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Physics of Vertically Integrated Waveguide Photodetectors and Amplifiers

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FINAL REPORT FOR SUBCONTRACT NO. B239593

**Physics of Vertically Integrated Waveguide
Photodetectors and Amplifiers**

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SUMMARY

This report describes the efforts supported by LLNL under the Subcontract No. B239593 at the University of Arizona during the Fiscal Year 1992.

A solid physical foundation has been developed for understanding the operation of vertically integrated photodetectors and amplifiers. This has been achieved through a combination of numerical simulation and development of simple coupled-mode theories.

Coupled-mode theory has been used to elucidate the physics underlying the operation of vertically integrated photodetectors. In particular, the relation between the spatial transients observed in experiments and numerical simulations, and the non-power orthogonality of the underlying modes has been clarified. The coupled-mode theory has been extended to the case of coupled waveguide-amplifiers.

Submitted to Dr. Raymond J. Hawkins
Lawrence Livermore National Laboratory
Computational Electronics and Electromagnetics Thrust Area

FINAL REPORT

The continued success of guided-wave optoelectronic circuits relies on the development of efficient photodetectors and coupled waveguide-amplifiers. Both of these integrated optical elements are multi-layer optical waveguiding structures, the photodetector containing an absorbing layer and the amplifier a gain layer. As such their analysis is complicated by the fact that the underlying wave equations for these systems are non-Hermitian. This means that the transverse modes of these systems are not power-orthogonal in the usual sense, and that our intuitive notions of waveguide propagation based on the Hermitian wave equation for lossless waveguides do not apply. It is therefore important for the future development of integrated optics that this gap in our knowledge is filled.

This document describes our efforts in this direction. In particular, here we describe our efforts to develop an understanding of the physics underlying the operation of vertically integrated waveguide/photodetectors. The approach used has been based on the development of a simple coupled-mode theory which highlights the underlying physics. The results of these studies have been submitted for publication, and presented at a technical conference, to excellent reception. The coupled-mode theory has also been extended to the case of coupled waveguide-amplifiers. Here the main interest is the fundamental noise properties of the system. This work is not quite at the publication stage, but our research notes in this area are appended to this report for completeness.

We have developed a combined numerical-analytical approach to propagation in integrated photodetectors and coupled waveguide-amplifiers. The numerical approach involves solving the paraxial wave equation for propagation in the system. A number of different numerical schemes were tested, and the finite-difference beam-propagation-method (FDBPM) was finally selected. The FDBPM was found to run about 10-20 times faster than the corresponding split-step BPM algorithm. The developed numerical simulation code models propagation in a general five-layer waveguide structure, three of the layers comprising the input waveguide. The input to the structure is the TE_0 mode of the input waveguide. The other two layers allow for the addition of an absorber, as in the photodetector, or an amplifying waveguide as in the coupled waveguide amplifier. The numerical code was tested extensively against known solutions for single waveguide propagation, and directional couplers. A listing of this numerical code, named `vim_fdm.f`, is appended to this final report.

The numerical package `vim_fdm.f` was used to perform extensive simulations of both integrated photodetectors and amplifiers. The main aim of this research, however, was to develop simple physical models of propagation in these structures. Since the key issue of this initial phase of the work was the integrated photodetectors, we now turn to the coupled-mode theory for this case.

Conventional photodetector designs employ an absorber grown adjacent to the waveguide core, and evanescent field coupling leads to light absorption. This strategy leads, however, to small detector absorption coefficients and correspondingly long detector lengths for high quantum efficiency. In response to the need for shorter detector lengths, Deri and Wada proposed the Vertically integrated Impedance-Matched waveguide/photodetector, or VIM diode, in which a thin matching-layer is inserted between the waveguide core and the absorber. The reduction of the diode length due to the addition of the matching-layer can be viewed as a two-step process: The field coupled into the waveguide is transferred to the matching-layer in the double-moded waveguide/ matching-layer structure prior to the absorber region. By judicious choice of the extended-matching-layer length M the optical power then can be concentrated very close to the absorber thereby greatly increasing the evanescent coupling to the absorber and reducing the device length.

The coupled-mode theory for the VIM diode is built upon the premise that only two dominant supermodes are excited, a fact that is supported by numerical simulations. Prior to the absorber the waveguide-matching layer system is assumed to support two supermodes described by the amplitudes b_1 and b_2 . Assuming that the combinations $b_{\pm} = (b_1 \pm b_2)/2$ correspond to fields concentrated in the waveguide (+) and the matching-layer (-), then for a matching-layer of length M the field at the input to the absorber is

$$b_+(M) = \cos(\pi M/2L_c) , \quad b_-(M) = i \sin(\pi M/2L_c) , \quad (1)$$

where L_c is the coupling length. In the absorbing region the coupled-mode equations are

$$i \frac{d}{dz} \vec{b}(z) = \begin{bmatrix} 0 & -\kappa \\ -\kappa & -i\alpha/2 \end{bmatrix} \vec{b}(z) = H \vec{b}(z) , \quad (2)$$

where $\vec{b} = \text{col}(b_+, b_-)$, α is the absorption coefficient of the matching-layer due to the absorber, and the 2×2 matrix H is non-Hermitian. Here we have assumed that the field suffers no absorption when it is concentrated in the waveguide, thus only a_- is subject to absorption. The CMT equations are to be solved subject to the initial conditions (1).

We have compared the predictions of the CMT (2) against the full numerical simulations, and obtained excellent agreement (the comparison is discussed in detail in the appended paper 1). In particular, the coupled-mode theory reproduced the spatial transients seen in the absorption curves obtained from the numerical simulations.

We now turn to the interpretation of the spatial transients in the framework of the CMT. In deriving the coupled-mode Eqs. (2) for the absorber region, the transverse modes of the unperturbed waveguide were used. These field profiles are power-orthogonal in the usual

$dxw_{\ell}^*(x)w_{\ell'}(x) = \delta_{\ell,\ell'}$ sense. However, the non-Hermitian character of the problem due to the absorber is still present in the coupled-mode Eqs. (2). Setting $\bar{b}(z) = \exp(\gamma z)\bar{a}$, $\bar{a} = \text{col}(a_1, a_2)$, we obtain the eigenvalues of H as

$$\gamma_{\pm} = -\frac{\alpha}{4} \pm i\sqrt{\kappa^2 - (\alpha/4)^2} , \quad (3)$$

The corresponding eigenvectors are $\bar{a}_{\pm}$, and are biorthogonal in the sense

$$\begin{aligned} \bar{a}_{+}^T \bar{a}_{+} &= \bar{a}_{-}^T \bar{a}_{-} = 1 \\ \bar{a}_{+}^T \bar{a}_{-} &= 0 , \end{aligned} \quad (4)$$

where T means transpose. Within this CMT framework there are two dominant transverse eigenmodes in the absorber region which are approximated by linear combinations of the sum and difference fields

$$U_{\pm}(x) = a_{1,\pm}w_1(x) + a_{2,\pm}w_2(x) . \quad (5)$$

From Eq. (5) we see that both transverse eigenmodes have the same absorption coefficient $\alpha_{\pm} = 2\text{Im}(\gamma_{\pm}) = \alpha/2$.

An arbitrary solution of Eq. (2) can always be expressed in terms of the eigenvectors of H

$$\bar{b}(M+L) = c_{+}e^{\gamma_{+}L}\bar{a}_{+} + c_{-}e^{\gamma_{-}L}\bar{a}_{-} , \quad (6)$$

where $c_{\pm} = \bar{a}_{\pm}^T \bar{b}(M)$. Since the absorption coefficients of the two eigenmodes are equal, $\alpha_{+} = \alpha_{-} = \alpha$, the solution (2) is generally a sum of two exponentially decaying components corresponding to the two biorthogonal eigenmodes, whose phase-difference $\Delta\phi(L) = 2\sqrt{\kappa^2 - (\alpha/4)^2}L$ varies along the absorber. The interference between these two biorthogonal modes is responsible for the observed spatial transients.

To summarize, we have developed a combined numerical-analytic approach to propagation in multi-layer waveguide structures with loss or gain. This first years work has concentrated primarily on the integrated photodetectors, the VIM diode, but work has also initiated on the coupled waveguide amplifier. Highlights of this years work include

1. Developed a general propagation program for multi-layer waveguides including loss or gain. This code performs significantly better than the methods previously used for this problem.

2. Developed a coupled-mode theory for VIM diodes which compares well with the numerical simulations. In particular, the coupled-mode theory correctly models the spatial absorption transients seen in numerical simulations and experiments.
3. The physics of the operation of VIM diodes has been elucidated on the basis of the coupled-mode theory. In particular, we have shown that the spatial transients are a consequence of the non-power orthogonality of the underlying modes.
4. We have also developed the connection between the coupled-mode theory and the supermodes of the multi-layer structure. The parameters for the coupled-mode theory can now be extracted from the supermode properties for an optimized structure.
5. The coupled-mode theory has been extended to the case of a coupled waveguide-amplifier.

These results and all of the associated technical details are contained in the attached papers. Also included are the research notes on the coupled waveguide-amplifier, and the listing of the numerical simulation code, `vim_fdm.f`.

FUTURE PLANS

In the next phase of this work we intend to continue development of the coupled-mode theory for the waveguide-amplifier system, and to couple this more strongly with numerical simulations. The coupled waveguide-amplifier is a key element in planar opto-electronic circuits, and Dr. R. J. Deri at LLNL is currently developing experiments in this area. In particular, our second year plans include

1. Develop the coupled-mode theory more fully in conjunction with the numerical simulations. In this way a reliable coupled-mode theory shall evolve.
2. Use the coupled-mode theory to elucidate the physics underlying propagation in the coupled waveguide-amplifier. In particular, investigate those features which arise from the non-power orthogonality of the modes.
3. Develop a reliable noise theory for the coupled waveguide-amplifier based on the coupled-mode theory, and compare with the existing numerical capability to calculate the amplified spontaneous emission.
4. Extend the numerical simulations and coupled-mode theory to incorporate realistic models of the gain medium.

These developments shall greatly enhance our basic understanding of coupled waveguide-amplifiers, and more generally, in multilayer waveguide systems with gain. It is anticipated that this work will continue to attract the attention of the integrated optics community, and to be of interest to LLNL.

PUBLICATIONS AND PRESENTATIONS

1. E. M. Wright, R. J. Hawkins, and R. J. Deri, "Coupled-mode theory of vertically integrated impedance-matched waveguide/photodetectors," submitted to IEEE J. Quant. Electron.
2. E. M. Wright, R. J. Hawkins, and R. J. Deri, "Coupled-mode theory of vertically integrated impedance-matched waveguide/photodetectors," Proceedings of the Integrated Photonics Research (IPR) Meeting, Palm Springs, CA, March 1993.

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RESEARCH NOTES ON COUPLED WAVEGUIDE-AMPLIFIER

THEORY OF WAVEGUIDE-AMPLIFIER COUPLING

The model consists of two *index-guided waveguides* which are identical in the absence of current pumping. The waveguides are coupled by their evanescent fields, giving directional coupler action. Only one waveguide is pumped, and therefore exhibits gain. The input is injected into the passive waveguide and we are interested primarily in the amplified signal exiting that waveguide.

1. Coupled-mode theory

We designate the field amplitude in the passive or input waveguide as "a" and that in the amplifier as "b". The coupled-mode equations are then

$$\frac{da}{dz} = i\kappa b ; \quad \frac{db}{dz} = i\kappa a + \frac{g}{2}(1-iR)b , \quad (1)$$

where g is the modal gain of the amplifier, and R is the linewidth enhancement factor. The coupling coefficient κ is related to the half-beat length by

$$\kappa = \pi / 2L_c . \quad (2)$$

The solution of this system of equations is

$$\bar{u}(z) = e^{gz(1-iR)/4} \left[I \cos(\Lambda(z)) + \frac{\sin(\Lambda(z))}{\Lambda(z)} \begin{pmatrix} -gz(1-iR)/4 & i\kappa z \\ i\kappa z & gz(1-iR)z/4 \end{pmatrix} \right] \bar{u}(0) , \quad (3)$$

where $\bar{u}(z) = \text{col}(a(z), b(z))$, I is the unit 2×2 matrix, and

$$\Lambda(z) = \sqrt{(\kappa z)^2 - (gz(1-iR)/4)^2} . \quad (4)$$

The initial condition is $\bar{u}(0) = \text{col}(\sqrt{P_{in}}, 0)$, where P_{in} is the input power. From this we can calculate the amplified fraction through the passive (input) waveguide

$$A.F.(z) = \frac{|a(z)|^2}{|a(z)|^2 + |b(z)|^2} , \quad (5)$$

and the gain $\mathcal{G}$ in dB

$$\mathcal{G}(z) = 10 \log_{10} [|a(z)|^2 / P_{in}] . \quad (6)$$

2. Noise properties

There is noise in the system due to the presence of amplification. Including the noise the coupled mode equations become

$$\frac{da}{dz} = i\kappa b ; \quad \frac{db}{dz} = i\kappa a + \frac{g}{2}(1-iR)b + \Gamma(z) , \quad (7)$$

where the noise source obeys

$$\langle \Gamma(z) \rangle = 0 , \quad \langle \Gamma(z) \Gamma^*(z') \rangle = g P_N \delta(z-z') , \quad (8)$$

P_N being the effective input noise power to waveguide b, and $\langle \dots \rangle$ denoting a bath average. This system of equations can be written in the form

$$\frac{d\bar{u}}{dz} = M\bar{u} + \bar{\Gamma}(z) , \quad (9)$$

where $\bar{u}(z) = \text{col}(a(z), b(z))$, $\bar{\Gamma}(z) = \text{col}(0, \Gamma(z))$, and

$$M = \begin{bmatrix} 0 & i\kappa \\ i\kappa & g(1-iR)/2 \end{bmatrix} . \quad (10)$$

The formal solution of this system of equations is given by

$$\bar{u}(z) = G(z)\bar{u}(0) + \int_0^z dz' G(z-z') \bar{\Gamma}(z') , \quad (11)$$

where $G(z-z')$ is the Green's matrix defined by

$$G(z-z') = \exp[M(z-z')] \equiv \begin{bmatrix} G_{aa}(z-z') & G_{ab}(z-z') \\ G_{ba}(z-z') & G_{bb}(z-z') \end{bmatrix} . \quad (12)$$

Note that since M in Eq. (10) is symmetric, so is G , $G_{ba} = G_{ab}$. The explicit form of G is

$$G(z) = e^{gz(1-iR)/4} \left[\text{Icos}(\Lambda(z)) + \frac{\sin(\Lambda(z))}{\Lambda(z)} \begin{bmatrix} -gz(1-iR)/4 & i\kappa z \\ i\kappa z & gz(1-iR)z/4 \end{bmatrix} \right] . \quad (13)$$

The first term in Eq. (11) corresponds to the deterministic evolution of an injected signal. This was dealt with in Sec. 1. In the present notation the incident power evolves in the two waveguides as

$$P_a(z) = |G_{aa}(z)|^2 P_{in} , \quad (14)$$

and

$$P_b(z) = |G_{ba}(z)|^2 P_{in} = |G_{ab}(z)|^2 P_{in} . \quad (15)$$

The noise is given by the second term. We then obtain for the noise

$$a(z) = \int_0^z dz' G_{ab}(z-z') \Gamma(z') , \quad (16)$$

and

$$b(z) = \int_0^z dz' G_{bb}(z-z') \Gamma(z') . \quad (17)$$

Using Eq. (8) we then obtain

$$P_{N,a}(z) = \langle |a^2(z)| \rangle = P_N \int_0^z dz' g |G_{ab}(z-z')|^2 , \quad (18)$$

and

$$P_{N,b}(z) = \langle |b^2(z)| \rangle = P_N \int_0^z dz' g |G_{bb}(z-z')|^2 . \quad (19)$$

Using Eqs. (12) it is straightforward to show that

$$\frac{\partial}{\partial z'} (|G_{aa}(z-z')|^2 + |G_{ab}(z-z')|^2) = -g |G_{ab}(z-z')|^2 , \quad (20)$$

and

$$\frac{\partial}{\partial z'} (|G_{bb}(z-z')|^2 + |G_{ab}(z-z')|^2) = -g |G_{bb}(z-z')|^2 . \quad (21)$$

Substituting Eqs. (20) and (21) into (18) and (19) yields

$$P_{N,a}(z) = P_N [|G_{aa}(z)|^2 + |G_{ab}(z)|^2 - 1] , \quad (22)$$

and

$$P_{N,b}(z) = P_N [|G_{bb}(z)|^2 + |G_{ab}(z)|^2 - 1] , \quad (23)$$

where we have used $G_{aa}(0) = G_{bb}(0) = 1$, and $G_{ab}(0) = 0$.

From these results we can calculate the cross-talk which is the power in the undesired waveguide (b) divided by that in the desired waveguide (a),

$$X.T.(dB) = 10 \log_{10}(P_b(z)/P_a(z)) = 10 \log_{10}(|G_{ba}(z)|^2 / |G_{aa}(z)|^2) , \quad (24)$$

and also the signal-to-noise in the amplifier

$$S.N.(z) = \frac{P_a(z)}{P_{N,a}(z)} = \eta \frac{|G_{aa}(z)|^2}{|G_{aa}(z)|^2 + |G_{ab}(z)|^2 - 1} , \quad (25)$$

where $\eta = P_{in}/P_N$ is the signal-to-noise ratio expected for an isolated index-guided amplifier.

3. Numerical Results

Figures 1-4 show numerical results for $\kappa L = 10$, $R = 3$, and $gL = 20\text{dB}$. Figure 1 shows the amplified fraction, which approaches unity for $\kappa z \approx 3, 6, 9$. As shown in Fig. 2, at these distances the cross-talk also becomes small, with an isolation of -10dB at $\kappa z = 9$. Furthermore, the signal to noise goes through local maxima at these distances as shown in Fig. 3. Note that $SN/\eta \approx 1$ for $\kappa z = 9$, so that the signal to noise performance is equivalent to that for an isolated amplifier. However, even for $\kappa z = 9$ the gain of the output signal is only $\mathcal{G} \approx 9\text{dB}$, roughly half that for an isolated amplifier. This is the penalty one must pay for amplifying a signal in this way: To obtain efficient waveguide-amplifier coupling the net gain of the signal cannot exceed half that of the isolated amplifier. Thus, to obtain 10dB gain the amplifier must supply at least 20dB of gain. Such high gains are dangerously close to those for which amplified spontaneous emission can saturate the gain.

Fig. 1

$$\chi L = 9, R = 3, gL = 60 \text{ dB}$$

Fig. 2

$$K_L = 9, R = 3, g_L = 60 \text{ dB}$$

Fig. 3
 $g_L = 9, R = 3, g_L = 60\text{dB}$

Fig. 4

$$\chi_L = 9, R = 3, g_L = 60 \text{ dB}$$

LISTING OF NUMERICAL SIMULATION CODE vim_fdm.f

1.6 0.4 0.0 1.3
-14.0 5.0 1024 26.0 500 -250
0
1
0.0
8.0
17.0
27.0
ns ng nimp nar nai neff
t timp d lambda (all in microns)
xwmnin xwmax nx L nz ncycle (xmax, M, and L in microns)
par0hell
nMpts < 100
M(1)
M(2)

```

C     *** PROGRAM VIM_FDM.F ***
C
C     This code obtains the response curves for VIM diodes
C     as a function of the matching layer length M
C     using the Finite-Difference-Method
C
C     implicit double precision (a-h,o-z)
C     complex*16 ai,e(2048),e0(2048),bm(2048),dM(2048),r(2048)
C     complex*16 a(2048),c(2048),bA(2048),dA(2048),bact(2048)
C     complex*16 aval,avac,dzeff,dzefc,nsq,olap
C     double precision xar(2048),Mval(100)
C     double precision M,L,k0,lambda,kappa,Intabs
C     double precision ns,ng,nimp,nar,nai,neff
C     double precision kcrit
C     ai = dcplx(0.0d0,1.0d0)
C
C     *** Input data ***
C
C     open(14,file='vim.dat')
C     read(14,*)ns,ng,nimp,nar,nai,neff
C     read(14,*)t,timp,d,lambda
C     read(14,*)xwmin,xwmax,nx,L,nz,ncycle
C     read(14,*)par0hell
C
C     *** par0hell = -1, paraxial,spectral filtering ***
C     *** par0hell = 0, paraxial approximation ***
C     *** par0hell = 1, (1,1) Pade approximation ***
C
C     read(14,*)nMpts
C     do 999 i = 1,nMpts
C     read(14,*)Mval(i)
999  continue
C     close(14)
C
C     *** Open output files ***
C
C     ncyc = abs(ncycle)
C     open(15,file='profiles_labs.dat')
C     open(16,file='profiles_field.dat')
C     open(17,file='profiles_proj.dat')
C     open(18,file='profiles_abs.dat')
C
C     *** Convert to dimensionless units ***
C
C     pi      = 4.0d0*datan(1.0d0)
C     k0      = 2.0d0*pi/lambda
C     xmax    = xwmax - xwmin
C     xwmin   = k0*xwmin
C     xwmax   = k0*xwmax
C     xmax    = k0*xmax
C     L       = k0*L
C     t       = k0*t
C     timp    = k0*timp
C     d       = k0*d
C     dx      = xmax/float(nx)
C     kcrit   = pi/dx
C     eps     = neff*abs(par0hell)/kcrit/kcrit
C
C     *** Calculate for each M ***
C
C     do 99 Mcount = 1,nMpts
C     M = Mval(Mcount)
C     M = M*k0
C     zmax = M + L
C     dz = zmax/float(nz)

```



```

      z = 0.0d0
C
C
C
      *** Setup calculation parameters ***

      dzeff = dz + ai/2.0d0/neff
      dzeffc = dz - ai/2.0d0/neff
      aval = dzeff/2.0d0/dx/dx
      avac = dzeffc/2.0d0/dx/dx
      if(par0hell1.le.0)then
      dzeff = dz
      dzeffc = dz
      aval = (dz/2.0d0/dx/dx)*cmplx(1.0d0,-eps)
      avac = (dz/2.0d0/dx/dx)*cmplx(1.0d0,-eps)
      endif
C
C
C
      *** Setup initial field ***

      anorm = 0.0d0
      gammas = dsqrt(neff*neff - ns*ns)
      gammac = dsqrt(neff*neff - 1.0d0)
      kappa = dsqrt(ng*ng - neff*neff)
      fac = gammac/kappa
      pre = dcos(kappa*t) + fac*dsin(kappa*t)
C
C
C
      *** Field calculation and waveguide profile ***

      do 1 i = 1,nx
      x = float(i - 1)*dx + xwmin
      xar(i) = lambda*x/2.0d0/pi
      if(x.lt.-t)e(i) = pre*dexp(gammas*(x+t))
      if(x.lt.0.0d0.and.x.gt.-t)e(i) = dcos(kappa*x) - fac*dsin(kappa*x)
      if(x.gt.0.0d0)e(i) = dexp(-gammac*x)
      e0(i) = e(i)
      anorm = anorm + abs(e(i))**2
      a(i) = aval
      c(i) = aval
      bM(i) = 2.0d0*ai*neff - 2.0d0*aval - dzeff*neff*neff/2.0
      dM(i) = 2.0d0*ai*neff + 2.0d0*avac + dzeffc*neff*neff/2.0
      bA(i) = bM(i)
      dA(i) = dM(i)
C
C
C
      *** Waveguide profile in matching layer ***

      if(x.lt.-t)nsq = ns*ns
      if(x.lt.0.0d0.and.x.gt.-t)nsq = ng*ng
      if(x.gt.0.0d0.and.x.lt.timp)nsq = nimp*nimp
      if(x.gt.timp)nsq = 1.0d0
      bM(i) = bM(i) + nsq*dzeff/2.0d0
      dM(i) = dM(i) - nsq*dzeffc/2.0d0
C
C
C
      *** Waveguide profile in absorber ***

      if(x.lt.-t)nsq = ns*ns
      if(x.lt.0.0d0.and.x.gt.-t)nsq = ng*ng
      if(x.gt.0.0d0.and.x.lt.timp)nsq = nimp*nimp
      if(x.gt.timp.and.x.lt.timp+d)nsq = (nar+ai*nai)**2
      if(x.gt.timp+d)nsq = 1.0d0
      bA(i) = bA(i) + nsq*dzeff/2.0d0
      dA(i) = dA(i) - nsq*dzeffc/2.0d0
1
      continue
      anorm = anorm*dx
      Pold = 1.0d0
C
C
C
      *** Start calculation loop for each M ***

      do 3 i = 1,nz

```

```

c
c The numerical scheme works by solving the tridiagonal
c system
c
c  $a(i)*e(i+1) + b(i)*e(i) + c(i)*e(i-1) = r(i)$ 
c
c where e(i), i = 1,nx, are the field values to be
c calculated and r(i) is a source term obtained from
c the field values at the previous step. The method is
c based on the Cranck-Nicholson Scheme.
c

    if(z.lt.M)then
    r(1) = dM(1)*e(1) - avac*e(2)
    r(nx) = dM(nx)*e(nx) - avac*e(nx-1)
    bact(1) = bM(1)
    bact(nx) = bM(nx)
    do 50 it = 2,nx-1
    r(it) = -avac*e(it-1) + dM(it)*e(it) - avac*e(it+1)
    bact(it) = bM(it)
50  continue
    endif
    if(z.ge.M)then
    r(1) = dA(1)*e(1) - avac*e(2)
    r(nx) = dA(nx)*e(nx) - avac*e(nx-1)
    bact(1) = bA(1)
    bact(nx) = bA(nx)
    do 51 it = 2,nx-1
    r(it) = -avac*e(it-1) + dA(it)*e(it) - avac*e(it+1)
    bact(it) = bA(it)
51  continue
    endif

c
c *** Solve tridiagonal system ***
c
    call tridag(a,bact,c,r,e,nx)
    z = z + dz
    if(mod(i,ncyc).eq.0.or.i.eq.1)then

c
c *** Calculate Output ***
c

    sum = 0.0d0
    olap = 0.0d0
    do 4 ii = 1,nx
    ans = abs(e(ii))
    sum = sum + ans**2
    olap = olap + e0(ii)*e(ii)*dx
    if(ncycle.lt.0.and.nMpts.eq.1)
& write(16,*)i,sngl(xar(ii)),sngl(ans)
4  continue
    Pnow = sum*dx/anorm
    Intabs = 1.0d0 - Pnow
    Proj = abs(olap)**2/anorm/anorm
    absorp = k0*log(Pold/Pnow)/dz/float(ncycle)
    Pold = Pnow
    zans = lambda*(z-M)/2.0d0/pi
    zer0 = 0.0d0
    pdum = 1.0d0
    if(zans.le.0.0d0)write(15,*)Mcount,sngl(zer0),sngl(zer0)
    if(zans.gt.0.0d0)write(15,*)Mcount,sngl(zans),sngl(Intabs)
    if(zans.le.0.0d0)write(17,*)Mcount,sngl(zer0),sngl(pdum)
    if(zans.gt.0.0d0)write(17,*)Mcount,sngl(zans),sngl(Proj)
    if(zans.le.0.0d0)write(18,*)Mcount,sngl(zer0),sngl(zer0)
    if(zans.gt.0.0d0)write(18,*)Mcount,sngl(zans),sngl(absorp)
    endif
    continue
3

```

```

      continue
C
C      *** Close output files ***
C
do 100 i = 15,18
100  close(i)
      stop
      end
C
C
C
      subroutine tridag(a,b,c,r,u,n)
      implicit double precision (a-h,o-z)
      complex*16 gam(10000),a(*),b(*),c(*),r(*),u(*)
      complex*16 bet
      bet = b(1)
      u(1) = r(1)/bet
      do 11 j = 2,n
      gam(j) = c(j-1)/bet
      bet = b(j) - a(j)*gam(j)
      u(j) = (r(j) - a(j)*u(j-1))/bet
11      continue
      do 12 j = n-1,1,-1
      u(j) = u(j) - gam(j+1)*u(j+1)
12      continue
      return
      end

```

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END