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Some Considerations for Modeling the Creep Behavior of Composite Materials

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ABSTRACT

A simple treatment of the mechanics of creep in unidirectional composites is presented with the objective of stressing the need for using appropriate constitutive equations when modeling the behavior of the constituents. It is demonstrated that pseudo-constitutive relations, such as those obtained from constant stress creep experiments for example, can not be used to describe the mechanical behavior of the constituents when modeling the creep of composites. By describing the mechanical behavior of the composite constituents using linear viscoelastic models, an analysis is presented to assess the type of errors which are incurred when inappropriate constitutive relations are used. Other considerations for modeling the creep behavior of ceramic matrix composites are discussed.

INTRODUCTION.

The main concerns when using a material in structural applications at high homologous temperatures are strength retention, service life and dimensional stability, also known as creep resistance. As ceramic matrix composites start to be considered for a number of long-life structural applications at elevated temperatures, it has become necessary to understand and model the mechanics of their creep deformation.

To date, the creep data for most structural ceramics have been obtained from constant stress experiments, and often the results are summarized with empirical correlations of the Bailey-Norton type, which provide the stress dependence of the rate of deformation during the so-called "steady state" regime[1,2]. In most data bases and analyses, the transients and deformation accumulated prior to the "steady state" creep regime are ignored without justification, and little or no data exist on the response of structural ceramics to more complex loading histories at elevated temperatures[1,2].

During the creep deformation of composites, the most relevant mechanism is that of time-dependent load re-distribution between the fibers and the matrix, and although the composite would be subjected to constant stress during creep, the phases will experience a state of continuously changing stress. This re-distribution of the load is a direct result of the difference in creep resistance of the phases, and the rate at which load is transferred between the phases is controlled by their relative stiffness and concentration. Other aspects of the mechanics of composites that must be considered include matrix and fiber cracking, fiber debonding and sliding and the effect of interphases and protective coatings. However, even if these aspects are considered in a mechanics analysis, a lack of realistic material models to describe the behavior of the constituents will make the most sophisticated models useless.

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There are several studies on the modeling of the creep of composites where the behavior of the phases was described using pseudo-constitutive equations, such as those obtained from creep experiments[3-7]. However, because these data were obtained from tests at constant stress, they do not represent the behavior of a material when subjected to non-constant stresses, and therefore, can not be used to model the behavior of the phases in the composite.

The purpose of this paper is to highlight the deficiencies in the formulation of existent composite creep analyses, particularly in the choice of the constitutive equations. By using the simple case of linear viscoelastic materials, the governing relations for composite creep are presented, and the effect of considering pseudo-constitutive equations is investigated.

ANALYSIS.

To convey the ideas in this paper, the Burgers model will be used to represent the behavior of the matrix in an idealized composite. The Burgers model is the simplest of the linear viscoelastic models that best reproduces, at least qualitatively, the mechanical behavior of real materials when loaded mechanically at high homologous temperatures[8].

Burgers Material. The schematic representation of this model is given in Figure 1 and its mathematical counterpart, or constitutive equation is given by

$$\ddot{\epsilon} + \frac{E_2}{\eta_2} \dot{\epsilon} = \frac{1}{E_1} \ddot{\sigma} + \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} + \frac{E_2}{E_1 \eta_2} \right) \dot{\sigma} + \frac{E_2}{\eta_1 \eta_2} \sigma \quad (1)$$

where ϵ denotes strain, σ stress, the dot denotes differentiation with respect to time, E_i are spring constants and η_i are coefficients of viscosity. The constants are associated with the different elements depicted in Figure 1. Although the Burgers model is linear and very simple, it reproduces the features of material behavior during creep, i. e. it exhibits an instantaneous elastic response followed by a transient stage (primary creep) and a stage of steady state deformation at constant strain rate after the exhaustion of the transient. No "tertiary" creep is predicted by this model.

Composite Creep.

Rigorous Constitutive Equations. Let us consider the creep behavior of a unidirectional composite containing long continuous fibers. Let us assume, without losing generality, that the matrix is a Burgers material and that the fibers are perfectly elastic and occupy a volume fraction v_f of the composite (See Figure 2.a). Furthermore, if end-effects are neglected and if it is assumed that the phases are bonded, a plane strain formulation may be adopted and the mechanical behavior of the composite can be described with the rheological model in Figure 2.b.

The equations governing the behavior of this composite are :

$$\epsilon_c = \epsilon_f = \epsilon_m \quad (2)$$

$$\dot{\epsilon}_c = \dot{\epsilon}_f = \dot{\epsilon}_m \quad (3)$$

$$\ddot{\epsilon}_c = \ddot{\epsilon}_f = \ddot{\epsilon}_m \quad (4)$$

and as a result of these conditions, the so-called rule of mixtures is obtained

$$\sigma_c = v_f \sigma_f + v_m \sigma_m \quad (5)$$

and

$$E_c = v_f E_f + v_m E_m \quad (6)$$

where the indices c, f and m refer to the composite, fibers and matrix respectively, and evidently

$$v_f + v_m = 1$$

If we assume that the composite is subjected to a constant stress σ_0 (creep), then Equation 5 can be re-written as follows:

$$\sigma_0 = v_f E_f \epsilon + v_m \sigma_m \quad (7)$$

and it follows from Equations 3 and 4 that

$$\dot{\sigma}_m = - \frac{v_f E_f}{v_m} \dot{\epsilon} \quad (8)$$

$$\ddot{\sigma}_m = - \frac{v_f E_f}{v_m} \ddot{\epsilon} \quad (9)$$

Equations 8 and 9 describe the most relevant feature of the creep of our composite, i. e. the stress acting on the phases will change continuously with time, and in this particular case in proportion to the rate of deformation.

To determine the creep strain of the composite as a function of time, it is necessary to combine the constitutive equations that describe the mechanical behavior of the phases with the governing equations for composite creep. Let us define the following terms:

$$\epsilon_{\text{equilibrium}} = \frac{\sigma_0}{E_f v_f} \quad (10)$$

$$u(t) = \frac{\epsilon(t)}{\epsilon_{\text{equilibrium}}} \quad (11)$$

Because the fibers are perfectly elastic, $\epsilon_{\text{equilibrium}}$ is the maximum attainable strain of the composite for the stress, σ_0 , while u represents a measure of

how much of the maximum attainable composite creep strain has been accomplished.

If we combine Equation 1, the constitutive equation for the matrix, with Equations 7-9 and 11, the following differential equation for u is obtained:

$$\ddot{u} + \left(\frac{E_1 E_2 v_m}{\eta_2 E_c} + \frac{E_1 E_f v_f}{E_c} \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} + \frac{E_2}{\eta_2 E_1} \right) \right) \dot{u} + \frac{E_1 E_2 E_f v_f}{\eta_1 \eta_2 E_c} u = \frac{E_1 E_2 E_f v_f}{\eta_1 \eta_2 E_c} \quad (12)$$

which has a solution of the following form

$$u(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} + 1 \quad (13)$$

where C_1 and C_2 are constants that can be determined from the initial conditions:

$$u(0) = \frac{v_f E_f}{E_c} \quad (14)$$

and

$$\dot{u}(0) = \frac{v_f E_f}{v_m} \left(1 - \frac{v_f E_f}{E_c} \right) \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) \quad (15)$$

Let us now investigate the characteristics of Equation 13. The results will be presented in normalized time using the retardation time of the Kelvin-Voigt element of the matrix as the time scale. The numerical values given in Table 1 will be used as the control set to investigate the effect of the different parameters in the solution.

Some important characteristics of the solution are immediately evident from the form of Equation 13. For example, because the combined effect of the exponential terms will decay with time, the creep deformation of the composite will be a long transient in which the rate of deformation continuously decreases, and will eventually become zero when u becomes equal to 1. As the composite deforms, the stress acting on the matrix will decrease with increasing composite deformation, as dictated by Equation 9, and because the stress acting on the matrix is the only driving force for the time-dependent deformation of the composite, the equilibrium strain will be reached at a continuously slower rate.

Figure 3 shows the predicted composite creep behavior as a function of the stiffness of the matrix (E_1). These results indicate that by increasing the matrix stiffness, the creep curves become steeper at the beginning of the deformation process. This occurs since the stress acting on the matrix is the only driving force for the creep deformation of this composite and by this stress being largest at the beginning of the creep process, most of the attainable deformation will be accumulated in a short period of time.

The effect of the matrix viscosity coefficient (η_1) on the creep response of the composite is illustrated in Figure 4. It is shown that because the resistance of the matrix to deformation decreases by decreasing its viscosity, softer matrices will result in composites that reach the equilibrium strain faster. This can be best understood if we consider that, as described by Equation 15, the instantaneous rate of change of u at time zero is inversely proportional to η_1 , i. e. for less viscous matrices, the creep curves will be steeper at the beginning approaching the equilibrium strain faster. Furthermore, similarly to the definition of the retardation time for the Kelvin-Voigt element, an appropriate time scale to determine the duration of the creep process in this composite will be best described by a composite "retardation time" η_1/E_f .

Approximate Constitutive Equations. The effect of using approximate constitutive equations to model the mechanical behavior of the composite constituents will now be investigated. Let us consider again the rheological model in Figure 2, which consists of a composite with a viscoelastic matrix and perfectly elastic fibers subjected to a constant stress. However, to model the mechanical behavior of the matrix we will use the following equation

$$\dot{\epsilon} = \sigma_m \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} e^{-\frac{E_2}{\eta_2} t} \right) + \frac{1}{E_1} \sigma_m \quad (16)$$

which describes the behavior of a Burgers material during creep. This approach of using reduced forms of the exact constitutive equation, particularly equations obtained from creep experiments, such as equations of the type of Equation 16, has been adopted by many authors [3-7]. Other variations of Equation 16 have been used as well, by considering only the first and third terms, for example[5-7]. However, because these are special cases of Equation 16, the conclusions from this exercise will be equally applicable to those cases.

Because the fibers are perfectly elastic in this composite, the maximum attainable strain for the composite will also be $\epsilon_{\text{equilibrium}}$ (Equation 10). Let us consider again u , which is the measure of the attainable composite creep strain as defined in Equation 11. By combining Equation 16 with the governing equations for composite creep, Equations 2-4 and 7-9, the following differential equation is obtained for u :

$$\dot{u} + \left(\frac{E_f v_f E_1}{E_c} \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} e^{-\frac{E_2}{\eta_2} t} \right) \right) u = \frac{E_f v_f E_1}{E_c} \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} e^{-\frac{E_2}{\eta_2} t} \right) \quad (17)$$

subject to the initial condition:

$$u(0) = \frac{v_f E_f}{E_c}$$

The solution to Equation 17 is :

$$u(t) = 1 + \left(\frac{E_f v_f}{E_c} - 1 \right) \exp \left(\frac{E_1 E_f v_f}{E_c} \left(-\frac{t}{\eta_1} + \frac{1}{E_2} \left(e^{-\frac{E_2}{\eta_2} t} - 1 \right) \right) \right) \quad (18)$$

Evidently, this equation, similarly to Equation 13, predicts the existence of an equilibrium strain that is reached as $t \rightarrow \infty$. However, this solution is intrinsically different to that in Equation 13. To assess these differences and to show the deficiencies of the latter derivation, Equations 13 and 18 are plotted as a ratio as a function of time normalized by the retardation time for the Kelvin-Voigt element of the matrix.

Figure 5 illustrates the effect of the matrix stiffness on the difference between the two solutions. Although the curves coincide at $t=0$, the solutions diverge rapidly at the start of the creep process, which is when the rate of stress transfer between the phases is highest. It is observed that for all conditions, the incorrect formulation overpredicts the creep strain of the composite. Eventually the solutions will converge again as $t \rightarrow \infty$, since both equations predict the same equilibrium strain. It is worth mentioning that when $E_f \gg E_1$ (e. g. Graphite Reinforced Plastics) the deviation from the correct solution is not very significant. However, as $E_f \approx E_1$, (e. g. Ceramic Matrix Composites) the deviation becomes substantial.

The effect of the matrix viscosity on the deviation from the exact solution is illustrated in Figure 6. It is shown that for a time scale equal to one retardation time of the Kelvin-Voigt element of the matrix, the viscosity has no effect on the magnitude of the deviation of the solutions, although the deviation is almost 50%. For longer times, the incorrect formulation will converge faster to the exact solution for those systems with smaller matrix coefficients of viscosity, while for matrices with higher viscosity coefficients, it is observed that the deviation from the exact solution will last much longer.

DISCUSSION.

The results obtained in the previous section demonstrate the necessity of using valid constitutive equations for modeling the mechanical behavior of the constituents during the creep of composites. This is particularly critical when the stiffness of the constituents is of the same order of magnitude, or when the mismatch in properties between the constituents is such that the stress transfer among them during the creep of the composite is significant. This effect can be best understood if we consider that the only driving force for time-dependent deformation in our composite is the stress acting on the matrix. For stiffer matrices, the stress acting on the matrix will be proportionally higher at the beginning of the creep process, resulting in faster rates of stress transfer to the fibers. Although for the particular case in which the fibers are perfectly elastic, both the correct and incorrect solutions will converge at long times¹, the solutions may not converge at all if the fibers exhibit time-dependent deformation.

It has been demonstrated that the major deficiency in the erroneous formulation is that the pseudo-constitutive equation employed to model the behavior of the matrix (Equation 16) lacks information to address the

¹since both solutions predict the same equilibrium strain

relaxation mechanisms that occur in the matrix as it is stress-relieved by the fibers. In other words, Equation 16 possesses less predictive power about the behavior of the material than Equation 1, which possesses all the information and predictive power about the behavior and response of the material under any set of loading conditions. Therefore, Equation 16 can not be used to predict the behavior of the material for conditions other than those for which it was derived, in this case constant stress histories.

Although the Burgers model material is linear and very simple, it possesses all the attributes of real materials. It is not intended to imply that the behavior of every material can be represented using linear viscoelastic models. Linear viscoelasticity was used here because the concepts leading to the formulation of exact constitutive equations are well defined and easy to understand. Furthermore, these concepts allowed for a simple formulation to transmit the message in this paper. Evidently, the nature of the primary creep process in real materials may be quite different from that of a simple Kelvin-Voigt element and it is very likely that materials will exhibit non-linear σ - $\dot{\epsilon}$ behavior. Although these facts are immaterial for the purpose of the present discussion, it is likely that the deviations indicated here, when inappropriate constitutive equations are used, will be magnified for non-linear material behavior.

The results obtained in the last section are important because as ceramic matrix composites start to be considered for a number of high temperature structural applications, it will be necessary to have access to data and material models for the design and development of components using these materials. However, with the lack of data bases for the mechanical behavior of ceramics under loading histories other than creep, new mechanical testing methodologies need to be devised to generate data that may lead eventually to the formulation of appropriate constitutive equations. This objective could be accomplished either by: (1) formulating constitutive equations based on physical principles and conducting mechanical tests to determine the constants and material functions in these equations, or (2) by developing phenomenological equations to summarize the results from different mechanical tests with various loading conditions.

CONCLUSIONS

It has been demonstrated that large deviations occur when pseudo constitutive equations are used to describe the mechanical behavior of constituents during composite creep. A number of studies exist in which equations of the Bailey-Norton type have been incorrectly used to model the mechanical behavior of the constituents. Because these relations were derived from constant stress experiments, they can not be used to model the behavior of the constituents as these experience varying stresses during composite creep. It has also been shown, by using a simple composite model consisting of a Burgers material for the matrix and perfectly elastic fibers, that severe overpredictions of the composite creep deformation occur when inappropriate forms of the constitutive equations are used to model the matrix behavior, and that the deviations are most severe at a short time scale, when stress transfer among the phases is fastest. Furthermore, the effect of this event is magnified when the stiffness of the phases is comparable. The main deficiency of pseudo constitutive equations, such as those obtained from constant stress experiments, is that they lack the power

to predict the behavior of the material under circumstances other than those for which they were derived.

The results and observations presented here call for new methodologies for the mechanical evaluation of composite constituents at elevated temperature and for the development of appropriate constitutive equations. The new methodologies should include mechanical tests to determine the constants or material functions in these equations. Evidently, because these constitutive equations should reduce to simpler forms under specific loading histories (e. g. creep, creep recovery, stress relaxation, monotonic loading) these tests may be used to evaluate the needed parameters .

Finally, efforts to model the creep deformation of composites using powerful computers and finite element analyses will be useless unless appropriate constitutive equations are used to describe the mechanical behavior of the materials.

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E_f	200 GPa
ν_f	0.5
E_1	100 GPa
E_2	10 GPa
η_1	1.0×10^{13} MPa sec
η_2	1.0×10^{11} MPa sec
σ_0	300 MPa

Table 1
Control Set

Figure 1. Burgers Model.

Figure 2.(a). Schematic representation of unidirectional composite containing a volume fraction of fibers ν_f .(b). Rheological representation of the composite material in Figure (a). The matrix is modeled with a Burgers material and the fibers are perfectly elastic.

Figure 3. Effect of the matrix stiffness (E_1) on the creep response of the composite material of Figure 2.b. (a) $E_1 = 100$ GPa, (b) $E_1 = 200$ GPa, (c) $E_1 = 400$ GPa.

Figure 4. Effect of the matrix viscosity coefficient (η_1) on the creep response of the composite material of Figure 2.b. (a) $\eta_1 = 1 \times 10^{12}$ Pa sec, (b) $\eta_1 = 1 \times 10^{13}$ Pa sec (c) $\eta_1 = 1 \times 10^{14}$ Pa sec.

Figure 5. Effect of matrix stiffness (E_1) in the deviation of the erroneous formulation from the exact solution. (a) $E_1 = 100$ GPa, (b) $E_1 = 200$ GPa, (c) $E_1 = 400$ GPa.

Figure 6. Effect of matrix viscosity coefficient (η_1) in the deviation of the erroneous formulation from the exact solution. (a) $\eta_1 = 1 \times 10^{12}$ Pa sec, (b) $\eta_1 = 1 \times 10^{13}$ Pa sec, (c) $\eta_1 = 1 \times 10^{14}$ Pa sec.

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