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Analytical Calculations of Multiple Scattering for High Energy Photons and Neutrons

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Analytical Calculations of Multiple Scattering for High Energy Photons and Neutrons

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ABSTRACT

Radiography of large dense objects often require the use of highly penetrating radiation. For example, a couple of centimeters of steel attenuates 50 keV x-rays by a factor of approximately 10^{-14} whereas this same amount of steel would attenuate a 500 keV photon beam by only a factor of about 0.25. However, this increase in penetrating power comes with a price. In the case of x-radiation there are two bills to pay: (1) For projection radiography, this increase in penetration directly causes a corresponding decrease in resolution. (2) This increase in penetration occurs in a region where the interaction of radiation and matter is changing from absorption to scattering. In the above example the fraction of scattering goes from about 0.1 at 50 keV to over 0.99 at 500 keV. These scattered photons can significantly degrade contrast. In order to overcome some of these difficulties, radiography using scattered photons has been studied by myself and numerous other authors. In all the above cases, calculation of the intensity of scattered radiation is of primary importance. In cases where scattering is probable, multiple scattering can also be probable. Calculations of multiple scattering are generally very difficult and usually require the use of extremely sophisticated Monte Carlo simulations. It is not unusual for these calculations to require several hours of CPU time on some of the worlds largest and fastest supercomputers. In this paper I will present an alternative approach. I will present an analytical solution to the equations of double scattering, and show how this solution can be extended to the case of higher order scattering. Finally, I will give numerical examples of these solutions and compare them to solutions obtained by Monte Carlo simulations.

2. INTRODUCTION

In this paper I wish to discuss a method of calculating the spectrum of multiply scattered gamma rays with particular emphasis on applications involving radiography or beam transport. Such applications typically involve the prediction and minimization of unwanted backgrounds produced by collimators, beam dumps, radiation shielding, etc. These calculations have typically been handled by running extremely sophisticated and computationally intensive Monte Carlo simulations^{1,2}. These codes explicitly follow the progress of a photon as it wanders through a the mass distributions specified by the particular problem. The calculation may be quite detailed involving not only Compton scattering, but also photo-ionization, fluorescence, coherent scattering, nuclear excitation, and pair production. The general problem, of course, must include all these effects since their relative importance changes strongly with energy. Although in principle, these simulations give the exact answer, in reality they possess many deficiencies.

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As might be expected, calculations such as these usually require a large platform and a considerable amount of cpu time. The sample problems I will present below required more than 1.5 hours of cpu time on the LLNL Cray YMP supercomputer and still only gave answers accurate to about 15 percent with very coarse energy resolution. Often the answer one receives gives very little insight into which physical processes are important. In order to examine the physics, what is necessary is to follow the trajectory of a typical photon in detail. The problem is that in a typical multiple scattering spectrum, hundreds of thousands of photons contribute to the spectrum. For example, for a spectrum with as few as 10 energy bins, at least 1000 photons must be followed to obtain a ten percent answer. Most Monte Carlo codes allow the user to obtain some information into these details by keeping track of the probabilities associated with each node in the trajectory of each detected photon and allowing the user to retrace the paths of those photons which undergo the most probable paths. They also may aid the user in understanding the physics by keeping tallies on how many collisions occur in each particular zone of the specified problem. Most of these statistical tallies are geared for looking at the most likely quantity. That is to say the largest contributions to an observed spectrum. What if one is interested in the unlikely quantity, that is a statistically small contribution to an observed spectrum?

There does not appear to be many techniques in the literature which seek to give analytical solutions to the multiple scattering problem. Watson³ has studied the spectra of scattered gamma rays as a function of various target densities in hopes of understanding their shape dependence on target structure. He defines a *spectral sensitivity matrix* which allows him to quantitatively relate the linear variations of the material densities through which the scattered photons travel to the variances in the intensities of each spectral component. That is he seeks to empirically relate the shape of the spectra to the variations in the target densities. Instead of deriving the spectra by following the trajectory of individual photons, he is able to empirically derive the functional of the spatial variations of the density and photoelectric factor which determines the shape of the spectra.

Anghele⁴ attacks the problem more directly. He is interested in using energy spectrum of Compton scattered radiation to directly obtain density information. His method of obtaining the spectrum from multiply scattered photons is to first determine where these photons originate. Next he assumes the spectra originates from single scatters alone. He then defines a scattering factor β given by the equation:

$$D(E) = \beta(E)D^1(E)$$

where: D is the observed energy spectrum and D^1 is the energy spectrum from single scattered photons; that is, $D(E) = D^1(E) + D^2(E)$. Using this spectrum he next calculates a density matrix, assuming $D^2(E) = 0$. Using this density matrix he now applies a correction to those densities that are in the region where multiple scattering can occur. These corrected densities are used to derive a new singles spectrum which when subtracted from the measured spectrum give the double scattered spectrum.

In this paper I will present details of how the spectrum from doubly Compton scattered photons may be explicitly calculated. The important constraint being that this method is useful only for problems where the scattered particle is distinguished from the incident particle on the basis of its angular deflection. In the specific case of Compton scattering this is accomplished by the energy shift as a function of scattering angle. The same would hold true for elastic neutron scattering.

If one examines the magnitude of the photons' interaction with matter for most elements one finds that the Compton cross section strongly dominates all other interactions throughout the region of from about 200 keV all the way up to 5 MeV. Therefore for a wide range of energies, the very region where scattering dominates, Compton scattering is very nearly the only significant interaction.

2.1 Outline of the Problem

Figure one is a sketch showing the scattering geometry I will use for these calculations. This geometry is sufficiently general to be useful for a wide range of specific applications. The photons are incident on a slab at angle α and confined to a cone of angle $d\Omega_0$. The slab extends vertically from Y_0 to Y_1 . The detector views the slab and detects photons if they are incident on it within its viewing cone of half angle ϕ inclined at angle β to the slab, and angle γ to the vector R . First I will describe how to calculate the signal due to single scattered photons only. Then I will extend this calculation to double scattered photons only. Finally I will show how to calculate the signal for any specified number of collisions.

2.2 Singly Scattered Photons

The signal scattered into the detector from singly scattered photons from a small volume dv is simply:

$$(1) dS(E)/dE = S_0 \left(\frac{d\Omega_0}{4\pi} \right) \left(\frac{\rho A_0}{A} \right) \left(\frac{d\sigma}{d\Omega} \right) dr d\Omega_d e^{-k_1 r_1 - k_2 R} \delta(\theta - f(E))$$

where:

S_0	= specific intensity of the source (photons/sec/solid angle)
$d\Omega_0$	= solid angle of the incident, collimated beam
$d\Omega_d$	= solid angle of the detector as viewed from the scattering volume
ρ	= density of volume element $\pi r_1^2 d\Omega_0 dr_1$
A_0	= Avogadro's number
A	= Atomic number
$\left(\frac{d\sigma}{d\Omega} \right)$	= Differential Compton scattering cross section
k_1	= Reciprocal mean free path of incident beam.

$\delta(\theta-f(E,\theta))$ = Delta function relating the energy of the scattered beam to its scattering angle. $f(E,\theta)$ = Compton equation: $E = E_0\mu/(\mu-E_0(1-\cos\theta))$. Where E_0 is the incident energy, μ is the restmass of the electron, and θ is the scattering angle.

R = distance from the scattering volume to the detector

r_1 = distance from the boundary of the slab to the scattering volume.

The delta function is necessary to relate the scattering angle to the energy of the scattered photons. This delta function also puts a constraint on the scattering volume which contributes to the spectrum at energy E . This volume is not arbitrary but determined by the solid angle of the incident beam and the width of the detector-- see figure 1. If the detector has a square aperture of width w , then the volume contributing to the spectrum at energy $E=E(\theta)$ is just $r_1^2 d\Omega_0 dr = r_1^2 d\Omega_0 w \cos\gamma / \sin\theta$. The solid angle of the detector can be written $w^2 \cos\gamma / R^2$. Therefore the explicit equation for the scattered spectrum detected for the geometry shown in figure 1 is:

$$(2) dS(E)/dE = S_0 \left(\frac{d\Omega_0}{4\pi} \right) \left(\frac{\rho A_0}{A} \right) \left(\frac{d\sigma}{d\Omega} \right) \frac{w^3 \cos^2\gamma}{R^2 \sin\theta} e^{-k_1 r_1 - k_2 R} \delta(\theta-f(E))$$

Integrating over the energies then, is equivalent to integrating over r_1 . Although it might be possible to evaluate this integral analytically, it is probably more useful to evaluate it numerically since most of the interesting problems will involve scattering from inhomogenous media. To evaluate this integral numerically simply express R , r_1 , and θ as a function of E and sum.

$$(3) dS(E)/dE = C \rho \left(\frac{d\sigma}{d\Omega} \right) \frac{\cos^2\gamma}{R^2 \sin\theta} e^{-k_1 r_1 - k_2 R}$$

where: C is the combined terms which are constant and the rest of the terms are functionals of E . It is seen that the intensity of the spectrum is proportional to the width of the detector to the third power. It is interesting to note that each position r_1 corresponds to a specific energy via the scattered angle θ . Therefore, the spatial resolution of this system is determined by the energy resolution of the detector. With regard to figure 1, it is also clear that this resolution is also directly dependent on w the width of the detector.

Scattering Geometry

Figure 1. This drawing shows the important quantities involved for calculating the spectrum for singly scattered photons.

2.3 Doubly Scattered Photons

In order to calculate the spectrum from doubly scattered photons, all that is necessary is to use the singly scattered photons as the source and calculate the spectrum of these photons as they are scattered into the detector. If this is attempted, the investigator will quickly discover that this is a formidable task. Since the source is no longer a point source the integration must be extended to include the location of all first scatterings i.e. over r_1 . Also for the second scattering, the source can no longer be considered collimated, therefore the integration must include all scattering angles. The effect of this is to increase the number of terms in the summation by a large factor of the order $(w/r_1)(E_0/dE)(w^3/V)$, where V is the total volume viewed by the detector, and dE is the energy resolution of the detector. Each one of these terms is typically greater than 100 for many problems of interest. Therefore, the number of calculations that must be performed will increase by a factor of about a million. This is the primary reason that Monte Carlo calculations have been so popular for determining spectra from multiply scattered photons.

The large increase in the number of terms to be summed implied above, however is not justified in the case of pure Compton scattering. In the Compton process there is an additional constraint imposed by the Dirac delta function $\delta(\theta - f(E))$. This function relates the energy of the scattered photon to the scattering angle. The effect of this constraint is radically

reduce the scattering volume. Let us examine the Compton equation as it applies to multiple scattering. For a single collision the final energy E_1 is related to the incident energy E_0 and the scattering angle θ by the equation.

$$(4) E_1 = E_0 \mu / (\mu - E_0 (1 - \cos \theta))$$

Then using E_1 for E_0 and specifying a new scattering angle θ_2 gives the equation for doubly scattered photons:

$$(5) E_2 = E_1 \mu / (\mu - E_1 (1 - \cos \theta_2))$$

Substituting E_1 in equation (4) into equation (5) gives:

$$(6) E_2 = E_0 \mu / (\mu - E_0 (2 - \cos \theta_1 - \cos \theta_2))$$

Comparing equation (6) with (4) we note that for E_1 to be equal to E_2 implies that $1 + \cos \theta = \cos \theta_1 + \cos \theta_2$. This equation in turn implies $\theta \neq \theta_1 + \theta_2$. Therefore, the singly scattered photons can never have the same final trajectory as doubly scattered photons. Figure 2 shows the a typical kinematically allowed collision volume for doubly scattered photons for the measurement depicted in figure 1.

Double Scattering Geometry

Figure 2 Same geometry as in figure 1 except the kinematically allowed volume for two collisions with θ_1 , θ_2 , and ψ is specified.

With the results of equation (6) and the aid of figure two it is possible to write down the intensity for the doubly scattered photon flux intercepted by the detector:

$$(7) \quad dS_2(E)/dE = S_0 \left(\frac{d\Omega_0}{4\pi} \right) \left(\frac{\rho_1 A_0}{A} \right) \left(\frac{\rho_2 A_0}{A} \right) \left(\frac{d\sigma}{d\Omega} \right)_1 \left(\frac{d\sigma}{d\Omega} \right)_2 \\ d\Omega_1 d\Omega_2 dr_1 dr_{12} e^{-k_1 r_1 - k_{12} r_{12} - k_2 R} \delta(\theta_1 - f(E_1)) \delta(\theta_2 - f(E_2))$$

Where in this equation we define the new quantities:

- $\rho_{1(2)}$ = The density for the first (second) collision.
- $\left(\frac{d\sigma}{d\Omega} \right)_{12}$ = Differential scattering cross section for the first(second) collision.
- $d\Omega_1$ = The solid angle of the second collision volume element as seen from the first collision volume element.
- $d\Omega_2$ = The solid angle of the detector as seen from the second collision volume element .
- k_{12} = The reciprocal mean free path between the two collisions.
- r_{12} = The distance between the first and second collisions.
- k_2 = The reciprocal mean free path between the second collision and the detector.
- r_2 = The distance between the second collision and the detector.

For a given energy E, all that must be done is to integrate over dr_1 , dr_{12} and θ_1 . Furthermore θ_1 must be between zero and θ , and r_{12} also has limits constrained by θ , θ_1 , and r_1 . As in the case of single scattering dr_{12} is not arbitrary but given by the projection of the detector's width w on the vector r_{12} . That is $dr_{12} = w \cos\gamma / \sin\theta$. Equation (7) may now be rewritten using the explicit expressions derived above:

$$(8) \quad dS_2(E)/dE = S_0 \left(\frac{d\Omega_0}{4\pi} \right) \left(\frac{\rho_1 A_0}{A} \right) \left(\frac{\rho_2 A_0}{A} \right) \left(\frac{d\sigma}{d\Omega} \right)_1 \left(\frac{d\sigma}{d\Omega} \right)_2 \\ d\Omega_1 dr_1 (w^2 \cos\gamma / R^2) (w \cos\gamma / \sin\theta_2) \\ e^{-k_1 r_1 - k_{12} r_{12} - k_2 R} \delta(\theta_1 + \theta_2 - f(E))$$

The integration reduces to an integral along r_1 and over a single angle θ_1 . Combining the terms that are constant into the constant C_2 gives the simplified equation:

$$(9) \quad dS_2(E)/dE = C_2 \rho_1 \rho_2 \left(\frac{d\sigma}{d\Omega} \right)_1 \left(\frac{d\sigma}{d\Omega} \right)_2 \\ d\Omega_1 dr_1 (\cos^2\gamma / R^2 \sin\theta_2) \\ e^{-k_1 r_1 - k_{12} r_{12} - k_2 R} \delta(\theta_1 + \theta_2 - f(E))$$

3. DOUBLY SCATTERED PHOTON RESULTS

In this section I will present the results of a numerical evaluation of equation (9) and compare them to the results obtained from using a detailed Monte Carlo code--Cog for the same geometry. The Cog results were obtained using the LLNL Cray YMP supercomputer. Equation (9) was evaluated using a Amiga 2000 desktop computer with a floating point co-processor. This Amiga runs at about 0.8 MFLOPS, the YMP can run fully vectorized code at speeds approaching 800 MFLOPS.

Many different integration techniques were tried in evaluating the integrals in equation (9); most of them gave very disappointing results. The real problem is the delta function which constrains the scattering angles. This constraint has the effect of reducing the volume of integration from the cone specified by r_{12} and θ_1 to the curve shown in figure 2.

It is extremely difficult to solve the transcendental equations that specify this curve. On the other hand, integrating over the solid angle $d\Omega_1$ numerically explicitly including the delta function is very inefficient. Since the integrand exists only over the small region (or several small regions) allowed by the delta function, most standard techniques of iteratively evaluating the integral and looking for convergence fail miserably. Therefore, I spent a considerable amount of time studying the convergence properties of these integrals.

The method I finally settled on was to simply integrate over r_1 from y_1 to y_2 and $d\Omega_1$ from θ_1 equals zero to θ_1 equals θ , and the azimuthal angle ϕ from zero to 2π and test to see that the delta function is satisfied at each volume element. This, of course, is extremely inefficient since a great many calculations are performed which do not contribute to the integral. Again the problem is to determine the allowed domain of r and θ_1 . During the integration over θ_1 The code keeps track of any regions where the integrand is non zero and uses these as the limits of integration for the next energy bin. Since each energy bin has its own unique domain for θ_1 , the code checks to verify that the current angular limits are outside this domain, and enlarges the domain if necessary. At every fifth energy bin the entire θ_1 space is integrated over and new domain boundaries are established. Figure three shows the domain of r for some arbitrary scattering angles θ_1 and θ_2 for collisions that are confined to a plane. For these scattering angles, the shaded area represents the only allowed values of r_{12} . The value of r can range from zero to $r_{\max}$, where $r_{\max}$ is the value of r at $y_{\max}$. In this figure two values of θ_1 are shown: The upper one is for $\phi = 0$, and the lower one is for $\phi = \pi$. Clearly $r_{\max}$ holds for all values of ϕ .

Double Scattering Geometry Limits

Figure 3. This figure shows the limits of r for scattering angles θ_1 and θ_2 for collisions that are confined to a plane. For these collisions, r_{12} may go from r_{max} at y_1 to zero at y_{max} .

After much experimentation trying various convergence methods, I have arrived at the following criteria: The upper angular limit is, of course, given by the angle corresponding to single scattering. The size of the differential $d\theta_2$ is simply the angular width of the detector which in turn determines the energy resolution. The width of the differential $d\theta_1$ is determined by the width of the acceptance angle of the detector (see figure 1). Finally the width of the differential dr is determined by the projected width of the detector on r . With these criteria the integrals all seem to be converged to better than a few percent.

Figure 4 shows the results of this calculation labeled 'exact' superimposed upon the corresponding calculation done by Cog. For this calculation the target was normal density iron, y_1 was 7 cm, y_2 was 14 cm, $x_0 = x_3 = 19.1$ cm, E_0 was 511 keV, α and β were both 45 degrees, and the energy resolution was 2.5 keV. Figure 4a is for a detector field of view of 1 degree half width, figures 4b and 4c are for 3 degrees and 7 degree respectively. The Cog calculation took about 110 minutes for all three detectors and traced the paths of 400,000 photons all launched into a half angle of 1 degree. The exact calculation took 69 minutes, 16 minutes, and 9 minutes for 4a, 4b, and 4c respectively. There has been no normalization between the two calculations.

It is interesting how good the two results agree especially at high energies. At the low energy end the exact curves all lie considerably below the Cog calculations. This is undoubtedly because of the presence of higher order scatterings which Cog calculates, but the exact result ignores at this time. Cog keeps track of the amount of scattering by order and outputs this result. The ratios of scatterings greater than two to double scattering are: 0.56 for 4a, 0.75 for 4b, and 0.90 for 4c. These figures imply high order scattering becomes increasingly important as the collimation is decreased and this high order scattering dominates mostly at the low end of the spectrum. I shall demonstrate that high order scattering also is important at very high energies.

Another defect of the exact calculation is the use of the Klein-Nishina equation to calculate both the mass attenuation coefficients and the scattering cross sections. The errors introduced by this approximation are larger at low energies and large scattering angles. I am in the process of adding a cross section library to the code which should not only improve the accuracy of this calculation, but also improve the speed of the program.

Finally, the cog results are for photons which were launched into a cone of 1 degree half angle. This range of angles degrades some of the structure shown in the exact curves. The exact curves assumed that the incident angles were all 45 degrees but that the area of the beam increased with distance as if it were collimated to a cone of one degree half-width.

Figure 4. This figure shows the results of three calculation for the geometry described in the text. The Cog Monte Carlo calculations are labeled 'cog', while the results of the direct double scattering calculation are labeled 'exact'. (a) is for the detector collimated to a cone of half angle 1 degree. The curve labeled (b) is for a detector collimated to a cone of half-angle 3 degrees. Curve (c) is for a detector collimated to a half angle of 7 degrees.

Figure 5. This figure shows the result in changing the axis of collimation of the detector from 30 degrees-part (a) to 60 degrees -part(f) in 5 degree increments. In each case the detector was collimated to a half angle of 3 degrees.

Figures 5 show what happens when the detectors field of view β is rotated from 30 degrees to 60 degrees and y_2 was decreased to 12 cm. In all cases the half angle was kept at 3 degrees. Each of the exact curves took from 12 to 15 minutes to run. As in figure 4 the corresponding Cog calculation took 110 minutes and traced 400,000 photons.

These results also seem in agreement with the Cog calculations with the exception of (f) which appears about a factor of two to small at the high energy knee. For theses

calculations Cog reports that the fraction of high order scatter to second order scatter is: 2.2, 1.43, 1.09, 0.54, 0.29, and 0.11 for curves (a) through (f) respectively. It is interesting that the best agreement appears for those calculations that contain the **most** higher order contributions. Since Cog is unable to keep track of the spectrum as a function of order, these conclusions are at this time uncertain.

4. HIGHER ORDER SCATTERING

In this section I will discuss some of the qualitative aspects of high order scattering and how the calculation described above for double scattering can be extended to include triple scattering.

If equation (5) is iterated another time one can write down the equation for the energy of the triply scattered photons..

$$(10) E_3 = E_2 \mu / (\mu - E_2 (1 - \cos \theta_3))$$

Substituting equation (6) for E_2 gives:

$$(11) E_3 = E_0 \mu / (\mu - E_0 (3 - \cos \theta_1 - \cos \theta_2 - \cos \theta_3))$$

In general for n collisions:

$$(12) E_n = E_0 \mu / (\mu - E_0 (n - \cos \theta_1 - \cos \theta_2 - \cos \theta_3 \dots - \cos \theta_n))$$

These equations tell us qualitatively quite a bit about the behavior of the spectra from each order of scattering. Each order, for example has a kinematically allowed maximum and minimum energy. For double scattering these extremum energy will occur when the variation in E_2 is zero. The maximum for double scattering occurs when $\theta_1 = \theta_2 = \vartheta / 2$. Where ϑ is corresponding angle for single scattered events. Likewise for triple scattered photons the maximum is for $\theta_1 = \theta_2 = \theta_3 = \vartheta / 3$. In general it can be shown that for n^{th} scattered photons, the maximum occurs for $\theta_1 = \theta_2 = \theta_3 \dots \theta_n = \vartheta / n$. Therefore (12) may be simplified to:

$$(13) E_n = E_0 \mu / (\mu - E_0 n (1 - \cos \theta / n))$$

For large n the corresponding maximum for n^{th} scattered photons approaches E_0 . For the Cog calculation shown in figure 4 these high energy limits occur at $E_2 = .322$ MeV for double scattering, $E_3 = 365$ MeV for triple scattering, and $E_4 = 0.391$ MeV for fourth order scattering. Although the statistics are very poor, there does appear to be edges in all the curves at these energies. This can be verified further by comparing with the calculated edges for the curves of figure 5. In these calculations the edges are shifted because of the rotation of the axis of collimation. The arrows labeled 2, 3, or 4 in each graph show the location of

each edge. Table one is a listing of these edges along with their corresponding single scatter angles.

Table 1
Multiple Scattering High Energy Limits

figure	single scatt angle	two scatters	three scatters	four scatters
5a	75	0.286	0.331	0.362
5b	80	0.298	0.342	0.371
5c	85	0.310	0.353	0.381
4a,b,c	90	0.322	0.364	0.392
5d	95	0.335	0.375	0.402
5e	100	0.348	0.387	0.411
5f	105	0.361	0.399	0.420

Finding the low energy edges is done in a similar manor, except that in this case more care must be taken to make sure that the kinematics is compatible with the geometry. Clearly, the lowest energy photons will be ones that suffer the largest angular deflections and still manage to enter the detector. For two collisions the equation to determine the minimum energy is similar to that for maximum energy except the angle ϑ must be replaced with $2\pi - \vartheta$. For the example of figure 4, this minimum energy for two collision photons is about 0.115 MeV. This energy is so low that it has little practical value.

In order to calculate the spectrum of third and higher order scattering what could be done is to convert the program for double scattering to a subroutine, and use it to calculate the doubly scattered spectrum for incident photons of energy $E(\theta_1)$ and integrate over θ_1 , ψ_1 , and r_1 . This calculation will take substantially longer than the corresponding calculation for double scattering because of the necessity of integrating over the incident angles. However, this increase may not be as bad as initially supposed if large regions of θ and ψ contribute negligibly to the integral. I believe that it is highly likely that this will be the case.

After the program for calculating third order spectra is complete, it can be converted to a subroutine and used to integrate over θ and ψ to obtain the spectrum for fourth order scattering.

5. CONCLUSIONS

In this paper I have demonstrated a practical method of calculating the spectrum from doubly scattered high energy photons. This technique can be applied using an inexpensive desktop computer of modest computing speed. The program I have written can in most cases obtain the spectrum of double scattered photons in just a few minutes.

By converting this program into a subroutine and using that subroutine to integrate over r and θ , it should be possible to calculate the spectrum of photons scattered more than twice.

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