graph paper on which the control limits are drawn in (see page 40 Appendix V for control chart examples). The 95% control limits on (w) are 0.61 and 7.22 (Appendix III).

Values of (w') are simply the average value of five consecutive (w) points, and can be plotted on the same charts, if desired. The control limits on (w') are 1.67 and 4.70.

B. Calculation of λ

The value of λ is computed using the general equation:

$$(11) \lambda_1 = q_1 \frac{\Delta RI_1}{t_1}$$

Where: λ = expected rupture rate in ruptures per calendar (or operating) day.

q = metal performance level multiplier.

ΔRI = change in Rupture Index during the time interval.

t = time interval in calendar (or operating) days.

For example, given the data:

1) $\Delta RI = 1.0 \times 10^{39}$

2) $t = 25$ days

3) $q = .380 \times 10^{-39}$

Compute λ as follows:

$$= \frac{(.380)(10^{-39})(1.0)(10^{39})}{25} = .015$$

Theoretically ΔRI , and hence λ , can be calculated for any time period. In practice, ΔRI is calculated monthly for each reactor. If λ can reasonably be assumed as being constant over a month, as is the case when data from several reactors are grouped together (e.g., the five old reactors), then it is convenient to express λ and t using calendar days. When calculations are being made for a single reactor, because of operating efficiency shifts from month to month, it is probably best to use operating days in expressing λ and t . At a single reactor, λ can shift considerably over the month. These shifts, intuitively, would have little or no effect on the calculation of (w) , when the time to accumulate the necessary ruptures is long, as will often be the case when a single reactor's experience is being examined. When the time to accumulate the necessary ruptures is short, say two to four weeks, shifts in λ could result in errors in (w) . Even in this case, however, the errors would probably be small. It is shown in Appendix I that if λ shifts as a

linear function of time during the interval in which the ruptures are accumulated, that the value of (w) computed using the average λ over the whole period gives the exact result obtained by breaking the interval up into smaller time segments with many different λ 's. The precision of the input data is such that if one can be assured that λ shifts in an essentially linear fashion, that corrections for non-linearity are not required.

Consider the case of an old reactor, on a variable goal plan, and operating with an expected rupture rate of about one rupture per calendar month. The reactor has just completed a rupture outage. Typically, after a normal discharge, the instantaneous rupture index in the I & E natural material at startup will be approximately 1.0×10^{38} . With $\lambda = .047$ ruptures per operating day (this corresponds to 1.1 ruptures per calendar month at 80% operating efficiency), by equation (i) the time to accumulate these ruptures from the index point (in this case, startup) would have to be less than $.61/.047 = 13$ operating days for (w) to go out of control on the low side. Assuming that the residual exposure of the I & E natural at startup is 200 MWD/T and that it changes at a rate of 10 MWD/T per operating day, the following data can be developed:

Operating Days Since Startup	Instantaneous Rupture Index*	Δ RI per 5 Operating Day Period	Average λ per 5 Operating Day Period**
0	1.0×10^{38}	---	---
5	2.8×10^{38}	0.18×10^{39}	.017
10	7.2×10^{38}	0.44×10^{39}	.042
15	1.6×10^{39}	0.88×10^{39}	.084

* assumes no discharging

**assumes $q = .476 \times 10^{-39}$

The average λ per "5 operating day period" is plotted below.

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Clearly, λ per "five operating day period" as a function of time can be well approximated by a linear function, in this case. From the above discussion it is intuitively clear that if the ruptures occur in much less than thirteen operating days that the (w) point will most assuredly be out-of-control, notwithstanding errors in λ .

Rupture Index data, i.e., Δ RI per calendar month, will be provided to the Reactor Engineers on a monthly basis. These data, and the current estimates of q, can be used for calculating λ -- preferably on an operating day basis for single reactor application. With this information, the Reactor Engineer will be able to maintain a control chart of the reactor's rupture experience in I & E natural material.

C. Current Metal Performance Levels

Values of q and corresponding values of C_R at the stated base conditions, in effect at the time of issue of this report, are as follows:

Type Material	Base Conditions			Tube C_R	q
	Inlet ($^{\circ}$ C)	Flow Rate (gpm)	Rupture Loss (hrs.)		
C-II-N	0	50	24	8.1×10^{-18}	2.03×10^{-39}
O-II-N	0	45	24	6.6×10^{-18}	0.476×10^{-39}
K-III-N	0	61	19	1.7×10^{-19}	0.136×10^{-39}

D. Treatment of Multiple Side Failure Lots

For purposes of calculating (w), all second failures in multiple failure lots should be excluded.

The rupture model that is used in calculating rupture index (and λ) implies that each tube has associated with it a set of reactor operating conditions under which the probability of incurring a rupture is fairly well established. There are tubes, more properly groups of tubes (lots), where this is not wholly true. By and large, these are lots which sustain more than one rupture, i.e. multiple failure lots. It can be shown that the proportion of lots which sustain multiple failures is significantly higher than would be expected due to chance alone, under the model. Unfortunately, one cannot positively assert, for any given multiple failure lot, that its performance was inconsistent with the rupture model. To remain objective, it is necessary, therefore, to decide beforehand how multiple failure lots should be treated in making the control chart calculations. The proportion of rupture-lots which sustain multiple failures is in the range of one in ten (1/10) to one in five (1/5), and experience has shown that this proportion is fairly stable. Clearly, the inclusion of all ruptures in the calculation of (w) will introduce an unknown bias because most of these ruptures are not consistent with the model and can unjustifiably cause points to go out-of-control -- this is over and above the chance effects that were accepted when the control limits were set. The eliminations of all ruptures in multiple failure lots when calculating (w) is, of course, also incorrect because some multiple failure lots are expected under the rupture model. Such a procedure would be impractical, as well, because one cannot consistently predict the occurrence of a second side failure on the basis of data associated with the first rupture sustained. There is, then, no basis for deciding beforehand whether or not any first rupture in a lot should be excluded from the calculation of (w).

The best course of action, which balances both the desirable and undesirable analytical aspects of multiple side failure lots, is to exclude all second failures is the calculation of (w) points.

E. Control Chart Calculation Records

The control chart calculations of themselves are simple once the required data have been assembled. To facilitate the calculations and the maintenance of adequate records, it is suggested that each reactor engineer use forms similar to those which are shown on the following page. Two sets of data should be kept up; the first, a working sheet where experience of the "current" month is tabulated; and second, a "final" or "historical" set using the more exact rupture index numbers. The "working" numbers will necessarily be based on projected values of λ , because the exact machine rupture index calculations are not available until sometime after the end of the "current" month. For single reactor calculations the Reactor Engineer is in the best position to project λ for use in the "working" sheet calculations. Correlations of λ with the parameters OSI and average goal exposure can be made and used for projecting λ , if necessary. A simple assumption that λ for the current month will be approximately as experienced during the preceeding month, or average of the preceeding two or three months, would often suffice. Whatever method is employed, it is probable that one or more of the λ values used in calculating a given control chart point will have been based on the "final" type calculations that preceeded the current month.

F. Adjustments to q and C_R

The metal performance level multiplier (and C_R) will be adjusted* on the basis of the following decision rule. As discussed in Appendix III, this rule is believed to be the "best" rule that can be developed on the basis of the investigations completed to date.

- a) When a "three-rupture" point (w) goes out of control, adjust q (and C_R) by a factor of 1.2, upward or downward as indicated by the data.
- b) When the average of five consecutive "three-rupture" points (w') falls out of control, notwithstanding that the (w) points are in control, adjust q (and C_R) by a factor of 1.5 in the direction indicated by the data.

When q shifts, subsequent control chart points must be calculated using the "new" value. The calculation of (w') points will be delayed then until five "new" (w) points have been calculated; do not combine "old" and "new" (w) points for this calculation.

*Adjustments for O-II-N material will be made on the basis of the combined experience of the five old reactors; adjustments for K-III-N material will be made on the basis of the combined experience of KE and KW Reactors; only C-II-N adjustments will be based on single reactor's experience.

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APPENDIX V

SAMPLING EXPERIMENT - DATA AND CONTROL CHARTS

The data presented in this section will serve to illustrate the extent to which chance variations alone influence the observed rupture frequency. The data that were generated in this sampling experiment were used to calculate values of the statistic (w), using the procedure in Appendix IV, and the control charts are plotted. Detailed and "questioning" inspection of these charts, and of the generated "rupture data" should prove to be very instructive to the reader. The following data were summarized from the rupture index history (Appendix II) of the B, D, DR, F, and H Reactors for CY-1959.

<u>Time Period</u>	<u>E(R)</u>	<u>λ</u>
(1) January - February	1.7	.028
(2) March - April	3.4	.057
(3) May - June	3.0	.050
(4) July - August	5.9	.098
(5) September - October	6.4	.107
(6) November - December	5.6	.093
TOTAL	26.0	

$E(R)$ is the expected number of ruptures calculated by $E(R)_i = q_i \Delta R I_i$ (q was erroneously taken as $.307 \times 10^{-39}$ rather than $.317 \times 10^{-39}$; hence the $E(R)$ values are slightly low and the total number of ruptures is 26 rather than the 27 actually observed).

λ is the rupture rate (ruptures per calendar day) assumed constant over the time period. A simplifying assumption that each time period is 60 days was also made.

Starting with these data (Note: the true total number of ruptures is 26, this is a known constant) and using tables of random numbers we will generate 20 sets of "CY-1959 O-II-N Rupture Incidence Data" as follows:

Let each three digit number in a table of random numbers correspond to one day. Entering the table at random, let the first sixty consecutive numbers represent time period (1); the next sixty; time period (2); and so on. For the first time period, let the numbers 000 - 027 represent days on which ruptures occur; for the next time period, let the numbers 000 - 056 represent days on which ruptures occur; and so on.

[In a table of random numbers all digits occur with the same frequency. For time period (1), for example we can choose beforehand a set of 28 three-digit numbers, the most convenient set being 000 - 027, and call them "days on which ruptures occur." Since there are exactly 1000 unique numbers (000-999) that could be selected from the table, the probability of any one of the numbers in the pre-determined set of 28 being selected on any one try (day) is $28/1000$ or .028, which is the value of λ for time period (1).]

The rupture data, in the order that they were generated, are summarized in Tables I(A) and I(B).

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TABLE I (A)

HYPOTHETICAL CY-1959 O-II-N RUPTURE INCIDENCE DATA
Sets 1 Through 10

Time Period	λ E(R)	1	2	3	4	5	6	7	8	9	10
(1) Jan. Feb.	.028 1.7	46 (1)	51 (1)	1,53 (2)	 (0)	47 (1)	19 (1)	20,45, 58 (3)	56 (1)	13 (1)	14,54 (2)
(2) Mar. Apr.	.057 3.4	47,56 (2)	16,30, 56 (3)	 (0)	31,36, 50,57 (4)	9,16, 18,48 (4)	13,17, 20,35 (4)	17,30, 40 (3)	9,10 14,18, 43 (5)	5 (1)	1,4, 6,8, 15,28,41 (7)
(3) May June	.050 3.0	4,16 (2)	19,41 (2)	36,38 (2)	5,12, 17,24, 38,40 (6)	6,25, 26,53 (4)	2,7, 12,34, 39 (5)	7,46, 49,60 (4)	13,16, 29,34 (4)	9,33, 35,37, 45 (5)	57,58 (2)
(4) July Aug.	.098 5.9	26,44, 53,54, 55 (5)	9,17, 24,37 (4)	5,42, 58 (3)	3,37 (2)	3,4, 11,17, 30,32, 42,43, 59 (9)	5,24, 34,38, 59 (5)	3,24, 28,32, 35,53, 59 (7)	2,13 16,26 28 (5)	8,9, 15,34, 36,46 49 (7)	25,26 43,56, 57 (5)
(5) Sept. Oct.	.107 6.4	6,7 26,38 43,44 54 (7)	23,60 (2)	22,36, 42,44 48,50 58 (7)	6,7, 26,38, 43,44, 54 (7)	21,25, 26,27, 52 (5)	10,13, 16,17, 21,25 45,54 (8)	1,7, 22,34, 42,46 60 (7)	4,18, 35 (3)	3,11, 22,25, 36,42, 45,46, 54,56 (10)	11,18, 22,48 (4)
(6) Nov. Dec.	.093 5.6	1,26, 28,32, 49 (5)	1,18, 27,28 (4)	3,45, 49,55 (4)	1,26, 28,31, 50 (5)	11,20, 36,38, 41,46, 51 (7)	5,8, 37,40, 47,53 (6)	5,8, 11,26, 34,44, 48 (7)	19,31, 37,45, 47,56, 57 (7)	13,31 37,55 (4)	4,6, 41,45 (4)
TOTAL R	26	22	16	18	24	30	29	31	25	28	24

In the main part of the above table the numbers in parenthesis are the total ruptures in each time period. The remaining number(s) represent the day(s) on which ruptures are sustained, e.g., 45 means a rupture on the 45th day in the time period.

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TABLE I (B)

HYPOTHETICAL CY-1959.0-II-N RUPTURE INCIDENCE DATA
Sets 11 Through 20

Time Period	λ E(R)	11	12	13	14	15	16	17	18	19	20
(1) Jan. Feb.	.028 1.7	53 (1)	39,42, 43 (3)	33,44 (2)	7 (1)	51 (1)	4,22, 60 (3)	9,12 29 (3)	 (0)	4,10 59 (3)	59 (1)
(2) Mar. Apr.	.057 3.4	25,26, 42,46, 52 (5)	2,8, 36,51 (4)	15,57 (2)	7,10, 14,41 (4)	42,60 (2)	2,17, 18,23, 29,46, 59 (7)	13,16, 25,30, 59,60 (6)	46,57, 59 (3)	52,55 (2)	8,12, 23,46 (4)
(3) May June	.050 3.0	9,45 (2)	10,17, 34 (3)	2,16, 26 (3)	33,57, 58 (3)	29 (1)	28,30, 31 (3)	16,56 (2)	45,46 (2)	11,41, 43 (3)	60 (1)
(4) July Aug.	.098 5.9	7,8, 19,56, 57 (5)	5,6, 12,20, 22,28, 33 (7)	9,14, 16,26, 38,44, 52,56 (8)	14,21, 23,24, 26,32, 50,52, 60 (9)	5,14, 24,27, 33,51 (6)	2,11, 36,42, 45,47, 50 (7)	3,31, 32,57 (4)	9,24, 26,27, 40,48, 56 (7)	3,13, 15,22, 40,52, 56,57 58 (9)	15,19, 30,37, 42,52, 53,59 (8)
(5) Sept. Oct.	.107 6.4	6,23, 29,39, 46,51 (6)	1,13, 15,28, 29,35, 39,43, 57 (9)	19,25, 29,38, 50 (5)	28,37, 47,52, 55 (5)	8,11, 13,17, 26,39, 54 (7)	12,31, 32,56 (4)	1,8, 14,33, 42,47, 48,54, 59,60 (10)	9,14, 19,21, 23,32, 40,43, 53,57 (10)	15,60 (2)	7,24, 32,40 (4)
(6) Nov. Dec.	.093 5.6	17,31, 42,51, 56 (5)	4 (1)	27,50, 53,55 (4)	4,13, 18,19, 37,38, 46,48, 49,53, 54 (11)	5,14 (2)	3,13, 28,30, 31,32, 45,52, 58 (9)	13,17, 23,26, 36,38 (6)	3,9, 14,22, 24,45, 50,60 (8)	5,15, 21,39, 41,49 (6)	8,28, 34,35, 39,40, 44,51, 59 (9)
TOTAL R	26	24	27	24	33	19	33	31	30	25	27

In the main part of the above table the numbers in parenthesis are the total ruptures in each time period. The remaining number(s) represent the day(s) on which ruptures are sustained, e.g., 45 means a rupture on the 45th day in the time period.

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As the tabulated data are being examined, keep in mind that they are observations from the same true underlying situation -- the expected total number of ruptures is twenty-six, in fact. The variation in the "observed" rupture frequency results solely from pure randomness. Observe, for example, the variation in the total number of ruptures between the sets of data. The totals varied from 16 to 33. The probability of observing 16 or less ruptures when the true value is 26 is quite small, 0.025, yet once in twenty times is surely not unusual. The interesting fact is that the total of 16 happened with the second set of data, not the tenth, or eleventh, or at some other point where one might be more "willing to believe."

There are numerous examples of chance variations within these data that can be used to illustrate the dangers of subjective analysis, especially when applied to a problem; such as rupture analysis, where the number of observations are very few. These examples will clearly illustrate that the subjective approach should be avoided whenever possible, and that when such an approach is necessary that (1) recognition of the subjectivity by the analyst is vital, and (2) that statistical inferences made from "selected" data have, in fact, little or no value. This is not to say that subjectivity is always improper. There are many times, particularly when theories and relationships are being developed, when a basically subjective approach is used in formulating hypothesis from a set of observations or data. Theories thus formed, and they are often weak in many respects, only acquire stature of reasonable degree when they are tested repeatedly, and found to hold true, with data other than those from which they were developed.

Analyses frequently become subjective when one "selects" the data that are to be analyzed while at the same time developing the null hypothesis to be tested, during the selection process. If the hypothesis developed is a "reasonable" one, and this is easily the case, the results will almost surely support or deny, according to the analyst's requirements, the thesis at hand.

For example, suppose that the actual old reactor rupture experience was Set #12 in Table I (B). As the data for the last six months are viewed "after the fact" one can hardly avoid an intuitive conclusion that the rupture rate really dropped off, particularly if the data are further broken down and one notes that no ruptures occurred during the last 56 days of this six-month period while during the preceeding 64 days ten ruptures were sustained.

Furthermore, there is no difficulty at all in choosing the data so as to calculate that there has been a "statistically significant" reduction in the rupture rate. These same data, however, when monitored by the proposed control chart scheme were in-control, not just for the last six months, but for the whole year (see chart 12, figure 2, to follow). Note also that the total number of "ruptures" during the year was 27, which is very close to the true value of 26.

Examine now the data of Set #19 for September - October. Note a total of two ruptures as compared with an expected value of 6.4. These data, even when tested "statistically", would not indicate that the rupture rate was "significantly" lower than expected, but note that if one simply "chose" fifty-nine days instead of sixty that (1) the observed number of ruptures drops to one, (2) the expected number to 6.3, and (3) the "difference" is "significant" at the four per cent level.

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At this time, the reader is asked to examine the control charts of (w) calculated from the generated data (see Figures 1 through 4 in this section). These control charts were calculated by the procedure in Appendix IV, using the data of Table I (A) and Table I (B), above. (The control charts of the hypothetical experience are labelled, correspondingly, ① through ②①. Chart ②① shows the actual experience for CY-1959 for O-II-N material.) Recall that nothing more than chance variations in (w) are reflected on these charts, and note the wide swings and perhaps intuitively unusual sequences of points that can occur. Examine chart 14, figure 3; note the strong downward trend ("indicating" a decline in metal performance) of the last five points, culminated by the last point being out-of-control. Examine the raw data in Table I (B) and note essentially the same trend. Examine chart 16, figure 3; note the "readily apparent" trend of improvement for the first eight or nine points. The rupture rate starts out "higher" than expected, gradually begins to show "good" agreement, then appears to be "lower" than expected; the intuitive feeling develops that the trend must be real, then suddenly the next point drops out-of-control indicating a decline in metal performance.

In both of the above instances, using the control limits and decisions rules recommended for adoption by this report, out-of-control points were observed and actions to alter reactor power and exposure conditions would be taken -- unnecessarily. It should be noted that in all 20 sets of data, involving 171 (w) points, that five were out-of-control. This, of course, is not inconsistent with the 95% control limits, since the expected number of out-of-control points is $(.05)(171) = 8.55$, due to chance alone. This risk is one that is assumed once the control limits have been decided upon.

The foregoing discussion serves to illustrate that in an area as important to the economic operation of the plant as is metal performance, that objective methods of analyzing rupture experience, utilizing to the fullest extent possible, the knowledge gained from past experience, are a real necessity. This is particularly true in the case of rupture analysis because, as is graphically illustrated by the control charts, the chance fluctuations in rupture frequency are of the same magnitude as the differences we wish to detect. It is the author's considered opinion that the adoption of the control chart technique, as proposed, is technically and economically prudent and correct.

Figure 1
Control Charts of (w)

Figure 2
Control Charts of (V)

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Figure 3

Control Charts of (w)

() Data Set Number
 () Total "Ruptures" for the "Year"

Figure 4
Control Charts of (w)

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