Configuration Space Faddeev Calculations

Progress Report

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102

TABLE OF CONTENTS

	Page
ABSTRACT	iii
 I. PERSONNEL	 1
Principal Investigators	1
Graduate Student Research Assistants	1
Time Devoted to Contract	1
 II. SUMMARY OF RESEARCH COMPLETED	 2
Proton-Deuteron Scattering and Reactions	2
Bound-State Calculations	4
Relativistic Mesonic Models	6
Vacuum Structures in Hamiltonian Light-Front Dynamics	7
Quantum Mechanical Interpretation of Bethe-Salpeter Models	8
Signatures of Quantum Chaos	9
Applications of Point Form Relativistic Quantum Mechanics	10
Collective Nuclear Models and the Symplectic Group $Sp(6, \mathbf{R})$	12
Anharmonic Oscillators and Quantum Mechanics Systems in Nonconstant Magnetic Fields	13
 III. PAPERS PUBLISHED, ACCEPTED OR SUBMITTED	 14
Papers Published (1 November 1992 – 31 October 1993)	14
Papers Accepted or Submitted (as of 31 October 1993)	15
Meetings Attended	16
Invited Talks	17
Visitors	17

ABSTRACT

The detailed study of few-body systems provides one of the most precise tools for studying the dynamics of nuclei and nucleons. Our research program consists of a careful theoretical study of few-body systems and methods for modeling these systems. During the past year we have completed several aspects of this program.

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Time Devoted to Contract

Gerald L. Payne	40%
William H. Klink	40%
Wayne N. Polyzou	40%

On the following pages is a summary of the theoretical research which has been carried out under Contract No. DE-FG02-86ER40286 at the Department of Physics and Astronomy of The University of Iowa. The requirements of the contract have been fulfilled.

II. SUMMARY OF RESEARCH COMPLETED

Proton-Deuteron Scattering and Reactions

G. L. PAYNE AND J. L. FRIAR

We are continuing our program to study p-d scattering, both below and above the breakup threshold. While there have been a number of calculations for n-d scattering, no complete continuum calculations for p-d scattering have been performed except at zero energy. It is expected that comparing accurate numerical calculations for p-d scattering processes to experimental data will reveal useful information about the nuclear force. While the numerical calculations for p-d scattering are similar to the n-d calculations, the boundary conditions are considerably different; and there have been some questions about the correct form of the boundary conditions for the configuration-space calculations. Last year we completed a review paper [1] on the present status of the use of the configuration-space Faddeev equations to solve three-body problems when long-range Coulomb interactions are present in the Hamiltonian. The review provided a detailed discussion on the numerical solution of both the two-body and three-body problems with Coulomb interactions. A careful derivation of the appropriate boundary conditions for both cases was presented, along with a discussion of the role of polarization potentials in the calculations. A new derivation of the boundary conditions for the three-body case was given which agreed with the boundary conditions obtained by Merkuriev [2] derived using Green's functions. We are presently modifying our n-d codes to include the boundary new conditions for p-d scattering.

We plan to use the Kohn variational principle to test the quality of the p-d scattering calculations. Thus, we needed to develop a code to do the Kohn calculations for three-nucleon calculations above the breakup threshold. This code is completed and has been used to verify the accuracy of our previous n-d calculations for scattering at energies above the breakup threshold. We are now modifying the n-d Faddeev codes to include the three-body Coulomb boundary conditions. Once this modification is completed, we will be able to perform the first complete numerical solutions for p-d reactions in configuration space. These solutions can be compared with the wealth of experimental data for p-d scattering, which should provide additional and new insights into the nature of the nuclear force. In particular, the role of the three-body force in scattering reactions can be studied.

In addition to the Coulomb calculations, we are continuing our work on n-d reactions. Using an improved numerical approximation for the breakup boundary conditions, we have obtained more accurate n-d breakup cross sections. Presently, we are comparing these new results to the momentum-space Faddeev calculations. This comparison will allow us to determine the accuracy of the present n-d calculations.

[1] J. L. Friar and G. L. Payne, "Proton-Deuteron Scattering and Reactions," *The Coulomb Problem*, edited by Frank Levin and David Micha [submitted]

[2] S. P. Merkuriev, *Sov. J. Nucl. Phys.* **24** (1976) 150.

Bound-State Calculations

G. L. PAYNE, J. L. FRIAR, AND B. F. GIBSON

Triton properties have been calculated using new nucleon-nucleon potentials which were fit to the world nucleon-nucleon data. In addition to the uncertainties from relativity and three-body force mechanisms, previous bound-state calculations had an uncertainty associated with the two-body nucleon-nucleon force. The various realistic two-nucleon forces all contain the one-pion-exchange potential for the long-range part of the potential; however, the χ^2 per degree of freedom for these potentials compared to the two-nucleon scattering data vary greatly. This is particularly true when a potential fit to the n-p data is compared to the p-p data. Also, there has been a large uncertainty in the measured value of the ϵ_1 mixing parameter, which leads to uncertainty in the two-body tensor force. These problems have been eliminated with the completion of the new and comprehensive Nijmegen partial-wave analysis. New Reid-like potentials have been constructed using an approach where each partial wave is parameterized independently. This procedure leads to potential models with an optimal χ^2 per datum. Using three new local model potentials derived in this manner, we have calculated the triton binding energy [1]. Using the results from these calculations, the triton binding energy for a realistic two-nucleon potential was estimated to be 7.62 ± 0.01 MeV. This is 900 keV lower than the experimental value. Thus, we have shown that local potentials which fit the two-body data bind the triton by a unique value of approximately 7.6 MeV. The additional binding energy in the triton could come from nonlocalities in the nuclear force, three-body forces, or relativistic effects. To study the effect of nonlocality in the two-nucleon potential, we have used inversion potentials generated from phase shifts corresponding to contemporary nucleon-nucleon potentials to calculate the triton binding energy. Inversion potentials which were fit to the phase shifts for the Nijmegen, Paris, and Bonn potentials yielded triton binding energies of 7.60 MeV,

7.47 MeV, and 7.84 MeV, respectively [2]. The result for the Bonn potential which has a large nonlocal tensor force is considerably different from the 8.14 MeV value obtained using the actual potential. This is a clear measure of the large effect that can be produced by a long-range nonlocality in the nucleon-nucleon interaction.

[1] J. L. Friar and G. L. Payne, V.G.J. Stoks, and J. J. DeSwart, Phys. Lett. B **311** (1993) 4.

[2] G. L. Payne and B. F. Gibson, Few-Body Systems **14** (1993) 117.

Relativistic Mesonic Models

W. N. POLYZOU

Research was conducted on the formulation of relativistic quantum mechanical models of nucleons and mesons with a model Hilbert space that is the space of square integrable functions of physical (rather than bare) nucleon and meson observables. The goal of this research is to construct relativistic quantum models of the meson-nucleon system that: (1) are given by a unitary representation of the inhomogeneous Lorentz group acting on the model Hilbert space, (2) have few and many-body problems that are mathematically decoupled, (3) satisfy cluster properties that fix most of the interactions of the many-body problem in terms of the interactions of the few-body problem, and (4) do not conserve meson number. The advantage of this type of model over models based on bare particle degrees of freedom is that there is no renormalization, and there is no need to refit the few-body interactions as the number of degrees of freedom is increased. The NN , $N\pi$, $\pi\pi$ and $NN+\pi$ sectors of the model were treated previously [1]. This year dynamical deuterons were included in the $NN+\pi$ sector and attempts were made to extend the results to the $NN+\pi+\pi$ sector. A complication encountered in the formulation of these models is that the dynamical generators (Hamiltonians) of the inhomogeneous Lorentz group for a system with a fixed maximum number of mesons are solutions of integral equations that use solutions of problems with a smaller maximum number of mesons as input. The resulting generators must still be diagonalized to solve for the dynamics. Because of this circumstance, any advantages obtained by using physical particle degrees of freedom are offset by the increased complexity in determining the generators. For this reason, I have abandoned this approach in favor of the more conventional one based on bare particle degrees of freedom.

[1] B. D. Keister and W. N. Polyzou, *Advances in Nuclear Physics*, Vol. 20, edited by J. W. Negele and E. Vogt (Plenum Press, New York, 1991).

Vacuum Structures in Hamiltonian Light-Front Dynamics

W. N. POLYZOU AND F. COESTER

We initiated a preliminary investigation of models of an infinite number of degrees of freedom that might provide systematic nonperturbative approximations to the observable consequences of a light-front field theory. The interactions in a Lagrangian field theory must be regularized in order that the Hamiltonian becomes an operator on the free-field Fock space. Although the regularization necessarily destroys rotational invariance, the invariance under the subgroup of the Poincaré group that leaves the light front, $z + ct = 0$, invariant can be preserved in many ways. Different regularizations lead to cut-off Hamiltonians that may have different vacuum structures than those of the free-field theory. In addition, because the cutoff theory is still a theory with an infinite number of degrees of freedom, different vacuum structures will be associated with inequivalent Hilbert space representations of the algebra of free fields restricted to the light front [1].

This year we began the first step in this investigation [2]. We established the existence of a large class of vacuum structures, each leading to a unitarily inequivalent representation of the algebra of free fields restricted to the light front and carrying a unitary representation of the subgroup of Poincaré group that leaves the light front invariant. Constraints on light-front Hamiltonians compatible with different vacua were established. The free-field light-front Hamiltonian is only consistent with the Fock representation of the Hilbert space. A large class of explicit examples of vacuum structures are presented. The role that these different vacuum structures play in the formulation of specific cutoff models remains to be studied.

[1] I. E. Segal, Trans. Am. Math. Soc., **88** (1958) 12.

[2] F. Coester and W. Polyzou, Vacuum Structures in Hamiltonian Light-Front Dynamics, Found. Phys. [to be published]

Quantum Mechanical Interpretation of Bethe-Salpeter Models

W. N. POLYZOU AND F. COESTER

We are studying constraints imposed by the existence of an underlying quantum theory on model Bethe-Salpeter equations and current vertex functions that are used to compute current matrix elements following the method in Ref. 1. The connection with an underlying quantum theory is provided by the relation to the Euclidean formulation of the theory. The existence of a Hilbert space with a four-momentum operator having a spectrum in the future-pointing light cone puts stringent constraints on the Euclidean Green functions. The Hilbert space is formulated in terms of an abstract scalar product [2] which must satisfy a positivity condition called Osterwalder-Schrader positivity [3]. The spectral condition requires that the Euclidean Green's function be analytic functions of the configuration space variables conjugate to the total momentum of each allowed type of physical intermediate state. Cluster properties and locality lead to additional constraints. The extent to which these constraints are realized in existing phenomenological models and the impact of relaxing specific constraints is being investigated. We also expect this work to lead to a quantum mechanical interpretation of Bethe-Salpeter models and a connection between the current operators used in Bethe-Salpeter formalisms and those used in quantum models based on either direct interactions or covariant constraints.

[1] S. Mandelstam, Proc. R. Soc. London **A233** (1955) 1425.

[2] W. Polyzou, Phys. Rev. **D32** (1985) 995.

[3] K. Osterwalder and R. Schrader, Comm. Math. Phys. **31** (1973) 83; **42** (1975) 281.

Signatures of Quantum Chaos

P. E. MOORE AND W. N. POLYZOU

We have studied different signatures of chaos in classical systems and determined the manifestations of these signatures in terms of quantum mechanical observables. Two definitions of quantum Lyapunov exponents, which provide a measure of the sensitivity of dynamical quantities to initial conditions, were studied. One is due to Vilela-Mendes [1] and a more sensitive measure was proposed by Mr. Moore [2]. In both cases the sensitivity is possible only for observables with an unbounded spectrum. The Hamiltonian and observable play complementary roles. Examples are constructed that illustrate that the combination of a simple Hamiltonian with a complicated observable or a complicated Hamiltonian with a simple observable can lead to a sensitive dependence on the initial condition.

We have concluded that the choice of quantum observables should be restricted to polynomials in the canonical variables in order to have a correspondence between classical chaos and a manifestation of this chaos in the corresponding quantum system. The relationship between the observable and the Hamiltonian in both the classical and quantum case has also been investigated by comparing properties of multiple commutators involving the Hamiltonian and observable and the corresponding expression involving brackets. In some instances the algebraic properties can be used to bound and/or compute Lyapunov exponents without having to solve the Schrödinger equation. We plan to apply this to the algebraic structures associated with some interacting boson models.

[1] R. Vilela Mendes, J. Phys. A **24** (1991) 4349.

[2] P. E. Moore and W. N. Polyzou, "Sensitive Dependence in Quantum Systems," J. Phys. A [submitted].

Applications of Point Form Relativistic Quantum Mechanics

W. H. KLINK

Point form relativistic quantum mechanics [1] offers a distinctive way of studying particle and nuclear spectroscopy, form factors, and structure functions, as well as dealing with production reactions. In contrast to other forms of dynamics, in the point form Lorentz transformations are kinematic so the theory is covariant, while interactions of the relativistic system are introduced into the four-momentum operator. In particular, the properties of the mass operator determine the bound and scattering states. In the following paragraph we sketch how the point form is being used to investigate the areas mentioned above.

We have formulated a relativistic $SU(6)$ model of hadrons [2] which is being used to fit spectroscopic data. (This is the honors project of J. Williams, an undergraduate physics major.) The $SU(6)$ model is a relativistic generalization of nonrelativistic $SU(6)$ models which have successfully fit hadronic data [3]. One advantage of our $SU(6)$ model, aside from giving a consistent relativistic formulation, is that semiempirical mass formulae such as the Gursey-Radicati mass formulae [4] can be derived from appropriate choices of the mass operator.

Second, point form relativistic quantum mechanics suggests a way of combining current operators with Poincaré generators, in which the current operators are viewed as raising and lowering operators acting on the four-momentum operators [5]. Form factors and structure functions, which are matrix elements of current operators on eigenstates of the four-momentum operators, can then be calculated algebraically. We are planning to test these ideas on deuteron form factors, as well as nucleon form factors using the hadronic wave functions of the previous paragraph.

Finally point form relativistic quantum mechanics can be used to analyze medium energy production reactions. Using separable potentials it is possible to find exact solutions of the relativistic Lippman-Schwinger equations describing production reactions [6]. The thesis project of M. Rogers is to apply this separable model to pion-nucleon production reactions, in the kinematic region where several pions are produced. This work is to be contrasted with the methods of Lee et al. [7] at Argonne, with whom we are collaborating.

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- [1] P.A.M. Dirac, Rev. Mod. Phys. **21** (1949) 392; W. H. Klink, "Relativistic Quantum Mechanics on Fock Space," in *Classical and Quantum Systems - Foundations and Symmetries* (World Scientific, Singapore, 1993).
 - [2] W. H. Klink, "Point Form Relativistic Quantum Mechanics and Relativistic $SU(6)$," in *Workshop on Harmonic Oscillators*, edited by D. Han, Y. S. Kim, and W. W. Zachary, NASA Conference Publication 3197, 1993, p. 219
 - [3] See, for example, A. Hey and R. Kelly, Phys. Rept. **96** (1983) 72, and references cited therein.
 - [4] F. Gursey and L. Radicati, Phys. Rev. Lett. **23** (1964) 173; see also the discussion given in Ref. 3.
 - [5] W. H. Klink, "Form Factors and Representations of the Poincaré Group" [to be submitted].
 - [6] W. H. Klink, Phys. Rev. C **41** (1990) 442.
 - [7] T.S.H. Lee and A. Matsuyama, Phys. Rev. C **36** (1987) 1459; J. A. Johnstone and T.S.H. Lee, Phys. Rev. C **34** (1986) 243.

Collective Nuclear Models and the Symplectic Group $Sp(6, \mathbf{R})$

W. H. KLINK AND G. ROSENSTEEL*

Many of the features of the Bohr-Mottelson collective model of nuclear rotations and quadrupole vibrations can be understood in terms of the symplectic shell model of nuclei [1]. In this model the subgroup chain $Sp(6, \mathbf{R}) \supset U(3) \supset SO(3)$ is used to include the Elliott $U(3)$ model which in turn realizes the orbital angular momentum as arising from $SO(3)$. Bound-state wave functions are related to the discrete series of representations of $Sp(6, \mathbf{R})$, and it is well known that the subgroup chain $Sp(6, \mathbf{R}) \supset U(3) \supset SO(3)$ is not multiplicity free, meaning that a given representation of $U(3)$ may occur many times in a given irreducible representation of $Sp(6, \mathbf{R})$.

In the past several years T. Ton-That and I have solved several mathematical problems [2] that are of particular interest in collective nuclear models. Using techniques of dual pairs of representations, we have shown how to generate invariant operators whose eigenvalues can be used to break the multiplicity and uniquely label the nuclear wave functions. In conjunction with G. Rosensteel, we propose to use these techniques to explicitly resolve the $U(3) \subset Sp(6, \mathbf{R})$ multiplicity problem.

*Department of Physics, Tulane University, New Orleans, Louisiana.

[1] G. Rosensteel and D. J. Rowe, *Ann. Phys.* **126** (1980) 343, and references cited therein.

[2] W. H. Klink, T. Ton-That, and R. G. Wills, *J. Phys. A* **26** (1993) 3229.

Anharmonic Oscillators and Quantum Mechanics Systems in Nonconstant Magnetic Fields

W. H. KLINK

All quantum mechanical anharmonic oscillators have an underlying nilpotent group structure that can be used to relate anharmonic oscillators to other quantum mechanical systems such as particles in nonconstant electric and magnetic fields [1]. We have analyzed the simplest such example, namely the quartic anharmonic oscillators with magnetic fields of the form $\vec{B} = (\beta_1 + \beta_2 x)\hat{z}$. We (including a graduate student, T. Allen) are analyzing the low-lying levels of the quartic anharmonic oscillator numerically and will use the resulting energy eigenfunctions to compute the wave function for a particle moving under the influence of the magnetic field given above.

[1] P.E.T. Jorgensen and W. H. Klink, Publ. RIMS 21 (1985) 969.

[2] W. H. Klink, "Nilpotent Groups and Anharmonic Oscillators" [to be published].

III. PAPERS PUBLISHED, ACCEPTED OR SUBMITTED

Papers Published

(1 November 1992 – 31 October 1993)

1. G. L. PAYNE and B. F. GIBSON
Variational Aspects of Faddeev Calculations
Few-Body Systems **14**, 117–125, 1993
2. J. L. FRIAR, G. L. PAYNE, V.G.J. STOKS, and J. J. DE SWART
Triton Calculations with the New Nijmegen Potentials
Phys. Lett. B **311**, 4–8, 1993
3. W. N. POLYZOU
A Theorem on Light Front Quantum Models
J. Math. Phys. **34**, 2852, 1993
4. W. H. KLINK
Relativistic Quantum Mechanics on Fock Space
in *Classical and Quantum Systems – Foundations and Symmetries*,
Proceedings of the II International Wigner Symposium
(World Scientific, Singapore, 1993), p. 584
5. W. H. KLINK
Point Form Relativistic Quantum Mechanics and Relativistic $SU(6)$
in *Workshop on Harmonic Oscillators*, edited by D. Han, Y. S. Kim,
and W. W. Zachary, NASA Conference Publication 3197, 1993, p. 219
6. W. H. KLINK, T. TON-THAT, and R. WILLS
Shift Operators and the $U(N)$ Multiplicity Problem
J. Phys. A **26**, 3229, 1993
7. W. H. KLINK
Relativistic Spin in Point Form Relativistic Quantum Mechanics
in *Group Theoretical Methods in Physics*, Proceedings of the
XIX International Colloquium, Salamanca, Spain, 1992
[and in *Anales de Fisica Monografias* **2**, edited by
M. A. del Olmo et al. (CIEMAT, Madrid, 1993), p. 466]

Papers Accepted or Submitted
(as of 31 October 1993)

1. H. C. JEAN, G. L. PAYNE, W. N. POLYZOU,
The Balian Brezin Method in Relativistic Quantum Mechanics
Few-Body Systems [in press]
2. F. COESTER and W. N. POLYZOU
Vacuum Structures in Hamiltonian Light Front Dynamics
Found. Phys. [to be published]
3. P. E. MOORE and W. POLYZOU
Sensitive Dependence in Quantum Systems
J. Phys. A [submitted]
4. A. S. LAHAMER and G. L. PAYNE
Numerical Techniques for the Two-Body Schrödinger Equation
J. Comput. Phys. [submitted]
5. J. L. FRIAR and G. L. PAYNE
Proton-Deuteron Scattering and Reactions
The Coulomb Problem, edited by Frank Levin and David Micha
[submitted]
6. W. H. KLINK
Nilpotent Groups and Anharmonic Oscillators
[to be published]
7. W. H. KLINK
Solutions of Quartic Anharmonic Oscillators
[submitted]
8. W. H. KLINK
Form Factors and Representations of the Poincaré Group
[submitted]

Meetings Attended

G. L. Payne

Physics Computing '93, Albuquerque, New Mexico, May 1993

W. H. Klink

NATO Conference on Noncompact Groups and Applications, San Antonio, Texas, January 1993

III International Wigner Symposium, Oxford, England, September 1993

W. N. Polyzou

Workshop on Relativistic Quantum Mechanics, C.E.B.A.F. Center, Newport News, VA, September 1993

Division of Nuclear Physics Meeting of the American Physical Society, Asilomar, CA, October 1993

C.E.B.A.F. Workshop, Snowshoe, West Virginia, June 1993

Invited Talks

G. L. Payne

"Numerical Methods for Three-Body Configuration-Space Calculations,"
Physics Computing '93, Albuquerque, New Mexico, May 1993

W. H. Klink

"Nilpotent Groups and Anharmonic Oscillators," invited talk at the NATO
Conference on Noncompact Groups and Applications, San Antonio, Texas,
January 1993

"Relativistic Quantum Mechanics," seminar talk at Tulane University,
January 1993

"Form Factors and Representations of the Poincaré Group," III International
Wigner Symposium, Oxford, England, September 1993

Visitors

Dr. Conrad Burden, The Australian National University

Dr. Mik Sawicki, Iowa State University

Dr. Hsien-Chih Jean, Florida State University

END

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