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NEW APPROACHES TO LINEAR
AND NONLINEAR PROGRAMMING

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1 Project Description

This report describes technical progress during the past eight months on DOE Contract DE-FG03-92ER25117. The project involves study of the theoretical properties and computational performance of algorithms that solve linear and nonlinear programs. Particular emphasis is placed on algorithms to solve large problems. Such problems are especially important in the energy area. For example, the safe and efficient distribution of electricity and the identification of the state of an electrical network are both large-scale nonlinearly constrained optimization problems. Other application areas in which we are actively involved include optimal trajectory calculations and optimal structural design.

2 Summary

Highlights of this year's research follow.

- A new second-derivative method for unconstrained optimization has been formulated [FGM93]. The method is based on finding a partial Cholesky factorization of the Hessian. The new method is one of the most efficient available for practical computation.
- An important question has been resolved concerning the stability of quasi-definite factorizations in barrier methods for linear programming [GSS93].
- A new SQP algorithm has been developed for large sparse problems [GMS93]. The application of this new algorithm to the important class of *trajectory optimization* problems has invoked considerable interest in the U.S. and European aerospace communities.

3 Review of Progress

Here we enlarge on each of the above items.

3.1 Second-derivative methods

With the advent of several successful automatic differentiation packages, second derivatives can be simply defined in many important applications. If second derivatives are available, Newton's method is extremely effective for finding a minimizer of a strictly convex twice-continuously differentiable function. This has prompted the proposal of various *modified* Newton methods for the nonconvex case. The ones we have studied utilize a linear combination of a descent direction and a direction of negative curvature. If these directions are *sufficient* in a certain sense, and a suitable linesearch is used, the resulting methods will generate limit points that satisfy the second-order necessary conditions for optimality.

In [FGM93], a new method for sufficient directions is proposed, based on a partial Cholesky factorization of the Hessian. The research was motivated by the need for an

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algorithm with the efficiency and simplicity of the Cholesky factorization, but with the guarantee of convergence when used in conjunction with a suitable linesearch. The proposed factorization is the first to provide theoretically satisfactory directions with the expense of only a *partial* pivoting strategy (i.e., the equivalent of only two rows of the Hessian need be examined at each step).

3.2 SQP methods for large-scale problems

Our work has continued on the development of new sequential quadratic programming (SQP) algorithms for large-scale nonlinear programming.

A key matrix in SQP methods is the so-called *transformed Hessian*, which is a (dense) matrix of the form Q^THQ where H is the Hessian of the Lagrangian and Q is nonsingular. Following on the work of Eldersveld [Eld91], partially funded by this grant, we have been concerned with methods that form a quasi-Newton approximation of a principal submatrix of the transformed Hessian.

In practice, many rows and columns of H are zero because certain *slack variables* appear only linearly in the Lagrangian. A crucial step in the formulation of efficient large-scale SQP methods has been identification of the structure of Q^THQ when linear variables are present. Knowledge of this structure makes it possible to store second-derivative information associated with the nonlinear variables only.

These ideas have been formulated in a new SQP algorithm that treats the Jacobian as a sparse matrix, and works with a submatrix of the transformed Hessian. The dimension of this submatrix can be chosen to reflect the amount of storage available. Under quite weak conditions, the method is globally convergent, i.e., the method will converge to an optimal point regardless of the starting point. An experimental implementation has solved some very difficult moderate-sized problems.

A paper describing the method was presented at the Fourth Stockholm Optimization Days, Royal Institute of Technology, August 1993, Stockholm.

The application of SQP methods to the important class of *trajectory optimization* problems has invoked considerable interest in both the U.S. and the European aerospace communities (see [GMS93]). SQP methods developed by the Investigators are in constant use by NASA and its various subcontractors. For example, our optimizer NPSOL was recently used to calculate the optimal reentry trajectory for the DC-Y single-stage-to-orbit vehicle being developed for NASA by McDonnell Douglas. The flight path included a 180° rotation of the vehicle just prior to landing. This is the first time that optimal trajectories have been computed for this type of maneuver. The computed trajectory resulted in an estimated reduction of 40% in the fuel needed for reentry and landing.

During the period covered by this progress report, lectures on the application of SQP methods to trajectory optimization were given at USAF Wright Laboratories, Wright-Patterson AFB, Dayton, Ohio; NASA Langley, Langley AFB, Virginia; McDonnell Douglas Aerospace, Huntington Beach, California; European Space Agency, Institute for Flight Mechanics, University of Stuttgart, Germany; and DVL, Oberpfaffenhofen, Germany.

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3.3 Interior methods for linear and quadratic programming

Almost all the work in interior methods lies in solving the linear system that defines the search direction. There has been significant investigation into how the equations may be solved efficiently. For linear programs, these KKT systems are usually reduced to smaller positive-definite systems $AH^{-1}A^Tq = s$, where H is a large principal submatrix of K . These systems can be solved more efficiently, but $AH^{-1}A^T$ is typically more ill-conditioned than K .

Two recent approaches define specially structured reduced systems $Ry = r$ that are not positive definite. The first, proposed by Vanderbei [Van91], is intended to provide a so-called *quasi-definite* system, which can be factorized with only 1×1 pivots and no row or column interchanges. The second, proposed by the Investigators [GMPS93], gives a reduced system whose dimension and condition lie somewhere in between those of K and $AH^{-1}A^T$.

For quasi-definite matrices R , the Cholesky factorization $PRP^T = LDL^T$ is known to exist for any permutation P , even though D has both positive and negative eigenvalues. In [GSS93] it is shown that the stability of this Cholesky factorization is naturally connected with the stability of the LDM^T factorization of a closely related unsymmetric positive-definite matrix. Conditions are given that allow the stable numerical solution by Gaussian elimination without further permutations.

This result was presented at the 12th Householder Symposium on Numerical Linear Algebra at Lake Arrowhead, California, 1993.

3.4 Other work

An intriguing new simplex-type method for linear programming was developed in [CPS93]. This bears resemblance to Lemke's method, but is at least four times as efficient in terms of work per iteration. A large-scale implementation seems warranted.

A revision of our large-scale optimization system MINOS was implemented and documented [MS93]. Important features include new Crash algorithms for obtaining starting solutions; use of a function-only linesearch when gradients are available (but expensive); and the ability to call MINOS as a subroutine. This version has been incorporated as the primary nonlinear optimizer in the algebraic modeling system AMPL [FGK93], which (like its earlier counterpart GAMS) opens the door to large-scale optimization throughout business and industry.

4 Publications

The following papers were referred to above. They have been published by the Principal Investigators during the review period.

- [CPS93] H. Chen, P. M. Pardalos and M. A. Saunders (1993). The simplex algorithm with a new primal and dual pivot rule, Report SOL 93-5, Dept of Operations Research, Stanford University, CA. To appear in *Operations Research Letters*.
- [FGM93] A. Forsgren, P. E. Gill and W. Murray (1993). Computing modified Newton directions using a partial Cholesky factorization, Report SOL 93-1, Dept of Operations Research, Stanford University, CA. To appear in *SIAM J. on Scientific Computing*.
- [GMS93] P. E. Gill, W. Murray and M. A. Saunders (1993). Large-scale SQP methods and their application in trajectory optimization, in R. Bulirsch and D. Kraft (eds.), *Control Applications of Optimization*, International Series of Numerical Mathematics, Birkhauser, Basel, Boston, Stuttgart.
- [GMPS93] P. E. Gill, W. Murray, D. B. Ponceleón and M. A. Saunders (1993). Solving reduced KKT systems in barrier methods for linear programming, *Proceedings of 15th Biennial Conference on Numerical Analysis*, Research Notes in Mathematics, Longmans Publishing Co.
- [GSS93] P. E. Gill, M. A. Saunders and J. R. Shinnerl (1993). On the stability of Cholesky factorization for quasi-definite systems, Report SOL 93-4, Dept of Operations Research, Stanford University, CA. To appear in *SIAM J. on Matrix Analysis and Applications*.
- [MS93] B. A. Murtagh and M. A. Saunders (1993). MINOS 5.4 User's Guide, Report SOL 83-20R, Dept of Operations Research, Stanford University, CA.

References

- [Eld91] S. K. Eldersveld (1991). *Large-scale Sequential Quadratic Programming*, Ph.D. thesis, Report SOL 92-4, Dept of Operations Research, Stanford University, CA.
- [FGK93] R. Fourer, D. M. Gay and B. W. Kernighan (1993). *AMPL: A Modeling Language for Mathematical Programming*, The Scientific Press, South San Francisco.
- [Van91] R. J. Vanderbei (1991). Symmetric quasi-definite matrices, Report SOR 91-10, Dept of Civil Engineering and Operations Research, Princeton University, NJ.

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