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**NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF EARTH SYSTEM SCIENCE**

**CLIMATE EVOLUTION WITH A COUPLED TWO DIMENSIONAL
ATMOSPHERE /OCEAN MODEL**

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Effects of Eddy Diffusivity Law on Global Warming in an Upwelling-Diffusion Ocean Climate Model

Martin Hoffert and Haroon Kheshgi

To test the impact of a density-dependent eddy diffusivity law on global warming predictions we did transient calculations with a standard upwelling-diffusion ocean/climate model driven by the IPCC historical anthropogenic greenhouse radiative forcing from 1860 to 1990 (Shine et al., 1990) for $\kappa_0 = 2000 \text{ m}^2 \text{ y}^{-1}$ and the $\kappa = a/N$ eddy diffusivity model of Hoffert and Kheshgi (1994). Boundary conditions bracketed various assumptions regarding mixing versus downwelling in the polar sea, and warming of the polar versus the surface mixed layer box.

FIG.1. Schematic diagram of upwelling-diffusing world ocean coupled to high-latitude polar sea.

The 1D World Ocean plus Polar Sea model is shown in FIG. 1. Here A_0 and A_p are surface areas of the World Ocean and Polar Sea, w is the mean upwelling velocity in the World Ocean and w_p the mean downwelling in the Polar Sea. From continuity, $A_0 w = A_p w_p$. Note that lateral flux between the World Oceans and Polar Seas is only allowed at the top (surface mixed layer) and bottom (benthic boundary layer) in this model.

In addition to different World Ocean diffusivity laws, we allow for a vertical diffusive heat flux in the Polar Sea box proportional to the difference between surface and deep polar temperatures, $T_p - T_b$. This is a departure from the purely downwelling Polar Sea of Hoffert et al. (1980) used by the IPCC (Bretherton et al., 1992). Net heat flux (downwelling plus mixing) to the deep polar ocean takes the form $F_b = \rho C A_p w_p [T_p + \gamma(T_p - T_b)] = \rho C A_0 w [T_p + \gamma(T_p - T_b)]$, where ρC is the

specific heat per unit volume and γ an empirical coefficient proportional to the ratio of mixing to downwelling heat flux in the Polar Seas. Downward heat flux at the sea floor $z = h$ in the Polar Box is equated to the net (upwelling plus diffusion) upward heat flux into the World Ocean, $F_b = \rho C A_0 [\kappa (\partial T / \partial z) + w T]_{z=h}$. Note that The corresponding lower boundary condition on the World Ocean is

$$@ z = h: \quad \kappa \left(\frac{\partial T}{\partial z} \right)_{z=h} + w T_b = w T_p + \gamma w (T_p - T_b), \quad (1)$$

with the *Polar Sea mixing coefficient*,

$$\gamma = -1 - \frac{\kappa (\partial T / \partial z)_{z=h}}{w (T_p - T_b)} \quad (0 \leq \gamma < \infty). \quad (2)$$

FIG. 2 Observed global mean surface temperature variation relative to the year 1890 compared with global warming from 1870-1990 predicted from an upwelling-diffusion ocean/climate model for different World Ocean diffusion laws and Polar Sea mixing and surface warming rates.

The limit of a nondiffusive Polar Sea ($\gamma = 0$) recovers the lower boundary condition of a purely downwelling Polar Sea (Hoffert et al., 1980): $\kappa (\partial T / \partial z)_{z=h} + w [T_b - T_p] = 0$. The opposite limit of an infinitely diffusing Polar Sea ($\gamma \rightarrow \infty$) keeps the temperature of the World Ocean

lower boundary equal to the Polar sea surface temperature (SST) as the system evolves: $T(h) = T_b = T_p$. Numerically, we set γ to an arbitrarily large value $\gg 1$ to compute the diffusion-dominated Polar Sea limit.

As previously, we parameterize the warming of the high-latitude SST to the World Ocean global mean SST by the *Polar Sea warming parameter*,

$$\Pi = \frac{T_p - T_{p0}}{T_s - T_{s0}}, \quad (3)$$

where subscript zero denotes initial conditions. The IPCC found $\Pi = 1$ matched coupled atmosphere/ocean GCM simulations reasonably well (Bretherton et al., 1992).

Global warming runs computed with the upwelling-diffusion model modified as described above are shown in FIG. 2. The thick solid line is a five-year running mean -- to smooth out interannual variations -- of observed global mean annual surface air temperature anomalies relative to the year 1890 corrected for the influence of the El Niño Southern Oscillation (ENSO) (Boden et al., 1991).

Despite residual "noise" in the signal there is a distinct global warming trend of 0.5 ± 0.2 °C over the past hundred years. Natural climate variability is a possible explanation, although century-long global warming (or cooling) trends of 0.5 °C occurred infrequently enough over the past several thousand years -- perhaps once or twice a millennium (Schneider, 1994) -- to support an anthropogenic greenhouse explanation. The problem has been that transient global climate models driven by greenhouse gas radiative forcing, including four of our six model runs shown in FIG. 2, produce *too much* global warming.

A possible explanation is that positive radiative forcing by anthropogenic greenhouse gases (CO_2 , CH_4 , N_2O and CFCs) is reduced by negative forcing from anthropogenic sulfate aerosols. Early order-of-magnitude estimates suggested that the $\sim 2 \text{ W m}^{-2}$ preindustrial-to-present radiative forcing from greenhouse gases is opposed by as much as $\sim -1 \text{ W m}^{-2}$ negative forcing by anthropogenic aerosols in the global mean. (The spatial distribution of sulfate is highly nonuniform and concentrated over industrial Northern Hemisphere land masses.) More recent estimates suggest weaker negative forcing from anthropogenic sulfate aerosols $\sim -0.3 \text{ W m}^{-2}$ (FIG. 3). We are in any event investigating whether more vigorous heat uptake by the oceans can explain the observed temperature history assuming a purely greenhouse forcing, recognizing that reality may involve a combination of oceanic uptake and aerosol effects.

Parameters of our model runs are defined in FIG. 2. All six runs had the same radiative greenhouse forcing history (FIG. 3) and the same atmospheric climate sensitivity ($\Delta T_{2x} = 2.2^\circ\text{C}$). Upwelling was also held constant at $w = 4 \text{ m y}^{-1}$.

FIG. 3 Greenhouse gas radiative forcing history used by IPCC and in the present calculations. Preindustrial to 1990 global mean radiative forcing estimates of Kiehl and Briegleb (1993) are also shown with and without compensatory sulfate aerosol scattering.

The predicted global warming over the hundred-year period from 1890 to 1990 ranged from 0.47 to 0.87°C depending on differences in the ocean model assumptions. The first two cases with $\Pi = 0$ resulted in the greatest warming because with the Polar Sea surface temperature held constant ($T_p = T_{p0}$) heat is constrained by the model to enter the deep sea though the main thermocline -- there is no "fast track" to the abyss bypassing thermocline bottleneck. When the Polar Sea route is suppressed, heat transfer to the deep oceans is diffusion-limited in the stably stratified World Ocean thermocline. A $K = a/N$ diffusivity law results in more warming than constant diffusivity because of the positive feedback from increases in the Brunt-Väisälä frequency $N = [(g/\rho)(\partial\rho/\partial z)]^{1/2}$ near the surface, as the surface warms, and temperature and density gradient $\partial T/\partial z$ and $\partial\rho/\partial z$ steepen (Lashof, 1989). Of the six cases considered, the penetration of heat to the oceans for $K = a/N$ with $\Pi = \gamma = 0$ (case 1) was the smallest, and the largest global warming -- 0.87°C per century.

Surprisingly, the effect of a stratification-dependent diffusivity law on the global warming rate can reverse if Polar Sea warming and diffusive mixing occur simultaneously. Cases 3 to 6 in FIG. 2 correspond

to high-latitude warming equal to that of the global mean, as assumed by the IPCC ($\Pi = 1$). So long as the Polar Sea pumps heat to the sea floor by downwelling ($\gamma = 0$) (cases 3 and 4) the $K = a/N$ diffusivity remains a positive feedback relative to the constant- κ law although both cases warm less than their $\Pi = 0$ counterparts. But oceanic heat uptake is greatest when the Polar Sea both warms and becomes diffusion-dominated ($\Pi = 1$ and $\gamma \rightarrow \infty$), as in cases 5 and 6. The stratification-dependent World Ocean diffusivity law (case 6) gave the largest oceanic heat penetration and the *smallest* global warming of any case studied -- 0.47 °C per century, the most consistent with the observed ~ 0.5 °C per century.

The apparently paradoxical result that $K = a/N$ produced both the greatest and smallest global warming is related to differences in the depth at which heat was injected to the World Ocean. When the high-latitude path to the abyss is suppressed (case 1), eddy diffusivity and heat penetration near the surface is reduced. When high-latitude heat transfer to the deep sea is most active (case 6), the temperature and density gradients $\partial T/\partial z$ and $\partial \rho/\partial z$, and the Brunt-Väisälä frequency N , become smaller at the base of the water column, where the diffusivity $K = a/N$ increases. Calculations show that temperature rises at the sea floor then propagate rapidly up the water column leading to even more vigorous heat uptake.

These calculations demonstrate the possibility of both less and more vigorous heat uptake by the oceans when the World Ocean vertical eddy diffusivity is stratification-dependent, depending on nature of the coupling with Polar Sea warming and mixing. A better fit to the ~ 0.5 °C per century trend of the instrumental temperature record than obtained by the IPCC is within the range of our model calculations. It remains to be seen whether the more vigorous oceanic heat uptake hypothesis investigated here is consistent with radiocarbon tracer constraints, and what the effect on global warming is of a $K = a/N$ diffusivity law in coupled atmosphere/ocean GCMs. We intend to explore these issues in forthcoming studies.

REFERENCES

- Boden, T.A., R.J. Sepanski and F.W. Stoss (eds.) (1991) *Trends '91: A Compendium of Data on Global Change*. ORNL/CDIAC-46, Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, Oak Ridge, Tennessee, pp. 516-517.
- Bretherton, F.P., K. Bryan and J.D. Woods (1990) Time-dependent greenhouse-gas-induced climate change. In J.T. Houghton et al. (eds.) *Climate Change: The IPCC Scientific Assessment*, Cambridge University Press, New York, pp. 173-193.

- Hoffert, M.I. and H.S. Kheshgi (1994) Internal wave-breaking turbulence in an upwelling-diffusion ocean: An analytic solution. Submitted to *Geophys. Res. Lett.*
- Hoffert, M.I., A.J. Callegari and C.-T. Hsieh (1980) The role of deep sea heat storage in the secular response to climatic forcing. *J. Geophys. Res.*, **85** (C11), 6667-6679.
- Kiehl, J.T., and B.P. Briegleb (1993) The relative roles of sulfate aerosols and greenhouse gases in climate forcing. *Science*, **260**, 311-314.
- Lashof, D.A. (1989) The dynamic greenhouse: feedback processes that may influence future concentrations of atmospheric trace gases and climatic change. *Climatic Change*, **14**, 213-242.
- Schneider, S. (1994) Detecting climatic change signals: Are there any fingerprints? *Science*, **263**, 341-347.
- Shine, K.P., R.G. Derwent, D.J. Wuebbles and J.-J. Morcrette (1990) Radiative forcing of climate. In J.T. Houghton et al. (eds.) *Climate Change: The IPCC Scientific Assessment*, Cambridge University Press, New York, pp. 39-68.

INTERNAL WAVE-BREAKING TURBULENCE IN AN UPWELLING-DIFFUSION OCEAN: AN ANALYTIC SOLUTION

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ABSTRACT

We obtain a coupled analytic solution for global mean potential temperature and density profiles for an upwelling-diffusion ocean model with the diffusivity law $\kappa = a N^{-1}$, where N is the Brunt-Väisälä frequency. This $\kappa(N)$ is appropriate to interior ocean turbulence produced by internal wave-breaking. A unique feature of our approach is the derivation of the coefficient $a = 6 \times 10^{-7} \text{ m}^2 \text{ s}^{-2}$ by matching the solution to observations of the global mean potential temperature profile -- as opposed to prior calibrations based on turbulence microstructure measurements. The model's vertical eddy scale ℓ increases with depth in range $6 < \ell < 190 \text{ cm}$ (well below the explicit resolution of current ocean circulation models); eddy diffusivity increases from $\sim 1,230 \text{ m}^2 \text{ y}^{-1}$ at the surface to $\sim 5,500 \text{ m}^2 \text{ y}^{-1}$ at 500 m to $\sim 39,200$ at 4000 m. The model's large deep diffusivity is consistent with ^{228}Ra -calibrated values measured near the sea floor, though surface values may be underestimates owing to our omission of turbulence from windstress and wintertime convection in the mixed layer. Mixed layer turbulence may explain the large surface diffusivities needed by prior models to match transient bomb-generated ^{14}C profiles. Ocean/climate and ocean/carbon cycle models should incorporate the combined effects of mixed layer and deep ocean turbulence on the diffusivity profile.

INTRODUCTION

One-dimensional ocean models in which the vertical potential temperature profile $T(z,t)$ is governed by an upwelling-diffusion (UD) conservation of energy equation,

$$\frac{\partial T}{\partial t} = w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left(\kappa \frac{\partial T}{\partial z} \right) \quad [1]$$

are often employed to estimate global warming and sea level rise from increased atmospheric greenhouse gas concentrations (Wigley and Raper, 1987, 1991; Bretherton et al., 1990, Schlesinger and Jiang, 1991). Here t is time, z ocean depth (positive downward), w upwelling velocity (positive upward) and $\kappa = \langle w'T' \rangle / (\partial T / \partial z)$ vertical (or diapycnal) eddy diffusivity.

The steady state solution to equation [1] for w and κ constant with depth is (Munk, 1966) $T(z) = T_p + (T_s - T_p) \exp(-z/\delta)$, where $\delta = \kappa/w$ is a thermocline depth scale, T_s the surface temperature and T_p a "polar sea" temperature attained asymptotically at infinite depth. Observed temperature profiles indicate an approximate depth scale of (Levitus, 1982) $\delta \sim 500$ m; For a typical upwelling velocity derived from ocean overturning rates (Gordon and Taylor, 1975), $w \sim 4$ m y^{-1} (1.27×10^{-7} m s^{-1}), the depth-independent eddy diffusivity that matches observed potential temperature profile is therefore (Hoffert et al., 1980) $\kappa = w\delta \sim 2000$ m² y^{-1} (6.3×10^{-5} m² s^{-1}).

UD models also describe world ocean concentration profiles of trace species for carbon cycle studies when appropriate internal source and sink terms are added to equation [1] -- for dissolved oxygen, nutrients, inorganic carbon, alkalinity, radiocarbon and tritium. In general, the vertical structure of steady tracer species (including prebomb radiocarbon) is reproduced reasonably well with κ and w calibrated on temperature profiles (Hoffert et al., 1981; Volk and Hoffert, 1985; Kheshgi et al., 1991). However, some recent studies suggest vertical variations of w and κ affect transient heat, radiocarbon and CO₂ uptake by the oceans (Lashof, 1989; Schlesinger and Jiang, 1990; Siegenthaler and Joos, 1992).

In this letter we derive an analytic steady solution to equation [1] coupled to an eddy diffusivity that varies inversely with the Brunt-Väisälä frequency, N . A numerical solution was found by Harvey and Schneider (1985) for a linear equation of state. An inverse dependence of mixing on density stratification is consistent with observations of intermittent gravity wave-breaking turbulence in oceanic interiors beneath the mixed layer and the scaling of the kinetic energy of internal gravity waves in the deep sea (Gargett, 1984, 1989; Monin and Ozmidov, 1985; Kraus, 1990). For simplicity,

and to clarify mechanisms, turbulence produced in the uppermost hundred meters of ocean generated by surface wind stirring and wintertime mixed layer overturning (Kraus, 1977) is and the effects of isopycnal mixing (Hoffert and Flannery, 1985) are omitted from our analysis. Another simplifying assumption is constancy of upwelling velocity with depth.

EDDY DIFFUSIVITY AND THE EQUATION OF STATE

Measurements of the turbulence dissipation rate in the oceans suggest an inverse relationship between the eddy diffusivity in oceanic interiors and $N \equiv [(g/\rho)\partial\rho/\partial z]^{1/2}$, where ρ is potential density and g is the gravitational acceleration (Gargett, 1984 -- also see later discussion). Any simultaneous solution to equation [1] for $T(z)$ with $\kappa \propto N^{-1}$ therefore requires an effective equation of state, $\rho = \rho(T)$, so that $\kappa = \kappa(T, \partial T/\partial z)$.

Standard equations of state relating seawater density to temperature, salinity and pressure are complex, nonlinear, and meant to apply to local water parcels (Bryan and Cox, 1972; Gill, 1982). We are interested in horizontally-average profiles for application to simple ocean/climate and carbon cycle models. Apart from complexity, standard seawater state equations do not recover world ocean density profiles when fed world ocean potential temperature and salinity profiles owing to nonlinear averaging effects. We are unaware of an appropriate equation of state in the published literature linking the world ocean $\rho(z)$ profile to $T(z)$ over the potential temperature range extending from the freezing point of sea ice to the warm surface ocean. ($-1.9^\circ\text{C} < T < 20^\circ\text{C}$). Linear density versus temperature equations of state sometimes used in simple ocean models are unacceptably inaccurate over this range.

Our approach is to parameterize a "world ocean" equation of state by curve-fitting the Levitus (1982) horizontally-averaged potential density versus potential temperature versus depth (FIG. 1):

$$\rho(T) \approx \rho_{fe} - \frac{b(T-T_f)^2}{2} \quad \text{for } T \geq T_f \quad [2]$$

Though equation [2] treats salinity effects only implicitly, FIG. 1 shows that it gives a close approximation to horizontally-averaged Levitus (ρ, T) pairs from the surface to an extrapolated freezing temperature of sea ice, $T_f = -1.88^\circ\text{C}$.

The corresponding coefficient of thermal expansion is a linear function of T : $\alpha \equiv - (1/\rho)(\partial\rho/\partial T) \approx b(T-T_f)$. The Brunt-Väisälä frequency is then $N = [-g\alpha\partial T/\partial z]^{1/2} \approx [-gb(T-T_f)\partial T/\partial z]^{1/2}$, and the eddy diffusivity,

$$\kappa = aN^{-1} = \frac{a}{[-gb(T-T_f)\partial T/\partial z]^{1/2}}, \quad [3]$$

where a is a constant to be determined.

COUPLED ANALYTIC SOLUTION

The first integral of equation [1] for $\partial T/\partial t = 0$ and constant w can be written

$$wT + \kappa \frac{dT}{dz} = wT^* \quad [4]$$

where T^* is a constant of integration. The steady state world ocean temperature profile $T(z)$ and eddy diffusivity distribution $\kappa(z)$ are defined by the simultaneous solution of [3] and [4] subject to boundary conditions at the surface and at the sea floor depth h :

$$@ z = 0: T = T_s \quad [5a]$$

$$@ z = h: T = T_b \quad [5b]$$

Surface and bottom boundary values from the Levitus (1982) data are given in TABLE 1.

Eliminating dT/dz between [3] and [4] gives

$$\kappa = \frac{a^2}{bgw(T - T_f)(T - T^*)}, \quad [6]$$

and substituting [6] back into [4] yields a first-order ODE:

$$\frac{dT}{dz} = -\frac{w(T - T^*)}{\kappa} = -\frac{bgw^2(T - T_f)(T - T^*)^2}{a^2} \quad [7]$$

To solve this, it is convenient to introduce the dimensionless potential temperature,

$$x = x(T) \equiv \frac{T - T_f}{T_s - T_f}, \quad [8]$$

so that $x_b = (T_b - T_f)/(T_s - T_f)$, $x^* = (T^* - T_f)/(T_s - T_f)$, $x_s = 1$ and $x_f = 0$.

A convenient alternative to direct solution of [7] for $T = T(z)$ is the equivalent inverse solution for $z = z(T) = z(x)$. Introducing the dimensionless temperature variable x and integrating [7] gives:

$$z(x, x^*) = d I(x, x^*), \quad [9]$$

where

$$d \equiv \frac{a^2}{bgw^2(T_s - T_f)^2} \quad [10]$$

is a thermocline depth scale and

$$I(x, x^*) \equiv \int_x^1 \frac{dt}{t(t - x^*)^2} = J(1, x^*) - J(x, x^*),$$

$$J(x, x^*) = \begin{cases} -\frac{1}{x^*(x - x^*)} - \frac{1}{x^{*2}} \ln\left(\frac{x - x^*}{x}\right) & \text{if } x^* \neq 0 \\ -\frac{1}{2x^2} & \text{if } x^* = 0 \end{cases} \quad [11]$$

We incorporate the surface temperature boundary condition of [5a] ($z(1) = 0$) into the definite integral $I(x, x^*)$, and satisfy the bottom boundary condition on temperature [5b] ($z(x_b) = h$) by computing the depth scale from

$$d = d(x^*) = h / I(x_b, x^*). \quad [12]$$

Each solution for $z(T)$ corresponds to a specific value of the parameter x^* and a specific value of $T^* = T^*(x^*)$, $d = d(x^*)$ and $a = a(x^*)$ (TABLE 1). We sought the value of a that best matches observed ocean profiles.

FIG. 2(a) and 2(b) show potential temperature and potential density profiles for three choices of the diffusivity constant a compared with Levitus (1982) world ocean data. All choices match the top and bottom boundary conditions with the largest diffusion constant producing the most uniform profile. The best fit at all depths is for $a = 6 \times 10^{-7} \text{ m}^2 \text{ s}^{-2}$ ($x^* = -0.085$), the value adopted henceforth. Both the potential temperature and potential density solutions fit the Levitus profiles quite well for this a -- a consequence of prior tuning of the state equation [3] (FIG. 1).

FIG. 2(c) shows the corresponding eddy diffusivity profiles for three values of a computed from $z = z(x, x^*)$ and (cf. equation [6])

$$\kappa(x, x^*) = \frac{wd(x^*)}{x(x - x^*)} \quad [13]$$

The $\kappa(z)$ profile for our temperature-calibrated calibrated value of a is shown as a the thick solid curve. Notice in FIG. 2(b) that the potential density profile becomes progressively more uniform with increasing depth. Accordingly, density stratification and $N = [(g/\rho)\partial\rho/\partial z]^{1/2}$ decrease with depth and $\kappa(z) = aN^{-1}$ increases to very large values at the sea floor.

COMPARISON OF RESULTS WITH HILDA MODEL

Our steady temperature-calibrated $\kappa = aN^{-1}$ is also compared in FIG. 2(c) with the constant $\kappa = 2000 \text{ m}^2 \text{ y}^{-1}$ of prior UD models and with the eddy diffusivity of the High-Latitude Exchange/Interior Diffusion-Advection (HILDA) model (Siegenthaler and Joos, 1992; Shaffer and Sarmiento, 1993) $\kappa = 465 + 7061 \exp(-(z-75 \text{ m})/253 \text{ m}) [\text{m}^2 \text{ y}^{-1}]$. While all simple ocean models are calibrated in some way on tracer data, model-to-model differences in diffusivity are large: The present κ profile (thick solid curve) increases dramatically with depth; from $\sim 1,230 \text{ m}^2 \text{ y}^{-1}$ at the surface to $\sim 5,500 \text{ m}^2 \text{ y}^{-1}$ at 500 m to $\sim 39,200$ at 4000 m. Our very large value near the sea floor is consistent with ^{228}Ra -calibrated diffusivities of $16,000 - 50,000 \text{ m}^2 \text{ y}^{-1}$ observed at 4000-5000 m depths (Broecker and Peng, 1982, Table 7-1), but almost two

orders of magnitude greater than HILDA's deep value. On the other hand, our near-surface diffusivity is some six times *smaller* than HILDA's.

This opposite depth dependence of $\kappa(z)$ requires an explanation. Firstly, The HILDA model differs from UD models (Hoffert et al., 1980, 1981) insofar as it allows for lateral transport with a polar sea and separate low-latitude and high-latitude surface ocean boxes. Besides upwelling (w) and diffusivity (κ), HILDA requires additional tunable coefficients for water exchange between its vertically-resolved world ocean reservoir and its polar sea and between its deep and surface polar boxes (u, q), so its transport coefficients are not directly comparable to the present model with only w and κ . For example, HILDA's constant upwelling is only $w = 0.7 \text{ m y}^{-1}$ as opposed to our $w = 4 \text{ m y}^{-1}$, although this may be partly augmented by lateral exchange with the polar sea.

A more fundamental problem that all simple ocean models face is simultaneously matching steady state and transient radiocarbon depth distributions. Bomb-generated ^{14}C seems to penetrate the oceans much faster than steady-state tracer calibrations imply: Even with four tunable coefficients, HILDA could not match both the top-to-bottom ^{14}C distribution of the prebomb steady state and the transient ^{14}C distribution of the upper oceans from atmospheric nuclear weapons testing without invoking a κ distribution > 15 times larger at the surface than at depth (Siegenthaler and Joos, 1992). The reconciliation of these results may lie in the combined effect of mixed layer turbulence in the upper hundred meters of the thermocline with the present interior turbulence model. We will not pursue this extension here. However, it is useful to examine the implications of the present world ocean temperature profile calibration for deep sea turbulence.

CONCLUDING REMARKS

The excellent agreement of the present UD model with world ocean temperature and density profile measurements supports a diffusivity law of the form $\kappa = aN^{-1}$ beneath the mixed layer. We conclude with some observations regarding the implications of our results for physically-based modeling of ocean mixing.

Our results are consistent with a model in which interior ocean turbulence is generated by internal gravity waves breaking intermittently into nonuniform patches of high shear (Gargett and Holloway, 1984). The mean fractional area occupied by turbulence at depth z is only $\sim 10^{-2}$ (Monin and

Ozmidov, 1985). Even so, internal wave instabilities appear to be the main source of vertical mixing in oceanic interiors; they produce turbulent kinetic energy per unit volume at the mean rate $\langle w'U'\partial U/\partial z \rangle$; a production very nearly balanced by the sum of the local buoyancy flux $\langle (g/\rho)w'\rho' \rangle \approx \kappa N^2$ and the turbulent energy dissipation rate ϵ (Garrett and Holloway, 1984);

$$\langle w'U'\partial U/\partial z \rangle \approx \langle (g/\rho)w'\rho' \rangle + \epsilon. \quad [14]$$

Here U is the horizontal velocity, angular brackets denote Reynolds averages, and primes are departures from the mean flow.

It is also often hypothesized that the deep oceans maintain a constant value of the flux Richardson number,

$$R_f \equiv \frac{\langle (g/\rho)w'\rho' \rangle}{\langle w'U'\partial U/\partial z \rangle} \approx \frac{\kappa N^2}{\langle w'U'\partial U/\partial z \rangle} \approx \text{constant}. \quad [15]$$

Substituting [14] into [15] gives

$$R_f \approx \frac{1}{1 + (\epsilon/\kappa N^2)}. \quad [16]$$

The flux Richardson number is therefore an (approximately) depth-independent constant if $(\epsilon/\kappa N^2)$ is independent of N . An independent finding is that, below the mixed layer, the energy per unit mass of the internal wave field is a universal -- to within a factor of two -- linear function of N throughout world ocean (Munk, 1981), $e/N = (e_0/N_0) \sim 0.6 \text{ m}^2 \text{ s}^{-1}$. We do not have comparably global samples of the dissipation rate. However, taking as typical of the world oceans' dissipation time scale for turbulent fluctuations a value measured in the main thermocline of the western Pacific (Moum and Osborn, 1986), $t_d = e/\epsilon \sim 2 \times 10^6 \text{ s}$, gives $\epsilon/N = (e_0/N_0) / t_d \sim 3 \times 10^{-7} \text{ m}^2 \text{ s}^{-2}$. Substituting this with $a = \kappa N \sim 6 \times 10^{-7} \text{ m}^2 \text{ s}^{-2}$ into [16] gives an effective flux Richardson number for the world ocean: $R_f = [1 + (e_0/N_0) / (a t_d)]^{-1} \sim 2/3$.

A characteristic eddy scale can also be written from dimensional considerations (Heisenberg, 1948),

$$\mathcal{L} \equiv \left(\frac{\kappa^3}{\epsilon} \right)^{1/4} = \left(\frac{R_f}{1 - R_f} \right)^{1/4} \frac{a^{1/2}}{N} = \left(\frac{R_f}{1 - R_f} \right)^{1/4} \frac{\kappa}{a^{1/2}} , \quad [17]$$

where last two expressions on the rhs of [17] are for the present gravity wave-breaking turbulence model. Inverse scaling of $\mathcal{L}$ with N beneath the mixed layer is intuitively reasonable as we expect density stratification to suppress the amplitude of vertical fluctuations. The eddy scale size by this model is proportional to κ , and increases with depth from $\mathcal{L} \sim 6$ cm at the surface to ~ 26 cm at 500 m to ~ 190 cm at 4000 m -- far too small for explicit resolution by current 3-D ocean general circulation models (OGCMs).

We conclude that stratification-dependent diffusivity models based on internal wave-breaking are potentially important components of ocean/climate and ocean/carbon cycle models, but that more work is needed to represent the combined effects of the surface mixed layer diffusivity with deeper layers of the thermocline.

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REFERENCES

- Bretherton, F.P., K. Bryan and J.D. Woods, Time-dependent greenhouse-gas-induced climate change, in *Climate Change: The IPCC Assessment*, edited by J.T. Houghton et al., pp. 173-193, Cambridge University Press, New York, 1990.
- Broecker, W.S., and T.-H. Peng, *Tracers in the Sea*, Eldigio Press, Palisades, New York, 1982.
- Bryan, K., and M.D. Cox, An approximate equation of state for numerical models of ocean circulation. *J. Phys. Oceanogr.*, **2**, 510-514, 1972.
- Gargett, A.E., Vertical eddy diffusivity in the ocean interior. *J. Mar. Res.*, **42**, 359-393, 1984.
- Gargett, A.E., Ocean turbulence. *Ann. Rev. Fluid Mech.*, **21**, 419-451, 1989.

- Gargett, A.E., and G. Holloway, Dissipation and diffusion by internal wave breaking. *J. Mar. Res.*, **42**, 15-27, 1984.
- Gill, A.E., *Atmosphere-Ocean Dynamics*, Academic Press, New York, pp. 599-603, 1982.
- Gordon, A.L., and H.W. Taylor, Heat and salt balance within the cold waters of the world oceans, in *Numerical Models of Ocean Circulation*, pp. 54-56, National Academy of Sciences, Washington, DC, 1975.
- Heisenberg, W., Zur statistischen theorie der turbulenz, *Z. Phys.*, **124**, 628-657, 1948.
- Hoffert, M.I., A.J. Callegari and C.-T. Hsieh, The role of deep sea storage in the secular response to climatic forcing. *J. Geophys. Res.*, **85**, 6667-6679, 1980.
- Hoffert, M.I., A.J. Callegari and C.-T. Hsieh, A box-diffusion carbon cycle model with upwelling, polar bottom water formation and a marine biosphere, in *Carbon Cycle Modeling, SCOPE 16*, edited by B. Bolin, pp. 287-305, John Wiley, New York, 1981.
- Hoffert, M.I., and B.P. Flannery, Model projections of the time-dependent response to increasing carbon dioxide. In *Projecting the Climatic Effects of Increasing Carbon Dioxide*, DOE/ER-0237, M.C. MacCracken and F.M. Luther (eds.), United States Department of Energy, Office of Energy Research, Washington, DC, pp. 149-190, 1985.
- Kheshgi, H., B.P. Flannery and M.I. Hoffert, Marine biota effects on the compositional structure of the world oceans. *J. Geophys. Res.*, **96**, 4957-4969, 1991.
- Kraus, E.B., editor, *Modelling and Prediction of the Upper Layers of the Ocean*, Pergamon Press, New York, 1977.
- Kraus, E.B., Diapycnal mixing, in *Climate-Ocean Interaction*, edited by M.E. Schlesinger, pp. 269-293, Kluwer, Dordrecht, The Netherlands, 1990.
- Lashof, D.A., The dynamic greenhouse: feedback processes that may influence future concentrations of atmospheric trace gases and climatic change. *Climatic Change*, **14**, 213-242, 1989.
- Levitus, S., *Climatological Atlas of the World Ocean*, NOAA Professional Paper 13, Geophysical Fluid dynamics Laboratory, Princeton, New Jersey, 1982.
- Monin, A.S., and R.V. Ozmidov, *Turbulence in the Ocean*, Reidel, Boston, 1985.

- Moum, J.N., and T.R. Osborn, Mixing in the main thermocline. *J. Phys. Oceanogr.*, **16**, 1250-1259, 1986.
- Munk, W.H. Abyssal recipes. *Deep Sea Res.*, **13**, 707-736, 1966.
- Munk, W.H., Internal waves and small-scale processes, in *Evolution of Physical Oceanography: Scientific Surveys in Honor of Henry Stommel*, edited by B.A. Warren and C. Wunsch, pp. 264-291, MIT Press, Cambridge, Mass., 1981.
- Schlesinger, M.E., and X. Jiang, Simple model representation of atmosphere-ocean GCMs and estimation of the time scale of CO₂-induced climate change. *J. Clim.*, **3**, 1297-1315, 1990.
- Schlesinger, M.E., and X. Jiang, Revised projection of global warming. *Nature*, **350**, 219-221, 1991.
- Shaffer, G., and J.L. Sarmiento, Biogeochemical cycling in the global ocean I: A new, analytical model with continuous vertical resolution and high latitude dynamics. *J. Geophys. Res.*, in press, 1993.
- Siegenthaler, U. and F. Joos, Use of a simple model for studying oceanic tracer distributions and the global carbon cycle. *Tellus*, **44B**, 186-207, 1992.
- Volk, T. and M.I. Hoffert, Ocean carbon pumps: Analysis of relative strengths and efficiencies in ocean-driven atmospheric CO₂ changes, in *The Carbon Cycle and Atmospheric CO₂: Natural Variations Archean to Present*, *Geophys. Monogr. Ser.* 32, edited by E.T. Sundquist and W.S. Broecker, pp. 91-110, AGU, Washington, DC, 1985.
- Wigley, T.M.L., and S.C.B. Raper, Thermal expansion of sea water associated with global warming. *Nature*, **330**, 127-131, 1987.
- Wigley, T.M.L., and S.C.B. Raper, Detection of enhanced greenhouse effect on climate, in *Climate Change: Science, Impacts and Policy*, Cambridge University Press, pp. 231-242, edited by J. Jäger and H.L. Ferguson, New York, 1991.

TABLE 1. Parameter values

a = diffusivity coefficient (see below)
 d = depth scale (see below)
 g = gravitational acceleration = 9.81 m s^{-2}
 h = oceanic depth = 4000 m
 w = upwelling velocity = $1.27 \times 10^{-7} \text{ m s}^{-1}$
 b = constant in equation of state = $1.657 \times 10^{-5} \text{ }^{\circ}\text{C}^{-2}$
 ρ_f = freezing point density = $1027.92 \text{ kg m}^{-3}$
 T_f = freezing point temperature = $-1.88 \text{ }^{\circ}\text{C}$
 $T_s = T(0)$ = surface potential temperature = $18.3 \text{ }^{\circ}\text{C}$
 $T_b = T(h)$ = bottom potential temperature = $1.08 \text{ }^{\circ}\text{C}$
 $T^* = T(\infty)$ = asymptotic deep temperature (see below)

$a \text{ [} 10^{-7} \text{ m}^2 \text{ s}^{-2}\text{]}$	$d \text{ [m]}$	$T^*[\text{ }^{\circ}\text{C}]$	x^*	comment
3	83	- 0.6	0.0634	
4.33	176	- 1.88	0.0	$T^* = T_f$
6	334	- 3.6	- 0.0852	"best fit"
12	1350	-10.8	- 0.442

FIG. 1. Equation of state based on empirical curve fit to Levitus (1982) world ocean data.

FIG. 2. Vertical ocean profiles of (a) potential temperature, (b) potential density and (c) eddy diffusivity. Analytic solutions for the present model matched to surface and 4 km depth potential temperature boundary conditions are shown for three values of the constant: $a = \kappa N$. The constant eddy diffusivity of Hoffer et al. (1980), the radiocarbon-calibrated HILDA diffusivity distribution of Siegenthaler and Joos (1992) and the SF₆ tracer measurements of dipycnal diffusivity 310 m deep at a station 1200 km west of the Canary Islands by Ledwell et al. (1993) are also shown in (c).

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