A DECAY SCHEME FOR $^{164}_{\text{Ho}}$

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ABSTRACT

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The present investigation has served to confirm the existing decay scheme for $^{164}$Ho. A high resolution Si(Li) detector is used to measure values for the relative intensities of the photon peaks. Based on the relative intensity measurements, the branching ratios are found to be 9.4% ± 0.6% to the 91.3 KeV state in $^{164}$Er, 18.7% ± 1.2% to the ground state in $^{164}$Er, 24.0% ± 3.4% to the 7.3 KeV state in $^{164}$Dy, and 48.0% ± 7.3% to the ground state in $^{164}$Dy. The half life of the isomeric state in $^{164}$Ho is measured to be 38.9 ± 0.4 minutes. The half life of the ground state of $^{164}$Ho is found to be 47.2 ± 0.5 minutes. The relative intensity measurements made in this investigation lead to a value for the $^{165}$Ho(n, 2n)$^{164}$Ho cross section at 16 MeV. The value for the total cross section is 3200 ± 200 mb. The cross section is 2370 ± 150 mb for the isomeric state and 829 ± 53 mb for the ground state of $^{164}$Ho.
A DECAY SCHEME FOR $^{164}_{\text{Ho}}$

THESIS

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CHAPTER I

INTRODUCTION

The decay of $^{164}$Ho has been studied previously by Brown and Becker$^1$; Jorgensen et al.$^2$; Sethi and Mukherjee$^3$; and Lear, Gray, and Windham$^4$. However all of these investigations were made with $\beta$-particle spectrometers, or with low resolution gamma-ray detectors, which could not completely resolve the photon spectrum. The present investigation was prompted by several considerations. In the previous studies there was considerable variance with regard to the reported values for the half-lives of the isomeric and ground states in $^{164}$Ho. There was also considerable variance with regard to the values reported for the branching ratios and the relative intensities of the transitions. Thus a further study of the problem was needed. A preliminary study of the decay of $^{164}$Ho with a high resolution Si(Li) detector revealed a photon energy spectrum which contained many more photon peaks than had been previously reported. This was due to the presence of twelve x-ray peaks, which had not been resolved in the previous investigations. The presence of these unresolved x-ray peaks affected the half life and intensity measurements made in these investigations.
In the present investigation energies and relative intensities are measured for all peaks in the photon energy spectrum. The values are reported in Table 1. Half life measurements are made on all resolved peaks. On the basis of these measurements half lives for the ground state and isomeric state of $^{164}$Ho are reported. This data, together with the results of a series of coincidence measurements, confirms the existing decay scheme for $^{164}$Ho. The relative intensities measured in this investigation lead to values for the branching ratios. A recalculation of the $^{165}$Ho(n, 2n)$^{164}$Ho cross section as reported by Lear, Gray, and Windham is made on the basis of these branching ratios.
CHAPTER II

EXPERIMENTAL PROCEDURES

The $^{164}\text{Ho}$ activity was produced by the $^{165}\text{Ho}(n, 2n)^{164}\text{Ho}$ reaction. The $^{165}\text{Ho}$ targets consisted of foils of 99.99% chemical purity, obtained from Alpha Inorganics Inc. The foils had a thickness of $1.35 \times 10^{-1}$ gm/cm$^2$. These foils were irradiated by 16 MeV neutrons. The neutrons were produced from the $^3\text{T}(d, n)^4\text{He}$ reaction, using the Regional Nuclear Physics Laboratory's 2 MeV Van de Graaff accelerator.

Photon spectra were taken with a high resolution Si(Li) x-ray spectrometer. The detector was a Kevex Unit, with dimensions of 30 mm$^2$ x 3 mm. The data was printed out on magnetic tape and analysed using the code SAMPO. The multi-channel analyser was calibrated to within ±0.15 KeV using known gamma-ray and x-ray energy values in $^{133}\text{Ba}$, Eu, Nd, Yb, Lu, Cu, Fe, Ge, and Pd. The resolution of the system was 405 eV full width half maximum at 37 KeV and 450 eV full width half maximum at 45 KeV. A typical energy spectrum is shown in figure 1.

The half life for each resolvable peak in the energy spectrum was measured using the same experimental arrangement, with the multi-channel analyser set in repetitive mode. The
counting interval was 800 seconds with a 12 second data-dump interval. The data was stored on magnetic tape for analysis by SAMPO. The results of the half life determination are shown in figure 2.

Once the various photon peaks in the spectrum had been identified, a series of coincidence spectra were taken in order to determine which peaks were in cascade. Due to efficiency and geometry considerations, most of the coincidence data was taken with a 25 cc coaxial Ge(Li) detector and a 4 cc Ge(Li) detector. Both detectors were used in conjunction with standard coincidence circuitry. The resolving time of the coincidence circuit was 240 nsec. Some coincidence data was recorded using the Si(Li) detector with the 25 cc Ge(Li) detector. Although the statistics were low for this data, the results were in agreement with the data from the two Ge(Li) detectors. Accidental coincidence rates were checked by inserting a 4 micro-second delay in one branch of the coincidence circuit. The results of this check showed that no accidental coincidences occurred in ten minutes. The coincidence spectra, as shown in figure 3, were typically sixty minutes in duration. Whenever coincidence data was taken, the Ho sample was placed between two 10 mil Fe foils. This was done to remove low energy $\beta$-particles from the energy region of interest.

Photon energy spectra which included the L x-rays were observed with the Si(Li) detector. A typical spectrum is
shown in figure 4. These spectra showed large effects due
to self absorption of the photons by the foil. In order to
reduce self absorption, a thin target was made by evaporating
$^{165}$Ho onto a 1 mil mylar backing. The thickness of this target
was of the order of 30 $\mu$g/m/cm$^2$. Photon energy spectra from
this target were observed with the Si(Li) detector. These
spectra were used in comparison with the thick target spectra.
Appropriate target thickness corrections were made to all
intensity data. The results of the measurements are given in
Table 1.
CHAPTER III
DATA ANALYSIS AND RESULTS

The photon spectra recorded the with high resolution Si(Li) detector showed far more detail than any of the previously reported $^{164}$Ho decay spectra. Sethi and Mukherjee had reported five photon peaks, with energies of 36 KeV, 46 KeV, 51 KeV, 72 KeV, and 90 KeV. In contrast to this, the spectra obtained in the present investigation clearly showed at least thirteen photon peaks. Photon spectra as taken with a 25 cc coaxial Ge(Li) detector and the Si(Li) detector are shown in Fig. 1. In addition to this, it was noticed that several of the peaks appeared to be broadened. The computer code showed these peaks to be composed of more than one photon peak, as seen in Fig. 5. Standard peak-shapes are shown in Fig. 6.

The results from this work are shown in Table 1. All intensities reported in this table have been corrected for self absorption and detector efficiency. In Table 2 is shown the corresponding data obtained by Brown and Becker, Sethi and Mukherjee, and Jorgensen et al. Sethi and Mukherjee, and Brown and Becker used Na(I) detectors to record the photon spectra. Jorgensen et al used a beta-ray spectrometer to record the internal conversion electron spectra. The gamma-ray
intensities which they report are calculated from the measured conversion electron intensities. The data from the present work shows that there are 16 photon peaks in the energy spectrum. The energies of twelve of these photon peaks correspond very closely to the known energies of x-rays in Holmium, Erbium, and Dysprosium. In almost all cases, the relative intensities of these peaks do correspond approximately to the reported relative intensities of the various $K\alpha$ and $K\beta$ x-rays. The remaining four photon peaks at 37.3 KeV, 56.6 KeV, 73.4 KeV, and 91.3 KeV must be attributed to gamma-ray transitions.

The half life measurements and the coincidence data verify the above interpretation and allow the sources of the gamma-rays to be identified. As can be seen from Fig. 2, two distinct half lives are present in the decay process. The 37.3 KeV gamma-ray, the 56.6 KeV gamma-ray, and the Holmium x-rays exhibit a $38.9 \pm 0.4$ minute half life. The 73.4 KeV gamma-ray, the 91.3 KeV gamma-ray, and the Dysprosium and Erbium x-rays exhibit a $47.2 \pm 0.5$ minute half life. This indicates that the 37.3 KeV gamma-ray and the 56.6 KeV gamma-ray are due to excited states in Holmium; while the 73.4 KeV gamma-ray and the 91.3 KeV gamma-ray are due to excited states in Dysprosium and Erbium. The 47.6 minute half life is attributed to the $^{164}$Ho ground state. These half life measurements were made from the peak intensities computed by the code SAMPO. This insured that the peaks were fitted uniformly and the background subtracted uniformly for each
time period. The half life measurement for the ground state, made in this investigation, does differ substantially from those previously reported. Sethi and Mukherjee report the excited states of $^{164}$Ho to have a 39 minute half life; and the ground state to have a 23.9 min. half life. Jorgensen et al report a 37.5 minute half life for the excited states and a 29 minute half life for the ground state.

The coincidence data as seen in Fig. 3 shows that the 37.3 KeV gamma-ray is in coincidence with the 56.6 KeV gamma-ray and the Ho x-rays. This confirms the assignment of these gamma rays to $^{164}$Ho. From the previous studies of $^{164}$Ho, it is known that the Holmium decays to Dysprosium by electron capture and to Erbium by beta-emission. Coincidences between the 73.4 KeV gamma-rays and the Dysprosium x-rays are seen in this investigation. This indicates that the 73 KeV gamma-ray is due to an excited state in Dysprosium. The 91.3 KeV gamma-ray is not in coincidence with any other gamma-rays. When the coincidence circuit was gated on the 91.3 KeV peak some coincidences were recorded. Although statistics were poor, these coincidences seem to be with Ho x-rays. This is probably due to self fluorescence of the foil by the $\beta$-decay of $^{164}$Ho. Thus the 91.3 KeV gamma-ray is assigned to Erbium in agreement with the previous investigations.

Sethi and Mukherjee, Brown and Becker, and Jorgensen et al all report the existence of a 45.9 KeV transition in $^{164}$Ho. Sethi and Mukherjee attribute the 46.0 KeV photon peak partially
to a gamma-transition in $^{164}$Ho and partially to Dysprosium x-rays. However, in the present investigation the half life measurements and the coincidence data indicate that the observed photon at 45.9 KeV does not belong to a direct gamma-transition in Holmium. In addition to this, the relative intensity of this peak with respect to the Dysprosium $K_{\alpha_2}$ is 1.85. The theoretical value for the $K_{\alpha_1}:K_{\alpha_2}$ ratio is 2:1. Thus there is little doubt that the 45.9 KeV photon is the Dysprosium $K_{\alpha_1}$ x-ray. However, Brown and Becker and Jorgensen et al observe a 45.9 KeV transition in the internal conversion electron-spectra. This they attribute to a 139 KeV state in $^{164}$Ho. They assign the level a multipolarity of E3.

Theoretical estimates of $\alpha_K$ for this transition are of order $\alpha_K = 1000$. Therefore there will be very few gamma transitions from this level. The data from the present work does confirm this interpretation for two reasons. First, the 37.6 min. half life of the excited states of $^{164}$Ho indicates that one of the excited levels should have a multipolarity of E3 or M3. However, if either the 37.3 KeV transition or the 56.6 KeV transition is assigned either of these multipolarities, then the level would also have an internal conversion coefficient of the order of $10^3$. Thus the gamma peak would be too weak to be seen. This is not the case. Secondly, the measured ratio of the intensity of the Ho L x-rays to the intensity of the 37.4 KeV gamma-ray was found to be 8.1:1. If there was not a 45.9 KeV transition in $^{164}$Ho this ratio should be 3.6:1.
(Based on theoretical values of $\alpha_K$, $\alpha_L$, and $\omega_L$). Whereas if the 45.9 KeV transition does exist the ratio should be approximately 7.5:1.10

Thus the present investigation confirms the $^{164}$Ho decay scheme previously predicted by Sethi and Mukherjee and Jorgensen et al, which is shown in Fig. 7. The multipolarity assignments shown are those reported by Jorgensen et al. The assignments for the transitions in $^{164}$Ho, were verified in the present investigation by showing the theoretical internal conversion coefficients, predicted for these multipolarities, to be consistent with the observed relative intensities of the gamma-ray transitions.11

As has been mentioned earlier, the half life measurements and the relative intensities obtained in the present investigation do differ from those previously reported. These relative intensity measurements lead to the following results for the branching ratios to the excited states and ground states of Dysprosium and Erbium.12 The ground state of $^{164}$Ho decays 9.4% ± 0.6% to the 91.3 KeV state in $^{164}$Dy and 24.0% ± 3.4% to the 7.3 KeV level in $^{164}$Dy. The values for the ground states are dependent upon the rate of L-electron capture in the decay branch to $^{164}$Dy. If the L-capture is presumed to be about 10% of the K-capture then the values obtained are 48.0% ± 7.3% to the ground state of $^{164}$Dy and 18.7% ± 1.2% to the ground state of $^{164}$Er. A theoretical calculation by Alaga, Alder, Bohr, and Mottelson, based on the unified model of the nucleus,
shows that the expected ratio of the branching ratios for
$2^+$ and $0^+$ states is 1/2. The branching ratios for $^{164}$Dy,
observed in the present investigation, agree with this prediction.

These values for the branching ratios lead to recalculation of the values for the $^{165}$Ho$(n, 2n)^{164}$Ho cross section
as reported by Lear, Windham, and Gray. The measured values
of the $^{165}$Ho$(n, 2n)^{164}$Ho cross section, based on the branching
ratios from this work, are 830 ± 53 mb for the ground state
and 2370 ± 150 mb for the metastable state at neutron energies
of approximately 16 MeV.13
APPENDIX A

Description of Experimental Apparatus

The photon spectra were taken with a Kevex Ray Si(Li) x-ray detector. The detector crystal was a disk with dimensions $30 \text{ mm}^2 \times 3 \text{ mm}$. This detector was used in conjunction with Kevex Ray 4600 power supply, 4500 x-ray amplifier, and 4590 pulse pileup rejector. The detector output was fed into a Nuclear Data 4096 multichannel analyser.

The coincidence spectra were taken with a 25 cc coaxial Ge(Li) detector and a 4 cc Ge(Li) detector. Both detectors were used in conjunction with Ortec 440A amplifiers, 420 timing single channel analysers, 411 pulse stretcher, and 414A fast coincidence circuit. The resolving time of the coincidence circuit was measured to be 240 nsec. The spectra were recorded by the Nuclear Data 4096 multichannel analyser. A block diagram of the circuit is shown in Fig. 8.
APPENDIX B

Calculation of Correction Factors for Self Absorption of Photons by a Thick Target

According to the theories of gamma-ray counting\textsuperscript{14}, the measured intensity of a photon transition is given by

\[ A_t = \frac{A_o[(1 - \exp(-\mu t))]}{\mu t} \]  

(1)

where \( A_o \) is the true intensity of the transition, \( t \) is the thickness of the target in gm/cm\(^2\), and \( \mu \) is the attenuation coefficient of the material. For a thick target the approximation

\[ e^{-\mu t} = 0 \]  

(2)

can be made. This reduces Eq. (1) to

\[ A_o = A_t \mu t \]  

(3)

In the present investigation the quantity of interest is the intensity of each gamma-transition relative to the intensity of the 37.3 KeV gamma-ray. If we denote the measured relative intensity by \( R \) then

\[ R = \frac{A_t(\gamma)}{A_t(37.3)} \]  

(4)
The value of $R$ for each peak is given in Table I. Let the true relative intensity be denoted by

$$R_0 = \frac{A_o(\gamma)}{A_o(37.3)}.$$  \hspace{1cm} (5)

This leads to the equation

$$R_0 = \frac{A_o(\gamma)}{A_o(37)} = \frac{A_{L}(\gamma)\mu(\gamma)t}{A_{L}(37.3)\mu(37.3)t}.$$ \hspace{1cm} (6)

This reduces to the desired equation for the corrected relative intensities,

$$R_0 = R \frac{\mu(\gamma)}{\mu(37.3)}.$$ \hspace{1cm} (7)
APPENDIX C

Calculation of the Intensity of the Ho L X-Rays Relative to the 37.3 KeV Gamma-Ray

The Si(Li) detector resolved the L x-rays into two peaks with energies of 6.75 KeV and 7.55 KeV. These peaks appear to consist mainly of the $L_a$ and $L_b$ x-rays from Holmium. There is a contribution from the Dysprosium and Erbium L x-rays but it should be small. The photon energy spectrum, including L x-rays was recorded for both thick and thin targets. Correction factors must be applied in both cases to account for the self absorption of the photons by the foil. The following notation will be used in the calculation of these corrected values. Let

$$A = \text{intensity of the } 37.3 \text{ KeV peak},$$
$$C = \text{intensity of the } 6.75 \text{ KeV peak},$$
$$D = \text{intensity of the } 7.55 \text{ KeV peak}.$$

For the thick target the attenuation coefficients and measured intensities are

$$A_t = 39673, \quad \mu_A = 2.5 \times 10^3 \text{ cm}^2/\text{gm},$$
$$C_t = 6887, \quad \mu_C = 5.23 \times 10^4 \text{ cm}^2/\text{gm},$$
$$D_t = 10927, \quad \mu_D = 4.05 \times 10^4 \text{ cm}^2/\text{gm}.$$
The quantity of interest is given by

\[
\frac{\text{Intensity of Ho L x-rays}}{\text{Intensity of 37.3 KeV gamma-ray}} = \frac{C_o + D_o}{A_o} . \tag{8}
\]

Using Eq. (3) one obtains

\[
\frac{D_o + C_o}{A_o} = \frac{C_t \mu_C + D_t \mu_D}{A_t \mu_A} . \tag{9}
\]

When the values given above are substituted into this equation the result for the thick target is

\[
\frac{C_o + D_o}{A_o} = 8.08 . \tag{10}
\]

For the thin target the term \(\exp(-\mu t)\) is not negligible. Therefore the equation for the initial intensity becomes

\[
I_o = \frac{I_t \mu t}{[1 - \exp(-\mu t)]} . \tag{11}
\]

If the same notation as was used for the thick target calculation is adopted, the appropriate values for the thin target are

\[
A_t = 1173, \quad \mu_A = 2.5 \times 10^3 \text{ cm}^2/\text{gm},
\]

\[
C_t = 2333, \quad \mu_C = 5.23 \times 10^4 \text{ cm}^2/\text{gm},
\]

\[
D_t = 2964, \quad \mu_D = 4.05 \times 10^4 \text{ cm}^2/\text{gm}.
\]

Once again the quantity of interest is given by Eq. (8). If Eq. (11) is substituted in Eq. (8) the resulting equation is

\[
\frac{C_o + D_o}{A_o} = \frac{[1-\exp(-\mu_A t)](C_t \mu_C)}{A_t \mu_A [1-\exp(-\mu_C t)]} + \frac{[1-\exp(-\mu_A t)](D_t \mu_D)}{A_t \mu_A [1-\exp(-\mu_D t)]} . \tag{12}
\]

For a thickness \(t = 3 \times 10^{-5} \text{ gm/cm}^2\) the above equation yields,
upon substitution of the thin target values,

\[
\frac{C_0 + D_0}{A_0} = \frac{8.54}{2.31} + \frac{8.40}{2.05} = 7.8 .
\]  \hspace{1cm} (13)

Thus for the thick target the ratio of the Intensity of the Ho L x-rays to the Intensity of the 37.3 KeV peak is 8.08. For the thin target the ratio is 7.8.

A theoretical estimate for this ratio can be obtained by considering the decay scheme and the internal conversion coefficients. If we assume that 1000 \(^{164}\)Ho atoms decay, starting at the 139 KeV state in \(^{164}\)Ho, then the 45.9 KeV transition should yield approximately 1000 L-shell transitions. This is true because \(\alpha = 1000\) for the 45.9 KeV transition. The values for the 56.6 KeV transition are \(\alpha_T = 11.3\) and \(\alpha_L = 1.4\). Using these values in the equation

\[
I_\gamma (1 + \alpha_T) = 1000 ,
\]  \hspace{1cm} (14)
leads to the result \(I_\gamma = 81.3\). This value is used in the equation

\[
I_L = I_\gamma \alpha_L .
\]  \hspace{1cm} (15)

The result is \(I_L = 114\). Thus there are 114 L-shell transitions from the 56.6 KeV transition. In a similar manner, it can be shown that the 37.3 KeV transition will yield 830 L shell transitions and 130 gamma-rays. This gives a total of 1944 L transitions. The L shell fluorescence yield is given as 0.51. This means that approximately 50% of the L shell
transitions will yield L x-rays. Thus 975 L x-rays should be observed. The number observed 37.3 KeV gamma-rays should be 130. This gives a ratio of 971/130 or 7.5. The unresolved Dysprosium x-rays will raise this ratio slightly.

If we assume that there is no 49.5 KeV transition in $^{164}$Ho then this ratio becomes 470/130 or 3.6.
APPENDIX D

Verification of the Multipolarity
Assignments for the $^{164}$Ho Transitions

The three transitions in $^{164}$Ho are assigned the following multipolarities by Jorgensen et al. The 37.3 KeV transition and the 56.6 KeV transition have Ml multipolarity. The 45.9 KeV transition has E3 multipolarity. The present investigation has shown the relative intensities of the gamma-rays to be 100 for the 37.3 KeV gamma-ray and 48 for the 56.6 KeV gamma-ray. The 45.9 KeV gamma-ray did not have a measurable intensity. Since these transitions are in cascade, the total intensities should be equal. The E3 multipolarity of the 45.9 KeV transition indicates an internal conversion coefficient of approximately 1000. Thus very few gamma-rays should be observed from this transition. The Ml multipolarity of the 37.3 KeV transition indicates an internal conversion coefficient of 4.9.

The total intensity of the transition is given by

$$I_T = I_\gamma (1 + \alpha) = 100(1 + 4.9) = 590 \quad .$$

The relative intensity of the 37.3 KeV transition is thus 590. The Ml multipolarity of the 56.6 KeV transition indicates an internal conversion coefficient of 11.3. Thus the total intensity of the 56.6 KeV transition is given by
\[ I_T = I_\gamma (1 + \alpha) = 48(1 + 11.3) = 590 \]  \hspace{1cm} (17)

This is exactly what is expected from the intensity of the 37.3 KeV transition.
APPENDIX E
Calculation of the Relative Intensity Correction
Factor Due to the Difference in the Half Lives of the Isomeric and Ground States in $^{164}$Ho

The relative intensities of the various photon peaks change with time due to the fact that the isomeric state and the ground state of $^{164}$Ho decay at different rates. All intensities reported are relative to the intensity of the 37.3 KeV gamma-ray. The number of decays from a transition in a time $t$ is given by

$$ N = N_0 [1 - \exp(-\frac{693}{\tau}t)] \quad . \quad (18) $$

$\tau$ represents the half life. Thus for a counting time $t$ the relative intensities are given by

$$ \frac{I(\gamma)}{I(37.3)} = \frac{I_0(\gamma) [1 - \exp(-\frac{693}{\tau(\gamma)}t)]}{I_0(37.3) [1 - \exp(-\frac{693}{\tau(37.3)}t)]} \quad . \quad (19) $$

The values for the two half lives in $^{164}$Ho and the counting time used are

$\tau(37.3) = 38.9$ min.,

$\tau(73.4) = \tau(91.3) = 47.2$ min., and

$t = 31$ min.

Using these values in Eq. (19), the result obtained is
\[
\frac{I(73.4)}{I(37.3)} = \frac{I_0(73.4)}{I_0(37.3)} \times \frac{.36}{.42} \quad \text{(20)}
\]

This reduces to the Equation

\[
\frac{I_0(73.4)}{I_0(37.3)} = 1.17 \frac{I(73.4)}{I(37.3)} \quad \text{(21)}
\]

Thus for a counting interval of 31 minutes taken after irradiation of the target, the correction factor for the relative intensities is 1.17
APPENDIX F

A Calculation of the Branching Ratios to the
Excited States and Ground States of $^{164}$Dy and $^{164}$Er

The following terminology is used in the branching ratio
calculations. $\beta_1$ is the branching ratio to the 91.3 KeV state
in $^{164}$Er. $\beta_2$ is the branching ratio to the 73.4 KeV state of
$^{164}$Dy. Internal conversion coefficients are denoted by $\alpha_i$. Fluorescence
yield coefficients are denoted by $\omega_i$. The following equations
are used to calculate the branching ratios:

$$\beta_1 + \beta_2 + \beta_2^* + \beta_2^{*'} = 1 \quad (22)$$

$$\frac{\beta_1}{\beta_2} = 2 \quad (23)$$

$$\frac{\beta_1}{\beta_2} = \frac{\text{Number of 91.3 KeV transitions}}{\text{Number of 73.4 KeV transitions}} = \frac{I_{91.3}(1 + \alpha_{91.3})}{I_{73.4}(1 + \alpha_{73.4})} \quad (24)$$

and

$$\frac{\beta_2}{\beta_2^*} = \frac{\text{Number of 73.4 KeV transitions}}{\text{Number of E.C. decays to the ground state of } ^{164}\text{Dy}} \quad (25)$$
Eq. (23) is based on a theoretical calculation by Alaga, Alder, Bohr, and Mottelson. The number of electron capture decays to the ground state of $^{164}$Dy is given by the total number of Dysprosium K x-rays, corrected for the number of K x-rays due to electron capture to the 73.4 KeV state and the number of K x-rays due to internal conversion of the 73.4 KeV state. There is also some L electron capture. This is estimated at approximately 10% of the K-capture. Correction is made for this effect. Thus Eq. (25) can be written as

$$\frac{\beta_2}{\beta'_2} = \frac{I_{73.4}(1 + \alpha_{73.4})}{[(I_{Dy\ x-rays}/\omega_K(Dy)) - I_{73.4}(1+\alpha_{73.4}) - I_{73.4}(\alpha_K)]1.1}$$  \hspace{1cm} (26)

Using the relative intensity values and internal conversion coefficients from Table I, and the value $\omega_K(Dy) = 0.943$, Eq. (26) gives the result

$$\frac{\beta_2}{\beta'_2} = 0.52$$  \hspace{1cm} (27)

Thus it is seen that $\beta'_2 \approx 2\beta_2$. Substitution of the values from Table I into Eq. (24) gives

$$\frac{\beta_1}{\beta_2} = 0.390$$  \hspace{1cm} (28)

This can be rewritten as

$$\beta_2 = 2.56 \beta_1$$  \hspace{1cm} (29)
Eq. (23) can be rewritten as

\[ \beta' = 2\beta_1 \]  \hspace{1cm} (30)

If the values from Eq. (27), Eq. (29), and Eq. (30) are substituted into Eq. (22), and the equations are solved simultaneously, the results are:

\[ \beta_1 = 9.4 \pm 0.6\% , \]
\[ \beta' = 18.7 \pm 1.2\% , \]
\[ \beta_2 = 24.0 \pm 3.4\% , \text{ and} \]
\[ \beta' = 48.0 \pm 7.3\% . \]  \hspace{1cm} (31)
APPENDIX G

Calculation of the Ratio of the \(^{165}\text{Ho}(n, 2n)^{164}\text{Ho}\) Cross Sections to the Isomeric and Ground States of \(^{164}\text{Ho}\)

When \(^{164}\text{Ho}\) is formed by the \((n, 2n)\) reaction on \(^{165}\text{Ho}\), some of the \(^{164}\text{Ho}\) is formed in the isomeric state and some in the ground state. The branching ratio to the 91.3 KeV state in \(^{164}\text{Er}\) is given by

\[
\beta_1 = \frac{I_{91.3}(1 + \alpha_{91.3})}{I_{37.3}(1 + \alpha_{37.3}) + I_G}.
\] (32)

\(I_G\) is the intensity of all transitions due to atoms created in the ground state of \(^{164}\text{Ho}\). The denominator of Eq. (32) can be written in the form

\[
I_{37.3}(1 + \alpha_{37.3}) + I_G = C I_{37.3}(1 + \alpha_{37.3}),
\] (33)

where \(C\) is a constant. This leads to the equation

\[
\beta_1 = \frac{I_{91.3}(1 + \alpha_{91.3})}{C I_{37.3}(1 + \alpha_{37.3})}.
\] (34)

If the relative intensities of the 91.3 KeV gamma-ray and the 37.3 KeV gamma-ray, corrected for the different half lives, are substituted into this equation, the result is

\[
\beta_1 = \frac{1}{C}(12.75).
\] (35)
The value of $\beta_1$, as calculated in Appendix 1, is 9.4%. Thus from Eq. (35) it is found that $C = 1.35$. Thus Eq. (33) can be written in the form

$$\frac{I_G}{I_{37.3}(1 + \alpha_{37.3})} = C - 1 = 0.35 \quad (36)$$

This is the ratio of the number of $^{164}$Ho atoms created in the ground state to the number created in the isomeric state. The cross sections will have this same ratio.
TABLES
### TABLE I

**RELATIVE INTENSITIES AND HALF LIVES OF THE PHOTON TRANSITIONS IN $^{164}$Ho AS MEASURED IN THE PRESENT INVESTIGATION**

<table>
<thead>
<tr>
<th>Photon Energy (KeV)</th>
<th>Source of Photon</th>
<th>Corrected Relative Intensity</th>
<th>Half Life (min)</th>
<th>Multipolarity</th>
<th>Internal Conversion Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.75</td>
<td>Ho L</td>
<td>360±36.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.55</td>
<td>Ho L</td>
<td>440±44.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.3</td>
<td>Ho $\gamma$</td>
<td>100±7.</td>
<td>38.9±0.4</td>
<td>M1</td>
<td>$\alpha_L=4.9$</td>
</tr>
<tr>
<td>45.1</td>
<td>Dy Ka$_2$</td>
<td>163±13.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.9</td>
<td>Dy Ka$_1$</td>
<td>297±22.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.6</td>
<td>Ho Ka$_2$</td>
<td>154±14.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47.4</td>
<td>Ho Ka$_1$</td>
<td>206±19.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47.9</td>
<td>Er Ka$_2$</td>
<td>49±7.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.1</td>
<td>Er Ka$_1$</td>
<td>10±1.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52.0</td>
<td>Dy K$\beta$$_1$</td>
<td>64±5.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.6</td>
<td>Dy K$\beta$$_2$</td>
<td>34±4.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.9</td>
<td>Ho K$\beta$$_1$</td>
<td>27±4.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>55.3</td>
<td>Ho K$\beta$$_2$</td>
<td>8.2±6.</td>
<td>38.9±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>55.6</td>
<td>Er K$\beta$$_1$</td>
<td>23±2.</td>
<td>47.2±0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.6</td>
<td>Ho $\gamma$</td>
<td>48±4.6</td>
<td>38.9±0.4</td>
<td>M1</td>
<td>$\alpha_L=1.4$, $\alpha_K=10.0$ (d)</td>
</tr>
<tr>
<td>57.0</td>
<td>Er K$\beta$$_2$</td>
<td>10±1.6</td>
<td>47.2±0.2</td>
<td>E2</td>
<td>$\alpha_L=2.15$, $\alpha_K=5.2$ (e)</td>
</tr>
<tr>
<td>73.4</td>
<td>Dy $\gamma$</td>
<td>23±2.</td>
<td>47.2±0.5</td>
<td>E2</td>
<td>$\alpha_L=1.4$, $\alpha_K=2.05$ (f)</td>
</tr>
<tr>
<td>91.3</td>
<td>Er $\gamma$</td>
<td>17±1.5</td>
<td>47.2±0.5</td>
<td>E2</td>
<td></td>
</tr>
</tbody>
</table>

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TABLE II
PHOTON INTENSITIES AND HALF LIVES MEASURED
IN PREVIOUS INVESTIGATIONS

<table>
<thead>
<tr>
<th>Reference</th>
<th>Photon Energy (KeV)</th>
<th>Source of Photon</th>
<th>Relative Intensity</th>
<th>Half Life (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown and Becker</td>
<td>37.3±.5</td>
<td>Dy ?</td>
<td>0.036±.01</td>
<td>·········</td>
</tr>
<tr>
<td></td>
<td>45.7±.5</td>
<td>Dy X-ray</td>
<td>0.90±.20</td>
<td>36.7±0.5</td>
</tr>
<tr>
<td></td>
<td>72.8±1.5</td>
<td>Dy γ</td>
<td>0.033±.007</td>
<td>36.7±0.5</td>
</tr>
<tr>
<td></td>
<td>90.5±.5</td>
<td>Er γ</td>
<td>0.035±.007</td>
<td>36.7±0.5</td>
</tr>
<tr>
<td>Jorgensen et al</td>
<td>37.0</td>
<td>Ho γ</td>
<td>130</td>
<td>37.5±1.5–0.5</td>
</tr>
<tr>
<td></td>
<td>45.9</td>
<td>Ho γ</td>
<td>0</td>
<td>37.5±1.5–0.5</td>
</tr>
<tr>
<td></td>
<td>56.1</td>
<td>Ho γ</td>
<td>57</td>
<td>37.5±1.5–0.5</td>
</tr>
<tr>
<td></td>
<td>73.0</td>
<td>Dy γ</td>
<td>46</td>
<td>·········</td>
</tr>
<tr>
<td></td>
<td>91.3</td>
<td>Er γ</td>
<td>48</td>
<td>·········</td>
</tr>
<tr>
<td>Sethi and Mukherjee</td>
<td>36.0</td>
<td>Ho γ</td>
<td>3.5±0.8</td>
<td>39.±3</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>Ho γ</td>
<td>100±8.</td>
<td>39.±1</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>Ho γ</td>
<td>29±5.</td>
<td>39.±1</td>
</tr>
<tr>
<td></td>
<td>72.0</td>
<td>Dy γ</td>
<td>4.5±0.5</td>
<td>39.±3</td>
</tr>
<tr>
<td></td>
<td>90.0</td>
<td>Er γ</td>
<td>5.4±0.5</td>
<td>39.±4</td>
</tr>
</tbody>
</table>

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ILLUSTRATIONS
Figure 1—$^{164}$Ho Photon Spectrum
Half-life Determinations $^{164}$Ho

Figure 2—Plot of Half Life Data for $^{164}$Ho
Figure 3—Photon Coincidence Spectra
Figure 4—L X-Ray Spectrum for $^{164}$Ho
Figure 5—Partial Photon Spectrum $^{164}_{Ho}$. Showing Computer Fit
Figure 6—Computer to Standard Peakshapes
Figure 7—Decay scheme for $^{164}$Ho
Figure 8—Coincidence Circuit
FOOTNOTES


5. See appendix A for a component description of the experimental apparatus.


7. See appendix A for a component description of the coincidence circuit.

8. The values of all internal conversion coefficients used in this investigation are found in: M. E. Rose, Internal Conversion Coefficients, (Interscience Publishers, New York, 1958) pp. 84-89.

9. The calculation of the correction factors for self absorption of the photons is shown in appendix B.

10. The theoretical and experimental ratios of the Ho L x-ray intensity to the 37.3 KeV gamma-ray intensity are calculated in appendix C.
11. The verification of the previous multipolarity assignments is developed in appendix D.

12. The branching ratios to the various states in $^{164}$Dy and $^{164}$Er are calculated in appendix F.

13. The ratio of the $^{165}$Ho(n, 2n)$^{164}$Ho cross sections to the isomeric and ground states of $^{164}$Ho is calculated in appendix G.


15. The values for the attenuation coefficients used in this investigation are those reported in: E. Storm and H. Israel, Nuclear Data Tables, Section A, 7, No. 6, 613 (1970).

16. Bernd Crasemann, University of Oregon, personal communication.


18. Bernd Crasemann, University of Oregon, personal communication.

FOOTNOTES FOR TABLE I

a. The observed relative intensities are corrected for detector efficiency, self absorption, and the difference in half lives between the isomeric and ground states of $^{164}$Ho. This last correction is discussed in appendix E.
b. The observed photon intensity at 45.9 KeV is attributed completely to the Dy Ka\textsubscript{2} x-ray. This assignment is based on the half life and relative intensity measurements. The 45.9 KeV transition in \textsuperscript{164}Ho is not observed due to the large internal conversion coefficient.


d. The value of 11.3 for the total internal conversion coefficient of this transition was calculated from the measured relative intensities and the theoretical value of $\alpha = 4.9$ for the 37.3 KeV transition. This measured value corresponds almost exactly to the value of 11.35 calculated theoretically by Rose.

e. These values are based on the accepted multipolarity assignments. These assignments are based on the following theoretical investigation: C. Alga, K. Alder, A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Phys. Medd. \textbf{29}, No. (1955).

f. See footnote e.

**FOOTNOTES FOR TABLE II**


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Books


Unpublished Materials

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Lear, Richard D., "Cross Section for the $^{165}\text{Ho}(n, 2n)^{164}\text{Ho}$ Reaction at 15.6 MeV", unpublished master's thesis, Department of Physics, North Texas State University, 1969.