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Designing SSC Quadrupole Supports To Minimize The Effects From Vibrational Noise *

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INTRODUCTION

G. Stupakov^[8] has shown theoretically that the emittance at the SSC should increase linearly with time and the seismic noise spectrum associated with quadrupole motion at the betatron frequency ~ 750 – $1,500$ Hz (depending on fractional machine tune). While the motion is also affected by overtones of the knockout frequencies, the frequencies are so high that the seismic noise becomes vanishingly small. Feedback control would be required to control emittance growth for a power spectrum in excess of 10^{-12} microns²/Hz, assuming unit transmission at the betatron knockout frequency through the quadrupole supports. At the 1991 Corpus Christi Workshop on Beam Dynamics, N. Dikanski predicted unacceptable emittance growths of minutes for the SSC collider in the absence of protective measures. This prediction was based on seismic measurements at UNK and INP^[6] showing an apparent peak in the seismic power spectrum in the 750–1,500 Hz range at a level of 10^{-8} – 10^{-9} microns²/Hz.

In view of this prediction a workshop^[1] was convened in February of 1992 to discuss vibrational issues. At this workshop G. Fischer referred participants to an early study^[2] based on the then best compilation from Aki and Richards^[3] of seismic measurements. Aki and Richards showed ambient ground noise for a generic site many orders of magnitude lower than the INP measurements for the 750–1,500 Hz range. Fischer referred to later extensive measurements in the US and USSR that had confirmed the Aki results and also showed that instrumental noise in the 750–1,500 Hz region could dominate measurement precision. Later measurements made by the Russian group at the SSC site^[7] measure quiet noise spectra of 10^{-13} – 10^{-14} microns²/Hz five orders of magnitude lower than the original values. Under noisy conditions measurements indicate that culturally induced vibrations^[9] might still lead to marginal emittance growth, assuming unit transmission in the relevant frequency range, and 100% efficient coupling of resonant modes to the beam. This is certainly an overestimate as relevant wavelengths are small compared with quadrupole dimensions.

We shall discuss simple methods of achieving transmissions substantially less than unity. The design criteria for quadrupole supports are twofold.

At frequencies below the betatron knockout frequency the induced beam motion is adiabatic and should not cause emittance growth. However if there is sizable seismic amplification within the structure there could be unacceptable movements of the beam centroids. Above ~ 20 Hz the noise spectrum drops rapidly and amplification at higher frequencies has minimal effect on centroid motion. The lowest vibration frequency should therefore be set higher than ~ 20 Hz to minimize adiabatic motions of the beams.

At the betatron knockout frequencies (750–1,500 Hz), vibrational noise causes emittance growth and it is desirable to minimize transfer functions subject to cost and design constraints.

Civil engineers associated with bridge or building designs have known for over 50 years that simple passive decouplers can qualitatively improve responses to vibrations. Such decouplers can in fact be springs fabricated of rubber or other material in the form of disks, or stacks of laminated disks. For the SSC we would propose the possibility of using hemispheric ball joints lined with a suitable material to provide the equivalent of a three dimensional spring.

The theory for transmission of vibrational noise through systems of decouplers, loads, and columns is analogous to the transmission of electric noise through networks of capacitors, resistors, inductances, and transmission lines. Commercial programs using finite element analysis are readily available to predict the performance of actual systems.

In principle, as with electrical AC-filter networks, it is possible to provide very substantial attenuation of vibrational noise. Perhaps the most extreme example of this is provided by the gravitational wave detector at Stanford University used in a search for gravitational wave-induced disturbances to a one-ton aluminum cylinder resonant at 1 kHz. The detector is located at ground level in the middle of the University campus. The mounting supports attenuate seismic and cultural noise

to a level below that caused by Brownian motion at a millidegree. This level of attenuation is tens of orders of magnitude lower than anything required at the SSC. The filter systems uses simple passive attenuators built from laminated steel and rubber disks.

At the SSC vibrational workshop it was suggested that the SSC provide rudimentary vibration isolation by connecting the quadrupoles to their mount structure and the mount structure to the ground through appropriate decouplers. A rule-of-thumb estimate^[9] based on the transmissibility of the power spectrum through a single oscillator gives

$$|Z(\omega)|^2 = \frac{\omega_0^4}{(\omega^2 - \omega_0^2)^2 + \omega^2 \Gamma^2}, \quad (1)$$

where $\omega_0/2\pi$ is the resonant frequency ν_0 of the system, $\omega/2\pi$ is the measurement frequency typically in the range of 10–100 Hz, and Γ is the damping factor.

For $\omega \gg \omega_0$ this goes over to

$$|Z(\omega)|^2 \simeq \frac{\omega_0^4}{\omega^4}, \quad (2)$$

and with ω corresponding to frequencies in the range 750–1,500 Hz this factor can be as small as 10^{-4} independent of Γ .

For transmission through n oscillators in the limit of high frequencies the fall off goes as $1/\omega^{2n}$.

Recent measurements at the SSC site^[7] comparing the noise power spectrum on the ground and at the cold mass appear to be in contradiction. The measurements show $|Z(\omega)|^2$ with amplification up to 16 converging to unit transmission at high frequencies. Possible explanations are that the high-frequency measurements are dominated by instrumental noise and therefore independent of physical location and/or the connections of the mounting structures to the ground are poorly

designed. Commercial equipment is readily available to induce strong variable-harmonic disturbances in structures and we suggest this be used in preference to reliance on the natural low-level white seismic noise spectrum. This would provide unequivocal transmission measurements and pinpoint design flaws in the mounting structures.

To understand the principles underlying the design of mounts we have analyzed simple models for vibration transmission. These models serve to simultaneously provide design criteria and scaling laws. Final actual designs should be validated both by computer modeling with a finite element code and by physical measurements.

EXAMPLES OF SIMPLE DECOUPLERS

Oscillators

We consider first the oscillator of Fig. 1. The oscillator is driven at an angular frequency ω , spring constant K , amplitude A_1 , and damping coefficient Γ .

Instantaneous displacements x_i are related to the A_i by

$$x_i = A_i e^{-i\omega t} . \quad (3)$$

The equation of motion of the mass m_2 is

$$m_2 \frac{d^2(x_2 - x_1)}{dt^2} + \Gamma \frac{d(x_2 - x_1)}{dt} + K(x_2 - x_1) = 0 , \quad (4)$$

or

$$-m_2 \omega^2 A_2 + K_1(A_2 - A_1) = 0 , \quad (5)$$

where $K_1 = K - i\omega\Gamma$.

This can be solved in terms of the transfer function $Z_2(\omega)$, defined as

$$Z_2(\omega) = \frac{A_2}{A_1} = \frac{K_1}{K_1 - m_2 \omega^2} . \quad (6)$$

The effective mass $m_1 + m_{2eff}$ seen by the drive force follows from Eq. (6) as

$$m_{2eff} \omega^2 A_1 = K_1(A_2 - A_1) , \quad (7)$$

giving

$$m_1 + m_{2eff} = m_1 + Z_2 m_2 . \quad (8)$$

Two or more linearly coupled oscillators

The problem of a series of coupled oscillators is most generally solved in matrix notation. The less general case of n oscillators stacked in series can be more conveniently solved in iterative steps as follows. Using Eq. (6) we find

$$Z_n(\omega) = \frac{K_n}{K_n - m_n \omega^2} , \quad (9)$$

with

$$m_{eff(n-1)}(\omega) = m_{n-1} + Z_n(\omega) m_n . \quad (10)$$

Then in successive steps

$$Z_{n-1}(\omega) = \frac{K_{n-1}}{K_{n-1} - m_{(n-1)eff} \omega^2} , \quad (11)$$

$$m_{eff(n-2)}(\omega) = m_{n-2} + Z_{n-1} m_{eff(n-1)}(\omega) , \quad (12)$$

and so on until the drive force at subscript 1 is reached.

The overall response function $Z(\omega)$ is given by

$$Z(\omega) = \frac{A_n}{A_1} = \frac{A_n}{A_{n-1}} \frac{A_{n-1}}{A_{n-2}} \dots \frac{A_2}{A_1}, \quad (13)$$

or

$$Z(\omega) = \prod_{i=2}^{i=n} Z_i(\omega). \quad (14)$$

Longitudinal Vibrations in columns

The SSC supporting structures for the triplet IR quads at the betatron knock-out frequencies are long compared to the wavelength of waves in the structure. The electrical analogue is a transmission line. The structures response to a driving force is fully described in terms of an effective mass $M_{eff}(\omega)$ and an overall amplification factor $Z(\omega)$. These quantities can again be iterated through a linear set of decouplers and resonant structures to obtain the overall system response.

To obtain $M_{eff}(\omega)$ and $Z(\omega)$ we first derive the traveling wave propagation in a column and then the standing wave solution with correct boundary conditions.

Propagation of longitudinal waves

For a column of mass-per-unit length of ρ , compressibility η , and longitudinal displacement x the force F generated at an interface at height s is

$$F = \eta \frac{\partial x}{\partial s}, \quad (15)$$

and the equation for wave propagation is

$$\rho \frac{\partial^2 x}{\partial t^2} = \eta \frac{\partial^2 x}{\partial s^2}, \quad (16)$$

equivalent to a wave propagation with velocity v_0 where

$$v = \sqrt{\frac{\eta}{\rho}}, \quad (17)$$

for steel $v \simeq 5,000$ m per second.

In the presence of damping we write^[10]

$$\eta = \eta_0 + i\omega\Gamma, \quad (18)$$

and in terms of the “undamped” velocity at low frequencies v_0 ,

$$\lambda = \frac{v_0}{2\pi\omega} (1 + i\omega\Gamma)^{1/2}. \quad (19)$$

Standing waves on a loaded column

Consider a column of height L loaded with a mass $RL\rho$. The column will have a standing wave pattern with the displacement at any point s given by

$$x(s) = A(s)e^{i\omega t} = A_u e^{i\omega t - \frac{2\pi s}{\lambda}} + A_d e^{i\omega t + \frac{2\pi s}{\lambda}}. \quad (20)$$

At the top of the column (at $s = 0$),

$$F = \eta \frac{\partial x}{\partial s} = RL\rho \frac{\partial^2 x}{\partial t^2}, \quad (21)$$

where R is the ratio of the load to column masses.

Normalizing to unit amplitude at the top of the column we get

$$A_d = \frac{1}{2} (1 + iC(\omega)), \quad (22)$$

and

$$A_u = \frac{1}{2} (1 - iC(\omega)), \quad (23)$$

with

$$C(\omega) = \frac{RL\lambda\omega^2}{2\pi v^2}. \quad (24)$$

In terms of these values

$$Z(\omega) = \frac{A_u + A_d}{A(L)} , \quad (25)$$

with

$$A(L) = A_u e^{-2\pi i L/\lambda} + A_d e^{2\pi i L/\lambda} , \quad (26)$$

and from definition $A_u + A_d = 1$.

Combining Eqs. (17), (21), (22) and (23) gives

$$M_{eff}(\omega) = i \frac{RL\rho Z(\omega)}{\eta(\omega)} (A_u e^{-2\pi i L/\lambda} - A_d e^{2\pi i L/\lambda}) . \quad (27)$$

The two complex quantities $Z(\omega)$ s and $M_{eff}(\omega)$ s are derived from an effective input load completely describe the response to a harmonic driving force.

RESULTS

Results scale as the ratio of load to weight of the supporting structure, and the length of the supporting structure (equivalent to the transit time through the structure).

From inspection of the accompanying figures we will show that the main requirement for qualitatively improved noise attenuation is the introduction of passive decoupling at the connection to ground and/or between the magnets and the support structure.

Wavelengths long compared to structure dimensions

The arc quads are supported by structures of the ~ 1 m, and at the betatron knockout frequencies (750–1,500 Hz) the wavelengths will be long compared to the structure sizes making transit time effects relatively unimportant. The mounts can then be modeled as a series of damped oscillators. Figure 2 shows three such

oscillators diagrammatically. For loads, M_{load} , supported by n steel and rubber disks with combined weights of M_0 per pair, n mode frequencies are distributed between ω_0 and ω_{max} .

For $M_{load} \gg nM_0$ the ω_0 of the lowest decoupler mode is

$$\omega_0 \simeq \sqrt{\frac{K}{nM_{load}}} , \quad (28)$$

where K is the decoupler constant of an individual disk and K/n is the effective decoupler constant for n decouplers in series.

The maximum mode frequency ω_{max} is given by

$$\omega_{max} \simeq \sqrt{\frac{2K}{M_0}} , \quad (29)$$

Thus

$$\frac{\omega_{max}}{\omega_0} \simeq \sqrt{\frac{2nM_0}{M_{load}}} . \quad (30)$$

Typical design constraints are for $\omega_0 > 20\text{Hz}$ and $\omega_{max} < 500\text{Hz}$ corresponding to

$$\frac{M_{load}}{M_{mount}} \simeq \frac{\omega_{max}^2}{2n^2\omega_0^2} < \frac{300}{n^2} . \quad (31)$$

For a relatively light mount with two damping decouplers at top and bottom

$$\frac{M_{load}}{M_{mount}} < 75 . \quad (32)$$

Figures 3 to 7 show representative cases with varying numbers of decouplers and load ratios consistent with maintaining $\omega_0 > 20$ Hz and substantial attenuations in the 750–1,500 Hz regions. The solid curves are for loaded mounts and the dashed curves are for unloaded but otherwise identical mounts.

As was expected, the response function falls as ω^{-2n} outside of the resonance region, where n is the number of decoupling stages. However, increasing the number of decoupling stages also moves the highest resonant frequency into the sensitive betatron knockout region. The best choice appears to be the use of two decouplers.

Wavelengths short compared to structure dimensions

The bridge support structures for the high-beta triplet quadrupoles will be ~ 10 m high. At the betatron knockout frequencies the system behaves like an unterminated electrical transmission line with resonances occurring beyond the highest frequencies of interest. However, base decoupling and/or decoupling between the quads and the top of the trestle still can provide large attenuations.

For vertically propagated longitudinal modes the lowest resonance is located at the first pole in $Z(\omega)$. The ratio of the amplitude of oscillation at the top of the column to the drive amplitude is of course $Z(\omega)$ and therefore from Eqs. (25) and (26) is given by

$$Z(\omega) = \frac{1}{A_u e^{-2\pi i L/\lambda} + A_d e^{2\pi i L/\lambda}} = \frac{1}{\cos(2\pi L/\lambda) - (RL\lambda\omega^2/2\pi v_0^2) \sin(2\pi L/\lambda)}, \quad (33)$$

and accordingly

$$\omega_{res} \simeq \frac{v}{(R + \frac{1}{2})^{1/2} L}. \quad (34)$$

Therefore R , the ratio of the load to column weight, must be less than a limiting value if the lowest resonant frequency is to be above a given value.

For steel $v_0 \simeq 5,000$ m/s and with $\nu_{res} > 30$ Hz and $L=10$ m $R < 7$. For $L=1$ m $R < 700$, a condition which is likely to be trivially met. For a structure through which both transverse and vertical vibrations are propagated, approximately one-third of the structure weight will be in vertical members, and the actual physical condition will be for $R < 2.5$, not 7.

Figures 8 through 12 show results for the response function for loaded columns of 10-m height. The dashed curves are evaluated without a decoupler between the

column and ground, and the solid curves are evaluated with a decoupler whose spring constant sets the lowest resonant frequency to 20 Hz. The results clearly show that attenuation in the 750–1,500 Hz region increases as R . Therefore to optimize the design R should be as large as possible, consistent with maintaining the lowest resonant frequency above 20 Hz, setting it in the range of 2–4.

For comparison Figs. 13 to 15 show results for 1-m arc quad mounts taking into account finite transit times. Again the solid curves are with base decoupling with a spring constant that is adjusted to put the lowest resonant frequency at 20 Hz. The dashed curves are without decoupling. The transmission line resonances are seen to be located at frequencies too high to cause significant emittance growth, and R can vary up to 700 (~ 200 for a practical structure). Again the higher the load ratio the better the attenuation in the 750–1,500 Hz range.

CONCLUSIONS

The physics underlying the response to vibration of quadrupole mounts imposes strong constraints on the optimized design.

Relevant wavelengths for the arc quadrupoles are large compared to the structure size, and their behavior is well-represented by the response of a stack of springs. It is desirable to choose decoupler spring constants so that the lowest system resonant frequency is above 10–20 Hz to prevent amplification in the range where the power spectrum is large. Then the number of decoupling stages that can be used is limited if resonances in the range 750–1,500 Hz are to be avoided. An optimum design would use single decouplers at the connections to ground and between the quadrupoles and support platforms.

The IR high-beta triplet quadrupoles are the components most sensitive to vibrational disturbance. Because the quadrupoles are mounted on a high bridge structure, transmission line resonances will occur above several hundred Hz. The sensitivity of the quadrupoles to movements, combined with the presence of resonances at high frequencies, will require careful design if the amplification or re-

sponse function $|Z|$ is to be kept below unity in the 750–1,500 Hz range. Again decouplers at the connections to ground and between the quadrupoles and mounting platform combined with a ratio of load to structure weights in the range of 2–4 will provide substantial attenuations at high frequencies. The use of higher load ratios would improve attenuations but would also unacceptably lower the lowest resonant frequency to below 20 Hz.

A comparatively low R , the ratio of load to structure weights, implies a relatively massive mounting structure that could be fabricated from steel girders or reinforced concrete girders.

A suggested design for the decouplers would use hemispheric ball joints lined with rubber or other suitable material to provide the required spring constants. These joints would serve to decouple motions in both the vertical and horizontal dimensions.

With the above parameters noise transmission in the 750–1,500 Hz range should be substantially less than unity for both arc and triplet quads, even while maintaining the lowest resonant frequency above 20 Hz.

If these design rules are followed in combination with standard good practice, cultural noise sources, and vibrational effects should affect SSC collider performance very minimally.

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10. This is the simplest form for damping. Damping terms proportional to ω/λ^{2n} may also be present.

FIGURE CAPTIONS

- 1) Diagram showing the conventions used for a single driven oscillator.
- 2) Diagram showing a decoupler consisting of three oscillators coupled in series.
- 3) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for one decoupler. The solid curve is for a ratio of M_{load} to M_{mount} of 100. The dashed curve is for no M_{load} . The solid curve falls as ω^{-2} .
- 4) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for two decouplers. The solid curve is for a ratio of M_{load} to M_{mount} of 100. The dashed curve is for no M_{load} .
- 5) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for four decouplers. The solid curve is for a ratio of M_{load} to M_{mount} of 10. The dashed curve is for no M_{load} .
- 6) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for nine decouplers. The solid curve is for a ratio of M_{load} to M_{mount} of 1. The dashed curve is for no M_{load} .
- 7) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for ten decouplers. The solid curve is for a ratio of M_{load} to M_{mount} of 1. The dashed curve is for no M_{load} .
- 8) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 10 m height for a ratio of M_{load} to M_{mount} of 0. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 9) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 10 m height for a ratio of M_{load} to M_{mount} of 1. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 10) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 10 m height for a ratio of M_{load} to M_{mount} of 7. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 11) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 10 m height for a ratio of M_{load} to M_{mount} of 25. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 12) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 10 m height for a ratio of M_{load} to M_{mount} of 100. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 13) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 1 m height for a ratio of M_{load} to M_{mount} of 10. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 14) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 1 m height for a ratio of M_{load} to M_{mount} of 100. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.
- 15) Response curves for $\log_{10}|Z|$ versus $\log_{10}\nu$ for a steel support column of 1 m height for a ratio of M_{load} to M_{mount} of 700. The solid curve is with a base decoupler and the dashed curve is without a base decoupler.

Fig. 1

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