TECHNICAL PROGRESS REPORT

by

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ELECTRON ACCELERATION BY LASER FIELDS IN A GAS

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In the period covered by this report, that is since the grant renewal date of June 1, 1992, work has proceeded as proposed in two closely related areas:

- (a) Collaboration with STI Optronics and with the Accelerator Test Facility at the Brookhaven National Laboratory. Both the 50 Mev electron accelerator and the short-pulse CO₂ laser have become operational, and preliminary tests have already been performed involving electrons and shaped laser beams inside a gas cell. The Principal Investigator of this grant has been in continuous interaction with STI and ATF staff and this collaboration has resulted in the publications and presentations listed below.
- (b) Independent theoretical work has been performed on the topic of interaction region synthesis and on FEL-type devices involving a gas medium and no wiggler structures. Appendixes I and II at the end of this report present some of the results.

Meetings attended and papers presented by the Principal Investigator:

- 1. Third Workshop on Advanced Accelerator Concepts, in Port Jefferson, Long Island, N.Y., June 14-20, 1992.
- 2. XV International Conference on High Energy Accelerators, in Hamburg, Germany, July 20-24, 1992. Paper presented there: "Latest Developments in Inverse Cherenkov Laser Accelerator Experiments and Theory," by J. R. Fontana, W. D. Kimura, I. Pogorelsky, and L. C. Steinhauer, published in Conference Proceedings, Int. J. Mod. Phys. A (Proc. Suppl.) 2A, (1993), p. 517-518.
- 3. Particle Accelerator Conference, Washington, D.C., May 17-20, 1993. Paper presented there: "Design Analysis for a 100 Mev Inverse Cherenkov Laser Accelerator," by J. R. Fontana, W. D. Kimura, I. Pogorelsky, and L. C. Steinhauer, to be published in the Conference Proceedings.
- 4. Center for Accelerator Physics Steering Committee Meeting, Brookhaven National Laboratory, May 24, 1993, and visit to the ATF facilities and staff.

STI Papers containing an acknowledgment to the Principal Investigator:

- 1. "A New Approach for Laser Particle Acceleration in Vacuum," by L. C. Steinhauer and W. D. Kimura, Jour. of Appl. Phys., 72 (8), October 15, 1992, pp. 3237-3245.
- 2. "Multistaging and E-Beam Trapping in Laser Particle Accelerators," by L. C. Steinhauer and W. D. Kimura, AIP Conf. Proc. 279, 1993, pp. 539-550.
- 3. "Multistaging in Free-Space Laser Particle Accelerators," by L. C. Steinhauer and W. D. Kimura, submitted to the Jour. of Appl. Phys.

APPENDIX I

CHERENKOV FREE ELECTRON LASERS

by J. R. Fontana, Principal Investigator

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Summary

In ordinary FEL's, electrons are made to interact with laser fields by a resonant transverse motion which creates a ponderomotive potential moving at a longitudinal velocity near the electron velocity, so energy transfer can occur. A similar equivalent potential can be produced in an optical waveguide propagating a longitudinal electric field mode and containing a dielectric to reduce the wave phase velocity to that of the electrons. Schemes involving solid dielectrics near the guide walls have fields at the electron position attenuated relative to those in the dielectric. The Cherenkov FEL (CFEL) described below uses instead an electron beam moving within a gaseous dielectric inside a guide propagating a mode with a longitudinal electric field highly concentrated on the guide axis where the beam is located.

The interaction process can be visualized in terms of the cone of Cherenkov radiation emitted by fast charged particles in a gas: the process is the same but stimulated by the waveguide fields which, for the mode chosen, consist of a cone of plane wave components converging on the electron beam. The effect of collisions between electrons and gas molecules is important in determining the available interaction region length, which in turn limits the realizable laser gain despite the fact that, for a given length, the gain of CFEL's is significantly larger than for wiggler FEL's.

I. Principles of Operation

In a gas-filled cylindrical waveguide propagating a TM_{0m} mode, i.e., a mode without circumferential variations, the fields can be written, in cylindrical coordinates (1):

$$E_z = E_0 J_0(nkr \sin \theta) \cos \psi \quad (1)$$

$$E_r = \frac{-E_0}{\tan \theta} J_1(nkr \sin \theta) \sin \psi \quad (2)$$

$$H_\phi = \frac{n}{\cos \theta} \sqrt{\frac{\epsilon_0}{\mu_0}} E_r \quad (3)$$

$$\psi = \omega t - (nk \cos \theta) z + \text{constant} \quad (4)$$

where θ depends on the mode, E_0 indicates its amplitude, $k = 2\pi/\lambda$ with λ the vacuum wavelength, and n is the refractive index of the gas at λ . The z -axis is the guide axis.

Physically, these fields can be interpreted as resulting from a continuous distribution of plane waves converging on the guide axis, all at the same angle θ . From Eq. (4), the phase velocity can be written as $\beta_w c$ with

$$c = (\epsilon_0 \mu_0)^{-1/2} \text{ and } \beta_w = (n \cos \theta)^{-1}. \quad (5)$$

An electron moving along the axis with velocity βc is acted on by only a longitudinal force

$$F_z = -e E_0 \cos \psi \quad (6)$$

$-e$ being the electron charge, so its energy, expressed in terms of $\gamma = (1 - \beta^2)^{-1/2}$ increases according to

$$mc^2 \frac{d\gamma}{dz} = F_z \quad (7)$$

with m the electron rest mass.

It is apparent from the last two Eqs. that the electrons see a ponderomotive potential similar to that in a planar or helical FEL:

$$V(\psi) = \int F_z(\psi) d\psi = -e E_0 \sin \psi \quad (8)$$

and that an energy exchange can happen.

To study this process, we consider first a uniform axial filamentary electron beam injected at $z = 0$ with velocity $\beta_0 c$ interacting with the TM_{0m} mode above. Following the standard FEL analysis procedure (2), we first consider the energy gain of the electrons assuming their velocity is unaffected by the fields. An electron entering the interaction region at $z = 0$ with initial phase ϕ will see at position z a field phase given by

$$\psi(z) = \left(\frac{1}{\beta_0} - n \cos \theta \right) kz + \phi. \quad (9)$$

So

$$\frac{d\gamma(z)}{dz} = -\frac{eE_0}{mc^2} \cos(\Delta kz + \phi) \quad (10)$$

where

$$\Delta k = \left(\frac{1}{\beta_0} - n \cos \theta \right) k = (\beta_0^{-1} - \beta_w^{-1}) k. \quad (11)$$

When the electron β_0 exactly matches the wave phase velocity β_w we have the Cherenkov condition

$$\beta_0 = (n \cos \theta)^{-1} \quad (12)$$

under which each electron sees the same phase along its path: $\Delta k = 0$. The θ given by Eq. (12) is the angle of the radiation cone emitted by an electron moving at $\beta_0 c$ in a gas of index n . For small θ and $\beta_0 \equiv 1$, Eq. (12) becomes

$$n - 1 \equiv \frac{\theta^2}{2} + \frac{1}{2\gamma_0^2} \quad (13)$$

and

$$\Delta k \equiv \left[\frac{\theta^2}{2} + \frac{1}{2\gamma_0^2} - (n - 1) \right] k \quad (14)$$

where $\gamma_0 = (1 - \beta_0^2)^{-1/2}$. With an unbunched electron beam, all phases ϕ are equally present and the overall beam energy gain at any z , obtained by averaging Eq. (10) over ϕ , is zero.

We now consider the evolution of the β of each electron due to the change in its γ :

$$\begin{aligned} \beta(z) &\equiv \beta_0 + \frac{\gamma(z) - \gamma_0}{\gamma_0^3} = \beta_0 + \frac{1}{\gamma_0^3} \int_0^z \frac{d\gamma(z)}{dz} dz \\ &= \beta_0 - \frac{eE_0}{mc^2 \gamma_0^3 \Delta k} [\sin(\Delta kz + \phi) - \sin \phi]. \end{aligned} \quad (15)$$

Because of the continuously changing β , the electron at any time advances over its formerly assumed position z by an amount

$$\Delta z = c \int_0^L [\beta(z) - \beta_0] dz = \frac{-eE_0}{mc^2 \gamma_0^3 (\Delta k)^3} [\cos \phi - \cos(\Delta k z + \phi) - (\Delta k \sin \phi) z] \quad (16)$$

which is equivalent to a phase increase

$$\Delta \phi = -nk \cos \theta \Delta z. \quad (17)$$

Entering this $\Delta \phi$ as a perturbation in Eq. (10), keeping only first order terms in E_0 and averaging over all phases ϕ gives the average electron energy gain $\langle \Delta \gamma \rangle_\phi$ at $z = L$, end of the interaction region. We obtain:

$$\langle \Delta \gamma \rangle_\phi \equiv - \frac{e^2 E_0^2 nk L^3}{\gamma (mc^2)^2 \beta_0^3 \gamma_0^3 \cos \theta} \frac{d}{d\alpha} \left(\frac{\sin^2 \alpha}{\alpha^2} \right) \Bigg|_{\alpha = \frac{\Delta k L}{2}} \quad (18)$$

The gain for an average beam current I during the pulse is given by:

$$G = \frac{-\langle \Delta \gamma \rangle_\phi mc^2 I / e}{P} \quad (19)$$

where P is the power propagated in the guide (3):

$$P = \frac{\lambda R (\epsilon_0 / \mu_0)^{1/2} E_0^2}{2\pi \tan \theta \sin^2 \theta} \quad (20)$$

with R the guide radius. So

$$G \equiv \left(\frac{4\pi^2 e}{\beta_0^4 c^4} \right) \left(\frac{\mu_0}{\epsilon_0} \right)^{1/2} \left(\frac{nk}{\cos \theta} \right) \left(\frac{IL^3}{R\lambda^2} \right) \frac{\sin^2 \theta \tan \theta}{\gamma_0^3} \times \frac{d}{d\alpha} \left(\frac{\sin^2 \alpha}{\alpha^2} \right) \Bigg|_{\alpha = \frac{\Delta k L}{2}} \quad (21)$$

For small θ , the maximum gain, obtained for $\Delta k L \cong 1.3$, can be written in MKS units:

$$G_{\max} \cong 2 \times 10^{-3} \left(\frac{IL^3}{R\lambda^2} \right) \left(\frac{\theta}{\gamma_0} \right)^3 \quad (22)$$

In a beam of finite thickness, off-axis electrons see smaller longitudinal forces because of the J_0 dependence in Eq. (1). The function is flat near the axis but changes sign at

$$r = r_0 = 0.383 \lambda / n \sin \theta. \quad (23)$$

In addition, there is a radial force:

$$F_r = -e(E_r - \beta c \mu_0 H_\phi) = e \tan^2 \theta E_r = -e E_0 \tan \theta J_1(nkr \sin \theta) \sin \psi. \quad (24)$$

Fig. 1 shows the dependence on ψ of F_z and F_r for electrons near the axis. In the region $0 < \psi < \pi$, i.e. around the bottom of the ponderomotive potential $V(\psi)$, these forces are focusing and tend to counteract the gas collision scattering discussed below. But outside this region they will contribute to beam divergence. However, for typical values of field amplitude, radial field forces are small compared with scatter and can be neglected.

At relativistic energies, elastic collision scattering is the predominant effect of the gas. After a distance L , this scattering produces a Gaussian angular distribution characterized by a mean square angle (4)

$$\langle \theta_s^2 \rangle = \left(\frac{E_s}{m \gamma \beta^2 c^2} \right)^2 \frac{L}{X_0} \quad (25)$$

where E_s is a constant and X_0 is the radiation length of the medium, inversely proportional to pressure for a gas.

The deflection of the electrons in any transverse direction x is also a Gaussian with

$$\langle x^2 \rangle = \frac{\langle \theta_s^2 \rangle L^2}{6}. \quad (26)$$

Taking as an effective electron beam radius $r_b = \langle x^2 \rangle^{1/2}$ we have that after a distance z within the gas

$$r_b \cong \frac{15z^2}{\gamma X_0^{1/2}} \quad (27)$$

in MKS units for $\beta \cong 1$.

The index of refraction $n(\nu)$ varies sharply with the frequency ν near the electronic resonances of the molecules, as shown schematically on Fig. 2. At frequencies just below a resonance ν_0 the Cherenkov condition of Eq. (12) can be met at reduced gas pressure due to the enhanced value of n , thus increasing X_0 and decreasing r_b .

The amplifier described above can be made into an oscillator by the use of a resonator. The simplest is obtained by shorting both ends of the cylindrical waveguide at $z = 0$ and $z = L$. At optical frequencies (i.e. for R and $L \gg \lambda$), the longitudinal modes are separated by

$$\Delta_z \nu = \frac{c}{2Ln \cos \theta} \quad (28)$$

and the transverse modes by

$$\Delta_r \nu = \frac{c}{2Rn \sin \theta} \quad (29)$$

so $\Delta_z \nu \ll \Delta_r \nu$ if $R \tan \theta \ll L$.

For an oscillator operating near a gas resonance, mode selection may be provided by the gas itself. At other frequencies, mode-selection optics would be required.

II. Analysis

The gain for a wiggler FEL is given by (5):

$$G = \left(\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\pi e}{4mc^2} \right) \frac{IL^3 a_w^2 (1 + a_w^2)}{A \lambda \gamma^5} [J_0(\zeta) - J_1(\zeta)]^2 \frac{d}{d\alpha} \left[\frac{\sin^2 \alpha}{\alpha^2} \right] \quad (30)$$

where $a_w = \frac{e \Lambda B}{2\pi mc}$ with Λ being the wiggler period and B its rms value, $\zeta = \frac{a_w^2}{2(1 + a_w^2)}$. A is the electron beam plus the optical beam area. The other symbols are as defined above.

Compared with a vacuum wiggler FEL, the Cherenkov FEL just described presents a gain expression similar in being directly proportional to $IL^3 d(\sin^2 \alpha / \alpha^2) / d\alpha$. But the γ -dependence is as γ^{-3} instead of γ^{-5} . There is also a denominator factor of λR instead of the beam cross-sections. Both signify an important advantage in CFEL gain.

A major limiting feature is the occurrence of beam spread due to scatter. As the electron deflections are also to some extent determined by radial field forces determined by their phase, only numerical calculations can give an accurate picture for each case. In the two examples below, radial field forces have been neglected but the parameters have been chosen to keep the freely scattered beam radius r_b below the limit radius r_0 of Eq. (23).

This choice drastically limits the interaction length L and thus the gain, which is nevertheless found to be reasonable over a very short length.

Example 1: $\lambda = 100$ nanometers
 $\theta = 1.4 \times 10^{-2}$
 $L = 5$ centimeters, $R = 5$ millimeters
 $\gamma = 70$

From the θ and γ chosen, $n - 1 = 2 \times 10^{-4}$, which at the wavelength of operation, located near the 95 nanometer resonance of the H_2 molecule, can be obtained at a pressure of only 0.17 atmospheres, corresponding to a radiation length of 41×10^3 meters.

The values calculated are as follows:

$$r_0 = 2.7 \times 10^{-6}, \quad r_b = 2.7 \times 10^{-6}$$

and $G_{\max} / I = 4\% / \text{amp.}$

Example 2: $\lambda = 500$ nanometers
 $\theta = 2 \times 10^{-2}$
 $L = 5$ centimeters, $R = 5$ millimeters
 $\gamma = 70$

Now $n - 1 = 3 \times 10^{-4}$, requiring at this wavelength a pressure of 2.15 atmospheres of H_2 , corresponding to a radiation length of only 3.2×10^3 meters.

We get now:

$$r_0 = 9.5 \times 10^{-6}, \quad r_b = 9.4 \times 10^{-6}$$

and $G_{\max} / I = 0.44\% / \text{amp.}$

III. Optical Klystron Operation

It results from Section II that devices of this sort are effective as bunchers. When the velocities of the fields and the injected electrons are the same, i.e. when $\Delta k = 0$, the beam becomes bunched due to the simultaneous operation of two distinct processes. Referring to Fig. 1, it can be seen that electrons will tend to drift towards a phase $\psi = \pi / 2$, corresponding to the bottom of the ponderomotive potential curve, under the longitudinal force F_z , as discussed in Section II. But another effect will also occur

provided the field amplitude is large enough and the interaction region sufficiently long. Electrons with $0 < \psi < \pi$ will be forced towards the axis by the radial force F_r , while the others will be pushed away and become lost to the interaction process. To be effective, the radial forces must be strong enough to counteract the scattering spreading of the beam, a condition very different from that assumed in the small signal examples of Section II. Computer simulations are necessary to study quantitatively the combined effect of collision scatter and radial forces, but the already available results of calculations performed for electron accelerators based on the same principle show that focusing forces can indeed constrain the beam over large distances despite scatter. (6)

Bunched beams obtained by this method can be used to generate power at the same wavelength by means of another gas-filled interaction region, also with $\Delta k = 0$ but phase-shifted so the electron bunches now are in the $-\pi/2 < \psi < \pi/2$ region. The $\psi < 0$ portion of the bunches will again be lost because of defocusing forces, but the remaining electrons will both be focused and decelerated, thus yielding energy to the fields. In the first order, the last mentioned energy exchange will be the only one to happen, as bunch formation by electron loss requires no net energy.

The power delivered to the fields can be calculated from the first order expression of Eq. (10) without invoking the phase correction of Eq. (17) by performing the averaging integration over ϕ only from 0 to $\frac{\pi}{2}$ instead of over the full 2π range. As $\Delta k = 0$,

$$\gamma(L) - \gamma(0) = \frac{-eE_0 L \cos \phi}{mc^2} \quad (31)$$

where L is the length of the electron-decelerating section.

So

$$\langle \gamma(L) - \gamma(0) \rangle_{0 < \phi < \pi/2} = \frac{-eE_0 L}{2\pi mc^2} \quad (32)$$

and the power gained by the fields is

$$\Delta P = -\langle \gamma(L) - \gamma(0) \rangle_{0 < \phi < \pi/2} mc^2 = \frac{E_0 L I}{2\pi}. \quad (33)$$

References

1. J. R. Fontana and R. H. Pantell, J. Appl. Phys. 54 (8), August, 1983.
2. See For Example Amnon Yariv, "Quantum Electronics," Third Edition, John Wiley, N.Y., 1989, Chapter 13.
3. See Reference 1, Eq. (13) with $L = 2R / \tan \theta$ for a waveguide.
4. B. Rossi, "High-Energy Particles," Prentice-Hall, N.Y., 1952, Chapter 2.
5. R. H. Pantell, "Free Electron Lasers," Beijing Institute of Modern Physics Series," Vol. 2, World Scientific Publishing Co., Singapore, 1989, p. 16.
6. J. R. Fontana, "Inverse Cherenkov Acceleration," AIP Conference Proceedings No. 130, 1985, and J. R. Fontana and G. S. Atukorala, "Simulation Studies in Gas Collision Effects in Inverse Cherenkov Accelerators," SPIE Conference Proceedings, Vol. 664, 1986.

Figure Captions

- Fig. 1: Longitudinal and transverse forces, F_z and F_r respectively, on an electron near the z-axis. The direction of motion of both the fields and the electrons (shown as dots) is towards the right, i.e. towards $+z$, so the fields phase ψ increases towards the left, as shown. The ponderomotive potential curve $V(\psi)$ moves to the right.
- Fig. 2: Index of refraction $n(\nu)$ of a gas near an electronic resonance ν_0 . Only in the range of frequencies where the curve is above β_0^{-1} can the Cherenkov condition be met.

FIGURE 1

FIGURE 2

APPENDIX II

INTERACTION REGION SYNTHESIS IN VACUUM ACCELERATORS

by J. R. Fontana, Principal Investigator

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Vacuum charged-particle accelerators of the far-field type must rely on fields made up of plane wave components, without low phase velocity evanescent waves. Furthermore, at very high energies, the particles travel at almost constant velocity and must keep to rectilinear trajectories to avoid synchrotron radiation loss. The design of suitable interaction fields to accelerate such particles is essentially an antenna problem. As a plane wave interacting with a constant velocity, straight trajectory particle cannot impart net energy over an infinite distance, the interaction region must be limited by terminating it with mirrors to reflect the fields or by equivalent arrangements. The general properties of antenna systems designed to give prescribed fields from a finite aperture have been studied by various authors. (1), (2), (3)

It is possible to synthesize a given longitudinal field distribution seen by a moving particle over a finite, straight trajectory by means of plane wave components having axial symmetry, all of the same wavelength but of varying intensity, phase and angle of incidence. An approach to generating such a pattern is by means of a holographic grating (4). The method described here uses instead a space Fourier integral to provide a general solution to the problem, as well as to determine which patterns are physically realizable.

Consider an electron with charge $-e$ moving parallel to the z -axis with approximately constant speed βc in vacuum. A laser field consisting of a set of plane waves converges on the z -axis with their k -vectors forming a cone at angle θ . As the phase velocity of this field is $c / \cos \theta > \beta c$, a dephasing relative to the electron will necessarily occur. If all the plane waves have their E -component in the plane of their k -vector and the z -axis, a longitudinal force will act on the electron, varying sinusoidally with the electron z -position.

We want to use a laser field consisting of a continuous distribution of such conical components, all with different angles θ . The electron will then see a force varying in z with a non-sinusoidal dependence. The purpose of this paper is to investigate the relationship between this force $F_z(z)$ acting on the electron at each position z and the amplitude and phase characteristics of the corresponding laser cone components as a function of their angle θ .

Each cone with θ will be seen by the electron with a phase

$$\psi(\theta, z) = \omega t - (k \cos \theta) z + \psi_0(\theta) = k \left(\frac{1}{\beta} - \cos \theta \right) z + \psi_0(\theta) \quad (1)$$

where $\psi_0(\theta)$ is the phase at $z = 0$. The force on the electron (in the z -direction) is:

$$F_z(\theta, z) = -eE_0(\theta)J_0(kr \sin \theta) \exp[i\psi(\theta, z)] \quad (2)$$

where $E_0(\theta)$ is the peak axial field on the z -axis due to the cone with θ . The space wavelength seen by the electron is:

$$\Lambda(\theta) = \frac{\lambda}{1/\beta - \cos \theta} \quad (3)$$

which corresponds to a space angular frequency (radians/meter)

$$\Omega(\theta) = k \left(\frac{1}{\beta} - \cos \theta \right). \quad (4)$$

Any realizable $F_z(z)$ is given by a superposition of cones with different θ 's having a suitable amplitude and phase distribution.

On the axis ($r = 0$):

$$F_z(z) = -e \int_0^\pi E_0(\theta) \exp \left[i \left(k \left(\frac{1}{\beta} - \cos \theta \right) + \psi_0(\theta) \right) \right] d\theta. \quad (5)$$

Let $F_z(\Omega)$ be the Fourier Transform of $F_z(z)$:

$$F_z(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_z(z) e^{-i\Omega z} dz \quad (6)$$

$$F_z(z) = \int_{\theta=0}^{\pi} F_z(\Omega) e^{i\Omega z} d\Omega \quad (7)$$

So

$$-eE_0(\theta) \exp i\psi_0(\theta) d\theta = F_z[\Omega(\theta)] d\Omega = F_z[\Omega(\theta)] k \sin \theta d\theta. \quad (8)$$

Knowing $F_z(z)$ we can thus determine $F_z(\Omega)$ and $E_0(\theta) \exp i\psi_0(\theta)$.

As $\Omega = k \left(\frac{1}{\beta} - \cos \theta \right)$ the limits of Ω must correspond to those of θ : 0 and π .

For $\theta = 0$ (waves in the $+z$ direction):

$$\Omega_{\min} = k \left(\frac{1}{\beta} - 1 \right) \cong \frac{k}{2\gamma^2} \text{ for } \gamma \gg 1. \quad (9)$$

For $\theta = \pi$ (waves in the -z direction):

$$\Omega_{\max} = k \left(\frac{1}{\beta} + 1 \right) \cong 2k. \quad (10)$$

As $\Omega = 0$ is not possible, $F_z(\Omega)|_{\Omega=0} = 0$ and so $\int_{-\infty}^{\infty} F_z(z) dz = 0$ which is in agreement with simple intuitive arguments. This is over an infinite range; if we limit the interaction region to a finite length L we can have a non-zero integral. It may be done by removing the electron from the fields by passing it through holes in mirrors, and limiting the laser waves range, which means dealing with diffraction but resulting in a finite laser power requirement. (It would be of course infinite for an infinite region).

In this case, the power required for the cone at θ is;

$$P(\theta) = \left[\frac{E_0(\theta)}{2(\mu_0/\epsilon_0)^{1/4} \pi^{1/2} \sin \theta} \right]^2 \lambda L = [\text{constants}] F_z[\Omega(\theta)] k^2 \lambda L. \quad (11)$$

The total power required is then:

$$P_{\text{tot}} = \int_0^\pi P(\theta) d\theta = \left(\frac{8.34 \times 10^{-3}}{e^2} \right) \left(\frac{L}{\lambda} \right) \int_0^\pi |F_z[\Omega(\theta)]|^2 d\theta \quad (12)$$

in MKS units.

A figure of merit for a given configuration relates this power to the electron energy gain:

$$\frac{\Delta W}{P_{\text{tot}}} = \frac{e^2 \int_0^L E_z(z) dz}{(8.34 \times 10^{-3}) (L/\lambda) \int_0^\pi |F_z[\Omega(\theta)]|^2 d\theta} \quad (13)$$

(MKS)

As an example, meant to illustrate this method, suppose we want

$$F_z(z) = F_0 e^{-(z/z_0)^2}. \quad (14)$$

Then

$$F_z(\Omega) = \frac{z_0}{2\pi^{1/2}} e^{-(z_0\Omega/2)^2}. \quad (15)$$

This $F_z(\Omega)$ is however impossible to obtain in view of the limits of Ω . Let us use an approximate one, equal to Eq. (15) within these limits and zero outside. The now realizable $F_z(z)$ will differ from Eq. (14) but not very much if some realistic conditions are met.

- (1) $F_z(\Omega) = 0$ for $\Omega > \Omega_{\max}$ will give a fast ripple with period $< \frac{\lambda}{2}$ on top of the Gaussian.
- (2) $F_z(\Omega) = 0$ for $\Omega < \Omega_{\min}$ will add a function

$$F'_z(z) \equiv \left(\frac{F_0 z_0 \Omega_{\min}}{\pi^{1/2}} \right) \frac{\sin(\Omega_{\min} z)}{\Omega_{\min} z}. \quad (16)$$

The effect will be small if $z_0 / \lambda \ll \gamma^2$ because the first zero will be larger than z_0 and the amplitude small.

Then, using the chopped-off $F_z(\Omega)$:

$$\Omega = k \left(\frac{1}{\beta} - \cos \theta \right) \equiv k \left(\frac{1}{2\gamma^2} + \frac{\theta^2}{2} \right) \quad (17)$$

$$d\Omega = k \sin \theta d\theta \equiv k \theta d\theta. \quad (18)$$

These equations require a small θ , but it is apparent from Eq. (15) that only small θ 's will give a significant contribution:

$$F_z[\Omega(\theta)] \equiv \frac{F_0 z_0}{2\pi^{1/2}} \exp \left\{ - \left[\pi \left(\frac{z_0}{\lambda} \right) \left(\frac{1}{2\gamma^2} + \frac{\theta^2}{2} \right) \right]^2 \right\}. \quad (19)$$

This gives:

$$E_0(\theta) = \frac{F_0 z_0}{-2e\pi^{1/2}} \exp \left\{ - \left[\frac{\pi}{2} \left(\frac{z_0}{\lambda} \right) \left(\frac{1}{\gamma^2} + \theta^2 \right) \right]^2 \right\} k \theta \quad (20)$$

and

$$\psi_0(\theta) = 0 \quad (\text{real function}) \quad (21)$$

So

$$P_{tot} \equiv (4.45 \times 10^{-4}) L z_0^{3/2} \lambda^{-1/2} \left(\frac{F_0}{e} \right)^2 \quad (\text{MKS}) \quad (22)$$

and

$$\Delta W_{(ev)} \equiv 84 \frac{(\lambda / z_0)^{1/4}}{(L / z_0)^{1/2}} P_{tot}^{1/2} \quad (\text{watts}) \quad (23)$$

With $\lambda = 10^{-5} m$, $P_{tot} = 10^{14}$ watts
 $z_0 = 10 cm$ and $L = 3z_0 = 30 cm$

$\Delta W \cong 48 \text{ Mev.}$

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