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Technical Progress Report

**HIGH BETA AND
SECOND STABILITY REGION
TRANSPORT AND STABILITY ANALYSIS**

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ABSTRACT

This report describes both ideal and resistive MHD stability studies of supershot plasmas in TFTR. In addition, we discuss some development effort which has significantly advanced our physics capabilities. In particular, MHD equilibria can be calculated with anisotropic pressure distributions which inevitably arise with intense neutral beam heating (or any other form of auxiliary heating). Preliminary work on the stability of such equilibria is included; at the present, this is confined to large toroidal mode number, ideal ballooning mode stability.

The MHD equilibrium and stability modeling of TFTR plasmas has concentrated on those experiments where the safety factor, q , profile was inferred from motional Stark effect measurements (MSE). These are conveniently divided into two groups, first where q is less than unity over some fraction of the plasma radius and, secondly, where q everywhere exceeds unity. In the first case, the experimental result that $q < 1$ together with the very large pressure gradients predicted by TRANSP analysis pose a major challenge to the validity of ideal MHD theory since these plasmas are usually observed to be free of any coherent 1/1 instability. We show that the stability of the ideal 1/1 internal kink mode is, in fact, sensitive to the precise details of the pressure profiles in the immediate vicinity of the $q=1$ surface and that plausible pressure profiles can be predicted where the 1/1 mode is stable.

In the second case where $q > 1$ we revisit the resistive stability properties of low β , high q_{edge} supershots again using MSL q -profiles as a starting point. We find that 2/1 and 3/2 tearing modes can be excited in some circumstances and that the MSE profiles are typically near marginal stability to each of these modes.

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I INTRODUCTION

This report describes work carried out at the Grumman Corporate Research Center during the period from March 1st 1993 to the present, and supported by the Department of Energy grant #DE-FG02-89ER51124. This grant was originally awarded in 1989 initially for a period of three years under the 'Transport Initiative in Tokamak Fusion Plasmas'. The agreement was renewed in early 1993 for a further two year period.

The renewal proposal submitted toward the end of 1992 emphasized theoretical and computational support for the D-T phase of operation in TFTR which is now in progress. The intention during the current year was to offer assistance in interpreting experimental data with the goal of establishing suitable target plasmas in TFTR optimized for D-T operation. Specifically, the two year program of work which we proposed was divided among three main tasks as follows:

Task 1: MHD Equilibrium and Stability Analysis of TFTR Experimental Data.

- (a) Ideal and Resistive MHD Stability Analysis of Recent TFTR Experimental Data.
- (b) Instability Suppression Studies.

Task 2: Transport Studies and MHD Analysis of Evolving Equilibria.

This task will investigate and analyze stable operating scenarios for D-T operation in TFTR.

Task 3: Implement Improvements in Equilibrium and Stability Models.

This task will improve our equilibrium and stability codes to better model specific TFTR experiments.

The work described in this document has concentrated mainly on Task 1 and Task 3 where significant progress is reported. Specifically, Task 1(a) and (b) are discussed in §II.A and II.B while topics comprising Task 3 are distributed between §II.C and §III. The topic of Task 2 has been initiated in collaboration with Robert Budny and others at the Princeton Plasma Physics Laboratory and will be described in a subsequent report.

The work summarized in this report has been described in two papers presented at the 1993 International Sherwood Meeting held in Newport, R.I. and three more papers given at the 1993 APS, Division of Plasma Physics Meeting held in St Louis, MO during November 1993. These presentations are enumerated in the Appendix to this document. It was also our pleasure to provide results and material to Dr. E. Strait for presentation in his invited talk¹ to the APS Meeting.

I.A Acknowledgments

Over a period of many years we have established excellent working relationships with many other groups in the Fusion Community both in the US and abroad and these continue to flourish. In view of our close proximity to the Princeton Plasma Physics Laboratory our main interaction has (inevitably) focused on that institution. It is a pleasure to acknowledge the cooperation of our colleagues in both the theoretical and experimental disciplines at PPPL, too many to enumerate individually. Also, we continue to interact successfully with the plasma physics groups at Columbia University, at General Atomics and, more recently, with colleagues at the University of Wisconsin.

II TFTR MODELING

Our previous MHD stability studies of TFTR have relied exclusively on pressure and current density profile data generated from TRANSP analysis of particular experimental shots. TRANSP employs measured temperature and density information to evaluate transport coefficients which are subsequently used to calculate the plasma evolution. During the course of such a transport simulation consistency with the experiment is monitored by comparing various calculated quantities (the neutron emission profile, for example) with the actual measurements. It was noted in ref. 2 that, while profiles obtained in this way are clearly preferable to arbitrary formulae plucked from the aether, TRANSP results depend sensitively on numerous model assumptions and are, therefore, subject to interpretation and uncertainty. This remark applied equally to the q -profiles employed previously *and* the pressure profile, since supershot plasmas in TFTR are both fueled and heated by intense neutral beam injection and these circulating energetic particles can contribute (typically) half the total pressure. The contribution of the beam pressure is obtained by calculation assuming only classical, or more precisely neo-classical, effects during thermalization of the beam particles along with *ad-hoc* models for particle diffusion. Since the calculation of the beam profiles, in particular, is a time consuming business consuming many tens of hours of computer time it is difficult to quantify the effects of uncertainties introduced by these models. Again, most previous MHD studies (with only very few exceptions), including our own, have treated the pressure as an isotropic, scalar quantity. For the most part, this assumption is retained here though some initial equilibrium and stability calculations involving anisotropic pressure distributions are reported.

While the pressure profiles required for MHD stability analysis still rely on TRANSP modeling the situation with regard to the current density profiles is now more satisfactory. Specifically, a motional Stark effect (MSE) diagnostic^{3,4} commissioned on TFTR during 1992 now routinely measures the pitch of the magnetic field from which the equilibrium configuration and the q -profile can be inferred iteratively. A substantial database is accumulating from this diagnostic measurement which yields both spatial and temporal information. Here, we shall study the MHD implications of these measurements bearing in mind the requirement and importance of mode free supershot plasmas which are the preferred target plasmas for D-T operation in TFTR. This requirement is driven by considerations of α -particle thermalization which demands mode free plasmas for times

significantly better than those achieved in the past to guarantee significant collective α physics results.

The MSE measurements of q show that often q is less than unity over a substantial fraction of the radius in supershot plasmas. Other times, elevated values of $q(0)$ are observed with $q(0)$ as large as 2 reported in some instances⁵ (though, these were not supershot plasmas). In the first case, the observation of $q < 1$ coupled with the steep pressure gradients thought to exist in the interior of supershot plasmas and the absence (usually) of the 1/1 sawtooth instability pose a potentially serious challenge to MHD theory. In fact, this observation has also inspired ingenious theoretical models^{6,7} developed specifically to explain these supershot results. Here we shall proffer a much simpler explanation. In particular, our approach here was to ask how uncertainties in the data might be exploited to yield consistency between the experimental facts and the predictions of ideal MHD theory. In view of the relative success of ideal MHD in the past and the importance still attached to the results of this model this seems to be a desirable undertaking. In this exercise of 1/1 mode stability the q -profile data obtained from MSE measurements were held fixed and only the pressure distributions were altered. As emphasized above, the justification for this procedure is that the large contribution to the pressure from the high energy injected beam particles is not measured.

In the case when $q > 1$, we revisit previous calculations of both the resistive tearing mode in low pressure plasmas and the pressure driven instabilities which occur at higher values of β . Each topic mentioned in this preamble is discussed in more detail in the following.

II.A Plasmas with $q < 1$

Here we shall concentrate attention on two specific TFTR plasmas, labeled #65611 and #65612, where q was experimentally determined from the MSE diagnostic and, in both cases, was shown to be less than unity. Moreover, in one plasma, namely #65612, 1/1 sawtooth oscillations were induced by injecting helium into the discharge to deliberately recover L-mode conditions while in the other, namely the supershot #65611, there was no coherent MHD activity. This is a particularly surprising result from the point of view of ideal MHD theory since the mode free case, #65611, had both the larger pressure ($\beta_p = 0.6$ compared with $\beta_p = 0.4$) and the larger (calculated) pressure gradients. Numerous other experiments exhibit this qualitative behavior. Fig. II.A-1 shows the measured values of q for shot #65611 at a number of major radius locations along the median plane of the

tokamak. The corresponding q -profile data for the other plasma, #65612, is very similar, though in this latter case the pressure distribution predicted by TRANSP was substantially different. This diagram, fig. II.A-1, shows how q can be less than unity over a large fraction (20-25%) of the plasma diameter. Note that the measurements do not extend to the plasma periphery where the value of q was 5.6.

fig. II.A-1 q -profile Obtained from MSE Measurements Along the Midplane Major Radius in Supershot #65611

Again referring to fig. II.A-1, for curve fitting purposes we have arbitrarily associated error bars of $\pm 10\%$ with the experimental values of q . The curve shown in fig. II.A-1 illustrates the result obtained from our equilibrium reconstruction of the q -profile data. The agreement between the measured and fitted profiles is regarded as satisfactory.

In the preamble to §II it was noted that the pressure distribution in supershot plasmas is not measured but calculated using the TRANSP analysis code on the basis of classical deposition and thermalization of the high energy neutral beam particles together with models for the particle transport. Using these calculated pressure profiles for the two

supershots of interest together with q profiles fitted to the experimental data we calculate the Bussac beta⁸,

$$\beta_{p1} \equiv (8\pi / I^2(r_s)) \int_0^{r_1} -r^2 \frac{dp}{dr} dr$$

shown in ref. 8 to be the relevant quantity is discussing the stability of the ideal 1/1 internal kink. Here, r_s is the radial location of the $q=1$ surface. Specifically, ideal MHD theory requires that $\beta_{p1} < 0.3$ for 1/1 stability⁸ which seems to be a good 'rule of thumb' even in complicated, strongly toroidal configurations such as those of interest here. The numerical value involved in Bussac's criterion is dependent on the current density profile. In practice, we find that the critical value is often smaller than 0.3 but, in our experience, is never larger than this value. We compare this maximum value with the result $\beta_{p1}=0.7$ obtained using the MSE q-data and pressure profiles given by TRANSP analysis of shot #65611 at $t=4$ sec. The corresponding data from #65612 yields $\beta_{p1}=0.29$. We would

fig. 11.A-2 Ideal, 1/1 mode Stability Diagram

expect from ideal MHD theory, therefore, that the shot #65611 would exhibit a 1/1 instability while the other, #65612, is possibly near marginal. This is the exact opposite of the experimental observation. In fact, we find computationally that both sets of data lead to unstable 1/1 ideal modes whose growth rates correspond to the largest values of β_{p1} shown in fig. II.A-2.

From the definition of β_{p1} it is clear that the numerical value of the integral is weighted by the pressure gradients in the vicinity of the rational surface, $q=1$, located at r_s . We can therefore change the value of β_{p1} by modifying the pressure profile in the immediate vicinity of r_s . Specifically, we can alter β_{p1} over a substantial range by inserting a 'flat spot' in the pressure profile near the rational surface. Physically, this might occur as the result of enhanced transport resulting from the localized resistive 1/1 mode which we find is always unstable when $q < 1$. We shall return to this topic later. Thus, we insert a flat spot with a given width and prescribed 'flatness' in the pressure profile. For technical reasons associated with the resolution of our equilibrium solver the width of the perturbation was limited to a minimum value. Keeping this width constant at the smallest possible value we alter β_{p1} by changing the flatness of the perturbation and calculate the 1/1 mode growth rates using the ARES⁹ stability code. Fig. II.A-2 illustrates the variation of the ideal 1/1 mode growth rates for the two supershots of interest as β_{p1} is altered. This diagram shows how the modes tend rapidly toward stability as β_{p1} approaches a limiting value which is essentially the same for the two cases studied. That $\beta_{p1} \approx 0.2$ represents a limiting value was subsequently confirmed using a second ideal MHD code, known as LOBAL², constructed specifically for the accurate location of marginal stability points.

Fig. II.A-3 shows the marginally stable pressure profile for shot #65611, which initially was the more unstable of the two cases. The pressure profile corresponding to the other case required a much shallower perturbation and is apparent only when the curve is differentiated. Also shown in fig. II.A-3 is the thermal contribution to the pressure, also obtained initially from TRANSP, but with a perturbation inserted which is similar to that applied to the total pressure. The difference of these two profiles yields the contribution of the beam pressure which, in this case, is comparable with the thermal component. The thermal component is, of course, simply the product of the plasma density with the electron and ion temperatures and each of these are measured. Very detailed studies carried out by B. Grek at TFTR have suggested the existence of flat spots located in the vicinity of low order rational surfaces. Similar conclusions from studies at JET and Tore Supra were described at the TTF Meeting held in Newport, RI during February 1993. The conclusions

from the experimental data are not entirely conclusive, however, as the result of experimental inaccuracies.

fig. II.A-3 Marginally Stable Pressure Profile for Shot #65611

The procedure adopted here of inserting a pressure perturbation in the vicinity of the $q=1$ surface was originally inspired, in fact, by the analysis of Grek and others as mentioned above. On the other hand, in his invited talk at the St Louis, APS Division of Plasma Physics Meeting, Zarnstorff¹⁰ described recent high spatial resolution experiments intended specifically to search for radial structures in the electron temperature profiles in the various operating modes of TFTR. He concluded that, in quiescent TFTR plasmas, there are no flat spots in the T_e profiles unless these are smaller than 1cm in radial extent. For reference, the perturbation illustrated in fig.II.A-3 is in the range of a few centimeters. These conflicting conclusions from experiments are still being hotly debated at PPPL. For our purposes, then, there is no clear experimental evidence either for or against the procedure adopted here. It is interesting, however, that the 1/1 ideal kink mode with its

distributed eigenfunction can undergo such a pronounced change under the influence of relatively small scale perturbations of the pressure in the vicinity of the $q=1$ surface.

To conclude this section on the stability of the 1/1 mode in TFTR we have shown that q -profiles obtained from MSE measurements combined with the very steep pressure profiles calculated by TRANSP for supersonic conditions are predicted to be highly unstable to the internal kink mode. On the other hand, we have shown that the stability of the 1/1 is sensitive to precise details of the pressure profile within, or in the immediate vicinity of, the $q=1$ surface. In particular, small scale perturbations in the pressure will suffice to stabilize the ideal 1/1 internal kink mode in supersonic plasmas without the need to invoke any further physics or, more importantly, without the need to abandon ideal MHD theory! This also suggests that the 1/1 internal kink mode could be stabilized, when required, by localized heating to alter the pressure distribution appropriately at $q=1$. The sawtooth stabilization by ECRH described in ref. 11 could well be a manifestation of this effect, although the explanation given in that paper suggested that the current distribution was altered by ECRH current drive.

These results evoked substantial interest when presented at the St. Louis, APS Meeting. In particular, Dr. Roscoe White of PPPL suggested further collaborative studies to elucidate the possible role of circulating non-thermal particles in the stabilization of the 1/1 mode in supersonic plasmas. This suggestion is currently being pursued.

II.B Supershots with $q > 1$

In this section we describe studies of plasmas where the experimentally derived q was found to everywhere exceed unity. In particular, we consider large β_p (> 1) plasmas where the central q increases significantly during the beam heating phase of the discharge. In ref. 5 this change in q was attributed to the 'bootstrap' current which tends to flow in the outer regions of the plasma resulting in a broadened net current profile. Here, it is convenient to separate these plasmas into low pressure ($\beta < 0.5\%$) and high pressure ($\beta > 0.5\%$) cases since the latter can be near marginal stability for small toroidal mode number ideal modes of the ballooning variety². Low pressure supershots, on the other hand, are predicted to be stable to ideal modes. The 'Troyon beta' for the plasmas considered in the next section is $\beta_N = \beta/(I/aB) \sim 2$ which is well below the limit for pressure driven ideal MHD² modes for the pressure profiles of interest. Despite this prediction of (ideal) MHD theory and computation it is an experimental fact that about half of these low pressure plasmas nevertheless exhibit instability. The most common instability has an $m/n=3/2$ structure though occasionally $2/1$ modes appear. More recent experiments include $m/n=4/3$ and $5/4$ instabilities. These modes have long growth times of order milliseconds and are assumed to be associated with the finite plasma conductivity. Our own studies suggest that high pressure supershot plasmas with $\beta \sim 1\%$ are precariously near ideal MHD instability, involving modes of the ballooning variety. These ideal ballooning modes themselves drive resistive instabilities,^{2,12} but these latter modes do not extend to the low pressure regime, $\beta \sim 0.5\%$. The possible excitation of resistive tearing modes in these low pressure plasmas has been studied previously but is revisited here in the light of MSE q -measurements, since these are not entirely consistent with our previous conclusions. Also, there is presently renewed interest in the role of the tearing mode in TFTR. In particular, the possibility of a collaborative studies of these instabilities involving Professor Callen's group at the University of Wisconsin and with L. Zakharov at PPPL is presently under discussion.

II.B.1 Finite Pressure Stabilization of the Tearing Mode.

The finite pressure stabilization of the tearing mode first discussed by Glasser *et al*¹³ was described in our last report. In that report we also discussed briefly the results of some recent analytic work in which we participated¹⁴ where linear resistive MHD theory is revisited and, not surprisingly, recovers the prediction of finite pressure stabilization of the tearing mode but questions the physical importance of the effect. During the period of this report that analytic work was extended to include the role of finite viscosity in the resistive

layer. We briefly recapitulate the previous computational result here both as a reminder and to demonstrate the improved results forthcoming with the use of a new, very fast and highly accurate equilibrium solver developed by ourselves and described in §III. This permits much better spatial resolution and enables the calculation of the very small growth rates and the correspondingly small spatial scale lengths that typically characterize resistive instabilities. Thus, fig. II.B.1-1 shows the growth rates of the 3/2 tearing instability in toroidal geometry as a function of the Lundquist number, S , in a tokamak with aspect ratio 3 and $\beta_p=1.2$. This figure shows clearly how, when S is small enough, there are two

Fig. II.B.1-1 Resistive Mode Growth Rate as Function of the Lundquist Number. This diagram illustrates the strong stabilizing influence of the pressure on the tearing mode when β_p is large ($\beta_p=1.2$ in this example)

unstable modes which approach each other as S is increased and eventually coalesce, in accordance with the qualitative predictions of analytic theory¹⁴. At the point of coalescence, S_B , a bifurcation occurs and the eigenvalues become complex (oscillatory growth); as S is further increased beyond S_B the mode rapidly stabilizes. In ref. 14 it is shown analytically that the tearing mode stabilizes at $S_C=5.5*S_B$. In this high $\beta_p=1.2$,

supershot, plasma the critical value of $S=S_C \sim \text{few} \times 10^5$ is several orders of magnitude smaller than the range of interest in TFTR ($>10^8$). Although this stabilization is a perfectly valid prediction of resistive MHD theory it is universally believed that, in practice, the mechanism is overwhelmed by hydrodynamic effects in the resistive layer and which are ignored in the simplest theoretical model. Again referring to our paper, ref. 14, it was there shown both analytically and computationally (in cylindrical geometry) that (parallel) viscous dissipation reduces the critical value Δ'_c of the resistive stability index, Δ' , with $\Delta'_c \propto \mu^{-1/4}$, where μ is the viscosity coefficient. When the magnitude of μ is roughly that predicted by Braginskii it was indicated in ref. 14 that the more appropriate criterion for instability is $\Delta' > 0$ rather than the expression $\Delta' > \Delta'_c$ derived by Glasser *et al.* As described in our previous report, this stabilization effect is conveniently removed and the tearing mode persists into the S-range of interest by the introduction of 'flat spots' in the pressure profile in a very narrow region centered around the rational surface of interest. This procedure is again adopted here purely as an artifice to remove this pressure stabilization.

In passing, we note that the arbitrary prescription sometimes used elsewhere of making the ratio of specific heats, $\Gamma=0$, to remove the finite pressure stabilization of the tearing mode is not an entirely valid procedure. It is shown in ref. 14 that while making $\Gamma=0$ indeed has a destabilizing effect on the tearing mode the net result is that the instability then persists with a growth rate scaling linearly with the resistivity. The conventional wisdom is that the only regimes of interest are those where the growth rate scales as some fractional power of the resistivity.

II.B.2 Tearing Mode Stability Criterion.

Before discussing recent calculations of the tearing instability in fully toroidal calculations it is instructive to first consider a simple model, at small β and large aspect ratio in cylindrical geometry, which serves to identify the conditions under which these instabilities are excited. A simple criterion which predicts the onset of the tearing mode with poloidal mode number m , given by Wesson¹⁵ and subsequently derived rigorously (see comment in Bibliography) by others,^{16,17} is

$$\frac{\mu_0}{(B_\theta/r)} \left| \frac{dJ_z}{dr} \right| > \frac{m}{q} \left| \frac{dq}{dr} \right| \quad (\text{for instability}) \quad \text{II.B.2-1.}$$

Fig. II.B.2-1 Tearing Mode Stability Diagrams and Current Density Profiles (Cylindrical Model, $q(0)=1$)
In the set of graphs to the right, the full curve shows the axial current density, J_z , while the dashed curve indicates its gradient.

Fig. II.B.2-2 Tearing Mode Stability Diagrams and Current Density Profiles (Cylindrical Model, $q(0)=1.1$)
In the set of graphs to the right, the full curve shows the axial current density, J_z , while the dashed curve indicates its gradient.

The LHS of this inequality, II.B.2-1, represents the driving effect of the current density gradient while the RHS represents the stabilizing influence of magnetic shear, $s = (r_s/q) \cdot (dq/dr)$. Each term in this criterion is evaluated at the rational surface of interest located at r_s where $m=nq$. Thus the criterion for instability can be cast in the more convenient form

$$\frac{r_s}{s} \cdot \left(\frac{1}{B_z/R} \right) \cdot \left| \frac{dJ_z}{dr} \right| > n \quad \text{II.B.2-2.}$$

In the following, we shall further assume that the safety factor can be described as

$$q(\chi) = q(0) \cdot e^{\alpha_q \chi} \cdot (1 + C \cdot \chi^{\beta_q}) \quad \text{II.B.2-3,}$$

where $0 \leq \chi \leq 1$ is the normalized poloidal flux. This convenient approximation for q is used extensively in this work and it is shown later that, with appropriate choice of parameters, this formula can often describe the q -profile data from both TRANSP and MSE measurements with substantial accuracy. Note that there are three independent parameters α_q , β_q and $q(0)$; the constant C is then determined from the edge value of q . Making $B_z = \text{constant}$ it is then a trivial matter to evaluate the RHS, say $n_{\max} = (r_s/s) \cdot (1/B_z/R) \cdot |dJ_z/dr|$, of the instability criterion, II.B.2-2, and compare this with the toroidal mode number of interest, (which must, of course, be an integer). Since the destabilizing effect of the current density gradient and the corresponding stabilizing influence of the shear are (obviously) not independent the prediction of the criterion is most easily displayed graphically. Thus fig. II.B.2-1 plots $n_{\max}$ as a function of $q(\chi)$ across the minor radius of the cylinder with $q(0)$ fixed at unity and with various values of α_q . In each graph of fig. II.B.2-1 the edge value of q was fixed at 8.9 and $\beta_q = 2.5$ corresponding to the TFTR plasma #67233 considered later. The next diagram, fig. II.B.2-2 illustrates the effect of a different, larger, value of $q(0) = 1.1$ and the same values of α_q . Each of these stability diagrams is accompanied by the current density and current density gradient distributions used to generate them.

Referring to the graphs shown in fig. II.B.2-1, rational surfaces lying in the shaded regions are stable. The curve delineating the shaded and white regions in these diagrams correspond to the locus of $n_{\max}$. Consider first the series of graphs II.B.2-1(a-c) where $q(0)$ is held fixed at unity and the shear distribution is varied by changing α_q . In the first graph, fig. II.B.2-1(a) where the shear parameter $\alpha_q = 1.2$, the 2/1 and 3/2 tearing modes are near marginal stability. Tearing modes with $n > 2$ are stable. Note that the largest toroidal mode number considered here is $n=4$; the statement 'all n -numbers' will then refer to $n \leq 4$.

In this series (a-c) any reduction of α_q , which in fact increases the shear since the rational surfaces move outward, makes the configuration more stable. At each stage of this sequence the current density profile is relatively peaked, though the reduction of α_q results in a narrower current channel and produces a more 'rounded' profile. To summarize, each case with $q=1$ is essentially stable or, at worst, near marginal.

On the other hand, when the calculation is repeated with a somewhat elevated value of $q(0)=1.1$ we obtain the results summarized in II.B.2-2. In this case, when $\alpha_q=1.2$ both the 2/1 and 3/2 tearing modes are predicted to be unstable; modes with $n \geq 3$ are stable (fig II.B.2-2). Now, however, as α_q is decreased the 2/1 mode approaches the marginal condition and eventually stabilizes (figs.II.B.2-2 (b) and (c)). The 3/2 mode remains unstable and ultimately the instability with $n=3$ is also destabilized (though somewhat marginal) when α_q is small enough (fig.II.B.2-2 (b)); in this sequence the $n=4$ mode remains stable or near marginal. The effect of elevating $q(0)$ is to shift the maximum value of dJ_z/dr relative to the location of the rational surface. Inspection of figures II.B.2-1 and II.B.2-2 shows how the magnitude of the current density gradient has increased at $q=3/2$. These diagrams illustrate clearly that, to drive 3/2 and higher order m/n tearing modes without exciting the 2/1, values of $q(0) \sim 1.1$ are required and the current density distribution together with the corresponding q -profile need to be relatively flat. The tearing modes with $n \geq 2$ are stabilized and the 2/1 mode is destabilized with more peaked current density profiles.

These results are qualitatively consistent with our previous (toroidal) studies of tearing modes in TFTR but there we required much larger values of $q(0) > 1.2$ before the instabilities were excited. The toroidal calculations described below are based on q -profiles derived from MSE measurements where $q(0) < 1.2$ and where resistive tearing modes appear under some conditions. These calculations are, in fact, more consistent with the simple criterion described above. The reasons for the discrepancies between the present calculations and those described previously are not clear and these are difficult to reconcile in view of the fact that we previously used numerical data obtained directly from TRANSP and where the shear profile was modified in an *ad hoc* fashion.

II.B.3 Tearing Mode Stability in Toroidal Geometry

The two supershots labeled #67233 and #67255 are low current ($q_{edge}=8.9$ and 11.4 respectively), low β plasmas where MSE q -data are available and which are similar to those studied in the past. As mentioned above, both plasmas have $\beta_N \sim 2$ and are

Fig. II.B.3-1 Variation of q Across the Equatorial Plane of TFTR Derived from MSE Measurements. The dotted curve was obtained from our equilibrium reconstruction.

Fig. II.B.3-2 q -profiles in Supershot Plasmas #67233 and #67255

Fig. II.B.3-3 Tearing Mode Stability of Supershot #67255
The vertical dashed line in the lower figure indicates the value of α_q representing the 'best fit' to the MSE data

robustly stable to ideal MHD modes. Experimentally, however, a 3/2 instability appeared in #67233 while a 2/1 mode was excited in #67255. Fig II.B.3-1 shows the variation of q from shot #67233 derived from MSE data along the median plane of the tokamak at three different time points. Again, we have associated error bars of $\pm 10\%$ with the measured q -data for the purpose of curve fitting. The dotted curve in fig. II.B.3-1 is the result of reconstructing the equilibrium using the simple analytic form for q already described in §II.B.2 viz.

$$q = q(0) * \exp(\alpha_q * \chi) * (1 + C * \chi^{\beta_q})$$

where χ is the normalized poloidal magnetic flux, $0 \leq \chi \leq 1$. Interestingly, the experimental data offers some guidance in the choice of exponents in this formula since the value of β_q largely determines the plasma inductance per unit length which can be estimated from the measured values of β_p and the shift of the magnetic axis. The exponent α_q clearly defines the shear in the vicinity of the magnetic axis and, given $q(0)$, (roughly) determines the radial location of a low order mode rational surface which can be estimated experimentally when instability occurs. The fitted curve in fig.II.B.3-1 has $q(0)=1.1$, $\alpha_q=1.2$ and $\beta_q=2.5$; the constant C is determined from the edge value of the safety factor, $q_{edge}=8.9$. The corresponding exponents for the second case, shot #67255, are $\alpha_q=1.9$ and $\beta_q=3$ (with $q_{edge}=11.4$). Fig. II.B.3-2 shows the radial profiles of q for the two cases of interest.

Analysis of the 'best fit' profiles to the MSE q -profiles show that these are stable to ideal modes as mentioned above and only feebly unstable to resistive tearing modes. Guided by the cylindrical geometry calculations described in §II.B.2 we deliberately make each configuration more unstable by reducing the value of α_q and varying this parameter keeping $q(0)=1.1$ and β_q constant. The resistive stability of each equilibrium configuration was subsequently tested using ARES. First, consider the tearing mode stability of shot #67255 (where a 2/1 mode appeared). Figs. II.B.3-3 (a) shows the prediction of the criterion II.B.2-2 calculated using the parameters given above. Referring to fig.II.B.3-3 (a) it can be seen that the simple cylindrical model predicts that only the 2/1 mode is unstable. Moreover, the results of our toroidal calculations, shown in fig.II.B.3-3 (b) confirms that this is, indeed, the case (i.e. when $\alpha_q=1.9$). As mentioned above, at this value of α_q corresponding to the best fit to the MSE data, the 2/1 instability is extremely feeble. For example, using typical TFTR supershot parameters the value $\gamma\tau_A/\epsilon = 10^{-4}$ corresponds to a mode growth time in the range of about 50msec when $\epsilon S \sim 10^8$. Note that, for computational economy fig.II.B.3-3 (b) plots the growth rates corresponding to $\epsilon S = 10^6$. The growth rates can then be extrapolated to the regime of practical interest since these scale

as $S^{-3/5}$. Thus, fig II.B.3-3 (b) shows that the q-profile derived from MSE measurements of supershot #67255 is (essentially) marginal to the resistive 2/1 tearing instability. As the exponent α_q is reduced the 2/1 mode growth rate increases rapidly and further, when α_q is roughly 1.6, the 3/2 tearing mode appears. Higher order m/n modes, with $n > 2$ are not excited at any value of α_q .

Fig. II.B.3-4 Tearing Mode Stability of Supershot #67233
The vertical dashed line indicates the value of α_q representing the 'best fit' to the MSE data

This toroidal analysis of shot #67255 is in rough accord with the prediction of the criterion II.B.2-2 and, indeed, with the experimental observation of a slowly growing 2/1 mode. Moreover, the study of this criterion summarized in §II.B.2 suggested that the 2/1 instability could be stabilized and higher order m/n modes excited when the current channel is more constricted with flatter current density, and corresponding safety factor profiles, near the plasma center. We note that the q-profile for the other case, #67233, is somewhat flatter than that for shot #67255 (fig. II.B.3-2). The application of criterion II.B.2-2 to this

case (fig.II.B.2-2) suggests that the 2/1 tearing mode should be stable and the 3/2 mode unstable when $\alpha_q \approx 0.6$, which is substantially smaller than the value $\alpha_q=1.2$ obtained from the MSE data for shot #67233. Moreover, toroidal calculations summarized in fig.II.B.3-4, which again plots the mode growth rates as a function of α_q , are not in accord with the simple cylindrical model. Fig.II.B.3-4 shows that, while the 3/2 mode is indeed excited when $\alpha_q < 1.0$, the 2/1 instability persists over the entire range albeit with smaller growth rates than those for the 3/2 mode when α_q is small.

The simple cylindrical model discussed in §II.B.2 has provided interesting insight into the conditions where tearing modes with various m/n structures are excited. Specifically, higher order m/n modes appear when the current channel is highly constricted with large current gradients in the vicinity of the mode rational surfaces of interest and where the current density at and outside the $q=2$ surface is very small. The exercise described in §II.B.2, indeed, suggested conditions where tearing modes with $n>1$ could be excited while the $n=1$ (specifically the 2/1) mode would be stable. Attempts to reproduce this behavior in fully toroidal geometry have not been entirely successful. In view of the simplicity of the model of §II.B.2 this is possibly not too surprising. Discussions are currently underway with colleagues at the University of Wisconsin, Madison to refine the criterion II.B.2-2 and to properly express this in toroidal, flux coordinates. A relatively straightforward, albeit accurate, criterion for the onset of tearing instabilities would be extremely useful to tokamak experimentalists.

In the meantime, it is interesting to note that our present resistive MHD computation predicts the excitation of tearing modes with both 2/1 and 3/2 structures with $q(0)$ in the range derived from experimental measurements. In particular, the experimentally derived q-profiles for the two supershot plasmas considered here are both near marginal to each of these modes. We have not yet, however, found conditions in toroidal geometry where the 2/1 mode is stable and 3/2 or higher order m/n are unstable. Thus, the experimental observation of single m/n structures remains a mystery.

II.C Anisotropic Pressure Distributions

All major tokamak facilities use some form of auxiliary heating in addition to ohmic heating of the plasma. Typically, when any kind of ion heating is used the pressure profiles tend to become anisotropic. This is because the ions do not thermalize as rapidly as electrons. The existence of anisotropic pressure profiles should affect all pressure driven MHD modes including the ballooning and internal kink modes. High beta and high $\epsilon\beta_{pol}$, tend to amplify the effect that differences in parallel and perpendicular pressure have. Hence, anisotropic pressure effects should be even more important in high performance tokamaks.

Anisotropic pressure profiles have been observed in TFTR supershots. At the beginning of the discharge the pressure profiles are fairly anisotropic. About half of the pressure in TFTR is due to the neutral beam ions. The beam particles are partially non-thermal with about a 40% difference between $p_{\parallel}$ and $p_{\perp}$. This makes the difference in the total pressures about 20%. As the discharge progresses the pressure profiles tend to become more isotropic and this difference decreases. However, in addition to total pressure difference, the pressure variation along a field line is also just as important. As will be demonstrated later, even when $p_{\parallel}$ and $p_{\perp}$ are balanced, the pressure variation along a field line results in significant changes in the ballooning unstable region.

Previous studies^{18,19,20,21,22} of anisotropic effects concentrated mainly on high n modes with model fields. Salberta et al.²³ developed anisotropic models from transport simulation data. However, a comprehensive picture of the effect of anisotropy on various MHD modes using realistic experimentally derived profiles still does not exist. Issues left to be resolved are the effects of anisotropy on low-n modes, the effect on the second region of stability, and to infer the degree of anisotropy from actual experimental data.

In this section we report on progress made in incorporating pressure anisotropy into our MHD analysis. A new equilibrium code was written which solves the anisotropic pressure equilibrium equations. A ballooning code was modified to solve the anisotropic form of the ballooning equations. A notable feature of this code is that local changes in the equilibrium are included which allows mapping out of both the first and second stability regions²⁴. Hence the effect of anisotropic pressure on the second region of stability can now be assessed.

Progress is also being made in addressing the other outstanding issues. To attempt to analyze the nature of the anisotropy in TFTR discharges, particle distributions for the neutral beam component of the pressure will be obtained from the TRANSP transport code simulation of selected shots and integrated to determine the pressure profile. A low-n MHD code is being adapted to solve a reduced form of the Chew-Goldberger-Low (CGL) fluid equations²⁵ to determine marginal stability points.

II.C.2 Anisotropic Equilibrium

In axisymmetric geometry, the anisotropic equilibrium configuration can be found by solving an equation similar to the Grad-Shafranov equation²⁶

$$x^2 \nabla \cdot \left(\frac{\sigma}{x^2} \nabla \chi \right) = - \frac{G}{\sigma} \frac{\partial G}{\partial \chi} - x^2 \frac{\partial p_{\parallel}}{\partial \chi} \Big|_B. \quad (\text{II.C-1}),$$

where $\sigma = 1 - \frac{p_{\parallel} - p_{\perp}}{B^2}$.

Here, B is the magnetic field, χ is the poloidal flux function, $p_{\parallel}$ is the parallel pressure and $p_{\perp}$ is the perpendicular pressure. Both $p_{\parallel}$ and $p_{\perp}$ are functions of χ and B only. The toroidal field function, G , is a function only of χ . The magnetic field representation used here is

$$\mathbf{B} = \nabla \phi \times \nabla \chi + \frac{G(\chi)}{\sigma} \nabla \phi \quad (\text{II.C-2}).$$

The pressures $p_{\parallel}$, $p_{\perp}$ are related by the expression.

$$\frac{\partial p_{\parallel}}{\partial B} \Big|_{\chi} = \frac{p_{\parallel} - p_{\perp}}{B} \quad (\text{II.C-3}).$$

Thus, when $p_{\parallel}$ is known then $p_{\perp}$ can be found by solving Eqn. II.C-3.

The equilibrium equation with anisotropy differs little from the Grad-Shafranov equation and the techniques used to solve the latter can still be employed. Our new equilibrium code (§III) was modified to solve the above equations, including the pressure anisotropy. Features of this code include

- Full asymmetric equilibria,

- Specified β ,
- Anisotropic pressure profile models.
- Current density profiles can be determined by specifying
 - the q profile,
 - the $\langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \phi \rangle$ profile, or
 - the G profile
- Fast Multigrid solution methods.

As a result of the improved numerical method much larger problems are amenable to solution than was possible in previous versions. This is a significant and essential improvement since some MHD modes, including resistive modes and internal or external kink modes, are often difficult to compute accurately and must be carefully converged.

The pressure profile model formulated by Salberta for $p_{\parallel}(\chi, B)$, $p_{\perp}(\chi, B)$ was implemented in the code. It is convenient to represent the pressure profiles as the sum of an isotropic background part and an anisotropic part

$$p_{\parallel}(\chi, B) = p_s(\chi) + p_{\parallel b}(\chi, B) \quad (\text{II.C-4a}),$$

$$p_{\perp}(\chi, B) = p_s(\chi) + p_{\perp b}(\chi, B) \quad (\text{II.C-4b}).$$

In this model $p_{\parallel}(\chi, B)$, $p_{\perp}(\chi, B)$ are given by the functions

$$p_{\parallel}(\chi, B) = p_s(\chi) + p_b(\chi) \exp \left\{ \int_{B_{\min}}^B [1 - h(\chi, B)] \frac{dB}{B} \right\} \quad (\text{II.C-5a}),$$

$$p_{\perp}(\chi, B) = p_s(\chi) + p_b(\chi) h(\chi, B) \exp \left\{ \int_{B_{\min}}^B [1 - h(\chi, B)] \frac{dB}{B} \right\} \quad (\text{II.C-5b}).$$

Here the function $h(\chi, B) = p_{\perp b}(\chi, B) / p_{\parallel b}(\chi, B)$ is the ratio of the anisotropic perpendicular and parallel components. The functional form of $h(\chi, B)$ used in the code is

$$h(\chi, B) = \sum_{i=1}^3 \sum_{j=1}^3 c_{ij} \bar{\chi}^i \bar{B}^j \quad (\text{II.C-6}),$$

where $\bar{\chi} = (\chi - \chi_0)/(\chi_{lim} - \chi_0)$, and $\bar{B} = (B - B_0)/(B_{lim\ max} - B_{lim\ min})$. Here χ_0 is χ at the magnetic axis and χ_{lim} is χ on the surface of the plasma. If $h(\chi, B) < 1$ the parallel component of the pressure is dominant. Conversely, if $h(\chi, B) > 1$ then the perpendicular component is dominant. Another reason for adopting this profile was to enable us to benchmark the code against known results.

New Q Solver Method

The equilibrium code uses a new method to solve for a given q profile. The previous method described by Delucia et al.²⁶ was found to be unstable using the multigrid method. For small grids, typically under 64 flux surfaces, the old method converged without difficulty. However, with more refined meshes (> 128 surfaces) this method often would become unstable, especially when running highly elongated and D- shaped plasma cross sections. The new method uses the local field line equilibrium condition to calculate q . This reduces the dependence on mesh errors. The following formula, which will be discussed in greater detail below, is used to solve for G' .

$$2\pi q' = G' \oint \frac{d\theta J}{x^2 \sigma} \frac{B^2}{B_p^2} + G \oint d\theta \frac{J}{\sigma^2 x^2 B_p^2} \frac{\partial p_{||}}{\partial \chi} \Big|_B - G \oint d\theta \frac{J}{\sigma x^3 B_p} \left[2\kappa_p + \frac{2}{x} \frac{z_\theta}{JB_p} \right] \quad (\text{II.C-7}),$$

$$\text{where } \kappa_p = \frac{x_{\theta\theta} z_\theta - x_\theta z_{\theta\theta}}{(x_\theta^2 + z_\theta^2)^{3/2}}.$$

The two q solving methods were found to agree very closely.

Current Profile Specification

Frequently it is desirable to specify the current profile more directly. The q solving method is particularly useful for studying modes where shear plays a crucial role as it allows direct manipulation of this quantity. Often though, when the q profile is specified the corresponding current density profile may become unrealistic, particularly near the edge of the plasma. One widely adopted method, which avoids this problem, is to specify the $\langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \phi \rangle$ profile. Here the brackets indicates the volume average on a flux surface. It can be shown that for an equilibrium with anisotropic pressure distributions that

$$\frac{\langle \mathbf{J} \cdot \mathbf{B} \rangle}{\langle \mathbf{B} \cdot \nabla \phi \rangle} = - \frac{G' \left\langle \frac{B^2}{\sigma} \right\rangle + \left\langle \frac{G}{\sigma} \frac{\partial p_{\parallel}}{\partial \chi} \right\rangle_B}{G \left\langle \frac{1}{\sigma \alpha^2} \right\rangle}. \quad (\text{II.C-8}).$$

An expression for GG' can be unfolded from Eqn. II.C-8 and used in Eqn. II.C-1.

II.C.3 Anisotropic Ballooning Stability

An anisotropic form of the ballooning equation can be derived by expanding the anisotropic CGL fluid equations²⁷ in powers of n and adopting the ballooning mode representation²⁸. This was the procedure followed by Cooper²⁹ and we use his derivation here. As in the isotropic case the higher the mode number the more unstable the mode. In the limit $n \rightarrow \infty$ the procedure leads to the ODE,

$$\begin{aligned} & \mathbf{B} \cdot \nabla \left(\sigma \frac{\mathbf{k}_{\perp}^2}{B^2} \mathbf{B} \cdot \nabla \Phi \right) + \frac{\sigma}{B^4} \left(\frac{1}{\tau} \frac{\partial p_{\perp}}{\partial \chi} \right)_B + \frac{1}{\sigma} \frac{\partial p_{\parallel}}{\partial \chi} \bigg|_B \left(\mathbf{k}_{\perp} \times \mathbf{B} \cdot \boldsymbol{\kappa} \right) (\mathbf{k}_{\perp} \times \mathbf{B} \cdot \nabla \chi) \Phi \\ & + \rho \omega^2 \frac{\mathbf{k}_{\perp}^2}{B^2} \Phi = 0 \end{aligned} \quad (\text{II.C-9}),$$

where $\mathbf{B} = \nabla \alpha \times \nabla \chi = \nabla \phi \times \nabla \chi + g \nabla \phi$ is the magnetic field, $\boldsymbol{\kappa} = (\mathbf{B} \cdot \nabla \mathbf{B})/B^2$ is the curvature, and $g = G(\chi)/\sigma$ is the toroidal field function. The first term is the field line bending term and is always stabilizing; the second term is the driving term for the instability. The second term represents the plasma expansion and is stabilizing in regions of good curvature and destabilizing in regions of bad curvature. The last term is the inertia with ρ the mass density and ω the eigenvalue. The wave vector $\mathbf{k}_{\perp}$ is given by

$$\mathbf{k}_{\perp} = n \left[\nabla \phi - q \nabla \theta - (\theta - \theta_k) \nabla q \right] = n \nabla \alpha \quad (\text{II.C-10}).$$

Here, $\theta = 2\pi \int_0^{\theta'} d\theta' g J / x^2 / \oint d\theta' g J / x^2$ and hence a particular Jacobian is assumed. It is shown here to illustrate the role of q in $\mathbf{k}_{\perp}$. In the following discussion the Jacobian, J , is assumed arbitrary. In Eqn. II.C-10 the safety factor, q , is defined as

$$2\pi q = G \oint \frac{d\theta J}{x^2 \sigma} \quad (\text{II.C-11}),$$

and α is defined by the equation for $\mathbf{B}$

$$\nabla\alpha = \frac{\nabla\chi \times B}{|\nabla\chi|^2} - I\nabla\chi \quad (\text{II.C-12}),$$

$$\text{where } I = -\frac{\nabla\alpha \cdot \nabla\chi}{|\nabla\chi|^2}. \quad (\text{II.C-13}).$$

The quantity I , the integrated local shear, is given by.

$$I = \frac{\nabla\chi \cdot \nabla}{|\nabla\chi|^2} \left(\int_{\theta_0}^{\theta} d\theta \frac{Jg}{x^2} \right) \quad (\text{II.C-14}),$$

where θ_0 is an arbitrary integration parameter which determines the location where $I = 0$.

Both the normal and geodesic curvature enter the driving term for the instability as can be noted by substituting the expression for $\nabla\alpha$ into the term involving curvature in Eqn. II.C-9.

$$\nabla\chi \times B \cdot \kappa = -\frac{B^2}{|\nabla\chi|^2} \kappa \cdot \nabla\chi + B \times \nabla\chi \cdot \kappa \quad (\text{II.C-15}).$$

The geodesic curvature term can be evaluated using

$$B \times \nabla\chi \cdot \kappa = \frac{g}{J} \frac{1}{\sigma B} \frac{\partial(\sigma B)}{\partial\theta} \quad (\text{II.C-16}).$$

The contravariant component of the normal curvature is evaluated using the expression

$$\nabla\chi \cdot \kappa = \nabla\chi \cdot (\hat{n} \cdot \nabla\hat{n}) = \frac{x B_p^3}{B^2} \kappa_p - \frac{B_t^2}{B} \frac{1}{J} z_\theta \quad (\text{II.C-17}),$$

$$\text{where } \kappa_p = \frac{x_{\theta\theta} z_\theta - x_\theta z_{\theta\theta}}{(x_\theta^2 + z_\theta^2)^{3/2}}, \quad (\text{II.C-18}).$$

Here, κ_p is the curvature of the poloidal cross section.

Integrated Local Shear

The integrated local shear can be computed directly given a numerical equilibrium using Eqn. II.C-14, however, an expression involving only local quantities can be found by expanding the equilibrium equations locally about a field line³⁰ or, alternatively, by expanding Eqn. II.C-14 analytically³¹ and using the equilibrium equation. The advantage

of using this procedure is that it can take into account the zeroth order changes in the equilibrium when quantities are varied. In our analysis the procedure has been extended to include local perturbations to the anisotropic equilibrium equation. The expression for integrated local shear then becomes

$$I = G' \int_{\theta_0}^{\theta} d\theta \frac{J}{x^2 \sigma} \frac{B^2}{B_p^2} + G \int_{\theta_0}^{\theta} d\theta' \frac{J}{\sigma^2 x^2 B_p^2} \frac{\partial p_{\parallel}}{\partial \chi} \Big|_B - G \int_{\theta_0}^{\theta} d\theta' \frac{J}{\sigma x^3 B_p} \left[2\kappa_p + \frac{2}{x} \frac{z_{\theta}}{JB_p} \right] + \frac{GJ}{\sigma x^2} \frac{\nabla \chi \cdot \nabla \theta}{|\nabla \chi|^2} \Big|_{\theta_0} \quad (\text{II.C-19}).$$

Expressions relating q' in term of G, G' can be found by evaluating Eqn. II.C-19 at $\theta = 2\pi$, where $I = 2\pi q'$.

$$2\pi q' = G' \oint \frac{d\theta J}{x^2 \sigma} \frac{B^2}{B_p^2} + G \oint d\theta \frac{J}{\sigma^2 x^2 B_p^2} \frac{\partial p_{\parallel}}{\partial \chi} \Big|_B - G \oint d\theta \frac{J}{\sigma x^3 B_p} \left[2\kappa_p + \frac{2}{x} \frac{z_{\theta}}{JB_p} \right] \quad (\text{II.C-20})$$

Including a local perturbation to the equilibrium is advantageous for determining the first and second regions of stability. The situation is somewhat more complicated in the anisotropic case in that, instead of simply p' , the ballooning equation has both $p'_{\parallel}$ and $p'_{\perp}$ evaluated at constant B . To define the second region boundary we can simplify matters somewhat by removing a factor of p'_s from the pressure profiles

$$p_{\perp} = p'_s(\chi) (1 + p'_{\perp b}(\chi, B) / p'_s(\chi)) \quad (\text{II.C-21a})$$

$$p_{\parallel} = p'_s(\chi) (1 + p'_{\parallel b}(\chi, B) / p'_s(\chi)). \quad (\text{II.C-21b}).$$

Solving for p'_s as the eigenvalue with $\omega = 0$ typically gives either two negative solutions or none. It is recognized that this approach might not be the most favorable since, in some situations, it might be desirable to have $p'_s = 0$ in the equilibrium. The two real negative eigenvalues correspond to the first and second region of critical pressure gradients. When the local value of p' is between these eigenvalues the equilibrium is unstable to ballooning modes on those flux surfaces. When the local value of p' (note: p' is usually negative) is greater than both eigenvalues the surface is stable and in the first region of stability. When the local value of p' is less than both eigenvalues the surface is stable and in the second

region of stability. The STBAL code has been revised to interface with the new equilibrium code and solve Eqn. II.C-9. Studies using this code are presented in the next section.

II.C.4 Cases Studied

Several studies were performed to investigate the general effect that anisotropic pressure has on equilibrium and ballooning stability of TFTR plasmas. The distribution between $p_{\parallel}$ and $p_{\perp}$ in TFTR is varied. The position of the neutral beams has been altered at various times since the machine began operation to obtain better beam penetration. At present the beam injection is almost perpendicular. The purpose of the present study is to determine what to expect for parallel, balanced and perpendicular dominant pressure distributions, rather than to analyze a specific shot.

The cases studied were all nominally TFTR supershots with $B_0 = 5.0$ T at the major radius, $R = 2.45$ m. The minor radius was $a = 0.8$ m with elongation $\epsilon = 0.95$. Three categories of equilibria were studied: 1) higher current supershots with $\beta = 1.0\%$, $\epsilon\beta_{pol} = 0.55$ and $I = 1.5$ MA; 2) moderate β_{pol} supershots with $\beta = 0.5\%$, $\epsilon\beta_{pol} = 0.78$ and $I = 0.87$ MA, and 3) moderate β_{pol} supershots with the same parameters as (2) but with a stronger B dependence in the pressure. In each case the "beam" pressure was adjusted to be equal to the isotropic pressure. For each category three different pressure distributions were studied: a) $p_{\parallel} \approx 3/4 p_{\perp}$; b) $p_{\parallel} \approx p_{\perp}$; c) $p_{\parallel} \approx 4/3 p_{\perp}$. The functional form of the isotropic part of the pressure was given by

$$p_s(\chi) = p_0 \exp(-p_1 \bar{\chi}) (1 - \bar{\chi}^{p_2}) \quad (\text{II.C-22a}),$$

$$p_b(\chi) = p_{b0} \exp(-p_1 \bar{\chi}) (1 - \bar{\chi}^{p_2}) \quad (\text{II.C-22b}).$$

This simple two parameter form of the pressure profile can be made to fit the majority of TRANSP cases studied. For these studies we chose $p_1 = 2.0$, $p_2 = 8.0$, which gives pressure profiles typical of supershots. For the equilibrium calculation the fixed q profile method was used. Ballooning stability depends directly on q' , or more specifically on the shear. The $\langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \phi \rangle$ profile, however, also depends on the pressure profile. When the $\langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \phi \rangle$ profile method is used for the equilibrium computation, the q profile usually changes as the pressure profile is altered. Using a fixed q profile thus allows the effects of the pressure profile on the stability to be isolated and compared. For the high current cases (1) the safety factor is given by

$$q(\chi) = 1.2 + 0.1 \bar{\chi} + 5.0 \bar{\chi}^4 \quad (\text{II.C-23}).$$

For the lower current (high $\epsilon\beta_{pol}$) cases (2 and 3) the safety factor is given by

$$q(\chi) = 1.2 + 0.1\bar{\chi} + 11.2\bar{\chi}^4 \quad (\text{II.C-24}).$$

Fig. II.C-1 Surfaces of constant poloidal flux (top) and a) surface of parallel pressure and b) surfaces of perpendicular pressure (bottom) for the high current supershot case (1).

Figure II.C-1 shows an equilibrium typical of a high current supershot. Depicted are the surfaces of constant poloidal flux (top) and constant pressure contours (bottom) for both: a) parallel pressure, $p_{||}$; and b) perpendicular pressure, $p_{\perp}$. The equilibrium in Fig. II.C-1 is for a case where the perpendicular pressure is dominant. It can be seen from the figure that both the parallel and perpendicular pressure surfaces are shifted outward (to the right in fig. II.C-1) from the equilibrium flux surfaces. The perpendicular pressure is shifted more than the parallel pressure.

The anisotropic pressure part of the pressure distribution is determined by the function $h(\chi, B)$. For the weak B dependence, case (1 and 2), the functional forms used were :

a) $h(\chi, B) = 2.0,$	$p_b(\chi) = 2/3 p_s(\chi)$	perpendicular dominant
b) $h(\chi, B) = 1.0,$	$p_b(\chi) = p_s(\chi)$	isotropic
c) $h(\chi, B) = 0.5,$	$p_b(\chi) = 4/3 p_s(\chi)$	parallel dominant.

In Fig. II.C-2 the total parallel pressure, total perpendicular pressure and isotropic pressure component are shown for the case with this weak B dependence for: a) perpendicular dominant; b) balanced and c) parallel dominant cases. Although it is difficult to detect visually when $p_{\perp} > p_{\parallel}$ (Fig II.C-2(a)) both the perpendicular and parallel pressure are shifted inwards slightly. When $p_{\parallel} = p_{\perp}$ (Fig II.C-2(b)) the pressure is isotropic for the weak B dependence form. When $p_{\parallel} > p_{\perp}$ (Fig II.C-2(c)) both the perpendicular and parallel pressure are shifted outwards slightly.

Fig. II.C-2 Total parallel pressure, total perpendicular pressure and isotropic pressure component for profiles with weak B dependence for a) perpendicular dominant, b) balanced and c) parallel dominant cases.

Figure II.C-3 shows the equilibrium contours for a case with moderate β_{pol} where the parallel pressure is stronger than the perpendicular pressure. This case also had a pressure profile with a stronger B dependence. Now the surfaces of constant pressure are shifted inwards with respect to the flux surfaces. The perpendicular pressure is shifted

much more than the parallel pressure distribution. The shift is much greater in this case because of the stronger B dependence.

Fig. II.C-3 Surfaces of constant poloidal flux (top) and a) surface of parallel pressure and b) surfaces of perpendicular pressure (bottom) for the supershot case (3), with moderate β_p .

For the strong B dependence case the functional forms of $h(\chi, B)$ were:

- a) $h(\chi, B) = 2 + 5\bar{B}(1 - \bar{\chi})$, $p_b(\chi) = 2/3p_s(\chi)$ perpendicular dominant
- b) $h(\chi, B) = 1 + 5\bar{B}(1 - \bar{\chi})$ $p_b(\chi) = p_s(\chi)$ balanced
- c) $h(\chi, B) = 0.5 + 5\bar{B}(1 - \bar{\chi})$ $p_b(\chi) = 4/3p_s(\chi)$ parallel dominant

The function $h(\chi, B)$ now has a specific B dependence. This means that even for the balanced case where $p_{||} = p_{\perp}$ on axis there is still some anisotropy along a field line. Constant flux surfaces are no longer constant pressure surfaces. This can be seen in Figs. II.C-4 which shows the total parallel pressure, total perpendicular pressure and the isotropic pressure component for the strong B dependence case. The inward shift of the $p_{\perp}$ is now readily apparent for the perpendicular dominant case, Fig. II.C-4(a), and the outward shift of $p_{\perp}$ for parallel dominant case, Figs. II.C-4(c). The parallel pressure also shifts inward for the perpendicular dominant case and outward for dominant parallel pressure, but much less than $p_{\perp}$.

Fig. II.C-4 Total parallel pressure, total perpendicular pressure and isotropic pressure component for profiles with strong B dependence for a) perpendicular dominant, b) balanced and c) parallel dominant cases.

It is expected, just on the basis of the equilibrium alone, that an equilibrium where the perpendicular pressure is dominant will have a much larger impact on stability than one where the parallel pressure dominates. This is because the surfaces of constant perpendicular pressure are shifted much more than those of parallel pressure. In addition, the stronger the B dependence of the pressure profile the greater the expected impact compared with the isotropic case. With no B dependence the beam pressure depends only on χ and, of course, the pressure profile is isotropic. The B dependence is the effect that produces a difference between $p_{\parallel}$ and $p_{\perp}$. The stronger the form of this dependence the larger the difference between anisotropic and isotropic equilibrium and stability.

The effect on ballooning stability was studied for each of the above categories by solving the ballooning equation for each of the three different pressure profiles. Fig. II.C-5 shows the critical pressure gradients for the higher current superset case. The equilibria

used in this study were all stable to ballooning modes. However, the instability boundaries determined from one equilibrium tend to remain fairly consistent, even for large changes in beta and in the pressure profile, because local effects of the equilibrium were included in the ballooning equation. As can be seen in Fig II.C-5 the perpendicular pressure anisotropy increases the size of the unstable region relative to the isotropic case. Parallel pressure tends to shrink the unstable region. Between 0.8 and 0.9 in the square root of relative χ , the parallel pressure dominant case has a slightly lower first stability boundary than the case where the perpendicular pressure dominates. The effect is very small. There is significant change, however, in the degree of access to the second region of stability in the center of the plasma.

Fig. II.C-5 Critical pressure gradients for ballooning mode stability depicting 1st and 2nd region boundaries and the intermediate unstable region for the high current supershot case and a pressure profile with weak B dependence.

Fig. II.C-6 shows the stability boundaries for moderate $\epsilon\beta_{pol}$ cases with a weak B dependence in the pressure profiles. Again, increased perpendicular pressure anisotropy increases the size of the unstable region. The relative size of this increase is not

significantly different from the higher current case. In both cases, the first region boundary is not significantly altered.

Fig. II.C-6 Critical pressure gradients for ballooning mode stability depicting 1st and 2nd region boundaries and the intermediate unstable region for moderate β_{pol} supershot case and a pressure profile with weak B dependence.

Fig. II.C-7 shows the stability boundaries for moderate $\varepsilon\beta_{pol}$ cases with a strong B dependence in the pressure profiles. With a stronger B dependence there is a greater relative difference between the perpendicular, balanced and parallel cases. Comparing the isotropic case in Fig. II.C-6 with the "balanced" case in Fig. II.C-7 shows that changing the B dependence alone makes a significant difference in the size of the unstable region. In all cases, the first region boundary barely moves and hence the effect of pressure anisotropy on the Troyon beta limit is probably very slight. Pressure anisotropy does significantly affect access to the second region in the center of the plasma. Previous studies showed that when the equilibrium pressure is in the transition region low-n infernal modes are likely. Hence, anisotropic pressure effects are likely to play an important role in these types of modes.

Fig. II.C-7 Critical pressure gradients for ballooning mode stability depicting 1st and 2nd region boundaries and the intermediate unstable region for a moderate β_{pol} supershot case and a pressure profile with strong B dependence.

II.C.5 Discussion - Anisotropic Equilibrium and Stability

Summarizing the general effects of anisotropic pressure, when $p_{||} > p_{\perp}$ there is an inward shift of the constant pressure surfaces. The perpendicular pressure distribution shifts inwards much more than the parallel pressure distribution with respect to the poloidal flux contours. This has a stabilizing effect on the ballooning mode. On the other hand, when $p_{\perp} > p_{||}$ there is an outward shift of the constant pressure surfaces. This has a destabilizing effect on ballooning modes.

For the cases considered here, changes to the equilibrium resulting from anisotropic pressure distributions had the greatest impact on the analysis. These effects enter the ballooning stability analysis through the second term in Eqn. II.C-9 where the parallel and perpendicular pressure profiles are summed before multiplying the curvature. At much larger beta values the quantity σ can become small which weakens the field line bending term. For the relatively low β cases considered here this was not a factor.

These results have several implications for current experiments. Since the first region boundary barely changes with pressure anisotropy, this is unlikely to have any significant effect on the Troyon beta limit for high current supershots. However, access to the second region is significantly affected. This has implications for the infernal mode. Previous studies of low- n stability in TFTR plasmas³² showed that the infernal modes are a likely candidates for some of the MHD activity observed. In these studies the current profiles were altered somewhat from those predicted from transport analysis of the data in order to excite the mode. The neutral particle beams in TFTR are oriented to give predominately perpendicular injection which produces profiles where $p_{\perp}$ exceeds $p_{\parallel}$. Since $p_{\perp} > p_{\parallel}$ closes the second region access it may increase the probability of exciting infernal modes.

The B dependence of the pressure profiles was also found to have a strong effect on stability. Even in the case where the total $p_{\perp}$ and $p_{\parallel}$ were the same, the variation of the pressure along a field line produced a strong effect on stability. This means that the fraction of trapped and passing particles is as important as total $p_{\perp}$ and $p_{\parallel}$ in determining the overall stability.

One intriguing possibility of this research may be to exploit anisotropic pressure effects to improve access to the second stable region. By using parallel neutral particle beam injection along with some method to decrease the trapped particle fraction, the unstable region may be made to disappear entirely thereby allowing full access to the second stability region.

In the near future we shall obtain particle distribution information from TRANSP runs for selected shots. This information is in the form of $f(v_{\perp}, v_{\parallel})$ which can be numerically integrated to obtain $p_{\parallel}$ and $p_{\perp}$. Work is progressing on adapting our low- n MHD code to solve a reduced form of the CGL equations in order to determine marginal stability points.

III CODE DEVELOPMENT

The goal of the code development task was to implement improvements to our equilibrium and stability codes to better model the experiments. Specifically, we wanted to add the capability of modeling the pressure anisotropy resulting from intense neutral beam heating and fueling in TFTR. The application of this model was discussed in §II.C In addition, several improvements were implemented which have significantly increased the accuracy of our equilibrium calculations and, at the same time, have reduced the computer time required to perform these calculations by an order of magnitude.

III.A Equilibrium Code.

A new MHD equilibrium code was developed with Grumman IR&D funding. The old equilibrium code required upgrading in several areas and our 1993 research plan required the introduction of anisotropic pressure effects into the code. We also wished to upgrade the code to run up-down asymmetric equilibria such as those expected in ITER. The final and decisive argument for a new code, however, was that in order to get reliable growth rates for resistive instabilities and internal or external kink modes, both the equilibrium and stability analysis require careful convergence. This requires the capability to calculate the equilibrium configuration using fine mesh sizes. While the previous equilibrium code was reliable and accurate it became major bottle-neck in our analysis because of its relatively slow computational speed. For example, a calculation involving mesh sizes larger than 128 flux surfaces required more than an hour of computer time. The old code used a successive line-over relaxation method to solve the partial differential equation. While this is adequate on a relatively coarse mesh it does not scale well to large mesh sizes. Thus, it was decided to replace this old solver with a new, faster code and which would permit running much finer mesh sizes. This was achieved by implementing a multi-grid numerical method. For typical runs with 64 flux surfaces the code is about ten times faster than the old code. As the grid size increased the code the relative improvement is even better. Now, for example, we typically run with 128 flux surfaces which is usually sufficient to get good convergence of all computed parameters. Converged numerical equilibria with mesh sizes up to 2000 by 4000 have been demonstrated. Although such large grid sizes are rarely needed the capability exists if required.

Using our DOE funding a capability for calculating the effects of anisotropic pressure distributions was added to the new equilibrium code. This involved changes to the partial differential equation to be solved and adding the profiles necessary to model $p_{\perp}$

and $p_{||}$. New methods for computing equilibria with fixed q and $\langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \phi \rangle$ profiles had to be developed and implemented. In addition, a new interface to the mapping code, which transfers data from the equilibrium solver to the stability codes, was developed based on netCDF. The netCDF library is an interface which is used to create, access and share scientific data in a form that is self-describing and network transparent. The library is in the public domain and is now widely used by the scientific community. Several features of this system are very useful to our application and facilitate sharing data. Since the data is network transparent this allows data to be shared among diverse platforms. Equilibria can be run on a local workstation and the data files produced can be read by stability codes on a CRAY or by analysis codes on personal computers. Several utilities are available which allow the data produced by netCDF to be viewed, edited and graphed. This has been extremely useful in debugging the code.

Our standard mapping code, which takes the equilibrium grid and maps it onto a new grid with a different Jacobian and grid size, was adapted to read in netCDF data. The code was further modified to output standard PEST-1 data. This project was done in collaboration with J. Manickam from PPPL. The equilibrium code was then used to benchmark several ITER equilibria and test their low- n stability with PEST-1.

III.B Stability Code

The STBAL ballooning code has been adapted to solve the anisotropic pressure form of the ballooning equation. This code solves for both the first and second region ballooning mode stability boundaries. It also has the capability of calculating the critical n -number of the ballooning mode and can trace out the local S - α diagram on each flux surface. A netCDF interface was added to the code allowing direct communication with the equilibrium code.

Finally, we note that, with Grumman funding, work is in progress to provide our stability codes with the capability of accurately modeling the vacuum region and discrete conducting boundaries outside the plasma. Improved boundary condition models should become available during 1994 as a result of this effort. The ultimate objective is to permit stability analysis of configurations with resistive walls.

IV CONCLUSIONS

The following briefly summarize the main conclusions from each section of this report:

IV.A Plasmas with $q < 1$ (§II.A)

The q -profiles derived from experimental MSE measurements show that, sometimes, q can be smaller than unity over a substantial fraction (20-25%) of the plasma radius. Typically, however, supershot plasmas with $q < 1$ are free of the 1/1 sawtooth instability. This, combined with TRANSP predictions of large pressure gradients in the interior of supershot plasmas, poses a major challenge to ideal MHD theory. In particular, the MSE q -profile together with pressure profile obtained from TRANSP analysis can violate the ideal MHD stability criterion for the 1/1 internal kink mode by factors of ~ 2 (or larger) in supershot plasmas. The stability of the 1/1, ideal internal kink mode has been shown to be highly sensitive, however, to precise details of the pressure gradient in the immediate vicinity of the $q=1$ surface. This mode could be stabilized by a narrow 'flat spot' feature in the pressure profile which, physically, might arise from enhanced transport resulting from the resistive 1/1 modes which (in our experience) are always unstable when $q < 1$. We note that there is no physics in the TRANSP code which would predict such a feature. At the present time, there seems to be no clear experimental evidence either for or against our hypothesis that the stability of the 1/1 mode in supershot plasmas is associated with small pressure gradients in the vicinity of $q=1$. There is a body of opinion, however, that such features exist in TFTR plasmas and, indeed, in other tokamak experiments.

Finally, there is a need to clarify the possible stabilizing role of circulating hot ions on the ideal 1/1 mode in supershot plasmas. This will be a topic of future work

IV.B Plasmas with $q > 1$ (§II.B)

Section IIB was divided into three related topics, each concerned with studies of resistive MHD stability. The conclusions are as follows:

IV.B.1 Finite Pressure Stabilization of the Tearing Mode (§II.B.1)

Our new, extremely fast and highly accurate equilibrium solver permits much better spatial resolution than was possible previously and enables the calculation of very small growth rates and the correspondingly narrow length scales associated with resistive tearing

modes. Analytic work carried out in collaboration with colleagues at Columbia University and General Atomics, inspired by computer studies of the tearing mode, has considered the role of parallel viscosity in the resistive layer. It was shown that the critical value, Δ'_c , of the resistive stability index, Δ' , where the tearing mode is stabilized by the finite pressure scales as $\Delta'_c \propto \mu^{-1/4}$, where μ is the viscosity coefficient. Values of μ with roughly the value predicted by Braginskii reduce the instability criterion, $\Delta' > \Delta'_c$, of Glasser, Greene and Johnson (where Δ'_c is a finite, positive definite quantity) to $\Delta' > 0$.

Our analysis of the role of viscosity adds to the consensus that the finite pressure stabilization of tearing modes, while a perfectly valid prediction of resistive MHD theory, is, in practice, overwhelmed by hydrodynamic effects in the singular layer which are ignored in the simplest model. This result justified the artifice in our toroidal calculations of introducing narrow flat spots in the pressure profile, centered at low order rational surfaces, to remove the finite pressure stabilization in large β_p , supershot, plasmas.

IV.B.2 Tearing Mode Stability Criterion (§II.B.2)

A simple criterion, first given by Wesson and subsequently derived by others, predicts the stability or instability of the tearing modes in periodic cylindrical geometry. For given model assumptions and a generic form for the q -profile, studies using this criterion give interesting insight into the conditions under which tearing modes with given m/n structures are excited. Simple cylindrical geometry calculations suggest conditions under which only the 2/1 tearing mode is unstable or, alternatively, where the 2/1 mode is stable and higher order m/n modes are excited. The latter situation, which is often observed experimentally to be the case in low pressure, high q supershot plasmas, is particularly interesting. Comparison between the simple cylindrical model used to evaluate Wesson's criterion and our own calculations in toroidal geometry suggest, perhaps not surprisingly, that some refinement of the analytic model is desirable. There is a clear requirement for a relatively simple, reliable criterion for predicting tearing mode stability or instability. Such a formula would be an useful asset to tokamak experimentalists.

IV.B.3 Tearing Mode Stability in Toroidal Geometry (§II.B.3)

The resistive tearing stability properties of large β_p (>1) supershot plasmas were studied where q -profiles derived from MSE measurements were available. Two specific plasmas were selected where the mode structures observed experimentally were different. In particular, one plasma exhibited a 3/2 mode while in the other a 2/1 instability appeared. The experimentally derived q -profiles contradict our previous assertion that tearing modes

are excited only when $q(0)$ is highly elevated (>1.2). For the plasmas of interest here the MSE diagnostic yields $1 < q(0) < 1.1$. Stability analysis of each plasma showed that 2/1 and 3/2 tearing modes could be excited in certain conditions with $q(0) \sim 1.1$. Specifically, only the 2/1 mode is excited when the current density profile is sufficiently peaked. The 3/2 mode appears when the current channel is more constricted with a relatively flat distribution near the center and large gradients in the vicinity of the rational surface of interest. We have so far been unable, however, to construct toroidal equilibria unstable to the 3/2 tearing mode alone. The 2/1 mode was always excited in these calculations albeit with substantially smaller growth rates in some situations. In particular, though, the q -profiles where 3/2 instabilities were excited were much flatter than the profile derived from the MSE diagnostic. The experimental observations of pure 3/2 or even higher order m/n modes remains a mystery, but we cannot rule out the tearing mode as a candidate. As mentioned above, these studies using strictly toroidal geometry are not entirely in agreement with Wesson's simple criterion for the stability of the tearing modes in periodic cylindrical geometry. The analysis leading to this criterion needs refinement to properly account for the toroidal curvature.

IV.B.4 Anisotropic Pressure Distributions (§IIC)

When the pressure distribution in a tokamak plasma is anisotropic and $p_{||} > p_{\perp}$ there is an inward shift of the constant pressure surfaces. This has a stabilizing influence on large toroidal mode number, ballooning modes. In this case, the perpendicular pressure distribution shifts inwards much more than the parallel pressure distribution with respect to the poloidal flux contours. On the other hand, when $p_{\perp} > p_{||}$ there is an outward shift of the constant pressure surfaces. This has a destabilizing effect of on ballooning modes. In these calculations the first region ballooning mode stability boundary barely changed as the result of the pressure anisotropy. Consequently pressure anisotropy is unlikely to have a significant effect on the Troyon beta limit for high current supershots. Access to the second stability region is, however, significantly affected. The magnetic field, B , dependence of the pressure profiles was found to have a strong influence on the stability of large toroidal mode number ballooning modes. Even in the case where the total $p_{\perp}$ and $p_{||}$ were the same, the variation of the pressure along a field line produced a strong effect on ballooning mode stability.

IV.C Code Development (§III)

Section III summarized both the physics and computing enhancements implemented during the period of this report. A new equilibrium solver, developed using Grumman IR&D funds, was introduced here since this has substantially improved the accuracy of some of our calculations. In addition, the impressively high speed of this code has significantly increased our productivity. This solver, together with our large toroidal mode number ballooning stability code have been modified to include the capability of describing anisotropic pressure distributions.

APPENDIX - ASSOCIATED PRESENTATIONS

(a) Refereed Publications

During the period of this report the following refereed publications have appeared in the literature from work carried out during 1992:

- (1) 'A Computer Model for Resistive MHD Analysis', M.H.Hughes, M.W.Phillips, and R.G.Storer, Computer Physics Communications, **72**, (1992) 76.
- (2) 'Magnetohydrodynamic Stability Studies of Supershot Plasmas', M.H.Hughes, M.W.Phillips and E.D.Fredrickson, Physics of Fluids B, **5**, (1993), 3267

The following article describing analytic studies of the resistive tearing mode, including new work on the effects of viscous dissipation, is also intended for submission in the near future:

- (3) 'Stability of Tearing Modes in Finite-Beta Plasmas', R.Iacono, A.Bhattacharjee, C.Ronchi, J.M.Greene and M.H.Hughes, to be submitted to Physics of Fluids B.

(b) Conference Papers

International Sherwood Fusion Theory Conference, Newport, RI, March 1993

- (1) 'MHD Stability Studies of TFTR Plasmas', M.H.Hughes and M.W.Phillips, Paper 1D55
- (2) 'A Fast, Highly Accurate MHD Equilibrium Solver', M.W.Phillips and M.H.Hughes, Paper 1D56

APS, Annual Meeting of the Division of Plasma Physics, St Louis, Mo, November 1993.

- (1) 'Mechanism of the Tokamak Sawtooth Oscillation', W.Stodiek and M.H.Hughes, Paper 2S1.

(2) 'Anisotropic Pressure Effects on High Beta Internal Modes in Tokamaks',
M.W.Phillips and M.H.Hughes, Paper 3Q10.

(3) 'MHD Stability Analysis of Experimentally Measured q-profiles in TFTR',
M.H.Hughes and M.W.Phillips, Paper 3Q11

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