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**Solving Linear Programs Under Uncertainty,
using
Decomposition, Importance Sampling and Parallel Processors**

Progress Report - March 1, 1994

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Planning under Uncertainty is a fundamental problem of decision science where solution could significantly advance man's ability to plan, schedule, design and control complex situations. Our goal is to develop efficient methods for solving an important class of planning problems, namely linear programs whose parameters (coefficients and right hand sides) are not known with certainty but are assumed to be known by their distributions. Stochastic Linear programs have greatest practical importance, especially in the field of energy planning, industry and finance. We have already solved large-scale practical planning problems of electric power planning, financial planning and industrial scheduling, taking into account millions of contingencies and demonstrated that our techniques led to more reliable and cost effective plans. Our research concentrated on theoretical tasks of decomposition and importance sampling techniques, implementation and software development issues and on applications. Major accomplishments of the second year of research are the following:

Dr. Gerd Infanger published his book "Planning under Uncertainty - Solving Large-Scale Stochastic Linear Programs", The Scientific Press Series, Boyd and Fraser.

David Morton finished his doctoral dissertation: "Algorithmic Advances in Stochastic Programming" in which he analyzed advanced decomposition techniques for a class of multi-stage stochastic programs which he applied to the hydro-electric system of PG&E. He also developed important convergence results and stopping rules for sampling based algorithms for stochastic programs.

Alamuru Krishna finished his doctoral dissertation: "Importance Sampling Techniques for Stochastic Linear Programming" in which he developed an importance sampling scheme based on a piece-wise linear approximation function and investigated an advanced Benders decomposition scheme for two-stage deterministic problems.

Professor George B. Dantzig and Dr. Gerd Infanger proposed new theories for a rigorous lower bound estimates which we are planning to develop further and to incorporate into our software for solving stochastic programs.

Gerd Infanger's book contains an extensive discussion of theory, implementation and numerical results of our methods for solving stochastic linear programs. It also shows how such programs can be solved using multiple parallel processors. Besides the two-stage case and a special class of multi-stage cases, Infanger developed the theory for solving a general class of multi-stage problems with dependency between the stochastic parameters within a stage and between stages. Two issues turned out to be critical for solving large-scale multi-stage problems with numerous scenarios per stage: Sharing of cuts between different scenarios of a stage and using a path importance sampling scheme for obtaining accurate upper bound estimates. The possibility of sharing of cuts is a necessary condition for being able to employ sampling techniques for solving the problem. Path sampling overcomes the difficulty associated with the exponential growth of scenarios with the number of stages in multi-stage problems. We are currently working on an implementation of our multi-stage results with the goal of developing a code for solving the most general class of multi-stage stochastic linear programming problems. The implementation is not yet finished, but is well under way. Its completion is a task of the ongoing research.

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Morton's and Krishna's results represent an important step in our ability of solving large-scale stochastic linear programs. The scope is demonstrated by numerical results obtained from various test problems.

In his thesis, David Morton investigated stopping rules and convergence properties for sampling based algorithms. He investigated the case where the difference between the upper and the lower bound is a normally distributed random variable or follows properties of the central limit theorem. His analysis provides conditions under which a desired confidence interval for the objective function value of the proposed solution is asymptotically valid, and a formula determining the size of the samples. Morton solved multi-stage stochastic hydro-electric scheduling problems that serve as subproblems in a stochastic hydro-thermal optimization model for Pacific Gas and Electric Company and investigated different solution (tree traversing) strategies for multi-stage stochastic linear programs. The best solution strategy included the use of warm starts, the recall of previous basis, advanced starting techniques (using cross sharing of cuts), and multiple cut formulations. The performance of several different strategies and enhancements has been explored. The computational results indicated the single most important enhancement is a warm start technique which utilizes optimal basis information from previous subproblem solutions. The "fastpass" strategy performed very well. However, a new class of "epsilon-cautious" tree traversing strategies produced comparable results to "fastpass". Further investigation of these strategies may be warranted for many stages.

In his thesis, Alamuru Krishna developed an importance sampling scheme based on a piecewise linear approximation function. His scheme works particularly well in situations where our current techniques based on an additive approximation function fail to produce large variance reductions. Using a piecewise linear function with only a few segments to approximate a piecewise linear function with numerous segments leads naturally to an excellent fit. However, the development of the importance sampling procedure and the implementation turned out to be very difficult. Because piecewise linear functions are not separable, the integration is difficult to do in carrying out the sampling from the non separable importance distribution. Krishna's scheme is based on using a piecewise linear approximation function for carrying out the importance sampling. He again uses importance sampling or an alternative control variates scheme for integrating the piecewise linear approximation function. The new scheme produced a significant variance reduction compared to the importance sampling scheme based on the additive approximation function developed earlier for electric power applications. Krishna also developed an advanced Benders decomposition scheme for deterministic two-stage problems which reduces the number of iterations significantly.

We have made significant improvements on the subjects of importance sampling and decomposition and have developed convergence results for sampling based algorithms. While the estimation of the upper bound distribution is rather straightforward, the estimation of the lower bound distribution is more difficult, and, because of its importance, we have given it our major attention. In our earlier work we assumed the sampling error to be constant with respect to the master problem variables. In Infanger's book we relaxed this assumption and were able to handle sampling error in the coefficients and the right hand sides of the cuts. Using an efficient way of computing the covariances of the error distributions of the coefficients and the right hand sides of the cuts, and based on local error analysis (assuming "small" sampling error) we are in the position to compute an unbiased normally distributed lower-bound estimate. We have found different ways of constructing lower bounds that are more conservative and do not depend on the "small error" assumption. We are currently writing-up the new rigorous theory of lower bound estimates.

We are continuing our research in the area of using parallel processors for solving stochastic programs in collaboration with Dr. John Tomlin at the IBM Almaden Research Center. On the basis of our parallel code running on the Hypercube multi-computer, Infanger and Tomlin have developed a parallel code running on a cluster of powerful parallel workstations. Using parallel workstations we are able to solve much larger problems than on the Hypercube Multicomputer. First tests have been successfully completed. We are currently working on an interface for using OSL as a subroutine for solving subproblems and are developing a version that will run on IBM's new powerful SP1 multicomputer. We intend to develop an asynchronous implementation and study load balancing issues.

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