1 of 3

*Time Reversal Invariance in
Polarized Neutron Decay*

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TIME REVERSAL INVARIANCE IN POLARIZED NEUTRON DECAY

by

Eric G. Wasserman

ABSTRACT

An experiment to measure the time reversal invariance violating (T-violating) triple correlation (D) in the decay of free polarized neutrons has been developed. The detector design incorporates a detector geometry that provides a significant improvement in the sensitivity over that used in the most sensitive of previous experiments. A prototype detector was tested in measurements with a cold neutron beam. Data resulting from the tests are presented. A detailed calculation of systematic effects has been performed and new diagnostic techniques that allow these effects to be measured have been developed. As the result of this work, a new experiment is under way that will improve the sensitivity to D to 3×10^{-4} or better. With higher neutron flux a statistical sensitivity of the order 3×10^{-5} is ultimately expected.

The decay of free polarized neutrons ($n \rightarrow p + e + \bar{\nu}_e$) is used to search for T-violation by measuring the triple correlation of the neutron spin polarization, and the electron and proton momenta ($\sigma_n \cdot p_p \times p_e$). This correlation changes sign under reversal of the motion. Since final state effects in neutron decay are small, a nonzero coefficient, D , of this correlation indicates the violation of time reversal invariance. D is measured by comparing the numbers of coincidences in electron and proton detectors arranged symmetrically about a longitudinally polarized neutron beam. Particular care must be taken to eliminate residual asymmetries in the detectors or beam as these can lead to significant false effects.

The Standard Model predicts negligible T-violating effects in neutron decay ($D_{SM} \leq 10^{-9}$). Extensions to the Standard Model include new interactions some of which include CP-violating components. Some of these make first order contributions to D . The discovery of a nonzero triple correlation would have a profound impact on our understanding of CP-violation and fundamental interactions in general, since it would require physics beyond the Standard Model.

Chapter 1: Introduction

1.1 Introduction

Understanding the adherence of physical processes to symmetries has become a central theme in modern physics. Of equal importance are the instances in which these symmetries are broken. Some symmetries appear to be inviolate. For example, rotational symmetry implies that angular momentum is conserved; no violation of the conservation of angular momentum has ever been observed. On the other hand, the discrete symmetries space inversion or parity (P), and charge conjugation (C), once thought to leave the dynamics of all systems unchanged, have each been observed to be violated by the weak interaction. After the downfall of C and P, they were briefly supplanted by a deeper symmetry, invariance under CP (simultaneous charge conjugation and parity inversion). However, the belief in CP as an absolute symmetry of nature was short-lived as its violation was observed less than a decade later [Chr64].

Nearly thirty years have passed since the discovery of the violation of the CP symmetry. Unlike C and P individually, CP has been found, despite intensive investigation, to be violated in only one case, neutral Kaon decay, and there at a very low level, 2 parts in 1000. As a result, our understanding of CP-violation is limited. The standard Kobayashi-Maskawa model accommodates CP-violation but neither predicts it nor explains its origin [Kob73]. Many alternative explanations remain viable with current experiments providing only weak constraints. A unique description of the origin of CP-violation will come only with continued searches for CP/T-violating processes. The fact that CP symmetry is obeyed in nearly every fundamental process and yet there is tiny violation in one system is puzzling. Whatever the solution to this puzzle, it will have a major impact on physics from the realm of fundamental subatomic interactions to cosmology.

Searches for violation of time reversal invariance (T-violation) provide an important alternative to direct CP-violation searches. This is possible because CP-violation cou-

pled with the conservation of the CPT symmetry (charge conjugation, parity, and time reversal) implies T-violation and vice versa. Violation of the CPT symmetry has never been observed and is prohibited in the modern quantum field theories. Despite this, the conservation of CPT must also be questioned in light of the observed violations of the C, P, and CP symmetries. The most sensitive tests come from comparisons of particle and antiparticle masses and lifetimes, which should be equal if CPT invariance holds. In fact, the most stringent test of CPT invariance comes from the comparison of the K^0 and $\bar{K}^0$ masses [PDG92]{Particle Data Group, 1992 #90}, the very system in which CP is violated. While CPT may not be inviolate, T-violation is implied by the CP-violation in the neutral kaons [Sac87], and in all likelihood it is equivalent to CP-violation. So searches for T-violation also provide important information about the origins of CP-violation.

Searches for T-violation span a wide variety of systems including atoms, nuclei and particles (for a review see [Hun91]) with a like variety in the sizes of the T-violation signals predicted by the various theoretical models. Understanding the origin of CP-violation will require continued observations in a number of complementary systems. One such system is the neutron. T-violating angular correlations among the neutron and its decay products can be used as a sensitive probe of time reversal invariance (TRI). The purpose of this work is to describe the design, development, and prototype testing of a new experiment that can measure one such correlation.

1.2 Background

The free neutron beta decays into a proton, electron and an electron antineutrino ($n \rightarrow p + e + \bar{\nu}_e$). Neutron decay can be used to search for T-violation by measuring the triple correlation of the neutron spin polarization, and the electron and proton momenta ($\sigma \times p_p \times p_e$). Measuring this correlation as a test of TRI was first proposed by Jackson, *et al.* [Jac57a]. Under the classical time reversal operation ($t \rightarrow -t$) all three of the factors in the triple product change sign, therefore the entire term is odd under time reversal. T-vio-

lation would be observed as a nonzero triple correlation when averaged over all decays. In particular, one would observe a difference between the number of decays for which the triple product is greater than zero vs. the number of decays in which it is less than zero. The neutron has an advantage over other systems because it is particularly insensitive to electromagnetic final state interactions which could mimic a T-violation signal [Cal67a]. In addition, this possible T-violating component in neutron decay conserves parity, which distinguishes it from other T-tests such as searches for a permanent electric dipole moment of the neutron.

The neutron triple correlation described above has been the subject of previous experiments. The two most recent and most sensitive were performed by Steinberg, et al. [Ste74] at the Institute Laue-Langevin (ILL) in Grenoble during the 1970's and by Erozkimskii, et al. [Ero78] at the IRT-M reactor of the Kurchatov Atomic Energy Institute in the 1960's and 70's. The two experiments exploited similar techniques and both achieved sensitivities to the triple correlation at the level of a few parts per thousand. Neither found evidence for a nonzero triple correlation. They observed the coincidence of electrons and protons produced by the decay of neutrons in a longitudinally polarized, cold (low energy) neutron beam. Pairs of proton and electron detectors were arranged at right angles to each other and in a plane perpendicular to the neutron polarization. This arrangement maximizes the triple product because the three vectors are, on average, mutually orthogonal. A schematic diagram of the detectors used in the previous experiments is shown in Figure 1-1.

The sensitivity of this apparatus to the triple correlation derives from the fact that it distinguishes between decays in which $\sigma \times \mathbf{p}_p \times \mathbf{p}_e < 0$, the case, for example, of the coincidence of electron detector **e1** and proton detector **p1**, and decays in which $\sigma \times \mathbf{p}_p \times \mathbf{p}_e > 0$, the case for the coincidence of detectors **e1** and **p2**. Under T-invariance, coincidence pairs arranged with sensitivity to opposite senses of the triple correlation would have the same count rate. By comparing these count rates the size of a T-violating

effect can be measured. The symmetry of the apparatus is designed to eliminate the effects of systematic differences in the detectors.

Figure 1-1: Schematic of detector geometries used in the most recent and sensitive experiments. The point of view is looking along the direction of the neutron beam with the neutron spin into the page.

1.3 The Current Work

The subject of this thesis is the design, development, and prototype testing of a new experiment for measuring the triple correlation. Improvements in neutron polarization and general detector technology, since the last experiments were performed, make a new study of TRI in neutron decay attractive. An additional potentially much more significant advancement can be made by adopting a new geometry for the particle detectors. This is discussed in detail in Chapter 3 but is also outlined here.

The ultimate sensitivity of this experiment is limited by the number of decays observed from a beam of free neutrons. Improvements in sensitivity come either from increases in neutron density or in the efficiency of the detection of the decays. The detector geometry used in the most recent experiments is not optimal. This arises from the fact that the electron and proton from a neutron decay tend to be emitted at a large angle (γ_{e-p}) with respect to one another. This anti-correlation results from the limited available phase-space (essentially from the requirement that energy and momentum be conserved in each decay). There are far fewer decays in which γ_{e-p} is near 90° than at larger angles. In fact, the phase-space factors, even when convoluted with the sine dependence of the cross

product on γ_{e-p} (n.b. $\mathbf{p}_p \times \mathbf{p}_e = |\mathbf{p}_e||\mathbf{p}_p|\sin\gamma_{e-p}$), skew the sensitivity to D toward large γ_{e-p} . Calculations show that the effect of a nonzero triple correlation term on the decay distribution is strongly peaked at about 160° . Once this fact is taken into consideration, it becomes apparent that sensitivity to the triple correlation can be greatly enhanced if the electron and proton detectors are placed at larger relative angles. The large-angle detector geometry described in Chapter 3 is more sensitive by the equivalent of a factor of approximately 6 in decay rate (or neutron flux) than a detector of the same size using the traditional 90° geometry.

A set of prototype electron and proton detectors in the large-angle geometry was constructed for a proof-of-principle run on a cold neutron beam at the National Institute of Standards and Technology's (NIST) Cold Neutron Research Facility (CNRF). The results of the test are discussed in Chapter 5.

It is equally important that systematic asymmetries in the experiment do not introduce false asymmetries that imitate a T-violation signal. To avoid false asymmetries, the detector design incorporates four-fold symmetry useful for reducing these effects. A detailed study of the susceptibility of the experiment to these effects has been performed and the magnitudes of systematic effects resulting from various sources are estimated. The sources must be controlled so they do not produce false effects comparable to the statistical uncertainty. Based on this study, practical measures against the false effects were taken in the detector design. In addition, several very useful diagnostic methods were found for determining the contribution of systematic effects to the results. (Chapter 6).

The combination of the new detector geometry and the other improvements will facilitate an improvement in sensitivity to T-violation by an order of magnitude over previous neutron decay experiments. This improvement may have a profound impact on our understanding of CP-violation.

Chapter 2: History and Theory

2.1 Introduction

In this chapter we consider the theoretical foundations of beta decay and the contributions to the beta decay observables. All theories of the weak interaction have their origins in Fermi's description of beta decay as a vector, four-fermion interaction. Although Fermi's original theory was generalized to allow for the possibility of additional interactions, most of these generalizations appear to be unnecessary. Thus Fermi's basic intuition has proved correct. The V-A description of beta decay can be derived from the Standard Model electroweak gauge theory. In this theory the discrete symmetries P, C, and T are all violated. The T-violation appears as a violation of CP-symmetry, which is produced by a complex phase in the quark mixing matrix, with the simultaneous conservation of CPT. There is no T-violation at tree-level in the Standard Model description of beta decay and higher order contributions would produce only minute effects. Thus, an observation of T-violation in beta decay is a clear indication of physics beyond the Standard Model. Extensions to the Standard Model remain viable, and several of these do predict measurable T-violation effects in beta decay. Among these effects is the nonzero triple correlation among the nuclear spin, and the electron and neutrino directions.

In this chapter, experiments searching for T-violation in the beta decay of the neutron and nuclei are reviewed. Particular attention is given to the implications of past experiments for improvements in the measurement of the T-violating triple correlation in the neutron. The theoretical implications of improved sensitivity in the measurement of this correlation are considered.

2.2 Beta Decay Theory

2.2.1 History of Beta Decay Theory

The simplest form of beta decay is the decay of the neutron into a proton, electron, and electron antineutrino $n \rightarrow p + e^- + \bar{\nu}_e$. The electrons in beta decay are observed to have a continuous spectrum of energies. Although the neutrino in this process is not generally observed, its existence was postulated by Pauli in 1931 on the basis of spin statistics and to allow simultaneously for the continuous electron spectrum and the conservation of energy and momentum.

The first theory of beta decay was proposed by Fermi in 1934 [Fer34] in analogy to the quantum theory of electromagnetic interactions, which describes the interaction between an electron and a photon as the contraction of two four-vectors

$$\mathcal{H}_{\text{EM}} = e J_\mu A^\mu \quad (2-1)$$

where A^μ is the four-vector potential of the electromagnetic field and $J_\mu = \bar{\psi}_e \gamma_\mu \psi_e$ is the electromagnetic current density of the electron field spinor ψ_e and its conjugate $\bar{\psi}_e$ [Man84]. A Feynman diagram of the basic interaction is shown in Figure 2-1. In Fermi's theory, the electron current is replaced by the nucleon current $\bar{\psi}_e \gamma_\mu \psi_e \rightarrow \bar{\psi}_p \gamma_\mu \psi_n$, the electromagnetic four-vector potential by the lepton current $A^\mu \rightarrow \bar{\psi}_e \gamma^\mu \psi_{\bar{\nu}_e}$, and the electromagnetic coupling constant by the Fermi coupling constant $e \rightarrow G_F / (\sqrt{2})$. With these replacements the interaction becomes

$$\mathcal{H}_\beta = \frac{G_F}{\sqrt{2}} (\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma^\mu \psi_{\bar{\nu}_e}) \quad (2-2)$$

where ψ_p , ψ_n , ψ_e , and $\psi_{\bar{\nu}_e}$ are the Dirac spinors for the proton, neutron, electron, and electron antineutrino respectively. A Feynman diagram for this four-fermion interaction is shown in Figure 2-2. This interaction allows for beta decays in which the nuclear spin is unchanged.

Figure 2-1: The electron-photon interaction.

Figure 2-2: The Fermi vector, four-fermion interaction.

While Fermi's basic intuition has proved correct, the interaction must at least be generalized to account for the observation of beta decays in which the nuclear spin changes $|\Delta J| = 1$. Gamow and Teller [Gam36] showed that these could be accommodated by the addition of axial-vector and/or tensor terms to the Fermi theory. The most general local four-fermion interaction invariant under Lorentz transformations that does not include derivative couplings is [Her90]

$$\begin{aligned}
 \mathcal{H}_\beta = & \bar{\psi}_e \gamma_\mu (C_V + C'_V \gamma_5) \psi_{\bar{\nu}_e} \bar{\psi}_p \gamma^\mu \psi_n \\
 & + \bar{\psi}_e \gamma_\mu \gamma_5 (C_A + C'_A \gamma_5) \psi_{\bar{\nu}_e} \bar{\psi}_p \gamma^\mu \gamma_5 \psi_n \\
 & + \bar{\psi}_e (C_S + C'_S \gamma_5) \psi_{\bar{\nu}_e} \bar{\psi}_p \psi_n + \bar{\psi}_e \gamma_5 (C_P + C'_P \gamma_5) \psi_{\bar{\nu}_e} \bar{\psi}_p \gamma_5 \psi_n \\
 & + \bar{\psi}_e \frac{\sigma_{\lambda\mu}}{\sqrt{2}} (C_T + C'_T \gamma_5) \psi_{\bar{\nu}_e} \bar{\psi}_p \frac{\sigma^{\lambda\mu}}{\sqrt{2}} \psi_n + \text{H. c.}
 \end{aligned} \tag{2-3}$$

where vector (V), axial-vector (A), scalar (S), pseudoscalar (P), and tensor (T) couplings are included. The primed coefficients are the parity-violating interactions proposed by Yang and Lee [Lee56]. The observation of these couplings in beta decay implied the fall of invariance under parity (P-violation). Invariance of the interaction under time reversal requires that the constants C_i , C'_i are real.

There is a large body of experimental work devoted to determining the 19 parameters (20 minus 1 overall phase) in Equation 2-3. A few early experiments were able to determine the key elements of the interaction [Com83]. See Table 2-1. The pseudoscalar couplings are negligible in the nonrelativistic limit. Frauenfelder *et al.* observed, via Mott scattering, that the electron from beta decay has helicity $P(e^-) = -v_e/c$, i.e. it is left-handed [Fra57], whereas positrons from β^+ decay (${}^AZ \rightarrow {}^A(Z-1) + e^+ + \nu_e$) have the opposite helicity. This implies that $C_V = -C'_V$, $C_A = -C'_A$, $C_S = C'_S$, and $C_T = C'_T$. By observing the circular polarization of γ rays emitted following beta decay, Goldhaber *et al.* were able to determine that the neutrino from β^+ decay has negative helicity $h(\nu_e) = -1$, i.e. it is left-handed [Gol58]. From observations of electron-neutrino angular correlations, the antineutrino from β^- decay was determined to have the opposite helicity, i.e. it is right-handed. These and other experiments (see ref. [Boo84] for a summary) show that the scalar and tensor couplings are small.

The absence of right(left)-handed neutrinos(antineutrinos) participating in the weak interaction show clearly that the interaction violates both the P and C symmetries. Under spatial inversion, a left-handed neutrino becomes a right-handed neutrino, which is not observed. Likewise, under charge conjugation, a left-handed neutrino becomes a left-handed antineutrino, which is also not observed. However, the helicity observations are consistent with invariance under the combination of C and P, i.e. under the CP transformation. For example, CP transforms a left(right)-handed neutrino(antineutrino) into a right(-left)-handed antineutrino(neutrino), which is observed.

The total decay rate of $J^\pi = 0^+ \rightarrow 0^+$ nuclear decays is inversely proportional to

the square of the vector coupling. From measurements of the decay rate of several of these nuclei we have

$$\frac{C_V}{G_F/\sqrt{2}} \approx 1. \quad (2-4)$$

The mean life of the free neutron, which is 889.1 ± 2.1 s [PDG92], depends on both the vector and axial-vector couplings. The neutron lifetime, with C_V from the $0^+ \rightarrow 0^+$ transitions, can be used to determine the magnitude of the ratio of the axial-vector and vector couplings

$$\lambda = -C_A/C_V = |\lambda|e^{i\phi}. \quad (2-5)$$

The currently accepted value is $|\lambda| = 1.2573 \pm 0.0028$ [PDG92]. The electron and neutrino asymmetries in polarized neutron decay give the relative phase of the vector and axial vector coupling constants $\phi \approx 180^\circ$. In the limit that the preceding observations are exact, the general beta decay Hamiltonian density reduces to its familiar V-A form

$$\mathcal{H}_\beta = \frac{G_F}{\sqrt{2}} \bar{\Psi}_e \gamma_\mu (1 - \gamma_5) \bar{\Psi}_{\bar{\nu}_e} \bar{\Psi}_p \gamma^\mu (1 + \lambda \gamma_5) \Psi_n + \text{H.c.} \quad (2-6)$$

which is completely specified by G_F and a single complex parameter λ .

This form is consistent with the generalization of Fermi's theory proposed by Feynman and Gell-Mann [Fey58] and Sudarshan and Marshak [Sud58] in which a "universal" self-coupled charged weak current forms the weak interaction. In this theory the weak current consists of the sum of individual hadronic and leptonic currents, each of which has the V-A, or $\gamma_\mu (1 - \gamma_5)$, structure. This universal current-current interaction describes neutron beta decay as well the other charged weak interactions, for example, muon decay ($\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ and $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$).

Observation	Implication
Nonrelativistic nucleon limit	$C_P \approx 0, C'_P \approx 0$
$P(e^-) = -v_e/c$ and $P(e^+) = +v_e/c$	$C_j \approx -C'_j$ for $j = V, A$ $C_j \approx C'_j$ for $j = S, T$
$h(v_e) = -1$ and $h(\bar{v}_e) = +1$ and Fierz interference term	$C_S \approx 0, C_T \approx 0$
$0^+ \rightarrow 0^+$ transition rate	$\frac{C_V}{G_F/\sqrt{2}} \approx 1$
$\tau_{1/2}$ (neutron)	$ C_V/C_A \approx 1.26$
Electron and neutrino asymmetries (A and B) in polarized neutron decay	$\phi \approx 180^\circ$

Table 2-1: Constraints on the beta decay couplings.

2.2.2 Beta Decay in the Standard Model

In the Standard Model neutron beta decay is described by the transition of a down quark into an up quark $d \rightarrow u + e^- + \bar{\nu}_e$. The transition is mediated by the exchange of a W^- boson between a (u, d, W^-) quark-boson vertex and a (e^-, ν_e, W^-) lepton-boson vertex. A Feynman diagram of the transition is shown in Figure 2-3. Because of the low momentum transfer in beta decay compared to the boson mass ($q^2 \ll m_W^2$), the momentum dependence of the W^- propagator can be neglected and the interaction becomes a local current-current interaction [Her90]

$$\mathcal{H}_{\text{SM}} = \frac{g^2}{8m_W^2} V_{ud} \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) \nu \quad (2-7)$$

where $\bar{u}$, d , e , and ν are the spinors for the up quark, down quark, electron, and neutrino fields respectively, g is the gauge coupling constant, m_W is the mass of the W^- boson. The factor V_{ud} is the ud -element of the Kobayashi-Maskawa (KM) [Kob73] quark mixing matrix

$$\begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d' \\ s' \\ b' \end{bmatrix}. \quad (2-8)$$

that relates the quark weak interaction eigenstates (unprimed) to the quark mass eigenstates (primed). The standard model interaction Equation 2-7 is left-handed ($V - A$), i.e. maximally parity violating, and T-conserving.

Figure 2-3: Neutron beta decay at the quark level.

Unlike complex nuclei, there are no nuclear structure effects that complicate the description of the neutron's decay. However, the interaction given in Equation 2-7 is at the quark level whereas in beta decay the quarks are, of course, bound in a baryon. To describe beta decay the quark current $J_{\mu \text{ quark}} = \bar{u} \gamma_{\mu} (1 - \gamma_5) d$ is replaced by a nucleon current that includes the effects of the nucleon structure. The effects are allowed for in the nucleon current by including in it all possible terms acting as vectors or axial vectors under the Lorentz transformations [Byr82]. Thus the nucleon current consists of vector and axial vector parts $J_{\mu \text{ nucleon}} = J_{\mu \text{ V}} + J_{\mu \text{ A}}$ whose the amplitudes in neutron decay are most generally given in terms of the momentum transfer q by [Com83]

$$\begin{aligned}
\langle p | J_{\mu V} | n \rangle &= \langle p | f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} \frac{q^\nu}{m_p} + f_3(q^2) \frac{q_\mu}{m_p} | n \rangle \\
\langle p | J_{\mu A} | n \rangle &= -\langle p | g_1(q^2) \gamma_\mu \gamma_5 + i g_2(q^2) \sigma_{\mu\nu} \gamma_5 \frac{q^\nu}{m_p} + g_3(q^2) \gamma_5 \frac{q_\mu}{m_p} | n \rangle
\end{aligned} \tag{2-9}$$

where the transition is between the baryon rather than quark states. The form factors describe the vector (f_1), induced tensor (f_2), induced scalar (f_3), axial vector (g_1), induced pseudotensor (g_2), and induced pseudoscalar (g_3) interactions, and p and n signify the proton and neutron respectively. These are described as “induced” to distinguish them from the explicit couplings in Equation 2-3, which are independent of q . The momentum transfer in beta decay is small (for the neutron $q/m_p \approx 10^{-3}$) so $q^2 \rightarrow 0$ in the form factors. The strong interaction is responsible for the deviation of $|\lambda|$ from unity in Equation 2-6. The induced interactions, proportional to the momentum transfer, are small in neutron beta decay. They are neglected in the so-called allowed approximation [Com83], so that only the terms proportional to f_1 and g_1 remain. The other form factors contribute forbidden corrections to this approximation. The pseudoscalar term (g_3) is negligible in the nonrelativistic limit appropriate to beta decay. The conserved vector current hypothesis (CVC) [Fey58], which relates the conserved electromagnetic current to the nucleon vector current, implies that $f_1(0) = C_V / (G_F / \sqrt{2}) = 1$, and that f_3 is zero. It also determines the size of the so-called “weak magnetism” form factor $f_2(0) = f_1(0) (\mu_p - \mu_n) / 2 \approx 3.70/2$ [Com83] in terms of the anomalous proton and neutron magnetic moments μ_p and μ_n . The form factors f_3 and g_2 are second class, in the sense defined by Weinberg [Wei58], since they transform differently under G-parity (i.e. charge conjugation and simultaneous rotation by π about the I_2 isospin axis) than the other, first class, currents in Equation 2-9. Second class currents have not been observed.

2.2.3 Beta Decay Observables

Deviations from the V-A picture given by the standard model remain a possibility

[Her90] given present experimental limits [Boo84] on scalar and tensor couplings in the general Hamiltonian density Equation 2-3. The probability distribution function (PDF) for the beta decay of a polarized nucleus was derived from the general Hamiltonian density by Jackson, Treiman, and Wyld (JTW) in 1957 [Jac57a]. The PDF for the beta decay of a spin 1/2 system (like the neutron) with polarization $\sigma_n = \langle \mathbf{J} \rangle / J$, including one possible time reversal violating term, given in terms of the electron and neutrino directions and the electron energy, is [Jac57a]

$$\frac{dw(\mathbf{p}_e, \mathbf{p}_\nu)}{d\Omega_e d\Omega_\nu dE_e} = G(E_e) \left[1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b \frac{m}{E_e} + \sigma_n \cdot \left(A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right) \right] \quad (2-10)$$

where a sum has been taken over the spins of the final particles, and natural units, in which $c = \hbar = 1$, are used. The function $G(E_e)$ contains phase-space factors and the Fermi function. The latter describes the distortion of the outgoing electron's wavefunction by the Coulomb repulsion of the recoil nucleus, slightly modifying the electron energy spectrum. The function $G(E_e)$ is given by Equations A-7 and A-8 of Appendix A. The coefficient a describes the electron-neutrino correlation. The coefficients A and B describe the beta ($\sigma_n \cdot \mathbf{p}_e$) and neutrino ($\sigma_n \cdot \mathbf{p}_\nu$) asymmetries, which are both odd under parity, while the coefficient D describes the triple correlation ($\sigma_n \cdot \mathbf{p}_e \times \mathbf{p}_\nu$), which is even under parity but odd under motion-reversal. The coefficient b describes the strength of the Fierz interference term which vanishes in the Standard Model.

2.2.3.1 T-violation in Neutron Beta Decay

The coefficient D multiplies the triple correlation term, proportional to ($\sigma_n \cdot \mathbf{p}_e \times \mathbf{p}_\nu$), the only term in the neutron decay distribution Equation 2-10 with time reversal noninvariant contributions. The triple correlation is odd under reversal of the motion ($t \rightarrow -t$) in which the particle spins and momenta are reversed [Sac87]. This is shown in Figure 2-4. The momenta of the electron, proton, and neutrino produced by the decay of a neutron at rest all lie in a plane. For illustrative purposes, the neutron spin can

be chosen to be normal to the decay plane as it is in the figure. If the motion-reversed case (Figure 2-4b) is rotated by 180° about the neutron's spin axis, it is identical to the original (Figure 2-4a) save the neutron's spin is inverted. This demonstrates that the sign of the triple correlation reverses under motion reversal. If neutron decay is invariant under motion reversal then the forward and motion reversed processes, shown in Figure 2-4 a and b respectively, should be equally likely. Since, as we have seen, the sign of the triple correlation is opposite for the two cases, then a nonzero value of its coefficient implies that invariance under motion reversal is violated. For example, if $D < 0$ then the process shown in Figure 2-4a would be more likely than the one in Figure 2-4b. Thus the coefficient D serves as a measure of the degree of motion reversal violation. Motion reversal and the time reversal operation are not equivalent so violation of motion reversal does not immediately imply T-violation. However, tests of motion reversal invariance are indirect tests of time reversal invariance.

Figure 2-4: Neutron decay under reversal of the motion. The single arrows indicate the proton, electron and neutrino momenta which all lie in a plane. The double arrow indicates the neutron spin direction.

The time reversal operator in quantum mechanics is antiunitary. It can be written as $T = UK$ where U is a unitary operator and K applies complex conjugation to the operator. The time reversal operation has the effect of exchanging the initial and final states as

well as reversing the motion. Reversal of motion must be related to the time reversal operation in order to use comparisons like the one shown in Figure 2-4 as a test of time reversal invariance. This can be accomplished [Com83] by considering the neutron decay transition in the S-matrix formalism. The decay is described by the matrix element

$$S_{fi} = \delta_{fi} - i \langle f | T^{(1)} | i \rangle + (i)^2 \langle f | T^{(2)} | i \rangle + \dots \quad (2-11)$$

where f and i represent the final and initial states respectively and the $\langle f | T^{(n)} | i \rangle$ are the partial amplitudes of the transition. The first-order invariant amplitude $\mathcal{M}$ for the decay process is proportional to the matrix element $\langle f | T^{(1)} | i \rangle$. Taking the complex conjugate of this matrix element and using the hermiticity of $T^{(1)}$, yields

$$\mathcal{M}^* \propto \langle f | T^{(1)} | i \rangle^* = \langle i | T^{(1)} | f \rangle \quad (2-12)$$

resulting in the exchange of the initial and final states but not the inversion of the spins or momenta. This is possible only at first order in perturbation theory since T is not Hermitian at higher order. The time reversal operation is applied to the new process, described by $\mathcal{M}^*$, again reversing the initial and final states as well as inverting the spins and momenta. The resulting process, described by the amplitude $\hat{\mathcal{M}}^*$, is equivalent to the motion reversed process, Figure 2-4b. By definition, time reversal invariance requires $\hat{\mathcal{M}} = \mathcal{M}^*$ [Sac87]. Observation of different rates for the usual decay process, described by $\mathcal{M}$, versus the motion reversed process, described by $\hat{\mathcal{M}}^*$, would imply $|\mathcal{M}| \neq |\hat{\mathcal{M}}|$ signaling violation of T-invariance. With the inclusion of higher order effects, like electromagnetic final state interactions, Equation 2-12 cannot be used to relate the motion reversed process to $\hat{\mathcal{M}}^*$. If final state interactions are included, then motion reversal can be violated even though all of the interactions are invariant under time reversal. This applies to the case at hand where T-conserving final state interactions mimic T-violation by contributing to the D coefficient. We can write the coefficient as $D = D_t + D_{fs}$ where D_t is the contribution to the coefficient by T-violating processes, and D_{fs} contains the contribu-

tion by T-conserving final state interactions. Fortunately, D_{fs} for the neutron is quite small, as we shall see the observation of $D \geq 10^{-4}$ would be unambiguous evidence for T-violation.

2.2.3.2 Contributions to D_t

Ignoring final state interactions for the moment, all the coefficients in Equation 2-10 are given by JTW in terms of the general coupling constants. For neutron decay we have [Jac57a]

$$D_t \xi = (2/\sqrt{3}) M_F M_{GT} \Im(C_S C_T^* + C'_S C'_T{}^* + C_V C_A^* + C'_V C'_A{}^*) \quad (2-13)$$

where

$$\begin{aligned} \xi = & |M_F|^2 (|C_S|^2 + |C'_S|^2 + |C_V|^2 + |C'_V|^2) \\ & + |M_{GT}|^2 (|C_A|^2 + |C'_A|^2 + |C_T|^2 + |C'_T|^2) \end{aligned} \quad (2-14)$$

and the couplings C_i, C'_i are those of the general interaction Hamiltonian density Equation 2-3¹. The Fermi and Gamow-Teller nuclear matrix elements are $M_F = 1$ and $M_{GT} = \sqrt{3}$ for the neutron. The Fierz interference term is given by

$$b\xi = \pm 2\Re \left[|M_F|^2 (C_S C_V^* + C'_S C'_V{}^*) - |M_{GT}|^2 (C_T C_A^* + C'_T C'_A{}^*) \right]. \quad (2-15)$$

Measurements of b are used to place limits on the size of the scalar and tensor couplings.

It will be useful later when we consider the effects of final state interactions.

Notice that D_t depends on the phase angle between the complex vector and axial-vector couplings. Allowing only the usual V-A coupling, the coefficient becomes [Byr82]

$$D_t = \frac{2|\lambda| \sin \phi}{1 + 3|\lambda|^2} \quad (2-16)$$

1. The C_i and C'_i are those used by Herczeg [Her90]. They differ from those of JTW only in that C'_V, C'_S, C'_P, C'_T and C_A have the opposite sign.

and the other angular correlations are

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2} \quad (2-17)$$

$$A = -2 \frac{(|\lambda|^2 + |\lambda| \cos \phi)}{1 + 3|\lambda|^2} \quad (2-18)$$

$$B = 2 \frac{(|\lambda|^2 - |\lambda| \cos \phi)}{1 + 3|\lambda|^2} \quad (2-19)$$

where λ and ϕ are defined by Equation 2-5. If T-invariance holds, C_V and C_A are relatively real so from Equation 2-16 $D_t = 0$.

JTW also give the distribution function for beta decay in terms of the electron polarization, which was summed over in Equation 2-10. This distribution contains a correlation $R \sigma_e \cdot \frac{\langle J \rangle}{J} \times p_e$ among the electron spin, the nuclear polarization, and the electron momentum that violates both parity and time reversal invariance. The coefficient R describing this correlation is given by

$$\begin{aligned} R\xi = & \pm 2\lambda_{J,J'} |M_{GT}|^2 \Im(C_T C_A'^* + C_T' C_A^*) \\ & + 2\delta_{J,J'} |M_F| |M_{GT}| \left(\frac{J}{J+1}\right)^{1/2} \Im(C_S C_A'^* \\ & + C_S' C_A^* + C_V C_T'^* + C_V' C_T^*) \end{aligned} \quad (2-20)$$

where the upper sign is for β^- decay and the lower for β^+ decay. The factor $\lambda_{J,J'}$ that depends on the angular momenta of the initial and final nuclei J and J' is defined by

$$\lambda_{J,J'} = \begin{pmatrix} 1 & J \rightarrow J' = J-1 \\ 1/(J+1) & J \rightarrow J' = J \\ -J/(J+1) & J \rightarrow J' = J+1 \end{pmatrix}. \quad (2-21)$$

Unlike the D coefficient, R is zero unless scalar or tensor couplings exist.

2.2.3.3 Contributions to D_{fs}

JTW have computed the first order corrections to the angular correlations that arise from Coulomb final state interactions. This contribution to D_{fs} , given to first order in $\alpha Z/p_e$ by [Jac57b]

$$D_{fs, \text{Coulomb}}^{(1)} \xi = \pm \frac{2\alpha Z m_e}{p_e} \Re(C_S C_A^* + C_V C_T^* + C'_S C'_A^* + C'_V C'_T^*), \quad (2-22)$$

vanishes in the absence of non-V,A couplings. This is advantageous because this final state contribution to the D coefficient does not violate time reversal invariance and thus would act as a background to a true T-violation signal. Following Calaprice *et al.* [Cal69], the contribution to D can be written in terms of the Fierz interference terms b_F and b_{GT} for Fermi and Gamow-Teller decays

$$D_{fs, \text{Coulomb}}^{(1)} \approx \frac{2}{\sqrt{3}} \frac{\rho}{1 + \rho^2} \left(\frac{\alpha Z}{p_e} \right) m_e (b_F + b_{GT}) \quad (2-23)$$

where $\rho = C_A M_{GT} / C_V M_F$. Using $\rho = 2.17$ and $(\alpha Z m_e) / \langle p_e \rangle \approx 107$ for the neutron, and present limits on the Fierz terms² [Boo84], we find $D_{fs, \text{Coulomb}}^{(1)} < 6 \times 10^{-5}$ which is well below the experimental sensitivity $D \approx 10^{-4}$ achievable in the immediate future and more than an order of magnitude below current experimental limits on D . Although higher order Coulomb corrections have not been calculated, they should be small. It should be noted that the $(\alpha Z/p)$ dependence makes this contribution about four times larger for the other interesting case of ^{19}Ne decay.

There are additional contributions to D_{fs} from final-state electromagnetic interac-

2. The results of experiments measuring the Fierz interference terms have been combined by Boothroyd *et al.* [Boo84]. The most stringent limit on $b_F = -0.001(6)$ comes from observations of the dependence on electron energy of ft values in $0^+ \rightarrow 0^+$ superallowed Fermi decays from which we derive a limit (at 95% confidence) of $b_F < 0.01$. From the limit on $b_{GT} = 0.0011(16)$ from $^{22}\text{Na} \beta^+$ decay we find $b_{GT} < 0.004$.

tions that depend on the nuclear recoil. They arise from the momentum transfer (q) dependent terms in Equation 2-9. Callan and Treiman [Cal67a] have considered these corrections assuming CVC and no second class currents. The most important effect they find is a T-conserving contribution to D_{fs} arising from the weak magnetism induced coupling. Unlike $D_{fs, \text{Coulomb}}^{(1)}$ above, this contribution is expected to be nonzero even for the pure V,A couplings in the Standard Model. The size of the contribution is proportional to the difference of the magnetic moments of the initial and final nuclei in the decay. Callan and Treiman find for ^{19}Ne , which is a $J^\pi = 1/2^+ \rightarrow 1/2^+$ mirror decay like the neutron, that this contribution is of the order

$$D_{fs, \text{Weak Magnetism}}^{(1)}(^{19}\text{Ne}) = 2.6 \times 10^{-4} \quad (2-24)$$

whereas for the neutron [Hol89] it is

$$D_{fs, \text{Weak Magnetism}}^{(1)}(n) = 1.1 \times 10^{-5} \quad (2-25)$$

about an order of magnitude smaller.

2.3 Previous Experiments

2.3.1 Neutron Sources and Properties

It will be useful to review some properties of neutrons and neutron sources before considering experiments studying neutron beta decay. High neutron density ρ is the primary requirement for experiments observing neutron beta decay since the detection rate is proportional to the number of neutrons inside the detector. This requirement implies that the only practical source of neutrons for these experiments is, and has been, beams from nuclear reactors. Neutrons emerge from the moderator of a reactor following (approximately) a Maxwellian energy distribution with mean de Broglie wavelength of about 1.8 Å. Some research reactors are equipped with “cold sources” that consist of volumes of cold (about 25°K) liquid deuterium, D_2O ice, or liquid hydrogen near the reactor core.

They are able to produce high intensity beams of “cold” neutrons whose wavelengths are predominantly between 1 and 10 Å.

The spectral density of neutrons $d\rho/dv$ in a beam is the number of neutrons per unit volume in a unit velocity range about v . The beam’s flux density written in terms of the spectral density is $\phi(v) = (d\rho/dv) v$. So the neutron density, the number of neutrons per unit volume in the beam, is related to these densities by

$$\rho = \int (d\rho/dv) dv = \int \frac{\phi(v)}{v} dv. \quad (2-26)$$

To a good approximation nuclear capture cross sections for low energy (<1 eV) neutrons vary inversely with the velocity. By convention, a “thermal” neutron has an kinetic energy of $T_T = 2.5 \times 10^{-2}$ eV, which is the mean energy of neutrons in thermal equilibrium with a heavy water moderator at 20°C. The thermal neutron velocity is $v_T = 2200$ m/s, and the thermal neutron de Broglie wavelength is $\lambda_T = 1.8$ Å. Since the capture cross section for cold neutrons on materials varies inversely as the neutron velocity, it is often defined in terms of the thermal velocity, as

$$\sigma(v) = \sigma_T (v_T/v) \quad (2-27)$$

where $\sigma_T = \sigma(v_T)$. Cold neutron beams are generally characterized by their “capture flux” defined by

$$\Phi_c = v_T \int (d\rho/dv) dv = v_T \rho. \quad (2-28)$$

This is a useful quantity for two reasons. First it is proportional to the neutron density. Second, if the thermal capture cross section σ_T is known for a target nucleus that obeys Equation 2-27, then the capture flux of a beam incident on the target is easily determined, regardless of the spectral shape of the flux density, by observing the capture reaction rate

$$w_C = C \int \phi(v) \sigma(v) dv = C \sigma_T v_T \int (d\rho/dv) dv = C \sigma_T v_T \rho = C \sigma_T \Phi_c, \quad (2-29)$$

where C is a constant determined by the target density and area.

The interaction of cold neutrons with matter can be described, in analogy to the transmission of light, by an index of refraction [Byr82]

$$n \approx 1 - \lambda^2 \frac{Na}{2\pi} \quad (2-30)$$

where N is the number of scattering centers per unit volume, and a is the scattering length. Neutrons incident from vacuum onto the surface of a material with $a > 0$ at a critical glancing angle below

$$\theta_c = \lambda \left(\frac{Na}{\pi} \right)^{1/2}, \quad (2-31)$$

will undergo total “external” reflection (i.e. they will be reflected by the surface back into the vacuum). Neutrons can be transported long distances (~ 100 m) away from a reactor core by evacuated rectangular neutron “guides” whose interiors are coated with materials that have $a > 0$ such as nickel or beryllium [Car82, Dub91]. This serves to almost entirely eliminate backgrounds produced near the core and to reduce fast neutron and γ ray fluxes in the beam.

2.3.2 Neutron Polarizers

In magnetized materials the scattering length, a , in Equation 2-30 is modified by a spin-dependent part, becoming [Byr82]

$$a = a_n \pm a_m \quad (2-32)$$

where a_n and a_m stand for the nuclear and magnetic portions of the scattering length respectively. The a_m term arises from the $\mu_n \cdot \mathbf{B}$ interaction between the neutron spin and the magnetic field in the material with its sign depending on the relative orientation of the neutron spin and the magnetization. Because of the spin dependence of the scattering length, neutrons of one spin state passing through magnetized material will be preferentially scattered [Blo43]. An unpolarized neutron beam transmitted through such material

will be partially polarized. This type of transmission filter is relatively inefficient since there is scattering of both spin states. Increasing the material's thickness increases the net polarization only at the cost of lower transmission.

The spin-dependence of the scattering length can be more effectively exploited to make neutron polarizers based on reflection from the surface of magnetized materials. Neutrons scattering from materials in which the nuclear and magnetic scattering lengths are nearly the same size will see two critical angles $\theta_c^\pm$, where the sign comes from Equation 2-32, one of which is nearly zero since $a \approx 0$. Neutrons with one spin-state will be reflected while those with the other will not. Using such materials polarizing “mirrors” [Hug51] and guides [Wil79] have been constructed.

Recently, polarizers utilizing so called “supermirrors” [Mez76] have been used to produce very highly polarized ($P > 95\%$) cold neutron beams [Mez77, Sch89a, Sch89b]. Supermirrors exploit interference by multilayers of magnetic materials to allow neutrons with a broader energy spectrum to be reflected efficiently. Supermirror-based polarizers typically transmit about 40% of the incident beam whereas an ideal polarizer would transmit 50%. Because all of the transmitted neutrons have been reflected, the polarizer may increase the beam divergence in one dimension, reducing the neutron density in the beam. The neutrons that are not transmitted, including the 50% of the neutrons with the “wrong” spin, must be captured in or near the polarizer. The (n,γ) capture process produces considerable γ backgrounds that must be shielded. Although supermirror polarizers are now in common use, none of the previous experiments measuring the D coefficient in neutron decay used them.

A promising new neutron polarizing technology currently under development is the ^3He spin filter using high density ^3He gas as proposed by Chupp [Chu90]. The very large capture cross section for thermal neutrons on ^3He (5300 barns) is strongly spin-dependent [Cou88]. Absorption of neutrons with their spins anti-parallel to the helium polarization is strongly preferred. Therefore, an unpolarized neutron beam that passes

through a volume of polarized ^3He will have the neutrons with spin anti-parallel to the helium polarization preferentially absorbed leaving the transmitted beam polarized along the helium polarization direction. This polarization method has two features that make it an attractive alternative to supermirror polarizers. First, the neutrons in the undesired spin state are removed by a reaction

that does not produce the large γ backgrounds that the supermirror polarizers do. Second, since the transmitted neutrons are not scattered, the divergence of the beam is not affected. Neutron polarizers of this type in which the helium was polarized by spin-exchange with optically pumped rubidium vapor have been made [Cou90, Tas92]. In order for this method to be competitive with the supermirror polarizers, refinements must be made to achieve, simultaneously, high ^3He polarizations ($\approx 80\%$) in large volumes (about 6 cm diameter by 10^{20} atom-cm $^{-2}$ thick) over long periods of time (≈ 1 month). Each of these requirements has been satisfied individually in other experiments [Chu90, Chu92].

2.3.3 The Neutron a , A , and B Coefficients

All of the angular correlations in Equation 2-10 have been measured for the neutron. A listing of the results of experiments measuring the a , A and B correlation coefficients is given in Table 2-2. Each of these coefficients is nonzero. The spin-electron ($\sigma_n \cdot p_e$) and spin-neutrino ($\sigma_n \cdot p_\nu$) angular correlations introduce large polarization-dependent asymmetries to the spatial distributions of the decay products. In measurements of the triple correlation, which is also polarization-dependent, the effects of these asymmetries must be eliminated or significantly reduced. Failure to do so can lead to the observation of a false nonzero triple correlation. In past experiments, this was done by placing the particle detectors so that the average electron and neutrino momenta, for the decays detected, lie in a plane perpendicular to the neutron polarization direction. Imperfections

in the cancellation of these polarization-dependent asymmetries are one of the leading sources of systematic error in the measurements. This will be discussed in Chapter 6.

a	A	B	Reference
0.07(12)			[Rob58]
	-0.11(2)	0.88(15)	[Bur60]
-0.06(13)			[Vla61]
		0.96(40)	[Cla60]
	-0.09(5)		[Cla61]
-0.091(39)			[Gri68]
		0.995(34)	[Ero70a]
	-0.115(8)	1.00(5)	[Chr70]
-0.099(11)			[Dob75]
	-0.113(6)		[Kro75]
	-0.115(6)		[Ero76]
-0.1017(51)			[Str78]
	-0.114(5)		[Ero79]
	-0.1146(19) ^a		[Bop86]
	-0.1131(14) ^a		[Ero90]

Table 2-2: Measured values of the angular correlation coefficients in neutron beta decay.

a. These values include a correction for recoil and weak magnetism effects of about 1%.

2.3.4 Previous Neutron D Coefficient Experiments

The electrons produced by neutron beta decay exhibit the classic beta energy spectrum (see. Equation A-7 of Appendix A) with endpoint 782 keV [Wil82]. While the majority of the electrons have sufficient energy to be easily detected by any of a number of common methods (e.g. scintillators, wire chambers, solid-state detectors), their spectrum does extend down to zero energy so some of them are necessarily lost. The protons have a maximum energy of only 0.75 keV precluding direct detection by conventional charged particle detectors. Therefore, the protons must be accelerated before they enter a particle detector.

The long mean life of the neutron implies that typically fewer than one in 10^6 of the neutrons in a beam will decay per meter as it passes through a decay detector. High intensity beams ($\approx 10^9$ neutrons/cm²/s) must be used to achieve sufficient counting statistics required to make precision measurements of the angular correlations. The lower average velocity of cold neutron beams gives the neutrons more time to decay within the detector. While the overwhelming majority of the neutrons produce no signal, they can produce considerable backgrounds by (n, γ) capture reactions in or near the detector. One fortunate consequence of the low proton energy is its relatively slow drift speed ($v < 3.7 \times 10^5$ m/s) in vacuum. If the proton is allowed to traverse a field free region of the order of 10 cm long before it is counted, then there will be a time delay between the detection of the electron and proton ≥ 0.26 μ s. This delayed coincidence is used to discriminate against processes, like (n, γ) neutron capture, that produce a prompt coincidence background in the detectors.

All measurements of the T-violating D coefficient in the neutron have been done by observing the in-flight decay of polarized neutrons produced by a reactor. The D coefficient has also been measured for the beta decay of ^{19}Ne . The results of all the D coefficient measurements are given in Table 2-2. The triple correlation term in Equation 2-10 is given in terms of the neutron polarization, electron velocity, and neutrino direction. Since the neutrino produced in the decay is essentially unobservable, energy and momentum conservation are used to eliminate the dependence on the neutrino direction in favor of the proton's momentum. Applying this substitution the triple correlation becomes

$$\sigma_n \cdot \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} = -\sigma_n \cdot \frac{\mathbf{p}_e}{E_e} \times \frac{\mathbf{p}_p}{(m_n - E_e - E_p)} \propto \sigma_n \cdot \mathbf{p}_p \times \mathbf{p}_e \quad (2-34)$$

where the decaying neutron has been assumed to be at rest. This approximation is valid because the kinetic energy of neutrons in a beam produced by a reactor is negligible (≈ 0.025 eV) compared to the 782 keV [Wil82] shared by the decay products. Coinci-

dences between the electron and proton produced by each decay are counted in a detector system that is able to distinguish “left-handed” decays (i.e. those with $\sigma_n \cdot p_p \times p_e < 0$) from “right-handed” decays (those with $\sigma_n \cdot p_p \times p_e > 0$). This is accomplished either by comparing the coincidence rates in a fixed pair of detectors as the neutron polarization is reversed, or by comparing the coincidence rates for detector pairs arranged to have equal and opposite sensitivity to the triple correlation, or both. Finally, the coefficient D , describing the strength of the triple correlation asymmetry, is determined from these comparisons of counting rates.

System	$D \times 10^{-3}$	Laboratory	Reference
neutron	40 ± 50	Argonne	[Bur60]
	-140 ± 200	Chalk River	[Cla60]
	10 ± 10	IAE	[Ero68]
	-2.7 ± 3.3	IAE	[Ero74]
	1.1 ± 1.7	ILL	[Ste74]
	2.2 ± 3.0	IAE	[Ero78]
^{19}Ne	2 ± 14	Berkeley	[Cal67b]
	2 ± 4	Berkeley	[Cal74]
	-0.5 ± 1	Princeton	[Bal77]
	0.4 ± 0.8	Princeton	[Hal84]

Table 2-3: Measurements of the D coefficient in beta decay.

The earliest experiments measuring the D coefficient for the neutron both counted electron-proton coincidences from the decay of neutrons in a transversely polarized beam. In both experiments proton and electron detectors were held at a fixed angle relative to each other in a detection plane normal to the polarization direction though they used different methods to distinguish the sign (handedness) of the triple correlation for each decay. Both detectors lacked the symmetry of later experiments. This symmetry reduces the residual contribution of the polarization-dependent A and B correlations to asymmetries in the coincidence count rates that simulate the effect of the T-violating triple correlation. In particular, both were susceptible to systematic asymmetries produced by misalignment

of the polarization direction relative to the detection plane. Neither found evidence for T-violation.

In the experiment of Burgy *et al.* [Bur58, Bur60] at Argonne, the beam, produced by the CP-5 reactor, was polarized to $87 \pm 7\%$ by reflection from a magnetized Co-Fe mirror at glancing incidence (8°). The experimental layout is shown in Figure 2-5. This produced a beam carrying 5×10^7 neutrons/s in a $20 \text{ cm} \times 0.6 \text{ cm}$ rectangular cross section polarized along the long (vertical) transverse dimension. The neutron polarization was maintained by a 2 to 5 gauss guide field over the evacuated detector region. The two particle detectors employed were arranged facing each other on either side of the beam with the axis between them orthogonal to both the beam direction and polarization. The proton detector consisted of an electron multiplier system fed by secondary electrons produced when a decay proton struck a silver-magnesium cathode. The electron current was multiplied by the dynodes of the electron multiplier to a detectable level. To achieve sufficient secondary electron emission at the cathode, a pair of flat grid electrodes produced a uniform (i.e. nonfocussing) electric field in the region of the beam that accelerated decay protons to about 10 keV and through the 15 cm square entrance of the proton detector. The opposing 13 cm diameter electron detector was constructed from a mosaic of square anthracene crystals 3.2 cm on a side by 0.5 cm thick. These were fitted to Lucite light guides terminating in a photomultiplier tube. A series of slits in front of the proton detector discriminated decays in which the proton's momentum had a component parallel to the beam from those with a component anti-parallel. Since the beam direction, polarization direction, and the axis between detectors were mutually orthogonal, the slits served to discriminate against one handedness of the triple correlation. The electron-proton delayed coincidence rate was compared to that when the beam was depolarized by inserting a $2.5 \mu\text{m}$ thick sheet of steel into the beam. Runs were also done with all the fields (and thereby the polarization) reversed. The raw count rate asymmetry had to be multiplied by correction factors to account for the finite detector sizes, kinematic effects, and imperfect

polarization. The final result was $D = 0.04 \pm 0.05$, consistent with T-invariance. Imperfect orthogonality of the slits and the polarization allow the spin-neutrino correlation to make a possible absolute (as opposed to multiplicative) false asymmetry contribution to the triple correlation term. This was estimated to contribute no more than 0.008 to D but constitutes a systematic limit to the sensitivity achievable by this method. The statistical uncertainty in the count rates dominated the uncertainty in the result.

Figure 2-5: The experiment of Burgy, *et al.*

The experiment of Clark and Robson [Cla58, Cla60] at Chalk River was similar to the Argonne experiment with some important differences. The experimental layout is shown in Figure 2-6. A neutron beam from the NRX reactor was transversely polarized to 27% by transmission through 1.27 cm of magnetized steel. The decay region within an evacuated tank was surrounded by an electrode at +7 kV. On one side of the beam, protons drifting into a 10° cone whose axis was perpendicular to both the beam and polarization directions passed through a wire grid and then were accelerated by 1 kV and focussed by a 6 kV cylindrical electrostatic lens onto the first dynode of an electron multiplier. Opposite

the proton detector were two electron detectors consisting of 12.7 cm diameter by 3 mm thick plastic scintillators held at $\pm 160^\circ$ from the proton direction in the plane whose normal was parallel to the polarization direction. The arrangement of the detectors is such that the handedness of a decay producing a detected electron-proton coincidence is determined by which of the two electron detectors is hit. The coincidence count rate of the proton detector and the two electron detectors were compared for both the polarized and unpolarized beams. The unpolarized beam was produced by turning off the magnetic field on the polarizer but not the magnetic guide field. The uncertainty on the final result $D = -0.14 \pm 0.20$ was reported to be primarily statistical but included contributions from uncertainties in the geometric and polarization corrections.

Figure 2-6: The experiment of Clark, *et al.*

The two most recent experiments have used a longitudinally polarized neutron beam allowing the electron and proton detectors to be arranged in a symmetric fashion in a detection plane orthogonal to the polarization direction. This symmetric arrangement serves to eliminate some of the systematic contributions to asymmetries in the coincidence count rates that simulate the triple correlation.

A series of measurements of the D coefficient have been carried out by Erokolimskii *et al.* at the Kurchatov Institute of Atomic Energy (IAE) [Ero68, Ero69, Ero70b, Ero74, Ero78]. Since a detector system with the same basic layout was used in all the measurements, only the latest and most sensitive experiment is discussed here. The detector system is shown in cross section in Figure 2-7. A neutron beam from the IRT-M reactor was polarized by reflection from a magnetized cobalt mirror to $68 \pm 3\%$ and $65 \pm 1\%$ in the two data sets taken before and after adjustments were made to the polarizer. Periodic nonadiabatic flipping of the polarization direction by 180° was facilitated by passing the beam through a current carrying foil. The initial transverse polarization of the beam was rotated adiabatically parallel to the beam direction by a gradual turn in the guide field. The beam had a 14 cm by 2 cm rectangular cross section and carried 10^9 neutrons/s into an evacuated chamber where electron-proton coincidences were counted. Neutrons decayed in an electric field free region inside a gridded cylindrical electrode held at 24.2 kV (labeled (1) in Figure 2-7) whose axis was colinear with the axis joining the opposing proton detectors. The protons produced by the decays drifted through the flat ends of the cylinder into a focussing field created by concentric electrodes. The outer spherical electrode (2) was held at 25 kV and the inner hemispherical gridded electrode (3) at ground. The protons were focussed and accelerated through the grounded electrode onto a 10 cm diameter by $1\mu\text{m}$ thick CsI(Tl) scintillator deposited onto the end of a light pipe that was coupled to a photomultiplier. A cylindrical electrode at 25 kV acted as an electrostatic "mirror" reflecting protons passing through the sides of the cylindrical decay region without altering their momentum component along the axis joining the proton detectors. Two electron detectors were arranged above and below the beam orthogonal to both the polarization direction and the axis joining the proton detectors. Each electron detector consisted of plastic scintillators coupled to photomultipliers. The symmetry of the detector arrangement was exploited by performing the experiment as a comparison between the coincidence count rates in the various electron and proton detector pairings. The coefficient D

was extracted from the ratio

$$x = KPD = \frac{N_1 - N_2}{N_1 + N_2} \quad (2-35)$$

where K accounts for geometric and kinematic averaging over the decay distribution, P is the beam polarization, and N_1 and N_2 are given by

$$\begin{aligned} N_1 &= N_{e_1 p_1}^{\uparrow} + N_{e_2 p_2}^{\uparrow} - N_{e_1 p_2}^{\uparrow} - N_{e_2 p_1}^{\uparrow} \\ N_2 &= N_{e_1 p_1}^{\downarrow} + N_{e_2 p_2}^{\downarrow} - N_{e_1 p_2}^{\downarrow} - N_{e_2 p_1}^{\downarrow} \end{aligned} \quad (2-36)$$

where $N_{e_i p_j}^{\sigma}$ is the number of coincidences detected between the i^{th} electron detector and the j^{th} proton detector with $\sigma = \uparrow(\downarrow)$ indicating the neutron polarization is parallel (anti-parallel) to beam direction.

Figure 2-7: Detector of the IAE experiment. The beam and polarization directions are into the page. 1: Cylindrical grid at 24.2 kV. 2: Spherical focussing electrode at 25 kV. 3—Hemispherical grid at ground.

Although the symmetric detector arrangement was shown [Ero70b] to eliminate the false asymmetry caused by overall misalignment between beam polarization and detectors that affected earlier experiments, two, more subtle, systematic asymmetries were observed [Ero78, Vor89]. The effects arise from two sources, nonuniform transverse com-

ponents of the polarization, and nonuniform detector efficiencies. These systematic effects will be considered in detail in Chapter 6. While these effects must be guarded against in any triple correlation measurement, the IAE experiment is particularly susceptible to them. Its large asymmetric beam shape contributes to the former effect, which scales with beam size. The large and complicated array of meshwork for proton focussing contributes to the latter. The result of the IAE experiments to date is $D = 0.0022 \pm 0.0030$ where the quoted error is purely statistical. The IAE group reported that the systematic errors were eliminated by performing several measurements under altered conditions. However, the systematic error removed is nearly three times the final uncertainty so decreasing the overall error in this experiment would require a simultaneous increase in statistics and improved elimination of the systematic error [Ero78]. Efforts in the IAE experiment continue [Mos93].

The experiment of Steinberg *et al.* [Ste74, Ste76], performed at the Institut Laue-Langevin (ILL) in Grenoble, was very similar to the IAE experiment. It also consisted of pairs of electron and proton detectors arranged at right angles to each other and the longitudinal beam polarization. The arrangement is shown in Figure 2-8. Two identical sections like the one shown were placed in series along the beam with each treated separately. The beam, originating from a liquid-deuterium cold source, was polarized to $70 \pm 7\%$ as it reflected from the walls of a curved magnetized section of Fe-Co neutron guide. The resulting transverse polarization could be selectively rotated either parallel or anti-parallel to the beam direction by a two coil spin flipper [Lia75]. In the field-free decay region, the beam's 5 cm by 8 cm rectangular cross section was surrounded by a rectangular electrode held at 20 kV. Gridded apertures in the electrode allowed decay protons to drift out of the box, through a flat accelerating gap, and onto an 8000 Å thick NaI(Tl) scintillator evaporated onto the end of a Lucite light guide. This arrangement differs from the IAE experiment in that there was a much smaller angular acceptance for the protons. In addition, no focussing of the protons was performed and the apertures defining their acceptance were

simple rectangles. The electrons were counted using plastic scintillators. The data were analyzed using a ratio of numbers of electron-proton coincidences. An extension to this method will be used in analyzing data from our detector and will be explained in Section 3.3.2. The systematic effect due to nonuniformity in detector efficiencies was examined and estimated to contribute less than 3×10^{-4} to D . Consideration of the systematic effect due to nonuniform transverse polarization components was not reported. The result $D = (-1.1 \pm 1.7) \times 10^{-3}$ was reported to be limited by the statistical uncertainty.

Figure 2-8: Detector of the ILL experiment. The neutron beam and polarization directions are into the page. The detector consisted of two identical banks of detectors like the one shown in series.

Important parameters from each of the experiments measuring the D coefficient for the neutron are summarized in Table 2-4. Included in this table are projections for experiments at the ILL and the NIST CNRF research reactors using the octagonal detector

described herein.

Experiment	Neutron Capture Flux (n/cm ² /s)	Neutron Polarization	Beam Size	Coincidence Detection Rate (s ⁻¹)	Signal to Background Ratio	Events
Burgy	4×10^6	87%	$20 \times 0.6 \text{ cm}^2$	5×10^{-3}	3/1	N/A ^a
Clark	N/A	27%	N/A	1.5×10^{-3}	1/2	N/A
IAE ^b	3.6×10^7	66% ^c	$14 \times 2 \text{ cm}^2$	0.8 ^d	2/1	2.5×10^6
ILL	2×10^9	70%	$5 \times 8 \text{ cm}^2$	1.5 ^d	4/1	5.9×10^6
Future, at CNRF ^e	4.4×10^7	95%	6 cm dia.	18	10/1	9×10^7
Future, at ILL	2×10^9	95%	6 cm dia.	140	10/1	9×10^9

Table 2-4: Important parameters of experiments measuring the D coefficient for the neutron.

a. N/A indicates the values are not available.

b. Parameters for the most recent IAE experiment [Ero78].

c. Adjustments to the polarizer were made between the acquisition of two data sets.

d. Rate after background subtraction.

e. The flux and coincidence rate are based on projections of an upgrade to the cold source scheduled for 1994 [Row93, Ric93].

2.3.5 ¹⁹Ne D Coefficient Experiments

The ¹⁹Ne nucleus is the only system besides the neutron for which the D coefficient has been measured. The short-lived ($t_{1/2} \approx 17 \text{ s}$) ¹⁹Ne nucleus beta decays, $^{19}\text{Ne} \rightarrow ^{19}\text{F}^- + e^+ + \nu_e$, into an energetic (0-2.2 MeV) positron, a very slow (0-0.2 keV) ion, and a neutrino. Like the neutron, its beta decay is a $J^\pi = 1/2^+ \rightarrow 1/2^+$ transition and the same distribution function Equation 2-10, including the triple correlation term, applies with the replacements $n \rightarrow ^{19}\text{Ne}$, $e^- \rightarrow e^+$, $p \rightarrow ^{19}\text{F}^-$, and $\bar{\nu}_e \rightarrow \nu_e$.

The first experiment, by Calaprice *et al.* [Cal67b, Cal69], employed a symmetric octagonal arrangement of positron and ion detectors, similar to those used in the early neutron experiments, about a longitudinally polarized beam of atomic ¹⁹Ne. The ¹⁹Ne was produced by bombardment of SF₆ by protons from the Berkeley 88" cyclotron and polarized to nearly 100% by deflection in a Stern-Gerlach magnet. The coincidence detection

rate was about 100 counts per hour. Using data analysis techniques similar to those used for the neutron, the result $D(^{19}\text{Ne}) = 0.002 \pm 0.014$, consistent with time reversal invariance, was obtained.

The subsequent measurements sought to avoid the low coincidence rates that come of using low density beams by capturing the polarized ^{19}Ne in a storage cell. It is possible to store much higher densities with near complete polarization inside a cell for several seconds while the decays are observed. There are two drawbacks to this storage method. First, the recoiling ^{19}F nucleus cannot penetrate the storage vessel so it must be detected within the cell. Second, the cell is large relative to the detector. This adds uncertainty to the determination of the angles between particle momenta reducing its sensitivity to D . The first storage cell experiment, by Calaprice *et al.* [Cal74] at Berkeley, employed eight ion detectors arrayed in an octagon inside a cell. The polarized ^{19}Ne beam was captured in the cell where the atoms remained for about four seconds. The decay positrons had to pass through these ion detectors, exiting the cell, into the cooled solid state Si(Li) detectors placed behind each ion detector. The higher coincidence rate, about 60 per second, more than offset the reduced sensitivity per decay due to the large source. The result, a four-fold improvement in sensitivity over the beam experiment, was $D = 0.002 \pm 0.004$.

The two storage cell experiments that followed were carried out at Princeton where the cyclotron there was used to produce the radioactive ^{19}Ne . The first experiment by Baltrusaitis *et al.* [Bal77] used a storage cell with a square cross section. Two opposing side walls of the cell held microchannel plate (MCP) detectors for ion detection. The other two walls held scintillators for detecting the positrons. The neon polarization was orthogonal to the axes joining the opposing walls. Although its coincidence detection efficiency was higher, the neon density and thus the decay rate in the cell was lower. The coincidence rate was about 50 per second. The result was $D = -0.0005 \pm 0.0010$. In the most recent experiment, by Hallin *et al.* [Hal84], the coincidence efficiency was increased to nearly unity using a cylindrical storage cell. See Figure 2-9. The ion was accelerated toward the

cell's end by an axial electric field. Secondary electrons produced by the ion striking the wall of the cell were collected in a segmented MCP at the end of the cell. The positrons were detected by a segmented cylindrical scintillator surrounding the cell. The result $D = 0.0004 \pm 0.0008$ remains the most sensitive measurement of the beta decay D coefficient.

Figure 2-9: Detector of the ^{19}Ne storage cell experiment by Hallin, *et al.*

In principle, the systematic effects described by Erokolimskii *et al.* in the neutron experiments could exist in the ^{19}Ne experiments. This is especially true of the effect due to nonuniform detection efficiency. These experiments continue to evolve and improvement in the sensitivity is expected [Cal91].

2.3.6 Other Searches for T-violation in Nuclear Beta Decay

The results of other tests of T-violation in beta decay are summarized in Table 2-5. They include two measurements of the T-violating term $(R\sigma_e \cdot (\langle J \rangle / J) \times p_e)$, described in Section 2.2. In both cases the transverse polarization of the positron (or electron) from the beta decay of a polarized nucleus was observed with the help of large angle Mott scattering. Neither the experiment in ^8Li [All92] nor the one in ^{19}Ne [Sch83] found evidence for T-violation. The coefficient R of this correlation, which has not been measured for the

Observable	Experiment	Value	Final State	Reference
R	^{19}Ne	-0.079 ± 0.053	1×10^{-3}	[Sch83]
	^8Li	0.004 ± 0.014	$\leq 10^{-3}$	[All92]
E1	^{58}Co	0.13 ± 0.30	N/A ^a	[Amb57]
	^{52}Mn	0.09 ± 0.16	N/A	[Amb58]
	^{56}Co	-0.011 ± 0.022	N/A	[Cal77]

Table 2-5: Measurements of the R coefficient in beta decay. The size of final T-conserving final state contributions to R , as computed by Vogel and Werner [Vog83], are included

a. N/A indicates the value is not available.

neutron, provides complementary information to the D coefficient as R depends on the existence of scalar or tensor couplings (see Equation 2-20). In addition, the R correlation is parity violating while the D correlation is not.

Also included are searches for β - γ correlations arising from T-violating second class currents. Such currents are kinematically suppressed in transitions within isospin multiplets, like neutron and ^{19}Ne decays [Hol72]. In the β - γ correlation experiments an oriented nucleus beta decays to a γ unstable daughter nucleus and the resulting γ radiation is observed rather than the recoiling daughter nucleus. The correlation observed is of the form $E_1 (\hat{J} \cdot \hat{k}) (\hat{J} \cdot \hat{p}_e \times \hat{k})$ where $\hat{J}$, $\hat{p}_e$, and $\hat{k}$ are unit vectors in the direction of the nuclear orientation, positron momentum, and photon momentum respectively. No evidence for T-violation was observed in any of these experiments.

Other tests of time reversal invariance in polarized nuclei have recently been reviewed by Boehm [Boe88] including searches for T-violating γ transitions due to T-violating components of the electromagnetic interaction and in nuclear states. None of these tests have shown evidence for T-violation.

2.3.7 Implications for Future Neutron Experiments

Some conclusions can be drawn from the previous experiments regarding performing more sensitive measurements of D for the neutron. Measuring the D coefficient

entails counting electron-proton coincidences. Significant reductions in the statistical uncertainty require either (a) new facilities to produce increased polarized neutron densities or (b) detectors with a higher useful coincidence detection efficiency per decay. At present higher unpolarized flux densities are not available, however, since the last of these experiments was done, neutron polarization technology has improved to the point where near perfect polarization is achievable. Despite this improvement, reducing the statistical uncertainty by as much as an order of magnitude would still require considerable improvements in (b). Improvements of this type, proposed and considered in detail in Chapter 3, are largely responsible for the high coincidence rates projected for the future experiments in Table 2-4.

As the statistical uncertainty is reduced, careful consideration must be given to eliminating systematic asymmetries that simulate a true triple correlation. As demonstrated by the experiments of the IAE group, these effects can easily become the limiting factor in these experiments. About two orders of magnitude improvement in sensitivity to D for the neutron is possible before the complication of T-conserving Standard Model final state effects need be considered.

2.4 Theoretical Implications

2.4.1 Experimental Constraints

The only observation of CP-violation has been in the neutral kaon system. The CP-violating decay of the long lived neutral kaon into two charged pions [Chr64]

$$\eta_{+-} = \frac{\text{Amplitude}(K_L \rightarrow \pi^+ \pi^-)}{\text{Amplitude}(K_S \rightarrow \pi^+ \pi^-)} \quad (2-37)$$

is observed at the level of $(2.268 \pm 0.023) \times 10^{-3}$ [PDG92]. A similar branching ratio η_{00} has been measured for its CP-violating decay into two neutral pions. The CP-violation in

the kaon system is generally described in terms of two complex parameters ϵ and ϵ' that can be written as a linear combination of η_{+-} and η_{00} (see, for example, [Cah91]). The parameter ϵ describes the CP-violating mixing between the short and long lived neutral kaon states. It is responsible for most, if not all, of the CP-violation observed in the kaon system. The parameter ϵ' describes the CP-violation due to the decay of a CP-eigenstate directly into a final state with opposite CP. It is known to be small compared to ϵ and to have nearly the same phase. The most recent experimental determinations at Fermilab and CERN give $\Re(\epsilon'/\epsilon) = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4}$ [Gib93], where the first uncertainty is statistical and the second systematic, and $\Re(\epsilon'/\epsilon) = (33 \pm 11) \times 10^{-4}$ [Bur88] respectively.

The violation of the CP symmetry is thought to imply T-violation because the simultaneous conservation of CP and T is required on very general grounds (see, for example, [Sac87]) by the CPT theorem [Lud57] and because no violation of CPT has been observed in any system, including the kaons. Thus the observation of CP-violation in Kaon decay implies that T is in fact violated by the weak interaction in at least one instance.

The observations in the kaon system set the scale for CP-violation in the weak interaction at about 10^{-3} . The measurements of the D coefficient are now at about this level of sensitivity. However, without knowing the exact mechanism for CP-violation, it is impossible to know at what level T-violation might be observed in beta decay. The mechanism responsible for producing CP-violation in the kaons could also produce T-violation in beta decay. On the other hand, other sources of CP-violation, as yet unobserved, may do so.

Both the kaon data and evidence showing a lack of CP/T-violation in other systems [PDG92], particularly in searches for permanent electric dipole moments (EDM) in the neutron and in atoms, set the most stringent limits on mechanisms for CP-violation in extensions to the standard model [Her91, Boe88, Hol91, Hun91]. A nonzero EDM implies violation of both parity and time reversal invariance. The neutron EDM d_n is sensitive to

T-violation predicted by a wide range of theories [Ell89]. The experiments searching for a neutron EDM are fantastically sensitive, with the most sensitive direct measurements giving a limit of $d_n < 12 \times 10^{-26} \text{ e cm}$ [PDG92] at the 95% confidence level. Theories accommodating CP-violation and their contributions to D and the neutron EDM are given in Table 2-6.

Theory	Predicted Nonzero D	Predicted Nonzero d_n (e-cm)	Reference
Standard Model	$\leq 10^{-10}$	$\approx 10^{-32}$	[Kob73]
QCD θ parameter	$\leq 10^{-9}$	$\leq \text{limit}$	[Sac87]
Superweak	No	No	[Wol64]
Higgs Exchange	No	$\approx 10^{-24}$	[Wei76]
Left-Right Symmetric	$< 10^{-3}$	$\leq \text{limit}$	[Her91]
Supersymmetry	$\leq 10^{-5}$	$\leq \text{limit}$	[Chr70]
Exotic Fermion	$\leq 10^{-3}$	$\leq \text{limit}$	[Her91]
Leptoquark	$< 10^{-3}$	No	[Her91]

Table 2-6: Theories accommodating CP-violation. The neutron EDM limits the parameters of some models.

2.4.2 Theoretical Sensitivity to the D Term

The minimal standard model based on the $SU(3)_c \times SU(2)_L \times U(1)$ gauge group contains two forms of CP-violation. The first of these is a CP-violating phase in the heavy quark sector of the KM quark mixing matrix that accommodates the CP-violation observed in the kaon system [Kob73]. The contribution to D from this phase comes only at higher order in the weak interaction so it is very small, of the order 10^{-10} [Her91]. Second, the degeneracy of the QCD vacuum leads to the possible existence of the CP-violating θ_{QCD} parameter. Experimental limits on the neutron EDM imply that θ_{QCD} is very small [Ell89]. As a result, it makes a negligible contribution $10^{-9} - 10^{-10}$ to D [Her91]. Thus, the observation of a nonzero triple correlation in beta decay, above the level of the Standard Model T-conserving final state interactions, would be unambiguous evidence for physics beyond the Standard Model.

In several models including CP-violation, besides the Standard Model, the contribution to D is negligible. The $\Delta S = 2$ superweak interaction proposed by Wolfenstein predicts that the CP-violation in the kaon system comes entirely from state mixing, i.e. $\epsilon' = 0$. It is essentially unobservable outside the kaon system. Weinberg pointed out that multiple Higgs doublets can provide a source of CP-violation [Wei76]. This model has been nearly ruled out as it predicts a relatively large neutron EDM ($d_n \approx 10^{-24}$ e cm).

Herczeg [Her90, Her91] has made a study of CP-violation in extensions of the Standard Model. He has determined that several models make a tree-level contribution to the D coefficient in beta decay. Left-right symmetric models of the electroweak interaction [Moh75] based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ contain two charged boson fields W_L and W_R , which couple to left and right handed fermion currents respectively. These models are attractive because, besides providing for CP-violation, they restore left-right symmetry at high energy. The boson fields, written in terms of their mass eigenstates W_1 and W_2 , are given by

$$\begin{aligned} W_L &= \cos\zeta W_1 + \sin\zeta W_2 \\ W_R &= e^{i\omega} (\cos\zeta W_2 - \sin\zeta W_1) \end{aligned} \quad (2-38)$$

where ζ is the mixing angle between the left and right-handed sectors, and ω is a CP-violating phase. There are additional CP-violating phases available in the right-handed analog of the KM matrix. A large difference in the boson masses accounts for the violation of parity. In models with manifest left-right symmetry, i.e. the left and right gauge couplings are equal $g_L = g_R = g$, the CP-violating effects of the right-handed currents in beta decay are unobservable [Her91]. However, in more general models there is a tree-level contribution to the D coefficient in beta decay from $W_{1,2}$ exchange [Her83, Her84]

$$D \approx -a_D \frac{g_R \cos\theta_1^R}{g_L \cos\theta_1^L} \zeta \sin(\alpha + \omega) \quad (2-39)$$

where the constant $a_D \approx 0.87$ for the neutron and $a_D \approx -1.03$ for ^{19}Ne , α is a CP-violating phase from ud element of the right-handed analog of the KM matrix, $(U_R)_{ud} = e^{i\alpha} \cos\theta_1^R$. The CP-violating phase $(\alpha + \omega)$ also contributes to the neutron EDM and ϵ'/ϵ in the kaons. Using the experimental values of these observables, a limit of $\approx 2 \times 10^{-5}$ on the contribution to D can be made [Her91]. However, because of the model-dependent assumptions required, this limit is not as reliable as the experimental limits on D [Her91].

Models including new “exotic” quarks and leptons that mix with the usual ones can add right-handed current contributions to the usual beta decay interaction [Kro75]. These models contribute at tree-level to the D coefficient in beta decay but the contribution here is again limited to $\approx 2 \times 10^{-5}$ by the neutron EDM and ϵ'/ϵ [Her91].

Many extensions to the Standard Model contain “leptoquarks”, spin-one or spin-zero bosons that induce quark to lepton transitions (e.g. [Kiz87, Buc86, Sha82]). The couplings of the leptoquarks to fermions may contain CP-violating phases. Herczeg [Her90, Her91] points out that first order leptoquark exchange can contribute to the D coefficient in beta decay but not to the neutron EDM or ϵ'/ϵ so there is no constraint on D in this case. It can be as large as the current experimental limit.

The minimal supersymmetric extension of the Standard Model contains complex coupling strengths that cannot be removed by redefinition of the fields. This can lead to T-violation beyond that provided for in the Standard Model. This additional T-violation has been shown by Christova and Fabbrichesi [Chr70] to provide a contribution to the D coefficient in beta decay at the one-loop level. They estimate the supersymmetric contribution is $D \leq 10^{-5}$ or about two orders of magnitude below the current experimental limits.

The origin of the observed CP/T-violation in the neutral kaon system remains unknown. The Standard Model description of the weak interaction has been tremendously successful. It is able to accommodate the occurrence of CP-violation in the kaons. Nonetheless, many theorists believe that new physics lies beyond the Standard Model. A num-

ber of extensions to the Standard Model include new CP-violating interactions. Whether or not one of these is responsible for the CP-violation in the kaon system, they may contribute to CP/T-violating observables in other systems, like the neutron, where the Standard Model contribution is vanishingly small. Thus experimental searches for CP/T-violation should be carried out in a number of systems and for a number of observables to achieve complementary sensitivity to the various extensions.

Chapter 3: Measurement Design

3.1 Introduction

As discussed in Chapter 2, the time-reversal violating term, proportional to the triple correlation $\sigma_n \cdot p_p \times p_e$, is expressed in terms of the neutron polarization, proton momentum, and the electron momentum observables. There are two methods, tracking and integral, for measuring the coefficient of this triple correlation. If the polarization is fixed and any five of the six components of the electron and proton momenta are measured for each decay, e.g. the electron and proton are tracked, then the decay can be completely reconstructed and the D coefficient extracted from a fit to the PDF given by Equation 2-10. In the limit of perfect detector momentum resolution, the tracking method provides maximal information.

In the integral method, electron and proton counters which subtend a limited solid angle are strategically placed about a beam of polarized neutrons. Coincidences between electron and proton detectors are then counted. The detectors are placed such that the electrons and protons collected in coincidence in a particular pair of detectors were all, or nearly all, produced by the decays that have same sign of the triple product. The sensitivity of the apparatus to the triple-correlation derives from the fact that it distinguishes decays in which $\sigma_n \cdot p_p \times p_e > 0$ from decays in which $\sigma_n \cdot p_p \times p_e < 0$. In addition, the detectors can be arranged such that, under T-invariance, coincidence pairs sensitive to opposite signs of the triple-correlation would have the same count rate. The magnitude of a T-violating effect can be measured by comparing the count rates in coincidence pairs sensitive to opposite signs of the triple correlation.

Previous experiments were discussed in Chapter 2, however, we remind the reader here of the general method they used as motivation for the new experimental design that will be presented. All previous neutron triple correlation experiments employed the integral method with their detectors acting to integrate the PDF over limited solid angles

[Bur60, Cla60, Ero78, Ste76]. The two most recent and sensitive experiments observed the coincidence of electrons and protons produced by the decay of neutrons in a longitudinally polarized, cold (low energy) neutron beam [Ero78, Ste76]. In the recent experiments, pairs of proton and electron detectors were arranged at right angles to each other in the plane perpendicular to the neutron polarization. Placing the detectors in the plane perpendicular to the neutron polarization makes them insensitive to the spin-electron and spin-neutrino correlations, whose coefficients in the PDF are A and B respectively. Placing the coincidence pairs at right angles to each other in the detection plane appears to maximize the effects of the triple-correlation term because a triple product of vectors is maximized when the three vectors are mutually orthogonal. However, as we shall demonstrate, maximizing the triple product does not maximize the sensitivity to the triple correlation term. The electron and proton from a neutron decay tend to be emitted at a large angle γ_{e-p} with respect to one another. This anti-correlation is a kinematic effect due to the limited available phase-space in the decay. Recall that the triple-product in the D term $\sigma_n \cdot p_p \times p_e = \cos\alpha |p_e| |p_p| \sin\gamma_{e-p}$ has an evident $\sin\gamma_{e-p}$ dependence. However, the phase-space factors in the PDF, even when convoluted with the triple-product's dependence on $\sin\gamma_{e-p}$, skew the effect of the triple-correlation on the decay distribution toward larger γ_{e-p} . Thus there are far fewer decays in which γ_{e-p} is near 90° , where the previous experiments placed their detectors, than at larger angles. Taking this fact into consideration, it becomes readily apparent that sensitivity to the triple-correlation can be greatly enhanced if the electron and proton detectors are placed at larger relative angles.

In this chapter, the probability distribution function describing polarized neutron decay in terms of the polarization of the neutron, and the momenta of the electron and proton produced is analyzed to determine the relative sensitivity of the two methods to the triple correlation coefficient D . In addition, the sensitivity to the arrangement and number of the counters employed in the integral method is studied. In order to do this, the product of the PDF and the resolution function of the detector being modeled must be integrated over

the detector solid angles. The integration of the PDF over the electron and proton momenta is most conveniently done numerically. The integrals involved become rather complicated using straight-forward numerical methods [Kof54] especially when performed over complicated beam or detector geometries. Instead, the integration is done by the Monte Carlo method.

A modification of the tracking method combines measurement of the electron's momentum in each decay with counting of the protons in coincidence in an integral type detector. This allows the decay vertex to be determined using intercept of the electron track with the neutron beam. While the event cannot be completely reconstructed using this method, it is considerably simpler technically in that it requires tracking only the relatively energetic electron. Limited information about the proton energy is available using its time of flight from the decay vertex to the integral detector. The added value in terms of sensitivity of reconstructing the decay vertex is evaluated.

3.2 Sensitivity to D

In this section, we present the results of studies using several detector models. We begin with a “perfect” detector in which the electron and proton momenta are reconstructed exactly. This detector represents the most sensitive, if impractical, case of the tracking method. It is used as a benchmark against which more practical detector models are compared. Next, models employing the integral method are studied and the features most important to sensitivity to the triple-correlation are determined. Finally, the results of these studies are applied to the design of a practical detector system.

3.2.1 Maximum Likelihood Calculations

An upper bound on the sensitivity of any detector to D is provided by a maximum likelihood estimation of the coefficient directly from a set of decay events generated by the Monte Carlo. This simulates a detector with perfect tracking and energy resolution

over 4π of solid angle for both the electron and proton. Because there is no binning of the data, there is no loss of information in the analysis. A useful by-product of the maximum likelihood estimation of the decay coefficients is that it provides a check of the validity of the Monte Carlo event generation. Verifying that the events from a Monte Carlo simulation follow the desired distribution is generally nontrivial. Details of the Monte Carlo event generation and its verification are given in Sections A.1 and A.3 of Appendix A.

The likelihood function is defined as the product of the value of the PDF for each of N decays

$$L(a, A, B, D) = \prod_{k=1}^N w(X_k | a, A, B, D) \quad (3-1)$$

where the vector X_k specifies the kinematic variables for the k^{th} decay and the explicit dependence of the PDF on the decay coefficients a, A, B and D is indicated. The vector X in the PDF, Equation 2-10, is specified in terms of the electron and neutrino momenta. Because energy and momentum are conserved, there only 5 kinematic variables. By convention, the electron energy E_e , and the directions ϕ_e , θ_e , ϕ_ν , and θ_ν of the electron and neutrino are used. However, a completely equivalent treatment would be to eliminate the neutrino variables in favor of the proton variables.

The maximum likelihood method is used to determine the most probable values of the decay coefficients (a, A, B and D) from the N data vectors X_k for the assumption that the PDF completely specifies the decay process. In the maximum likelihood method, $\ln L(a, A, B, D)$ is maximized with respect to the decay coefficients. The values of the coefficients at the maximum are known as the maximum likelihood estimates of the coefficients. They will be distinguished from the true values by a caret.

In the asymptotic limit of large N , the maximum likelihood parameter estimates are normally distributed with minimum variance [Ead71]. For a normal distribution the variance is the square of the standard error. Since the parameters are estimated with mini-

mal variance (error) there is no more sensitive method for determining the decay parameters from the vectors X_k . This ensures that the maximum likelihood estimation represents a bound on detector sensitivity. Other methods, for example, those that include binning of the data, as an integral type detector would, can be no more sensitive to D than the maximum likelihood method, assuming the asymptotic limit applies. In this limit, the variance $V(\hat{\theta})$ of a parameter θ can be estimated from the data by

$$\hat{V}(\hat{\theta}) = \left\{ \left(-\frac{\partial^2}{\partial \theta^2} \ln L \right) \Big|_{\theta = \hat{\theta}} \right\}^{-1} \quad (3-2)$$

where $\hat{\theta}$ is the maximum likelihood estimate value of the parameter θ . In a multiple parameter fit, the inverse of the second derivative matrix

$$(D_2)_{ij} = \left(-\frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln L \right) \Big|_{\theta = \hat{\theta}} \quad (3-3)$$

is the covariance or error matrix

$$E = D_2^{-1}. \quad (3-4)$$

The standard error in each fitted parameter θ_k is given by the square-root of the k^{th} diagonal element of the covariance matrix.

The maximum likelihood method estimates the parameters with minimum variance only in the asymptotic limit. For a finite sample the asymptotic approximation must be verified. This has been done in two ways. First, in the asymptotic limit $\ln L$ is parabolic as a function of the parameters. This has been tested for Monte Carlo data sets with as few as one thousand decays. In all cases and for all the parameters (the decay coefficients a , A , B and D) $\ln L$ is well approximated by a parabola in the decay coefficients. The log likelihood is shown as a function of the D coefficient for a one thousand event run in Figure 3-1 in which the decay coefficients used as input to the Monte Carlo were given their nominal

values $a = -0.1$, $A = -0.118$, $B = 0.995$, and $D = 0.0$. The 68% confidence interval in the parameters is given by the contour in a, A, B, D space for which [Ead71]

$$\ln L(a, A, B, D) = \ln L(\hat{a}, \hat{A}, \hat{B}, \hat{D}) - \frac{1}{2}. \quad (3-5)$$

This interval is indicated in the figure by the dashed lines.

Figure 3-1: The log of the likelihood function as a function of the D coefficient for 1000 Monte Carlo events. The circles are $\ln L$ evaluated from the Monte Carlo data and the solid line is a parabolic fit. The other coefficients are held fixed at their optimum fit values. The dashed lines indicate the standard error, i.e. a 68% confidence interval, that is $D = 0.03 \pm 0.07$. The value of D input to the Monte Carlo was 0.0, consistent with the maximum likelihood estimate.

As a second test that the asymptotic limit holds in the present case, we computed the uncertainty in the maximum likelihood parameter estimates as a function of the number of simulated decays. In the asymptotic limit the uncertainties should vary inversely as the square-root of the number of decays [Ead71]. Let us define the parameter G in terms of the uncertainty estimates $\hat{\sigma}$ as

$$G = \hat{\sigma} \sqrt{N}. \quad (3-6)$$

In the asymptotic limit, G should be independent of N . This behavior was observed. Sets of ten runs of 10^3 , 10^4 , 5×10^4 , 2×10^5 , and 3×10^5 decay events each were generated by the Monte Carlo simulation and fit by the maximum likelihood method. The uncertainty in $\hat{D}$ was estimated from the diagonal elements of the error matrix E in Equation 3-4. The parameter G is plotted versus the number of decays N in Figure 3-2. This plot demonstrates the independence of the parameter G from the number of decays, as expected in the asymptotic limit.

Figure 3-2: The parameter $G = \hat{\sigma}/\sqrt{N}$ versus the number of events in each run. The circles are G computed from the average of the maximum likelihood uncertainty estimates over ten runs with error bars given by the standard deviation of the mean. The horizontal dashed line is a weighted average of these G 's. In the asymptotic limit, G is independent of N .

As a further check, maximum likelihood parameter estimates were made for many runs each with a fixed number of events and with the decay parameters at their nominal values except D was set to -0.011. The D coefficient was given a value larger than $G/\sqrt{N}$ so that the sensitivity to D could be demonstrated with data runs of manageable size. The mean of the parameter estimates should reproduce the input values to the Monte Carlo.

The uncertainty in the mean should be the standard deviation of the mean (SDOM) given by $\hat{\sigma} = \sigma/\sqrt{n}$, where σ is the standard deviation of the parameter estimates for n runs. The results are summarized in Table 3-1. All the parameters are recovered within the uncertainty demonstrating the validity of both the Monte Carlo event generation and the maximum likelihood parameter estimation. The parameter uncertainties estimated from E_{ij} were compared to the standard deviation of each set of parameter estimates. In the asymptotic limit the maximum likelihood estimates of the parameters are normally distributed so they should equal the standard deviation for a sufficiently large sample of runs. With only 26 runs the standard deviation of the maximum likelihood estimates provides a rough check of the uncertainties estimated from E_{ij} .

Input Parameters	Mean of MLH Estimates	SDOM of the MLH Estimates	MLH Uncertainty Estimate ($E_{ij}^{1/2}$)	Standard Deviation of MLH Estimates
$a = -0.100$	-0.0991	0.0010	0.0040	0.005
$A = -0.118$	-0.1199	0.0012	0.0040	0.006
$B = 0.995$	0.9954	0.00040	0.0014	0.002
$D = -0.011$	-0.0105	0.0011	0.0052	0.006

Table 3-1: Behavior of the maximum likelihood (MLH) parameter estimates for 26 Monte Carlo data sets each with 200,000 decay events and identical input parameters. The input parameters (column 1) should be reproduced by the mean of the MLH estimates (column 2) within the uncertainty given by the SDOM (column 3). The uncertainty on the parameter estimate for each run is given by $(E_{ij})^{1/2}$ (column 4) which should be approximately the same as the standard deviation in the estimates for each run (column 5).

If some of the parameters in a fit are known *a priori*, then the contribution of their covariance to the variance of the remaining parameters can be removed [Jam80b]. The rows and columns in the second derivative matrix involving the known parameters are eliminated and the smaller resulting matrix is inverted to produce a new covariance matrix. The diagonal elements of the new smaller covariance matrix will be smaller reflecting the reduced uncertainty due to the information added by specifying some of the parameters. Although all of the decay coefficients were fit simultaneously, the contribution of the other parameters to the uncertainty in D is found by this reduction process to be

negligible. In other words the covariance of D with the other parameters is small.

As the result of these tests we conclude that for a runs with more than one thousand events the asymptotic limit applies. Since in the asymptotic limit the maximum likelihood parameter estimation is optimal, we will use the maximum likelihood uncertainty estimates as a bound to the sensitivity of any detector to the D coefficient. This bound, written in terms of the parameter G defined in Equation 3-6, was found, from the weighted mean shown in Figure 3-2, to be

$$G_{\text{Opt}} = 2.34 (1) \quad (3-7)$$

where “Opt” (for optimal) indicates that the result is for a maximum likelihood fit of the decay events directly. The uncertainty in G_{Opt} is due solely to finite statistics in the simulations. This case also represents the case of a tracking detector in the limit that it is capable of measuring the electron and proton momenta exactly.

Finally, we will define a figure of merit

$$M = \frac{1}{G^2} = \frac{1}{\sigma_D^2 N_{\text{decays}}} \quad (3-8)$$

which describes the sensitivity of any detector based on the uncertainty on the coefficient D , σ_D , achieved from a given number of decays, N_{decays} . In the usual case, in which the neutron decay rate in the detector is constant, the figure of merit M is inversely proportional to the running time required to reach a fixed uncertainty σ_D . The maximum likelihood bound G_{Opt} gives the upper bound on M

$$M_{\text{Opt}} = 0.183 (2) \quad (3-9)$$

representing an optimal detector. The figure of merit M can be broken down into several contributing factors

$$M = \varepsilon_{\text{coinc}} (KP)^2 \quad (3-10)$$

where ϵ_{coinc} is the fraction of decays which are detected, P is the neutron polarization (generally assumed to be 100% in this chapter), and K is a detector-dependent constant that can be calculated. The nature of K will be considered in Section 3.3.4.1. From this definition, K is a measure of the weight of each detected decay for extracting the D coefficient. In this chapter, unless otherwise noted, K is computed from M and ϵ_{coinc} using Equation 3-10. In the maximum likelihood fit ϵ_{coinc} and P are both unity so

$$K_{\text{Opt}} = \sqrt{M_{\text{Opt}}} = 0.428 (2) . \quad (3-11)$$

It will be shown later that the value of K for a detector depends primarily whether the detector can distinguish the sign of the triple product $\sigma_n \cdot p_p \times p_e$ for each decay.

3.2.2 Binning Calculations

3.2.2.1 General Procedure

In the integral method coincidences are simply counted in finite sized electron and proton detectors. Some information is necessarily lost using this detection scheme. The energies of the electron and proton are not used. The particles are not tracked so the particle directions are only approximately known and event reconstruction is not possible. Each counted decay is registered in a coincidence bin. Each bin is given a unique index corresponding to a particular pairing of an electron and proton detector. Since the integral of the PDF over the solid angles of coincidence pair and all electron energies is linear in D , we can express the number of counts in each coincidence bin as

$$N_i = N_{\text{decays}} (O_i + m_i D) \quad (3-12)$$

where N_{decays} is the total number of decays, O_i is the number of counts in the i^{th} coincidence bin due to the terms in the PDF not involving the D coefficient, and m_i is the slope of N_i as a function of D . Solving this expression for D we have

$$D_i = \frac{1}{m_i} \left(\frac{N_i}{N_{\text{decays}}} - O_i \right). \quad (3-13)$$

The values for O_i and m_i can be calculated (by Monte Carlo integration) for given values of the other decay coefficients, a , A , and B .¹ The statistical uncertainty in D_i is, from straight-forward error propagation in Equation 3-13,

$$\sigma_i = \left| \frac{\sqrt{N_i}}{m_i N_{\text{decays}}} \right| \quad (3-14)$$

and from the definition of M , Equation 3-8, the figure of merit for the i^{th} bin is

$$M_i = \frac{m_i^2 N_{\text{decays}}}{N_i}. \quad (3-15)$$

The overall statistical uncertainty in D for all N_{bin} coincidence bins in a detector is given by the uncertainty in the weighted average of D_i for each bin

$$\sigma = \left[\sum_{i=1}^{N_{\text{bin}}} (1/\sigma_i^2) \right]^{-1/2}. \quad (3-16)$$

From this expression and Equation 3-14, the figure of merit for the entire detector is simply the sum of the figures-of merit for each bin

$$M = \frac{1}{\sigma^2 N_{\text{decays}}} = \sum_{i=1}^{N_{\text{bin}}} M_i. \quad (3-17)$$

1. In principle, Equation 3-13 could be used to extract D from real experimental observations of the number of counts in the i^{th} coincidence bin N_i . This straight-forward approach is not feasible given the systematic difficulties it admits in practice. A more practical, though less general, data analysis method is presented in Section 3.3.2 that exploits symmetries of the detector to reduce possible systematic effects. The present method is nonetheless sufficient for calculating the statistical sensitivity of detectors that employ coincidence binning in our mathematical models.

Determining the figure of merit M for a detector requires that m_i and O_i be calculated for each coincidence bin in the detector. This was done using Monte Carlo decay simulation and Equation 3-12 by performing a linear fit to N_i across seven runs in which the coefficient D was fixed at the each of the values $\{-0.2, -0.1, -0.05, 0.0, 0.05, 0.1, 0.2\}$ while the other decay coefficients were held at their nominal values. Since the true value of $|D| \ll 1$, then from Equation 3-12 $N_i \approx O_i N_{\text{decays}}$ so $M_i \approx m_i^2 / O_i$. The wide range of D is needed to produce a significant change in N_i for runs with $N_{\text{decays}} = 1.6 \times 10^7$ decays. Maximum likelihood fits of runs with D set to ± 0.3 recover all the decay parameters to within the uncertainties. This and the simple linear form of the PDF in D justify the use of Monte Carlo events with the input value of D far outside the experimental limit on its magnitude of about 10^{-3} .

3.2.2.2 Spherical Detector Models

A simple spherical detector geometry was modeled to determine the significance of event reconstruction to a detector's sensitivity to D . In this model the proton and electron energy information is not used, the finite sized detectors simply counted coincidences. This is in contrast to the maximum likelihood fits above which used "reconstructed" decays.

The model consisted of a sphere divided into sections by lines of longitude and latitude. The north pole of the sphere was aligned with the neutron polarization direction. The longitude lines were equally spaced and the latitude lines were spaced so as to preserve equal areas in each of the sections. The latitude lines divide the sphere into "slices" and the longitude lines divide the sphere into "wedges". Each section of the sphere was modeled as acting simultaneously as an electron and proton detector. The spherical model thus shares the maximum likelihood fit's complete angular coverage, so ϵ_{coinc} is unity. The decaying neutrons were modeled as a point source at the center of the sphere, making the results independent of the radius of the spherical detector. The figure of merit and its

component factors are given for several segmentations of a sphere in Table 3-2. The sensitivity of a given model depends most strongly on the degree of longitudinal segmentation (number of wedges). There is also a weaker dependence on latitudinal segmentation (number of slices).

Segmentation		M	M_{Opt}/M	ϵ_{coinc}	K
Slices	Wedges				
8	8	0.0823(4)	2.22	1	0.287(1)
8	4	0.0441(3)	4.15	1	0.210(1)
8	2	0.00016(2)	1100	1	0.012(1)
8	1	0.00004(1)	4500	1	0.006(1)
4	8	0.0759(4)	2.41	1	0.276(1)
2	8	0.0669(4)	2.74	1	0.259(1)
1	8	0.0587(4)	3.12	1	0.242(1)
4	4	0.0410(3)	4.46	1	0.203(1)
Maximum Likelihood		0.183(2)	1.00	1	0.428(1)

Table 3-2: The figure of merit for the spherical detector models. The ratio M_{Opt}/M gives the increase in running time required for the spherical geometry to achieve the same sensitivity as the optimal case, given by M_{Opt} . The parameters from the maximum likelihood fit are given for comparison.

While none of these spherical detectors models would be practical to implement, they serve to demonstrate the relative value of event reconstruction, which requires particle tracking. The sensitivity lost by simply binning coincidences rather than measuring the momenta of the decay products and reconstructing the decay is not great, i.e. about a factor of two in M , even for a relatively coarsely divided spherical detector with 8 slices and 8 wedges. This integral type detector would take only about twice as long as an ideal tracking detector to reach the same sensitivity in D . Considering the tremendous complexity and costs avoided by opting for the integral method it does not appear that event reconstruction, even if it were possible, is warranted on the basis of sensitivity to D . In addition, a real tracking detector would have finite particle energy and momentum resolution, reducing its factor of two advantage in M . Since the spherical models all employ a point source of decaying neutrons, they effectively use the information carried by perfect

knowledge of the decay vertex. Models with distributed neutron sources will be considered in subsequent sections.

3.2.2.3 Cylindrical Detector Models

Presently a beam of neutrons from either a reactor or a spallation source is the only practical source for experiments studying neutron beta decay. Any real detector should be symmetric about the neutron beam axis with the ideal case being that of a cylindrical detector. In this section cylindrical detector models were used to determine three things: the value of decay vertex reconstruction, the importance of the detector's diameter relative to the beam's, and the optimal placement of the detector elements.

The amount of statistical strength in discerning D gained by reconstructing each decay vertex will be determined. In practice a long narrow beam of neutrons must be used, so information about the position of the decay along the length of the beam may be important even if the event is not reconstructed. While information about the transverse position of the vertex might also prove useful, reconstructing it's position to within a small fraction of a narrow beam's diameter would be very difficult in practice. The information content of the transverse position of the decay vertex is not determined directly. However, an upper limit on it's contribution to sensitivity to D is determined by comparing beams with zero diameter to larger beams. In the spherical models perfect information about the decay vertex was assumed, since they employed a point source of neutrons. In the cylindrical models, comparing point to extended neutron sources is effectively a comparison of the reconstructed versus nonreconstructed cases.

The positioning of the detector elements within a cylindrical geometry that optimizes the sensitivity to the D coefficient will be determined. Additional considerations of feasibility and systematic effects will be used to constrain some aspects of the detector geometries.

Several cylindrical geometries were modeled, all of which are derived from a sin-

gle basic geometry. In all cases the obvious cylindrical coordinate system (r, θ, z) will be used, with the z axis on the axis of the cylinder. In the basic geometry an array of detectors is modeled by a cylinder divided into “wedges” by equally-spaced longitude lines which run along its surface parallel to its axis. The cylinder is further subdivided into “slices” by equally-spaced latitude lines which encircle the cylinder. Each section of the cylinder is uniquely numbered and may act as a detector of electrons, protons, both, or neither depending on the particular geometry being modeled. The ratio of the cylinder’s diameter to its length was fixed at $1/5$ for two reasons. First, the ratio is loosely bounded below by the practical considerations of beam size and divergence from typical cold neutron sources. Second, and more importantly, this ratio is sufficiently small that relatively few decay products originating from the center of the cylinder will exit its ends so the results will not be dominated by these losses. The neutrons were assumed to be completely polarized along the cylinder’s axis (i.e. along $+\hat{z}$). In the cases where a point source was used it was located at the geometrical center of the cylinder. Likewise, the various line and cylindrical neutron sources were configured to lie on the cylinder’s axis centered about its midpoint. The figure of merit for each geometry was calculated by the method described in Section 3.2.2.1.

Vertex Reconstruction

To determine the relative merit of reconstructing the decay vertex for each event the figure of merit for a cylindrical geometry with a point source of neutrons was compared to that for several line sources, and several cylindrical sources. The detector modeled for these comparisons consisted of the 64 sections of cylinder divided by 8 longitude and 8 latitude lines. The results are summarized in Table 3-3. The generally high coincidence efficiency ($\epsilon_{\text{coinc}} > 92\%$) indicates that relatively few of the decay products ($< 8\%$) exit the ends of the cylinder so the results are not dominated by these losses.

Comparing the runs with line sources to the run with a point source, there is appar-

Neutron Source			M	M_{Opt}/M	ϵ_{coinc}	K
Geometry	Length	Diameter				
Point	0	0	0.0765(4)	2.39(2)	0.9634(2)	0.282(1)
Line	0.50	0	0.0674(4)	2.72(3)	0.9521(2)	0.266(1)
Line	0.75	0	0.0670(4)	2.73(3)	0.9245(2)	0.339(1)
Cylinder	0.50	0.15	0.0638(4)	2.87(3)	0.9520(2)	0.259(1)
Cylinder	0.50	0.30	0.0543(4)	3.37(3)	0.9521(2)	0.239(1)
Cylinder	0.50	0.60	0.0356(3)	5.14(5)	0.9525(2)	0.193(1)
Cylinder	0.50	0.90	0.0221(2)	8.3(1)	0.9532(2)	0.152(1)

Table 3-3: The figure of merit for the cylindrical detector models. The length of the sources is given as a fraction of the length of the detector. The diameter is given as a fraction of the detector's diameter.

ently little benefit in resolving the longitudinal position of the vertex in a cylindrical geometry. Comparing the point to the line source $1/2$ the detector length and correcting for the slight decrease in coincidence efficiency in the latter, only about an 11% decrease in M (or about 12% increase in running time) is attributable to the loss of the vertex information. On the other hand, the figure of merit does decrease as the diameter of the cylindrical neutron source is increased. This observation is easily understood. As the diameter of the source increases, it is increasingly possible for decays with opposite signs of the triple product $\sigma_n \cdot p_p \times p_e$ to enter a given coincidence pair of detectors. This possibility is shown in Figure 3-3. The effect of the triple product term is reduced when the detector fails to distinguish the sign of the triple product for each decay, thus reducing the detectors sensitivity to D .

Detector Sensitivity and Azimuthal Segmentation

As we have seen the relative insensitivity of the figure of merit M to the length of the neutron source implies that vertex reconstruction is of little value in resolving D . It also implies that the segmentation of a detector into slices along its length adds relatively little to its sensitivity. This will be shown later for several specific geometries (e.g. in Table 3-6). The primary benefit of segmentation apparently comes from the azimuthal

Figure 3-3: Demonstration of decays with opposite signs of the triple product $\sigma_n \cdot p_p \times p_e$ from a large diameter beam entering the same coincidence pair in the cylindrical detector. The detector is shown in cross-section with the neutron beam running along its axis polarized into the page. Each pair of arrows signifies the trajectories of the electron and proton produced by a neutron decay. Decays away from the center of the beam, like the one shown by the dashed lines, can enter a coincidence pair with the “wrong” sign of the triple product.

wedges. This fact can be exploited in constructing a model that compares the sensitivity of the coincidence detector pairs as a function of azimuthal angle. This will aid in determining the optimal placement of the detectors. Because of the cylindrical symmetry of the beam and detector, only relative azimuthal angles are relevant. Let the angles θ_e and θ_p be the angular coordinates of the intersection of the electron and proton trajectories respectively with the surface of the cylindrical detector. Define the relative angle $\Delta\theta$ to be difference of these angles, $\Delta\theta = \theta_e - \theta_p$. The definition of $\Delta\theta$ is shown schematically in Figure 3-4. Notice that $\Delta\theta$ is *not* the angle between the electron and proton momenta. That is, it is not $\gamma_{e-p} = \arccos(p_e \cdot p_p / (p_e p_p))$. For a line source at the center of the cylinder $\Delta\theta$ is equal to the projection of γ_{e-p} into the (r, θ) plane. This is not true for a

source which extends off the cylindrical detector's axis since $\Delta\theta$ is defined relative to the cylinder not the decay vertex.

Figure 3-4: The definition of $\Delta\theta$.

To compute the figure of merit M as a function of $\Delta\theta$ the Monte Carlo simulation was used to make histogram, with 100 bins, of decay events versus the intervals $\Delta\theta$. Note, this case is entirely equivalent to an infinitely long cylindrical detector divided into 100 azimuthal wedges but with no segmentation along its length. The usual procedure for determining the figure of merit M described in Section 3.2.2.1 was used. That is, the bins of these histograms were fit across several runs in which the input value of D in the simulation was varied. The figure of merit M_i for each bin was computed and the sum of the M_i over all the bins represents the overall figure of merit M . The results for a few cases are shown in Table 3-4. Notice the overall figure of merit M for this model with a line source is even higher than for the most sensitive spherical detector with a point source (see Table 3-2). This is due to the increased azimuthal segmentation of the present model. The differential distribution of M from a fit to the resulting histogram is the curve shown in

Figure 3-5. The most striking feature of this curve is that it is strongly peaked at large relative electron-proton angles ($\Delta\theta \approx 160^\circ$). This implies that arrangements that place the electron and proton detectors forming coincidence pairs at large relative azimuthal angles will be more sensitive to the D coefficient.

Neutron Source Diameter	M	M_{Opt}/M	ϵ_{coinc}	K
0	0.0947(5)	1.93(2)	1	0.308(1)
0.15	0.0807(4)	2.27(2)	1	0.284(1)
0.30	0.0611(4)	3.00(3)	1	0.247(1)
0.60	0.0353(3)	5.12(5)	1	0.188(1)
0.90	0.0205(2)	8.9(1)	1	0.143(1)

Table 3-4: Overall figure of merit for models with 100 $\Delta\theta$ bins as a function of source diameter. The diameter is given as a fraction of the detector's diameter.

Figure 3-5: The differential figure of merit versus the relative azimuthal angle between the electron and proton. The neutron source had zero diameter.

This is not what one would naively expect given the form of the triple product term

$$\sigma_n \cdot p_p \times p_e \propto \sin\gamma_{e-p} \quad (3-18)$$

where γ_{e-p} is the angle between the electron and proton momenta. Since it is proportional to $\sin\gamma_{e-p}$, one might suppose that the effect of the term would be greatest at $\gamma_{e-p} = 90^\circ$ where the sine function peaks rather than at larger angles. In the two most sensitive experiments measuring the D coefficient for the neutron, the electron and proton detectors were arranged at right angles to each other in an effort to exploit this. However, as Figure 3-5 demonstrates, M , and thereby the sensitivity to D , is actually skewed toward large relative electron-proton angles by the strong anti-correlation of the electron and proton momenta due to the limited phase space available in the decay. In fact, the entire PDF is similarly skewed toward large γ_{e-p} so there are few decays with γ_{e-p} near 90° compared to larger angles. This is demonstrated in Figure 3-6 which shows the coincidence detection efficiency ϵ_{coinc} as a function of $\Delta\theta$. This figure also shows K derived from M and ϵ_{coinc} using Equation 3-10.

Figure 3-6: The factors ϵ_{coinc} (solid line) and K (dashed line), which make up the figure of merit, versus the relative azimuthal angle between the electron and proton. The coincidence efficiency ϵ_{coinc} is shown as a differential distribution. The neutron source had zero diameter.

Practical Designs Based on the Cylindrical Model

Two factors must be considered in designing a practical detector. First, removing the requirement that each detector section be sensitive to both types of particles simultaneously is a significant simplification. In addition, the low energy (< 751 eV) of the recoil proton requires especially sensitive detectors and attendant acceleration electrodes, so reducing the area of these detectors is also beneficial. Second, since the absolute value of the D coefficient is known to be small ($< 10^{-3}$), it is important that systematic asymmetries in the detector be small lest they dominated the measurement (see Chapter 6). This is best accomplished by performing the experiment as a series of comparisons between numbers of counts in coincidence pairs of detectors which are sensitive to decays with opposite signs of the triple product ($\sigma_n \cdot p_p \times p_e$) but are otherwise identical. Reversing the direction of the neutron polarization during data taking, changes the sign of the triple product. The number of counts in a coincidence pair for one polarization can then be compared to the number for the opposite polarization. In addition, the detector design must incorporate the appropriate azimuthal symmetry. In the plane perpendicular to the neutron polarization, a symmetry axis must exist about each line which passes through the center of the neutron source and the center of the face of each detector section. An example of this is shown in Figure 3-7. By construction, each coincidence pair is then accompanied by another pairing that has equal and opposite sensitivity to the triple-product. For example, in Figure 3-7 the number of coincidences between electron detector e_1 and proton detector p_2 can be compared to the number of coincidences in the pair e_1 - p_3 . Similarly, the number in e_1 - p_1 can be compared to the number in e_1 - p_4 . For a given polarization, the number of counts in these complementary pairs can be compared. The method by which these detector symmetries and the neutron polarization reversal are exploited is described in detail in a later section.

In an effort to determine the optimal positions for the individual electron and proton detectors, several cylindrical geometries were simulated. Each of these is derived from

Figure 3-7: Demonstration of the detector symmetry required. As an example, Geometry B, which has the required symmetry, is shown here in cross-section. The dashed line is one of the required axes symmetry. The n^{th} electron detector is designated " e_n " and the m^{th} proton detector " p_m ".

the basic cylindrical geometry described above. In the derived geometries detector sections were merged, eliminated, or were made sensitive to only one of the two particles from the neutron decay. Schematic drawings of these geometries are shown in Section A.2 of Appendix A. Not all of the geometries described in Appendix A adhere to the two practicality considerations described above although the geometries discussed in this section do. The exception is Geometry A, which is given only as a reference.

The results for four particularly interesting geometries are given here in Table 3-5. The geometries are compared for a line neutron source whose length was $1/2$ of the detector's length. Geometry A₁ is simply the basic cylindrical geometry described above in which the cylinder has been divided by eight latitude and eight longitude lines into sixty-four sections each of which acts as both an electron and proton detector. It uses the maximal information for this level of segmentation of a cylinder. It is given as a reference. The wedges in Geometry B₁ alternate between acting as electron and proton detectors so it covers $(1/2)^2 = 1/4$ the coincidence solid angle that Geometry A₁ does. Half of the electron-proton coincidence pairs are at a large (135° average) relative azimuthal angles where the plot of $dM/d\Delta\theta$ in Figure 3-5 shows greater sensitivity to D lies. Very little of this

detector's sensitivity comes from the coincidence pairs that are 22.5° apart. Geometry C_1 simulates the traditional detector geometries which placed the electron and proton detectors at right angles to each other in the plane perpendicular to the neutron polarization. The electron and proton detectors are the same size as those used in Geometry B_1 though half as many are used. A pair of detector assemblies with the C_1 geometry would have the same number and size detector sections as Geometry B_1 . However, even the pair would still be about six times less sensitive than Geometry B_1 . This demonstrates the great advantage in placing the electron and proton detectors at large relative angles (135°) over the right-angle placement used in previous measurements. Geometry D_1 also employs right angle detector arrangement. It represents an attempt to improve the traditional right-angle geometry, modeled by Geometry C_1 , by increasing the size of the detector sections so that it accepts decays with larger relative electron-proton azimuthal angle (up to 135°). It's detectors cover the same amount of solid angle that Geometry B_1 's do. Despite the improvements, this detector remains less sensitive than Geometry B_1 . Its reduced segmentation also make it more susceptible to systematic effects produced by nonuniform detection efficiencies. These effects will be discussed in detail in Chapter 6.

Geometry Name	Electron		Proton		M	ϵ_{coinc}	K
	Slices	Wedges	Slices	Wedges			
A_1	8	8	8	8	0.0674(4)	0.9521(2)	0.266(1)
B_1	8	4	8	4	0.0281(2)	0.2068(1)	0.369(1)
C_1	8	2	8	2	0.0027(1)	0.0260(1)	0.323(5)
D_1	8	2	8	2	0.0183(2)	0.1555(1)	0.343(2)

Table 3-5: The figure of merit for several cylindrical geometries. The neutron source was a line 1/2 the length of the detector. Drawings of the geometries are given in Appendix A. The degree of azimuthal and longitudinal segmentation is indicated by the number of slices and wedges into which the electron and proton detection has been divided.

It is interesting to note that K varies by only 14% among the B, C, and D geometries while ϵ_{coinc} varies by up to a factor of about eight. The difference in K between, for example, Geometries B_1 and C_1 , accounts for only 12% of the factor of ten difference in

the figure of merit between those geometries. Recall that K is interpreted as the measure of the value of each detected decay in determining the D coefficient. These results indicate that the sensitivity of various geometries is primarily attributable to their coincidence efficiency ϵ_{coinc} rather than the value of each decay detected. One concludes that for detectors that are able to distinguish the sign of the triple correlation for each decay, the more sensitive detectors place their component coincidence detector pairs where the largest number of coincidences will be detected, namely at large relative electron-proton azimuthal angles.

The different geometries depend differently on the diameter of the neutron source. See Figure 3-8. In particular, the sensitivity of Geometry B₁ decreases rapidly as the diameter of the neutron source is increased. At the same time the sensitivity of Geometry C₁ actually increases slightly with the diameter of the neutron source due its coincidence efficiency ϵ_{coinc} increasing. This behavior is consistent with the explanation accompanying Figure 3-3. In Geometry C₁ the handedness of the decays cannot be confused unless they occur outside a diameter that is a sizable fraction (about 1/3) of the detector's diameter. On the other hand, Geometry B₁ is far more susceptible to this effect. The handedness of even some decays from the center of the beam (radially) can be confused. We conclude that the superiority of detector geometries like Geometry B₁, which accept electron-proton coincidences with large relative azimuthal angle, relies on the neutron source diameter being small compared to the detector's diameter. While, as will be shown below, this requirement is acceptable in an neutron beta decay experiment, triple correlation experiments using nuclei have different requirements (as well as different kinematics). An experiment measuring the D coefficient in ^{19}Ne beta decay used a gaseous source with the same effective diameter as the detector [Hal84]. Presumably because of a dependence of detector sensitivity on the relative source diameter analogous to that shown here, it used a detector geometry whose azimuthal segmentation more closely resembles the right angle geometry of Geometry D₁ than the large angle Geometry B₁.

Figure 3-8: The figure of merit M as a function of the neutron source diameter for three different cylindrical detector geometries. The neutron source was a beam 1/2 the length of the detectors with a diameter given as a fraction of the detectors' diameter.

These same four geometries were modified to remove the slice segmentation. Geometries in which this has been done are distinguished by the subscript "2" (i.e. Geometry A_1 differs from Geometry A_2 in that the latter is not divided into slices.). The results of the simulations for these geometries are given in Table 3-6. These results, when compared to those in Table 3-5, show that segmentation of the cylindrical detectors into slices has little effect on the sensitivity of the detector when an extended neutron source is used. The figure of merit M is reduced by only about 10% when the segmentation into slices is eliminated. This segmentation is nonetheless useful for reducing the systematic effect caused by nonuniform detection efficiencies discussed in Chapter 6.

In practice the length of the neutron source is comparable to or longer than the detector. To simulate this the four detector models described above were modified so that only the center two slices (the central 25%) of the detector were sensitive. The resulting geometries are distinguished by the subscript "3". The results, shown in Table 3-7, show that the relative merit of the basic models is not significantly affected by this change

Geometry Name	Electron		Proton		M	ϵ_{coinc}	K
	Slices	Wedges	Slices	Wedges			
A ₂	1	8	1	8	0.0602(4)	0.9521(2)	0.251(1)
B ₂	1	4	1	4	0.0255(2)	0.2068(1)	0.351(2)
C ₂	1	2	1	2	0.0024(1)	0.0260(1)	0.302(5)
D ₂	1	2	1	2	0.0166(2)	0.1555(1)	0.326(2)

Table 3-6: The figure of merit for several cylindrical geometries with no segmentation along their length. The neutron source was a line 1/2 the length of the detector. The degree of azimuthal and longitudinal segmentation is indicated by the number of slices and wedges into which the electron and proton detection has been divided.

toward a more realistic beam and detector. In this case the Geometry B₃ is still about twelve times more sensitive than the right-angle geometry, C₃. The neutrons were modeled by a line source which was twice the length of the shortened detector. The overall figure of merit for these geometries is much lower due to the reduction in the coincidence efficiency ϵ_{coinc} caused by the relatively long beam.

Geometry Name	Electron		Proton		M	ϵ_{coinc}	K
	Slices	Wedges	Slices	Wedges			
A ₃	2	8	2	8	0.0199(2)	0.2403(1)	0.288(2)
B ₃	2	4	2	4	0.0085(3)	0.0503(1)	0.412(3)
C ₃	2	2	2	2	0.00069(4)	0.00537(2)	0.36(1)
D ₃	2	2	2	2	0.0054(1)	0.0359(1)	0.387(4)

Table 3-7: The figure of merit for several cylindrical geometries which include only the center two longitudinal slices. The neutron source was a line twice the length of the two-slice detector. The degree of azimuthal and longitudinal segmentation is indicated by the number of slices and wedges into which the electron and proton detection has been divided.

3.3 Octagonal Detector Model

3.3.1 Design of the Model

A realistic detector model has been designed based on the simulations of the previous section. The model is based on the Geometry B cylindrical models which have separated electron and proton detection. This simplifies construction and provides the four-fold

symmetry required to reduce systematic effects. The symmetry of the detector geometry is exploited by the analysis method given in Section 3.3.2 below. The detector modeled consists of flat detectors panels arranged in an alternating octagonal geometry and is shown schematically in Figure 3-9. The electron and proton detectors are held at the relatively large average azimuthal angle of 135° which, as was shown in the previous section, enhances the detector's sensitivity to the D coefficient. Candidate apparatus for the proton and electron detector panels will be discussed in Chapter 4.

This detector accepts the use of a long beam of longitudinally polarized neutrons as its source. Existing high-flux neutron facilities, like the ILL and CNRF, can produce cold (low energy) neutron beams with circular cross-sections with a maximum diameter of about 6 cm. A large beam is required to produce sufficient decay rate within the detector to achieve sensitivity to the D coefficient superior to previous experiments. The diameter of the neutron beam sets the size of the required detector.

Figure 3-9: Cross-sectional schematic diagram of the octagonal geometry showing the detector numbering scheme. The neutron beam direction and its polarization are into the page. Proton detectors are designated p_n and electron detectors e_n where “n” is the detector number. An example decay is shown producing an coincidence in detectors e_1 and p_2 .

The neutron beam and the electron detector panels are 50 cm long and the proton detector panels are 24 cm long. The radial distance between the centers of the inner surfaces of the detector panels and the center of the beam is 10 cm. For a 6 cm diameter beam, the beam's diameter is 0.30 of the detector's diameter. This factor compares to the relative beam diameter used in the cylindrical detector models, e.g. in Table 3-3.

Some position resolution along the lengths of the detector panels (i.e. segmentation into slices) is assumed. The details of how this is achieved will be discussed in Chapter 4 for the specific detector apparatus. While in Section 3.2.2.3 it was demonstrated that segmentation of the detector along its length adds little to its sensitivity, there are systematic effects, caused by nonuniform detection efficiencies, that are mitigated by this segmentation. The systematic effects will be discussed in Chapter 6. For purposes of determining the sensitivity of this detector model, the proton and electron detectors panels are segmented into six slices.

The electron detector panels were chosen to be longer than the proton detector panels because the low kinetic energy of the protons requires that they be electrostatically accelerated before detection. Consequently, the proton detectors are generally more difficult to construct with sufficiently uniform efficiency as a function of position in addition to being more expensive per unit length. Making the electron detector panels longer enhances the probability that the electron produced with a detected proton will also be detected, thus the coincidence efficiency is enhanced.

There is an added benefit to making electron detector panels long compared to the proton detector panels. The distribution of protons, detected in coincidence with electrons, is made more uniform as a function of position along the length of the proton detector panels. The enhanced uniformity results from the reduction of end effects. For electron detector panels of finite length, the coincidence efficiency depends on their length. If the electron panels (and the beam) were made infinitely long then the distribution of protons as a function of position along the length of proton detectors would become completely

uniform. This situation is approximated by making the electron detector panels long compared to the proton detector panels. Conversely, the short proton detector panels reduce the importance of the ends of the electron panels since rarely will a proton, produced with an electron that is detected near the end of an electron detector panel, be detected in one of the shorter proton detector panels. Nor will many detected coincidences result from decays occurring near an end of the beam, i.e. where it enters or exits the decay region, since the ends are far from the short proton detector panel. Explicit calculations of distribution of protons along the length of the proton detector panels and its importance for systematic effects is discussed in Section 6.3.3.2 of Chapter 6.

In the previous models we were concerned only with sensitivity to the D coefficient. This approach was sufficient for the simulations where there are no systematic asymmetries in the detectors modeled. In the next section, the sensitivity of the octagonal detector design to the D coefficient will be computed in the context of a data analysis method which also serves to eliminate or reduce systematic effects arising from deviations from the perfectly symmetric ideal. The method presented is suitable for use with experimental data as well as in simulations.

3.3.2 The Slice-by-Slice Ratio Analysis Method

The foundation of the analysis method presented here is a ratio of numbers of electron-proton coincidences. All of the coincidences to which the triple correlation term contributes positively are in the numerator while those to which it contributes with the opposite sign are in the denominator. This ratio reduces systematic effects because the contribution of the triple correlation term to the number of coincidences falling in a particular detector pair changes sign with the reversal of the neutron polarization. Furthermore, for a given polarization, each proton and electron detector individually has two complementary detectors with which it makes coincidence pairs sensitive to opposite signs of the triple correlation. This is possible due to the four-fold symmetry of the detector. As a

result, the overall detector efficiencies and solid angles cancel for each detector for each polarization direction.

The analysis method described here is analogous to that employed by Steinberg, *et al.* [Ste76] but differs in that there are twice as many electron and proton detectors arranged azimuthally about the beam and they are segmented longitudinally into six "slices". (See Figure 3-10.) While the former difference necessitates only a trivial extension of the analysis, there is no simple way of incorporating the longitudinal segmentation besides treating each slice essentially as a separate experiment. That is, one can make the described ratio for each longitudinal slice separately, disregarding any coincidences in which the proton is detected in one slice but the electron in another. Longitudinal segmentation is advantageous because one of the systematic effects, which will be discussed in Section 6.3.3, is thereby reduced.

Dividing the detector into slices has one obvious drawback. The probability of a decay resulting in a useful coincidence is reduced, since decays in which the electron and proton are detected in different slices are discarded. This is mitigated by allowing the definition of the slices to include overlapping longitudinal sections of the electron detector panels. The physical lengths of the electron and proton detection within a single slice are different. The proton detector panel is divided longitudinally into six pairs of focussing cells each of which contributes to the proton detection for a different slice. A pair of focussing cells is 4 cm long by 8 cm wide. The electron detector for a slice is 30 cm long by 8 cm wide with its center defined relative to the center of the pair of proton focussing cells in the slice.

Since each longitudinal slice is treated equivalently, it will suffice to describe the method for analyzing data from a single slice. A ratio is formed for each slice and is used to derive a separate measurement of D . The measurements for all six slices are then combined to determine the final result. If we label each particle detector within a longitudinal slice according to the scheme shown in Figure 3-9, the ratio of numbers of coincidences

Figure 3-10: Illustration of a detector "slice". The gray regions are those portions of the detector panels assigned to a particular slice. For simplicity only one each of the proton detector and electron detector panels is shown. The neutron beam runs along the z axis.

takes the following form:

$$R = \frac{N_{e_1 p_3}^{\uparrow} N_{e_2 p_4}^{\uparrow} N_{e_3 p_1}^{\uparrow} N_{e_4 p_2}^{\uparrow} N_{e_1 p_2}^{\downarrow} N_{e_2 p_3}^{\downarrow} N_{e_3 p_4}^{\downarrow} N_{e_4 p_1}^{\downarrow}}{N_{e_1 p_2}^{\uparrow} N_{e_2 p_3}^{\uparrow} N_{e_3 p_4}^{\uparrow} N_{e_4 p_1}^{\uparrow} N_{e_1 p_3}^{\downarrow} N_{e_2 p_4}^{\downarrow} N_{e_3 p_1}^{\downarrow} N_{e_4 p_2}^{\downarrow}} \quad (3-19)$$

where $N_{e_i p_j}^{\sigma}$ is the number of coincidences detected between the e_i electron detector and the p_j proton detector with $\sigma = \uparrow(\downarrow)$ indicating the neutron polarization is parallel (anti-parallel) to the z-axis. Only the coincidence pairs in which the electron and proton detectors are at large relative angles (e.g. e_1 and p_2) are included since there are few coincidences in the pairs that are adjacent (e.g. e_1 and p_1). The $N_{e_i p_j}^{\sigma}$ can be written as

$$N_{e_i p_j}^{\sigma} = C^{\sigma} \Omega_{e_i}^{\sigma} \Omega_{p_j}^{\sigma} \epsilon_{e_i}^{\sigma} \epsilon_{p_j}^{\sigma} (1 \pm \bar{K}PD) \quad (3-20)$$

where C^σ is proportional to the beam flux for a given neutron polarization, the Ω 's are the solid angles subtended by the detectors, the ϵ 's are the overall efficiencies of the detectors, P is the neutron polarization, $\tilde{K}$ is an instrumental constant dependent on the geometry which will be discussed in Section 3.3.4, and D is the coefficient of the triple correlation. A more complete treatment is given in Section 6.3.3. The N_{ϵ, P_j}^σ in the analysis ratio have been arranged so that the sign of the $\tilde{K}PD$ term is $+$ ($-$) for all of the factors in the numerator (denominator). The constant $\tilde{K}$ will be shown later to be equivalent to K defined by Equation 3-10.

3.3.3 Determination of the Triple Correlation

We can extract D using Equations 3-19 and 3-20 by noting that all of the C 's, Ω 's, and ϵ 's cancel in the ratio. Since $\tilde{K}$ is assumed, by the symmetry of the detector, to be the same for each coincidence pair, the ratio reduces to

$$R = \frac{(1 + \tilde{K}PD)^8}{(1 - \tilde{K}PD)^8}. \quad (3-21)$$

Solving this for D and estimating the uncertainty in D by straightforward error propagation,

$$D = \frac{1}{\tilde{K}P} \frac{(R^{1/8} - 1)}{(R^{1/8} + 1)}, \quad (3-22)$$

$$\sigma_D = \left\{ \left(\frac{1}{4\tilde{K}P} \frac{R^{1/8}}{(R^{1/8} + 1)^2} \frac{\sigma_R}{R} \right)^2 + \left(D \frac{\sigma_{\tilde{K}}}{\tilde{K}} \right)^2 + \left(D \frac{\sigma_P}{P} \right)^2 \right\}^{1/2} \quad (3-23)$$

The overall value of D and σ_D for the entire detector can be determined from a weighted average of the results for each slice. Notice that the terms containing the uncertainties $\sigma_{\tilde{K}}$

and σ_p are proportional to D , which is known to be small. These uncertainties are thus relatively unimportant.

Assuming that the only contribution to σ_R in Equation 3-23 is statistical, σ_D can be simplified. From previous measurements it is known that D is small so from Equation 3-21 $R \approx 1$. Accordingly, all the $N_{e\mu_j}^\sigma$ of Equation 3-19 are approximately equal so $N_{e\mu_j}^\sigma = (1/16) N_T$, where N_T is the sum over all the $N_{e\mu_j}^\sigma$. Using Poisson uncertainties for the $N_{e\mu_j}^\sigma$, the statistical contribution to σ_D becomes

$$\sigma_D(\text{stat.}) = \frac{1}{\tilde{K}P} \frac{1}{\sqrt{N_T}}, \quad (3-24)$$

exhibiting the usual inverse square root dependence on the number of counts.

The equivalence between $\tilde{K}$ and K in used Section 3.2 can now be established. Combining the definition of the figure of merit M Equation 3-8 and its components Equation 3-10

$$\frac{1}{\sigma_D^2 N_{\text{decays}}} = \epsilon_{\text{coinc}} (KP)^2. \quad (3-25)$$

Solving Equation 3-25 for σ_D yields

$$\sigma_D = \frac{1}{KP} \frac{1}{\sqrt{\epsilon_{\text{coinc}} N_{\text{decays}}}} = \frac{1}{KP} \frac{1}{\sqrt{N_T}} \quad (3-26)$$

where the definition of ϵ_{coinc} has been used to make the substitution $N_T = \epsilon_{\text{coinc}} N_{\text{decays}}$ in the last step. The equivalence of $\tilde{K}$ and K is established by inspection of Equations 3-24 and 3-26 and recalling that σ_D in Section 3.2 referred to the statistical uncertainty in D . Since $\tilde{K}$ and K are equivalent, the tilde will be dropped on the former.

3.3.4 The Instrumental Constant K

3.3.4.1 The Nature of K

Equation 3-20 hides the effects of all of the decay coefficients except D . The instrumental constant K can be understood by considering the neutron decay probability distribution function written in terms of the neutron polarization $\sigma_n = \langle J \rangle / J$ and the momenta of the electron and neutrino, given by Equation 2-10. The neutron, whose kinetic energy is of the order 0.025 eV, is assumed to be at rest, so, using energy and momentum conservation, the neutrino's momentum p_ν and energy E_ν can be replaced by those of the recoil proton. After integrating the resulting distribution over the solid angles of a particular coincidence pair of detectors, the number of coincidences in that pair is

$$N = \left(\sum_{i = \{1, a, A, B\}} I_i \right) + D\gamma_D \quad (3-27)$$

where the I_i are the integrals over the first four terms in Equation 2-10, and γ_D is the integral over the term D multiplies. Defining K as

$$K = \frac{\gamma_D}{\sum_{i = \{1, a, A, B\}} I_i} \quad (3-28)$$

then N can be rewritten to have the form used in Equation 3-20 with $P = 1$

$$N = \left(\sum_{i = \{1, a, A, B\}} I_i \right) (1 + KD) . \quad (3-29)$$

Thus KD is essentially the ratio of the number of counts thrown into a coincidence pair by the triple correlation term to the number of counts thrown into it by all of the other terms.

3.3.4.2 Determination of K

In Section 3-2 the constant K was determined for each geometry from M and ϵ_{coinc} using Equation 3-10. In this section, a more direct approach is used. The constant K

was determined by generating sets of simulated decays with several input values of D and calculating K from an equation similar to Equation 3-22. Since reversing the polarization of the neutrons serves only to eliminate systematic effects that do not exist in the Monte Carlo, only the $N_{e,p_j}^\uparrow$ are used in the ratio equation. Consequently, modified expressions for D and σ_D

$$KD = \frac{(R^{1/4} - 1)}{(R^{1/4} + 1)}, \quad (3-30)$$

$$\sigma_D = \left| \frac{1}{2K} \frac{R^{1/4}}{(R^{1/4} + 1)^2} \frac{\sigma_R}{R} \right| \quad (3-31)$$

are used. In these equations $P = 1$ and the dependence of σ_D on σ_K and σ_P has been dropped since these are zero in the Monte Carlo. By symmetry, K is the same for each coincidence detector pair within a slice. Since the simulated beam has finite length, K is not necessarily the same for the different detector slices which are distributed along the beam's length.²

Within the calculation's statistical uncertainty, the values of KD were observed to be the same for all six detector slices. For simplicity, a weighted average of the KD 's over all slices was computed. The average KD 's were fit as a function of the input values of D to determine K . The linear fit was found to be good in all cases. The results obtained for several beam diameters are given in Table 3-8.

K was also calculated in this way for the case of a diverging beam. The beam was

2. The value of K for a given beam diameter was determined by generating seven runs of 10^7 decays each in which the input D was varied over the values $D = \{-0.2, -0.1, -0.01, 0.0, 0.01, 0.1, 0.2\}$ while the other decay coefficients were held at their nominal values: $a = -0.1, A = -0.118, B = 0.995$. Events in which the electron kinetic energy was below 100 keV were cut. The positions of the decaying neutrons were distributed uniformly within a 50 cm long cylindrical beam. The values of KD for each slice were computed for each run from Equation 3-30.

Beam Diameter (cm)	Calculated K
0.02	0.444(9)
3.0	0.426(4)
4.2	0.409(4)
6.0	0.372(4)
8.0	0.319(4)

Table 3-8: Values of the instrumental constant K calculated by Monte Carlo simulation with a 50 cm long uniformly populated cylindrical beam for several beam diameters.

a uniformly populated section of a cone with beginning diameter 5 cm and ending diameter 7 cm. The resulting value $K = 0.378(4)$ is not significantly different from the cylindrical case with a 6 cm diameter (the cone's average diameter). The neutron distribution in an actual beam also diverges with distance from the collimation. However, unlike the uniformly populated conical beam simulated, the neutron population is lower at larger radii. Thus an actual neutron beam would fall between the extremes of a uniformly populated cylinder and a uniformly populated section of a cone.

While the Monte Carlo calculation of K is required to determine the magnitude of D from the experimental data, the uncertainty in the determination of K is relatively unimportant. K , like the polarization P , always appears as a factor of D so it alone cannot cause the observation of a false nonzero triple correlation. We are confident that K can be calculated with an accuracy of 5%. From Equation 3-23 we see that for a true value of D as high 10^{-3} (approximately the current experimental limit on D) the contribution σ_K makes to σ_D would be comparable to a statistical uncertainty $\sigma_D(\text{stat.}) = 5 \times 10^{-5}$. If the contribution of final state effects to D were comparable to or larger than the statistical uncertainty, then a precise determination of K would be required to distinguish this effect (presuming its size were sufficiently well known) from a time-reversal violating contribution.

3.3.4.3 Dependence of K on Beam Diameter

The sensitivity of the detector depends on the diameter of the neutron beam. As the beam diameter increases, there is a corresponding decrease in K that reflects a reduced sensitivity of the detector to D . The origin of this phenomenon was considered in the discussion leading up to Figure 3-3. From the results in Table 3-8, K is observed to decrease with the beam's cross-sectional area. The values of K as a function of beam radius are an excellent fit to

$$K = \alpha - \beta r^2 \quad (3-32)$$

where r is the beam radius, $\alpha = 0.444$, and $\beta = 7.8 \times 10^{-3} \text{ cm}^{-2}$.

3.3.5 Detector Sensitivity

The sensitivity of the detector to the D coefficient depends on both the efficiency for detecting an electron-proton coincidence from a decay in the neutron beam, ϵ_{coinc} , and the instrumental constant K . The sensitivity to D in terms of a one sigma statistical uncertainty is given by Equation 3-24. The coincidence detection efficiency is easily included in this equation by rewriting the total number of coincidences counted in the detector in terms of (1) R , the decay rate in the volume of the beam where the detector has sensitivity, (2) ϵ_{coinc} , the probability of detecting one coincidence per decay in that volume of the beam, and (3) t , the run time. This gives $N_T = tR\epsilon_{\text{coinc}}$ and the statistical uncertainty becomes

$$\sigma_D = \frac{1}{KP\sqrt{tR\epsilon_{\text{coinc}}}} \quad (3-33)$$

The weak dependence of ϵ_{coinc} on the beam radius has been determined from the Monte Carlo simulation to have the form

$$\epsilon_{\text{coinc}} = a + br + cr^2 \quad (3-34)$$

where $a = 0.0441$, $b = 8.2 \times 10^{-4} \text{ cm}^{-1}$, and $c = 4.1 \times 10^{-4} \text{ cm}^{-2}$.

The figure of merit M defined in Section 3.2 was purposefully constructed to be independent of the number of decays N_{decays} so different detector geometries could be compared without regard to the intensity of the neutron source. M can be modified slightly to make it a more appropriate measure of the sensitivity of the entire experiment, including the beam. We define a new figure of merit

$$\tilde{M} = \frac{1}{\sigma_D^2} = MN_{\text{decays}} = MtR \quad (3-35)$$

which depends on the source strength and running time.

An optimum beam radius can be derived from Equations 3-32, 3-34, and 3-35 by assuming a neutron beam of uniform density across its circular cross-section so that the decay rate R is proportional to the square of the beam radius. The figure of merit $\tilde{M}$ as a function of beam radius is shown in Figure 3-11. However, this optimization neglects the limits on beam radius imposed by systematic effects, beam produced backgrounds, and neutron guide size. Because of these considerations, a beam with a 3 cm average radius is used, smaller than that prescribed by the maximum of this curve (about 4.6 cm), but yielding only a slightly larger relative uncertainty ($\sigma_D / (\sigma_D)_{\min}$) (21% over optimal).

3.3.6 Conclusion

The octagonal detector has a geometry superior to those used in previous neutron D measurements because the large (135°) angle between electron and proton detectors enhances its sensitivity. At the same time, its symmetry allows for the cancellation of most systematic effects. The statistical sensitivity to D can be estimated using the projections shown in Table 2-4 for the ILL and the upgraded CNRF research reactors. Assuming a 60 days of continuous running with the octagonal detector, about an order of magnitude improvement in sensitivity over previous experiments

Figure 3-11: Figure of merit $\tilde{M}$ versus beam radius. The density of neutrons in the beam is ρ , the running time is t , and neutron lifetime τ_n .

$$\sigma_D (\text{stat.})_{\text{CNRf}} \approx 3 \times 10^{-4} \quad (3-36)$$

could be achieved at the CNRF. This could be further improved by running at the ILL (where the most sensitive previous neutron D experiment was run). There a sensitivity of

$$\sigma_D (\text{stat.})_{\text{ILL}} \approx 5 \times 10^{-5} \quad (3-37)$$

could be achieved.

3.4 Conclusion

The purpose of this chapter was to investigate the factors that influence a detector's sensitivity to the D coefficient. Two general types of detectors, tracking and integral, were considered. The former allows for the kinematic reconstruction of each decay, however, this was found to contribute only modestly to improving the detector's sensitivity (about a factor $1/\sqrt{2}$ improvement in σ_D). Likewise, in more practical cylindrical geom-

etries, reconstruction of the decay vertex appears to produce almost no benefit in sensitivity. While kinematic reconstruction is apparently not very important, the positioning the integral electron and proton detectors is. Contrary to appearances, the greatest sensitivity to D comes at large relative electron-proton angles. Several cylindrical geometries were considered with the best placing the two types of detectors 135° apart. This design was developed into the octagonal detector design. It promises to allow improvement in the sensitivity to D over previous experiments by an order of magnitude or more.

2 of 3

Chapter 4: Apparatus

4.1 Design Considerations

Barring the development of new facilities to produce higher unpolarized neutron densities, improvements in experimental sensitivity to D can only be achieved with improved coincidence detection efficiencies and longer running time. The octagonal detector system developed and modeled in Section 3.3 incorporates improvements to ϵ_{coinc} . At least an order of magnitude improvement in statistical sensitivity over recent experiments measuring D for the neutron can be achieved using the same intensity neutron beams. Since no fundamental design improvements to the neutron source are required for the success of an experiment using the octagonal detector, the beam production and transport will be given little attention here.

4.1.1 Polarization Production and Transport

The production and transport of polarized neutrons is a well-established technology in which only modest gains over existing techniques can be expected. A possible exception to this is the ^3He neutron polarizer, described in Section 2.3.1, that is currently under development. Currently, supermirror polarizers are used to produce highly polarized beams with high efficiency. Once produced, the beam polarization is maintained by a weak (≈ 3 gauss) magnetic guide field produced by conventional magnet coils. Its direction can be manipulated by introducing gradual turns to the guide field that the neutron polarization will follow adiabatically. Finally, the direction of the polarization can be reversed with high efficiency through the use of commonly used current sheet spin flippers. These devices are used to make the neutron polarization anti-parallel to the guide field by rapid (nonadiabatic) transmission of the polarization between regions with opposing magnetic fields. Alternatively, spin flippers based on the NMR-technique of adiabatic

fast passage [Wei89] may prove superior in reducing polarization nonuniformity since they can be used directly on longitudinally polarized beams.

4.1.2 Background

Some consideration must be given to backgrounds produced in the detector system by neutrons from the beam. The delayed coincidence between the proton and electron provides a powerful tool for rejecting backgrounds. However, the beam-associated γ backgrounds produced by neutron capture in material near the detector can be considerable. The best means of reducing this background is by preventing neutrons from striking material near the detectors. This is accomplished by keeping sufficient distance between the beam and the detectors and detector support structure, by transporting the neutron beam in a vacuum, and by moving any material the beam must pass through, like vacuum windows, as far from the detection region as possible.

4.1.3 Detector

Implementation of the octagonal detector design requires four electron sensitive detectors panels and four proton sensitive detector panels arranged in an alternating octagonal geometry. There are several requirements for these detector panels. (1) The efficiency of the individual detector panels must be high in order to achieve sufficient counting statistics, even with the superior ϵ_{coinc} of the octagonal design. (2) The apertures defining the entrances to the detector panels must have the geometric symmetry called for by the octagonal design to facilitate the cancellation of detector efficiencies and solid angles as described in Section 3.3.2. This symmetry requirement is satisfied by flat rectangular apertures like those modeled in the octagonal detector simulation. If the actual detector panels deviate from this ideal, a false effect, due to nonuniform detection efficiency, can arise. The allowable size of such deviations will be considered in Chapter 6. (3) The octagonal decay region must be free of electric fields so the trajectories of the low energy recoil pro-

tons are not disturbed and they are allowed to drift from the decay region to the proton detectors. Electric fields in the drift region would complicate the coincidence acceptance of the detector and possibly introduce systematic asymmetries.

4.2 Proton Detection

Prototype electron and proton detector panels were constructed for a detector test run with a cold neutron beam at the NIST CNRF. The run was carried out from March through June of 1992.

4.2.1 Introduction

The low kinetic energy of the recoil protons makes their detection the most technically challenging aspect of the detector. The proton energy is at most 0.75 keV so they must be accelerated in order to be detected in conventional charged particle detectors. In addition, each of the proton detector panels must cover a relatively large area, about 8 cm by 24 cm. The size of the false asymmetry due to nonuniform detection efficiencies is reduced by segmentation of the proton detection. Thus some proton position information is useful.

Each of the four proton detector panels consists of an array of 12 rectangular focussing cells each backed by a PIN diode solid state detector. Each focussing cell consists of a grounded box with the top covered by a grounded wire mesh (94% open) through which the decay protons enter. In this chapter, the coordinates used for describing the proton detector panel will be (u, v, w) . This coordinate system is the same as that of Figure 3-10 with a rotation by $+135^\circ$ about the z -axis so $\hat{u}$ is normal and $\hat{v}$ is parallel to the face of the detector. A schematic diagram of the p_2 proton detector panel is shown in Figure 4-1. Once inside a cell, a proton is focussed and accelerated down a cylindrical tube held at a high negative potential (-30 to -40 kV). Behind each tube, also at high voltage, is a PIN diode detector in which the accelerated protons are stopped and their energy

measured. A schematic diagram of a single focussing cell is shown in Figure 4-2. The position sensitivity of the detector panel is provided by the cell in which proton is detected.

Figure 4-1: Proton detector focussing array.

4.2.2 Proton Focussing System

4.2.2.1 Design Study

In arriving at this design for the focussing system several constraints were considered. First, accelerating voltages above 40 kV were thought to present undue difficulties with regard to the required high voltage feedthroughs and insulation, as well as high voltage breakdown between the focussing electrodes. Second, the focussing efficiency, the probability that a proton entering a cell is focussed into the spot at the back, was required to be very high ($> 90\%$). This is principally to avoid nonuniformities in detection efficiency caused by losses in the focussing. It was found that focussing efficiency could be made nearly unity by masking the edges of the entrance to the focussing array. Since protons striking the mask never enter the focussing cell, they do not contribute to the over-

Figure 4-2: A single proton focussing cell shown in cross section.

all focussing efficiency or the uniformity of the focussing efficiency. Finally, the size of the solid state detectors is limited since they must have sufficiently low noise thresholds to detect 30-40 keV protons. Because phase space is conserved in the focussing, there is an upper limit on the size of the focussing cells. From Liouville's theorem, the largest factor by which focussing can reduce the active detector area required is

$$\frac{A_1}{A_2} \leq \sqrt{T_2/T_1} = \sqrt{(qV + T_1)/T_1} \quad (4-1)$$

where A_1 is the area of the entrance aperture of the cell, A_2 is the area of the focussed spot on the detector, T_1 (T_2) is the proton kinetic energy before (after) acceleration, q is the charge of the proton, and V is the accelerating potential. For the most energetic protons, $V = 35$ kV gives

$$\frac{A_1}{A_2} = \sqrt{\frac{(qV + 0.75\text{keV})}{0.75\text{keV}}} \approx 7. \quad (4-2)$$

This provides an optimum focussing cell size for a given detector size: $A_1 \leq 7A_2$.

Solid state detectors with sufficiently low noise were found to be available in sizes up to $A_2 \approx 3.2\text{cm}^2$. Since high focussing efficiency was required, the $8.4\text{cm} \times 24\text{cm}$ proton detector panel was divided into the twelve sections each with a rectangular entrance $4.2\text{cm} \times 4\text{cm}$ and area $A_1 = 16.4\text{cm}^2$ for which Equation 4-1 gives a focussed proton spot size of $A_2 \geq 2.3\text{cm}^2$.

The size and shape of the focussing electrodes were determined with the help of the Relax3D and Track3D computer simulations [Ric90]. The Relax3D code uses iterative relaxation to compute the potential created by an arrangement of electrodes at fixed potentials. The benefit of this over other codes is that no particular symmetry is required for the volume or electrodes. The Relax3D code produces an array containing the value of the potential at each point on a three dimensional rectangular lattice of points. The potential between lattice points is computed by linear interpolation. The Track3D program tracks ions using the potential array created by the Relax3D code. In the calculations described below the source of protons was taken to be a 50 cm long beam 6 cm in diameter. The radial distance between the center of the beam and the entrance to the focussing cells is $u = 10\text{ cm}$.

A large number of electrode shapes were considered. In general, more complicated designs involving electrodes at several different potentials were found to be no more efficient than the simple cylindrical lens shown in Figure 4-2. In the best design the cylindrical electrode is centered in the rectangular cell and its diameter is 1.9 cm. Its top edge is 3.0 cm from the mesh forming the entrance plane. Simulation of this geometry revealed that over 90% of the protons entering a focussing cell were successfully focussed down through the cylindrical electrode to form a spot on the solid state detector. A proton was "lost" if it entered the focussing cell but did not terminate in the focussed spot. Nearly all

of the proton losses came from protons entering the focussing cell near one of its grounded walls. The focussing efficiency was enhanced by allowing protons entering the array of focussing cells near one of the interior cell dividing walls to cross into the next cell.

A focussing array allowing crossing between cells was simulated by an 8 cm by 8.4 cm array of four cells like the one shown in Figure 4-2. A cross sectional view of one pair of the focussing cells in this model is shown in Figure 4-3. The viewpoint is looking along the direction of the simulated neutron beam. Included in the figure are the electrodes modeled, the resulting lines of equipotential, and several sample proton trajectories. The second pair of focussing cells is adjacent to the ones shown in the $\hat{w}$ direction.

The effect of the focussing cells on simulated decay protons was characterized in several ways. The position of each proton after it had been focussed onto the back of a focussing cell was recorded. A contour plot of the fraction of protons successfully focussed onto the back plane (i.e. the plane containing the solid state detectors) versus their positions in that plane is given in Figure 4-4. The trajectories of the focussed protons were not allowed to pass through any electrodes except the inner walls dividing the cells. Some protons entered the array but were not successfully focussed. They were "lost" in collisions with the exterior walls of the array. Figure 4-5 shows a contour plot of the initial positions of these lost protons. Finally, the angle of incidence of each focussed proton on the back plane was accumulated in a histogram. See Figure 4-6. From this figure it is clear that nearly all the protons (> 99% for the case shown) are incident on the back plane with an angle less than 30° from normal. This is important because the energy loss in the insensitive surface layer of the solid state detectors is dependent on the angle of incidence. At 30° the effective thickness of the surface layer is increased by $1/\cos(30^\circ)$ or about 15%.

Figures 4-4, 4-5, and 4-6 show the results for a particular set of conditions. Namely, the protons incident on the focussing array were assumed to have half their maximum kinetic energy (0.375 keV) and the focussing electrodes had a 30 kV potential dif-

Figure 4-3: Calculated focussing potential and sample proton trajectories. A pair of focussing cells is shown in cross section. The simulation was done in three dimensions. Thick lines indicate the points held fixed in the calculation of the potential. The top, left and right edges are at 0 V as is the dividing electrode extending down from the top edge. The electrodes near the bottom are at high negative potential. The dashed lines indicate the resulting equipotential contours. The curved lines are sample proton trajectories. The protons drift through the top plane into the focussing field, down the throats of the focussing cylinders, onto the solid state detectors at the bottom.

Figure 4-4: Proton intensity contours on the detectors after focussing in the simulation of the four focussing cells. The contours are equally spaced by 20% showing the fraction of protons hitting a unit area. The results shown are for protons with 0.375 keV of kinetic energy. The potential difference on the focussing electrodes was 30 kV.

ference between them. Additional simulations were performed for which the proton energy and the potential difference on the focussing electrodes were varied. The results were similar to those for the case shown. The size of the proton spot produced on the back plane of the focussing array is given in Figure 4-7 as a function of incident proton energy for two different electrode potential differences. Similarly, in Figure 4-8 the fraction of protons successfully focussed onto the back plane is shown. All of the proton losses come

Figure 4-5: Intensity contours of protons lost in the simulation of four focussing cells. The contours are equally spaced by 20% showing the fraction of protons lost as a function of their point of entry into the focussing array. The results shown are for protons with 0.375 keV of kinetic energy. The potential difference on the focussing electrodes was 30 kV.

from protons entering near one of the four exterior (nontransmitting) walls. It should be noted that the focussing losses are higher for the four-cell model than for the full twelve-cell focussing array because in the latter there is less exterior wall, on average, per cell. The losses shown at the top and bottom of Figure 4-5 would only occur in the end pairs of focussing cells in the full twelve-cell array.

The detection efficiency can be made more uniform by placing an aperture over

Figure 4-6: Calculated angle of incidence of the focussed protons on the solid state detectors. The results shown are for protons with 0.375 keV of kinetic energy. The potential difference on the focussing electrodes was 30 kV.

the entrance to the focussing array. Most of the lost protons enter the focussing array near an exterior wall. By masking out all protons entering the array near such a wall, the focussing efficiency can be made nearly unity. Losses to the walls transverse to the beam direction, those at the top and bottom of Figure 4-5, are less important for the full twelve-cell array. Effects due to losses on these end walls can be eliminated from the data by cutting events in which the proton is detected in one of the end cells. In future designs, uninstrumented "passive" pairs of focussing cells on both ends of the focussing array will be added so these cuts on the data can be avoided. Losses to the long side walls of the focussing array can be eliminated by masking all protons entering near the side walls. While this has the desired effect of making the focussing efficiency more uniform, it reduces the overall acceptance of the proton detector paddle. Only a very narrow (2 to 4 mm) portion of the entrance need be masked near these walls to eliminate nearly all of the losses. Figure 4-9 shows the fraction of protons that would have been lost in the side walls that

Figure 4-7: Calculated size of focussed proton spot on solid state detectors. The maximum spot dimensions transverse and parallel to the beam are shown. Two electrode potential differences, 30 and 40 kV, were used.

Figure 4-8: Calculated fraction of protons entering the focussing array successfully focussed on the detectors. Results for both 30 and 40 kV potential differences on the electrodes are shown. A higher focussing efficiency can be achieved by reducing the size of the entrance aperture slightly.

are eliminated by masking the focussing array entrance near these walls. This fraction is given as a function of the width of the mask along each wall.

Figure 4-9: Fraction of unsuccessfully focussed protons eliminated by masking the edges of the entrance of the focussing array that run parallel to the beam.

4.2.2.2 High Voltage Electrode Tests

The electrodes that create the proton focussing and acceleration field must hold the required high voltages without vacuum breakdown. In order to ensure this an electrode test setup was constructed. It is shown in Figure 4-10. The apparatus consisted of a vacuum vessel constructed from an 8" conflat Tee, an adjustable stand for the grounded rectangular cell wall structure, and a high voltage vacuum feedthrough that supported the cylindrical focussing tube. During the tests a vacuum at or below 1.5×10^{-7} torr was maintained in the chamber with the help of a 360 l/s turbomolecular pump. The pressure was measured by an ionization gauge mounted on a port on the body of the tee.

Two sizes of focussing electrodes were tested. The cell walls of the smaller set were 2.8 cm apart while the larger set had the 4 cm by 4.2 cm separation shown in Figure 4-1. The smaller set was tested since it would produce a smaller proton spot allow-

Figure 4-10: Experimental setup for focussing electrode tests.

ing the use of smaller, lower noise solid state detectors. Twenty-seven of the smaller cells could be used, three abreast, to make up the focussing array. After these tests were completed, solid state detectors large enough to be used with the 4 cm×4.2 cm electrode structure were found so they were used in the final apparatus.

The role of the mesh, normally assumed to make up the entrance of the grounded cell, was played by a solid rectangular block. The electric field near the mesh is relatively small so this did not compromise the validity of the test. The tube electrodes were

machined from bar stock to have an outer diameter of 1.5 cm and 1.9 cm for the small and large electrode sets respectively. For both sets, the tube walls were 0.1 cm thick and the ends of the tubes were simply ground to a 0.05 cm radius. Initially, all the electrodes were made from 6061 aluminum alloy. They were hand-polished using a slurry of jewelers rouge and machine oil. The apparatus held the tube centered in the cell with the end of the tube 2.3 cm (3.0 cm) from the top of the small (large) cell. A large negative potential was applied to the tube by a high voltage power supply through the high voltage vacuum feedthrough. The potential of the cell walls was maintained by connection to the grounded vacuum vessel.

The electrodes were conditioned by raising the voltage in steps of a few kilovolts pausing for about a minute between steps. During this procedure occasional breakdowns were observed as sudden increases in current accompanied by a drop in the supplied voltage and increases in the pressure. Using this procedure a maximum potential of 41kV (47kV) was applied to the small (large) electrode set. It was not possible to maintain these maximum potentials for more than a few minutes before a breakdown occurred.

Prior to longer term tests, the electrodes were conditioned to a voltage about 5 kV above the desired test voltage. Neither the pressure in the vessel (up to 1×10^{-5} torr) nor running the ion gauge during tests were observed to contribute to high voltage vacuum breakdowns. While high voltages could be applied for periods of a few minutes with all-aluminum electrodes, they could not be maintained for several hours. In several instances the occurrence of a breakdown subsequently prevented these electrodes from withstanding high voltages at all (in one case no more than 18 kV). This situation could only be remedied by repolishing the tube electrode.

In later tests the large aluminum tube electrode was replaced by one made of 304 stainless steel. With the new electrode the conditioning process was found to produce a higher maintainable voltage. Voltages as high as 33 kV were maintained for more than a day without breakdown. The conclusion of these tests was that the electrode design was

feasible if electrodes at negative high voltage were made from stainless steel.

4.2.3 Solid State Detectors

The focussing/acceleration design requires large area (about 12 mm by 16 mm) charged particle detector that can detect 30–40 keV protons with high efficiency. Consideration was given to organic and inorganic scintillators, and several types of solid state detectors including silicon surface barrier detectors. Silicon solid state detectors were preferred for their high efficiency, low backscattering ($\approx 1\%$), ease of use, and ability to run at room temperature. The best candidate based on its size and noise properties was found to be the silicon PIN photodiode. The designation PIN refers to the *p*-type, intrinsic, and *n*-type semiconductor layers of which the diode is made. They are available in large sizes with the low capacitance and low room temperature leakage current required to resolve energies of about 20 keV. They are also relatively inexpensive. The important requirement for these detectors is that their insensitive surface layer be thin enough to leave low energy protons passing through it with sufficient energy to be detected. Tests of the detectors with low energy protons were performed to measure the energy loss in the dead layer and thereby its effective thickness.

Several models of detectors manufactured by Hamamatsu Corp. [HAM01] were tested including rectangular PIN diode detectors in two sizes, 18 mm×18 mm (model S3204) and 10 mm×10 mm (model S3590), each in two thicknesses 300 and 500 μm . [Note: the manufacturer claims the active area of the thicker model S3590 is only 9 mm×9 mm.] The S3204 detectors are large enough to be used with the twelve cell focussing array described above. Although 40 kV protons have a range in silicon of less than 5000 Å, the thicker PIN diode detectors (500 μm) were preferred for their lower junction capacitance (≈ 80 pF in the S3204 PIN diodes) and resulting lower noise. A 10 mm×10 mm×300 μm planar diffusion diode (model S1337-1010BQ) was also tested. While it was not available in large enough sizes to be used in the focussing array, it was

reported by the manufacturer to have a thinner surface layer than the PIN diode detectors.

The resolution of the large PIN diode detector was compared for several preamplifiers. The electronics used for the tests are shown in Figure 4-11. The energy calibration of the detectors was determined using photoelectron peaks produced by the γ rays from ^{119m}Sn ($E_\gamma = 23.87$ keV) and ^{241}Am (59.54 keV). Based on these tests, the RL-789/3 hybrid preamp manufactured by Rel-Labs, Inc. was selected [REL01]. It was chosen for its compatibility with the high capacitance of the large PIN diodes, low noise, low cost ($\approx \$100$ for a 3-channel package), and small size. The latter point is important because the twelve channels of preamplification electronics must be located inside the focussing array assembly at high voltage. The 24 keV γ rays, which effectively simulates the energy deposited in the PIN diodes by ≈ 35 keV protons, were detected with a FWHM of 9.2 keV (6.4 keV) in the large (small) 500 μm thick PIN diode detector.

Figure 4-11: Electronics for PIN diode detector tests.

The energy loss in the dead layer of the PIN diode detectors was first measured in December 1990 using a proton beam at the Los Alamos National Laboratory (LANL). The planar diffusion diode was tested in March 1991. The experimental setup is shown in Figure 4-12. The protons in the beam were produced by ionizing hydrogen gas by electrons from a hot filament that had been accelerated through a few tens of volts. The protons were accelerated across an electrostatic gap to an energy that was varied between 30 and 45 keV. After acceleration all other species of ions were removed from the beam by velocity selection in crossed electric and magnetic fields (a Wien filter). The beam exiting the Wien filter was found to have a significant neutral component so the electric field in

the filter was adjusted to spatially separate the proton beam from the neutral one. One of the problems encountered with this setup was achieving a beam with a low enough intensity so as not to damage the detector. This was achieved by reducing or stopping the hydrogen flow in the source and intentionally using fringing fields of the filter to diffuse the remaining beam. Care was taken to ensure that the diffused beam used was monoenergetic and not degraded by scattering in the filter.

Except where noted, each detector tested was coupled to an Ortec 242 preamplifier. The remaining electronics were the same as those used in the preamplifier tests. The results of these tests for several proton energies are given in Table 4-1. The proton energy loss table of Janni [Jan82] was used to calculate the thickness of the dead layer from the measured energy loss. For the purpose of computing the dead layer's thickness, it was assumed to be composed entirely of silicon. The differential energy loss dE/dx (the stopping power) of the protons in the dead layer varies with energy. Since the energy lost in the dead layer is a significant fraction of their total energy, the inverse of the stopping power given by the table was numerically integrated to determine dead layer's thickness. The dead layer thickness is given as a density thickness product ρx . Its uncertainty includes the uncertainty in the tabulated stopping power. Our results, Table 4-1, are consistent with the result of another group, C. Weinheimer *et al.* [Wei92], who have measured the dead layer of a model S3590-06 Hamamatsu PIN diode. They determined the dead layer by measuring the energy loss of monoenergetic electrons. They find the dead layer is $45.1 \pm 6.8 \mu\text{g}/\text{cm}^2$.

Additional tests at higher energy were performed in April 1991 on the BLAZE atomic physics beamline at Argonne National Lab. The protons were produced by a radio frequency dissociator floated at high voltage. The energy of the protons was analyzed using a 90° bending magnet. Protons from the beam were Rutherford scattered 90° by a $10\mu\text{g}/\text{cm}^2$ Au foil into the detector. The energy calibration was determined from photope-

Figure 4-12: Proton beam setup at LANL.

Detector	Size (mm×mm×μm)	Incident Proton Energy (keV)	Deposited Energy (keV)	Dead Layer (μg/cm ²)
PIN S3590-06	10×10×500	30	14.0	38(4)
		35	17.3	39(4)
		40	21.8	38(5)
PIN S3204-06	18×18×500	30 ^a	12.5	42(4)
		40	20.4	41(5)
Planar Diffusion S1337- 1010BQ	10×10×300	30	24.8	11(3)
		35	29.0	12(4)
		40	34.1	12(4)
		45	38.7	12(4)

Table 4-1: Results of proton beam tests in December 1990 and March 1991 at LANL.

a. The proton peak was not completely resolved from the noise at low energy.

aks produced by several γ lines from an ^{243}Am source. In the environment of the accelerator, significant electronic noise was introduced into our amplification electronics. The resulting increase in the noise threshold precluded measurements at lower proton energies. These measurements, summarized in Table 4-2, confirm the observations made at lower energies at LANL.

Detector	Size (mm×mm×μm)	Incident Proton Energy (keV)	Deposited Energy (keV)	Dead Layer (μg/cm ²)
PIN S3590-06	10×10×500	49.6	29.2	40(5)
		59.5	37.9	41(6)
		70.2	47.7	43(7)
PIN S3204-06	18×18×500	59.2	35.0	46(6)
		69.3	44.0	48(7)
Planar Diffusion S1337- 1010BQ	10×10×300	49.6	42.0	14(5)
		59.6	51.5	15(6)
		69.3	61.0	16(7)

Table 4-2: Results of proton beam tests in April 1990 at Argonne.

The initial tests at LANL and Argonne demonstrated that the proton energy loss in the dead layer of the standard PIN diode detectors would require undesirably high acceleration voltages. An acceleration voltage of nearly 40 kV was required to resolve protons in the larger PIN diode from the noise. However, the energy loss in the planar diffusion diode was found to be much less. Unfortunately, it was not available in the larger sizes and thickness we required. We contracted Hamamatsu to make a special PIN diode detector based on their S3204-06 PIN diode but with, at most, one half their standard PIN diode's dead layer. These detectors were tested with the LANL proton beam in December 1991. The results of these tests using the RL-789/3 preamplifier are given in Table 4-3. The dead layer of these "thin windowed" detectors was found to be about one half that of the standard PIN diode. With these detectors we were able to detect 30 keV protons. Spectra from runs with 30 and 35 keV protons are shown in Figure 4-13. The ^{119m}Sn calibration run is shown for comparison.

In the course of testing the thin windowed PIN diodes, the energy loss in the original PIN and planar diffusion diodes was remeasured on the LANL proton beam. The PIN diodes showed an increased energy loss corresponding to an increase in dead layer thickness of 30%. The planar diffusion diode showed a 13% increase. The increase was attributed to surface contamination accumulated during the year between tests. Prior to this

Detector	Size (mm×mm×μm)	Incident Proton Energy (keV)	Deposited Energy (keV)	Dead Layer (μg/cm ²)
Thin PIN (thin win- dowed version of S3204-06)	18×18×500	30.0	21.0	20(4)
		32.5	23.1	20(4)
		35.0	25.3	20(4)
		37.5	27.5	20(4)
		40.0	30.0	20(4)
		42.5	32.6	19(4)
		45.0	34.9	20(4)

Table 4-3: Results of proton beam tests with the special "thin windowed" PIN diode detectors in December 1991 at LANL.

Figure 4-13: Energy spectra taken with the special "thin windowed" PIN diode detectors.

finding the detectors had been stored in air. Subsequently, they were stored in an evacu-

ated desiccator.

4.2.4 Proton Detector Panel

The focussing electrode array and solid state detectors were combined to form a proton detector panel. The proton panel, shown in Figure 4-14, is one of four required by the octagonal detector design. It consists of three components: a vacuum housing, a grounded focussing cell divider array, and a detector paddle at high negative potential that supports the cylindrical focussing tubes.

The detector paddle, shown in cross section in Figure 4-15, is composed of a 27 cm long by 7 cm diameter semi-cylindrical stainless steel shell whose long flat side supports a plate containing the array of twelve cylindrical focussing tubes. Each end of the shell is closed by a flat wall that has a hole in it to accommodate the passage of a long straight tube lengthwise through the paddle. This tube is used to suspend the paddle, which is at negative high voltage, inside the vacuum housing. One end of this tube is supported by three quartz rods extending radially inward from the vacuum housing. The quartz rods stand off the high voltage between the vacuum housing and the paddle. The other end of the tube is supported by a stainless steel "spider", an adjustable cylindrical clamp. The suspension tube passes through the spider a short distance and is firmly clamped by the inside of the spider. The outside of the spider bears against the inside of a portion of the vacuum housing to which the high potential is applied. This portion of the vacuum housing is electrically isolated from the rest of the vacuum vessel by a commercial high voltage vacuum break. The spider connection fixes the position of the paddle and establishes the high voltage electrical contact to the paddle.

Besides supporting the detector paddle via its suspension tube, the vacuum housing also provides support for the grounded cell divider array. The interior walls of the cell array are made of solid vanes and, near the mesh that covers the entrance to the cells, wire planes. The wire planes allow protons entering a focussing cell near one of the interior

Figure 4-14: Proton detector panel. The proton detector panel consists of three major parts, a vacuum housing, the grounded focussing cell divider array, and the detector paddle at high negative potential that supports the cylindrical focussing tubes. The detector paddle is suspended on a long tube. The tube is supported by a "spider" on one end of the vacuum housing and by three radial quartz rods on the other. The detector paddle is aligned so that the focussing tubes are centered in the cells formed by the dividing walls of the cell array. The PIN diode detectors and their preamplification electronics are inside the detector paddle at high voltage.

Figure 4-15: Cross sectional view of the proton detector panel. Note the chamfer on the ends of the focussing tubes shown in the detail. The 45° angle reduces the electric field at the surface.

dividing walls to pass into the next cell. The wire planes consist of twelve 0.003" diameter gold-plated tungsten wires equally spaced by 1 mm between centers. The proton focussing simulation shows that protons do not cross the dividing walls below the 1.2 cm formed by the wire planes. The rectangular entrance of the cell array is 7.6 cm by 30 cm. It is covered by a mesh 94% open and 0.001" thick manufactured by Buckbee-Mears [BUC01]. The mesh provides a ground plane defining the entrance to the cells and preventing the focussing fields from entering the octagonal decay region. The long exterior walls of the cell array includes a 3.8 mm lip that prevents protons from entering near these walls. The lip is shown in Figure 4-15. From the graphs in Figures 4-8 and 4-9, the calculated focussing efficiency for protons entering the cell is greater than

$$1 - (1 - 0.9) (1 - 0.95) = 99.5\% . \quad (4-3)$$

Such a high focussing efficiency precludes the focussing from contributing significantly to possible variations in detection efficiency.

The potential calculated by the Relax3D code was also used to estimate the maximum electric fields on the surfaces of the electrodes. Care was taken to reduce the possibility of electrical breakdown between electrodes at different potentials in the cell. At each point on the cathode's surface the electric field must be less than a certain value characteristic of the material from which it is made. All the electrodes at high negative potential were made from stainless steel, one of the materials best able to resist high voltage breakdown [Haw68]. Based on the breakdown criterion of Kilpatrick [Kil57], the maximum field sustainable on the cathode surface was estimated to be about 100 kV/cm. The shapes of the electrodes were carefully designed to minimize the surface fields on the cathodes. The highest field is located at the end of the cylindrical focussing tubes. This can be seen from the equipotential lines in Figure 4-4. The field is directed at about 45° from the axis of the tube toward the entrance to the cell. The calculated maximum field depends on the shape of the tube's end. A tube whose end is chamfered by 45° has a maximum surface

field of 95 kV/cm for a -40 kV applied potential. The chamfer is shown in the detail in Figure 4-15. By way of comparison, if this tube were simply rounded off at the end, the maximum field would be about 130 kV/cm. The maximum field is below 80 kV/cm on the surface of the rest of the proton paddle for a -40 kV applied voltage. As a final precaution, all of the parts at high voltage were electropolished to remove microscopic protrusions on their surfaces that might contribute to breakdowns. The assembled focussing cell array was successfully conditioned up to -46 kV and tested for long times at voltages up to -40 kV.

The paddle is aligned via radial adjustment screws in the spider and on the three quartz rods that support the ends of the suspension tube. A mating pair of jigs were constructed that replace the cell array and the plate containing the focussing tubes. Using these jigs, the alignment of the paddle is set relative to the cell array accurate to ≈ 0.1 mm. The alignment of the cell array is in turn fixed with respect to the vacuum housing by a pair of pins. Once the paddle is aligned, the jigs are removed. The alignment is retained even if the entire detector panel must be moved or if access to the interior of the paddle via the removable focussing tube plate is required.

A rectangular printed circuit board is mounted inside the detector paddle. It supports the PIN diode detectors and their preamplification electronics. When the paddle is assembled, there is one detector backing each of the twelve focussing tubes. The detectors and preamplifiers float at the same potential as the focussing tubes and the rest of the paddle. They are protected from high voltage breakdowns by the surrounding paddle shell. Despite numerous breakdowns that occurred during high voltage conditioning of the focussing electrodes, none of the detectors or preamplifiers were damaged. Cables carrying the electronic signals from and power to the preamplifiers are brought out through the suspension tube to the high voltage isolated end of the detector panel's vacuum housing. The suspension tube is slotted inside the paddle so the cables can be brought inside of it. The signals are brought out of the high voltage end of the vacuum housing by an elec-

tronic vacuum feedthrough. They pass from the feedthrough into an insulated shielded cable finally ending at an isolated electronics rack that also floats at the high potential.

Each of the preamplifier chips inside the detector paddle dissipates 70 mW of power per channel for a total of 840 mW. Since the preamplifiers are at high voltage and in vacuum, special provision had to be made to carry away this heat. Thermal contact to each of the four preamplifier chips was made by sandwiching them between a pair of copper plates. Each pair of plates was soldered to a copper braid and the braid connected to the end of a copper rod. The rod runs through the paddle suspension tube from the center of the paddle to the end of the tube near the three quartz standoffs. Figure 4-16 shows how this rod is connected to the cooling bath which is at 0 V. The rod's end is clamped to a sapphire rod, which in turn is connected to the end of another copper rod. Thin indium shims were used to ensure good thermal contact between the copper and sapphire. This second copper rod, encased in a vacuum jacket, passes out of the vacuum housing to a dewar containing the cooling bath. Both liquid nitrogen (-196°C) and an ethanol-dry ice bath (-78°C) were used for cooling. The sapphire stands off the high voltage and conducts heat between the copper rods. The temperature was monitored at three points in the cooling apparatus by thermocouples. Two were connected near the low voltage side of the sapphire rod. The other was connected to one of the preamplifier chips on the high voltage side.

4.3 Electron Detection

Detection of the electrons is significantly simpler than detection of the protons. Since the electrons have energies as high as 782 keV, conventional charged particle detectors can be used. The electron energy spectrum is shown in Figure 4-17. Since the electron energies extend down to zero, some events are necessarily lost. In the test run, two forms of electron detectors were employed: a simple plastic scintillator paddle and a prototype time projection chamber (TPC) backed by a plastic scintillator. A TPC is a type of position

Figure 4-16: Proton detector high voltage cooling link.

sensitive wire chamber. Both detectors were smaller than the 8.3 cm by 50 cm active area called for in the octagonal detector design. Instead their sizes in the test run were dictated by expediency. However, it would be a simple matter to modify the designs of either of them to the called for dimensions.

Figure 4-17: The electron energy spectrum for neutron beta decay. The solid line is the complete theoretical spectrum. The dashed line is the spectrum when coincidence is required between an electron and a proton detector in the octagonal detector design. The relative normalization of the two curves is arbitrary.

Primarily because they employ plastic scintillator, these detectors meet the criteria: large area, fast timing, low electron backscattering, and high efficiency. Since measurement of the energy of the electron is not required for the measurement of the D coefficient, the energy resolution of the scintillator is primarily useful for the rejection of higher energy backgrounds. The scintillator-backed TPC provides the possibility of enhanced γ background rejection by requiring a charged particle track in the wire chamber in coincidence with the proper energy signal in the scintillator.

The TPC also provides tracking of the electron. The octagonal detector design does not require that the electron be tracked in order to extract D but this information

could prove useful for studying systematic effects. The vertex of the decay can be approximately reconstructed from the electron track by determining the track's intersection with the neutron beam. The approximate proton momentum could then be determined from the proton time-of-flight, the position of the decay vertex, and the focussing cell the proton entered. Knowledge of the angles and positions of the particles incident on the detectors would be useful for studying the systematic effects due to nonuniform detector efficiencies discussed in Chapter 6. In addition, the systematic effect due to nonuniform beam polarization could be studied by determining the portion of the beam in which certain decays occurred.

4.3.1 Simple Scintillator Detector Paddle

The scintillator paddle is made of an 8 mm thick slab of Nuclear Enterprises NE110 plastic scintillator coupled on each end to a 44 mm diameter Amperex model XP2262 photomultiplier tube (PMT) via an adiabatic¹ light guide [AMP01]. The 8 mm thickness is more than sufficient to stop the highest energy electrons from neutron decay. Since minimum ionizing particles deposit about 1.2 MeV in the paddle, the extra thickness is useful for discriminating against backgrounds produced by through-going particles. The octagonal detector design calls for electron detectors with a 50 cm by 8.3 cm active area. The detector paddle actually used in the detector test run was 12 cm wide by 42 cm long. The entrance of decay electrons was defined by a 10.2 cm by 35 cm rectangular aperture in the vacuum vessel that supported the paddle. Its size was dictated by the ready availability of scintillator in 12 cm widths and by the length of the vacuum vessel. The scintillator paddle was wrapped in aluminized Mylar® and directly formed the interface between the chamber vacuum and atmosphere. Crazeing of the scintillator, probably caused

1. A light guide is adiabatic if its cross sectional area is constant and sharp bends are avoided. This type of guide theoretically can transmit all of the light entering its end within the acceptance angle.

by the mechanical stress placed on it, was observed after the test run. In the future the scintillator should be placed inside the vacuum to alleviate the stress.

By comparing the timing between the signals from the PMT's at either end of a scintillator paddle, the position of the detected particle along the length of the paddle can be determined. The same scintillator paddles in 1 m lengths were tested for energy and position resolution using the setup shown in Figure 4-18. The position resolution was measured by collimating γ ray sources so the Compton electrons they produced in the scintillator were localized. Using this method, a position resolution of about 10 cm (FWHM) was obtained. Electrons produced by the Compton scattering of γ 's with energy E_γ have a well-defined maximum energy [Kno89]

$$E_e \leq \frac{2E_\gamma^2}{(0.511 + 2E_\gamma)} \quad (4-4)$$

where all energies are in MeV. The energy calibration of the scintillator was obtained by measuring the position of the sharp upper edge of the Compton energy spectrum for γ lines of several energies. The position of each edge was taken to be at half its maximum as per the prescription of Dietze and Klein [Die82].

The effect of the energy threshold imposed by the scintillator electronics can be determined from the convolution of the scintillator energy resolution function and the electron spectrum. This is useful for determining what fraction of electrons are cut by the threshold. The formula of Dietz and Klein for estimating the resolution

$$\frac{\Delta E}{E}(\text{FWHM}) \approx 1.5 \frac{(E_{1/2} - E_{\max})}{E_{1/2}}, \quad (4-5)$$

based on the energies of maximum ($E_{\max}$) and half-height ($E_{1/2}$) of the edge in the Compton spectrum, includes the effects of photon multiple scattering. It was used to estimate the energy resolution of the system. It was found to be best fit by the linear model

$$\frac{\Delta E}{E}(\text{FWHM}) \approx 0.39 - \left(\frac{0.23}{\text{MeV}} \right) E. \quad (4-6)$$

Based on tests of several materials for wrapping the scintillators and light guides, aluminized Mylar was chosen because it produced the best energy resolution. Although tests of the spatial and energy resolutions were not made for the shorter lengths called for in the octagonal detector design, the results are expected to be at least as good as those for the 1 m paddles.

4.3.2 Tracking TPC and Scintillator

4.3.2.1 Introduction

A bench top prototype TPC was constructed to investigate the possibility of recovering information about the electron track for each decay. Charged particles entering the gas-filled interior of the TPC produce ionization electrons that drift at a constant velocity under the influence of a uniform electric field toward a set of crossed wire planes. As the ionization reaches the wire planes it creates an avalanche on anode wires in one of the planes. The position of the avalanche in two dimensions (x and y) is determined from its position in the crossed wire planes. The timing of the avalanche is also recorded. The time information and the known ionization drift velocity are used to recover the third (z) position coordinate. Since a decay electron creates ionization at points all along its path through the gas volume, its trajectory can be reconstructed from the coordinate triplet measured for each of the ionization points.

A TPC was chosen over the more conventional solution of a stack of crossed wire chambers (multiwire proportional chambers or drift chambers) primarily because all the wire planes in the TPC can be placed at the backs of the electron detector panels, where the front is the side closest to the beam. See Figure 4-19. Thus, very little space is required on the sides of the electron detector panels, which are very close to the adjacent proton detector panels. If several layers of crossed wire chambers were used, some detection

Figure 4-18: Data acquisition setup used for scintillator timing tests. The solid lines are analog signals the dashed lines are logic signals.

solid angle would be lost in accommodating the support and signal extraction from wire planes transverse to the beam. The TPC also provides measurement of the track all along its length while avoiding the added multiple scattering of the decay electrons that would be produced by the windows between multiple sets of wire planes.

There are several drawbacks to using any form of wire chamber in the detector. The large number of wire signals that must be read out for each event complicates the data

Figure 4-19: Conceptual design of an electron detector panel in the octagonal design based on a TPC. The heavy arrow is a sample decay electron trajectory.

acquisition considerably. In addition, a gas to vacuum window is required between the octagonal decay region and the body of the wire chamber. A vacuum window was constructed so the prototype TPC could be used in the detector test run at NIST. However, thickness of the window material required was several times greater than would be acceptable from the standpoint of multiple scattering of the decay electrons in the material. To date this problem has not been solved.

4.3.2.2 Prototype TPC

The TPC consists of a 20 cm by 20 cm by 11.7 cm gas volume with a stack of three crossed wire planes on one of the square sides. The wire planes are backed by a 1.0 cm thick slab of plastic scintillator 26.7 cm by 24.1 cm. Light from the scintillator is transported by a pair of adiabatic light guides on each end into four photomultiplier tubes. The same PMT's were used as for the simple scintillator paddle. The entire assembly is shown

in Figure 4-20.

The wire planes are mounted on printed circuit board frames with 1.5 mm center-to-center spacing between planes and between wires within each plane. All but the anode wires are 0.003" diameter gold-plated aluminum. The anode wires, where the avalanche occurs, are 0.0008" diameter gold-plated tungsten. The central wire plane, called the "anode" plane, is composed of alternating field and anode wires. The uppermost plane, in the figure, is the "cathode" plane and the lowermost the "grid" plane. The wires in the anode and grid planes run into the page. The wires in the cathode plane run perpendicular to the wires in the other planes. In the cathode and grid planes, pairs of adjacent wires are connected to ground through a $1\text{ M}\Omega$ resistor and to the input of a preamplifier through a $0.0027\text{ }\mu\text{F}$ (3 kV) decoupling capacitor. The anode wires are connected through a $1\text{ M}\Omega$ resistor to a bus held at positive high voltage by a power supply. The field wires were held at a lower positive voltage by a similar bussing system and a second power supply. Signals are induced on wires in the grid and cathode planes near an avalanche on an anode wire. This process has been described by Charpak and Sauli [Cha79]. The wires in the cathode plane provide the transverse coordinate of the avalanche and those in the grid plane the z-coordinate. The position resolution is simply the spacing of the instrumented wire pairs, 3 mm.

The wires in the grid plane define the potential of one side of the drift volume. A thin aluminized Mylar "drift" window ($210\text{ }\mu\text{g}/\text{cm}^2$) held at about +2.2 kV, provides the potential for the other side. The four sides of the drift volume joining the drift window and grid wire plane hold 2 mm wide copper field shaping rings spaced by 11.1 mm. The field shaping rings are on a flexible Kapton® film backing that was glued to the inside of the walls of the TPC. The potential on each ring is determined by a resistor chain that divides the drift voltage. Together, the drift window, grid plane, and field shaping rings create a uniform electric field $\approx 200\text{ V}/\text{cm}$ in the drift volume under the influence of which the ionization electrons drift at about $1\text{ cm}/\mu\text{s}$ into the wire planes.

Figure 4-20: Prototype TPC and scintillator. An example decay electron trajectory is shown producing ionization electrons. The direction of the uniform drift electric field is indicated by the heavy vertical arrow.

The walls of the TPC are made from 2.5 cm thick acrylic. The positioning of the wire planes is maintained via registration dowels. The gas seals among the TPC body, wire planes, and the various window frames are made with a special silicone rubber sealant [GE01] that does not outgas acetic acid, which would poison the chamber gas. A mixture of 90% helium and 10% isobutane is flowed through the TPC at a rate of about 0.5 l/min. using the gas handling system shown in Figure 4-21. A high percentage of helium is used to reduce multiple scattering of the decay electrons in the gas. Other gas mixtures including helium-propane were investigated but they either produced inferior tracks or made the TPC more susceptible to high voltage breakdown. The helium purity was specified at 99.999%. The specified isobutane purity was only 99.5%, the highest available. The oxygen content of the exhaust from the TPC is monitored in a model 311-1 Teledyne Trace Oxygen Analyzer [TEL01]. Typically, the oxygen level is less than 45 ppm. The fraction of oxidant in the gas must be kept low to prevent the loss of the ionization electrons as they drift.

Figure 4-21: TPC gas handling system.

The potentials on the field and anode wires were adjusted to produce tracks that

were solid but not too thick. Typical voltages were +1630 V and +310 V for the anode and field wires respectively. The position resolution of the TPC is determined by the wire spacing. It was verified by observing the tracks of cosmic rays through the TPC. Coincidence between small scintillator paddles above and below the TPC were used to trigger on through-going particles. A sample cosmic ray track is shown in Figure 4-22.

Figure 4-22: A typical track produced by a cosmic ray in the TPC. Each small square indicates a hit on a wire within a time bucket.

The TPC was designed as a bench top prototype to investigate its viability as a

component of the electron detector in octagonal design. It was retrofitted to be used in the detector test run at NIST. Although the acceptance of the refitted TPC-based detector for decay electrons was about five times less than that of the simple scintillator paddle, it was included in the test so its usefulness as a means of rejecting beam-related backgrounds could be studied. By requiring coincidence between the TPC wires and scintillator, the background singles rate was reduced by about a factor of ten.

Using the TPC in the detector test run at NIST required the addition of a window to serve as a gas to vacuum interface. It consisted of a thin Mylar foil supported by a screen. The screen was constructed from a sheet of 0.022" thick stainless steel into which an array of 5 mm square holes separated by 0.25 mm were cut by a laser milling machine. After fabrication, the screen was electropolished to remove any sharp points and the aluminized Mylar foil was stretched over it and glued to its edges using epoxy. The transmitting portion of the screen is 9 cm wide by 19.4 cm long. A 2.5 mm wide strip was left in the middle, across the short dimension, to increase the screen's strength. The screen was found to have sufficient strength to support atmospheric pressure. In doing so it was deformed considerably remaining permanently bowed out by about 1 cm in the center. Aluminized Mylar foils as thin as $210 \mu\text{g}/\text{cm}^2$ used with the screen were sufficient to hold rough vacuum, however, foils thinner than $1600 \mu\text{g}/\text{cm}^2$ developed tiny leaks that prevented high vacuum from being achieved in the vacuum vessel. Even the thicker foil degraded the vacuum somewhat. The multiple scattering of decay electrons in this thickness of Mylar is too great to reconstruct the positions of decay vertices across the beam.

The prototype TPC was, by necessity of its construction, run with the gas at atmospheric pressure. In retrospect, if it were run with a low pressure gas (e.g. 60 torr of pure isobutane) the window could likely be made thinner since it would need to withstand less pressure.

4.4 Detector Chamber

A vacuum vessel was constructed for the detector test run at NIST. It is shown in cross section in Figure 4-23. It served to support the detector panels at the relative 135° orientation called for in the octagonal detector design. There are ports on each end to allow the beam pass through the vessel. A 1400 l/s turbomolecular pump with a nitrogen trap provided the vacuum. The pressure in the system was measured by an ionization gauge.

Figure 4-23: Triangular vacuum chamber with detector panels. The circle near the center of the vessel indicates the neutron beam. During the test run the beam direction was into the page.

4.5 Data Acquisition System

For each event the data acquisition system recorded the decay electron's energy from scintillator while the decay proton's position and time-of-flight were given by the channel number and timing of the PIN diode that fired. The energy of the proton after acceleration into the PIN diode detector was also recorded. In addition, when the TPC was used, information about the electron's track was read out from the TPC wires. Coincidence data were obtained by requiring that a PIN diode in the proton detector panel fire within a few microseconds of the scintillator. Several channels of a C.A.E.N. v260 VME scaler were used to record various singles and coincidence rates [CAE03]. Diagrams of the electronics are given in Figures 4-24 and 4-25. In addition, provision was made to allow the singles energy spectrum from each detector to be measured.

The proton detector electronics were maintained at the same high negative potential as proton detector paddle in an isolated electronics rack. An optical computer link was used to transfer data between the electronics at high voltage and ground. In addition, a pair of optical fibers carried timing signals between the high and low voltage electronics. The data for each event were acquired by a Motorola MVME147S single-board computer based on the Motorola 68030 microprocessor. The computer communicated with the data acquisition modules via a VME bus. The computer contained sufficient random access memory to buffer the events until they could be read out and stored by an Apple Macintosh IIfx computer that was interfaced to the VME bus by a Bit-3 controller [BIT01]. A second Macintosh, connected by ethernet to the first, was used to examine the data as it was acquired.

4.5.1 Scintillator Electronics

The same electronics were used for scintillator backing the TPC and the simple scintillator paddle, so data could not be accumulated from both these detectors simultaneously. The scintillator electronics are shown in the top part of Figure 4-25. The energy

Figure 4-24: Proton detector data acquisition electronics. The solid lines are analog connections, the dashed lines are logic connections, and the dotted lines are digital connections over optical fibers.

deposited in the scintillator was measured by an integrating the PMT signals in a C.A.E.N. model V265 ADC [CAE01]. In the case of the TPC's scintillator, the signals from the pair of PMT's on each end were summed in a LeCroy model 428F linear fan-in before being fed into the ADC. The scintillator signal was used as a fast $t = 0$ start for acquisition of TPC wire data. The same signal was delayed and used as the stop for the TAC that measured the time delay between the electron and proton detector signals. The TAC output was read out by a C.A.E.N. v419 peak sensing ADC [CAE02]. The relative timing between the signals from the PMT's on either end of the scintillators was not recorded during the test run.

The energy resolution of the scintillator system was degraded by the use of the C.A.E.N integrating ADC over what had previously been observed with a LeCroy 2249 ADC. The C.A.E.N. ADC was used because it was compatible with the fast VME bus required by the TPC data acquisition electronics. Although measurement of the energy of the decay electrons is not required, this module will be replaced in the future as it also has some other unfavorable characteristics that contribute to the dead time of the system.

4.5.2 Proton Paddle Electronics

The proton paddle electronics are shown in Figure 4-24. The signals from the PIN diodes' preamplifiers were brought by a insulated cable from the vacuum system to an electronics rack held at negative high voltage by a high voltage power supply. There the preamplified signals were fed into a biased summing amplifier. The amplifier was originally designed for use with a silicon strip detector that is part of the Los Alamos neutrino mass experiment [Rob91]. The principle of the amplifier's operation is to bias each of the twelve input signals by a fixed offset and sum only those portions of the signals that exceed the offset. In this way the amplifier acts to multiplex the twelve analog signals down to one. The amplifier outputs the analog sum of the biased signals and the encoded channel number of the detector exceeding a discriminator threshold.

Bias to the PIN diodes was supplied by a 90 V battery. The energy deposited in the PIN diode was determined from the biased amplifier output by a C.A.E.N. v419 peak sensing ADC. The channel number of the detector that fired was encoded by a Xycom model XVME-200 I/O module [XYC01]. Both modules were connected to a VME bus biased at high voltage and were read out by the main data acquisition computer via a pair of optically linked VME computers. A fast optical signal sent to the low voltage electronics provided the start for the TAC that measured the electron-proton time delay.

The biased amplifier proved to be the most troublesome portion of the proton detector data acquisition system because this type of signal multiplexing is not well-suited for use with signals so close to the detector noise threshold. Attempts were made to calibrate the twelve PIN diode detectors at room temperature using the γ 's from $^{119\text{m}}\text{Sn}$ and ^{241}Am sources as had been done for a single detector in the dead layer measurements. However, the increased noise threshold from the amplifier multiplexing made it impossible to obtain an accurate energy calibration. The noise threshold was reduced substantially when the detector panel was placed under vacuum and cooling was applied to the preamplifiers. The PIN diodes themselves were incidentally cooled by radiative coupling to the cold finger as evidenced by the observed reduction in the collective PIN diode leakage current from over 70 nA (full-scale of our meter) to negligible levels. The manufacturer specifies 15 nA per diode as the typical leakage current at 25°C. The lower leakage current also helped reduce the noise. Despite the reduced noise level, an undetermined fraction of the pulses produced by successfully focussed protons still remained below the noise threshold and had to be discriminated against. Part of the problem was due to some additional energy loss by the protons in a layer of condensation (presumably water) on the surface of the cooled PIN diodes. Buildup of this condensation was observed as a gradual decrease in coincidence detection rate as a function of time after cooling down.

New ways of multiplexing the PIN diode signals are being investigated and the biased amplifier will be replaced in the future. In addition, separate temperature control

for the diodes will be implemented to try to avoid condensation on the detectors. Most of the noise reduction benefits from cooling the diodes is expected to be achieved by about 0° C [Wei92]. Provision should also be made for in situ energy calibration of the PIN diode detectors.

4.5.3 TPC Electronics

The TPC electronics are shown in the lower portion of Figure 4-25. There are 128 channels of wire electronics corresponding to 64 cathode plane wire pairs and 64 grid plane wire pairs. Each channel consists of an amplifier and discriminator. The discriminators latch and hold for up to 2 μ s or until they are cleared. The system is triggered by the fast signal coming from the scintillator backing the TPC. Once the system is triggered, the outputs of all 128 discriminators are recorded every 750 ns by 128 channels of first-in-first-out (FIFO) digital storage. The 750 ns time intervals, or “buckets”, correspond to a z coordinate resolution of about 7.5 mm. After each bucket is read out, the discriminators are cleared and re-enabled. The amplifier and discriminator electronics had been previously used for a multiwire proportional chamber build by members of the Physics Division at LANL [Lil86]. The FIFO's used were developed by a group in the LANL Space Sciences and Technology (SST-10) division for use in digital communications. They are based on the IDT 72104 FIFO chip [IDT01]. We adapted them to our use because of their high speed, low cost per channel, and compatibility with the VME computer and data acquisition bus standard. The FIFO's were designed to meet the bus-independent Industry-Pack™ (IP) daughter board standard. They were mounted on IP carriers manufactured by Green Spring [GRE01]. With the FIFO's as designed, it was not possible to collect wire data before a trigger. This prevented us from using track information as part of an event trigger. Instead a wire “trigger” was constructed from the coincidence of the grand OR of the cathode plane wires the grand OR of the grid plane wires. Pretrigger capability is being designed into future versions of the FIFO boards.

If development of a TPC for the octagonal design is continued, it would be advantageous to instrument the anode wires as well. The signals on these wires are typically several times larger than the signals induced on the grid and cathode planes. They were not instrumented in the prototype because the available amplifier electronics were of the wrong polarity.

Chapter 5: Test Run

5.1 Introduction, Goals of Test

From April through June of 1992 a proof-of-principle test run was carried out at the NIST CNRF. This run had several goals. Foremost, we hoped to prove the feasibility of the most difficult aspect of the experiment, the proton detection scheme, by testing the proton detector panel under experimental conditions and measuring its efficiency. It was particularly important to measure the efficiency of the proton panel in the test run, since no other source of low energy protons was available to test its focussing and acceleration features. This run would also establish whether a plastic scintillator paddle alone would suffice for the proton detection or whether the addition of a wire chamber would be required to reject γ background. The test run was successful, proving that the proton detector panel design works well and that a sufficiently large signal to background ratio can be achieved without wire chambers in the electron detection (e.g. TPC's). In addition, we gained valuable knowledge regarding beam optics and background reduction.

5.2 Beamline

5.2.1 Description

As a preface to this section we remind the reader of the general discussion of neutron sources and properties in Section 2.3.1. The neutrons produced for the NIST CNRF by the NBS reactor were moderated by a D₂O ice cold source kept at about 30–40 K [Ari89]. They were transported in the NG-6 guide tube over about 60 m to the experimental area. The guide tube has a 6 cm by 15 cm rectangular cross section and its reflecting inner surface is coated with ⁵⁸Ni. A shutter at the end of the guide allowed the flux of cold neutrons to be interrupted. The beam in the test run was not polarized. The section of the beamline near the detector is shown in Figure 5-1. After exiting the guide, the beam

passed through a polycrystalline bismuth filter that reduced the flux of γ 's in the beam. The diameter and divergence of the beam was defined by a pair of circular ^6LiF collimators separated by a helium-filled flight tube. ^6Li has a high capture cross section for cold neutrons and the captures produce no penetrating radiation. Inside the flight tube, a series of circular ^6LiF skimming apertures removed neutrons too divergent to pass through the second (downstream) collimator. The tube was filled with helium because ^4He has a very low neutron scattering cross section. After the collimation system, the beam passed through a thin window into the vacuum system. The window was constructed from an 0.56 mm thick single crystal silicon wafer. For most of the runs, a ^6LiF disk in the vacuum chamber was used as a beam stop.

The upstream collimator, which was 6.0 cm in diameter, was placed as close to the bismuth filter cryostat as possible. Several downstream collimators were used at different times. The largest was 4.2 cm in diameter. The separation of 2.8 m between the collimators and the collimator diameters determined the maximum divergence of the beam.

The capture flux (see Equation 2-28) was measured with a fission chamber at two points. It was $(1.54 \pm 0.05) \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}$ just upstream of the first collimator, and $(8.9 \pm 0.3) \times 10^7 \text{ cm}^{-2} \text{ s}^{-1}$ just after the second collimator. The neutron decay rate in the 40.6 cm long decay region was calculated from the measured capture flux by simulation of the transport of the neutrons through the collimation system. With the 4.2 cm second collimator, the decay rate in the decay region was 220 s^{-1} . The simulation also gives the size of the beam. To a good approximation the beam in the decay region was a cylinder with the diameter of the second collimator.

The initial beamline for the test apparatus included a circular neutron guide tube after the collimation, the purpose of which was to confine the beam spatially until it entered the decay volume. The guide tube is shown in Figure 5-2. The idea was to increase the neutron flux through the detector by accepting more divergent neutrons. The more divergent neutrons are valuable because they tend to have lower velocities so they spend

Figure 5-1: Beamline used in the test run.

more time in the decay region. The secondary guide tube was abandoned because it created large γ backgrounds in the electron detectors' scintillators. The γ 's were created by neutron capture in the guide walls and in the aluminum and stainless steel components near the scintillators. This problem was accentuated by the fact that the guide tube's natural Ni coating has a smaller critical angle than the ^{58}Ni coating of the NG-6 guide. The mismatch lead to neutron losses of several percent. While the resulting reduction in beam flux was small, the number of "lost" neutrons was sufficient to produce unacceptably high backgrounds. In retrospect, even with a ^{58}Ni -coated secondary guide tube the losses due to imperfect guide alignment, surface roughness, etc. would likely scatter an unacceptably large fraction of the neutrons. Secondary guides may prove useful in future runs but only if they are kept away from the detectors where they can be adequately shielded.

5.2.2 Beam Associated Backgrounds

We found that the general environment of the guide-hall at the CNRF had a negligible impact on the detector backgrounds. With the beam stopped by the guide-end shutter the backgrounds in the detectors were essentially the same as those observed when the reactor was shut down. We did observe significant beam-related background in the plastic scintillators most of which appeared to be γ 's produced by the capture of neutrons that had been scattered out of the beam by the vacuum windows, collimators, and the air in the gaps in the beamline near the collimators. For the test run we employed little shielding against these backgrounds, preferring instead to reduce these backgrounds at their source. The singles rate in the scintillator paddle was used as the measure of the beam related background in our studies.

A γ energy spectrum taken with a germanium detector near the vacuum chamber is shown in Figure 5-3. Much of γ background was produced by scattered neutrons that were captured in the aluminum and stainless material near the detectors. Peaks from neutron capture on these materials are visible in the γ energy spectrum. There was also a signifi-

Figure 5-2: Beamline with guide tube. The guide tube was removed from the beamline after a few tests because it produced large backgrounds in the detectors.

cant component produced by captures in the silicon entrance window, which was exposed to the entire flux of the beam. An additional contribution came from neutrons captured in the borated plastic sheets surrounding the helium-filled flight tube. These sheets were used to stop neutrons that were scattered in the flight tube as far from the detectors as possible.

The beamline was surveyed with a ^3He proportional counter to determine the source of the scattered neutrons. They were observed in large numbers near the collimators and in lesser numbers near the beam stop. The beam passed through air gaps at both collimators. The number of scattered neutrons was reduced significantly when a helium filled bag was placed around the gap occupied by the downstream collimator. However, this yielded only a modest ($\approx 9\%$) reduction in the scintillator background rates.

Additional tests revealed that most of the γ background in the detectors was produced near the detectors. The contributions to the background rate in the paddle scintillator by the various sections of the beamline are given in Table 5-1. About $3/4$ of the background was produced by the beam after it had passed the second collimator. We determined that this background was proportional to the beam's area rather than its circumference, scattering from the edges of the circular collimators was not the dominant source of background. Instead, most of the background appears to have been produced by the passage of the beam through material in the beamline. The effects of directly placing material in the beam is shown in Table 5-2. It appears that much of the background could be eliminated by transporting the beam in vacuum. The places where materials must be placed in the beam, the entrance window and beam stop, should be kept as far from the detector as possible. We estimate that the background in the scintillator could be reduced by at least a factor of three by these measures and the incorporation of more shielding along the beamline.

Figure 5-3: Beam associated γ energy spectrum. The label " $B(n, \alpha)$ " indicates the peak from decay of the excited ${}^7\text{Li}$ state produced by the ${}^{10}\text{B}(n, \alpha){}^7\text{Li}^*$ reaction. The 511 keV positron annihilation photon peak is also labeled. The other labels indicate the material producing the peaks by prompt (n, γ) reactions. Single and double escape peaks are given the designation "S" and "D" respectively. The germanium detector was located about 30 cm radially from the center of the beam in the decay region.

Component of Background Rate	Paddle Scintillator Rate (counts/s)	Marginal Contribution to Beam Related Background Rate
Beam off	54	0%
Beam blocked with ^6LiF at 1st collimator	196	8%
Beam blocked with ^6LiF at 2nd collimator	520	18%
Normal beam	1855	74%

Table 5-1: Effects of passage of beam through sections of the beamline on the background rates in the scintillator paddle. The nominal beam on background rate is about 45% lower than in the coincidence runs because the face scintillator was partially ($\approx 1/3$) covered by lead bricks over the ends of the paddle.

Material	Increase in Scintillator Paddle Singles Rate
Additional Si window near 2nd collimator	+ 8%
Additional Si window in decay region	+ 300%
He-filled bag around 2nd collimator	- 9%
^6LiF beam stop in decay region	+ 8000%
Air-filled polarizer cell near 2nd collimator	+ 300%

Table 5-2: Effects of materials in the beam on background rates in the scintillator paddle. The increase in rate is given relative to the background produced by the normal beam.

5.3 Data Acquisition

Singles and coincidence data were taken with each detector under a variety of conditions. The conditions were varied to determine the sources of the detector backgrounds and to search for the neutron decay signal. By setting the focussing/acceleration voltage to zero, signals produced by decay protons in the PIN diodes were eliminated. Runs were taken for all four combinations of beam on/off and high voltage on/off. By comparing these runs, both the neutron beam and the acceleration voltage were observed to produce background in the detectors. Typical singles background rates are given in Table 5-3. The expected singles rates in each detector from neutron decay are given for comparison.

The beam-related background in the scintillator panel was discussed in

Source		Contribution to Singles Rate (counts/s)		
		Proton Panel	Scintillator Paddle	TPC Scintillator
Background	Detector Noise	30	90	140
	Focussing HV	50 ^a	0	0
	Beam	50	2300	3730
Signal	Neutron Decays ^b	5	15	3
Total (beam on, HV on)		135	2405	3873

Table 5-3: Contributions to the typical singles rates.

a. The rate was strongly dependent on the magnitude of the acceleration voltage and the electrode conditioning.

b. Calculated by the detector simulation for a beam activity of 220 decays/s.

Section 5.2.2. The beam also produced background in the scintillator backing the TPC and in the proton detector. While the singles rates in both scintillators were completely dominated by the beam-related γ background, the rate in the proton detector panel was more evenly divided among several sources: electronic noise, high voltage induced background, and beam related background. As expected, the proton detector alone was susceptible to backgrounds produced by the acceleration high voltage. This background was strongly dependent on the magnitude of the high voltage and the state of conditioning of the electrodes.

Nine of the proton panel's twelve PIN diodes were sensitive to protons from neutron decay. The data from two diodes could not be acquired due to failure of the readout electronics. The remaining diode was covered by a clear plastic sheet that carried a ^{119m}Sn calibration source. Either liquid nitrogen (-196°C) or a dry ice-ethanol bath (-78°C) was used to provide cooling to the proton detector preamplifiers. With liquid nitrogen cooling, the preamplifiers were maintained at about -130°C , and with the dry ice-ethanol bath they were kept at -40°C . It is believed that the PIN diodes and the rest of the paddle assembly were also cooled significantly. The diodes, in particular, were probably at about the same temperature as the preamplifiers. The leakage current in the PIN diodes is an indirect measure of their temperature. The current dropped to negligible levels ($< 1 \text{ nA}$) after the cool-

ing was applied. From manufacturer's literature, this implies the diodes were below -20°C [HAM01]. Only a minor reduction in noise was observed in the PIN diodes in passing from the dry ice to liquid nitrogen cooling.

In the test run, the TPC was used to examine its utility for reducing background in the electron detector. About a factor of ten reduction in the background singles rate was achieved when coincidence was required between the TPC wires and the scintillator backing the TPC. The prototype TPC was not used in the electron-proton coincidence measurements because of its low acceptance for decay electrons and the inability to acquire pre-trigger wire data.

5.4 Analysis of Singles Data

Energy spectra from both scintillators and the proton panel were taken under varying conditions. From Table 5-3, we see that the expected singles signal rates from neutron decays are low compared to the observed background rates. In the scintillators, the signal to background ratio is so low (≈ 0.006) that no attempt was made to extract the singles neutron decay signal from the background. This was, however, attempted for the proton detector. The rate in the detector was assumed to be composed of four independent rates

$$R = R_N + R_B + R_{HV} + R_P \quad (5-1)$$

where R_N is the rate due to detector noise, R_B is due to beam background, R_{HV} is associated with the focussing/acceleration voltage, and R_P is the signal rate due to protons from neutron decay. The signal can then be extracted by forming a linear combination of the rates for four data sets taken under different conditions

$$R_P = R_{11} - R_{01} - R_{10} + R_{00} \quad (5-2)$$

where R_{ij} on the right hand side is the rate with the beam on(off) for $i = 1 (0)$ and the acceleration voltage on(off) for $j = 1 (0)$. This analysis was attempted for each channel

of the proton panel for several sets of runs. The results showed variations in R_p , from Equation 5-2, between channels, and between different sets, far outside those expected from counting statistics alone. We conclude that there were variations from run to run in the experimental conditions that are not accounted for in Equations 5-1 and 5-2. Applying a high energy cut on the proton detector did not remedy this situation. This cut will be discussed below. Thus it appears that the signal rate cannot be extracted reliably from the proton detector singles runs.

The proton panel singles runs did provide some other useful information. The rate in the PIN diode covered by the plastic sheet was found to be independent of the acceleration voltage. The rates in the other channels increased rapidly as the potential was raised above about 30 kV. It was therefore determined that the background caused by the high voltage was from ions accelerated onto the PIN diodes by the focussing electrodes. In addition, the rate increase varied significantly from channel to channel indicating that the ions were produced by anomalies in the focussing electrodes rather than elsewhere in the system (e.g. by the vacuum ionization gauge).

5.5 Coincidence Data

A large number of runs were devoted to the study of coincidences between the electron and proton detector panels. A measurement of D would, of course, be based on this type of data. For each event the energies and relative timing in the electron and proton detectors were recorded. The delayed coincidence between the detection of the electron and proton from true neutron decay events proved valuable for distinguishing neutron decays from background. The calculated proton time-of-flight spectrum from a simulation of the test run detector geometry is shown in Figure 5-4. This spectrum does not include the effects of finite time resolution of the electronics, energy slewing caused by shaping in the biased amplifier, or the acceleration/focussing time.

During data taking, the event trigger required that the proton detector fire within a

5 μs time window opened by the firing of the scintillator. A histogram of the time delays between the scintillator paddle and the proton panel observed during one 2.60 hour run is shown in Figure 5-5. There are three interesting features in the histogram. At zero time delay, there is a peak corresponding to prompt coincidences. Since the highest energy recoil protons take at least 0.23 μs to reach the proton detector panel (see Figure 5-4), these coincidences are due entirely to background. The prompt events are produced mainly by beam-associated backgrounds, e.g., electromagnetic showers from high energy γ radiation, that produce signals in the electron and proton detectors simultaneously. The finite time resolution and energy slewing of the proton detector electronics are evident in the width and shape of the peak. The prompt events, indicated by the PR window in Figure 5-5, are easily rejected by placing time cuts on the data. The second feature is a broader peak due primarily to the detection of electrons and protons from neutron decays, indicated by PT in Figure 5-5. Finally, there is a flat background due to uncorrelated coincidences in the detectors, BG in the figure. The number of events in BG is significant since the true decay events in PT lie on top of the same background of uncorrelated coincidences.

Figure 5-4: Proton time-of-flight spectrum from a simulation.

Figure 5-5: Time delay spectrum for the scintillator paddle and proton panel. This spectrum represents 2.6 hours of data taking. No background subtraction or energy cuts have been applied. The time intervals below the graph represent the three classes of events, those in PR are prompt coincidences, many of those in PT are produced by neutron decays, and those in BG are from uncorrelated coincidences.

5.5.1 Data Reduction

The time windows in Figure 5-5 were used to place cuts on the data. In all cases, when cuts were applied to the data, the primary concern was to avoid rejecting any of the events produced by neutron decay. The primary goal of the coincidence runs was to determine the efficiency of the proton detector panel. Thus no attempt was made to optimize the signal to background ratio as one would in a measurement of D . (See Appendix B.) The positions and widths of the time windows were chosen based on the features of the data. The true signal rate is assumed to be given by the difference

$$R_S = R_{PT} - R_{BG} (t_{PT}/t_{BG}) \quad (5-3)$$

where $R_{PT}(R_{BG})$ is the integrated rate in the PT(BG) time window which is $t_{PT}(t_{BG})$ wide. The PT and BG windows shown have equal widths so $(t_{PT}/t_{BG}) = 1$.

Additional cuts based on the energy deposited in the scintillator and the pulse-height in the proton detector were applied to the data. The scintillator paddle and proton panel electronics each included a lower level threshold. While no additional rejection of events with low electron or proton energy was required, upper level energy cuts were applied in the analysis. The positioning of these cuts was determined from the energy spectra after background subtraction. The spectra before and after background subtraction for one run are shown in Figures 5-6 and 5-7. Each background-subtracted energy spectrum was produced by subtracting the spectrum of events with a time delay within the BG window, from the spectrum of events in the PT window. The arrows in the figures indicate the positions of the upper-level cuts. Events rejected by the electron upper-level energy cut have electron energies well past the beta endpoint (782 keV) so this cut should not eliminate any of the neutron decay signal. Although an accurate energy calibration of the PIN diodes was not possible, the proton upper-level energy cut is well past the peak produced by the 24 keV γ line from the calibration source on one of the diodes. Neither of these upper level cuts appear to reject true decay events. They are valuable for reducing the

amount of background in the PT window of the time spectrum. A time spectrum for a one hour run, with energy cuts, is given in Figure 5-8. Note that the peak corresponding to prompt coincidences is nearly eliminated by these cuts. Of more significance, the number of counts in the BG window is reduced, thereby increasing the signal-to-background (S/B) ratio, $r = R_S/R_{BG}$, from $r = 1.6$ to $r = 3.0$.

Figure 5-6: Proton pulse-height spectrum. The solid line is the spectrum after background subtraction. The dotted line is the uncut spectrum. The peak in the uncut spectrum near channel 3100 was caused by saturation of the amplifier. These spectra represent 2.6 hours of data taking.

5.5.2 Evidence for Condensation on the PIN Diodes

The cooling applied to the preamplifiers in the proton panel also reduced the temperature of the PIN diodes. This cooling caused condensation, presumably water, to accumulate on the surface of the diodes, increasing the energy loss of the protons as they entered the diodes. The buildup of the condensation was observed as a gradual reduction in R_S over time. See Figure 5-9. A large decrease in signal was also observed when the source of the preamplifier cooling was changed from the dry ice/ethanol bath to the liquid nitrogen bath. The signal rate in these runs was lower than that in the best runs because the

Figure 5-7: Scintillator paddle energy spectrum. The solid line is the spectrum after background subtraction. The dotted line is the uncut spectrum. These spectra represent 2.6 hours of data taking.

Figure 5-8: Time delay spectrum after energy cuts. The solid line is the time spectrum after upper-level energy cuts were applied to the proton and electron energies. The dotted line is the spectrum with no cuts. These spectra represent one hour of data taking.

acceleration voltage was insufficient to put all of the protons' signals above the energy

threshold. This was advantageous for this test because a small increase in proton energy loss would produce a relatively large decrease in signal rate.

Figure 5-9: Coincidence rate as a function of time under cooling. The type of cooling applied is shown by the time intervals near the bottom of the graph.

5.5.3 Determination of the Proton Panel Focussing Efficiency

A lower limit on the detection efficiency of the proton panel was determined from the measured signal rate R_S . Although many coincidence data runs were performed, only the one, taken under the conditions most favorable to producing the largest R_S , was considered in the determination of the proton panel's efficiency. Two measures were taken in the attempt to maximize the signal. The acceleration voltage was set as high as possible (35 kV) and the run was taken shortly after (about 7 hours) the preamplifier cooling was applied. The latter was an effort to minimize the amount of condensation on the surfaces of the cool PIN diode. The resulting 1 hour run yielded

$$R_S = 0.616(17) \text{ s}^{-1} \quad (5-4)$$

with a S/B ratio of 3.0(2).

This rate is compared to the expected coincidence rate to determine the proton panel efficiency. The expected rate R_E is the product of several contributing factors,

$$R_E = R_{\text{decay}} \epsilon_{\text{coinc}} \epsilon_e \epsilon_p C_{\text{electronic}} \quad (5-5)$$

where R_{decay} is the decay rate in the detector, ϵ_{coinc} is the geometric coincidence efficiency, $\epsilon_e (\epsilon_p)$ is the electron(proton) panel singles detection efficiency, and $C_{\text{electronic}}$ accounts for deadtime and other losses in the electronics. Values for these factors are given in Table 5-4. The electron detector efficiency can be further decomposed as

$$\epsilon_e \approx (1 - f_{e\text{-thresh}}) (1 - f_{e\text{-BS}}) \quad (5-6)$$

where f_{BS} is the fraction of electrons backscattered by the scintillator, and f_{thresh} is the fraction of coincidence electrons cut by the energy threshold. Similarly, the proton panel efficiency is given by

$$\epsilon_p \approx (1 - f_{\text{mesh}}) (1 - f_{p\text{-thresh}}) (1 - f_{p\text{-BS}}) \epsilon_{\text{focus}} \quad (5-7)$$

where f_{mesh} is the fraction of decay protons lost in traversing panel's entrance mesh, $f_{p\text{-thresh}}$ is the fraction of protons cut by the energy threshold, $f_{p\text{-BS}}$ is the fraction of protons backscattered from the PIN diodes, and ϵ_{focus} is the focussing efficiency.

Many of the factors contributing to Equation 5-5 were determined for the test run setup. The decay rate in the beam, R_{decay} , was determined from the measured beam capture flux and a simulation of the neutron beam transport through the collimation and detector. The geometric coincidence efficiency, ϵ_{coinc} , was calculated from a simulation of the test run detector. The correction for deadtime and other losses due to the data acquisition electronics was determined from the scaler data. Some of the factors contributing to the

Contributions to Expected Coincidence Rate	Factor
Decay rate ^a , R_{decay}	$220(23) \text{ s}^{-1}$
Coincidence efficiency ^b , ϵ_{coinc}	$6.3(2) \times 10^{-3}$
Paddle scintillator efficiency, ϵ_e :	
Electron energy threshold, $(1 - f_{e\text{-thresh}})$	0.75(8)
Backscattering from Scintillator, $(1 - f_{e\text{-BS}})$	—
Proton panel efficiency, ϵ_p :	
Proton panel entrance mesh, $(1 - f_{\text{mesh}})$	0.96(1)
Proton energy threshold ^c , $(1 - f_{p\text{-thresh}})$	—
Backscattering from PIN diodes, $(1 - f_{p\text{-BS}})$	—
Electronics (deadtime, etc.), $C_{\text{electronics}}$	0.810(6)
Expected coincidence rate ^d	$0.81(16) \text{ s}^{-1}$

Table 5-4: Determination of the expected coincidence rate.

- a. Uncertainty includes a 10% allowance for fluctuation in beam intensity over the reactor cycle.
- b. Includes the effects of the three missing detector channels.
- c. Accurate energy calibration of the PIN diodes could not be obtained due to problems with the electronics.
- d. The unknown factors, designated by “—”, were not included.

individual detector efficiencies were measured. Among these is $f_{e\text{-thresh}}$ which accounts for losses due to the scintillator paddle energy threshold. It was determined from the measured energy threshold, and the simulated coincidence electron energy spectrum shown in Figure 5-10. The approximate energy resolution function was convoluted with this spectrum in the calculation, although doing so was found to produce a negligible effect. The factor f_{mesh} , accounting for the loss of protons in the mesh covering the entrance of the proton detector panel, was estimated from the measured fraction of open to filled area. Some of the factors were not accounted for. They include backscattering of protons from the PIN diodes, $f_{p\text{-BS}}$, and of electrons from the scintillator paddle, $f_{e\text{-BS}}$. Both of these are believed to be small, with $f_{p\text{-BS}}$ less than 2% [Zei91], and $f_{e\text{-BS}}$ a few percent [Tit70],

but we did not measure them. The effect of the proton energy threshold $f_{p\text{-thresh}}$ could not be determined without an accurate energy calibration.

Figure 5-10: Calculated coincidence electron energy spectrum. The electrons are detected in coincidence with protons in the test run detector.

The known factors in Equation 5-5 can be used to place limits on the unknown factors, in particular, on those contributing to the proton segment efficiency. The expected rate, including only the known factors, is

$$R_E = 0.81 \pm 0.16 \text{ s}^{-1}. \quad (5-8)$$

Comparing this rate to measured coincidence rate we have

$$\frac{R_S}{R_E} \approx 0.76 (20). \quad (5-9)$$

If we assume that the difference between R_S/R_E and 1 is due entirely to unaccounted for inefficiencies or losses in the proton panel, we have

$$\epsilon_{\text{focus}} (1 - f_{p\text{-BS}}) (1 - f_{p\text{-thresh}}) = 0.76 (20) \quad (5-10)$$

and the important result

$$\epsilon_{\text{focus}} \geq 0.76 \quad (20) \quad (5-11)$$

which is consistent with the 99.5% focussing efficiency calculated in Chapter 4.

The signal rate in a run taken under the same conditions but two hours after the one yielding the result in Equation 5-7 was

$$R_S = 0.544 (10) \text{ s}^{-1}. \quad (5-12)$$

The 12% reduction in signal rate was due to additional condensation on the PIN diodes. This indicates that some fraction of the proton signal was likely below the energy threshold in the first run as well. Thus, from Equation 5-10, we see that ϵ_{focus} was likely closer to 1.0 than 0.76. In fact, the majority of the unaccounted for inefficiency in the proton detector panel may have been the result of $f_{\text{p-thresh}}$. In the future, it is expected that $f_{\text{p-thresh}}$ can be eliminated by reducing the proton detector energy threshold. This will require lowering the noise in the detectors and the energy loss due to the condensation on the PIN diodes. The noise will be lowered by replacing the biased amplifier and by using better preamplifiers. The condensation will be reduced or eliminated by controlling the temperature of the diodes.

5.6 Conclusion

The test run produced many useful results. Valuable information that will be used in designing the neutron transport for future runs was obtained. Of particular importance was the determination that a reasonably high coincidence S/B ratio (≈ 3) could be obtained with a minimum of shielding using only plastic scintillator for the electron detection. While the achieved S/B ratio would be sufficient for a measurement, our beamline studies indicate that a reduction in the beam related γ background in the scintillators by a factor of 3 is possible. This would bring the S/B ratio to over 10, assuming a proportional reduction

in beam associated proton detector background. Such a reduction in the S/B ratio would be sufficient to nearly eliminate the sensitivity advantage of including a wire chamber component in the electron detector for background rejection. Thus we have decided that for the initial runs a detector employing scintillator alone will be used.

In order to achieve the desired increase in the S/B ratio several steps will be taken in the design of the beamline in future runs. The background studies confirmed that materials should be kept out of the direct beam. The places where this is unavoidable, e.g. the window on the entrance to the vacuum system and the beam stop, should be kept as far from the detectors as possible. In the future, the beam will be transported as far as possible in vacuum. The collimators will be inside the vacuum. As in the test run, the collimators and beam stop will be made from ^6Li material because neutrons captured in ^6Li do not produce background in the detectors. A further goal is to coat any surface that has a line of sight to material in the beam with ^6Li material as well. In addition, provision will be made for reducing the γ backgrounds by placing shielding walls near the detectors and annular shields behind the collimators.

The detector system performed well. However, the test run revealed several places where improvements should be made to both the electron and proton detectors. The most troublesome aspect of the proton detector panel implementation for the test run was the amplification electronics. Happily, the problems there should be relatively easy to remedy. We found that using a biased amplifier to multiplex the PIN diode signals was unsatisfactory. Either each channel will be given separate amplification electronics, or a new multiplexing scheme, more appropriate for signals close to the noise threshold, will be employed. Reducing the noise by improving the PIN diode preamplifiers will also allow the energy thresholds on the PIN diodes to be lowered. In this way, it should be possible to enhance the proton panel detection efficiency by reducing (or eliminating) the number of focussed protons rejected by the threshold.

Also of concern is the observed buildup of condensation on the surfaces of the PIN

diodes. Because the protons lose some of their energy to the layer of condensate, the signal in the diodes is reduced. We are considering two means of reducing the rate of condensation build up. First, the temperature of the PIN diodes will be controlled. In the test run they were allowed to become quite cold through passive connection to the preamplifier cooling. By keeping the diodes at a higher temperature the rate of condensation buildup will be reduced. In addition, they could be warmed periodically to drive off the condensate. Second, the vacuum inside the detector system will be improved by adding cryopumping and improving the vacuum compatibility of the proton panel electronics and other materials that reside inside the vacuum system. Special attention will be given to eliminating water from the system. It is hoped that a UV lamp can be used to speed the desorption of water from the vacuum vessel, since baking of the system is not feasible.

The lack of an in situ energy calibration system prevented us from determining how much energy the focussed protons were depositing in the diodes during the test run. Such a system will be added. In addition, the system will include α sources that will be used to measure the energy loss in the dead layer (including any condensate) on the surfaces of the diodes.

There are a few improvements that will be made to the scintillator paddle design over the one tested in the run. Position sensitivity along the paddle's length will be achieved by measuring the relative timing between the signals from either end of the paddle. The scintillator paddles will be removed from their test run role as a gas-to-vacuum interface. The stresses this produced were found to cause crazing of the scintillator material with a consequent degradation of the resolution. In future runs, the scintillator will reside inside the vacuum chamber and light guides will be used pipe the light out of the vacuum vessel. The integrating ADC used to measure the energy will also be replaced as it contributed unduly to the deadtime of the electronics.

The test run was successful in demonstrating the viability of both the general measurement design and the designs of the individual detectors. With a few modifications, the

designs of the tested scintillator paddle and proton panel will be used to implement eight detectors panels required in the octagonal design to measure D .

Chapter 6: Systematic Effects

6.1 Introduction

The systematic effects in this experiment fall into two categories: (1) effects that are proportional to D and (2) false effects that can arise even if the true value of D is zero. Since D is known to be small, effects of type (1) are of lesser importance. They contribute to the uncertainty in D but will not cause a false nonzero D to be observed. Effects of the second type must be carefully guarded against.

In this chapter the octagonal geometry and analysis method described in Chapter 3 are assumed. The scheme for analyzing the data from the experiment exploits the four-fold symmetry of the detector in eliminating or reducing the effects of systematic differences in the individual detectors. The analysis method is based on the formation of a ratio of numbers of coincidences, R in Equation 3-19. Using R , an estimation of the types and sizes of the systematic effects that remain is made. Experimental indications of the presence of false effects are also given.

In Section 3.3.3 it was noted that the beam intensities, individual detector efficiencies, and solid angles cancel in R for each of the two neutron polarization directions. Consequently, many factors, such as unequal detector efficiencies, which one might think would contribute to the observation of a false nonzero value of D , are insignificant. However, this analysis assumed a model in which, for example, the detector efficiencies are uniform, the distribution of beam polarization direction is azimuthally symmetric, etc. Deviations from this ideal are possible in practice. In most cases these deviations make a negligible contribution to false effects in the measurement.

Systematic effects eliminated by the symmetry of the detector and analysis method as well as some of the less important deviations from the ideal model will be considered in Section 6.2. The two particular instances in which deviations from the ideal model could make a significant contribution are discussed in subsequent sections. These effects were

observed in a previous experiment [Ero78].

6.2 Effects Proportional to D

From Equation 3-23 we see that the uncertainties σ_K and σ_P , by virtue of the known smallness of the coefficient D that multiplies them, make a negligible contribution to the overall uncertainty σ_D . They contribute only if the coefficient D is nonzero and therefore cannot produce the observation of a false nonzero D coefficient.

6.2.1 Contributions to σ_K

Contributions to σ_K include:

1. Statistical uncertainty in the Monte Carlo calculation of K .
2. Determination of the decay volume. Including misalignment of the beam with respect to the detector axis or nonuniform neutron density within the beam.
3. Backscattering from the electron and proton detectors.
4. Energy-dependent detector efficiencies.
5. Distortions of the particle trajectories due to magnetic and electric fields.

(1.) The Monte Carlo is sufficiently fast that the statistical uncertainty can be reduced to 1% relatively easily.

(2.) In general, Monte Carlo calculations of K show that it is rather insensitive to even gross deviations of the overall detector geometry from the idealized octagonal detector. Comparison of K for the different detector slices discussed in Section 3.3.4.2 show that decay volume end effects are negligible. Likewise displacing and misaligning the beam with respect to the detector axis over reasonable ranges ($\approx 1\text{cm}$) produces no change in the calculated K to within the few percent accuracy of the calculations. Thus this contribution to σ_K/K is at most a few percent.

(3.) Since backscattering from the detectors has not been explicitly accounted for in the Monte Carlo simulation, it can contribute to the uncertainty in K but only insofar as

the backscattering is angle or energy dependent. An overall backscattering fraction can be accounted for within the overall detector efficiencies that cancel in the analysis ratio Equation 3-19. The backscattering of the accelerated protons from the PIN diode detectors used in the prototype proton detector has been calculated by the TRIM90 code of Ziegler [Zei91] to be less than 2%. Some of the backscattered protons are lost in collisions with the walls of the focussing tube. The energies of protons reflected back into the PIN diode by the focussing field are degraded by multiple encounters with the diode's surface dead layer so many are cut by the energy threshold. The wire mesh that defined the entrance to the proton detector focussing cells was 94% open. Together we estimate that at most 8% of the protons incident on the proton focussing cell within the edge masking could be lost. Based on experiments measuring the backscattering of electrons by plastic scintillator [Tit70] we estimate that only a few percent of the incident electrons are lost to backscattering. K as given by Equation 3-28 is essentially a ratio of numbers of coincidences. The greatest possible influence backscattering could have on K would be for either the numerator or denominator alone to be decreased by the backscatter fraction. Thus we can place an upper limit of 8% on this contribution to the uncertainty in K .

(4.) and (5.) are estimated to contribute at most 10% to the uncertainty in K though they are difficult to calculate exactly. The effects of finite energy resolution and thresholds were simulated by varying the lower level energy cuts on both the electrons and protons in the Monte Carlo simulation. Variations of ± 100 keV for the electrons and ± 100 eV for the protons produce changes in K smaller than 10%.

Thus we estimate σ_K/K is at most 10%. With present limits indicating $D < 10^{-3}$ the contribution of the uncertainty in K to σ_D in Equation 3-23 is below 10^{-4} and not important unless D of the order 10^{-4} is found.

6.2.2 Contributions to σ_P

Contributions to σ_P include:

1. Drifts in polarization or spin flipping efficiency.
2. Nonuniform polarization or spin flipping efficiency (not including transverse components of the polarization).
3. Uncertainty in the determination of the average beam polarization P .
4. Imperfect spin flipping.

(1.), (2.) and (3.) depend on the type of polarizer used. The current state-of-the-art in neutron spin polarization are “supermirror” polarizers which typically produce over 98% neutron polarization [Sch89a]. As a result of this high degree of polarization, uncertainties due to drifts, nonuniformities and uncertainty in the determination of the degree of polarization are limited to a few percent.

(4.) Practical devices for reversing the direction of the polarization of a beam of cold neutrons, known as “spin flippers”, are highly efficient. For example, Liaud *et al.* constructed a flipper with an efficiency of 97% [Lia75]. However, a flipping efficiency differing from unity implies that there are two values $P^\uparrow$ and $P^\downarrow$ for the magnitude of the mean polarization in Equation 3-21 which, following Steinberg *et al.* [Ste76], becomes

$$R = \frac{(1 + KP^\uparrow D)^4 (1 + KP^\downarrow D)^4}{(1 - KP^\downarrow D)^4 (1 - KP^\uparrow D)^4}. \quad (6-1)$$

Neglecting terms of higher order in D and defining

$$\bar{P} = \frac{1}{2} (P^\uparrow + P^\downarrow) \quad (6-2)$$

Equation 6-3 can be solved for D

$$D = \frac{1}{2K\bar{P}} \frac{(R^{1/4} - 1)}{(R^{1/4} + 1)}. \quad (6-3)$$

The smallness of D implies that R is near unity and may be written as $R = 1 + \delta$ where $|\delta| \ll 1$. Applying the binomial expansion to the right-hand side of Equation 6-3 yields

$$D \approx \frac{\delta}{16K\bar{P}}. \quad (6-4)$$

The same expansion applied to the right-hand side of the usual expression for D Equation 3-22 gives the same result except that $\bar{P}$ on the right-hand side of Equation 6-4 is replaced by P . Thus this contribution is eliminated by using $\bar{P}$ instead of $P = P^\uparrow$ and $P^\downarrow$ in Equation 3-22.

In summary, σ_P/P is less than 5% and is not important unless D of the order 5×10^{-5} is found.

6.3 False Effects

In this section we describe the sources of possible false effects. Since these effects are not proportional to the true value of D , they can arise even if no T-violation is present. We also estimate the characteristic size of each source that would produce a false D of the order 10^{-4} .

6.3.1 Minor Effects: Contributions to σ_R

There are a number of systematic contributions to R that are not proportional to D but are, nonetheless, zero or very small. They will be considered here for completeness. They can be characterized as a contributions to σ_R in Equation 3-23.

Contributions to σ_R include:

1. Counting statistics.
2. Dependence of the detector efficiencies or beam intensity on time, or beam polarization flipping.
3. Unequal counting times for the two beam polarization states.
4. Overall misalignment of the beam polarization with respect to the detector symmetry axis.

5. Radial components of the beam polarization (i. e. “Diverging” or “converging” beam polarization).

(1.) As shown in Equation 3-24 this contribution is proportional to the inverse of the square root of the total number of coincidences counted. This contribution to σ_R is not the result of a systematic uncertainty.

(2.) Each of the individual detector efficiencies cancels exactly for each of the polarization states in the analysis ratio Equation 3-19 so the effects of unequal overall efficiencies do not contribute to a systematic error.

(3.) Unequal counting times for the two polarization states can be subsumed within the polarization dependent constants proportional to the beam intensity $C^\uparrow$ and $C^\downarrow$ which also cancel in the analysis ratio thereby eliminating this contribution.

(4.) and (5.) have been investigated in the course of the Monte Carlo calculations of Section 6.3.2 and were found to produce no false nonzero D even when they are grossly exaggerated. The largest contribution to σ_R/R from the factors listed above is therefore statistical.

None of the uncertainties discussed in this section will lead to the observation of a false nonzero value of the D coefficient. Of the factors listed in this section, the statistical uncertainty dominates the uncertainty in the measured D coefficient.

6.3.2 Polarization Nonuniformities

6.3.2.1 False Effect Due to Asymmetric Transverse Components of the Neutron Polarization (ATP Effect)

Nonuniformities in beam polarization direction, specifically asymmetric distributed transverse components of the polarization, can lead to the observation of a false nonzero value of D . This effect, which we will designate the ATP (for Asymmetric Transverse Polarization) effect, arises as the result of the combination of two factors.

First, imperfect alignment between the neutron polarization and the direction of

the detector axis (the z axis in Figure 3-10) breaks the azimuthal symmetry in the detection of decay protons. The large neutrino-spin correlation ($B \approx 1$ in Equation 2-10) causes neutrinos from polarized neutron decay to be emitted preferentially along the direction of the polarization. This, combined with the limited available phase space, causes the proton momenta to be anti-correlated with polarization direction. Thus a component of the neutron polarization transverse to the beam direction, e.g. toward electron detector e_2 in Figure 6-1, increases the count rate in proton detectors p_4 and p_3 while decreasing the count rate in detectors p_1 and p_2 . However, because of the symmetry of the detector, an *overall* polarization misalignment such as this will not cause a false effect.

Figure 6-1: Detector with a neutron beam that has a uniform transverse component to the neutron polarization. The neutron polarization is misaligned by a small angle ξ from the direction of the detector axis (into the page).

For the ATP effect to arise, a second, confederate, factor must be present. The distribution of neutrons with the transverse polarization component must be asymmetric across the cross section of the beam. More precisely, the polarization density

$$M(x) = \rho(x)P(x), \quad (6-5)$$

where $\rho(x)$ is the neutron number density and $P(x)$ is the neutron polarization, must break azimuthal symmetry.

Naturally, different detectors present different solid angles to decay products originating from different parts of the neutron beam. The importance of this fact is best illustrated through an example. Suppose that only the top half of the beam has a transverse polarization component and that it is pointing toward the electron detector e_2 . (See Figure 6-2.) More electrons from decays in the top half of the beam will be detected in electron detector e_1 than in e_3 . The proton-spin anti-correlation described above will cause more e_1-p_3 than e_1-p_2 coincidences. At the same time, the number of e_3-p_4 coincidences similarly increases, but, because of the lesser solid angle that e_3 presents to the top half of the beam, the increase will be smaller. Thus the ratio $N_{e_1 p_3}^\sigma / N_{e_3 p_4}^\sigma$ does not completely cancel in Equation 3-19 even if $D = 0$.

Figure 6-2: Detector with a neutron beam that has an asymmetric transverse component to the neutron polarization. The neutron polarization in one half of the beam is misaligned by a small angle ξ from the direction of the detector axis (into the page).

Several things should be noted about this somewhat subtle effect. If the transverse polarization component remains constant when the overall neutron polarization is

reversed, then the effect disappears. Returning to our example, when the neutrons are polarized $\sigma = \uparrow$, the number of e_1-p_3 coincidences $N_{e_1 p_3}^\uparrow$ appears in the numerator of the ratio in Equation 3-19, but the e_1-p_3 coincidences appear in the denominator for the reverse polarization. An increase in e_1-p_3 coincidences independent of σ would cancel in the ratio.

The magnitude of the ATP effect depends on the diameter of the beam. A larger beam allows regions of transverse polarization to exist farther from the axis of the detector. The further these regions are from the axis the greater the difference in solid angle opposing detectors present to the decay products from those regions. In addition, transverse polarization components tend to be larger off axis due to neutron polarization transport effects.

The magnitude of the ATP effect depends upon the magnitude of the transverse polarization components. More precisely it depends on the sine of the angle made by the direction of the detector axis and the neutron polarization. However, the effect cannot arise unless the density of transverse polarization components across the beam is asymmetric. For example, a distribution of polarization with a radial component, such as that shown in Figure 6-3, would not produce a false nonzero triple correlation. Such a distribution might be produced by the neutron polarization following the diverging or converging of the magnetic "guide" field that maintains the neutron polarization direction.

6.3.2.2 Estimate of the Magnitude of the False Asymmetry Caused by the ATP Effect

Monte Carlo simulation has been used to estimate the magnitude of the false nonzero value of D that could be produced by the ATP effect. In order to perform these calculations, we must first define the distributions for which we wish to calculate the magnitude of the effect.

In general, the neutron polarization can depend on position in the beam. In the ideal case the polarization is along the z -axis in Figure 3-10, i.e. $P(x) = P\hat{z}$. More gener-

Figure 6-3: A distribution of polarization with a radial component, as shown here, would not cause the ATP effect.

ally, the polarization makes an angle ξ with the direction of the detector axis so the polarization has a transverse component, $P \sin \xi$. The ATP effect can arise when the distribution of polarization directions across the beam's cross section has a particular asymmetry. We can define a quantity, call it L_z since it is analogous to angular momentum, that expresses the existence of the required asymmetry. First, define the distribution

$$\Lambda(\mathbf{r}) = \mathbf{r} \times \mathbf{M} = \mathbf{r} \times (\rho \mathbf{P}) \quad (6-6)$$

where $\mathbf{r}$ is the position vector. The origin of the coordinate system lies on the detector axis so the z -coordinate is co-axial with the axis of symmetry of the detector. The coordinate system is shown in Figure 3-10. Defining L_z to be the z component of the integral of the distribution Λ over all position space we have

$$L_z = \hat{z} \cdot \int (\mathbf{r} \times \mathbf{P}) \rho d\mathbf{r} . \quad (6-7)$$

A neutron beam that has a nonzero L_z breaks symmetry in the way that produces a false D through the ATP effect.

Polarization Twist Model

There is a simple distribution of transverse polarization components that would produce a particularly large false D coefficient via the ATP effect. The beam is divided into semi-cylinders. The polarization in the half of the beam closest to one of the electron detector panels has a transverse component parallel to the face of that electron detector. The polarization in the opposite half of the beam has a transverse component in the opposite direction. An example is shown in Figure 6-4. This polarization “twist” represents an extreme case so it is valuable as a means of placing an upper bound on the size of the resulting systematic effect. There are other distributions of transverse components that could also make a contribution, albeit a smaller one, to the ATP effect.

Figure 6-4: The pathological case of a polarization “twist”. The A and B halves of the beam have opposing transverse components of the neutron polarization shown by the arrows.

The magnitude of the systematic effect produced by the polarization twist has been estimated via Monte Carlo simulation. The Monte Carlo was used to simulate decays from the probability distribution in Equation 2-10 with D set to zero and the neutron polarization parallel to $\hat{z}$. As the events were generated the momenta of the proton and electron were rotated by an angle ξ away from $\hat{z}$. The direction of the rotation depended on

whether the decay occurred in the top or bottom half of the beam. The electron and proton were then tracked normally and the usual analysis ratio was computed. The size of the systematic effect is the value for D , which we will call D_{ATP} , extracted from Equation 3-30 using the value computed for K in Section 3.3.4.2.

As demonstrated by Equation 3-24, the statistical uncertainty in the value of D calculated by this method is inversely proportional to the square root of the number of decays tracked. The computer calculation time required to reduce the statistical uncertainty below 10^{-3} rapidly becomes prohibitive. The factors that contribute to a systematic value of D must be exaggerated in the simulations. Consequently, it is important to understand the functional dependence of D_{ATP} on those factors and verify this in the simulation.

As discussed above, the size of the ATP effect is expected to vary linearly with the magnitude of the transverse polarization component, which is proportional to $\sin\xi$. This has been tested in the simulation for several values of ξ ranging from 36° to -36° with a uniformly populated cylindrical beam 6 cm in diameter. The resulting values of D_{ATP} are a good fit to a line with slope 0.130 ± 0.006 . Thus, this “worst case” can produce $D_{\text{ATP}} = 10^{-4}$ for a twist angle ξ of the order of 1 mrad.

The simulation of the polarization twist was performed for several other beam diameters. The difference in solid angle subtended by an electron detector (e.g. e_1 in Figure 6-4) for the half of the beam nearest to it (A) as compared to the far half (B) is approximately proportional to the beam’s diameter. This difference in solid angle for parts of the beam with different transverse components is, as discussed above, a necessary component of the systematic effect. The results of simulations with a polarization twist with $\xi = \pm 36^\circ$ for several beam diameters bears this out showing that the size of the D_{ATP} is $4.5 \times 10^{-3} \text{ cm}^{-1}$ times the beam diameter, vanishing as the diameter goes to zero.

Uniform Transverse Component and Asymmetric Beam

The asymmetry of transverse beam polarization required to produce the ATP effect can result from a misshapen or displaced beam that has an overall transverse polarization component. In Equation 6-7 this case is represented by a constant polarization misalignment so that $\mathbf{r} \times \mathbf{P} \propto \hat{\mathbf{u}} \sin \xi$, where $\hat{\mathbf{u}}$ is a constant vector that makes an angle ξ with $\hat{\mathbf{z}}$, and a neutron number density $\rho(\mathbf{x})$ that is asymmetric about the detector axis. Examples are shown in Figures 6-5A and 6-5B. The size of the systematic effect produced is again proportional to the magnitude of the transverse component of the neutron polarization $P \sin \xi$. It also depends on the degree to which the beam is asymmetric about the detector axis. This dependence arises from the difference in solid angle presented by opposing electron detectors to the asymmetric beam.

Figure 6-5: Examples of neutron beams that are not symmetric about the detector axis and have a uniform transverse polarization component. In (B) the asymmetry is the result of the beam being displaced from the detector axis.

The effects of general beam asymmetries can be reasonably estimated simply by

displacing the usual cylindrical beam away from the detector axis in the Monte Carlo Simulation. The functional dependence of D_{ATP} on beam displacement is difficult to calculate analytically so we will not assume that large extrapolations of the size of the effect as a function of the beam displacement can be made. It is possible to calculate the solid angle an electron detector presents to a point in the beam. The difference in solid angle presented by opposing electron detectors to a point displaced from the detector axis toward one of the detectors is, for displacements of a few centimeters, nearly proportional to the displacement. The magnitude of D_{ATP} was likewise found to be proportional to the beam displacement. Several extreme cases of beam displacement were simulated to place an upper limit on the size of this contribution to the ATP effect. The polarization was rotated 36° away from the detector axis toward one of the electron detectors. The beam was displaced both perpendicular and parallel to the resulting transverse polarization component by several distances. By symmetry, displacements parallel to the transverse component of the polarization should not produce the effect, though perpendicular displacements should. This has been confirmed in the simulations. (See Table 6-1.)

Beam Displacement (cm)		$D_{\text{ATP}} (\times 10^{-4})$	
Perpendicular	Parallel	$\xi = 36^\circ$	$\xi = 1^\circ$
0.1	0.0	35 ± 13	1.0 ± 0.4
0.5	0.0	59 ± 13	1.7 ± 0.4
1.0	0.0	164 ± 13	4.8 ± 0.4
2.0	0.0	320 ± 40	10.0 ± 1
0.0	1.0	50 ± 40	1.0 ± 1
1.0	1.0	220 ± 40	7.0 ± 1

Table 6-1: False value of the D coefficient induced by displacing a 6 cm diameter beam that has an overall polarization misalignment angle ξ away from the direction of the detector axis. The beam was displaced both perpendicular and parallel to the transverse polarization component resulting from the polarization misalignment. The false value of D is also given scaled down to a polarization misalignment of 1° . The uncertainties are from the limited number of events tracked in the Monte Carlo.

A large 36° polarization misalignment was used in order to facilitate the Monte Carlo simulation of this contribution to the ATP effect. Since the size of D_{ATP} is propor-

tional to the sine of the misalignment angle, the effect can be scaled down to smaller more probable misalignments. The values of D_{ATP} scaled down to $\xi = 1^\circ$ are given in the right-most column of Table 6-1. The size of a possible false D is kept to the 10^{-4} level for misalignments of 1° or less with displacements less than 5 mm.

Many other simulations were performed in which varying transverse components were introduced to the two halves of the beam. The results of all the simulations were consistent with the description of the source of the ATP effect described in Section 6.3.2.1 and the size of the effect scaling with $\sin \xi$. None of the distributions other than those related to the examples presented above were found to induce significant systematic effects. In particular a symmetric beam with uniform transverse polarization component did not produce a nonzero D_{ATP} .

6.3.2.3 Possible Sources of Asymmetric Transverse Polarization Components

Unless there is a transverse component to the neutron polarization the ATP systematic effect cannot arise. There are mechanisms by which the polarization could take on a transverse component. The neutron polarization is held by a magnetic "guide" field of a few gauss parallel to the axis of the detector. A misalignment between the detector and the guide field by an angle ξ would produce an overall transverse component of the field that in turn would produce an overall transverse component of the neutron polarization. In order to produce the systematic effect there would also have to be a nonuniform neutron distribution or displacement of the beam. As shown in the Section 6.3.2.2, even if ξ were as large as 1° the displacement of the beam would have to be at least a few millimeters for the effect to approach the level of 10^{-4} in D . In the experiment we expect to achieve $\xi < 1^\circ$ and displacements of less than 1 mm.

In Section 6.3.2.2 it was shown that variations in the polarization direction across the beam can lead to systematic contributions to the D coefficient. This in turn could be caused by nonuniformities at the 1 mrad level in the direction of the magnetic guide field.

Stray fields or distortions of the guide field by magnetic material that vary by a few milligauss over the diameter of the beam could produce these nonuniformities. In general, these distortions fall off very rapidly with distance from the source of the distortion. In the experiment the use of materials that could distort the field will be minimized and those that are required will be kept as far as possible from the beam. Contributions by the earth's field or other external fields will be reduced through the use of magnetic shielding.

It is possible for the neutron polarization to deviate from the direction of the guiding magnetic field if the field rapidly changes direction. A rotation of the guide field with angular frequency ω_G , in the neutron's rest frame, will produce an angle between the guide field and the polarization of order ω_G/ω_L where ω_L is the Larmor frequency of the neutron spin precession in the field. The distribution of the neutron velocities in the beam causes the average precession angle induced by such a rotation to gradually dissipate since the phase of the precession depends on the neutron velocity. Reversal of the guide field direction will reverse the direction of the precession and thereby the sign of any systematic contribution to the D coefficient. The guide field can be reversed occasionally during data taking to cancel this possible contribution to the ATP effect.

As stated above, the ATP effect is proportional to the beam diameter while the true value of the triple correlation is, of course, independent of this parameter. An increase in the measured asymmetry as the beam diameter was increased would signal the presence of the ATP effect.

The Monte Carlo calculations in Section 6.3.2.2 could be checked by artificially creating an asymmetric transverse component to the neutron polarization. For example, the ATP effect could be intentionally produced by misaligning the guide field from the detector axis by an angle $\xi = 10^\circ$ and displacing the beam from the detector axis by 1 cm. This should produce $D_{\text{ATP}} = 0.05$, which would be easily observable in a short time.

6.3.3 Detector Efficiency Nonuniformities

As discussed in Section 6.3.2.1, the momentum of the neutrino is correlated with the neutron spin through the B term in Equation 2-10 and anti-correlated with the proton momentum by phase space. When coincidence between the proton and electron is required, these correlations skew the distributions of both the positions and angles of incidence of the electrons and protons on the faces of the particle detectors. If the detectors' efficiencies are position or angle dependent, then the additional particle correlations can give to a false nonzero value of D .

To see this recall that in moving from Equation 3-19 to Equation 3-21 complete cancellation of the individual detector efficiencies was assumed. The cancellation was possible because, although the detectors' efficiencies were assumed to depend on the polarization direction, σ , the efficiencies were assumed to be independent. Nonuniform detection efficiency of the particle detectors coupled with the phase space anti-correlation between the electron and proton momenta would spoil the cancellation of the detector efficiencies by making them depend on identities of *both* of the detectors counting the coincidence. In other words the individual detector efficiencies $\epsilon_{e_i}^\sigma$ or $\epsilon_{p_j}^\sigma$ become correlated through a factor $\xi_{e_i p_j}^\sigma$.

6.3.3.1 False Asymmetry Due to Nonuniform Detection Efficiencies (NDE Effect)

The form for the number of coincidences $N_{e_i p_j}^\sigma$ assumed in Equation 3-20 ignores the possibility of efficiency correlations. We shall demonstrate that if either or both the detector efficiencies is nonuniform, then the simple model given by Equation 3-20 is not sufficient. In this case we must include a factor representing an efficiency correlation $\xi_{e_i p_j}^\sigma$ in the expression for the number of coincidences detected in a pair

$$N_{e_i p_j}^\sigma = C^\sigma \Omega_{e_i}^\sigma \Omega_{p_j}^\sigma \epsilon_{e_i}^\sigma \epsilon_{p_j}^\sigma \xi_{e_i p_j}^\sigma (1 \pm KPD) \quad (6-8)$$

where $\varepsilon_{e_i}^\sigma$ and $\varepsilon_{p_j}^\sigma$ represent constant overall electron and proton detector efficiencies respectively.

Although the ξ_{e,p_j}^σ are all near unity, they do not cancel exactly in the analysis ratio (Equation 3-19). It becomes

$$R = \frac{\xi_{e_1 p_3}^\uparrow \xi_{e_2 p_4}^\uparrow \xi_{e_3 p_1}^\uparrow \xi_{e_4 p_2}^\uparrow \xi_{e_1 p_2}^\downarrow \xi_{e_2 p_3}^\downarrow \xi_{e_3 p_4}^\downarrow \xi_{e_4 p_1}^\downarrow (1 + KPD)^8}{\xi_{e_1 p_2}^\uparrow \xi_{e_2 p_3}^\uparrow \xi_{e_3 p_4}^\uparrow \xi_{e_4 p_1}^\uparrow \xi_{e_1 p_3}^\downarrow \xi_{e_2 p_4}^\downarrow \xi_{e_3 p_1}^\downarrow \xi_{e_4 p_2}^\downarrow (1 - KPD)^8} \quad (6-9)$$

so the ratio R does not reduce to unity, even if $D = 0$. The fact that $R \neq 1$ even if $D = 0$ shows that efficiency correlations can produce a false effect.

We make a simplification that will be useful later by defining

$$r_{ij} = \frac{\xi_{e_i p_j}^\downarrow}{\xi_{e_i p_j}^\uparrow} \quad (6-10)$$

so the ratio becomes

$$R = \frac{r_{12} r_{23} r_{34} r_{41} (1 + KPD)^8}{r_{13} r_{24} r_{31} r_{42} (1 - KPD)^8}. \quad (6-11)$$

We now show that efficiency correlations arise from nonuniform detector efficiencies. We express N_{e,p_j}^σ as convolution of a decay probability density $\Upsilon^\sigma(\mathbf{x}_e, \mathbf{p}_e, \mathbf{x}_p, \mathbf{p}_p)$ and the position and momentum dependent detector efficiencies,

$$N_{e,p_j}^\sigma = C^\sigma \int_{S_{e_i}} \int_{S_{p_j}} \Upsilon^\sigma(\mathbf{x}_e, \mathbf{p}_e, \mathbf{x}_p, \mathbf{p}_p) \varepsilon_{e_i}^\sigma(\mathbf{x}_e, \mathbf{p}_e) \varepsilon_{p_j}^\sigma(\mathbf{x}_p, \mathbf{p}_p) d^2 x_p d\mathbf{p}_p d^2 x_e d\mathbf{p}_e \quad (6-12)$$

where the integrations of the electron (proton) position, $\mathbf{x}_e$ ($\mathbf{x}_p$), and momentum, $\mathbf{p}_e$ ($\mathbf{p}_p$), are carried out over the face of the e_i (p_j) electron (proton) detector, S_{e_i} (S_{p_j}). The distribution Υ^σ depends on the spatial distribution of neutrons in the beam. The overall inten-

sity of the beam and counting time are, as usual, reflected in the constant C^σ . The normalization of Υ^σ is defined such that

$$\int_{S_{e_i}} \int_{S_{p_j}} \Upsilon^\sigma(x_e, p_e, x_p, p_p) dx_p dp_p dx_e dp_e = \Omega_{e_i}^\sigma \Omega_{p_j}^\sigma \quad (6-13)$$

Notice that the expression for N_{ep_j} given in Equation 3-20 is recovered from Equation 6-12 if both detector efficiencies are constant over the integration ranges (i.e. $\epsilon_{e_i}^\sigma(x_e, p_e) = \epsilon_{e_i}^\sigma$ and $\epsilon_{p_j}^\sigma(x_p, p_p) = \epsilon_{p_j}^\sigma$). From Equations 6-8 and 6-12 we see that

$$\xi_{e_i p_j}^\sigma (1 \pm KPD) = \int_{S_{e_i}} \int_{S_{p_j}} \frac{\Upsilon^\sigma(x_e, p_e, x_p, p_p)}{\Omega_{e_i}^\sigma \Omega_{p_j}^\sigma} \frac{\epsilon_{e_i}^\sigma(x_e, p_e)}{\epsilon_{e_i}^\sigma} \frac{\epsilon_{p_j}^\sigma(x_p, p_p)}{\epsilon_{p_j}^\sigma} d^2 x_p dp_p d^2 x_e dp_e. \quad (6-14)$$

The detector efficiencies are at least approximately constant over the integration (i.e. $\epsilon_{e_i}^\sigma(x_e, p_e) \approx \epsilon_{e_i}^\sigma$ and $\epsilon_{p_j}^\sigma(x_p, p_p) \approx \epsilon_{p_j}^\sigma$), so, using Equation 6-13, we have $\xi_{e_i p_j}^\sigma \approx 1$. However, even if $D = 0$ the $\xi_{e_i p_j}^\sigma$ are not exactly unity unless detector efficiencies are constant (and thereby uniform). Thus nonuniform detection efficiencies can lead to the efficiency correlations that cause a false effect.

Effect of a Single Nonuniform Detector Efficiency

In order to estimate the size of the false effect created by position and momentum dependent efficiencies, we will assume that the true value of D is zero and all but one of the detectors has constant efficiency, say proton detector p_j whose efficiency is $\epsilon_{p_j}^\sigma(x_p, p_p)$. [Note: The following analysis, with trivial modifications, also applies if we chose one of the electron detectors.] Under these assumptions only the efficiency correlations involving p_j differ from unity and Equation 6-9 becomes

$$R = \frac{\xi_{e_m p_j}^{\uparrow} \xi_{e_n p_j}^{\downarrow}}{\xi_{e_m p_j}^{\downarrow} \xi_{e_n p_j}^{\uparrow}} = \frac{r_{nj}}{r_{mj}} \quad (6-15)$$

where e_m and e_n are the electron detectors paired with the p_j proton detector in Equation 3-19.

The expression for the efficiency correlations involving the p_j detector given by Equation 6-14 can be integrated over the electron variables, becoming

$$\xi_{e_m p_j}^\sigma = \int_{S_{p_j}} \frac{\chi^\sigma(e_m; \mathbf{x}_p, \mathbf{p}_p)}{\Omega_{p_j}^\sigma} \frac{\epsilon_{p_j}^\sigma(\mathbf{x}_p, \mathbf{p}_p)}{\epsilon_{p_j}^\sigma} d^2 x_p d\mathbf{p}_p \quad (6-16)$$

where $\chi^\sigma(e_m; \mathbf{x}_p, \mathbf{p}_p)$ is the probability distribution after the electron variables have been integrating over the surface of the e_m electron detector. Assuming that the p_j detector is not misaligned or misshapen, then by the symmetry of the detector arrangement the normalization of χ^σ is given by

$$\int_{S_{p_j}} \chi^\sigma(e_i; \mathbf{x}_p, \mathbf{p}_p) d\mathbf{x}_p d\mathbf{p}_p = \Omega_{p_j}^\sigma \quad (6-17)$$

for both values of σ and for either $\chi^\sigma(e_m; \mathbf{x}_p, \mathbf{p}_p)$ or $\chi^\sigma(e_n; \mathbf{x}_p, \mathbf{p}_p)$. [Note: a misaligned or misshapen detector is easily modeled by using the position dependence of detector efficiency.] The polarization-proton anti-correlation, which is induced by the B coefficient in Equation 3-19 and the limited phase space, ensures that

$$\chi^\uparrow(e_i; \mathbf{x}_p, \mathbf{p}_p) \neq \chi^\downarrow(e_i; \mathbf{x}_p, \mathbf{p}_p) \quad (6-18)$$

and the electron-proton anti-correlation ensures that

$$\chi^\sigma(e_j; \mathbf{x}_p, \mathbf{p}_p) \neq \chi^\sigma(e_k; \mathbf{x}_p, \mathbf{p}_p) \quad (6-19)$$

for $(j \neq k)$. Thus the four $\xi_{e p_j}^\sigma$ in Equation 6-15 are all different so they do not cancel. This failure to cancel leads to $R \neq 1$ and results in the signal of a false nonzero value for D . This is the essence of the systematic effect that we will designate the NDE (for Non-uniform Detection Efficiency) effect.

Effect of Multiple Detectors with Nonuniform Efficiency

In the previous section we considered the case a single detector with nonuniform detection efficiency. These results can be extended to the case where multiple detectors have nonuniform efficiency. First we will consider the case of multiple proton detector nonuniformities. If we define

$$1 + \delta_{p_j} = \frac{r_{nj}}{r_{mj}}, \quad (6-20)$$

then from Equations 6-11 and 6-15 we find

$$R = \prod_{j=1}^4 (1 + \delta_{p_j}) \approx 1 + \sum_{j=1}^4 \delta_{p_j} \quad (6-21)$$

where we have use the fact that $\delta_{p_j} \ll 1$ and kept only quantities first order in the δ_{p_j} 's. Whether δ_{p_j} is positive or negative depends on the nature of the nonuniformity of the p_j proton detector. For example, in the model in which the efficiency is divided into quadrants, the sign depends on which quadrant has a different efficiency. A similar relation holds for the electron detectors.

6.3.3.2 Estimate of the Magnitude of the False Asymmetry Due to the NDE Effect

The size of the NDE effect can be estimated by modeling the detector efficiency $\epsilon_{p_j}^\sigma(\mathbf{x}_p, \mathbf{p}_p)$ as piecewise constant over regions of $(\mathbf{x}_p, \mathbf{p}_p)$ space. For simplicity, we divide the calculations into two parts. In the first, we ignore the momentum dependence of the detector efficiency and divide the detector face into sections, assigning each a different, constant, efficiency. In the second, the position dependence is ignored and constant efficiencies are assigned to ranges of angles of incidence of the particles on the detector faces.

In order to estimate the size of the NDE effect we must calculate the position and momentum dependent distributions, $\chi^\sigma(e_j; \mathbf{x}_p, \mathbf{p}_p)$. Since we are assuming the nonuniform detection efficiencies can be modeled by constants over finite ranges of $\mathbf{x}_p$ and $\mathbf{p}_p$, we

need only determine the average values of the χ^σ 's over each section. Under these assumptions the integral in the expressions for $\xi_{e_p j}^\sigma$ given by Equation 6-16 can be simplified,

$$\xi_{e_p j}^\sigma = \sum_{u=1}^N f_u(\uparrow, e_m) \alpha_u = f(\uparrow, e_m) \cdot \alpha \quad (6-22)$$

where u designates which of the N regions has the constant efficiency α_u , and f_u is the fraction of protons that are incident on that region. The sum has been expressed as a scalar product. The normalization of the f_u 's is given by

$$\sum_{u=1}^N f_u(\sigma, e_j) = 1. \quad (6-23)$$

The f_u 's depend on e_j and σ because they distort the distribution of the protons. The explicit dependence of the detector's efficiency (the α_u) on the polarization direction has been dropped although an additional overall polarization dependent factor would cancel in the ratio.

Notice that if all the α_u are equal, then, using Equations 6-22 and 6-23, all the $\xi_{e_p j}^\sigma$ in Equation 6-15 are equal so $R = 1$ and there is no systematic effect. This fact demonstrates that we are concerned with *variations* in the detector efficiency. We will assume that one of the efficiencies α_v is different from the others by a factor $(1 - \Delta)$ so that

$$\alpha_u = \begin{cases} \epsilon_0 & (u \neq v) \\ \epsilon_0 (1 - \Delta) & (u = v) \end{cases} \quad (6-24)$$

Using this expression for the α_u , in Equation 6-22, we have,

$$\xi_{e_p j}^\sigma = 1 - \Delta f_v(\sigma, e_m) \quad (6-25)$$

so

$$\Delta f_v(\sigma, e_m) = (1 - \xi_{e_m p_j}^\sigma). \quad (6-26)$$

Using these relation in Equation 6-15 we have

$$R = \frac{r_{nj}}{r_{mj}} = \frac{[1 - \Delta f_v(\uparrow, e_m)] [1 - \Delta f_v(\downarrow, e_n)]}{[1 - \Delta f_v(\downarrow, e_m)] [1 - \Delta f_v(\uparrow, e_n)]}. \quad (6-27)$$

For small Δ the ratio can be expanded in powers of Δ about $\Delta = 0$. To first order in Δ we have

$$R = \frac{r_{nj}}{r_{mj}} \approx 1 + \Delta [f_v(\downarrow, e_m) - f_v(\uparrow, e_m) - f_v(\downarrow, e_n) + f_v(\uparrow, e_n)]. \quad (6-28)$$

Small deviations of R from unity affect the value of D extracted. With $R = 1 + \tau$ where $|\tau| \ll 1$, Equation 3-22, which relates D and R , becomes, to first order in τ ,

$$D \approx \frac{\tau}{16 KP} = \frac{(R - 1)}{16 KP} \quad (6-29)$$

so the efficiency correlations produce a false D

$$D_{\text{NDE}} \approx \frac{(r_{nj}/r_{mj} - 1)}{16 KP}. \quad (6-30)$$

$$D_{\text{NDE}} \approx \frac{\Delta}{16 KP} [f_v(\downarrow, e_m) - f_v(\uparrow, e_m) - f_v(\downarrow, e_n) + f_v(\uparrow, e_n)]. \quad (6-31)$$

We return to the method of Monte Carlo simulation to calculate the fractions $f_u(\sigma, e_i)$. The electrons and protons from decay events are tracked in the usual way until they reach the faces of the detectors. Then, the positions and angles of incidence the particles make on the detector faces are binned and recorded in histograms. The value of a fraction $f_u(\sigma, e_i)$ (or $f_u(\sigma, p_i)$) for an area or range of angles is simply the number of events falling in the corresponding histogram bin divided by the sum of events in all the bins.

Position Dependent Detector Efficiency Nonuniformities

The magnitude of the NDE effect as function of position dependent detector efficiency nonuniformities is modeled by dividing the face of the detector in question into quadrants. Each of the quadrants is assigned a constant efficiency, the α_u of Equation 6-22. The naming of the constants is shown in Figure 6-6. The corresponding values of the fractions f_u were computed by Monte Carlo simulation. The four-fold symmetry of the detector geometry in the Monte Carlo leads to the relations,

$$\begin{aligned} f_1(\sigma, e_m) &= f_3(\sigma, e_n) \\ f_2(\sigma, e_m) &= f_4(\sigma, e_n) \\ f_3(\sigma, e_m) &= f_1(\sigma, e_n) \\ f_4(\sigma, e_m) &= f_2(\sigma, e_n) \end{aligned} \quad (6-32)$$

This allows us to simplify the notation by eliminating the f_u 's involving e_n and writing the f_u 's as $f_u^\sigma = f_u(\sigma, e_m)$. Using these relations in Equations 6-28 and 6-31 gives

$$R = \begin{cases} 1+(-)\Delta[f_1^\downarrow - f_1^\uparrow - f_3^\downarrow + f_3^\uparrow] & \text{for } \alpha_1 (\alpha_3) \\ 1+(-)\Delta[f_2^\downarrow - f_2^\uparrow - f_4^\downarrow + f_4^\uparrow] & \text{for } \alpha_2 (\alpha_4) \end{cases} \quad (6-33)$$

and

$$D_{\text{NDE}} = \begin{cases} \frac{+(-)\Delta}{16KP}[f_1^\downarrow - f_1^\uparrow - f_3^\downarrow + f_3^\uparrow] & \text{for } \alpha_1 (\alpha_3) \\ \frac{+(-)\Delta}{16KP}[f_2^\downarrow - f_2^\uparrow - f_4^\downarrow + f_4^\uparrow] & \text{for } \alpha_2 (\alpha_4) \end{cases} \quad (6-34)$$

where the efficiency in α_v is different from the other three by a factor $(1 - \Delta)$ as indicated. The sign on the Δ depends on which quadrant has different efficiency.

Each proton detector (in a slice) is 8 cm $\times$ 4 cm long where the latter dimension is along the beam. Each detector is divided in the simulation into quadrants 4 cm wide by 2 cm long. The f_u^σ , computed by Monte Carlo simulation, do not vary significantly from one slice to the next. As a result, the values of D_{NDE} from Equation 6-34 are approxi-

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Figure 6-6: Naming scheme for the constant efficiencies assigned to the quadrants of the face of a detector. The face of the detector is viewed from the decay region.

mately the same for each detector slice so we use the averages over all the slices. The f_u^σ averaged over the slices are given in Table 6-2 for both a cylindrical and “diverging” beam. [Rather than use the averaged fractions in Equation 6-34 directly, it is preferable to use Equation 6-34 to calculate D_{NDE} for each slice and then average them over all the slices. This is done below.] The neutron density in an actual beam would lie somewhere between these uniformly populated cylindrical and diverging beam models. The results, given below, are the same for both cases, demonstrating that beam divergence has no significant impact on the magnitude of the NDE effect.

An identical procedure is followed to estimate the effects of position dependent electron detector efficiencies. The electron detector slices are 30 cm long and about 8 cm wide so the electron detector quadrants are 15 cm by 4 cm. The averaged quadrant fractions for the electron positions for a cylindrical and a diverging beam are given in Table 6-3.

The size of the systematic effect resulting from the difference of a single quadrant’s efficiency from that of the other quadrants is determined for each detector slice using Equation 6-34. The estimate of the size of D_{NDE} for each case is summarized in

Average Proton Position Fractions for a Cylindrical Beam			
$f_1^{\uparrow}$	0.340 ± 0.001	$f_1^{\downarrow}$	0.337 ± 0.001
$f_2^{\uparrow}$	0.337 ± 0.001	$f_2^{\downarrow}$	0.340 ± 0.001
$f_3^{\uparrow}$	0.162 ± 0.001	$f_3^{\downarrow}$	0.161 ± 0.001
$f_4^{\uparrow}$	0.161 ± 0.001	$f_4^{\downarrow}$	0.162 ± 0.001
Average Proton Position Fractions for a Diverging Beam			
$f_1^{\uparrow}$	0.341 ± 0.001	$f_1^{\downarrow}$	0.338 ± 0.001
$f_2^{\uparrow}$	0.336 ± 0.001	$f_2^{\downarrow}$	0.342 ± 0.001
$f_3^{\uparrow}$	0.163 ± 0.001	$f_3^{\downarrow}$	0.159 ± 0.001
$f_4^{\uparrow}$	0.161 ± 0.001	$f_4^{\downarrow}$	0.161 ± 0.001

Table 6-2: Slice averaged proton position fractions. The fraction of coincidences in which the proton falls in the u position quadrant for a 6 cm diameter cylindrical beam and for a beam diverging from 5 to 7 cm in diameter. The naming scheme for the quadrants is given in Figure 6-6.

Average Electron Position Fractions for a Cylindrical Beam			
$f_1^{\uparrow}$	0.163 ± 0.002	$f_1^{\downarrow}$	0.178 ± 0.002
$f_2^{\uparrow}$	0.178 ± 0.002	$f_2^{\downarrow}$	0.163 ± 0.002
$f_3^{\uparrow}$	0.312 ± 0.002	$f_3^{\downarrow}$	0.347 ± 0.002
$f_4^{\uparrow}$	0.347 ± 0.002	$f_4^{\downarrow}$	0.313 ± 0.002
Average Electron Position Fractions for a Diverging Beam			
$f_1^{\uparrow}$	0.163 ± 0.002	$f_1^{\downarrow}$	0.175 ± 0.002
$f_2^{\uparrow}$	0.179 ± 0.002	$f_2^{\downarrow}$	0.165 ± 0.002
$f_3^{\uparrow}$	0.311 ± 0.002	$f_3^{\downarrow}$	0.348 ± 0.002
$f_4^{\uparrow}$	0.347 ± 0.002	$f_4^{\downarrow}$	0.313 ± 0.002

Table 6-3: Slice averaged electron position fractions. The fraction of coincidences in which the electron falls in the u position quadrant for a 6 cm diameter cylindrical beam and for a beam diverging from 5 to 7 cm in diameter. The naming scheme for the quadrants is given in Figure 6-6.

Table 6-4. From the worst case in Table 6-4, a single proton detector quadrant efficiency

jected vector at the center of the detector, its head lies within one of the quadrants shown in Figure 6-6. Let the subscript in this figure denote the angular quadrant the particle entered. The angles of incidence cover a considerable range. Along the beam direction the angles range from nearly $+90^\circ$ to -90° . Since the beam is only about 6 cm across, the range is more limited along the transverse dimension, from about -35° to $+35^\circ$. The slice-averaged fractions for proton and electrons detectors for both a cylindrical and diverging beam are given in Tables 6-5, and 6-6. The sizes of resulting D_{NDE} are summarized in Table 6-7. A single proton detector quadrant efficiency differing from the others by $\Delta = 1/2\%$ produces a D_{NDE} of about 10^{-4} while a quadrant electron detector efficiency need differ by about 1% to produce the same effect.

Average Proton Angular Fractions for a Cylindrical Beam			
$f_1^\uparrow$	0.474 ± 0.001	$f_1^\downarrow$	0.284 ± 0.001
$f_2^\uparrow$	0.284 ± 0.001	$f_2^\downarrow$	0.473 ± 0.001
$f_3^\uparrow$	0.149 ± 0.001	$f_3^\downarrow$	0.091 ± 0.001
$f_4^\uparrow$	0.093 ± 0.001	$f_4^\downarrow$	0.152 ± 0.001
Average Proton Angular Fractions for a Diverging Beam			
$f_1^\uparrow$	0.476 ± 0.001	$f_1^\downarrow$	0.287 ± 0.001
$f_2^\uparrow$	0.283 ± 0.001	$f_2^\downarrow$	0.472 ± 0.001
$f_3^\uparrow$	0.147 ± 0.001	$f_3^\downarrow$	0.091 ± 0.001
$f_4^\uparrow$	0.094 ± 0.001	$f_4^\downarrow$	0.151 ± 0.001

Table 6-5: Slice averaged proton angle fractions. The fraction of coincidences in which the proton falls in the μ angular quadrant for a 6 cm diameter cylindrical beam and for a beam diverging from 5 to 7 cm in diameter. The naming scheme for the quadrants is given in Figure 6-6.

The results for the angular quadrant efficiencies can be somewhat misleading because the four angular quadrants represent such large ranges of incidence angles. Another calculation employing finer segmentation of the angular ranges has been done. The $\pm 90^\circ$ range of incidence angles along the beam direction was divided into eight equal

Average Electron Angular Fractions for a Cylindrical Beam			
$f_1^\uparrow$	0.145 ± 0.002	$f_1^\downarrow$	0.101 ± 0.002
$f_2^\uparrow$	0.102 ± 0.002	$f_2^\downarrow$	0.145 ± 0.002
$f_3^\uparrow$	0.431 ± 0.002	$f_3^\downarrow$	0.322 ± 0.002
$f_4^\uparrow$	0.322 ± 0.002	$f_4^\downarrow$	0.433 ± 0.002
Average Electron Angular Fractions for a Diverging Beam			
$f_1^\uparrow$	0.144 ± 0.002	$f_1^\downarrow$	0.101 ± 0.002
$f_2^\uparrow$	0.102 ± 0.002	$f_2^\downarrow$	0.146 ± 0.002
$f_3^\uparrow$	0.431 ± 0.002	$f_3^\downarrow$	0.320 ± 0.002
$f_4^\uparrow$	0.323 ± 0.002	$f_4^\downarrow$	0.433 ± 0.002

Table 6-6: Slice averaged electron angle fractions. The fraction of coincidences in which the electron falls in the u angular quadrant for a 6 cm diameter cylindrical beam and for a beam diverging from 5 to 7 cm in diameter. The naming scheme for the quadrants is given in Figure 6-6.

Detector Type	Beam Geometry	Differing Quadrant Efficiency	(R-1) ($\times 10^{-3}$)	D_{NDE} ($\times 10^{-4}$)
Proton	Cylindrical	$\alpha_1 (\alpha_3)$	$- (+) (133 \pm 2) \Delta$	$- (+) (224 \pm 3) \Delta$
		$\alpha_2 (\alpha_4)$	$+ (-) (131 \pm 2) \Delta$	$+ (-) (220 \pm 3) \Delta$
	Diverging	$\alpha_1 (\alpha_3)$	$- (+) (133 \pm 2) \Delta$	$- (+) (223 \pm 3) \Delta$
		$\alpha_2 (\alpha_4)$	$+ (-) (132 \pm 2) \Delta$	$+ (-) (221 \pm 3) \Delta$
Electron	Cylindrical	$\alpha_1 (\alpha_3)$	$+ (-) (65 \pm 4) \Delta$	$+ (-) (110 \pm 7) \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) (67 \pm 4) \Delta$	$- (+) (113 \pm 7) \Delta$
	Diverging	$\alpha_1 (\alpha_3)$	$+ (-) (68 \pm 4) \Delta$	$+ (-) (114 \pm 7) \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) (66 \pm 4) \Delta$	$- (+) (111 \pm 7) \Delta$

Table 6-7: Size of the systematic NDE effect caused by a variation, Δ , in the angle dependent detector efficiency of one quadrant of one detector in a single slice as given by Equation 6-34. Two uniformly populated beam geometries were simulated, a 6 cm diameter cylindrical beam, and a beam diverging from 5 cm to 7 cm in diameter. The sign depends on which quadrant efficiency differs from the others.

sections. The $\pm 35^\circ$ range of transverse angles was divided into four equal sections. The numbering of resulting 32 sections of angular detector acceptance with efficiencies α_u and fractions f_u^σ is shown in Figure 6-7. The fractions were calculated by the Monte Carlo sim-

ulation and used in Equation 6-31 to estimate the size of the systematic effect. The NDE effect resulting from one of the α_u differing from the others by a factor $(1 - \Delta)$ produces *at most*

$$|D_{\text{NDE}}| \leq 8 \times 10^{-3} \Delta \quad (6-35)$$

for the proton detectors or

$$|D_{\text{NDE}}| \leq 4 \times 10^{-3} \Delta \quad (6-36)$$

for the electron detectors. [Sections 3, 6, 11, 14, 19, 22, 27, and 30 produce the largest effects when their efficiencies differ from the rest. (See Figure 6-7.)] So a difference in efficiency of the order 1% for a particular 22.5° by 17.5° angular section of a proton detector's acceptance could produce at most $D_{\text{NDE}} = 10^{-4}$. The same size effect would be produced by electron detector efficiency differences that were about twice as large. Although the sign of the D_{NDE} depends on which particular α_u differs from the others, all of the α_u that fall within the same quadrant (e.g. α_1 through α_4 and α_9 through α_{12} in Figure 6-7) produce the same sign.

6.3.3.3 Measurement of the Factors Contributing to the NDE Effect

It is possible to indirectly measure the factors that contribute to the NDE effect by forming ratios of coincidences similar to the analysis ratio of Equation 3-19. Certain ratios can be constructed so that they are insensitive to a true triple correlation while retaining sensitivity to the NDE effect. The notation of the efficiency correlations developed in Section 6.3.3.1 is convenient for comparing the influence of the NDE effect on different ratios of coincidences. In the following sections we will consider several complementary ratios of coincidences that are insensitive to D but whose susceptibility to the NDE effect closely parallels that of the analysis ratio.

α_{32}	α_{24}	α_{16}	α_8	$+90^\circ$ z -90°
α_{31}	α_{23}	α_{15}	α_7	
α_{30}	α_{22}	α_{14}	α_6	
α_{29}	α_{21}	α_{13}	α_5	
α_{28}	α_{20}	α_{12}	α_4	
α_{27}	α_{19}	α_{11}	α_3	
α_{26}	α_{18}	α_{10}	α_2	
α_{25}	α_{17}	α_9	α_1	
$+35^\circ$			-35°	

Figure 6-7: Naming scheme for a finer segmentation of a detector efficiency into piecewise constant sections. The face of the detector is viewed from the decay region. The direction of increasing azimuth is a positive rotation about the beam direction. The angles indicate the range divided when the segmentation refers to the space of angles of incidence on the detector face.

Pairing Left and Right-Handed Numbers of Coincidences

The coincidences in the numerator of R (Equation 3-19) are “right-handed” while those in the denominator are “left-handed”. The coincidences can be rearranged to form new ratios. There are six diagnostic ratios that can be formed by pairing off the left and right-handed coincidences for a given polarization. By “paring off” we mean the coincidences appear together in either the numerator or denominator of the new ratio. For example, from Figure 3-9 and Equation 6-8, we see that pairing $N_{e_1 p_2}^\uparrow$ with $N_{e_4 p_2}^\uparrow$ in the numerator puts the factors $(1 + KPD)$ and $(1 - KPD)$ both in the numerator of the new ratio. Making similar pairings for the remaining detectors cancels the dependence of the ratio on the D coefficient, but the efficiency correlations do not cancel. In addition, the detector solid angles ($\Omega_{e_i}^\sigma$ and $\Omega_{p_j}^\sigma$), and the constant overall efficiencies ($\epsilon_{e_i}^\sigma$ and $\epsilon_{p_j}^\sigma$) can-

cel only if they are independent of polarization. For the remainder of this section we assume that any dependence of these solid angles and efficiencies on the polarization state can be absorbed into the efficiency correlations ($\xi_{e_i}^\sigma$ and $\xi_{p_j}^\sigma$). The beam-dependent constants C^σ cancel without this simplification.

There are six unique diagnostic ratios of this type. We refer to the group as the LR ratios because they come from pairing left and right-handed (LR) numbers of coincidences. The first three LR ratios come from pairing the left and right-handed coincidences involving the same electron detector. They are:

$$R^{(\text{LRA})} = \frac{N_{e_2 p_3}^\uparrow N_{e_2 p_4}^\uparrow N_{e_4 p_1}^\uparrow N_{e_4 p_2}^\uparrow N_{e_1 p_2}^\downarrow N_{e_1 p_3}^\downarrow N_{e_3 p_1}^\downarrow N_{e_3 p_4}^\downarrow}{N_{e_1 p_2}^\uparrow N_{e_1 p_3}^\uparrow N_{e_3 p_1}^\uparrow N_{e_3 p_4}^\uparrow N_{e_2 p_3}^\downarrow N_{e_2 p_4}^\downarrow N_{e_4 p_1}^\downarrow N_{e_4 p_2}^\downarrow}, \quad (6-37)$$

$$R^{(\text{LRB})} = \frac{N_{e_3 p_1}^\uparrow N_{e_3 p_4}^\uparrow N_{e_4 p_1}^\uparrow N_{e_4 p_2}^\uparrow N_{e_1 p_2}^\downarrow N_{e_1 p_3}^\downarrow N_{e_2 p_3}^\downarrow N_{e_2 p_4}^\downarrow}{N_{e_1 p_2}^\uparrow N_{e_1 p_3}^\uparrow N_{e_2 p_3}^\uparrow N_{e_2 p_4}^\uparrow N_{e_3 p_1}^\downarrow N_{e_3 p_4}^\downarrow N_{e_4 p_1}^\downarrow N_{e_4 p_2}^\downarrow}, \quad (6-38)$$

and

$$R^{(\text{LRC})} = \frac{N_{e_2 p_3}^\uparrow N_{e_2 p_4}^\uparrow N_{e_3 p_1}^\uparrow N_{e_3 p_4}^\uparrow N_{e_1 p_2}^\downarrow N_{e_1 p_3}^\downarrow N_{e_4 p_1}^\downarrow N_{e_4 p_2}^\downarrow}{N_{e_1 p_2}^\uparrow N_{e_1 p_3}^\uparrow N_{e_4 p_1}^\uparrow N_{e_4 p_2}^\uparrow N_{e_2 p_3}^\downarrow N_{e_2 p_4}^\downarrow N_{e_3 p_1}^\downarrow N_{e_3 p_4}^\downarrow}. \quad (6-39)$$

The other three ratios come from pairing the coincidences involving the same proton detector. They are:

$$R^{(\text{LRU})} = \frac{N_{e_1 p_3}^\uparrow N_{e_2 p_3}^\uparrow N_{e_2 p_4}^\uparrow N_{e_3 p_4}^\uparrow N_{e_3 p_1}^\downarrow N_{e_4 p_1}^\downarrow N_{e_1 p_2}^\downarrow N_{e_4 p_2}^\downarrow}{N_{e_3 p_1}^\uparrow N_{e_4 p_1}^\uparrow N_{e_1 p_2}^\uparrow N_{e_4 p_2}^\uparrow N_{e_1 p_3}^\downarrow N_{e_2 p_3}^\downarrow N_{e_2 p_4}^\downarrow N_{e_3 p_4}^\downarrow}, \quad (6-40)$$

$$R^{(\text{LRV})} = \frac{N_{e_1 p_2}^\uparrow N_{e_4 p_2}^\uparrow N_{e_2 p_4}^\uparrow N_{e_3 p_4}^\uparrow N_{e_3 p_1}^\downarrow N_{e_4 p_1}^\downarrow N_{e_1 p_3}^\downarrow N_{e_2 p_3}^\downarrow}{N_{e_3 p_1}^\uparrow N_{e_4 p_1}^\uparrow N_{e_1 p_3}^\uparrow N_{e_2 p_3}^\uparrow N_{e_1 p_2}^\downarrow N_{e_4 p_2}^\downarrow N_{e_2 p_4}^\downarrow N_{e_3 p_4}^\downarrow}, \quad (6-41)$$

and

$$R^{(LRW)} = \frac{N_{e_1 p_2}^{\uparrow} N_{e_4 p_2}^{\uparrow} N_{e_1 p_3}^{\uparrow} N_{e_2 p_3}^{\uparrow} N_{e_3 p_1}^{\downarrow} N_{e_4 p_1}^{\downarrow} N_{e_2 p_4}^{\downarrow} N_{e_3 p_4}^{\downarrow}}{N_{e_3 p_1}^{\uparrow} N_{e_4 p_1}^{\uparrow} N_{e_2 p_4}^{\uparrow} N_{e_3 p_4}^{\uparrow} N_{e_1 p_2}^{\downarrow} N_{e_4 p_2}^{\downarrow} N_{e_1 p_3}^{\downarrow} N_{e_2 p_3}^{\downarrow}}. \quad (6-42)$$

A diagnostic ratio and its reciprocal are considered identical since this inversion merely changes the sign of the resulting NDE effect. Using Equation 6-8 to write the LR ratios in terms of the efficiency correlations they become:

$$\begin{aligned} R^{(LRA)} &= \frac{\xi_{e_2 p_3}^{\uparrow} \xi_{e_2 p_4}^{\uparrow} \xi_{e_4 p_1}^{\uparrow} \xi_{e_4 p_2}^{\uparrow} \xi_{e_1 p_2}^{\downarrow} \xi_{e_1 p_3}^{\downarrow} \xi_{e_3 p_1}^{\downarrow} \xi_{e_3 p_4}^{\downarrow}}{\xi_{e_1 p_2}^{\uparrow} \xi_{e_1 p_3}^{\uparrow} \xi_{e_3 p_1}^{\uparrow} \xi_{e_3 p_4}^{\uparrow} \xi_{e_2 p_3}^{\downarrow} \xi_{e_2 p_4}^{\downarrow} \xi_{e_4 p_1}^{\downarrow} \xi_{e_4 p_2}^{\downarrow}}, \\ &= \frac{r_{12} r_{13} r_{31} r_{34}}{r_{23} r_{24} r_{41} r_{42}} \end{aligned} \quad (6-43)$$

$$R^{(LRB)} = \frac{r_{12} r_{13} r_{23} r_{24}}{r_{31} r_{34} r_{41} r_{42}}, \quad (6-44)$$

$$R^{(LRC)} = \frac{r_{13} r_{42} r_{12} r_{41}}{r_{34} r_{24} r_{23} r_{31}}, \quad (6-45)$$

$$R^{(LRU)} = \frac{r_{31} r_{41} r_{12} r_{42}}{r_{13} r_{23} r_{24} r_{34}}, \quad (6-46)$$

$$R^{(LRV)} = \frac{r_{31} r_{41} r_{13} r_{23}}{r_{12} r_{42} r_{24} r_{34}}, \quad (6-47)$$

and

$$R^{(LRW)} = \frac{r_{31} r_{41} r_{24} r_{34}}{r_{12} r_{42} r_{13} r_{23}}. \quad (6-48)$$

The usual procedure is to assume that one detector has a nonuniform efficiency while all the rest have constant efficiencies. Suppose the p_2 proton detector has the nonuniform

efficiency. Then all of the efficiency correlations except those involving the p_2 detector are unity. The LR ratios reduce to

$$R^{(\text{LRA})} = R^{(\text{LRB})} = \frac{r_{12}}{r_{42}}, \quad (6-49)$$

$$R^{(\text{LRC})} = R^{(\text{LRU})} = r_{42}r_{12}, \quad (6-50)$$

and

$$R^{(\text{LRV})} = R^{(\text{LRW})} = \frac{1}{r_{42}r_{12}} \quad (6-51)$$

so the analysis ratio becomes, from Equation 6-15,

$$R = \frac{r_{12} (1 + KPD)^8}{r_{42} (1 - KPD)^8}. \quad (6-52)$$

For this particular example, $R^{(\text{LRA})}$ and $R^{(\text{LRB})}$ have exactly the same dependence on the NDE effect as the analysis ratio so they are a direct measure of the effect. The other four LR ratios have a slightly different dependence.

In general, regardless of the particular type, electron or proton, or number of the detector that has the nonuniformity, two of the six LR ratios have exactly the same dependence on the NDE effect that the analysis ratio does. Namely, they go as the ratio of two r 's. The other four LR ratios have slightly different dependence. They depend on the product of two r 's. Whether a *given* LR ratio has the same dependence on the NDE effect as the analysis ratio depends on the type and number of the nonuniform detector.

For a single detector with a nonuniform efficiency, four LR ratios have a different dependence on the NDE effect than the analysis ratio. The size of the effect on these ratios can be estimated using the piecewise constant efficiency model described in Section 6.3.3.2. Assuming the p_j proton detector's efficiency is nonuniform, then four LR ratios depend on the product $r_{mj}r_{nj}$, where m and n denote the electron detectors paired

with p_j in the analysis ratio Equation 3-19. Some of the four LR ratios may depend on the reciprocal of this product. This merely changes the sign of the resulting effect. This product, written in terms of the piecewise constant efficiency model using Equation 6-25, becomes

$$r_{mj}r_{nj} = \frac{[1 - \Delta f_v(\downarrow, e_m)] [1 - \Delta f_v(\downarrow, e_n)]}{[1 - \Delta f_v(\uparrow, e_m)] [1 - \Delta f_v(\uparrow, e_n)]} \quad (6-53)$$

expanding this in powers of Δ to first order in Δ

$$r_{mj}r_{nj} \approx 1 + \Delta [-f_v(\downarrow, e_m) + f_v(\uparrow, e_m) - f_v(\downarrow, e_n) + f_v(\uparrow, e_n)] . \quad (6-54)$$

This is slightly different from the corresponding expression Equation 6-28 for the analysis ratio.

We will directly compare the size of the NDE effect on the analysis ratio and the product by assuming the detector efficiency is divided into quadrants. From Equation 6-54 and the relations 6-32 the product becomes

$$r_{mj}r_{nj} \approx \begin{cases} 1 + (-)\Delta (-f_1^\downarrow + f_1^\uparrow - f_3^\downarrow + f_3^\uparrow) & \text{for } \alpha_1 (\alpha_3) \\ 1 + (-)\Delta (-f_2^\downarrow + f_2^\uparrow - f_4^\downarrow + f_4^\uparrow) & \text{for } \alpha_2 (\alpha_4) \end{cases} . \quad (6-55)$$

The f_v 's are the same as those computed previously by the Monte Carlo simulation for both electron and proton detectors with quadrant positional and angular efficiencies. The slice-averaged f_v 's from Tables 6-2, 6-3, 6-5, and 6-6 are used. The results for both positional and angular efficiency variations are summarized in Table 6-8. In all cases the absolute value of NDE effect is about a factor of two larger for the four diagnostic ratios that depend on the product $r_{mj}r_{nj}$, than for R . Some of the four diagnostic ratios go as the reciprocal of the product $r_{mj}r_{nj}$ in which case the NDE effect enters with the opposite sign.

The results for a single detector with nonuniform efficiency can be extended to the case where multiple detectors have nonuniform efficiency. The expression for R when

Type of Efficiency Nonuniformity	Detector Type	Differing Quadrant Efficiency	$(r_{mj}r_{nj} - 1) (x10^{-3})$
Positional	Proton	$\alpha_1 (\alpha_3)$	$+ (-) (7 \pm 2) \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) (6 \pm 2) \Delta$
	Electron	$\alpha_1 (\alpha_3)$	$- (+) (49 \pm 4) \Delta$
		$\alpha_2 (\alpha_4)$	$+ (-) (48 \pm 4) \Delta$
Angular	Proton	$\alpha_1 (\alpha_3)$	$+ (-) (248 \pm 2) \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) (246 \pm 2) \Delta$
	Electron	$\alpha_1 (\alpha_3)$	$+ (-) (154 \pm 4) \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) (154 \pm 4) \Delta$

Table 6-8: Size of the systematic NDE effect on the product $r_{mj}r_{nj}$ caused by a variation, Δ , in the detector efficiency of one quadrant of one detector in a single slice as given by Equation 6-55. A uniformly populated beam diverging from 5 cm to 7 cm in diameter was simulated. The sign depends on which quadrant efficiency differs from the others.

multiple proton detectors are nonuniform is given by Equation 6-21. The six LR ratios have similar relations. To express these we first define

$$\kappa_{p_j} = r_{nj}r_{mj} \quad (6-56)$$

in analogy to the definition of δ_{p_j} given by Equation 6-20. Then we have from Equations 6-43 through 6-48:

$$R^{(LRA)} \approx 1 - \delta_{p_1} + \delta_{p_2} - \delta_{p_3} + \delta_{p_4}, \quad (6-57)$$

$$R^{(LRB)} \approx 1 - \kappa_{p_1} + \delta_{p_2} + \kappa_{p_3} - \delta_{p_4}, \quad (6-58)$$

$$R^{(LRC)} \approx 1 + \delta_{p_1} + \kappa_{p_2} - \delta_{p_3} - \kappa_{p_4}, \quad (6-59)$$

$$R^{(LRU)} \approx 1 + \kappa_{p_1} + \kappa_{p_2} - \kappa_{p_3} - \kappa_{p_4}, \quad (6-60)$$

$$R^{(LRV)} \approx 1 + \kappa_{p_1} - \kappa_{p_2} + \kappa_{p_3} - \kappa_{p_4}, \quad (6-61)$$

and

$$R^{(LRW)} \approx 1 + \kappa_{p_1} - \kappa_{p_2} - \kappa_{p_3} + \kappa_{p_4}. \quad (6-62)$$

In the quadrant efficiency model, we arrive at the simple relation

$$\kappa_{p_j} \approx -2\delta_{p_j} \quad (6-63)$$

by comparing the values of $(R - 1)$ and $(r_{mj}r_{nj} - 1)$ given in Tables 6-4, 6-7, and 6-8. This allows us to express the LR ratios in terms of the δ_{p_j} 's that contribute to R . Thus sums and differences of the LR ratios can be used to determine the contribution of the NDE effect to R . A similar analysis applies for the electron detectors.¹

These diagnostic ratios provide a sensitive check for the existence of D_{NDE} . At the same time, they are insensitive to a true nonzero D . If the false effect arises, it can be detected using these ratios using the same data that are used to measure D .

Pairing Numbers of Coincidences with Parallel and Anti-Parallel Polarization

Consider the following ratio of numbers of coincidences

$$R^{(1)} = \frac{N_{e_1p_3}^{\uparrow} N_{e_1p_3}^{\downarrow} N_{e_2p_4}^{\uparrow} N_{e_2p_4}^{\downarrow} N_{e_3p_1}^{\uparrow} N_{e_3p_1}^{\downarrow} N_{e_4p_2}^{\uparrow} N_{e_4p_2}^{\downarrow}}{N_{e_1p_2}^{\uparrow} N_{e_1p_2}^{\downarrow} N_{e_2p_3}^{\uparrow} N_{e_2p_3}^{\downarrow} N_{e_3p_4}^{\uparrow} N_{e_3p_4}^{\downarrow} N_{e_4p_1}^{\uparrow} N_{e_4p_1}^{\downarrow}} \quad (6-64)$$

in which the number of coincidences counted in a particular pair of detectors for one polarization is always multiplied by the number for the same pair with the opposite polarization. This configuration is insensitive to D . At the same time all of the overall detector efficiencies, solid angles, and beam related constants exactly cancel. This is demonstrated by rewriting the numbers of coincidences using Equation 6-8 thereby finding that the ratio

$$R^{(1)} = \frac{\xi_{e_1p_3}^{\uparrow} \xi_{e_1p_3}^{\downarrow} \xi_{e_2p_4}^{\uparrow} \xi_{e_2p_4}^{\downarrow} \xi_{e_3p_1}^{\uparrow} \xi_{e_3p_1}^{\downarrow} \xi_{e_4p_2}^{\uparrow} \xi_{e_4p_2}^{\downarrow}}{\xi_{e_1p_2}^{\uparrow} \xi_{e_1p_2}^{\downarrow} \xi_{e_2p_3}^{\uparrow} \xi_{e_2p_3}^{\downarrow} \xi_{e_3p_4}^{\uparrow} \xi_{e_3p_4}^{\downarrow} \xi_{e_4p_1}^{\uparrow} \xi_{e_4p_1}^{\downarrow}} \quad (6-65)$$

1. There is no sign in the equation corresponding to Equation 6-63 for the electrons.

depends only on the efficiency correlations. If one particular detector, p_j , has a nonuniform efficiency, then the ratio becomes

$$R^{(1)} = \frac{\xi_{e_m p_j}^{\uparrow} \xi_{e_m p_j}^{\downarrow}}{\xi_{e_n p_j}^{\uparrow} \xi_{e_n p_j}^{\downarrow}} \quad (6-66)$$

which we see from Equation 6-15 has similar but not identical dependence on the efficiency correlations that R has.

Since there is no direct relation between the ratios of efficiency correlations found in R compared to those in $R^{(1)}$, the nonuniformity of efficiency has a different effect on each of them. The size of the effect on $R^{(1)}$ can be compared to the effect on R by expressing the nonuniformity in terms of the piecewise constant efficiency model using Equation 6-25

$$R^{(1)} = \frac{[1 - \Delta f_v(\uparrow, e_m)] [1 - \Delta f_v(\downarrow, e_m)]}{[1 - \Delta f_v(\uparrow, e_n)] [1 - \Delta f_v(\downarrow, e_n)]} \quad (6-67)$$

Expanding the right-hand side in powers of Δ to first order in Δ we have

$$R^{(1)} \approx 1 + \Delta [-f_v(\downarrow, e_m) - f_v(\uparrow, e_m) + f_v(\downarrow, e_n) + f_v(\uparrow, e_n)] \quad (6-68)$$

which is slightly different from the corresponding expression, Equation 6-28, for R .

We can directly compare the size of the NDE effect on R and $R^{(1)}$ by assuming the detector efficiency is divided into quadrants. From Equation 6-68 and the relations 6-32, $R^{(1)}$ becomes

$$R^{(1)} \approx \begin{cases} 1 + (-)\Delta (-f_1^{\downarrow} - f_1^{\uparrow} + f_3^{\downarrow} + f_3^{\uparrow}) & \text{for } \alpha_1 (\alpha_3) \\ 1 + (-)\Delta (-f_2^{\downarrow} - f_2^{\uparrow} + f_4^{\downarrow} + f_4^{\uparrow}) & \text{for } \alpha_2 (\alpha_4) \end{cases} \quad (6-69)$$

The f_v 's are the same as those computed by the Monte Carlo for both electron and proton detectors with positional and angular efficiencies differing by quadrant. We use the slice-

averaged f_v 's from Tables 6-2, 6-3, 6-5, and 6-6 to compute $R^{(1)}$. The results for both positional and angular efficiency variations are summarized in Table 6-9. These results

Type of Efficiency Nonuniformity	Detector Type	Differing Quadrant Efficiency	$(R^{(1)} - 1) (\times 10^{-3})$
Positional	Proton	$\alpha_1 (\alpha_3)$	$- (+) 360 \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) 360 \Delta$
	Electron	$\alpha_1 (\alpha_3)$	$+ (-) 320 \Delta$
		$\alpha_2 (\alpha_4)$	$+ (-) 320 \Delta$
Angular	Proton	$\alpha_1 (\alpha_3)$	$- (+) 530 \Delta$
		$\alpha_2 (\alpha_4)$	$- (+) 510 \Delta$
	Electron	$\alpha_1 (\alpha_3)$	$+ (-) 510 \Delta$
		$\alpha_2 (\alpha_4)$	$+ (-) 510 \Delta$

Table 6-9: Size of the systematic NDE effect on the diagnostic ratio $R^{(1)}$ caused by a variation, Δ , in the detector efficiency of one quadrant of one detector in a single slice as given by Equation 6-69. A uniformly populated beam diverging from 5 cm to 7 cm in diameter was simulated. The sign depends on which quadrant efficiency differs from the others.

should be compared to those for R given in Tables 6-4 and 6-7 to see the relative impact of the NDE effect on the ratios. In all cases the absolute value of NDE effect is larger for the $R^{(1)}$ than R . At the same time, $R^{(1)}$ is insensitive to $D \neq 0$.

The sign of the systematic effect produced by the differing of a particular α_u is not generally the same for $R^{(1)}$ and R . Thus it would, in principle, be possible for the NDE effect resulting from a particular combination of differing detector section efficiencies α_u (perhaps in several different detectors within one slice) to cancel in the unpolarized case yet reinforce in R . It is still possible, through the conspiracy of several nonuniformities, that $(R^{(1)} - 1)$ vanishes, even if the NDE effect makes $(R - 1) \neq 0$.

Unpolarized Beam

For an unpolarized beam, it is possible to form a diagnostic ratio that is, of course, insensitive to D but sensitive to the NDE effect

$$R^{(0)} = \frac{N_{e_1 p_3}^0 N_{e_2 p_4}^0 N_{e_3 p_1}^0 N_{e_4 p_2}^0}{N_{e_1 p_2}^0 N_{e_2 p_3}^0 N_{e_3 p_4}^0 N_{e_4 p_1}^0} \quad (6-70)$$

where $N_{e_i p_j}^0$ is the number of coincidences counted in the e_i electron detector and the p_j proton detector. The superscript 0 indicates that the beam is unpolarized. By analogy to Equation 6-8, $N_{e_i p_j}^0$ can be written as

$$N_{e_i p_j}^0 = C^0 \Omega_{e_i}^0 \Omega_{p_j}^0 \epsilon_{e_i}^0 \epsilon_{p_j}^0 \xi_{e_i p_j}^0 \quad (6-71)$$

where KPD has been dropped since $P = 0$ for an unpolarized beam. The individual over-all detector efficiencies and solid angles cancel in $R^{(0)}$ as they do in R . Substituting Equation 6-71 into Equation 6-70 we have

$$R^{(0)} = \frac{\xi_{e_1 p_3}^0 \xi_{e_2 p_4}^0 \xi_{e_3 p_1}^0 \xi_{e_4 p_2}^0}{\xi_{e_1 p_2}^0 \xi_{e_2 p_3}^0 \xi_{e_3 p_4}^0 \xi_{e_4 p_1}^0} \quad (6-72)$$

demonstrating its susceptibility to the NDE effect. With a trivial modification of the steps leading to Equation 6-28, this ratio can be written in terms of the piecewise constant efficiency model

$$R^{(0)} = \frac{\xi_{e_m p_j}^0}{\xi_{e_n p_j}^0} \approx 1 + \Delta [-f_v(0, e_m) + f_v(0, e_n)] \quad (6-73)$$

where, as usual, it has been assumed that the efficiency of the v^{th} section of the proton detector p_j differs from that of the other sections by a factor $1 - \Delta$. The fractions for an unpolarized beam can be rewritten as the average of the fractions for the two polarizations² (e. g. $f_v(0, e_m) = 1/2 [f_v(\uparrow, e_m) + f_v(\downarrow, e_m)]$) so Equation 6-73 becomes

2. The polarization-dependent terms in the PDF (Equation 2-10) reverse sign under a polarization reversal ($\sigma_n \rightarrow -\sigma_n$). Thus the average of the PDF with $\sigma_n = \hat{z}$ and the PDF with $\sigma_n = -\hat{z}$ equals the PDF where $\sigma_n = 0$, i.e. the unpolarized case.

$$R^{(0)} \approx 1 + \frac{\Delta}{2} [-f_v(\downarrow, e_m) - f_v(\uparrow, e_m) + f_v(\downarrow, e_n) + f_v(\uparrow, e_n)] . \quad (6-74)$$

Comparing this to Equation 6-68 we see that $R^{(0)}$ receives only half the deviation from unity that $R^{(1)}$ does for the same detector nonuniformity. Although the unpolarized diagnostic ratio $R^{(0)}$ provides the same information about the NDE effect that $R^{(1)}$ does, it is generally possible to achieve a considerably higher unpolarized flux through the detector, thereby reducing the statistical uncertainty, so this method may ultimately prove more sensitive.

Consistency of the Asymmetry

Each detector slice is treated essentially as a separate experiment so another possible indication of the presence of the NDE effect would be for different slices to give different values of D . A true triple correlation would, within statistical uncertainties, give the same value of D for each slice. It is, however, possible for the same nonuniformity of detection efficiency to be present in several or all of the slices, so it is possible for the NDE effect to be the same in all slices. It should be noted that while the proton detection for different slices is separate, there is some overlap in the electron detection between slices. (See Figure 3-10.).

Conclusions Regarding Measurement of NDE Effect

All but one of the diagnostic ratios is formed from the same data which are used to perform the measurement of D . For these tests no additional data taking is necessary, nor are any changes required in the detector system. The basic principle is to avoid the NDE effect by making the detector efficiencies sufficiently uniform. Variations in the detector efficiencies should be sufficiently constrained so that the NDE effect does not arise at a level that would impact the result of the experiment. At the same time, the combination of diagnostic ratios provide a powerful test for the presence of the NDE effect. They would

reveal the presence of the NDE effect should it arise at a significant level

6.3.3.4 Possible Sources of the Nonuniform Efficiencies

The investigation of the NDE effect reveals two important principles which should be adhered to in the design and construction of the particle detectors. First, the efficiency for detection of the electrons and protons should be made as high as possible, since variations from near unit efficiency are necessarily small. Second, the remaining loss mechanisms should be made as independent as possible from the position or angle of incidence of the decay products. More precisely, the dependence of the loss mechanisms on position or angle should be made to have the appropriate symmetry to avoid the NDE effect. To keep $D_{\text{NDE}} < 10^{-4}$ we require the quadrant positional (angular) detector efficiencies have $\Delta < 10\%$ ($\Delta < 1/2\%$) for the protons and $\Delta < 2\%$ ($\Delta < 1\%$) for the electrons.

Every effort in the design of the detector must be made to minimize the NDE effect through these principles. The application of these principles is evident in the particle detector designs. The particle detection efficiencies are made as uniform as possible by defining the detector acceptances by simple geometric apertures. The entrance to the electron detectors is defined by a flat rectangular aperture in the vacuum system. The proton detector entrances are defined by rectangular apertures spanned by a flat 94% open mesh. Ideally the symmetry of the apertures and their arrangement precludes their contributing to the NDE effect. The region inside the right octagonal prism formed by a full array of detector panels is free from electric fields that could distort the trajectories of the low energy recoil protons.

Since large fractional deviations in detector efficiency are, of course, not possible if the detectors have near unit efficiency, the efficiency for detecting an electron or proton that passes through the appropriate aperture is made as high as possible. No focusing or other distortion of the proton trajectories is done until after they have passed through the well-defined proton detector entrance apertures into the focussing cells. Modeling of the

electrostatic proton focussing inside the cells indicates that a high percentage of decay protons entering a focussing cell (90-95%) are successfully focussed onto the face of the PIN diode at the back of the cell. This percentage can be made almost 100% by masking 2-3 mm along the sides of the focussing cells. Even without masking, the symmetry of the cells is such that the losses along the sides do break symmetry in a way that produces the NDE effect. In the test run the fine wire mesh that covered the fronts of the focussing cells was 94% open. Fewer than 2% of the accelerated protons are backscattered by the PIN diodes so the detection efficiency for successfully focussed protons is very high. None of these known loss mechanisms appears sufficient to produce the 10% variations in efficiency as a function of proton position required to produce the NDE effect. The variations in angular efficiency are likewise made small by the high focussing and detection efficiency. In addition, none of the loss mechanisms have the appropriate asymmetry to produce the NDE effect.

The electron detectors are considerably simpler than proton detectors, since no acceleration or focussing is required. An example is a simple plastic scintillator paddle like that used in the test run detector. As has been noted above the backscatter fraction of electrons from plastic scintillator is low, a few percent. In addition the simple geometric regularity of the scintillator precludes, to first order, the type of asymmetric losses which are required to produce the NDE effect. However, care must be taken in the detection and collection of the scintillation light to avoid introducing these asymmetries. This can be at least partially achieved through the use of light guides which mix the light as it is transported to the photomultiplier tube.

6.4 Conclusions

The purpose of this chapter was to present a method for estimating the types and sizes of systematic effects which could be present in the measurement. An ideal detector system would exhibit no systematic effects. By virtue of the four-fold symmetry of the

detector and the analysis method, the majority of reasonably-sized deviations from the ideal also lead to no systematic effects. There are two types of false effects, the ATP and NDE effects, that can arise at a significant level. The ATP effect arises from asymmetric transverse components of the neutron polarization. The NDE effect arises from asymmetric nonuniformities in the position or angle dependent detector efficiencies. Relations between the magnitude of the nonuniformities that give rise to these effects and their contribution to a false D have been estimated. Clear indications for the presence of these two effects from experimental data have been given. Of particular usefulness are a number of diagnostic ratios, which are not sensitive to D , that can be used to reveal the presence of the NDE effect.

Chapter 7: Conclusion

7.1 Results

As the result of the work presented in this thesis, the major components of a new experiment for measuring the T-violating D coefficient of the triple-correlation in neutron decay have been developed. A new detector geometry was found that provides a significant advancement in sensitivity to D without requiring a corresponding increase the intensity of the neutron source. This geometry exploits the large enhancement in sensitivity to D available at larger relative electron-proton angles.

The implementation of the detector system requires the employment of both electron and proton detector panels. We have established that the electron detection can be accomplished using relatively simple scintillator paddles. The tracking information and more sophisticated background rejection provided by wire chamber components, like the TPC tested, in the electron panels were found to be nonessential to the experiment. The more technically challenging requirement of detecting the very low energy protons with high efficiency is satisfied by the proton detector panel that we developed. Since it includes a focussing and acceleration stage, the panel's active area is over five times larger than the area of the PIN diode particle detectors. At the same time, the panel's high focussing and detection efficiency and its simple geometric acceptance are valuable for reducing the size of its contribution to systematic effects such as the NDE false effect.

The test run on a cold neutron beam at the NIST CNRF demonstrated the viability of the detector designs. This was particularly important in the case of the proton detector panel since the run was the first test of the combination of the panel's focussing/acceleration stage with its PIN diode detectors. The test measurement also provided us with important insights into the rates and sources of backgrounds that we can expect in future runs as well as the means of avoiding some of the backgrounds.

As in any measurement of a small asymmetry, care must be taken that asymmetries

in the experimental setup neither produce a false effect nor mask a true effect. The sources of the two important false effects are nonuniform transverse components to the neutron polarization (ATP) and nonuniform detector efficiencies (NDE). In Chapter 6 the size of each of these false effects was related to the size of the nonuniformity that produces it.

The most stringent requirement that must be met to ensure that the false asymmetry $D_{\text{ATP}} < 10^{-4}$ is that the alignment of the neutron polarization must not be allowed to vary by more than about 1 mrad across the beam. A false effect arises only if these variations break azimuthal symmetry in the way described in Chapter 6. To ensure $D_{\text{NDE}} < 10^{-4}$ variations Δ of the positional (angular) detector efficiencies must be $\Delta < 10\%$ ($\Delta < 1/2\%$) for the protons and $\Delta < 2\%$ ($\Delta < 1\%$) for the electrons. These variations too must be asymmetric for a false effect to arise. We are confident that asymmetries in the beam and detectors can be controlled to this level where they will contribute no more than 10^{-4} to the measured D .

In the event that these expectations are not fulfilled, it is of paramount importance that we have some indication of this failure. Thus the diagnostic techniques outlined in Chapter 6 are a very important component in assuring the reliability of the results of the experiment. They are designed to reveal the presence of the ATP and NDE effects while remaining insensitive to the signal from a true nonzero D . The ATP effect is distinguished by its dependence on the beam radius. The size of NDE effect can be determined from the diagnostic ratios developed in Chapter 6. Most of these ratios are formed from the same data used to determine D so additional data taking is not required.

With the octagonal detector and the cold neutron beam designated for fundamental physics at the NIST CNRF, it should be possible to achieve nearly an order of magnitude improvement over previous experiments searching for T-violation in free neutron decay. A sensitivity to D of 3×10^{-4} should be achievable in about two months of beam-time as compared to over 12 months for a traditional right-angle geometry. This estimate is based on the expected beam flux distribution after an upgrade of the CNRF cold source sched-

uled for 1994. The detector could be used effectively with the higher decay rate that would come with the more intense neutron beams at the ILL. The reactor at the ILL, where the most sensitive of the previous neutron D experiments was performed, is currently shut down for repair. With the higher neutron fluxes produced at the ILL before the shutdown, the experimental sensitivity would be enhanced even further, by as much as another order of magnitude.

7.2 Future Directions For the Experiment

Progress so far indicates that improvements of an order of magnitude or more in sensitivity to D is possible with the new detector. That this improvement is possible without requiring more intense neutron beams than used in the past is its most important feature. There are a number of improvements to the detector system that should be made to enable it to fulfill its potential of providing the most sensitive measurement of D . In addition, there are a few of other aspects of the experiment that, until the detector was proved in the test run, were left unaddressed. Probably the most important of these unaddressed tasks is the production and transport of the neutron polarization. There are also improvements to be made in the beam transport and in the shielding of the detector from beam related backgrounds.

The test run showed several places in which the proton detector panel can be improved. The problems with the amplification electronics, condensation buildup, and energy calibration will all be addressed. These problems should be relatively simple to remedy. There are also a few improvements that will be made to the scintillator paddle design over the one tested in the run. Instrumentation that will give position sensitivity along the paddle's length will be added and the problem with crazing will be addressed. Again these improvements should be relatively simple to implement.

Improvements in the beam transport should reduce the γ backgrounds in the scintillators and PIN diodes sufficiently that the signal to noise ratio will be 10 or greater. If

this is achieved, the background due to uncorrelated coincidences will make only a minor negative impact on the experimental sensitivity. The remaining factors limiting sensitivity would be the decay rate and running time, and the possible presence of false effects.

If false effects are ultimately found to limit the experiment, tracking may be added to the electron detection so that these effects could be further studied and possibly eliminated. Also, the addition of tracking may enable the detector system to be used for measuring the B coefficient in Equation 2-10. This parameter is important in the determination of the ratio of the vector and axial vector coupling constants. Further study is required to determine if such an experiment would be feasible and what other modifications of the apparatus would be required.

Finally, the polarization production and transport system must be designed and built. For the initial measurements we plan to use a supermirror polarizer obtained from the ILL. A conventional current-sheet spin flipper will likely be used to reverse the polarization. The beam polarization must be rotated from transverse to longitudinal and then maintained by a guide field until it has passed through the detector. In principle, each of these components is a well-established technology. However, we know from our study of the ATP effect that care must be taken to ensure that the beam polarization is uniform in the decay region as small deviations can produce a false effect. A detailed study of the possible sources of the troublesome transverse polarization components has begun. The preliminary results indicate that this source of a false D can be kept at or below 10^{-4} . However, special care must be given to assure that nonadiabaticities in the polarization transport do not produce these deviations. Development of the ^3He polarizer continues and it may be used in future runs. It has the advantages of producing less background than a supermirror, and less spatial and angular disruption of the neutron distribution in the resulting polarized beam.

There are two orders of magnitude improvement in the sensitivity to D that can be made before Standard Model final state interactions ($D_{\text{fs, Weak Magnetism}}^{(1)}(n) = 1.1 \times 10^{-5}$)

begin to complicate the interpretation of the results as a test of time reversal invariance. This experiment will likely improve the sensitivity to D to 3×10^{-4} with additional improvement of perhaps an additional order of magnitude using the most intense cold neutron beams available today. These improvements will contribute to our understanding of CP/T-violation. If the experiment finds a nonzero D at this level, it would reveal the existence of new physics beyond the Standard Model.

Appendix A: Monte Carlo Simulation

A.1 Decay Event Generation

The polarized neutron beta decay probability distribution function given by Equation 2-10 is written in terms of five independent variable quantities, the electron energy, and the electron and neutrino directions relative to the neutron polarization. In spherical polar coordinates, with the neutron polarization along the polar axis (i.e. $\sigma_n = P\hat{z}$), the momenta are

$$\begin{aligned} p_e &= p_e (\sin\theta_e \cos\phi_e, \sin\theta_e \sin\phi_e, \cos\theta_e) \\ p_\nu &= p_\nu (\sin\theta_\nu \cos\phi_\nu, \sin\theta_\nu \sin\phi_\nu, \cos\theta_\nu) \end{aligned} \quad (\text{A-1})$$

and the probability distribution function becomes, in units where $\hbar = c = 1$,

$$\begin{aligned} w(E_e, \Omega_e, \Omega_\nu) &= G(E_e) [1 + an_e (\sin\theta_e \sin\theta_\nu \cos(\phi_e - \phi_\nu) + \cos\theta_e \cos\theta_\nu) \\ &\quad + P(A n_e \cos\theta_e + B \cos\theta_\nu + D n_e \sin\theta_e \sin\theta_\nu \sin(\phi_\nu - \phi_e))] \end{aligned} \quad (\text{A-2})$$

where we have defined $n_e = p_e/E_e$. The joint probability distribution function $w(E_e, \Omega_e, \Omega_\nu)$ is a probability density proportional to the probability of observing the electron from a decay with energy E_e , polar angle θ_e , azimuthal angle ϕ_e , and the neutrino with polar angle θ_ν and azimuthal angle ϕ_ν . To generate events according to this distribution it is useful to utilize conditional probabilities to allow each random variable to be generated in turn.

Following Eadie, *et al.* [Ead71], the projection of the joint probability density $f(X, Y)$ onto the X axis is the marginal density function of X ,

$$g(X) = \int_{Y_{\min}}^{Y_{\max}} f(X, Y) dY, \quad (\text{A-3})$$

where $Y_{\min}$ and $Y_{\max}$ denote the full range of the random variable Y . The conditional probability density function of Y given that $X = X_0$ is written $f(Y|X_0)$ and is defined through the relation

$$f(Y|X_0) = \frac{f(X_0, Y)}{\int f(X_0, Y) dY} = \frac{f(X_0, Y)}{g(X_0)}. \quad (\text{A-4})$$

It gives the probability of observing Y given that X is fixed at $X = X_0$.

Applying this to the case at hand we can write the joint probability distribution as the product of conditional probabilities

$$w(E_e, \Omega_e, \Omega_\nu) = f(E_e) f(\theta_e | E_e) f(\phi_e | E_e, \theta_e) f(\theta_\nu | E_e, \theta_e, \phi_e) f(\phi_\nu | E_e, \theta_e, \phi_e, \theta_\nu). \quad (\text{A-5})$$

The individual factors are probability distribution functions from which the individual random variables $E_e, \theta_e, \phi_e, \theta_\nu, \phi_\nu$ can be generated. Although, for simplicity, they have all been called f , the conditional probability distributions are distinct and can be distinguished from one another by their arguments.

The probability density of a decay producing an electron with energy E_e is

$$f(E_e) = \int \int_{\Omega_e, \Omega_\nu} w(E_e, \Omega_e, \Omega_\nu) d\Omega_\nu d\Omega_e \quad (\text{A-6})$$

where the integrals are taken over all electron and neutrino angles. The terms involving the angular correlations (those proportional to a, A, B , and D) integrate to zero leaving only

$$f(E_e) = (4\pi)^2 G(E_e) = (4\pi)^2 C F(E_e, Z) p_e E_e (E_0 - E)^2 \quad (\text{A-7})$$

where C is an overall constant which can be neglected, $F(E_e, Z)$, the Fermi function discussed in Chapter 2, corrects for the distortion of the outgoing electron wave by the Coulomb field of the residual nucleus. For neutron beta decay $Z = 1$. In the limit $(\alpha Z)^2 \ll 1$ the relativistic representation of $F(E_e, Z)$ reduces to the simpler classical Fermi function

$$F(E_e, Z) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}. \quad (\text{A-8})$$

This form is sufficient here since it deviates from the relativistic form in free neutron decay by only about 0.02% [Wil82]. The electron energy E_e for a given decay is generated according to the distribution given by Equation A-7.

For a given E_e the conditional probability density of polar angle of the electron θ_e from this same decay is

$$\begin{aligned} f(\theta_e | E_e) &= \frac{\int w(E_e, \Omega_e, \Omega_\nu) d\Omega_\nu d\phi_e}{f(E_e)} \\ &= \frac{1}{2} [1 + APn_e \cos\theta_e] \end{aligned} \quad (\text{A-9})$$

Again the overall normalization of this distribution is unimportant. The value of θ_e is generated according to this distribution where E_e has already been fixed for this decay by the distribution given in Equation A-7.

Now with E_e and θ_e fixed, the distribution of the azimuthal electron angle ϕ_e is

$$f(\phi_e | E_e, \theta_e) = \frac{\int w(E_e, \Omega_e, \Omega_\nu) d\Omega_\nu}{f(E_e, \theta_e)} = \frac{\int w(E_e, \Omega_e, \Omega_\nu) d\Omega_\nu}{f(E_e)f(\theta_e | E_e)}$$

where in the last step the relation shown in Equation A-4 has been used. This gives trivially

$$f(\phi_e | E_e, \theta_e) = (2\pi)^{-1}. \quad (\text{A-10})$$

Since its distribution is constant, ϕ_e is distributed uniformly over the interval $0 \leq \phi_e \leq 2\pi$.

The distributions for the neutrino variables can be found similarly. The distribution of θ_ν is

$$f(\theta_v | E_e, \theta_e, \phi_e) = \frac{\int_0^{2\pi} w(E_e, \Omega_e, \Omega_v) d\phi_v}{f(E_e)f(\theta_e | E_e)f(\phi_e | E_e, \theta_e)} \quad (A-11)$$

$$= \frac{1}{2} \left[1 + \frac{an_e \cos \theta_e \cos \theta_v + B\sigma_n \cos \theta_v}{1 + APn_e \cos \theta_e} \right]$$

The distribution of ϕ_v is

$$f(\phi_v | E_e, \theta_e, \phi_e, \theta_v) = \frac{w(E_e, \Omega_e, \Omega_v)}{f(E_e)f(\theta_e | E_e)f(\phi_e | E_e, \theta_e)f(\theta_v | E_e, \theta_e, \phi_e)} \quad (A-12)$$

$$= \frac{1}{2\pi} \left[1 + \frac{(a \cos(\phi_v - \phi_e) + DP \sin(\phi_v - \phi_e)) n_e \sin \theta_e \sin \theta_v}{1 + an_e \cos \theta_e \cos \theta_v + P(An_e \cos \theta_e + B \cos \theta_v)} \right]$$

Each decay event is produced in Monte Carlo simulation by successively generating the random variables $E_e, \theta_e, \phi_e, \theta_v$ and ϕ_v from their respective distributions A-7, A-9, A-10, A-11, and A-12. Except for the trivial case of ϕ_e , the distributions are sampled using the rejection method [Jam80a]. Say the random variable x from the distribution $f(x)$ where $a \leq x \leq b$ is being sampled. Further let us denote the maximum of $f(x)$ over the interval as $f_{\max}$. In this method, two random numbers r_1 and r_2 , chosen from a uniform distribution between zero and one, are used to pick the random deviate x from the distribution in two steps. First, we choose a candidate x denoted x' and given by $x' = r_1(b - a) + a$. Second, if $r_2 < f(x')/f_{\max}$ then x' is accepted otherwise new random numbers r_1 and r_2 are generated and the process is repeated. The x' 's selected in this way follow the distribution $f(x)$.

Each decay thus produced by the Monte Carlo gives the event in terms of the electron energy, electron direction, and the neutrino direction. In general, the electron and proton vector momenta are desired. The proton momentum is computed using energy and momentum conservation. The decaying neutron is at rest. Since the kinetic energy of the recoiling proton is negligible relative to the total energy released in the decay, the neutrino

momentum is given by

$$p_v = E_v = m_n - m_p - E_e \quad (\text{A-13})$$

where m_n and m_p are the masses of the neutron and proton respectively. This approximation is invalid for small volume of phase space. However, since the approximation has no bias for left versus right-handed decays, it has a negligible effect on the triple-correlation. The proton momentum is given by

$$p_p = -(p_e + p_v) \quad (\text{A-14})$$

and its energy by

$$E_p = \sqrt{p_p^2 - m_p^2}. \quad (\text{A-15})$$

A.2 Cylindrical Geometries

Several cylindrical detector geometries were modeled so their relative sensitivities to the D coefficient could be determined. The geometries are all based on the basic model of a cylinder divided into sections by longitude and latitude lines. See Section 3.2.2.3. Each section of the cylinder so subdivided acts as an electron detector, a proton detector, or both. The division of the cylinder into sections is independent for the two detector types. That is, in some cases the division of the cylinder into electron detector sections was different from its division into proton detector sections. In the following figures the sensitivity for electrons of a particular section of the cylinder is designated by "e", proton sensitivity by "p", and both by "e,p". End and side views of each geometry are shown. In cases where the cylinder was divided differently into electron and proton sensitive sections the cylinder is shown twice, once for each detector type. Each basic derived geometry is given a letter designation with the subscript "1". These geometries are shown in the figures below. Variations on these basic derived geometries are distinguished by a different subscript. The subscript "2" indicates that all the longitudinal slices in the basic geom-

etry have been combined. These geometries demonstrate the weak dependence of the figure-of-merit M on the degree of longitudinal segmentation. Geometries with the subscript “3” utilize only the center two slices. These geometries were used when simulating a neutron beam longer than the detector. Schematic drawings of the geometries and the results of Monte Carlo simulations using them are in the following sections. The geometry designation scheme is summarized in Table A-1

Label Subscript	Description
1	Basic derived geometry
2	Longitudinal slices are combined
3	Only center two slices are used

Table A-1: Designation scheme for the detector geometries.

A.2.1 Geometry A₁

Figure A-1: Geometry A₁. The basic cylindrical geometry from which all the other cylindrical geometries were derived. This detector design is unrealistic in that it requires sensitivity to both electrons and protons in each section. It is provided as a reference.

A.2.2 Geometry B₁

Figure A-2: Geometry B₁. This geometry is practical in that it each section is sensitive to only one of the two particle types. Its sensitivity is enhanced because it includes electron and proton coincidence pairs at large (135°) relative azimuthal angles. Almost none of this geometry's sensitivity comes from the coincidence pairs at small (22.5°) relative azimuthal angles. The symmetry of the arrangement readily allows for comparisons of count rates in coincidence pairs with opposite handedness. This symmetry is useful for reducing or eliminating systematic effects.

A.2.3 Geometry C₁

Figure A-3: Geometry C₁. An approximation of the traditional geometry in which the electron and proton detectors are placed at right angles to each other. In addition, it incorporates segmentation into slices.

A.2.4 Geometry D₁

Figure A-4: Geometry D₁. This geometry is an attempt to improve on the traditional geometries by accepting decays with large electron-proton azimuthal angle ($\Delta\theta$ up to 180°). It is similar to the traditional designs in that it places the electron and proton detectors at an average right angles to one another. In addition, it incorporates segmentation into slices.

A.2.5 Geometry E₁

Figure A-5: Geometry E₁. This geometry places the electron and proton detection at average right angles. It requires overlap between the electron and proton detection.

A.2.6 Geometry F1

Figure A-6: Geometry F₁. This geometry employs fine segmentation for the electron detection but only two proton wedges per slice. It approximates a detector in which the protons are focussed either left or right. Detectors which employ this type of proton focussing would be highly susceptible to systematic effects due to nonuniform focussing efficiency. See Chapter 6.

A.2.7 Geometry G_1

Figure A-7: Geometry G_1 . A derivative of Geometry F_1 in which the segmentation of the proton detection proton into slices is removed. This more closely approximates strong left-right proton focussing in which the information about proton's longitudinal position would be lost due to the focussing. Detectors which employ this type of proton focussing would be highly susceptible to systematic effects due to nonuniform focussing efficiency. See Chapter 6.

A.2.8 Geometry H₁

Figure A-8: Geometry H₁. This geometry differs from the traditional right-angle geometries in its increased segmentation. Unfortunately, the increased segmentation cannot be usefully incorporated into an analysis scheme which compares coincidence pairs of with primary sensitivity to opposite signs of the triple product, $\sigma_n \cdot p_e \times p_p$.

A.2.9 Results for a Point Neutron Source

A.2.10 Results for Line Neutron Sources

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0765(4)	0.9634(2)	0.282(1)
2	B ₁	8	4	8	4	0.0323(3)	0.2098(1)	0.392(2)
7	C ₁	8	2	8	2	0.0028(1)	0.0266(1)	0.324(5)
4	D ₁	8	2	8	2	0.0204(2)	0.1580(1)	0.359(2)
12	E ₁	8	2	8	2	0.0134(2)	0.9634(2)	0.118(1)
5	F ₁	8	8	8	2	0.0258(2)	0.9634(2)	0.164(1)
6	G ₁	8	8	1	2	0.0211(2)	0.9634(2)	0.148(1)
3	H ₁	8	4	8	4	0.0216(2)	0.1580(1)	0.369(2)
17	A ₂	1	8	1	8	0.0601(4)	0.9634(2)	0.250(1)
9	B ₂	1	4	1	4	0.0257(2)	0.2098(1)	0.350(2)
8	C ₂	1	2	1	2	0.0021(1)	0.0266(1)	0.281(5)
11	D ₂	1	2	1	2	0.0163(2)	0.1580(1)	0.321(2)
13	E ₂	1	2	1	2	0.0108(2)	0.9634(2)	0.106(1)
10	H ₂	1	4	1	4	0.0170(2)	0.1580(1)	0.328(2)
18	A ₃	2	8	2	8	0.0636(4)	0.6674(2)	0.309(1)
14	B ₃	2	4	2	4	0.0277(3)	0.1390(1)	0.447(2)
16	C ₃	2	2	2	2	0.0020(1)	0.0142(1)	0.376(1)
15	D ₃	2	2	2	2	0.0171(2)	0.0977(1)	0.418(2)
Maximum Likelihood						0.183(2)	1.00	0.428(1)

Table A-2: Figure-of-merit for various cylindrical detector models. The neutron source was a point.

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0674(4)	0.9521(2)	0.266(1)
2	B ₁	8	4	8	4	0.0281(2)	0.2068(1)	0.369(1)
7	C ₁	8	2	8	2	0.0027(1)	0.0260(1)	0.323(5)
4	D ₁	8	2	8	2	0.0183(2)	0.1555(1)	0.343(2)
12	E ₁	8	2	8	2	0.0116(2)	0.9521(2)	0.111(1)
5	F ₁	8	8	8	2	0.0222(2)	0.9521(2)	0.153(1)
6	G ₁	8	8	1	2	0.0202(2)	0.9521(2)	0.146(1)
3	H ₁	8	4	8	4	0.0193(2)	0.1555(1)	0.352(2)
17	A ₂	1	8	1	8	0.0602(4)	0.9521(2)	0.251(1)
9	B ₂	1	4	1	4	0.0255(2)	0.2068(1)	0.351(2)
8	C ₂	1	2	1	2	0.0024(1)	0.0260(1)	0.302(5)
11	D ₂	1	2	1	2	0.0166(2)	0.1555(1)	0.326(2)
13	E ₂	1	2	1	2	0.0109(1)	0.9521(2)	0.107(1)
10	H ₂	1	4	1	4	0.0173(2)	0.1555(1)	0.333(2)
18	A ₃	2	8	2	8	0.0199(2)	0.2403(1)	0.288(2)
14	B ₃	2	4	2	4	0.0085(3)	0.0503(1)	0.412(3)
16	C ₃	2	2	2	2	0.00069(4)	0.00537(2)	0.36(1)
15	D ₃	2	2	2	2	0.0054(1)	0.0359(1)	0.387(4)

Table A-3: Figure-of-merit for various cylindrical detector models. The neutron source was a line 0.50 the length of the detector.

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0670(4)	0.9245(2)	0.269(1)
2	B ₁	8	4	8	4	0.0283(3)	0.2004(1)	0.376(2)
7	C ₁	8	2	8	2	0.0024(1)	0.0248(1)	0.310(5)
4	D ₁	8	2	8	2	0.0178(2)	0.1497(1)	0.345(2)
12	E ₁	8	2	8	2	0.0113(2)	0.9245(2)	0.110(1)
5	F ₁	8	8	8	2	0.0221(2)	0.9245(2)	0.155(1)
6	G ₁	8	8	1	2	0.0202(2)	0.9245(2)	0.148(1)
3	H ₁	8	4	8	4	0.0188(2)	0.1497(1)	0.354(2)
17	A ₂	1	8	1	8	0.0606(4)	0.9245(2)	0.256(1)
9	B ₂	1	4	1	4	0.0260(2)	0.2004(1)	0.360(2)
8	C ₂	1	2	1	2	0.0021(1)	0.0248(1)	0.289(5)
11	D ₂	1	2	1	2	0.0164(2)	0.1497(1)	0.331(2)
13	E ₂	1	2	1	2	0.0107(2)	0.9245(2)	0.107(1)
10	H ₂	1	4	1	4	0.0171(2)	0.1497(1)	0.338(2)
18	A ₃	2	8	2	8	0.0133(2)	0.1604(1)	0.288(2)
14	B ₃	2	4	2	4	0.0058(1)	0.0336(1)	0.414(4)
16	C ₃	2	2	2	2	0.00047(3)	0.00359(2)	0.36(1)
15	D ₃	2	2	2	2	0.0036(1)	0.0240(1)	0.388(5)

Table A-4: Figure-of-merit for various cylindrical detector models. The neutron source was a line 0.75 the length of the detector.

A.2.11 Results for Cylindrical Neutron Sources

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0638(4)	0.9520(2)	0.259(1)
2	B ₁	8	4	8	4	0.0263(3)	0.2127(1)	0.352(2)
7	C ₁	8	2	8	2	0.0027(1)	0.0268(1)	0.319(5)
4	D ₁	8	2	8	2	0.0175(2)	0.1601(1)	0.331(2)
12	E ₁	8	2	8	2	0.0116(2)	0.9520(2)	0.110(1)
5	F ₁	8	8	8	2	0.0215(2)	0.9520(2)	0.150(1)
6	G ₁	8	8	1	2	0.0196(2)	0.9520(2)	0.143(1)
3	H ₁	8	4	8	4	0.0183(2)	0.1601(1)	0.338(2)
17	A ₂	1	8	1	8	0.0566(4)	0.9520(2)	0.244(1)
9	B ₂	1	4	1	4	0.0237(2)	0.2127(1)	0.334(2)
8	C ₂	1	2	1	2	0.0023(1)	0.0268(1)	0.294(5)
11	D ₂	1	2	1	2	0.0159(2)	0.1601(1)	0.315(2)
13	E ₂	1	2	1	2	0.0108(2)	0.9520(2)	0.107(1)
10	H ₂	1	4	1	4	0.0164(2)	0.1601(1)	0.320(2)
18	A ₃	2	8	2	8	0.0187(2)	0.2408(1)	0.279(2)
14	B ₃	2	4	2	4	0.0079(1)	0.0523(1)	0.388(3)
16	C ₃	2	2	2	2	0.00073(4)	0.0056(2)	0.36(1)
15	D ₃	2	2	2	2	0.0052(1)	0.0373(1)	0.373(4)

Table A-5: Figure-of-merit for various cylindrical detector models. The neutron source was a cylinder 0.50 the length of the detector by 0.15 of its diameter.

A.3 Verification of the Monte Carlo Simulation

Verification of the Monte Carlo event generation alone comes from the recovery of the input decay coefficients from a maximum likelihood fit of a set of decay events generated by the Monte Carlo. This is described in Section 3.2.1.

The decay events are completely determined by any *two* of the electron, proton, and neutrino momentum vectors. The Monte Carlo generates the electron and neutrino momenta according to Equation 2-10 and determines the proton momentum using energy and momentum conservation. A trivial change in the Monte Carlo detector simulation makes it possible to integrate the electron and neutrino momenta rather than the electron and proton momenta over finite solid angles. The validity of the Monte Carlo was checked

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0543(4)	0.9521(2)	0.239(1)
2	B ₁	8	4	8	4	0.0206(2)	0.2256(1)	0.302(2)
7	C ₁	8	2	8	2	0.0032(1)	0.0296(1)	0.328(4)
4	D ₁	8	2	8	2	0.0159(2)	0.1720(1)	0.304(2)
12	E ₁	8	2	8	2	0.0114(2)	0.9521(2)	0.109(1)
5	F ₁	8	8	8	2	0.0197(2)	0.9521(2)	0.144(1)
6	G ₁	8	8	1	2	0.0179(2)	0.9521(2)	0.137(1)
3	H ₁	8	4	8	4	0.0165(2)	0.1720(1)	0.310(2)
17	A ₂	1	8	1	8	0.0481(3)	0.9521(2)	0.225(1)
9	B ₂	1	4	1	4	0.0187(2)	0.2256(1)	0.289(2)
8	C ₂	1	2	1	2	0.0028(1)	0.0296(1)	0.308(4)
11	D ₂	1	2	1	2	0.0146(2)	0.1720(1)	0.291(2)
13	E ₂	1	2	1	2	0.0105(2)	0.9521(2)	0.105(1)
10	H ₂	1	4	1	4	0.0148(2)	0.1720(1)	0.294(2)
18	A ₃	2	8	2	8	0.0157(2)	0.2424(1)	0.255(2)
14	B ₃	2	4	2	4	0.0061(1)	0.05676(6)	0.329(3)
16	C ₃	2	2	2	2	0.00088(5)	0.00647(2)	0.37(1)
15	D ₃	2	2	2	2	0.0048(1)	0.04135(5)	0.341(4)

Table A-6: Figure-of-merit for various cylindrical detector models. The neutron source was a cylinder 0.50 the length of the detector by 0.30 of its diameter.

by comparing its results for electron-neutrino integrations to analytic integrations of Equation 2-10 over many simple electron and neutrino momentum ranges. The probability distribution function Equation 2-10 written in spherical coordinates with the neutron polarization along the polar axis and integrated over finite electron and neutrino angular intervals and the electron energy from E_{low} to the beta endpoint energy E_0 is

$$\begin{aligned}
& \int_{E_{\text{low}}}^{E_0} dE_e \int_{\cos \theta_{e1}}^{\cos \theta_{e2}} d(\cos \theta_e) \int_{\phi_{e1}}^{\phi_{e2}} d\phi_e \int_{\cos \theta_{v1}}^{\cos \theta_{v2}} d(\cos \theta_v) \int_{\phi_{v1}}^{\phi_{v2}} d\phi_v w \\
& = C \{ \Delta_{\cos \theta_e} \Delta_{\cos \theta_v} \Delta_{\phi_e} \Delta_{\phi_v} \\
& + \frac{a\eta}{16} [(2\Delta_{\theta_e} - \Delta_{\sin 2\theta_e}) (2\Delta_{\theta_v} - \Delta_{\sin 2\theta_v}) (\Delta_{\cos \phi_e} \Delta_{\cos \phi_v} + \Delta_{\sin \phi_e} \Delta_{\sin \phi_v}) + \Delta_{\phi_e} \Delta_{\phi_v} \Delta_{\cos 2\theta_e} \Delta_{\cos 2\theta_v}] \quad (\text{A-16}) \\
& + \frac{A\eta}{4} [\Delta_{\phi_e} \Delta_{\phi_v} \Delta_{\cos 2\theta_e} \Delta_{\cos \theta_v}] + \frac{B}{4} [\Delta_{\phi_e} \Delta_{\phi_v} \Delta_{\cos \theta_e} \Delta_{\cos 2\theta_v}] \\
& + \frac{D\eta}{16} [(2\Delta_{\theta_e} - \Delta_{\sin 2\theta_e}) (2\Delta_{\theta_v} - \Delta_{\sin 2\theta_v}) (\Delta_{\sin \phi_v} \Delta_{\cos \phi_e} - \Delta_{\cos \phi_v} \Delta_{\sin \phi_e})] \}
\end{aligned}$$

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0356(3)	0.9525(2)	0.193(1)
2	B ₁	8	4	8	4	0.0083(1)	0.2413(1)	0.185(2)
7	C ₁	8	2	8	2	0.0048(1)	0.0434(1)	0.332(4)
4	D ₁	8	2	8	2	0.0119(2)	0.2075(1)	0.240(2)
12	E ₁	8	2	8	2	0.0106(2)	0.9525(2)	0.105(1)
5	F ₁	8	8	8	2	0.0143(2)	0.9525(2)	0.123(1)
6	G ₁	8	8	1	2	0.0130(2)	0.9525(2)	0.117(1)
3	H ₁	8	4	8	4	0.0136(2)	0.2075(1)	0.256(2)
17	A ₂	1	8	1	8	0.0311(3)	0.9525(2)	0.181(1)
9	B ₂	1	4	1	4	0.0073(1)	0.2413(1)	0.174(2)
8	C ₂	1	2	1	2	0.0043(1)	0.0434(1)	0.315(4)
11	D ₂	1	2	1	2	0.0110(2)	0.2075(1)	0.231(2)
13	E ₂	1	2	1	2	0.0098(2)	0.9525(2)	0.102(1)
10	H ₂	1	4	1	4	0.0120(2)	0.2075(1)	0.241(2)
18	A ₃	2	8	2	8	0.0105(2)	0.2493(1)	0.205(2)
14	B ₃	2	4	2	4	0.0023(1)	0.0634(1)	0.192(3)
16	C ₃	2	2	2	2	0.0014(1)	0.0111(1)	0.35(1)
15	D ₃	2	2	2	2	0.0036(1)	0.0539(1)	0.259(3)

Table A-7: Figure-of-merit for various cylindrical detector models. The neutron source was a cylinder 0.50 the length of the detector by 0.60 of its diameter.

where the notation $\Delta_{g(x)} = g(x_2) - g(x_1)$ for any function g has been used, C is an overall constant, and η is the ratio

$$\eta = \frac{\int_{E_{\text{low}}}^{E_0} G(E_e) (p_e/E_e) dE_e}{\int_{E_{\text{low}}}^{E_0} G(E_e) dE_e}$$

where $E_{\text{low}} = m_e + T_{\text{thresh}}$. The ratio η is obtained by numerical integration and is shown as a function of a lower level kinetic energy threshold on the electron T_{thresh} in Figure A-9. If T_{thresh} is zero, i.e. there is no cut on the electron energy, then $\eta = 0.729$.

#	Model Name	Electron		Proton		M	ϵ_{coinc}	K
		Slices	Wedges	Slices	Wedges			
1	A ₁	8	8	8	8	0.0221(2)	0.9532(2)	0.152(1)
2	B ₁	8	4	8	4	0.0060(1)	0.2376(1)	0.159(2)
7	C ₁	8	2	8	2	0.0026(1)	0.0590(1)	0.209(3)
4	D ₁	8	2	8	2	0.0066(1)	0.2368(1)	0.167(2)
12	E ₁	8	2	8	2	0.0068(1)	0.9532(2)	0.084(1)
5	F ₁	8	8	8	2	0.0081(1)	0.9532(2)	0.092(1)
6	G ₁	8	8	1	2	0.0072(1)	0.9532(2)	0.087(1)
3	H ₁	8	4	8	4	0.0079(1)	0.2368(1)	0.183(2)
17	A ₂	1	8	1	8	0.0186(2)	0.9532(2)	0.140(1)
9	B ₂	1	4	1	4	0.0050(1)	0.2376(1)	0.145(2)
8	C ₂	1	2	1	2	0.0023(1)	0.0590(1)	0.198(3)
11	D ₂	1	2	1	2	0.0061(1)	0.2368(1)	0.161(2)
13	E ₂	1	2	1	2	0.0063(1)	0.9532(2)	0.081(1)
10	H ₂	1	4	1	4	0.0069(1)	0.2368(1)	0.171(2)
18	A ₃	2	8	2	8	0.0068(1)	0.2642(1)	0.160(2)
14	B ₃	2	4	2	4	0.0020(1)	0.0658(1)	0.173(3)
16	C ₃	2	2	2	2	0.00073(4)	0.01693(3)	0.207(6)
15	D ₃	2	2	2	2	0.0020(1)	0.0668(1)	0.174(3)

Table A-8: Figure-of-merit for various cylindrical detector models. The neutron source was a cylinder 0.50 the length of the detector by 0.90 of its diameter.

This parameter is responsible for the dependence of the instrumental constant K on the

electron energy threshold.

Figure A-9: The parameter η as a function of the electron kinetic energy threshold. The a , A , and D terms of the PDF are reduced by this factor relative to the other terms.

Each term in Equation A-16 has been evaluated for each coincidence bin of a cylindrical geometry assuming a point source of neutrons at the center of the cylinder with the neutrons polarized along its axis. A coincidence bin merely denotes the range of electron angles (θ_{e1} to θ_{e2} and ϕ_{e1} to ϕ_{e2}) and neutrino angles (θ_{v1} to θ_{v2} and ϕ_{v1} to ϕ_{v2}) which subtend a single section of Geometry A₁. The resulting five arrays represent the fraction of coincidences which fall into each coincidence bin due to each term in the electron-neutrino PDF. A linear combination of these five arrays has elements which are proportional to the number of events which should fall into each coincidence bin due to the full distribution. Thus the number of counts N_e^i expected analytically in the i^{th} bin can be written as linear function of the four decay coefficients a , A , B , and D .

In order to compare the calculation to the Monte Carlo, electron-neutrino coincidences are binned in the basic cylindrical detector model, Geometry A₁ defined in Section A.2. As always, the Monte Carlo produces an array of numbers of coincidences

N_{MC}^i which fell into each detector pair. The accuracy of the Monte Carlo is tested by fitting the N_e^i to the N_{MC}^i for all the coincidence bins as a function of the decay coefficients. This has been done for several cases in which the values of the decay coefficients (a , A , B and D) were varied. The results are summarized in Table A-9.

Input To Monte Carlo				Fit To Analytic Calculation			
a	A	B	D	a	A	B	D
0.3	0.0	0.0	0.0	0.300(2)	-0.001(2)	0.003(2)	-0.005(2)
0.0	0.3	0.0	0.0	0.000(2)	0.293(2)	-0.003(2)	0.002(2)
0.0	0.0	0.3	0.0	-0.007(2)	0.001(2)	0.301(2)	0.003(2)
0.0	0.0	0.0	0.3	-0.002(2)	0.002(2)	0.002(2)	0.304(2)
0.3	0.3	0.3	0.3	0.301(2)	0.297(2)	0.300(2)	0.303(2)

Table A-9: Comparison of Monte Carlo and analytic integration for the cylindrical geometry.

These results provide a verification of the Monte Carlo event generation and a direct check of the coincidence binning routines. The vertex distribution and binning procedures in the Monte Carlo are identical whether electron-neutrino or electron-proton coincidences are being counted. Therefore, assuming the trivial conversion from proton to neutrino momenta is done correctly, the usual electron-proton Monte Carlo results are verified as well.

A.4 Method Used in the Advanced Monte Carlo Simulation

The Monte Carlo detector simulations beyond the simple spherical and cylindrical models were performed using an object-oriented particle tracking computer program. It is written in a subset of the C++ programming language on the Macintosh II® and Quadra® series of computers manufactured by Apple Computer Corporation.

The simulation operates as follows. A detector is simulated by an arbitrary number of different types of detector "components" each of which has an arbitrary shape, position and orientation in a global coordinate system. Each component is an object responsible for determining whether or not a given point in three-dimensional global coordinates is within

its volume, and tracking particles through its volume while changing the dynamical variables of the particle and producing a record of the simulated detector response as it does so. The main simulation program reads a Macintosh resource file which defines the detector components. It then produces neutron decay events which are distributed in position and orientation to simulate a polarized neutron beam. Each detector component is interrogated to find which component contains the decay vertex. Once found, control is passed to that component which is responsible for tracking the particle until it is either stopped or it exits the component's volume. If the particle exits a component's volume, the main program again interrogates each detector component until the component which the particle entered is found. This process is repeated until the particle is stopped within a component or is lost by exiting the detector geometry altogether.

The unique feature of this simulation is that no global information about the relative shapes, orientations, and positions of the detector components is required. This allows the shape, orientation, and position of any detector component in the geometry to be easily modified without affecting the other components or requiring modifications the main program. As the first few decay events are tracked by the described method, each detector components "learns" which components neighbor it in space. For subsequent events in which a particle exits the component, its neighbors are checked first to see if they now contain the particle. Generally, after a short time, the simulation has adapted to the makeup of the current geometry and subsequent decay events are tracked efficiently. This adaptive technique allows changes in the geometry to be implemented with a minimum of human effort at little cost in computational power.

A.5 Detector Geometry used Systematic Effects Calculations

Details of the geometry simulated for the calculation of systematic effects are given here. The geometry consists of four rectangular electron sensitive panels and four rectangular proton sensitive panels arranged in an octagon about a neutron beam. The

electron detector panels are 50 cm long by 8.284 cm wide. The proton detector panels are 24 cm long by 8.4 cm wide. The panels are arranged so that their centers, lengthwise, are coincident. The center of each panel is 10 cm radially from the axis of the octagonal prism the panels lie on. The positions and orientations are given in Table A-10. Each detector panel is subdivided along its length into slices. The proton panels are divided into six 4 cm lengths. The electron panels are divided into six overlapping 30 cm lengths. The centers of the electron slices (lengthwise) are centered relative to the centers of the proton detector sections in the same slice.

Detector Panel		Position (cm)			Rotation (degrees)		
Type	Number	x	y	z	ϕ	θ	ψ
Electron	1	10.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	10.0	0.0	90.0	0.0	0.0
	3	-10.0	0.0	0.0	180.0	0.0	0.0
	4	0.0	-10.0	0.0	-90.0	0.0	0.0
Proton	101	7.071	7.071	13.0	45.0	0.0	0.0
	201	-7.071	7.071	13.0	135.0	0.0	0.0
	301	-7.071	-7.071	13.0	-135.0	0.0	0.0
	401	7.071	-7.071	13.0	-45.0	0.0	0.0

Table A-10: Positions and orientations of the eight detector panels. The positions are given in the global coordinate system of the simulation. The orientations are given by the Euler angles defining the rotation required to orient the panel from its local coordinate system into the global coordinate system. In the local coordinate system, each of the panels lies in the y-z plane with one end centered on the origin. The euler angles define the rotation of the panel. The order of application of the rotations, which do not commute, is specified by the euler angles as: positive rotation ϕ about the z-axis, positive rotation θ about the resulting x-axis, positive rotation ψ about the resulting z-axis.

Appendix B: Effect of Background on Sensitivity

The effect of a constant background on the statistical sensitivity to D can be computed in the framework of the analysis method given in Section 3.3. We consider the case in which the number of counts in each coincidence pair is increased by a fixed number of background counts N_{BG} so the number of measured counts is $N_M = N_S + N_{BG}$. The ratio R in Equation 3-19 is formed from the number of signal counts N_S in each coincidence pair so it consists of factors of the form $(N_M - N_{BG})$. Since D is small, N_S is approximately the same for each coincidence pair and for each polarization. We define $N_T \approx 16N_S$ the total number of counts after the background has been subtracted. Using Poisson uncertainties for N_M and N_{BG} for each of the sixteen $N_{eip_j}^\sigma$ in R ,

$$\left(\frac{\sigma_R}{R}\right)^2 = \sum_{i,j,\sigma} \left(\frac{\sigma_{N_S}}{N_S}\right)^2 \approx 16 \left(\frac{\sqrt{N_M + N_{BG}}}{N_S}\right)^2 \quad (\text{B-1})$$

where the sum is over the 16 N 's on the right hand side of Equation 3-19. From Equations B-1 and 3-23 and using $R \approx 1$, the statistical uncertainty in D with background is

$$\sigma_D(\text{stat.}) = \frac{1}{KP} \frac{1}{\sqrt{N_T}} \sqrt{1 + 2/r} \quad (\text{B-2})$$

where $r = N_S/N_{BG}$. Comparing to Equation 3-24, we find that for a fixed running time (i.e. fixed N_T) the uncertainty in D is increased by the factor $\sqrt{1 + 2/r}$ or, equivalently, the running time to reach a particular uncertainty is increased by $(1 + 2/r)$. This is shown graphically in Figure B-1.

Figure B-1: The washout factor versus the signal to background ratio.

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