Microwave Interferometry of Laser Induced Air Plasmas Formed by Short Laser Pulses

Patrick W. Jungwirth

**A Thesis Presented to the
Faculty of California Polytechnic State University San Luis Obispo
In Partial Fulfillment of the Requirements for the Degree
Master of Science**

Manuscript date: August 1993

LAWRENCE LIVERMORE NATIONAL LABORATORY
University of California • Livermore, California • 94551

MASTER

ds

APPROVAL PAGE

Title: Microwave Interferometry of Laser Induced Air Plasmas Formed by Short Laser Pulses

Author: Patrick W. Jungwirth

Date Submitted: August 1993

Dr. Cheng Sun
Thesis/Project Advisor

Signature

Dr. Micheal Wollman
Committee Member

Signature

Dr. Gary Granneman
Committee Member

Signature

Dr. Saul Goldberg
Department Chair

Signature

ABSTRACT

MICROWAVE INTERFEROMETRY OF LASER INDUCED AIR PLASMAS
FORMED BY SHORT LASER PULSES

Patrick W. Jungwirth

August 1993

Applications for the interaction of laser induced plasmas with electromagnetic probes requires time varying complex conductivity data for specific laser/electromagnetic probe geometries. Applications for this data include plasma switching (Q switching) and the study of ionization fronts.

The plasmas were created in laboratory air by 100 ps laser pulses at a wavelength of 1 μm (Janus Laser Facility at Lawrence Livermore National Laboratory). A long focal length lens focused the laser pulse into WR90 (X band) rectangular waveguide. Two different laser beam/electromagnetic probe geometries were investigated. For the longitudinal geometry, the laser pulse and the microwave counterpropagated inside the waveguide. For the transverse geometry, the laser created a plasma "post" inside the waveguide. The effects of the laser beam deliberately hitting the waveguide were also investigated. Each geometry exhibits its own characteristics. This research project focused on the longitudinal geometry.

Since the laser beam intensity varies inside the waveguide, the charge distribution inside the waveguide also varies. A 10 GHz CW microwave probe traveled through the laser induced plasma. From the magnitude and phase of the microwave probe, a spatially integrated complex conductivity was calculated. No measurements of the temporal or

spatial variation of the laser induced plasma were made. For the "plasma post," the electron density is more uniform.

TABLE OF CONTENTS

List of Tables	ix
List of Figures	x
Chapter 1: Introduction	1
Chapter 2: Plasma Physics and Microwave Interferometry Theory	4
2.1 Plasma Physics	4
2.1.1 Plasma Frequency	4
2.1.2 Langevin Equation	5
2.1.3 Complex Conductivity	5
2.1.4 Collisionality	6
2.1.5 Propagation Function in Rectangular Waveguide	7
2.1.6 Plasma Complex Conductivity in Rectangular Waveguide	9
2.2 Microwave Interferometry	11
2.2.1 Phase Angle Measurement	11
2.2.2 Approximate Accuracy of the Phase Shift	11
2.3 Equation Summary	12
Chapter 3: Microwave and Optical Systems	14
3.1 Introduction	14
3.2 Microwave System	15
3.2.1 Forward and Reverse Wave Calibration	18
3.2.2 Phase Angle Calibration	19
3.2.3 Diode Detector Calibrations	21
3.3 Optical System	24
3.3.1 Janus Laser System	24
3.3.2 Optical Beam Path	24

3.3.3 Optical and Microwave Timing	27
3.4 Measurement Procedure	29
Chapter 4: Results	31
4.1 Introduction	31
4.2 Group 1: Longitudinal Not Hitting	34
4.3 Group 2: Longitudinal Not Hitting	35
4.4 Group 3: Longitudinal Hitting	35
4.5 Group 4: Transverse Not Hitting	36
4.6 Group 5: Transverse Hitting	37
4.7 Results Comparision	37
Chapter 5: Conclusion	79
Notes	84
Bibliography	85
Appendix A: Interferometer	86
Appendix B: Estimate of Phase Error	89
Appendix C: WR90 Rectangular Waveguide	91
Appendix D: Propagation Function in Rectangular Waveguide	92
Appendix E: Complex Conductivity Derivation	95
Appendix F: Diode Calibration Curves at 10 GHz	98
Appendix G: Propagation Delay Measurements	100
Appendix H: Breakdown Threshold for 1 Atmosphere Air	104
Appendix I: Estimate of the Overlap Integral	108

Appendix J: Low Directivity Couplers in Plasmas	112
Appendix K: Results for the Transverse Case Not Hitting the Waveguide Wall	115
Appendix L: Frequency Response of the Microwave Test Set	123
Appendix M: Transmitted and Incident Laser Energy	125

LIST OF TABLES

Table 2.1	Complex Conductivity and Complex Dielectric Function	6
Table 2.2	Propagation Functions for Free Space and Rectangular Waveguide	8
Table 2.3	Equation Summary	13
Table 3.1	Port Definitions	16
Table 3.2	Return Loss	18
Table 3.3	S Parameters for the Forward and Reverse Waves	18
Table 3.4	Reference Port Calibration	20
Table 3.5	Power Correction Factors	21
Table 3.6	Inverse Diode Equations	23
Table 3.7	Janus Laser Facility Parameters	25
Table 3.8	Lens Parameters at 1.064 μm	25
Table 3.9	Propagation Delays	27
Table 4.1	Laser Energy Ranges Covered for Each Geometry	32
Table 4.2	Results Presentation Order	32
Table 4.3	Comparison of the Peak Values at 1 J	38
Table 4.4	Comparison of Slopes	38
Table 4.5	Peak Attenuation and Phase Shift	78
Table 5.1	Forward and Reverse Power Flow Ranges	80
Table 5.2	Ratio Comparison	81
Table 5.3	Electric Field Potential	82
Table A.1	Scatter Parameters for the 5 Port Network at 10.0 GHz	87
Table C.1	WR90 Waveguide Specifications	91
Table F.1	MA/COM Diode Input Powers as a Function of Output Voltages	99
Table F.2	Narda Diode Input Power as a Function of Output Voltage	99
Table G.1	Propagation Delays	100
Table G.2	Propagation Delays from the APRP to the Test, Interference, and Reflection Ports	101
Table G.3	Timing Correction Factors	101
Table I.1	Example Values	111
Table J.1	Coupler Frequency Responses	113
Table J.2	Coupler Directivities	113

LIST OF FIGURES

Figure 1.1	Waveguide and Laser Beam Geometries	2
Figure 2.1	Collision Frequency	5
Figure 2.2	Electric Field Distribution for TE ₁₀ Mode in Rectangular Waveguide	7
Figure 2.3	Phasor Diagrams and Interferometry Theory	12
Figure 3.1	Simplified Diagram of the Microwave System	14
Figure 3.2	Microwave System without Diode Detectors	16
Figure 3.3	Magic Tee Port Notation and Scatter Matrix	16
Figure 3.4	Microwave Hardware Including Detectors	17
Figure 3.5	Forward and Reverse Wave Scatter Parameter Measurements	19
Figure 3.6	Interferometer Phasor Diagram	20
Figure 3.7	Coupler Calibration	22
Figure 3.8a	MA/COM Diode Calibration	22
Figure 3.8b	Narda Diode Calibration	22
Figure 3.9	Narda Diode Output Voltage versus Input Power	23
Figure 3.10	Narda Diode Input Power versus Output Voltage	24
Figure 3.11	Optical Beam Path	25
Figure 3.12	Complete Optical and Microwave Timing Diagram	26
Figure 3.13	Typical Oscilloscope Trace	26
Figure 3.14	Microwave Timing	26
Figure 3.15	Timing Corrections Applied to Figure 3.14	28
Figure 3.16	Measurement Procedure Summary	30
Figure 3.17	Complete Microwave and Optical Systems Diagram	30
Figure 4.1	Diode Voltages	33
Figure 4.2	Timing Diagram	33
Figure 4.1.1	Absolute Forward Power in mW	39
Figure 4.1.2	Absolute Reverse Power in mW	40
Figure 4.1.3	Forward Wave Attenuation in dB	41
Figure 4.1.4	Forward Wave Phase Shift in Degrees	42
Figure 4.1.5	Real Part of the Spatially Integrated Complex Conductivity	43
Figure 4.1.6	Imaginary Part of the Spatially Integrated Complex Conductivity	44
Figure 4.1.7	Real Part of the Spatially Integrated Dielectric Function	45
Figure 4.1.8	Imaginary Part of the Spatially Integrated Dielectric Function	46
Figure 4.1.9	Collisionality from the Spatially Integrated Complex Conductivity	47
Figure 4.1.10	Spatially Integrated Electron Density	48
Figure 4.2.1	Absolute Forward Power in mW	49
Figure 4.2.2	Absolute Reverse Power in mW	50

Figure 4.2.3	Forward Wave Attenuation in dB	51
Figure 4.2.4	Forward Wave Phase Shift in Degrees	52
Figure 4.2.5	Real Part of the Spatially Integrated Complex Conductivity	53
Figure 4.2.6	Imaginary Part of the Spatially Integrated Complex Conductivity	54
Figure 4.2.7	Real Part of the Spatially Integrated Dielectric Function	55
Figure 4.2.8	Imaginary Part of the Spatially Integrated Dielectric Function	56
Figure 4.2.9	Collisionality from the Spatially Integrated Complex Conductivity	57
Figure 4.2.10	Spatially Integrated Electron Density	58
Figure 4.3.1	Absolute Forward Power in mW	59
Figure 4.3.2	Absolute Reverse Power in mW	60
Figure 4.3.3	Forward Wave Attenuation in dB	61
Figure 4.3.4	Forward Wave Phase Shift in Degrees	62
Figure 4.3.5	Real Part of the Spatially Integrated Complex Conductivity	63
Figure 4.3.6	Imaginary Part of the Spatially Integrated Complex Conductivity	64
Figure 4.3.7	Real Part of the Spatially Integrated Dielectric Function	65
Figure 4.3.8	Imaginary Part of the Spatially Integrated Dielectric Function	66
Figure 4.3.9	Collisionality from the Spatially Integrated Complex Conductivity	67
Figure 4.3.10	Spatially Integrated Electron Density	68
Figure 4.4.1	Absolute Forward Power in mW	69
Figure 4.4.2	Absolute Reverse Power in mW	69
Figure 4.4.3	Forward Wave Attenuation in dB	70
Figure 4.4.4	Forward Wave Phase Shift in Degrees	70
Figure 4.5.1	Absolute Forward and Reverse Power in mW	71
Figure 4.5.2	Absolute Reverse Power in mW	71
Figure 4.5.3	Forward Wave Attenuation in dB	72
Figure 4.5.4	Forward Wave Phase Shift in Degrees	72
Figure 4.6	Line Styles for the Four Different Geometries	73
Figure 4.6.1	Absolute Forward Power in mW	73
Figure 4.6.2	Absolute Reverse Power in mW	74
Figure 4.6.3	Forward Wave Attenuation in dB	74
Figure 4.6.4	Forward Wave Phase Shift in Degrees	75
Figure 4.7	Peak Attenuation and Peak Phase Shift versus Laser Energy	76
Figure 4.8	Attenuation to Phase Shift Ratio as a Function of Laser Energy	77
Figure 4.9	Peak Attenuation versus Laser Energy	78
Figure A.1	Main Part of the Interferometer	86
Figure A.2	S Parameters for the Interferometer	88
Figure A.3	Interferometer Bandwidth	88
Figure B.1	Exact Surface Plot in Degrees	89
Figure B.2	Estimated Surface Plot in Degrees	89
Figure B.3	Error Surface Plot in Degrees	90
Figure C.1	WR90 Dimensions	91
Figure C.2	Magic Tee Port Notation and Scatter Matrix	91
Figure F.1	MA/COM Diode Output Voltages as a Function of Input Powers	98

Figure F.2	MA/COM Diode Input Powers as a Function of Output Voltages	98
Figure F.3	Narda Diode Output Voltage as a Function of Input Power	99
Figure F.4	Narda Diode Input Power as a Function of Output Voltage	99
Figure G.1	Relative Timing Measurement	102
Figure G.2	Absolute Timing Measurement	102
Figure G.3	Impulse Generator	102
Figure G.4	Relative Timing Diagram	102
Figure G.5	Absolute Timing Diagram	102
Figure H.1	Threshold versus Caustic Length	105
Figure H.2	Laser Breakdown	106
Figure I.1	Electron Density inside Rectangular Waveguide	108
Figure J.1	Directivity Limited Coupler	112
Figure J.2	Coupler Frequency Responses	114
Figure K.1.1	Absolute Forward Power in mW	115
Figure K.1.2	Absolute Reverse Power in mW	115
Figure K.1.3	Forward Wave Attenuation in dB	116
Figure K.1.4	Forward Wave Phase Shift in Degrees	116
Figure K.1.5	Attenuation Divided by the Interaction Length	116
Figure K.1.6	Real Part of the Spatially Integrated Complex Conductivity	117
Figure K.1.7	Imaginary Part of the Spatially Integrated Complex Conductivity	117
Figure K.1.8	Real Part of the Spatially Integrated Dielectric Function	117
Figure K.1.9	Imaginary Part of the Spatially Integrated Dielectric Function	118
Figure K.1.10	Collisionality from the Spatially Integrated Complex Conductivity	118
Figure K.1.11	Spatially Integrated Electron Density	118
Figure K.2.1	Absolute Forward Power in mW	119
Figure K.2.2	Absolute Reverse Power in mW	119
Figure K.2.3	Forward Wave Attenuation in dB	119
Figure K.2.4	Forward Wave Phase Shift in Degrees	120
Figure K.2.5	Attenuation Divided by the Interaction Length	120
Figure K.2.6	Real Part of the Spatially Integrated Complex Conductivity	120
Figure K.2.7	Imaginary Part of the Spatially Integrated Complex Conductivity	121
Figure K.2.8	Real Part of the Spatially Integrated Dielectric Function	121
Figure K.2.9	Imaginary Part of the Spatially Integrated Dielectric Function	121
Figure K.2.10	Collisionality from the Spatially Integrated Complex Conductivity	122
Figure K.2.11	Spatially Integrated Electron Density	122
Figure L.1	Ports Numbers for Scatter Parameter Measurements	123
Figure L.2	S Parameters for the Microwave Test Set	124
Figure M.1	Transmitted Laser Energy versus Incident Laser Energy	125
Figure M.2	Transmitted Laser Energy versus Incident Laser Energy	126
Figure M.3	Transmitted Laser Energy versus Incident Laser Energy	126
Figure M.4	Transmitted Laser Energy versus Incident Laser Energy	127

CHAPTER 1

Introduction

Gaseous electronics requires complex conductivity information for design applications. The applications for the complex conductivity data include plasma switching, Q switching of microwave cavities, and the study of ionization fronts. The spatially integrated complex conductivity was calculated from the magnitude and phase shift of a 10 GHz CW microwave probe traveling through a gaseous plasma.

The plasmas studied were created in 1 atmosphere laboratory air by 100 ps (full width half maximum) laser pulses inside WR90 rectangular waveguide. Figure 1.1a through 1.1d shows the four different geometries studied. The results from the longitudinal case were compared to the more familiar transverse case in Figure 1.1b. Figure 1.1c and 1.1d show the laser deliberately hitting the waveguide wall.

As shown in Figure 1.1a and 1.1c, the microwave and the laser beam counter-propagate. The forward wave travels through the laser induced plasma and the reverse wave (the scattered wave) travels in the opposite direction. Both the longitudinal and transverse geometries are applicable to plasma switching. The longitudinal geometry may be applied to the study of relativistic ionization fronts.

The results for the longitudinal case are spatially integrated over several microwave wavelengths and spatially averaged over the cross section of the waveguide. Unlike the longitudinal case, the length of the plasma region is small for the transverse case. For Figure 1.1a, 1.1b, and 1.1d, the laser beam is approximately at the maximum value of the electric field. For Figure 1.1c, the laser beam is near the minimum value of the electric field. In Chapter 4, the forward and reverse wave powers as a function of time are shown.

Essential concepts in plasma physics and interferometry theory are covered in Chapter 2. The complex conductivity of a uniform plasma is defined in terms of an amplitude change and a phase shift experienced by a probe wave form. The phase shift accuracy is estimated.

Chapter 3 describes the microwave and optical systems used to measure the microwave characteristics of laser induced plasmas. Chapter 3 includes amplitude and timing calibrations along with a summary of the measurement procedure.

The results of this research project are presented in Chapter 4. For the longitudinal case, the results are spatially integrated over several microwave wavelengths (approximately 14 guide wavelengths). The forward and reverse wave powers are calculated from measured diode voltages and calibration results. The spatially integrated

complex conductivity, spatially integrated complex dielectric function, spatially integrated collisionality, and the spatially integrated or effective electron density are shown as functions of time. The results cover a laser energy range of approximately 0.3 to 15 Joules for the case shown in Figure 1.1a and 0.5 to 1.1 Joules for the cases in Figures 1.1b, and 1.1c. The results for the geometry in Figure 1.1d cover a single 1.1 J laser shot.

CHAPTER 2

Plasma Physics and Microwave Interferometry Theory

2.1 Plasma Physics

The plasma frequency is defined in terms of the electron density. The complex conductivity is derived from the *Langevin Equation* and collisionality is defined. From Equation A4¹, the propagation functions for three different plasma frequency regions were determined. The complex conductivity is defined in terms of the forward wave attenuation and phase shift for a uniform plasma inside a rectangular waveguide².

2.1.1 Plasma Frequency

An important quantity that characterizes a uniform plasma is the *plasma frequency*. The plasma frequency is the natural frequency at which the electron plasma oscillates when subjected to an electric field disturbance. It is therefore, a measure of the plasma response time to this disturbance. For the one dimensional case in a uniform plasma, the plasma [angular] frequency is defined in Equation 2.1.

$$\omega_p \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad (2.1)$$

n_e =electron density in $1/m^3$

ϵ_0 =permittivity of free space $=8.85 \times 10^{-12}$ F/m

m_e =electron's rest mass $=9.11 \times 10^{-31}$ kg e =unit of charge $=1.602 \times 10^{-19}$ coulomb

For example if $n_e = 10^{12}$ $1/cm^3$ then $\omega_p = 5.64 \times 10^{10}$ rad/sec and $f_p = 8.98$ GHz. The electron density is proportional to the square of the plasma frequency.

2.1.2 Langevin Equation

The Langevin Equation models a plasma as an electron gas. The model simply adds an additional momentum loss term to the classical Lorentz force. It is assumed that only the average energy (velocity) of the electrons in the presence of an electric field is important. The averaging is over some distribution. The following discussion is for the "average" electron.

$$\frac{d\mathbf{u}}{dt} = -e[\mathbf{E} + \mathbf{u} \times \mathbf{B}] - \mathbf{u} \nu$$

Langevin Equation (2.2)

$\mathbf{u}$ = average drift velocity [m/s] ν = collision frequency [1/s] $\mathbf{E}$ = electric field [V/m]
 m = electron's rest mass = 9.11×10^{-31} kg $\mathbf{B}$ = magnetic field = 0

The coordinate system was selected so that $-\mathbf{x}$ is the direction the "average" electron is traveling. The collision frequency ν is the average number of times per second an electron collides with an atom and is redirected from the $-\mathbf{x}$ direction to a path parallel to the yz plane as shown in Figure 2.1 (all momentum in the $-\mathbf{x}$ direction is lost). By definition, Equation 2.2 describes the momentum altering collisions. The collision frequency, ν , monotonically increases with the pressure or particle density. Multiple possible paths are shown after the collision.

2.1.3 Complex Conductivity

By assuming a $e^{j\omega t}$ time dependence, the steady state drift velocity $\mathbf{u}$ is determined

from the Langevin Equation. The steady state velocity is defined as $\mathbf{u} = \frac{-e\mathbf{E}}{m_e(\nu + j\omega)}$.

The current density is $J = -ne\mathbf{v}$, which gives the complex conductivity as

$$\sigma \equiv \frac{J}{E} = \frac{n_e e^2}{m(\nu - j\omega)} = \frac{\omega_p^2 \epsilon_0}{\nu + j\omega}. \quad \text{The contribution of this conductivity to the dielectric}$$

function is $\bar{\epsilon} = \epsilon_0 - j \frac{\bar{\sigma}}{\omega}$. Table 2.1 shows how the real and imaginary parts of the complex conductivity and complex dielectric function are related.

Table 2.1

Complex Conductivity and Complex Dielectric Function

			Electric Field Phasor
Re{sigma} corresponds to	Im{epsilon}	loss term due to collisions	attenuation
Im{sigma} corresponds to	Re{epsilon}	reactive term	phase shift

2.1.4 Collisionality

The collisionality, k , is defined as the ratio of the collision frequency to the probe frequency. For the simple conductivity model used here (i.e. Drude model), it is also the negative ratio of the real part of the complex conductivity to the imaginary part.

According to Equation A7³, the $\text{Re}\{\sigma\}$ and the $\text{Im}\{\sigma\}$ must have opposite signs.

Equation 2.3 defines the collisionality.

$$k \equiv \frac{\nu}{\omega} = \frac{-\text{Re}[\bar{\sigma}]}{\text{Im}[\bar{\sigma}]} \quad (2.3)$$

The collisionality and the collision frequency are linearly related. By decreasing the pressure, k will monotonically decrease and the plasma will become less absorptive.

2.1.5 Propagation Function in Rectangular Waveguide

The propagation function in rectangular waveguide for the TE_{10} mode for 3 different plasma frequency regions was calculated using $\sigma=f(\omega_p^2)$ and Equation A4⁴. Figure 2.2 shows the electric field distribution for the TE_{10} mode in rectangular waveguide. The derivation for the waveguide propagation functions in Table 2.2 is in Appendix D.

In case I, the plasma behaves as a "metallic conductor" with $\alpha \approx \beta$. The skin depth for a plasma in free space or rectangular waveguide is approximately $\delta \approx \sqrt{\frac{2\nu}{\omega}} \left(\frac{c}{\omega_p} \right)$ or in terms of the collisionality, k , $\delta \approx \sqrt{2k} \left(\frac{c}{\omega_p} \right)$. The propagation function for case II is an evanescent wave (decaying exponential). For case III, the propagation function is purely imaginary. The dielectric constant for this case is slightly less than 1. Table 2.2 also shows that for probe frequencies many times the cutoff frequency, the propagation functions for free space are a good estimate for rectangular waveguide.

Table 2.2

Propagation Functions for Free Space and Rectangular Waveguide

CASE	Free Space ⁵	Rectangular Waveguide
Case I $\omega \ll v \ll \omega_p$	$\gamma \cong (1+j)\sqrt{\frac{\omega}{2v}}\left(\frac{\omega_p}{c}\right) = \alpha + j\beta$ <p>Note: $\alpha \cong \beta$ implies a good conductor skin depth</p> $\delta = \sqrt{\frac{2v}{\omega}}\left(\frac{\omega_p}{c}\right) = \sqrt{\frac{v\omega\lambda^2}{2\pi^2\omega_p^2}}$	$\gamma^2 \cong j\frac{\omega}{v}\left(\frac{\omega_p}{c}\right)^2\left[1 - \frac{\omega v}{\omega_p^2}\left(1 - \left(\frac{\lambda}{\lambda_c}\right)^2\right)\right]$ $\gamma \cong (1+j)\sqrt{\frac{\omega}{2v}}\left(\frac{\omega_p}{c}\right) = \alpha + j\beta$ <p>Note: $\alpha \cong \beta$</p>
Case II $v \ll \omega \ll \omega_p$	$\gamma \cong \frac{\omega_p}{c}$ <p>High Loss</p> $\text{loss} = 10\log(e^{2\alpha\ell}) = 8.686\alpha\ell \text{ dB}$ $= 8.686\frac{\omega_p}{c}\ell \text{ dB}$	$\gamma \cong \frac{\omega_p}{c}\left[1 + \left(\frac{1}{2}\right)\left(\frac{\lambda}{\lambda_c}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\right]$ $\gamma \cong \frac{\omega_p}{c} \quad \text{since } \omega_p \gg \omega$
Case III $\omega \gg \omega_p$ $\omega \gg v$	$\gamma \cong j\frac{\omega}{c}\left[1 - \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2\right]$ $\gamma \cong j\frac{\omega}{c}\left[1 - \frac{1}{2}\left(\frac{\lambda}{\lambda_p}\right)^2\right] \cong j\frac{\omega}{c}$	$\gamma \cong j\frac{\omega}{c}\left\{1 - \left(\frac{1}{2}\right)\left[\left(\frac{\omega_p}{\omega}\right)^2 + \left(\frac{\lambda}{\lambda_c}\right)^2\right]\right\}$ $\gamma \cong j\frac{\omega}{c}\left[1 - \frac{\lambda^2}{2}\left(\frac{1}{\lambda_p^2} + \frac{1}{\lambda_c^2}\right)\right]$ $\gamma \cong j\frac{\omega}{c}\left[1 - \frac{\lambda^2}{2\lambda_c^2}\right] \quad \text{for } \omega \gg \omega_p$ $\gamma \cong j\frac{\omega}{c} \quad \text{for } \lambda \ll \lambda_c$

2.1.6 Plasma Complex Conductivity in Rectangular Waveguide

J. B. Gerado and M. A. Gusinov outlined a derivation for the complex conductivity of a uniform plasma inside a rectangular waveguide⁶. They derive the complex conductivity in terms of the forward wave attenuation and phase shift. The derivation is repeated here with corrections for typographical errors in their work. In Appendix E, all of the intermediate steps for the derivation are included.

The authors assume: plasma uniformly fills the waveguide, TE₁₀ is the propagation mode in the rectangular waveguide, the complex conductivity completely describes the filling medium and is constant over a given volume, and the nonzero conductivity does not perturb the electric field inside the waveguide. Equation A6 does not consider a decrease in forward power caused by an increase in reflected (reverse) power. A6 only considers power loss due to true absorption.

Equation A4 is the propagation parameter inside rectangular waveguide.

$$\gamma^2 = j\omega \mu_0 (\bar{\sigma} + j\omega\epsilon) + \left(\frac{\pi}{a}\right)^2 = (\alpha + j\beta)^2 \quad (2.4) \text{ A4}$$

Equation A4 reduces to the free space case as "a" becomes large. The real and imaginary parts are separated in A5.

$$\omega \mu_0 \sigma_r = 2\alpha\beta \quad \omega \mu_0 \sigma_i = \beta^2 - \alpha^2 - \beta_o^2 \quad (2.5) \text{ A5}$$

β_o is defined as the propagation parameter when $\bar{\sigma} = \bar{0}$. The correct equation for β_o is shown below along with β_o in terms of the guide wavelength λ_{go} .

$$\beta_o \equiv \beta|_{\bar{\sigma}=\bar{0}} = \sqrt{\omega^2 \mu_0 \epsilon_0 - \frac{\pi^2}{a^2}} \quad (2.6)$$

$$\beta_o = \frac{2\pi}{\lambda_{go}}$$

The attenuation and phase shift are

$$\text{Attenuation} \quad \Delta \text{dB} = 10 \log(e^{2\alpha \ell}) = 8.686 \alpha \ell \quad \text{dB} \quad \text{note correction.} \quad (2.7a) \quad A6$$

$$\begin{aligned} \text{Phase Shift} \\ \text{in Degrees} \quad \Delta \Phi^\circ = \frac{180}{\pi} (\beta_o - \beta) \ell \geq 0 \end{aligned} \quad (2.7b)$$

Where ℓ is defined as the interaction length (the longest dimension of the ionized region).

The attenuation and phase shift describe the temporal variation of the forward wave through the plasma.

Substituting A6 into A5 gives the real and imaginary parts of the complex conductivity. There are two corrections in the imaginary part.

$$\sigma_r = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}^2} (0.115) \frac{\Delta \text{dB}}{\pi \ell} [1 - \Theta] \quad (2.8a) \quad A7$$

$$\sigma_i = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta \text{dB} \lambda_{go}}{8.686 \ell 2\pi} \right)^2 \right] \quad (2.8b)$$

$$\text{where } \Theta \equiv \frac{\Delta \Phi^\circ \lambda_{go}}{360 \ell} \quad (2.8c)$$

For the longitudinal cases, $1 - \Theta \approx 1$, and $\Theta^2 \approx 0$ because of the long interaction length. The $\frac{\Delta \text{dB}}{\ell}$ was not large enough to dominate over the Θ term. For the transverse case not hitting the waveguide wall, the Θ term is below the noise floor. It is assumed for the laser beam not hitting the waveguide wall that Θ for the transverse case is on the same order of magnitude as the longitudinal case. Because $\frac{\Delta \text{dB}}{\ell}$ is large for the transverse case not hitting the waveguide wall, the effect of the Θ is negligible. For the calculations where Θ is below the noise floor, Θ was assumed to be zero.

2.2 Microwave Interferometry

Microwave Interferometry Theory is briefly covered in Section 2.2. The phase shift of the forward wave was measured with a microwave interferometer. The phase shift accuracy is estimated in Appendix B. The phase shift accuracy is presented in section 2.2.1. Section 2.2.3 combines the complex conductivity equations⁷ with the phase angle from 2.2.1.

2.2.1 Phase Angle Measurement

The phase angle , ϕ , of the forward wave was measured with a microwave interferometer. The interference phasor is the sum of the test and reference phasors. For the initial null (balanced) condition on the interferometer, the test and reference phasors are equal in magnitude and 180° out of phase. The interference phasor equals $\bar{0}$.

The interference phasor is sensitive to amplitude and phase changes in the test phasor. Figure 2.3a shows a phasor diagram under null (balanced) conditions ($\phi = 180^\circ$). Figure 2.3b shows $\bar{V}_{inter}$ not equal to $\bar{0}$. The change in phase angle is $\Delta\Phi^\circ = \phi_2 - \phi_1 \geq 0$. The principal region for $\cos\theta$ is $0 \leq \theta \leq 180$, therefore $\Delta\Phi^\circ = 180 - \phi_{calculated}$ as shown in Figure 2.3c. Included in Figure 2.3 is a short derivation for the phase angle.

2.2.2 Approximate Accuracy of the Phase Shift

A simple mathematical model was used to estimate the phase shift accuracy. For small phase shifts, the phase shift is within approximately $\pm 10^\circ$. The rms phase shift error is approximately 16° . Appendix B contains more information on the accuracy of the phase shift measurements.

$$\bar{V}_{\text{inter}} = \bar{V}_{\text{refer}} + \bar{V}_{\text{test}} = V_{\text{refer}} + V_{\text{test}} e^{j\phi}$$

$$P_{\text{inter}} = P_{\text{refer}} + P_{\text{test}} + 2\sqrt{P_{\text{refer}} P_{\text{test}}} \cos\phi_{\text{calc}}$$

$$\cos\phi_{\text{calc}} = \frac{P_{\text{inter}} - P_{\text{test}} - P_{\text{refer}}}{2\sqrt{P_{\text{test}} P_{\text{refer}}}} \quad (2.9)$$

Figure 2.3

Phasor Diagrams and Interferometry Theory

2.3 Equation Summary

Summarized in section 2.3 are the equations used for determining the phase shift and the complex conductivity for the results presented in Chapter 4.

Table 2.3

Equation Summary

$$\phi_{\text{calculated}} = \cos^{-1} \left[\frac{P_{\text{inter}} - P_{\text{test}} - P_{\text{reference}}}{2\sqrt{P_{\text{test}} P_{\text{reference}}}} \right] \quad (2.9)$$

$$\Delta\Phi^{\circ} = 180 - \phi_{\text{calculated}} \quad (2.10)$$

$$\sigma_r = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}} (0.115) \frac{\Delta\text{dB}}{\pi\ell} [1 - \Theta] \quad (2.8a) \text{ A7}$$

$$\sigma_i = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta\text{dB}\lambda_{go}}{8.686\ell 2\pi} \right)^2 \right] \quad (2.8b) \text{ A7}$$

$$\text{where } \Theta \equiv \frac{\Delta\Phi^{\circ}\lambda_{go}}{360\ell} \quad (2.8c) \text{ A7}$$

ω = probe angular frequency λ = free space wavelength of the probe frequency

λ_{go} = guide wavelength ℓ = interaction length σ = complex conductivity

CHAPTER 3

Microwave and Optical Systems

3.1 Introduction

Chapter 3 describes the microwave and optical systems used to measure the microwave characteristics of laser induced air plasmas. The object is to measure the amplitude and phase shift of a 10 GHz CW probe through a plasma for the longitudinal case and compare to the transverse case. From the amplitude and phase shift, the spatially integrated complex conductivity is determined. The plasmas were created in 1 atmosphere laboratory air inside WR90 waveguide. The laser pulse and forward going microwave waveform counterpropagate. The FORWARD coupler samples the forward wave and the REVERSE coupler samples the reverse or reflected wave. Both couplers are 10 dB waveguide directional couplers.

All components are coaxial type unless stated otherwise. The forward wave, reverse wave, and laser beam propagation directions are shown in Figure 3.1. The laser beam entrance and exit points are open ended to the microwaves (i.e. connected to free space).

The magnitude and phase of the forward wave were measured along with the magnitude of the reverse wave. The phase of the forward wave was measured with a microwave interferometer. The waveguide is the test section of the interferometer. *Due to the boundary conditions of the waveguide, the scattered wave must remain inside the waveguide.* The calculated complex conductivity shows temporal variation; however, it is *spatially averaged* over the cross section and *spatially integrated* along the length of the waveguide.

3.2 Microwave System

Figure 3.2 shows the microwave system without the detector diodes. The three subsections of the microwave system are shown: waveguide components with light solid lines, 10 GHz signal source and power amplifier with dotted lines, and the interferometer with dark solid lines. Figure 3.3 shows the port notation and the scatter matrix for the magic Tee's. Figure 3.4 shows the complete microwave setup including the diode detectors. Appendix A contains more information on the interferometer. The *Absolute Power Reference Plane (APRP)* is at the center of the waveguide. Absolute power flow inside the waveguide is referenced to this plane. The *Interference Reference Plane (IRP)* is used for all phase angle calculations. The laser beam enters the waveguide at magic Tee 1 port 1 and travels through the forward coupler, the reverse coupler, magic Tee's 2 and 3 and exits at port 2 on Tee 3. Table 3.1 defines the measurement ports in Figure 3.2.

An HP83623A microwave signal source was attenuated and fed into a Varian TWT amplifier. At 10.0 GHz, approximately 7 W of power was generated. The output power was monitored using a 10 dB coupler followed by a 20 dB attenuator connected to a HP438A power meter. The noise floor on the amplifier was 32 mW (23 dB down from 7 W). This limits the dynamic range on the forward wave to about 23 dB.

Figure 3.2

Microwave System Without Diode Detectors

$$[S] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Figure 3.3

Magic Tee Port Notation and Scatter Matrix

Note: All waveguide components are shown with thin solid lines. All coax connectors are SMA or N.

Table 3.1

Port Definitions

Port Name	Definition
Test Port	monitoring point for the Forward Wave Power.
Reference Port	monitoring point for the Reference Power.
Reflection Port	(Reverse Wave Port) monitoring point for the Reverse Wave.
Interference Port	sum of Test Phasor and Reference Phasor

The microwave power is split equally between ports 1 and 2 on Tee 2. Half of the power is absorbed by the loads on ports 3 and 4 of Tee 3 (port 2 is isolated). The other half is the forward going wave. The loads on Tee 1 ports 3 and 4 absorb the forward wave (port 1 is isolated).

To allow the laser beam to pass through magic Tee 1 without defocusing, the tuning stub inside the Tee was removed. Without the tuning stub, the reverse wave was 13 dB down from the forward wave under static conditions (Table 3.2). A change in the reverse wave power that is small compared to 40 mW is undetectable with this system. With a pulsed microwave probe, the reflected wave due to Tee 1 could be separated in time from a reflection off of a step change in the dielectric constant. By adding a time delay before the pulse starts, the afterglow could also be studied.

Table 3.2

Return Loss

Forward Wave Power	300 mW
Reverse Wave Power without tuning stub	40 mW
Reverse Wave Power with tuning stub	<15 mW

3.2.1 Forward and Reverse Wave Calibration

Forward and Reverse power flow inside the waveguide is referenced to the Absolute Power Reference Plane (APRP). Figures 3.5a and 3.5b show how the attenuation was determined from the APRP to the measurement ports. By disconnecting the right half (forward wave) from the left half (reverse wave) at the APRP, the correction factors due to attenuation were measured. During the measurements in Figures 3.5a, and 3.5b, the ports not connected to the HP8756A network analyzer were connected to the appropriate diode detectors terminated into 50 Ω . Table 3.3 lists the S parameters for the Forward Wave and Reverse Wave (Reflected Wave) at 10.0 GHz. By adding 31 dB, $|S_{46}|$, to the forward power measured at port 4 (Test Port), the forward power at the Absolute Power Reference Plane was determined. By adding 16 dB ($|S_{46}| - |S_{56}|$), the power at port 4 referenced to the Interference Reference Plane was determined.

Table 3.3

S Parameters for the Forward and Reverse Waves

S Parameter	Magnitude in dB	Description
S_{46}	-31	attenuation from 6 to 4
S_{56}	-15	attenuation from 6 to 5
S_{87}	-21	attenuation from 7 to 8

3.2.2 Phase Angle Calibration

The phase angle calculation requires the TEST, REFERENCE, and INTERFERENCE powers to be referenced to the same reference plane. The attenuator and phase shifter were disconnected from port 1. Applying power to port 1 in Figure 3.5, S_{11} , S_{31} , and S_{51} were measured. Table 3.4 lists the S parameters at 10.0 GHz. By measuring power at port 3 (Reference Port), an effective power at the Interference Reference Plane may be determined by

$$\begin{aligned} \text{Power at Port 3 Referenced to the IRP} &= \text{power at port 3} + |S_{31}| - |S_{51}| \quad (3.1) \\ &= \text{power at port 3} + 20 - 3.7 \text{ dB} \\ &= \text{power at port 3} + 16 \text{ dB} \end{aligned}$$

Table 3.4

Reference Port Calibration

S_{11}	-25 dB
S_{31}	-20 dB
S_{51}	-3.7 dB

The phasor diagram used to determine the phase angle is shown in Figure 3.6. The power correction factors for the Interference Reference Plane give the equivalent powers from the Test Port, Reference Port, and Interference Port at the Interference Reference Plane. From the powers referenced to the Interference Reference Plane, the phase angle of the forward wave through the plasma was calculated from Equation 3.2. Appendix L shows the S parameters for the measurements in 3.2.1 and 3.2.2. Table 3.5 lists the power correction factors for the APRP and the IRP.

$$\phi = \cos^{-1} \left[\frac{P_{\text{interference}} - P_{\text{test}} - P_{\text{reference}}}{2\sqrt{P_{\text{test}} P_{\text{reference}}}} \right] \quad 0 \leq \phi \leq 180^\circ \quad (3.2)$$

Table 3.5
Power Correction Factors

Port Name	Correction to Interference Reference Plane (IRP)	Correction to Absolute Power Reference Plane (APRP)
Test Port (port 4)	16 dB	31 dB
Reference Port (port 3)	16 dB	N/A
Interference Port (port 5)	0	N/A
Reflection Port (port 8)	N/A	21 dB

3.2.3 Diode Detector Calibrations

Diode detectors were used to measure the Test, Reference, Interference, and Reflection ports. Three MA/COM 2085-6017-00 diodes and one Narda #503 were used. MA/COM diodes were used on the ports 3 (Reference) and 4 (Test) on the interferometer. The other MA/COM diode was used to measure the reverse (reflected) power. The more rugged Narda diode was directly connected to port 5 (Interference) on the interferometer. All of the diodes were operated in their small signal linear range.

The diode detectors are limited by their dynamic range and rise time. The maximum output voltage from the Narda Diode loaded into $50\ \Omega$ was 150 mV. This makes viewing the diode voltage on a Tektronics 7250 6 GHz scope with a fixed 5 volts full scale resolution difficult. The measured rise times are $<0.6\ \text{ns}$ for the MA/COM diodes and $<0.8\ \text{ns}$ for the Narda diode. The fast rise times justify calibrating the diodes with a CW source. The dynamic range of the Narda diode is approximately 30 dB and 25 dB for the MA/COM diodes (1 mV output voltage to maximum output voltage into a $50\ \Omega$ load).

A 10 dB coupler (Narda #4246B-10) was calibrated as shown in Figure 3.7a and 3.7b. The output of the Wiltron oscillator was held constant for Figures 3.7a and 3.7b.

The measured direct power was 17.07 dBm and the coupled power was 7.46 dBm. The coupled power is 9.61 dB less than the direct power. Using the setup in Figure 3.8a, the MA/COM diodes were calibrated. The Narda diode was calibrated as shown in Figure 3.8b.

The output voltages for all the diodes were measured in 1 dB power steps by changing the attenuation. By adding 9.61 dB to the coupled power, the input power to the Narda diode was determined. By subtracting 9.61 dB from the direct power, the input powers to the MA/COM diodes were determined. Figure 3.9 shows the input power in dBm and the output voltage in mV for the Narda #503 diode at 10.0 GHz. Using a curve fitting program, the input power as a function of diode output voltage was found for each one of the diodes.

The inverse diode curves (input power as a function of output voltage) along with the power corrections (Table 3.5) were used for all power flow calculations. Figure 3.10 shows the inverse curve for the Narda diode. Equation 3.3 gives the INPUT power *in*

dBm for the Narda #503 as a function of output voltage *in mV*. Table 3.6 shows the inverse diode equations for all the diodes. The complete set of calibration graphs may be found in Appendix F.

$$P(v) = -15.48 + 10.59 \log(v) - 0.0902v + 4.53 \times 10^{-4} v^2 + 9.95 \times 10^{-7} v^3 \text{ dBm} \quad (3.3)$$

Table 3.6

Inverse Diode Equations

Diode	Input Power in dBm as a function of Output Voltage in mV	R
MA/COM #1 Reflection	$P(v) = -24.08 + 9.688 \log(v) + 8.397 \times 10^{-3} v^2 - 5.518 \times 10^{-6} v^3$	0.99997
MA/COM #3 Reference	$P(v) = -24.51 + 9.550 \log(v) + 8.493 \times 10^{-3} v^2 - 5.112 \times 10^{-6} v^3$	0.99999
MA/COM #4 Test	$P(v) = -24.21 + 9.720 \log(v) + 8.895 \times 10^{-3} v^2 - 5.550 \times 10^{-6} v^3$	0.99997
Narda #503 Interference	$P(v) = -15.5 + 10.6 \log(v) - 0.0902v + 4.53 \times 10^{-4} v^2 + 9.95 \times 10^{-7} v^3$	0.99998

Figure 3.9 Narda Diode Output Voltage versus Input Power

Figure 3.10 Narda Diode Input Power versus Output Voltage

3.3 Optical System

The laser and the focusing lens are described in Section 3.3. The optical and microwave timing are explained together. Figure 3.12 shows a complete optical and microwave timing diagram. The timing corrections for propagation delays are shown in Table 3.9.

3.3.1 Janus Laser System

The Janus Laser Facility at Lawrence Livermore National Laboratory provided 100 ps (full width half maximum) laser pulses from approximately 0.2 J to 15 J. The laser operates at a wavelength of 1.064 μm . Table 3.7 summarizes the Janus Laser Facility Parameters. A Scientech Energy meter was compared to the Janus System Calorimeter. The Janus System Calorimeter was found to be systematically 1 % low. The Laser Pulse Energy is given by Equation 3.2.

$$E = 0.99 \times \text{Janus System Calorimeter} \quad (3.2)$$

3.3.2 Optical Beam Path

Figure 3.11 shows the optical beam path and the complete optical system. The laser beam diameter is approximately 10 cm. The inner dimensions of WR90 are 2.286 cm by 1.016 cm. For the laser beam to approximate a plane wave and direct all of the laser energy inside the waveguide, a long focal length lens was used. Table 3.8 shows the lens parameters.

Table 3.7

Janus Laser Facility Parameters

Wavelength	1.064 μm
Unfocused Beam Diameter	10 cm
Polarization	Vertical
Energy Range	0.2 J to 15 J
FWHM pulse width	100 ps

Table 3.8

Lens Parameters at 1.064 μm

Lens Focal Length	11.2 m
Lens Diameter	12.5 cm
Lens F/#	88

Figure 3.12
Complete Optical and Microwave Timing Diagram

Figure 3.13
Typical Oscilloscope Trace

Figure 3.14
Microwave Timing

The laser beam is 1 cm in diameter when it reaches port 1 on Tee 1. The focal point is near port 1 on Tee 2. The focal diameter is approximately 1.5 times the diffraction limit (360 μm). For low energy laser shots (laser energy < 1.2 J), a Scientech Energy meter measured the energy exiting the waveguide. For more energetic shots (energy > 1.3 J), an aluminum plate beam dump was used. The difference between the incident laser energy and the energy exiting the waveguide is the total energy dissipated. This dissipated energy is the sum of the absorption and off axis scattering effects. Absorption accounts for initial ionization and acceleration of free electrons.

3.3.3 Optical and Microwave Timing

The propagation delays for the different ports on the microwave system were determined by two separate propagation delay measurements. The propagation delays in Table 3.9 are from Appendix G on microwave timing.

Table 3.9

Propagation Delays

Port Name	Absolute Power Reference Plane	Interference Reference Plane*
Interference Port	6.0 ns	0 ns
Test Port	5.1 ns	-1.0 ns
Reflection Port	4.5 ns	-1.5 ns

* Data Rounded off to Nearest 0.5 ns

The propagation delay from the absolute reference plane to the test port is 5.1 ns. With the interference port defined to be $t=0$, the output from the test port occurs 1.0 ns before the interference port.

The Janus Laser Facility provides a pre-trigger pulse. The pre-trigger occurs 18.7 ns before the laser pulse reaches the lens. The pre-trigger signal was delayed by 50 ns to give the timing fiducials and trigger pulses for the Tektronix 7104 scopes (Figure 3.16). A simplified diagram of the optical system and a timing diagram are shown in Figure 3.12. Combining the microwave and optical timing with the fiducial on the interference port as a timing reference, gives the timing corrections (Figure 3.14 and Table 3.9). Figure 3.13 shows a typical scope trace where the fiducial on channel 1 is added to channel 2. Figure 3.16 shows the interference, test and reflection scope traces. The fiducial on the interference trace was defined as $t=0$. Shifting the reflection (reverse wave) and test diode voltage traces removes the difference in propagation delays. Figure 3.14 shows before the timing corrections and Figure 3.15 shows after the timing corrections.

3.4 Measurement Procedure

Figure 3.16 shows the measurement procedure including the attenuation and timing corrections. Figure 3.17 shows all the hardware used in the experiment: including the oscilloscopes, detectors, and the optical system.

The reference power was held constant for each laser shot. The reference power was measured with MA/COM diode #3 (reference diode) connected to a Fluke 77 voltmeter. Scopes 1, 2, and 3 recorded the interference, test, and reflection diode voltage traces. The fiducials on all three of the diode voltage traces were aligned. The fiducials were removed and the timing corrections were applied (Table 3.9). Using the inverse diode equations (Table 3.4), the measured diode voltages were converted to relative power. Applying the power corrections (Table 3.5), the absolute forward and reverse power as a function of time were calculated. Absolute power flow is referenced to the Absolute Power Reference Plane. Using the Interference Reference Plane power corrections (Table 3.4), the phase shift of the forward wave was calculated. From the forward wave power and phase shift, the following **SPATIALLY INTEGRATED** quantities were calculated:

<complex conductivity>,

<complex permittivity>,

<collisionality>, and

<electron density> i.e. bulk effect on a 10 GHz microwave probe.

CHAPTER 4

Results

4.1 Introduction

The results of the microwave interferometry experiment are presented in this chapter. The four different geometries for the experiment are listed in Table 4.1. The laser energy range from approximately 0.2 to 1.2 Joules is covered by low energy shots. The energy range from approximately 5 to 15 Joules is covered by high energy shots. The explanation in Section 4.2 uses the low energy shots for the longitudinal geometry as the canonical case.

The oscilloscopes bandwidth for all cases is 400 MHz unless stated otherwise. The conversions for absolute power may be found in Chapter 3. Absolute power is reference to the Absolute Power Reference Plane (Figure 3.2). The accuracy of the phase shift was approximated in Section 2.2.1. The phase shift is within 10° for small shifts. The noise floor limits attenuation measurements to less than approximately 23 dB. Each digitized data trace consisted of approximately 50 to 100 data points.

The complex conductivity, complex dielectric function, collisionality, and electron density are spatially integrated over the interaction length and spatially averaged over the cross section of the waveguide. Table 4.2 shows the order in which the results are presented. The X in the figure number is for results Group 1, 2, 3, 4, or 5 in Table 4.1. For the results in Groups 1, 2 and 3, a full time scale is shown in Figure 4.X.Ya and an expanded scale is shown in Figure 4.X.Yb. All results are shown as a function of time unless stated otherwise.

Table 4.1

Laser Energy Ranges Covered for Each Geometry

	Laser Beam Not Hitting the Waveguide Wall	Laser Beam Hitting the Waveguide Wall
Longitudinal (parallel to z direction)	Low Energy Figure Numbers 4.1.1 to 4.1.10 Low Energy and High Energy Figure Numbers 4.2.1 to 4.2.10 Scope Bandwidth 1 GHz	High Energy Figure Numbers 4.3.1 to 4.3.10
Transverse (Parallel to Electric Field)	Low Energy Figure Numbers 4.4.1 to 4.4.4	Low Energy Figure Numbers 4.5.1 to 4.5.4

Table 4.2

Results Presentation Order

Figure Number	Order of Figures in Each Group
4.X.1	Absolute Forward Power in mW
4.X.2	Absolute Reverse Power in mW
4.X.3	Forward Wave Attenuation in dB
4.X.4	Forward Wave Phase Shift in Degrees
4.X.5 and 6	Spatially Integrated Complex Conductivity
4.X.7 and 8	Spatially Integrated Dielectric Function
4.X.9	Collisionality from the Spatially Integrated Complex Conductivity
4.X.10	Spatially Integrated Electron Density

The 0 ns point on each one of the figures is the peak of the fiducial on the interference diode voltage trace (Chapter 3). With this reference point, the laser reaches the center of the waveguide (APRP) at 6.2 ns, and the microwave signal reaches the oscilloscopes at 10.5 ns for the Reflection Port, 11 ns for the Test Port and 12.2 ns for the Interference Port. The Reflection Diode Voltage Trace and the Test Diode Voltage Trace were shifted in time to line up with the Interference Diode Trace (3.3.3 Optical and Microwave Timing). As the laser pulse enters the waveguide, the data show up at approximately 11 ns on all the graphs. As the laser pulse crosses the waveguide center

(APRP), the data show up at 12.2 ns on all the graphs. Figure 4.1 shows a typical plot for the interference, test, and reflection diode voltage traces with the fiducials removed.

Figure 4.2 shows when the laser pulse reaches the waveguide center (APRP) and when the microwave signals reach the oscilloscopes.

Figure 4.1 Diode Voltages

Figure 4.2 Timing Diagram

4.2 Group 1: Longitudinal Not Hitting

Figure 4.1.1 and Figure 4.1.2 show the absolute forward and reverse power for five different laser energies. The forward measurement is limited by a dynamic range of 23 dB. The reverse measurement is limited by a 40 mW signal caused by a mismatch in the system (i.e. return loss limitation). The reflected power due to a mismatch for the transverse case is less than 15 mW. Magic Tee 1 limits the return loss for the longitudinal case (Figure 3.4). For the transverse case, Tee 1 was replaced with a termination.

Figure 4.1.3 shows the Forward Wave Attenuation in dB. As shown in Figure 4.1.3, the rising edges occur earlier for the higher energy laser shots. The occurrence of laser induced air breakdown is a function of position. For the higher energy laser shots, breakdown occurs earlier in time (closer to the laser beam entrance point). This causes the Δ dB curve to rise earlier in time for higher laser energies since a CW microwave probe is used.

Attenuation measurements larger than 23 dB are below the noise floor. This causes sharp peaks in the real part of the complex conductivity (Figure 4.1.5) and the imaginary part of the dielectric function (Figure 4.1.8). The attenuation per unit length, $\alpha = \frac{\Delta \text{dB}}{\ell}$, is approximately 4/3 times the attenuation.

The phase shift shows about 5 degrees of phase noise. The accuracy of the phase shift is presented in Chapter 2 and Appendix B. This also effects the imaginary part of the complex conductivity and the real part of the dielectric function.

The collisionality was calculated from the ratio: $\frac{-\text{Re}\{\bar{\sigma}\}}{\text{Im}\{\bar{\sigma}\}}$. Figure 4.1.10 shows the spatially integrated electron density or the "effective" electron density. The microwave probe is not sensitive to electron densities less than 10^{15} 1/m^3 . The critical density for a 10 GHz probe is approximately 10^{18} 1/m^3 .

4.3 Group 2: Longitudinal Not Hitting

Figures 4.2.1 through 4.2.10 cover laser energies of 0.45, 1.0, 11, 12, and 15 Joules. The results are similar to Group 1. The electron recombination after the laser beam exits the waveguide is called the afterglow. The afterglow decay modes are easily seen in Figure 4.2.3. The 12 Joule laser energy shot shows 3 different decay modes. The Reverse Wave Power in Figures 4.2.2a and 4.2.2b shows an increase. This increase manifests itself as a short spike. This "increase" in reverse wave power is due to the spatial extent of the charge distribution and the low directivity of the coupler. The short spike is evidence of a coherent effect operating on a few ns time scale unlike plasma reflectivity. Appendix J explains this effect.

The sharp peaks in Figure 4.2.3 are from the limited dynamic range explained in Group 1. A 10 Joule laser shot was fired with the RF off to search for any plasma generated microwave noise. The test and reflection diode voltages were almost negligible. The peak diode voltage was approximately 4 mV for the test diode and 8 mV for the reflection diode. This corresponds to a peak forward wave power of approximately 20 mW and a peak reverse wave power of 4 mW. Noise powers as large as 300 mW⁸ for a 14 J laser shot were measured with an S band waveguide to coax transistion and a diode detector. The transition was placed near the focal point of the Janus Laser Beam. This experiment was performed in laboratory air at a laser wavelength of 1.064 μm .

4.4 Group 3: Longitudinal Hitting

Group 3 covers a laser energy from 0.3 to 1 Joule. For the geometry in Group 3, a large percentage of the ionized region is near the electric field minimum located at the waveguide wall. The results in Group 3 are similar to Groups 1 and 2. The reflected wave for the 0.31 Joule shot shows an "increase" in reverse wave power. This increase is due to the limited directivity of the reverse coupler combined with the large spatial extent

of the plasma. The rising edge in Figure 4.3.3 for the 1 Joule laser shot does not fit the trend in Groups 1, 2, and 3. The cause of this discrepancy is uncertain.

4.5 Group 4: Transverse Not Hitting

Unlike Groups 1, 2, and 3, estimating the interaction length for the transverse geometry is difficult and highly imprecise (errors $\geq 100\%$). The interaction length for the longitudinal case is 72 cm (+0% -20%). The interaction length for the transverse case is approximately 0.5 mm or about 3 times the diffraction limited spot size due to the F/# 88 lens. The $\frac{\Delta\text{dB}}{\ell}$ in the complex conductivity equations dominates for this series. The squared term in the complex conductivity equation (Equation 2.8b) magnifies even small errors in the interaction length. The peak $\frac{\Delta\text{dB}}{\ell}$ for a 1 J laser shot is 26 dB/m for the longitudinal not hitting, and 15 dB/m for hitting the waveguide wall. The peak $\frac{\Delta\text{dB}}{\ell}$ for a 1 J laser shot transverse and not hitting the waveguide wall is on the order of 10^3 dB/m.

For the transverse case, the interaction length is much less than a guide wavelength. The phase shift for the transverse case not hitting the waveguide wall is near zero and phase noise limited. This justifies setting the phase shift equal to zero to simplify calculational estimates since the $\frac{\Delta\text{dB}}{\ell}$ term in Equations 2.8a and 2.8b dominate.

The results for Group 4 cover only Figures 4.4.1 to 4.4.4. In Appendix K, the length dependent quantities are shown for interaction lengths of 1 and 2 mm. For interaction lengths of 1 or 2 mm, the results from Group 4 show the same trends as Groups 1, 2, and 3. The forward wave shows the same energy dependence as Groups 1, 2, and 3. The leading edge on the Forward Wave Attenuation, Figure 4.4.3, shows the same energy dependence as Groups 1, 2, and 3.

Figure 4.4.2 shows an increase in reverse wave power. Unfortunately, no laser shots were fired with the RF power off to look for plasma generated noise. Assuming the noise for the transverse case is small like the longitudinal case, then Figure 4.4.2 shows an increase (enhancement) in reverse wave power. This increase in reverse wave power is *not* attributed to the spurious coherent effects in waveguide couplers with limited directivities.

4.6 Group 5: Transverse Hitting

For the transverse case hitting the waveguide wall, data for a 1.1 Joule shot is presented. The forward and reverse power flow are shown in Figures 4.5.1 and 4.5.2. The forward wave attenuation curve is shown in Figure 4.5.3. The reverse wave power was not subtracted from the incident wave power (input power at the APRP) in Group 5. For an incident wave power of 800 mW and a reverse power of 110 mW, the actual attenuation is about 0.6 dB less than shown in Figure 4.5.3 (Section 2.1.6). The phase shift is shown in Figure 4.5.4. The phase shift and attenuation for the transverse case hitting the wall are much larger than the not hitting case. More information on the charge distribution is required to calculate the length dependent quantities. Figures 4.5.X show that deliberately hitting the waveguide wall is significantly different from the not hitting case.

4.7 Results Comparision

Figures 4.6.X compare the 4 different geometries at a laser energy of approximately 1 Joule. Table 4.3 shows the peak values of the Reverse Power, Attenuation, and Phase Shift for the 4 geometries. The peak values show that each geometry is quite different. Figure 4.7 shows the peak attenuation and phase shift for the longitudinal case hitting and not hitting the waveguide wall. The approximate slope of the lines is also shown for both geometries.

Figure 4.8 shows the attenuation per phase shift ratio as a function of laser energy for the longitudinal cases. For the laser beam hitting the wall, the ratio is approximately 0.3 dB/degree and 0.22 dB/degree for the laser beam not hitting the wall.

Figure 4.9 shows the peak attenuation versus the laser energy for the transverse geometry not hitting the waveguide wall. The slope of Figure 4.9 is approximately 2 dB/J. Table 4.4 compares the slopes of the peak amplitude and phase shifts for all four laser geometries. Table 4.5 lists the peak amplitude and phase shift for the laser beam transverse and hitting the waveguide wall.

Table 4.3

Comparison of Peak Values at Approximately 1 J

Geometry:	Peak Attenuation in dB	Peak Reverse Wave Power in mW	Peak Phase Shift in Degrees
Longitudinal Hitting	11	N/A	32
Not Hitting	19	N/A	85
Transverse Hitting	>23	110	65
Not Hitting	2.5	50	$\cong 0$

Table 4.4

Comparison of Slopes

Geometry:	Slope of Peak Amp	Slope of Peak Phase
Longitudinal Not Hitting	20 dB/J	90 °/J
Hitting	10 dB/J	30 °/J
Transverse Not Hitting	2 dB/J	----
Hitting	----	----

Figure 4.1.1a Absolute Forward Power in mW

Figure 4.1.1b Absolute Forward Power in mW (Expanded Scale)

Figure 4.1.2a Absolute Reverse Power in mW

Figure 4.1.2b Absolute Reverse Power in mW (Expanded Scale)

Figure 4.1.3a Forward Wave Attenuation in dB

Figure 4.1.3b Forward Wave Attenuation in dB (Expanded Scale)

Figure 4.1.4a Forward Wave Phase Shift in Degrees

Figure 4.1.4b Forward Wave Phase Shift in Degrees (Expanded Scale)

Figure 4.1.5b Real Part of the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.1.5a Real Part of the Spatially Integrated Complex Conductivity

Figure 4.1.7a Real Part of the Spatially Integrated Dielectric Function

Figure 4.1.7b Real Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.1.8a Imaginary Part of the Spatially Integrated Dielectric Function

Figure 4.1.8b Imaginary Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.1.9a Collisionality from the Spatially Integrated Complex Conductivity

Figure 4.1.9b Collisionality from the Spatially Integrated Complex Conductivity (Expanded Scale)

4.1.10a Spatially Integrated Electron Density

4.1.10b Spatially Integrated Electron Density (Expanded Scale)

Figure 4.2.1a Absolute Forward Power in mW

Figure 4.2.1b Absolute Forward Power in mW (Expanded Scale)

Figure 4.2.2a Absolute Reverse Power in mW

Figure 4.2.2b Absolute Reverse Power in mW (Expanded Scale)

Figure 4.2.3a Forward Wave Attenuation in dB

Figure 4.2.3b Forward Wave Attenuation in dB (Expanded Scale)

Figure 4.2.4a Forward Wave Phase Shift in Degrees

Figure 4.2.4b Forward Wave Phase Shift in Degrees (Expanded Scale)

Figure 4.2.5a Real Part of the Spatially Integrated Complex Conductivity

Figure 4.2.5b Real Part of the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.2.6a Imaginary Part of the Spatially Integrated Complex Conductivity

Figure 4.2.6b Imaginary Part of the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.2.7a Real Part of the Spatially Integrated Dielectric Function

Figure 4.2.7b Real Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.2.8a Imaginary Part of the Spatially Integrated Dielectric Function

Figure 4.2.8b Imaginary Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.2.9a Collisionality from the Spatially Integrated Complex Conductivity

Figure 4.2.9b Collisionality from the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.2.10a Spatially Integrated Electron Density

Figure 4.2.10b Spatially Integrated Electron Density (Expanded Scale)

Figure 4.3.1a Absolute Forward Power in mW

Figure 4.3.1b Absolute Forward Power in mW (Expanded Scale)

Figure 4.3.2a Absolute Reverse Power in mW

Figure 4.3.2b Absolute Reverse Power in mW (Expanded Scale)

Figure 4.3.3a Forward Wave Attenuation in dB

Figure 4.3.3b Forward Wave Attenuation in dB (Expanded Scale)

Figure 4.3.4a Forward Wave Phase Shift in Degrees

Figure 4.3.4b Forward Wave Phase Shift in Degrees (Expanded Scale)

Figure 4.3.5a Real Part of the Spatially Integrated Complex Conductivity

Figure 4.3.5b Real Part of the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.3.6a Imaginary Part of the Spatially Integrated Complex Conductivity

Figure 4.3.6b Imaginary Part of the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.3.7b Real Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.3.7a Real Part of the Spatially Integrated Dielectric Function

Figure 4.3.8a Imaginary Part of the Spatially Integrated Dielectric Function

Figure 4.3.8b Imaginary Part of the Spatially Integrated Dielectric Function (Expanded Scale)

Figure 4.3.9a Collisionality from the Spatially Integrated Complex Conductivity

Figure 4.3.9b Collisionality from the Spatially Integrated Complex Conductivity (Expanded Scale)

Figure 4.3.10a Spatially Integrated Electron Density

Figure 4.3.10b Spatially Integrated Electron Density (Expanded Scale)

Figure 4.4.1 Absolute Forward Power in mW

Figure 4.4.2 Absolute Reverse Power in mW

Figure 4.4.3 Forward Wave Attenuation in dB

Figure 4.4.4 Forward Wave Phase Shift in Degrees

Figure 4.5.1 Absolute Forward and Reverse Power in mW

Figure 4.5.2 Absolute Reverse Power in mW

Figure 4.5.3 Forward Wave Attenuation in dB

Figure 4.5.4 Forward Wave Phase Shift in Degrees

Figure 4.6.1 Absolute Forward Power in mW

Figure 4.6.2 Absolute Reverse Power in mW

Figure 4.6.3 Forward Wave Attenuation in dB

Figure 4.6.4 Forward Wave Phase Shift in Degrees

Figure 4.7 Peak Attenuation and Peak Phase Shift versus Laser Energy

Figure 4.8 Attenuation to Phase Shift Ratio as a Function of Laser Energy

Figure 4.9 Peak Attenuation versus Laser Energy

Table 4.5

Peak Attenuation and Phase Shift

Laser Beam Transverse and Hitting the Waveguide

Peak Attenuation	>23 dB
Peak Phase Shift	65°

CHAPTER 5

Conclusion

For all 4 of the geometries shown in Figure 1.1, the absolute forward and reverse power flows as a function of time were calculated. From interferometric data, the phase shift of the forward wave was determined. The spatially integrated complex conductivity is calculated from the amplitude and phase shift of the forward wave. From this information the complex dielectric function and the effective electron density were determined.

From the time varying spatially integrated complex conductivity data, components using laser induced air plasmas may be numerically modeled and designed. Designs for plasmas switches (on to off and off to on) are possible with this data. Another use for this data is for the design of high speed Q switched microwave cavities. More research is required to study the effects of varying the atmospheric pressure and composition. By decreasing the pressure, the plasma should become less absorptive.

The forward and reverse wave powers for all four geometries were large. Powers range from approximately 0 mW to 800 mW for the laser longitudinal to the waveguide. For the laser transverse to the waveguide, powers range from approximately 40 mW to 800 mW. The reverse wave powers for the laser beam longitudinal to the waveguide are return loss limited. The 40 mW reverse wave power is due to an impedance mismatch. For ionization fronts a system with a higher return loss is required. For the laser transverse to the waveguide, reverse wave powers range from 10 mW to 50 mW for the laser not hitting the waveguide and from 30 mW to 110 mW for the laser hitting the waveguide wall. Table 5.1 summarizes the power ranges for each geometry.

Table 5.1
Forward and Reverse Power Flow Ranges

Geometry	Forward Range mW		Reverse Range mW	
Longitudinal Not Hitting	800	$\cong 0$	40	<10
Hitting the Waveguide	800	80	40	<10
Transverse Not Hitting	800	430	50	<10
Hitting the Waveguide	800	40	110	<30

The transverse geometry has a much higher peak reflected power than the longitudinal geometry. The plasmas for the longitudinal geometries did not exhibit an increase in reverse wave power except for the directivity limited coupler effect explained in Appendix J.

A 10 Joule laser shot was fired longitudinal to the waveguide and not hitting the waveguide wall. The peak plasma generated noise power in the forward direction is 20 mW. The peak plasma generated noise in the reverse direction is 4 mW. The peak noise powers are useful for theoretical signal to noise ratio calculations for ionization fronts.

Comparing the ratio of the real part of the propagation function to the imaginary part (α to β) shows significant differences between the four geometries. Table 5.2 shows the ratios for the four geometries. For a metallic conductor, the ratio is approximately 1 (Table 2.2 Case I). For the laser beam longitudinal to the waveguide and not hitting the waveguide wall, the laser induced plasma has nearly the same ratio. For the laser beam transverse to the waveguide and not hitting the waveguide wall, α is much greater than β . When α is much greater than β , the plasma is lossy (Case II in Table 2.2).

Table 5.2
Ratio Comparision

Geometry	α/β Ratio 1/Rad
Longitudinal	
Not Hitting the Waveguide	1.5
Hitting the Waveguide	2
Transverse	
Not Hitting the Waveguide	>20
Hitting the Waveguide	2.3

Table 5.3 shows the peak values of phase shift and attenuation for the four geometries at approximately 1 Joule. When the laser beam hits the waveguide wall, the electric field potential has a strong effect on the attenuation and phase shift. For the laser beam transverse to the waveguide, the electric field potential is a maximum for both the laser beam hitting the waveguide wall and not hitting the wall. Table 5.3 shows a large increase in both attenuation and phase shift for the transverse geometry. For the laser beam longitudinal to the waveguide, the attenuation and phase shift both decrease. This effect is attributed to the electric field potential minimum.

The laser induced plasmas were compared by attenuation per unit length. The attenuation per unit length for the laser beam longitudinal to the waveguide is 26 dB/m for the laser beam not hitting the wall and 15 dB/m for hitting the wall. The laser induced plasma is near the maximum electric field potential for the not hitting case and near the minimum for the laser beam hitting the waveguide wall. For the laser beam transverse to

the waveguide and not hitting the waveguide wall, the attenuation per unit length is greater than 10^3 dB/m. No reliable attenuation per unit length is available for the transverse geometry hitting the waveguide wall due to the great uncertainty in the plasma length and distribution.

Table 5.3
Electric Field Potential

Geometry	E-field Potential	Peak Attenuation	Difference	Peak Phase Shift	Difference
Longitudinal		dB	dB	Degree	Degree
Not Hitting	maximum	19	8	85	53
Hitting	minimum	11		32	
Transverse					
Hitting	maximum	>23	>20	65	$\cong 65$
Not Hitting	maximum	2.5		0	

The slopes of the peak attenuation and phase shift plots were compared for the laser beam longitudinal to the waveguide, hitting the waveguide wall and not hitting the wall. For the not hitting case from 0.3 to 1 J, the slope of the peak attenuation curve is approximately 20 dB/J. The slope of the peak phase shift curve is approximately $90^\circ/\text{J}$. For the laser beam hitting the waveguide near the minimum electric field potential, the slope of the peak attenuation curve is approximately 10 dB/J. The slope of the peak phase shift is approximately $35^\circ/\text{J}$.

For the laser beam transverse to the waveguide and not hitting the waveguide wall, the slope of the peak attenuation curve is approximately 2 dB/J. No information is

available on the slope of the peak phase shifts (phase noise limitation). Only the peak amplitude and peak phase of one laser shot are available for the transverse case hitting the waveguide wall (Figure 4.5). Therefore, no reliable slope is available.

The microwave interferometer experiment on laser induced plasmas has shown that the plasma generated noise for the laser beam longitudinal to the waveguide and not hitting the waveguide wall is small. The forward and reverse power flows are large. Small wave powers are return loss limited. To improve low level power measurements, a higher system return loss is required and/or a pulsed microwave probe is required. Small phase shift measurements are limited by the system noise floor. Using a low noise high power solid state amplifier instead of a TWT amplifier would improve the phase shift accuracy.

NOTES

¹ J. B. Gerado and M. A. Gusinow. Electronic Recombination of He_3^+ , Phys. Rev. A225-6 (1971).

² Ibid.

³ Ibid.

⁴ Ibid.

⁵ William A. Elmore and Mark A. Heald. The Physics of Waves, New York. Dover, 1985.

⁶ J. B. Gerado. Op. cit.

⁷ Ibid.

⁸ Paul Bolton. Microwave Generation from Ionization Induced by 100 ps Laser Pulses, Presentation, LLNL. 1991.

BIBLIOGRAPHY

- Bekefi, George. Principles of Laser Plasmas. New York. Wiley, 1976.
- Elmore, William A. and Mark A. Heald. The Physics of Waves. New York. Dover, 1985.
- Gerado, J. B. and M. A. Gusinow. Electronic Recombination of He_3^+ . Phys. Rev. A225-6 (1971).
- Ghandi, Om P. Microwave Engineering and Applications. Elmsford. Pergamon Press, 1981.
- Grigor'ev, F. V. et al. Modeling the Mechanism for Air Breakdown by 1 Micro Laser Radiation. Sov. Phys. Tech Phys. 31(1) (January 1988).
- Liboff, Richard L. and G. Conrad Dalman. Transmission Lines, Waveguides, and Smith Charts. New York. Macmillan, 1985.
- Marshall, Staney V. and Gabriel G. Skitek. Electromagnetic Concepts and Applications. Englewood Cliffs. Printice Hall, 1990.
- Schadowitz, Albert. The Electromagnetic Field. New York: Dover, 1988.

APPENDIX A

Interferometer

The main part of the interferometer used two 20 dB couplers (Narda #4246B-20) and a 2 to 18 GHz Omni Spectra Splitter/Combiner. Before the laser was fired, the amplitude and phase of the reference signal were adjusted to null the interferometer (i.e. interference power set to zero).

For the attenuation correction factors used in this experiment, the appropriate [S] parameters were measured with the interferometer connected to the waveguide test section as shown in Figure 3.3. Figure A.1 shows the interferometer disconnected from

the waveguide test section. The S parameters in Table A.1 are only for the five port network in Figure A.1. All unused ports on the network were terminated into 50 Ω . Figure A.2 shows the S parameters in dB from 8 to 12 GHz for the Network in Figure A.1.

Table A.1

Scatter Parameters for the 5 Port Network at 10.0 GHz

Frequency in GHz	S ₅₁	S ₅₂	S ₅₅	S ₄₂	S ₃₁	S ₁₁	S ₂₂
8	-3.9	-3.6	-24	-20	-20	-23	-24
8.25	-3.9	-3.6	-25	-20	-20	-18	-22
8.5	-3.9	-3.6	-22	-20	-20	-17	-17
8.75	-3.9	-3.6	-32	-20	-20	-16	-22
9	-4.0	-3.7	-17	-20	-20	-16	-28
9.25	-4.0	-3.7	-12	-20	-20	-23	-29
9.5	-3.8	-3.7	-15	-20	-20	-26	-33
9.75	-3.8	-3.8	-32	-20	-20	-25	-30
10	-3.8	-3.8	-20	-20	-20	-23	-24
10.25	-3.8	-3.7	-17	-20	-19	-17	-24
10.5	-3.7	-3.7	-20	-20	-20	-16	-30
10.75	-3.8	-3.8	-15	-20	-19	-21	-27
11	-4.2	-3.8	-16	-20	-20	-24	-23
11.25	-3.8	-4.0	-20	-20	-20	-22	-27
11.5	-3.7	-3.7	-15	-20	-20	-20	-45
11.75	-3.9	-3.8	-35	-20	-20	-21	-30
12	-3.9	-3.9	-22	-20	-20	-22	-23

The bandwidth of the interferometer in Figure 3.1 (connected to the microwave test set) was measured using a HP8756A network analyzer. By comparing a reference line to the transmitted power in Figure A.3, the bandwidth of the interferometer was measured. The bandwidth was found from the two frequency points 3 dB below the reference line. The 345 MHz measured bandwidth limits the rise time to about 2 ns. To increase the bandwidth of the interferometer, an equal length of WR90 waveguide is required in the reference channel to balance out the dispersion in the test channel.

Figure A.2 S Parameters for the Interferometer

Figure A.3 Interferometer Bandwidth

APPENDIX B

Estimate of Phase Error

The phase angle was estimated by calculating the difference between the exact phase angle and the estimated phase angle with white noise. The exact phase angle was calculated as a function of the interference voltage and test voltage with the reference voltage constant. The interference voltage was varied from -17.8 to +17.8 volts. The test voltage was varied from -8.9 to +8.9 volts (Equations 2.9 and 2.10). The reference voltage was held constant at 8.9 volts. The estimated phase angle was calculated with a noise term (rms voltage of 1.26 volts, i.e. a noise power of 32 mW) added to both the interference and test voltages. The rms error between the exact and the estimated phase angle is 16 Degrees.

Figure B.1 shows the exact phase angle and B.2 shows the estimated phase angle. Figure B.3 shows the phase error. For small phase shifts, the error is less than 10° .

Figure B.1
Exact Surface Plot
in Degrees

Figure B.2
Estimated Surface Plot
in Degrees

APPENDIX C

WR90 Rectangular Waveguide

Appendix C gives the specifications for WR90 rectangular waveguide. Figure C.1 shows the actual size of WR90 waveguide. Figure C.2 shows the S matrix for a Magic Tee and the port notation.

Table C.1

WR90 Waveguide Specifications

Frequency Range in GHz	8.2 to 12.4
Dimensions in cm	2.286 by 1.016
Cutoff Frequencies for the two lowest modes in GHz	$f_{10} = 6.56$ $f_{20} = 13.1$

Figure C.1

WR90 Dimensions

$$[S] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Figure C.2

Magic Tee Port Notation and Scatter Matrix

APPENDIX D

Propagation Function in Rectangular Waveguide^{1, 2}

The derivation for the propagation function for the three frequency regions defined in Chapter 2 Table 2.2 is contained in Appendix D. The difference between free space and rectangular waveguide is the addition of the cutoff frequency term.

CASE I $\omega_p \gg v \gg \omega$

$$\gamma^2 = j\omega\mu_0\sigma - \omega\mu_0\epsilon_0 + \left(\frac{\pi}{a}\right)^2$$

$$\gamma^2 = j\omega\mu_0\left(\frac{\omega_p^2\epsilon_0}{v + j\omega}\right) - \omega^2\mu_0\epsilon_0 + \left(\frac{\pi}{a}\right)^2$$

$$\gamma^2 = j\omega\frac{\omega_p^2}{c^2(v + j\omega)} - \frac{\omega^2}{c^2} + \left(\frac{\pi}{a}\right)^2$$

$$\gamma^2 \cong j\frac{\omega\omega_p^2}{c^2v} - \frac{(2\pi)^2}{\lambda^2} + \frac{(2\pi)^2}{(2a)^2} = j\frac{\omega\omega_p^2}{c^2v} - \frac{(2\pi)^2}{\lambda^2} + \frac{(2\pi)^2}{(\lambda_c)^2}$$

$$\gamma^2 \cong j\frac{\omega\omega_p^2}{c^2v} + \left[-\frac{(2\pi)^2}{\lambda^2} + \frac{(2\pi)^2}{\lambda_c^2}\right] = j\frac{\omega\omega_p^2}{c^2v} + \left[\frac{\omega_c^2}{c^2} - \frac{\omega^2}{c^2}\right]$$

$$\gamma^2 \cong j\frac{\omega\omega_p^2}{c^2v} - \left(\frac{2\pi}{\lambda}\right)^2 \left[1 - \left(\frac{\lambda}{\lambda_c}\right)^2\right]$$

¹William A. Elmore and Mark A. Heald. The Physics of Waves, New York. Dover, 1985 page 302.

²J. B. Gerardo and M. A. Gusinow. Electronic Recombination of He⁺₃, Phys. Rev. A225-6 (1971).

$$\gamma^2 \cong j \frac{\omega \omega_p^2}{c^2 v} \left[1 - \frac{\omega v}{\omega_p^2} \left(1 - \left(\frac{\lambda}{\lambda_c} \right)^2 \right) \right]$$

$$\gamma^2 \cong j \frac{\omega \omega_p^2}{c^2 v} [1]$$

Therefore $\boxed{\gamma \cong \sqrt{\frac{\omega}{2v}} (1+j) \left(\frac{\omega_p}{c} \right)}$

CASE II $v \ll \omega \ll \omega_p$

$$\gamma^2 = j\omega\mu_o\sigma - \omega\mu_o\epsilon_o + \left(\frac{\pi}{a} \right)^2$$

$$\gamma^2 = j\omega\mu_o \left(\frac{\omega_p^2 \epsilon_o}{v + j\omega} \right) - \omega^2 \mu_o \epsilon_o + \left(\frac{\pi}{a} \right)^2 \cong \omega_p^2 \epsilon_o \mu_o - \omega^2 \epsilon_o \mu_o + \left(\frac{\pi}{a} \right)^2$$

$$\gamma^2 \cong \frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2} + \left(\frac{2\pi}{2a} \right)^2 = \frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2} + \frac{\omega_c^2}{c^2}$$

$$\gamma^2 \cong \frac{\omega_p^2}{c^2} + \frac{\omega_c^2}{c^2} \quad \text{since } \omega_p \gg \omega > \omega_c$$

Therefore $\gamma \cong \frac{\omega_p}{c}$

CASE III $\omega \gg \omega_p$ and $\omega \gg v$

$$\gamma^2 = j\omega\mu_o\sigma - \omega\mu_o\epsilon_o + \left(\frac{\pi}{a} \right)^2$$

$$\gamma^2 = j\omega\mu_o \left(\frac{\omega_p^2 \epsilon_o}{v + j\omega} \right) - \omega^2 \mu_o \epsilon_o + \left(\frac{\pi}{a} \right)^2 \cong \omega_p^2 \epsilon_o \mu_o - \omega^2 \epsilon_o \mu_o + \left(\frac{\pi}{a} \right)^2$$

$$\gamma^2 \cong \frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2} + \left(\frac{2\pi}{2a} \right)^2 = \frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2} + \frac{\omega_c^2}{c^2}$$

$$\gamma^2 \cong \frac{-\omega^2}{c^2} [1 - \omega_p^2 - \omega_c^2] \quad \text{since } \omega \gg \omega_p \text{ and } \omega \gg \omega_c$$

Therefore

$$\boxed{\gamma \cong j \frac{\omega}{c}}$$

APPENDIX E

Complex Conductivity Derivation¹

The complete derivation for the complex conductivity inside a rectangular waveguide is given in Appendix E. The authors assume: TE₁₀ mode, uniform plasma, complex conductivity completely describes the filling medium, and the nonzero complex conductivity does not perturb the electric field distribution inside the waveguide.

$$\Gamma^2 = \left[j\omega\mu_o(\bar{\sigma} + j\omega\epsilon_o) + \left(\frac{\pi}{a}\right)^2 \right] = (\alpha + \beta)^2 \quad (A4)$$

$$j\omega\mu_o\sigma_r - \omega^2\mu_o\sigma_i - \omega^2\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

$$j\omega\mu_o\bar{\sigma} - \omega^2\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

$$\text{Re} \left[j\omega\mu_o\bar{\sigma} - \omega^2\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 \right] = \alpha^2 - \beta^2 \quad \text{Im} \left[j\omega\mu_o\bar{\sigma} - \omega^2\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 \right] = j\alpha\beta$$

$$\omega\mu_o\sigma_i = \beta^2 - \alpha^2 - \omega\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 = \beta^2 - \alpha^2 - \left(\omega^2\mu_o\epsilon_o - \left(\frac{\pi}{a}\right)^2 \right)$$

$$\text{define } \beta_o \equiv \beta|_{\bar{\sigma}=0} \quad \text{gives} \quad j\omega\mu_o\sigma_r - \omega\mu_o\sigma_i - \omega^2\mu_o\epsilon_o + \left(\frac{\pi}{a}\right)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

$$\beta_o \equiv \beta|_{\bar{\sigma}=0} = \sqrt{\omega^2\mu_o\epsilon_o - \left(\frac{\pi}{a}\right)^2}$$

$$\text{Therefore} \quad \omega\mu_o\sigma_i = \beta^2 - \alpha^2 - \left(\omega^2\mu_o\epsilon_o - \left(\frac{\pi}{a}\right)^2 \right) = \beta^2 - \alpha^2 - \beta_o^2 \quad (A5)$$

¹ J. B. Gerado and M. A. Gusinow. Electronic Recombination of He⁺₃, Phys. Rev. A225-6 (1971).

Imaginary Part of A4

$$\text{Im} \left(j\omega\mu_o\sigma_r - \omega\mu_o\sigma_i - \omega^2\mu_o\varepsilon_o + \left(\frac{\pi}{a}\right)^2 \right) = \omega\mu_o\sigma_r = 2\alpha\beta$$

Define $\Delta\Phi^\circ$ as the phase shift measured by the interferometer in degrees.

$$\Delta\Phi^\circ = \frac{180}{\pi}(\beta_o - \beta)\ell \quad (\text{A6})$$

Define ΔdB as the measured attenuation in dB.

$$\Delta\text{dB} \equiv 10\log(e^{2\alpha\ell}) = 8.686\alpha\ell \Rightarrow \alpha = \frac{\Delta\text{dB}}{8.686\ell}$$

Substituting A6 into A5 gives $\omega\mu\sigma_r = 2\alpha\beta$

Define Θ as
$$\Theta \equiv \frac{\Delta\Phi^\circ \lambda_{go}}{360\ell} \quad \text{where } \lambda_{go} \text{ is the TE}_{10} \text{ guide wavelength}$$

Therefore
$$\beta = \frac{2\pi}{\lambda_{go}} \left(1 - \frac{2\pi\Delta\Phi^\circ}{360} \lambda_{go} \right) = \frac{2\pi}{\lambda_{go}} (1 - \Theta)$$

Substituting into $\text{Im}\{A4\}$ gives $\text{Re}\{\sigma\}$

$$\sigma_r = \frac{2\alpha\beta}{\omega\mu_o} = \frac{2}{\omega\mu_o} \left(\frac{\Delta\text{dB}}{8.686\ell} \right) \left(\frac{2\pi}{\lambda_{go}} \right) (1 - \Theta)$$

$$\sigma_r = \frac{4\pi^2\varepsilon_o}{\omega\mu_o\varepsilon_o} \left(\frac{0.115}{\lambda_{go}} \right) \left(\frac{\Delta\text{dB}}{\pi\ell} \right) (1 - \Theta)$$

$$\sigma_r = \frac{2\pi\varepsilon_o c^2}{c} \frac{\lambda}{\lambda_{go}} (0.115) \left(\frac{\Delta\text{dB}}{\pi\ell} \right) (1 - \Theta)$$

$$\sigma_r = 2\pi\varepsilon_o f\lambda \frac{\lambda}{\lambda_{go}} (0.115) \frac{\Delta\text{dB}}{\pi\ell} (1 - \Theta)$$

$$\sigma_r = \omega\varepsilon_o \frac{\lambda^2}{\lambda_{go}} (0.115) \left(\frac{\Delta\text{dB}}{\pi\ell} \right) (1 - \Theta)$$

$$\sigma_r = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}} (0.115) \left(\frac{\Delta dB}{\pi \ell} \right) (1 - \Theta) \quad (A7)$$

To determine the imaginary part of the complex conductivity.

$$\omega \mu_o \sigma_i = \beta^2 - \alpha^2 - \beta_o^2 = (\beta - \beta_o)(\beta + \beta_o) - \alpha^2$$

$$\text{from } \Delta \Phi^\circ = \frac{180}{\pi} (\beta - \beta_o) \ell \Rightarrow (\beta - \beta_o) = \frac{\pi}{180} \Delta \Phi^\circ = \frac{-2\pi}{360\ell} \Delta \Phi^\circ \frac{\lambda_{go}}{\lambda_{go}} = \frac{-2\pi}{\lambda_{go}} \Theta$$

$$\beta + \beta_o = \beta - \beta_o + 2\beta_o = \frac{-2\pi}{\lambda_{go}} \Theta + 2 \left(\frac{2\pi}{\lambda_{go}} \right) = \left(\frac{2\pi}{\lambda_{go}} \right) (-\Theta + 2)$$

$$\omega \mu_o \sigma_i = \beta^2 - \alpha^2 - \left(\omega^2 \mu_o \epsilon_o - \left(\frac{\pi}{a} \right)^2 \right) = \beta^2 - \alpha^2 - \beta_o^2 = (\beta - \beta_o)(\beta + \beta_o) - \alpha^2 \quad (A5)$$

$$\omega \mu_o \sigma_i = \left(\frac{-2\pi}{\lambda_{go}} \Theta \right) \frac{2\pi}{\lambda_{go}} (-\Theta + 2) - \left(\frac{\Delta dB}{8.686\ell} \right)^2$$

$$\sigma_i = \frac{\frac{4\pi^2}{\lambda_{go}^2} (\Theta - 2\Theta) - \left(\frac{\Delta dB}{8.686\ell} \right)^2}{\omega \mu_o} = \frac{4\pi^2 \epsilon_o}{\omega \mu_o \epsilon_o^2 \lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta dB \lambda_{go}}{8.686\ell 2\pi} \right)^2 \right]$$

$$\sigma_i = \frac{4\pi^2 \epsilon_o c^2}{2\pi f \lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta dB \lambda_{go}}{8.686\ell 2\pi} \right)^2 \right] = \frac{4\pi^2 \epsilon_o c^2 f}{2\pi f^2 \lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta dB \lambda_{go}}{8.686\ell 2\pi} \right)^2 \right]$$

$$\sigma_i = \frac{2\pi f \epsilon_o c^2}{f^2 \lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta dB \lambda_{go}}{8.686\ell 2\pi} \right)^2 \right]$$

$$\sigma_i = \omega \epsilon_o \frac{\lambda^2}{\lambda_{go}^2} \left[\Theta^2 - 2\Theta - \left(\frac{\Delta dB \lambda_{go}}{8.686\ell 2\pi} \right)^2 \right] \quad (A7)$$

APPENDIX F

Diode Calibration Curves at 10 GHz

Appendix F contains the calibration curves for the MA/COM diodes and the Narda diode. The input powers as a function of output voltages are given in Tables F.1 and F.2. The diodes were all calibrated at 10.0 GHz.

Figure F.1 MA/COM Diode Output Voltages as a Function of Input Powers

Figure F.2 MA/COM Diode Input Powers as a Function of Output Voltages

Table F.1
MA/COM Diode Input Powers as a Function of Output Voltages

$$\text{test}(v) := -24.2089 + 9.72000 \log(v) + 8.895 \cdot 10^{-4} \cdot v^2 - 5.5500 \cdot 10^{-6} \cdot v^3 \text{ dBm}$$

$$\text{reflect}(v) := -24.08335 + 9.6879 \log(v) + 8.3967 \cdot 10^{-4} \cdot v^2 - 5.18266 \cdot 10^{-6} \cdot v^3 \text{ dBm}$$

$$\text{reference}(v) := -24.5143 + 9.5498 \log(v) + 8.4936 \cdot 10^{-4} \cdot v^2 - 5.112064 \cdot 10^{-6} \cdot v^3 \text{ dBm}$$

Figure F.3 Narda Diode Output Voltage as a Function of Input Power

Figure F.4 Narda Diode Input Power as a Function of Output Voltage

Table F.2
Narda Diode Input Power as a Function of Output Voltage

$$\text{inter}(v) := -15.4863 + 10.5919 \log(v) + 0.090158v - 4.5337 \cdot 10^{-4} \cdot v^2 + 9.9473 \cdot 10^{-7} \cdot v^3 \text{ dBm}$$

APPENDIX G

Propagation Delay Measurements

The propagation delays to the INTERFERENCE, TEST, and REFLECTION ports were measured. Figure G.1 shows the set up to measure the time delay from the interference port to the reflection and test ports. Figure G.3 shows the hardware used to generate impulses at a low repetition rate. Table G.1 shows the propagation delay from the fiducial to the test and interference ports. Output from the test port occurs 1 ns before the interference port.

Table G.1

Propagation Delays

Port Name	Fiducial to Pulse Output	Time Difference from Interference Port
Test Port	6.8 ns	-1.0 ns
Interference Port	7.8 ns	0.0 ns
Reflection Port (see text)	6.2 ns	-1.6 ns

Figure G.2 shows the waveguide separated at the Absolute Power Reference Plane (the center of the waveguide). An impulse was fed into port 6 in Figure G.2. The propagation delay from the absolute power reference plane to the test and interference ports was measured. An impulse was also fed into port 7 in Figure G.2 to measure the propagation delay from the Absolute Power Reference Plane to the reflection port. Table G.2 shows the results. The delay for the reflection port in Table G.2 is only for a wave traveling from the APRP to the reflection port. This does not take into account the round

trip for the reflected wave (forward wave traveling through the waveguide, scattering, changing directions, and traveling to the reflection port). If the forward wave were to scatter off of a moving step change in the dielectric constant, the correction for the propagation delay to the reflection port would be a function of time. By inserting a shorting bar at the APRP in Figure G.1, a 4.5 ns delay was measured. By inserting a shorting bar between magic Tee 1 and the forward coupler, a 7.9 ns delay was measured. The average of 7.9 and 4.5 is 6.2 ns. Using 6.2 ns as the delay, the output from the reflection port occurs 1.6 ns before the interference port.

Table G.3 summarizes the propagation delays and the timing corrections used for the longitudinal cases (rounded off to the nearest 0.5 ns). Table G.2 summarizes the delays for the transverse cases. The value given for the reflection port is only an approximation.

Table G.2

Propagation Delays from the APRP to the Test, Interference, and Reflection Ports

Port Name	APRP to Output	Delay from Interference to Port
Test	5.0	-1.0
Reflection	2.8	-3.2
Interference	6.0	0

Table G.3

Timing Correction Factors

Port Name	Delay from the Interference to Port in ns
Test	-1.0
Interference	0.0
Reflection	-1.6

Figure G.1 Relative Timing Measurement

Figure G.4 Relative Timing Diagram

Figure G.2 Absolute Timing Measurement

Figure G.3 Impulse Generator

Figure G.5 Absolute Timing Diagram

The time delay from the APRP to the interference port is 6.0 ns (Table G.2).

Since the laser pulse arrives at the APRP at 6.2 ns, the output signal will not reach the interference port until 12.2 ns. Figure 3.12, 3.14, and 3.15 show this in more detail. For the laser beam transverse to the waveguide, the output from the reverse wave (reflection) port occurs 3.2 ns before the interference port.

APPENDIX H

Breakdown Threshold for 1 Atmosphere Air

The lower limit for laser induced ionization is estimated. The mechanism for the threshold in air with aerosols is modeled by Grigor'ev *et al.*

We now examine the mechanism proposed in Ref. 9 for the breakdown of air by laser radiation with $\lambda \approx 1 \mu\text{m}$ in more detail. According to this mechanism, vaporization of suspended particles and ionization of the molecules of which they are composed are caused by laser radiation. At the initial moment the product electrons have energies $\leq 1 \text{ eV}$ and cannot immediately ionize the gas molecules in the air. Accelerated in the field of the light wave, the electrons efficiently excite molecular vibrations in nitrogen and oxygen. Excited molecules have a higher polarizability than the unexcited ones, so that the refractive index of the gas in the focal region is enhanced with a gradient directed toward the axis of the beam. This causes self focusing of the laser light, which leads to breakdown of the air.¹

For an interaction length (caustic) of approximately 1 m, the threshold for breakdown² is approximately $4 \times 10^9 \text{ W/cm}^2$ or $4 \times 10^{13} \text{ W/m}^2$. For dry air³, the breakdown is approximately $7 \times 10^{16} \text{ W/m}^2$. The breakdown in laboratory air is approximately 10^3 times lower than dry air.

According to Grigor'ev *et al.*, the ionization threshold in laboratory air is low. For interaction lengths of approximately 10 cm or more, the threshold is $< 10^{10} \text{ W/cm}^2$. Figure H.1 shows the breakdown threshold according to Grigor'ev *et al.* The intensities for 5 different laser energies are shown in Figure H.2 as a function of laser beam diameter. The intensity as a function of the distance from the focal point is also shown in Figure H.2.

¹ F. V. Grigor'ev, et al, "Modeling the Mechanism for Air Breakdown by 1 Micron Laser Radiation," Sov. Phys. Tech Phys. 31 1 (1986): 70.

² Ibid.

³ George Bekefi. Principles of Laser Plasmas. (New York: Wiley, 1976) 462-466.

Figure H.1 Threshold versus Caustic Length⁴

The aerosols provide the seed electrons for the ionization. The seed electrons and the self focusing effect, seed the avalanche process⁵. The threshold for multi-photon ionization is much greater than the avalanching process.

The threshold for dry air is calculated below for 1.064 μm ⁶. A 1 Joule Laser Energy is used for the calculations below. The pulse width is 100 ps full width half maximum. Equation G.1 gives the threshold, I_{Bd} , for avalanche ionization. Equation G.2 gives the ionization rate.

$$I_{\text{Bd}} = \frac{800}{P t_p \lambda^2} (1 + 4.5 \times 10^{-6} P^2 \lambda^2) (1 + 2 \times 10^8 t_p P) \quad (\text{G.1}) (11.14)$$

$$I_{\text{Bd}} \cong \frac{707}{10^{-10}} (1 + 4.5 \times 10^{-6} (1.064)^2) (1 + 2 \times 10^8 \times 10^{-10}) \cong 7 \times 10^{12} \text{ W/cm}^2 = 7 \times 10^{16} \text{ W/m}^2$$

$$\text{ionization rate } \nu_i \cong \frac{e^2 I}{\epsilon_0 c m_e \phi \left[1 + \left(\frac{\omega}{\nu_c} \right)^2 \right]} \text{ 1/s} \cong 3.4 \times 10^{11} \text{ 1/s} \quad (\text{G.2}) (11.12)$$

⁴ Grigor'ev, et al. op. cit.

⁵ Ibid.

⁶ George Bekefi. op. cit.

Figure H.2 Laser Breakdown

Assuming :

$\nu_e \cong 10^{12}$ 1/s in laboratory air,
 $\omega = 1.4 \times 10^{15}$ Rad/s (laser frequency), and
 $\phi = 10$ eV ionization potential for Nitrogen

the natural log of the electron density enhancement is 34.

$$\ln\left(\frac{n}{n_0}\right) \cong \nu_i t_p = 34 \quad (\text{G.3})$$

The electron density enhancement is $\frac{n}{n_0} \cong 6 \times 10^{14}$. For moderately low seed densities, electron densities on the order of 10^{17} to 10^{18} 1/m³ may be produced.

APPENDIX I

Estimate of the Overlap Integral

For a non-uniform charge distribution, the Equations A7a and A7b give the spatially integrated complex conductivity¹. The overlap integral allows the complex conductivity at a point to be approximated from the charge distribution and the electric field distribution inside the waveguide. No measurements of the electron density were made in this experiment. To approximate the order of magnitude of the overlap integral, a simple charge distribution is assumed. The distribution is assumed to have a skin depth on the order of $10\text{ }\mu\text{m}$. The charge distribution for this model² is shown in Figure I.1.

Figure I.1

Electron Density inside Rectangular Waveguide

¹J. B. Gerado and M. A. Gusinow. "Electronic Recombination of He_3^+ ." Phys. Rev. A265-6 (1971).

²P. Bolton. Private Communication. LLNL, 1993.

Equation I.1 shows the overlap integral³.

$$\langle \sigma_{r,i} \rangle = \frac{\iint_{\text{plasma}} \sigma_{r,i} E_y^2 dx dy}{\iint_{\text{waveguide}} E_y^2 dx dy} \quad (\text{I.1}) \text{ A11}$$

Assume $\sigma_{r,i}$ is uniform at some fixed position z_0 over the cross section of the plasma.

$$\text{then } \langle \sigma_{r,i} \rangle \equiv \sigma_{r,i} \frac{\int_{\text{plasma}} E_y^2 dx}{\int_{\text{waveguide}} E_y^2 dx} \cdot \frac{d}{b} \quad (\text{I.2})$$

Assume the electric field inside the plasma has the form in Equation I.3.

$$E_{y \text{ plasma}} \equiv E_{y0 \text{ wg}} e^{\frac{-x}{\delta}} \quad \text{for} \quad \frac{a}{2} \geq x \geq \frac{a}{2} - \frac{d}{2} \quad (\text{I.3})$$

The skin depth was calculated from Equation I.4.

$$\delta \equiv \frac{1}{\sqrt{f \mu_0 \sigma_r}} \quad \text{where} \quad \sigma_r = \frac{n_e e^2}{v^2 + \omega^2} v \quad e = 1.602 \times 10^{-19} \text{ coul} \quad (\text{I.4})$$

Assuming $d = 500 \mu\text{m}$, and E_{y0} is constant over 'd', the overlap integral was estimated.

³J. B. Gerardo. op cit.

$$\int_{\text{plasma}} E_{y0}^2 dx \cong 2E_{\text{max}}^2 \int_0^{\frac{d}{2}} e^{\frac{-2x}{\delta}} dx = E_{\text{max}}^2 \delta \left(1 - e^{\frac{-d}{\delta}} \right) \quad (\text{I.5})$$

$$\int_{\text{waveguide}} E_{\text{max}}^2 \sin^2 \left(\frac{\pi}{a} x \right) dx = E_{\text{max}}^2 \frac{a}{2} \quad (\text{I.6})$$

$$\langle \sigma_{r,i} \rangle \cong \sigma_{r,i} \frac{\int_{\text{plasma}} E_y^2 dx}{\int_{\text{waveguide}} E_y^2 dx} \cdot \frac{d}{b} \cong \sigma_{r,i} \frac{\frac{d}{b} \delta \left(1 - e^{\frac{-d}{\delta}} \right)}{\frac{a}{2}} = \sigma_{r,i} \frac{2d}{ab} \delta \left(1 - e^{\frac{-d}{\delta}} \right) \quad (\text{I.7})$$

For this model, the complex conductivity is approximated by Equation I.8.

$$\sigma_{r,i} \cong \frac{ab}{2d\delta} \frac{\langle \sigma_{r,i} \rangle}{\left(1 - e^{\frac{-d}{\delta}} \right)} \quad (\text{I.8})$$

$$\text{OI} \equiv \frac{\langle \sigma_{r,i} \rangle}{\sigma_{r,i}} \cong \frac{2d\delta}{ab} \left(1 - e^{\frac{-d}{\delta}} \right) \cong \frac{2(500 \times 10^{-6})(10^{-5})(1 - e^{\frac{-500 \times 10^{-6}}{10 \times 10^{-6}}})}{(0.0228)(0.0102)} \cong 3.3 \times 10^{-6} \quad (\text{I.9})$$

$$\text{EF} = \frac{1}{\text{OI}} \cong \frac{ab}{2d\delta} \frac{1}{\left(1 - e^{\frac{-d}{\delta}} \right)} \cong \frac{(0.0228)(0.0102)}{(2)(500 \times 10^{-6})(10^{-5})(1 - e^{\frac{-500 \times 10^{-6}}{10 \times 10^{-6}}})} = 3 \times 10^5 \quad (\text{I.10})$$

Therefore the Overlap Integral is on the order of 10^{-5} for the example values in Table I.1. The enhancement factor is $1/\text{OI}$. For the assumed charge distribution, the complex conductivity is on the order of 10^5 times the spatially integrated complex conductivity from Equations 2.8a and 2.8b.

Table I.1
Example Values

electron density	$n_e = 10^{25} \text{ 1/m}^3$
collision frequency	$\nu \cong 5 \times 10^{11} \text{ 1/s}$
skin depth	$\delta \cong 10^{-5} \text{ m}$
width 'd'	$d = 500 \text{ }\mu\text{m}$

APPENDIX J

Low Directivity Couplers in Plasmas

Figure J.1 shows the reverse wave or reflected wave coupler from the microwave test set. The directivity is limited to 20 dB. A plasma inside the waveguide can decrease only the forward wave power. If the forward and reverse wave powers are momentarily equal and in phase, a sudden increase in coupled power will occur. If the forward and reverse waves are out of phase, a sudden drop in power will occur¹. These coherent interference effects are exhibited as short spikes on the diode signals.

A HP8756A network analyzer was used to measure the forward and reverse coupling values. The forward and reverse coupling values as a function of frequency are shown in Figure J.2 and Table J.1. A pulsed directivity measurement was made using a 1 ns pulse multiplied by a 10 GHz sine wave. Table J.2 shows the "pulsed" directivities for a low directivity coupler and a high directivity coupler. Figure J.2 and Table J.1 also show the coupling values for the high directivity coupler.

¹P. Bolton. Private Communication., 1993

Table J.1
Coupler Frequency Responses

Frequency GHz	High Directivity (HD) Forward dB	High Directivity (HD) Reverse dB	Low Directivity (LD) Forward dB	Low Directivity (LD) Reverse dB
8	-9	-27	-4	-13
8.4	-10	-31	-6	-17
8.8	-10	-33	-8	-21
9.2	-11	-34	-12	-20
9.6	-11	-35	-9	-27
10	-11	-43	-10	-31
10.4	-11	-44	-15	-27
10.8	-10	-36	-10	-23
11.2	-10	-34	-10	-16
11.6	-11	-32	-7	-15
12	-11	-27	-11	-14

Table J.2
Coupler Directivities

	HP8756A Network Analyzer @ 10.0 GHz	Pulse Measurement
Low Directivity Coupler (Reverse Wave Coupler in the microwave system)	21 dB	9 dB
High Directivity Coupler (Forward Wave Coupler in the microwave system)	32 dB	>> 10 dB*

*Reverse coupled pulse was below the noise floor.

Figure J.2 Coupler Frequency Responses

APPENDIX K

Results for the Transverse Case Not Hitting the Waveguide Wall

The length dependent results for the transverse case not hitting the waveguide wall are presented in Appendix K. The Figures K.1.X are for an interaction length of 1 mm. The Figures K.2.X are for an interaction length of 2 mm. The first four graphs for the 1mm and 2mm geometries are NOT length dependent.

Figure K.1.1 Absolute Forward Power in mW

Figure K.1.2 Absolute Reverse Power in mW

Figure K.1.3 Forward Wave Attenuation in dB

Figure K.1.4 Forward Wave Phase Shift in Degrees

Figure K.1.5 Attenuation Divided by the Interaction Length

Figure K.1.6 Real Part of the Spatially Integrated Complex Conductivity

Figure K.1.7 Imaginary Part of the Spatially Integrated Complex Conductivity

Figure K.1.8 Real Part of the Spatially Integrated Dielectric Function

Figure K.1.9 Imaginary Part of the Spatially Integrated Dielectric Function

Figure K.1.10 Collisionality from the Spatially Integrated Complex Conductivity

Figure K.1.11 Spatially Integrated Electron Density

Figure K.2.1 Absolute Forward Power in mW

Figure K.2.2 Absolute Reverse Power in mW

Figure K.2.3 Forward Wave Attenuation in dB

Figure K.2.4 Forward Wave Phase Shift in Degrees

Figure K.2.5 Attenuation Divided by the Interaction Length

Figure K.2.6 Real Part of the Spatially Integrated Complex Conductivity

Figure K.2.7 Imaginary Part of the Spatially Integrated Complex Conductivity

Figure K.2.8 Real Part of the Spatially Integrated Dielectric Function

Figure K.2.9 Imaginary Part of the Spatially Integrated Dielectric Function

Figure K.2.10 Collisionality from the Spatially Integrated Complex Conductivity

Figure K.2.11 Spatially Integrated Electron Density

APPENDIX L

Frequency Response of the Microwave Test Set

Figure L.1 shows the S parameter Port definitions for the microwave test set.

Figure L.2 shows the S parameters for the ports in Figure L.1 over 8 to 12 GHz.

Figure L.2 S Parameters for the Microwave Test Set

APPENDIX M

Transmitted and Incident Laser Energy

The incident laser energy is the input energy into the waveguide. The transmitted laser energy is the energy exiting the waveguide. The laser wavelength and the full width half maximum pulse width are $1.064\text{ }\mu\text{m}$ and 100 ps respectively. The difference in laser energy is the total energy dissipated. This dissipated laser energy is the sum of the absorption and off axis scattering effects. Absorption accounts for the initial ionization and acceleration of the free electrons. Figures M.1 and M.2 show the transmitted energy versus the incident laser energy for the laser beam longitudinal to the waveguide and not hitting the waveguide wall. Figures M.3 and M.4 are for the laser beam transverse to the waveguide and not hitting the waveguide wall. The different symbols are used to show several transmitted laser energies for the same incident laser energy.

Figure M.1 Transmitted Laser Energy versus Incident Laser Energy

Figure M.2 Transmitted Laser Energy versus Incident Laser Energy

Figure M.3 Transmitted Laser Energy versus Incident Laser Energy

Figure M.4 Transmitted Laser Energy versus Incident Laser Energy

END

DATE

FILMED

5 / 4 / 94