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STABILITY AND DYNAMICS OF SPATIO-TEMPORAL STRUCTURES

Progress Report
for Period September 15, 1993 - September 14, 1994

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May 1994

Prepared for
THE U.S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-FG02-92ER14303

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1 Abstract

The main goal of the project supported in this grant is to contribute to the understanding of *localized* spatial and spatio-temporal structures far from thermodynamic equilibrium. Here we report on our progress in the study of three classes of systems.

1) We have studied cellular flame structures arising in a circular burner. Using numerical computations we have found a number of traveling-wave structures in which different cells undergo different motion. Most strikingly, we have found a localized wave traveling through the array of steady cells. The results are interpreted using various asymptotic approaches. They are in qualitative agreement with recent experiments.

2) We have continued our investigation of localized waves in binary-mixture convection. Starting from the extended Ginzburg-Landau equations we had introduced earlier, we have derived equations of motion for interacting fronts connecting the conductive and the convective state. These equations reveal a repulsive interaction between the fronts which implies a new localization mechanism for waves. It is solely due to the long-wavelength mode specific to the extended Ginzburg-Landau equations. The stability properties of the resulting localized waves are in qualitative agreement with very recent experiments.

3) We have extended our investigation of domain structures to include their temporal evolution.

2 Description of Progress

2.1 Structures in Combustion

In a collaboration with A. Bayliss and B.J. Matkowsky (Northwestern U.) we have investigated the dynamics of flames in gaseous combustion [1-3]. The system investigated consists of an axisymmetric line source of premixed fuel. Neglecting the feed-back of the flame on the fluid flow, the system can be described by reaction-diffusion-advection equations for the deficient components of the fuel and for the temperature. The reaction is assumed to be of Arrhenius type. In the simplest case one considers only a single deficient component. We have solved the resulting equations numerically using an adaptive pseudo-spectral method which is able to resolve the large gradients in the narrow reaction zone very efficiently even though the location of the reaction zone is changing with time. For the interpretation of the results we employ suitable amplitude equations based on the symmetries of the system.

The main results can be summarized as follows. Decreasing the Lewis number $\mathcal{L}$, which is given by the ratio of mass diffusion of the deficient component to heat diffusion, a whole sequence of structures is found. First the axisymmetric flame front is destabilized by a steady flame consisting of 4 reflection-symmetric cells. The steady 4-cell state becomes unstable to a traveling 4-cell state. This transition breaks the reflection symmetry of the cells and is therefore called a parity-breaking bifurcation. Interestingly, the state consisting of 4 equal traveling cells is unstable at its onset although it arises in a supercritical bifurcation of pitch-fork type. Instead one finds in a narrow window of parameters a stable state in which all 4 cells have different shapes as shown in fig.1. Some cells are almost reflection symmetric and stationary whereas others are asymmetric traveling cells. With time the asymmetric cells become more symmetric and come to rest whereas the steady cells start traveling. Due to its appearance we call this state 'Pushmi-Pullyu' (PMPY). It can also be considered as a certain type of modulated traveling wave. Further decrease of the Lewis number leads to additional types of modulated traveling waves. Their symmetries can be characterized as different types of 'Ponies-on-a-merry-go-round' (POM), a term which was first coined in the context of coupled Josephson junctions [4]. In such a state the different cells undergo the same type of motion albeit phase-shifted with respect other. Finally, for very small Lewis numbers we find irregular dynamics which has the appearance of spatio-temporal chaos in which cells are created and destroyed in chaotic way.

The essence of the first few transitions can be understood analytically. The appearance of the steady 4-cell state has been treated in the limit of high activation energy which reduces the flame front to a flame sheet [5]. The parity-breaking bifurcation to traveling waves can be understood within the frame work of a resonant interaction of the 4-cell state and the 8-cell state, which due to the axisymmetry of the system can be described by two complex equations for the respective amplitudes (e.g. [6, 7]).

Having established the origin of the parity-breaking bifurcation we have studied

Figure 1: ‘Pushmi-Pullyu’-state for $\mathcal{L} = 0.42$.

the waves arising from it using two coupled equations for the small deviation of the cells from reflection symmetry and for the phase of the cells, which essentially gives their position. This allows the description of the PMPY as a localized wave of asymmetry drifting through a background of reflection-symmetric cells. It also allows an understanding of the instability of the extended traveling waves if one allows for the possibility that the parity-breaking bifurcation can arise very close to the Eckhaus instability. One obtains then the interesting result that the traveling waves can bifurcate off the unstable steady cells and nevertheless gain stability for somewhat larger amplitudes although the steady cells remain unstable for the same parameter values.

Related phenomena have been observed in experiments with axisymmetric porous-plug burners. In particular, under various conditions a motion of the cellular flames corresponding to the PMPY has been found [8].

2.2 Localized Structures in Binary-Mixture Convection

In collaboration with H. Herrero (U. Navarra, Pamplona, Spain) we have continued our investigation of the extended Ginzburg-Landau equations which we had derived previously for the description of oscillatory convection in binary liquid mixtures. In comparison with the usual complex Ginzburg-Landau equations the new equations are characterized by a second field which corresponds to a long-wavelength concentration mode. We have concentrated on the localized waves (‘pulses’) of large length which do not resemble solitons in the nonlinear Schrödinger equation but rather two stably

coupled fronts [9–11].

In the presence of weak dispersion the interaction of fronts in a subcritical complex Ginzburg-Landau equation is reasonably well understood. We therefore focus on the effect due to the additional concentration mode introduced in [12] and neglect dispersion for the time being. First, we have investigated the dynamics of single fronts [9]. Considering the concentration mode as a perturbation we have determined their velocity. Strikingly, it shows hysteretic behavior as a function of parameters like the Rayleigh number which measures the temperature difference between the top and the bottom plate of the convection cell. This can be understood intuitively if one keeps in mind that the concentration mode tends to decrease the front velocity in that regime. Since the concentration mode is generated by the front it reaches larger values if the front moves slowly, which in turn slows down the front further. If, however, the front is fast, the concentration mode remains weak and decreases the velocity only by a small amount.

To investigate the interaction between fronts we have considered the limit of steep fronts. We have derived coupled equations for the velocities of two interacting fronts. In general, these equations are integro-differential-delay equations. However, in the limit of weak diffusion considered here they reduce to ordinary differential equations. These equations reveal a simple, robust mechanism that leads to a repulsive interaction between the fronts and therefore to stable pulses. Most strikingly, this interaction strongly depends on the velocity of the pulse and is indeed repulsive only for *backward* traveling pulses. For forward traveling pulses the interaction is attractive and does not allow the fronts to form a stably bound pair.

These results are in qualitative agreement with very recent experiments [13]. There the length and velocity of pulses have been studied for a range of values of the separation ratio ψ which measures the alcohol concentration of the mixture. Specifically, a strong correlation between the direction of propagation and the stability of the pulses has been found for long pulses with the stable pulses traveling backward. Thus, the extended Ginzburg-Landau equations do not only explain the strongly reduced propagation velocity of short pulses, which had motivated the introduction of the concentration mode [12], but also the stability of long pulses. Neither is possible with the complex Ginzburg-Landau equation alone.

For weak separation ratio, the experiments show the coexistence of two pulses with different lengths. Only one of them is stable. Strikingly, it is the shorter pulse which is found to be stable. In the complex Ginzburg-Landau equation, however, it is always the longer pulse which is stable. Presently, in a collaboration with W.-J. Rappel (ENS, Paris) we are investigating this question within the extended Ginzburg-Landau equations. Preliminary calculations indicate that the experimental stability behavior can be understood if both, dispersion and the concentration mode are taken into account. In fig.2 the length of numerically determined pulses is shown as a function of the reduced Rayleigh number. For the parameters chosen the pulses travel forward. Clearly, in the

Figure 2: Dependence of the pulse length on the reduced Rayleigh number.

range $(-0.96, -0.925)$ for the reduced Rayleigh number one obtains two coexisting pulses. In agreement with the experiments, it is now the longer one which is unstable and the shorter one which is stable. In the figure this can essentially be read off the slope of the curve: if the length of the pulse increases with the Rayleigh number it is stable [10]. We are currently investigating the combined effect of dispersion and the concentration mode in detail.

Since the localization mechanism provided by the concentration mode is very robust one may expect that it would allow even the localization of chaotic waves arising from a Benjamin-Feir instability. This will be one of the topics to be investigated in the coming budget period. Others will be a detailed study of the velocity of the soliton-like pulses and the interaction of pulses.

2.3 Evolution of Domain Structures

In collaboration with my graduate student David Raitt we have extended our analysis of domain structures to include also dynamical aspects. Previously we had investigated the existence regimes of domain structures, i.e. of patterns which consist of coexisting domains with different wave numbers. Within the frame work of a Ginzburg-Landau equation describing the competition of patterns with two different wave numbers $q_{1,2}$, we had found that the domain walls separating the domains can lock into each other and lead to stable domains. Their size is quantized with the possible lengths characterized by the number of spatial oscillations in the wave number [14, 15].

Figure 3: Evolution of a domain structure involving domains of different lengths. Time is running from bottom to top (measured in units of 10,000). Between (b) and (d) as well as between (d) and (f) Δq is reduced.⁸

We have now studied the temporal evolution of an array of such domains when the parameters are varied such that domains of a certain length cease to exist. More specifically, when $\Delta q \equiv q_2 - q_1$ is reduced, states with 1, 4 and 2 minima disappear successively. The state to survive longest is that with 3 minima. An example of such an evolution is shown in fig.3. An array of states with different lengths is chosen as initial condition. Figs.3a,c,e,g show the pattern with its local wave number while figs.3b,d,f give the location of the zeroes of the reduced wave number as a function of time (running from bottom to top). In fig.3b the locking of the domain walls is clearly visible for the short domain state (which has a single minimum) marked by *A*; both walls move with their distance fixed. After a change of parameters the 1-states cease to exist and disappear through a merging of two domain walls as can be seen at *B*. Through further increases all locked states disappear successively leaving only long domains, the length of which is not quantized (fig.3g). Eventually, these domains are expected to annihilate each other until only two domain walls remain. Since the interaction between the walls decays exponentially with their distance the time-scales for this process become, however, extremely long.

As discussed in [14,15] the results for the evolution of the domain structures are expected to be relevant also for systems exhibiting two-dimensional roll structures. There, a very general instability is the zig-zag instability which involves the competition of opposite wave numbers in the direction along the rolls and can lead to zig-zag-type structures (e.g. [16]). Domain structures are one-dimensional analogs of zig-zag-structures and therefore give insight into some of their dynamical properties.

During this budget period the P.I. will have taken two months in salary during the summer 1994. It is expected that the funds provided in the budget year 1993/1994 will be completely used by the beginning of the third budget year.

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